Reconsidering The Relationship between Inflation and Relative Price Variability*

Chi-Young Choi

Department of Economics

University of Texas at Arlington

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Abstract

The relationship between inflation and relative price variability (RPV) is widely believed to be positive and stable. Using disaggregated CPI data for the U.S. and Japan, however, this study finds that the relationship is neither linear nor stable over time. The overall relationship is approximately U-shaped and symmetric around a certain threshold: a positive correlation on one side but a trade-off relation on the other. More importantly, the relationship is by no means stable over time but varies significantly in a way that coincides with regime changes of inflation or monetary policy. It was positive in the high inflation environment of the 1970s and the early 1980s as often reported by numerous earlier studies, but changes to U-shape after the great moderation period under a low and stable inflation environment. The results are robust to the use of core inflation that precludes the traditionally volatile prices of food and energy. This paper then presents a simple version of the Calvo-type sticky price model to describe the observed empirical characteristics within a setting of sectoral heterogeneity. According to my simulation results, the nature of the relationship hinges upon the degree of price rigidity such that it is close to U-shape if firms adjust prices more slowly, whereas the U-shape disappears when firms adjust prices more frequently.

Keywords: Inflation, Relative price variability (RPV), Time-varying behavior, U-shape, Calvo model, Sectoral heterogeneity, Degree of price rigidity, Monetary policy.

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1 Introduction

This paper reinvestigates the relationship between inflation and relative price variability (hereafter, RPV) which has been a central theme of modern macroeconomics. With important implications for the welfare cost of inflation and the theorem of monetary neutrality, the relation has received substantial attention from both theoretical and empirical research. To this date, popular theoretical models predict a positive link between the two variables and it is broadly supported by a large body of empirical evidence in various countries for different time periods.¹

For all of its theoretical appeal and empirical support, however, the connection is not fully understood on a couple of grounds. First, there is little consensus on the underlying functional form of the relation. The prima facie evidence on the positive association is largely established by the empirical work utilizing linear regression model. The linearity assumption, however, is often challenged by subsequent studies that put forth evidence in favor of quadratic or piecewise linear relation [e.g. Parks (1978), Hartman (1991), Reinsdorf (1994)]. More recently, Fielding and Mizen (2008) find a U-shaped relationship using the quarterly U.S. Personal Consumer Expenditure (PCE) data over the period 1967-2003. Identifying correct functional form of the relationship bears a crucial implication for monetary policy. If the true relation is positive, monetary authorities can reduce RPV simply by lowering inflation via disinflationary policy, whereas this is no longer the case if the relation is not linear. Second and more important, relatively little attention has been devoted to addressing the stability of the relationship chiefly because the emphasis has been predominantly placed on linear regression analysis that postulates time-invariant marginal impact of inflation on RPV. In view of the ample empirical evidence on the structural changes of inflation series in numerous countries [e.g. Cogley and Sargent (2005), Levin and Piger (2004)], however, it is reasonable to suspect that the relation remains to be stable over time particularly when structural shifts in inflation exert a non-negligible impact on the link. Some recent studies by Dabús (2000) and Caglayan and Filiztekin (2003) indeed report that the relationship between inflation and RPV varies

¹Popular economic models, such as the menu cost and the Lucas-type incomplete information approaches, typically predict the positive relationship and it is generally supported by a considerable empirical literature since the work of Vining and Elwertowski (1976). For a few dissenting views, however, the reader is referred to the studies by Konieczny and Skrzypacz (2005), Reinsdorf (1994), and Silver and Ioannidis (2001) who report a negative relation.
over different regimes of inflation in Argentine and Turkey, respectively.²

The primary objective of this paper is to reexamine the connection between inflation and RPV with an emphasis on its functional form and stability over time. Instead of exploring functional form and stability of the relationship separately, I analyze them jointly as the interface between them is believed to be essential for the proper understanding of the inflation-RPV nexus. To my knowledge, no systematic attempt in this direction has been undertaken to date. To this end, I consider the case of two major economies, the U.S. and Japan, that have maintained relatively stable and low inflation since the great moderation in the mid-1980s. My data set starts from the 1970s and hence covers diverse regimes of inflation and monetary policy, including the deflationary episode of Japan in the mid-1990s that has been rarely studied in the literature.

This study offers interesting new insights on the relationship between inflation and RPV on both empirical and theoretical sides. At the empirical level, I find that the overall relationship takes a U-shape profile as in Fielding and Mizen (2008). More notably, the relationship is not stable over time but varies significantly across sample periods. Figure 1 displays the scatterplots of inflation and RPV in the U.S. and Japan for the full sample period along with three sub-sample periods, maintaining 1984 and 1997 as break points.³ A quick glance at Figure 1 hints that in most cases considered nonlinear specification provides a superior characterization of the association between inflation and RPV over usual linear models. Specifically, the overall relationship is best described by U-shape as RPV dips with inflation initially but rises back when inflation increases gradually after passing a certain threshold. The U-shaped relation implies different effect on RPV at different levels of inflation: a positive impact on RPV if inflation is above the threshold level, while a negative effect on RPV if inflation is below the threshold. From a policy perspective, this means that disinflation efforts pursued by monetary authorities may not necessarily bring about welfare improvement if the benefit from lower

²Dabús (2000) find that the relation between inflation and relative prices in Argentine exhibits structural changes across different levels of inflation. Using price data in Turkey, Caglayan and Filiztekin (2003) also show that the association between inflation and price variability is significantly different between low and high inflationary periods.

³Year 1984 marks the onset of the so-called ‘Great Moderation’ period when the volatility of aggregate economic activity including inflation has declined drastically, whereas 1998 is the year when the Japanese CPI got into a downward trend. Though Japan experienced declining prices between the middle of 1995 and the end of 1996, the continuous price decline did not set out until 1997. The choice of break points seems somewhat arbitrary, but defended by more formal econometric analysis in Section 3.
inflation is outweighed by the cost of increased volatility of relative prices. The U-shaped relationship is observed not just in the U.S., in accordance with Fielding and Mizen (2008), but also in Japan particularly during deflationary period. The results are unaltered by the use of core inflation that strips out the traditionally volatile prices, such as food and energy related items.4

However, it is very important to note that the evidence of U-shaped relationship is not sturdy across sub-sample periods. Whilst the U-shape pattern is apparent in both countries for the sample period after the great moderation when inflation gets stabilized substantially, it is not easy to draw such a characterization for high inflation episodes before the great moderation period. In Japan, for example, the relationship is strongly positive before 1984, reminiscent of the results from a good deal of earlier research in the literature. This is not stunning in light of the fact that the near consensus view on positive relationship was largely built by the vast majority of earlier studies that have been focused on the period of high inflation in the late 1970s and early 1980s when most industrial countries have experienced a double-digit inflation. Taken together, it is fair to argue that the relationship between inflation and RPV is not stable over time but rather varies across sample periods. The overall U-shaped relationship in full sample periods might have been driven by the U-shaped property formulated after the great moderation period. To gain additional information on the issue of stability, several econometric techniques are applied, such as semi-parametric regression, rolling regression approach, and the multiple structural break test due to Bai and Perron (1998, 2003). The formal econometric tools yield qualitatively similar results regarding the key conclusions of this study. Not only the relationship varies over time, but also the time-varying pattern coincides well with the changes in inflation dynamics or monetary policy regime, possibly via changes in the perceived central-bank target for inflation or inflationary expectations of the public. Despite its important implications for monetary policy, not much attention has yet been paid to this time-varying behavior of the relationship, and far less attention to its driving force.5

4Numerous earlier studies which found positive relation between inflation and RPV have been conducted on the period of high inflation, such as oil shock periods in the mid 1970s through early 1980s. Some authors [e.g. Fischer (1981), Hartman (1991)] attribute the positive relationship to an artifact of higher inflation episode during this period that may have been resulted from common supply shocks, such as food and energy prices.

5Fielding and Mizen (2008) also note that their results are sensitive to changes in the sample period. But they did not explore the possibility of time-varying behavior of the relation simply by ascribing parameter
Figure 1: Scatterplots of RPV (vertical axes) and inflation (horizontal axes) in U.S. and Japan
On the theoretical side, this study introduces an economic model to describe the observed empirical characteristics. Given that models are judged on their ability to match the data, traditionally popular models, such as menu cost or Lucas-type imperfect information, are of reduced appeal as they typically predict a positive correlation between inflation and RPV. Their variants, however, that incorporate multi-sector forward-looking components into a model are postulated to be capable of generating the key empirical regularities within a setting of ‘sectoral heterogeneity’. In this paper, I present a simple version of the Calvo-type sticky price model to demonstrate that the U-shaped pattern of the relationship can be generated in an economic environment roughly matching the real data. Sectoral heterogeneity embedded in a Calvo-type sticky price model contributes to its relatively good fit of the observed empirical regularities.

The intuition behind this is that in the presence of sectoral heterogeneity sectors with relatively flexible prices responds to an external shock much more than sectors with relatively sticky price and hence this wedge across sectors gives rise to RPV. When high and persistent growth of wages is expected, firms in the flexible sectors front-load prices a lot because price adjustment is not allowed between the exogenously determined opportunities to adjust prices. According to my simulation experiments based on the Calvo model, the nature of the relationship hinges critically upon the degree of price rigidity. Under an environment of more rigid price setting, the relationship takes a U-shape profile, while the U-shape disappears when price adjustment is highly flexible. Provided that the degree of price stickiness varies with trend inflation as often documented in the literature, a varying degree of price rigidity across inflation regimes could have led to the changes in the relationship between inflation and RPV.

This paper is structured as follows. After this introduction, section 2 presents a brief description of the data used in the current study. Section 3 contains the core of econometric analysis with diverse econometric tools. Section 4 introduces a simple Calvo model of price adjustment to describe the important empirical regularities obtained in this paper. This section also probes to find what is behind the structural change of the relationship by means of simulation experiments. Section 5 concludes the paper.
2 Data

My data set comprises monthly consumer price indices for the U.S. and Japan at the second level of disaggregation. As summarized in Table 1, the resulting series are available for 36 product categories in the U.S. beginning from 1978.M1 and for 46 categories in Japan starting from 1970.M1. The U.S. price data are downloaded from the website of the Bureau of Labor Statistics (BLS) and the Japanese CPI data are collected from the Statistical Library of the Statistics Bureau. Both headline CPI and core CPI are considered in each country in an effort to assess the role of traditionally volatile CPI items related to food and energy. Bold faced items listed in Table 1 represent the food and energy related items that are subtracted from aggregate inflation measure to calculate core inflation.

The principal measure of inflation used here is the monthly log-difference of the CPI which are computed from the seasonally adjusted price indices using the Census X12-method. RPV is then constructed by the weighted average of subaggregate inflation for double-digit consumer price sectors using the standard deviation (s.d.)

$$\text{s.d.}(\pi_t) = \sqrt{\sum_{i=1}^{N} \omega_i (\pi_{it} - \pi_t)^2}$$

where \(\pi_{it} = \ln P_{it} - \ln P_{i,t-1}\), \(\pi_t = \sum_{i=1}^{N} \omega_i \pi_{it}\), \(\omega_i\) denotes the fixed expenditure weight of \(i_{th}\) product that sums to unity, and \(P_{it}\) represents the price index of \(i_{th}\) good at time \(t\).\(^7\)

\(^6\)Compared with the prices of specific individual commodities that have been adopted by some earlier studies in the literature [e.g. magazine prices by Cecchetti (1986) and food prices by Lach and Tsiddon (1992) and Reinsdorf (1994)], disaggregated price indices contain much more information on the variability of relative prices by virtue of the coverage of a wider variety of CPI categories.

\(^7\)Given the nature of index data, RPV measure adopted here should be read as relative inflation variability. Throughout the paper, however, we follow the tradition in the literature to call this measure as relative price variability (RPV). Another common formulation for RPV is the coefficients of variation (CV) which is defined as \(CV(\pi_t) = \frac{s.d.(\pi_t)}{\pi_t}\). This measure of dispersion is not equivalent to s.d. especially when the mean level of inflation exhibits certain structural changes. In fact, while a large number of studies using variance or standard deviation as dispersion measure tend to provide evidence of a positive association between price dispersion and inflation level, studies based on CV [e.g. Reinsdorf (1994) and Silver and Ioannidis (2001)] tend to find a negative relationship. Here I stick to s.d. as RPV measure for a couple of reasons. First, the overwhelming majority of extant studies employ s.d. as the measure of RPV and hence comparison with the previous studies can be facilitated. Second and more important, CV is not easily defined when inflation is close to zero or even negative which is quite common in Japan during the period of deflation.
3 Econometric analysis

Notwithstanding their vintages, visual inspection from scatterplots could be of reduced merit in evaluating the underlying relationship between inflation and RPV, particularly when possible presence of structural changes is suspected. In this section, more formal econometric tools are utilized to have a better sense of the underlying relationship. Toward this end, three econometric methods are adopted: (i) a semi-parametric regression approach; (ii) a rolling regression analysis; and (iii) the multivariate multiple break test due to Bai and Perron (1998, 2003).

3.1 Semiparametric regression analysis

Although the scatterplots in Figure 1 convincingly point toward a quadratic form of the RPV-inflation nexus, it would be helpful not to place any restriction on specific form especially when economic theory does not provide any concrete guidance to the functional form. Parametric models are attractive as they can be estimated accurately and the fitted parametric models can be easily interpreted, but they may give a misleading picture of the relationship if the underlying assumptions are violated. In this vein, nonparametric approach bears a clear advantage as it can avoid the restrictive assumptions on the functional form of regression. But, nonparametric approach is not without its own weakness. Major drawbacks of the nonparametric approach are that it is not easy to interpret and it may yield inaccurate estimates when the number of regressors is large. To compromise, I employ a semi-parametric approach which blends the attractive features of both parametric and nonparametric models by keeping the easy interpretability of the former while retaining some of the flexibility of the latter.8

Here I follow Fielding and Mizen (2008) to consider the following partially linear regression model,

\[ RPV_t = X'_t \beta + g(\pi_t) + \varepsilon_t, \]

where \( X_t \) is a \((p + q) \times 1\) vector of the regressors that includes the lagged terms of RPV and inflation, \( X'_t = \{RPV_{t-1}, ..., RPV_{t-p}, \pi_{t-1}, ..., \pi_{t-q}\} \). \( g(\cdot) \) is an unknown smooth differential

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8 Fully nonparametric regression estimators are more flexible and robust against functional form misspecifications, but their statistical precision decreases substantially if the regression model includes several explanatory variables. By combining attractive features of parametric and nonparametric techniques, semiparametric approach is known to get around the so-called ‘curse of dimensionality’ problem while allowing for flexibility in functional form.
function which captures contemporary effect of inflation on RPV.\textsuperscript{9}

Figure 2 plots the semiparametric estimates of $g'(\cdot)$ function for various values of inflation ($\pi_t$). In most cases considered, the fitted $g'(\cdot)$ function is approximately linear and upward sloping, indicating that $g(\cdot)$ function is nonlinear and U-shaped. Notice the point where the estimated $g'(\cdot)$ function crosses the x-axis. At that point, $g'(\cdot) = 0$ and hence inflation rate corresponds to the threshold inflation level. If inflation rate is below the threshold level, then $g'(\cdot) < 0$ and $g(\cdot)$ function is downward-sloping, while $g'(\cdot) > 0$ and $g(\cdot)$ function is upward-sloping if inflation rate is above the threshold level. This transition of $g'(\cdot)$ function from positive to negative values convincingly implies the quadratic form of $g(\cdot)$ function. In view of the smooth transition of the $g'(\cdot)$ function without any discontinuity around the threshold level, the underlying $g(\cdot)$ function is closer to U-shape than to V-shape.\textsuperscript{10} As pointed out by Fielding and Mizen (2008), the fitted $g(\cdot)$ function is approximately U-shaped because the estimated $g'(\cdot)$ function is not a completely straight line but shows a little curvature at the upper (and lower) ends.

Additional advantage of semiparametric regression method is to enable us to track the stability of $g(\cdot)$ function by examining whether the fitted $g'(\cdot)$ function shifts across samples. Figure 2 exhibits that the fitted $g'(\cdot)$ function varies significantly across sample periods. In Japan, for instance, the estimated $g'(\cdot)$ function lies consistently above the x-axis for the first subsample, suggestive of the positive relationship, while it crosses the X-axis in the subsequent subsamples. Moreover, the estimated threshold inflation rate shifts over time in both countries. In the U.S., it shifts from 0.5% in the first sub-sample to around 0.1% in the third sub-sample, averaging around 0.25% for the full sample period. The shift is also pronounced in Japan which has experienced a transition from trend inflation to trend deflation around the mid-1990s. The estimated threshold inflation rate is around 0.05% in the second sub-sample but down to -0.05% in the third sample when the Japanese economy falls into deflation. The results remain much the same when we use core inflation that strips out the traditionally volatile prices of food and energy related items. In Japan, exclusion of the more volatile price items has little

\textsuperscript{9}If the true functional form is quadratic, for instance, $g(\cdot)$ will take the form of $g(\pi_t) = \alpha + \gamma_1 \pi_t + \gamma_2 \pi_t^2$. Details on the procedure of semi-parametric approach are contained in Appendix A.

\textsuperscript{10}While some previous studies [e.g. Tommassi (1993), Debelle and Lamont (1997)] find V-shaped relation by regressing RPV onto the absolute values of price changes, other studies [e.g. Parks (1978), Bomberger and Makinen (1993), and Konieczny and Skrzypacz (2005)] advocate employing a quadratic functional form in compatible with the U-shaped relation witnessed here.
Figure 2: Derivatives of the $g(\cdot)$ function for different values of inflation ($\pi_t$)
effects on the results. In the U.S., however, the slope of the fitted $g'(\cdot)$ function looks a bit flatter in the post great moderation period, implying that RPV becomes less responsive to the changes of inflation level when more volatile food and energy related prices are excluded. This point calls attention to the potentially important role of the food and energy related prices in the inflation-RPV nexus under the environment of low and stable inflation after 1984.

To sum, the results from semi-parametric analysis generally confirm the visual evidence shown in Figure 1. The upward sloping line of fitted $g'(\cdot)$ function suggests a convincing evidence of U-shaped relationship and its shift across subsample periods indicates a time-varying pattern of the relationship.

### 3.2 Rolling regression analysis

So far the intriguing evidence of time-varying pattern of the relationship between inflation and RPV has been drawn from subsample analysis in which samples are split by cut-off points that are maintained rather than explored. To ensure that the results are robust to the choice of cut-off points, I appeal to the method of rolling regression that captures time variation of the relation without imposing any prior on the timing of break points. The value of this approach is that it can entertain the advantage of greater flexibility in detecting structural changes over time by allowing for each rolling sample to have a completely different estimate.11

I estimate the following parametric model that accommodates both inflation level ($\pi$) and inflation volatility ($\pi^2$) as regressor, together with the lagged terms of both RPV and inflation.

$$RPV_t = \alpha_0 + \sum_{h=1}^{p} \alpha_h RPV_{t-h} + \beta_1 \pi_t + \beta_2 \pi_t^2 + \sum_{j=1}^{q} \gamma_j \pi_{t-j} + \varepsilon_t. \quad (2)$$

Inclusion of the square term of inflation is not just motivated by the evidence of U-shaped relationship obtained from Figures 1 and 2, but also supported by some earlier studies [e.g. Parks (1978), Hartman (1991), Van Hoomissen (1988)] that report inflation volatility as a significant explanatory variable of RPV. Be aware that this model specification allows for different degrees of response of RPV to changes in inflation level ($\pi_t$) than to changes in inflation volatility ($\pi_t^2$). Consequently, time-varying behavior of the relation between inflation and RPV can be traced by the stability of the key parameters, $\beta_1$ and $\beta_2$, over rolling samples.

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11 This feature of rolling regression is attractive especially when one suspects that the full-sample estimates are vulnerable to time variation in the conditional mean of the inflation process (O’Reilly and Whelan, 2005).
Figure 3: Rolling 12-year estimates of $\beta_1$ (top panels) and $\beta_2$ (bottom panels) with 95% confidence intervals

Figure 3 presents the estimates of $\beta_1$ and $\beta_2$ from a sequence of rolling samples. Two panels on the left are for the estimates of the U.S., while the other two panels on the right are for Japan. In each panel, solid line represents coefficient estimates for $\beta_1$ (top panels) or $\beta_2$ (bottom panels) at $t$ which are obtained using data from $t-144$ to $t$ with a window of 12 years\(^{12}\), while two dashed lines represent the 95% significance level based on the heteroskedasticity-autocorrelation robust standard errors with prewhitening. The numbers on the horizontal axis therefore represent the ending year of each 12-year window. For instance, 1990 captures the subsample period of 1979-1990, and so on.

As is obvious from the plots, the coefficient estimates for $\beta_1$ and $\beta_2$ exhibit significant variation over time in both countries, implying that the marginal effects of inflation and inflation volatility on RPV are far from stable throughout the estimation period. Moreover, comparison of the two estimates unveils a marked difference in the impacts between inflation level

\(^{12}\)The choice of 12 years is mainly guided by the need to have long enough samples to assess dynamic relation. Similar results are obtained using the rolling windows of 10 and 15 years separately, although the variability of the estimates are inversely related to the length of rolling sample window.
and inflation volatility. Both inflation level and inflation volatility have statistically significant effects on RPV, but in opposite directions. While the impact of inflation volatility \((\pi_t^2)\) on RPV is positive in both countries, that of inflation level \((\pi_t)\) is negative.\(^{13}\) Nevertheless, the two coefficient estimates display similar timing of break points in each country. Simple visual inspection shows that there are a couple of obvious jumps, one in 1994 and the other in 1998 for the U.S. and in 1988 and 1998 for Japan. The break point of 1998 in both countries is very close to the break point of 1997 used in our sub-sample analysis of preceding sections. It bears further noting that the width of the confidence intervals does not change in an important way. The 95% confidence intervals of coefficient estimates are of similar magnitude across sample period and the upper and lower ends move quite closely with the coefficient estimates. This result could be interpreted as vindicating my argument that the variability of the coefficient estimates is to a more degree driven by structural change rather than by model misspecification or sampling errors.

3.3 Test for multiple structural breaks

To gain further insights into the structural change issue, I now implement a form test for structural changes developed by Bai and Perron (1998, 2003). An appealing feature of this popular testing methodology is that it allows us to locate multiple structural breaks endogenously from the data, without assuming any prior knowledge about the potential break dates and the number of breaks. The number of breaks, their timing, and the constant are all estimated from a series of sequential Wald tests. Here I consider a multivariate setting similar to eq. (2) with a general error process, in which both conditional heteroskedasticity and autocorrelation are allowed. Following the guidelines from Bai and Perron, the break is assumed not to occur during the initial 15 percent nor the final 15 percent of the sample period in testing for structural breaks. Maximum number of breaks is set to five and minimum regime size to 5 percent of sample.

Table 2 reports the estimated dates for structural breaks in the relationship between inflation and RPV. The 95 percent confidence intervals are also computed using robust standard errors based on a quadratic spectral kernel HAC estimator with AR(1) prewhitening filters.

\(^{13}\)Our results closely resemble the findings of Hartman (1991) and Reinsdorf (1994), but run counter to Parks (1978), Bomberger (1987), Lach and Tsiddon (1992) who find positive signs for both level and volatility inflation terms.
Bai-Perron’s multivariate break-point analysis identifies two break points (three regimes) in each country and the identified break dates are 1982 and 1999 for U.S. and 1986 and 1998 for Japan. The estimated break dates are not much far from the breakpoints of 1984 and 1998 maintained in our subsample analysis and the 95% confidence intervals are well inclusive of the maintained breakpoints. In this sense, the results can be viewed as consistent with the findings of previous sections as well as with the maintained breakpoints in the subsample analysis.

4 Linking the empirical findings to theoretical model

The central empirical finding of this study is that the relationship between inflation and RPV is approximately U-shaped but varies significantly over time. The key aspect of this time-varying property is that it is consistent with U-shaped relationship as well as with positive relationship. This empirical feature, however, appears to be difficult to capture in the setting of traditional workhorse models in the literature - e.g. menu cost or Lucas-type imperfect information - because they typically predict a positive connection between inflation and RPV. Instead, it is claimed here that forward looking models with nominal rigidity can effectively model these important empirical findings after embedding sectoral heterogeneity into the model.

In the presence of sectoral heterogeneity in price adjustment, RPV increases with inflation because price changes are not perfectly synchronized due to a wedge between prices with higher flexibility and prices with lower flexibility. As illustrated shortly, modeling sectoral heterogeneity of price rigidity has particular relevance for the time-varying behavior of the relationship.

Consider a Calvo model of sticky prices with multiple sectors, in which firms produce differentiated goods across sectors with labor as the only production factor and a technology linear in labor such that

\[ Y_{it} = N_{it}, \quad i = 1, ..., N, \]

See Fischer (1981), Lach and Tsiddon (1992), and Reinsdorf (1994), for further discussion on the traditionally popular theoretical models.

The literature now abounds with micro and macro evidence of significant heterogeneity of price stickiness across sectors [e.g. Blinder et al. (1998), Bils and Klenow (2004), Dhyne et al. (2006) and Nakamura and Steinsson (2008), to cite just a few]. The literature provides several explanations for the sectoral heterogeneity of price rigidity across: (i) structure of market and industry concentration; (ii) contractual arrangements and the number of stages of processing; (iii) variability of the input costs; (iv) the history or persistence of local shocks and/or supply and demand elasticities; and (v) demand-side variations between durable and nondurable goods.
where $i$ denotes the $i_{th}$ sector. In this setting, the representative firm in sector $i$ updates its nominal prices with probability of $1 - \lambda_i$ each period, so that $\lambda_i$ measures the degree of nominal rigidity at sector $i$. It is further assumed that the only uncertainty in this economy is embedded in the evolution of nominal wage ($W_t$) that follows the stochastic process,

$$\log W_t - \log W_{t-1} = \frac{\varepsilon_t}{1 - \rho L},$$

or equivalently,

$$\log \frac{W_t}{W_{t-1}} = \rho \log \frac{W_{t-1}}{W_{t-2}} + \varepsilon_t,$$

where $\varepsilon_t$ is a white noise, $L$ denotes the lag operator, and $\rho$ represents the persistence of wage growth.

The sectoral price then evolves over time according to

$$\log \left( \frac{P_{it}}{W_t} \right) = \lambda_i \log \left( \frac{P_{i,t-1}}{W_{t-1}} \right) - \lambda_i \frac{(1 - \beta \rho)}{(1 - \lambda_i \beta \rho)} \log \frac{W_t}{W_{t-1}},$$

(3)

where $\beta$ is the usual discount factor. The detailed derivation of equation (3) is relegated to Appendix B.

To assess qualitative and quantitative properties of this simple forward looking model, I run a series of simulation exercises at the range of economically meaningful parameterization. In each simulation, I consider an environment of 30 sectors ($N = 30$) and $\beta = 0.99$ which is quite typical of those in the literature. Remaining structural parameter values for the simulation experiments are specified in Table 3.

In the first two simulations, the probability that firms do not reoptimize their nominal price each period is assumed to be uniformly distributed from 0.5 to 0.9 ($\lambda_i \in \{0.5, 0.9\}$), roughly matching the 4-5 months of price stickiness of Bils and Klenow (2004). The sole difference

\footnote{The overall economic environment of the simple Calvo sticky price model considered here is similar to that of Woodford (2003). The author is very grateful to an anonymous referee for suggesting this model. Fielding and Mizen (2008) use the Rotemberg-Danziger model with a quadratic adjustment cost function to explain the U-shaped relation found in the U.S. data. In their model, however, since the U-shaped relation exists only when inflation rate is between the lower bound ($\pi_L$) and the upper bound ($\pi_H$), it is not clear whether it can still account for the U-shaped relationship found in the period of deflation in Japan, not to mention the time-varying pattern.}

\footnote{Qualitatively similar results are obtained with $N = 50$ and 100. Though attention has been restricted to a simple Calvo-type model of price stickiness, the nature of the results is likely to extend to more general environments discussed in recent literature [e.g. Carvalho (2006), Midrigan (2006) and Nakamura and Steinsson (2008)].}

\footnote{To understand this, note that duration of price rigidity is calculated by $d = \frac{1}{\ln(\lambda_i)}$. When $\lambda_i \in \{0.5, 0.9\}$, the corresponding durations are $d \in \{1.4, 9.5\}$ in months. As $\lambda$ decreases, therefore, the probability that firms adjust their prices increases.}
between simulation 1 and simulation 2 rests on the inflationary environment, a fairly high positive inflation environment for simulation 1, while a low and deflationary environment for simulation 2. The top two panels of Figure 4 plot the resulting simulated relationship between aggregate inflation and RPV. As is clear from the figure, the simulated relationship is very close to U-shape in both high and low inflation environments. The basic idea that this simple model can generate the U-shape relation is that in the presence of sectoral heterogeneity, sectors with relatively flexible prices responds a lot while sectors with relatively sticky price respond a little in the face of wage growth. Expectations of high and persistent wage growth lead firms in the flexible sectors to front-load prices substantively when they reset their prices. This is because with sticky prices firm’s prices are not permitted to adjust against the changes in marginal cost between the exogenously determined opportunities.

While it is crucial for valid models to characterize the U-shaped relation, it is equally important to show its time-varying feature. The bottom two panels of Figure 4 are meant to probe this issue by presenting simulated relationships under different environments of price stickiness. To convey an idea of what the degree of price rigidity implies for the relationship,
\( \lambda_i \) is set in simulation 3 at the narrow range of \( \{0.9, 0.95\} \) for a stickier price environment, whereas in simulation 4 \( \lambda_i \) is set at \( \{0.1, 0.35\} \) for a much more flexible price adjustment. As shown in the bottom-left panel, the U-shaped relationship is much more obvious in a more rigid price setting environment. By stark contrast, the U-shape property disappears completely in a flexible price adjustment environment as displayed in the bottom-right panel. It can be therefore deduced that the degree of price rigidity exerts an important influence upon the relationship between inflation and RPV and its time-varying behavior. To the extent that degree of price rigidity varies with trend inflation as noted by Fischer (1981) and Kiley (2000), it is plausible that changes in the degree of price rigidity resulted from structural changes of inflation process can lead to time-varying pattern of the relationship between inflation and RPV. If firms prefer to set flexible prices in high inflation settings, while maintaining a sticky price in low inflation environment, the relationship takes a U-shape profile under low inflation environment as prices become more sticky. But the U-shaped link breaks down at high inflation environment when prices are more flexibly adjusted. In light of the ample evidence on the structural changes of inflation process documented in the literature, the time-varying behavior of the relationship could have been driven by the change of inflation regime via changes in the degree of price rigidity. This point is consistent with the visual evidence shown in Figure 1 where the relationship is best characterized by U-shape in low inflation environment after the great moderation, while no such compelling evidence of U-shape is found in the period of high inflation of the 1970s.

5 Concluding remarks

For a long time, it has been popularly believed that RPV is positively correlated with inflation such that high inflation is associated with increased cross-sectional dispersion of relative price. The present study yields a novel view on the relationship between inflation and RPV at both empirical and theoretical levels. Using disaggregated CPI data for the U.S. and Japan, this study finds that the overall relationship is not linear but approximately U-shaped: RPV is positively related to inflation on one side, but negatively correlated on the other. This finding reinforces the recent empirical result of Fielding and Mizen (2008), but runs counter to most other extant findings in the literature. The disagreement, however, is not hard to reconcile from
the perspective of subsample analysis. Econometric analysis based on diverse econometric tools convincingly suggests that the relationship has not been stable but varied significantly over time in a way coinciding with regime changes of inflation or monetary policy. The relationship is nearly positive in the period of high inflation of the 1970s and the early 1980s as documented by a number of previous studies, whereas it takes the U-shape profile after the great moderation period. The overall U-shaped relationship is construed to be driven by the U-shaped profile formulated in the post great moderation period.

In this sense, traditionally popular theoretical models, such as standard menu cost or imperfect information models, do not make a good fit of the data as they typically predict a positive association between inflation and RPV. This paper introduces an alternative model for an accurate description of the empirical characteristics. Specifically, the current study presents a simple version of the Calvo-type sticky price model within a setting of sectoral heterogeneity of price rigidity. In the presence of sectoral heterogeneity, the model could successfully produce not just U-shape but also time-varying behavior of the relationship. Simulation results based on the Calvo-type model suggest that structural change in the relationship between inflation and RPV might have been driven by the changes in the extent of price rigidity. So far as the degree of price stickiness varies with inflation trend such that it decreases with the average inflation rate, the relationship takes a U-shape in a low inflation environment as price adjustment is more sticky, while the U-shaped profile vanishes in a high inflation environment when price setting is more flexible.

Important policy implications follow from this discussion. In a high inflation environment where RPV is positively correlated with inflation, monetary authorities can achieve a welfare gain simply by lowering RPV via a reduction in inflation. But, pursuing disinflationary policy will not necessarily lead to a welfare gain if the relationship takes a U-shape profile under low inflation environment. As such, time-varying property of the relationship makes the central bank’s task no easier particularly when the degree of price rigidity varies with inflation regimes and monetary policy framework. A proper understanding of the relationship between inflation and RPV is therefore of great importance for policy making and the current paper is believed to provide an advance over the existing literature in that regard.
References


Appendix A: Procedure of semi-parametric analysis

The function in equation (1) is estimated semi-parametrically in the following two-stages. First, the parameter vector $\beta$ is estimated from a regression equation of the form

$$RPV_t = \hat{X}_t' \beta + \eta_t,$$

where $\hat{X}_t$ is the residual series from a non-parametric regression of $X_t$ onto $\pi_t$. Next, the function $g(\cdot)$ is estimated nonparametrically from the regression

$$\hat{\eta}_t = g(\pi_t) + u_t,$$

where $\hat{\eta}_t = RPV_t - \hat{X}_t' \beta$. The object of our ultimate interest is the derivative of the $g(\pi_t)$ function, $g'(\pi_t) = \frac{\partial g(\cdot)}{\partial \pi_t}$, which captures the marginal effect of $\pi_t$ on $g(\cdot)$, or the sensitivity of RPV to marginal increase in inflation. If $g'(\pi_t) > 0$ ($g'(\pi_t) < 0$), then RPV is increasing (decreasing) with inflation. The level of inflation that ensures $g'(\pi_t) = 0$ therefore pertains to the threshold level of inflation at which RPV is minimized. To estimate the $g(\cdot)$ function, we adopt the Nadaraya-Watson kernel regression estimator using the Gaussian kernel based on automatic bandwidth selection method. As is widely agreed in the literature, the results of nonparametric regression are more sensitive to the bandwidth parameter than to the choice of kernel function. In our analysis, however, we find that changing the bandwidth is not much consequential to the general shape of the function.

Appendix B: Derivation of the Calvo model of sticky price

Assume that with the probability of $1 - \lambda$, firm $i$ can change its price, $P_{it}^*$. Let $MC_{i,t}$ be the nominal marginal cost of production ($MC_{i,t} = W_t$ in our case where we assume a simple production function with labor as the only production factor and technology linear in labor) and $P_t$ be the aggregate price index given by,

$$P_t = \left[ \int_0^1 P_{it}^{1-\eta} \, dt \right]^{\frac{1}{1-\eta}}.$$ 

Profit maximization problem is

$$\max_{P_{it}} V_{it} = E_t \sum_{k=0}^{\infty} Q_{t+k|t} \left( \frac{P_t}{P_{t+k}} \right)^\lambda (P_{it}^* - MC_{i,t+k}) \left( \frac{P_{it}^*}{P_t} \right)^{-\eta} D_{t+k},$$

where $V_{it}$ is the nominal value of firm $i$; $Q_{t+k|t}$ is the cumulative real discount factor

$$Q_{t+k|t} = Q_{t+k-1|t} \beta_{t+k}, \text{ for } k > 1,$$
$$Q_{t|t} = 1,$$

where $\beta_{t+k} = (1 + r_{t+k})^{-1}$ is the period-by-period real discount factor, $P_{it}^*$ is the optimal choice of the price set by the firm; $D_{t+k}$ is the aggregate factors of the economy which are composed of the sectoral total demand, economy wide total demand, and aggregate price indices. We can then think of $Q_{t+k|t} \left( \frac{P_t}{P_{t+k}} \right)$ as the cumulative nominal discount factor.

The first order condition is given by

$$\frac{\partial V_{it}}{P_{it}^*} = 0 = E_t \sum_{k=0}^{\infty} Q_{t+k} \left( \frac{P_t}{P_{t+k}} \right)^\lambda (P_{it}^* \left( \frac{P_{it}^*}{P_t} \right)^{-\eta} D_{t+k} \left[ 1 - \eta + \eta \left( \frac{MC_{i,t+k}}{P_{it}^*} \right) \right].$$

20
Rewriting it, we have
\[ E_t \sum_{k=0}^{\infty} Q_{t+k} \lambda^k P_{it}^{\eta-1} P_{t+k}^{\eta-1} P_t D_{t+k} = \left( \frac{\eta}{\eta - 1} \right) E_t \sum_{k=0}^{\infty} Q_{t+k} \lambda^k P_{it}^{\eta-1} P_{t+k}^{\eta-1} P_t MC_{i,t+k} D_{t+k}. \] (4)

As 1 − λ fraction of firms can change prices and select price \( P_{it}^* \), the price index follows
\[ P_t^{1-\eta} = \lambda P_{t-1}^{1-\eta} + (1 - \lambda) P_{it}^{1-\eta}. \] (5)

Let’s assume that the inflation rate is zero in the steady state. With this assumption, we have
\[ P = P_i^* = \left( \frac{\eta}{\eta - 1} \right) MC \] in the steady state.

Now, we use the log-linearization to (4) and (5). From (4), we have
\[ p_{it}^* = (1 - \beta \lambda)^\infty \sum_{j=0}^\infty (\beta \lambda)^j E_t mc_{t+j}, \] (6)
where \( x = (X_t - X)/X \). From (5), we have
\[ p_t = \lambda p_{t-1} + (1 - \lambda) p_{it}^*. \] (7)

Now, we assume \( MC_t = W_t \), and \( W_t \) follows
\[ \log \frac{W_t}{W_{t-1}} = \rho \left( \log \frac{W_{t-1}}{W_{t-2}} \right) + u_t. \] (8)

We can rewrite it as
\[ w_{t+j} = w_{t+j-1} + \frac{u_{t+j}}{1 - \rho L}, \]
where \( L \) is the lag operator. With repeated iteration, we have
\[ w_{t+j} = w_t + \left( \frac{1}{1 - \rho L} \right) (u_{t+j} + u_{t+j-1} + \ldots + u_{t+1}). \]

Since \( \Delta w_{t+j} \equiv w_{t+j} - w_{t+j-1} = u_{t+j} / (1 - \rho L) \), and \( E_t \Delta w_{t+j} = \rho^j \Delta w_t \), we have
\[ E_t w_{t+j} = w_t + \rho (1 + \rho + \ldots + \rho^{j-1}) \Delta w_t \]
\[ = w_t + \frac{\rho (1 - \rho^j)}{1 - \rho} \Delta w_t. \] (9)

Applying (9) to (6) with \( mc_t = w_t \), we have
\[ p_{it}^* = (1 - \beta \lambda)^\infty \sum_{j=0}^\infty (\beta \lambda)^j \left[ w_t + \frac{\rho (1 - \rho^j)}{1 - \rho} \Delta w_t \right] \]
\[ = (1 - \beta \lambda)^\infty \sum_{j=0}^\infty (\beta \lambda)^j w_t + \rho \left( \frac{1 - \beta \lambda}{1 - \rho} \right) \sum_{j=0}^\infty (\beta \lambda)^j (1 - \rho^j) \Delta w_t \]
\[ = w_t + \rho \left( \frac{1 - \beta \lambda}{1 - \rho} \right) \left[ \left( \frac{1}{1 - \beta \lambda} \right) - \left( \frac{1}{1 - \beta \lambda \rho} \right) \right] \Delta w_t \]
\[ = w_t + \frac{\beta \lambda \rho}{1 - \beta \lambda \rho} \Delta w_t. \]
From (7) and (10), we have

\[ p_t = \lambda p_{t-1} + (1 - \lambda) \left[ w_t + \frac{\beta \lambda \rho}{1 - \beta \lambda \rho} \Delta w_t \right]. \]

Rearranging it, we have

\[
\begin{align*}
pt - wt &= \lambda (pt-1 - wt-1) - \lambda \Delta w_t + \frac{\beta \lambda \rho (1 - \lambda)}{1 - \beta \lambda \rho} \Delta w_t, \\
&= \lambda (pt-1 - wt-1) - \frac{\lambda (1 - \beta \rho)}{1 - \beta \lambda \rho} \Delta w_t.
\end{align*}
\]

In level terms, we have

\[
\log \frac{P_t}{W_t} = \lambda \log \left( \frac{P_{t-1}}{W_{t-1}} \right) - \frac{\lambda (1 - \beta \rho)}{1 - \beta \lambda \rho} \log \left( \frac{W_t}{W_{t-1}} \right).
\]
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Note: Bold-faced items represent the food and energy related items that are subtracted from aggregate inflation measure to calculate core inflation.
Table 2: Results of Bai-Perron tests

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Note: Maximum number of breaks set to five and minimum regime size to 5 percent of sample. Robust standard errors used based on a quadratic spectral kernel HAC estimator with AR(1) prewhitening filters.

Table 3: Parameter values used in simulation

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<td>$\beta$ (time-discount factor)</td>
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Note: Duration of price rigidity is calculated by $d = -\frac{1}{\ln(\lambda_i)}$. 