Real Exchange Rates and Time-Varying Trade Costs

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WORK IN PROGRESS

Abstract

In a recent paper Jacks et al. (2008) advocate a micro-founded measure for bilateral trade costs. We employ this measure and extend two nonlinear models for the real exchange rate so as to re-examine the Purchasing Power Parity hypothesis in the presence of time-varying trade costs. Using data for the dollar-sterling real exchange rate from 1830 to 2005, we provide significant evidence in favor of a positive relation between the level of trade costs and the degree of persistence of the real exchange rate.

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1 Introduction

Trade costs can exhibit significant economic magnitudes and can play an essential role in addressing several major puzzles in international economics (Obstfeld and Rogoff, 2000; Anderson and van Wincoop, 2004). In the Purchasing Power Parity (PPP) framework, equilibrium models of real exchange rate determination demonstrate how trade costs induce nonlinear but mean reverting adjustment toward PPP and, hence, provide a possible explanation for the well-documented persistence in the real exchange rate (Dumas, 1992; O’Connell and Wei, 2002; Taylor and Taylor, 2004). For example, O’Connell and Wei (2002) extend the iceberg model of trade to allow for fixed as well as proportional costs of arbitrage. As a consequence, the tendency of the real exchange rate to return to the equilibrium rate will become apparent only for misalignments which cover the level of transactions costs and imply arbitrage opportunities. Small misalignments, close to equilibrium and within the transactions band, will be left uncorrected so that the real exchange rate will exhibit near unit root behavior.

In a number of empirical contributions trade costs are assumed constant and the implied type of nonlinear behavior of the real exchange rate is modeled by the Exponential Smooth Transition Autoregressive (ESTAR) model (see, e.g., Michael et al., 1997; Kilian and Taylor, 2003; Taylor, Peel and Sarno, 2001). However, it can be argued that this assumption is too restrictive over long time periods. In a recent contribution, inspired by the gravity literature, Jacks et al. (2008) present an aggregate micro-founded model which allows the construction of long span trade costs series. The authors illustrate that trade costs related to the exchange of goods across countries, far from being constant, have exhibited substantial and nonmonotonic changes from 1870 to 2000. This finding has potentially important implications concerning the behavior of the real exchange rate. Because trade costs vary in time so does the speed of mean reversion for a given PPP deviation (see, e.g., Dumas, 1992; Sercu et al., 1995). Intuitively, when trade costs increase (decrease) the trade costs band—in which no trade takes place—widens (narrow) and the real exchange rate process becomes more (less) persistent. Hence, the persistence of the real exchange rate does not only depend on the size of the deviation but also on the level of trade costs at each particular point in time. Neglecting significant changes in trade costs leads to underestimating/overestimating the degree of persistence and the time required for the process to absorb shocks at specific periods.

The contribution of this paper is to report estimates and the properties of two smooth transi-

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2 Clemens and Williamson (2001) and Mohammed and Williamson (2004) among others illustrate that tariffs and global freight rates have fluctuated substantially in the last century. These studies focus on specific impediments of trade costs and, therefore, provide indirect evidence of time-varying trade costs. A survey on recent developments in the measurement of total trade costs and their components is provided by Anderson and van Wincoop (2004).

3 Consequently, the effect of trade costs cannot be approximated by deterministic trends.
tion regression models of the real exchange rate which incorporate time-varying trade costs. The models are fitted to a long span of data (1830-2005) for the dollar-sterling real exchange rate and the trade costs index for the United Kingdom-United States country pair. Our choice is based on the fact that the relationship between trade frictions and the persistence of the real exchange rate should become apparent over long time periods in which large fluctuations of trade costs occur.

The rest of the paper is structured as follows. In Section 2 we present the trade costs measure of Jacks et al. (2008). Section 3 outlines our nonlinear models of the real exchange rate. Section 4 deals with the description of the data and the empirical results. A summary and concluding comments are offered in the last section.

2 Trade Costs

“Trade costs, broadly defined, include all costs incurred in getting a good to a final user other than the marginal cost of producing the good itself” (Anderson and van Wincoop, 2004, p. 691). Obviously, trade costs break down into a vast number of components such as transportation costs (freight rates and time costs), policy barriers (tariffs and nontariff barriers), informational costs and costs associated with the use of different currencies. The fact that several of these components are unobservable and data limitations pose serious problems in obtaining accurate estimates of the magnitude of total trade costs by direct atheoretical measures. The gravity literature circumvents this obstacle on the basis of theoretical models which enable measuring the degree of trade restrictiveness by extracting information from trade flows.

In this framework, Jacks et al. (2008) present a micro-founded measure of aggregate bilateral trade costs that captures trade frictions. The key idea in the derivation of their measure is that changes in trade barriers have an effect on both international and intranational trade. By establishing a relationship between countries’ average international trade barriers and intranational trade, trade costs can be obtained directly from observable trade data without imposing a particular trade cost function (Novy, 2008).

Consider a world consisting of $N$ countries and a continuum of differentiated goods. Anderson and van Wincoop (2003) derive the following gravity equation of international trade

$$ x_{i,j} = \frac{y_i y_j}{y_w} \left( \frac{t_{i,j} (1 - \sigma)}{\Pi_i P_j} \right), $$

where $x_{i,j}$ are nominal exports from country $i$ to $j$. Income levels of country $i$, country $j$ and world income are denoted by $y_i$, $y_j$ and $y_w$, respectively. The elasticity of substitution, $\sigma$, is assumed
to be constant and greater than unity. The cost of importing a good or, equivalently, the trade cost barrier (one plus the tariff equivalent) is $t_{i,j} \geq 1$. Finally, the price indices (or outward and inward multilateral resistance variables) $\Pi_i$ and $P_j$ for countries $i$ and $j$ represent the average trade restrictiveness of the countries. Novy (2008) uses Equation (1) to obtain a bidirectional gravity equation, which includes inward and outward multilateral resistance variables for both countries,

$$x_{i,j}x_{j,i} = \left( \frac{y_i y_j}{y_w} \right)^2 \left( \frac{t_{i,j} t_{j,i}}{\Pi_i P_j \Pi_j P_i} \right)^{1-\sigma}.$$  

(2)

In turn, the author makes use of the fact that intranational trade, like international trade, depends on the magnitude of trade barriers, $x_{i,i} = \left( \frac{(y_i y_i)}{y_w} \right) \left( \frac{t_{i,i}}{\Pi_i P_i} \right)^{1-\sigma}$, so as to control for multilateral resistance. Substituting into the bidirectional gravity equation yields

$$x_{i,j}x_{j,i} = x_{i,i}x_{j,j} \left( \frac{t_{i,j} t_{j,i}}{t_{i,i} t_{j,j}} \right)^{1-\sigma}.$$  

(3)

The geometric average of the tariff equivalent can now be obtained by

$$\tau \equiv \left( \frac{t_{i,j} t_{j,i}}{t_{i,i} t_{j,j}} \right)^{\frac{1}{2}} - 1 = \left( \frac{x_{i,i} x_{j,j}}{x_{i,j} x_{j,i}} \right)^{\frac{1}{2(\sigma-1)}} - 1.$$  

(4)

The above equation states that a drop in trade flows between countries with respect to trade flows within countries is associated with higher trade costs. Note that the micro-founded measure evaluates bilateral trade costs against the domestic trade cost benchmark. Further, it enables the construction of long span trade costs series since its estimation only requires data for bilateral exports and intranational trade. The latter variable can be approximated by subtracting aggregate exports from a country’s Gross Domestic Product (GDP) (Jacks et al., 2008).

3 Nonlinear Adjustment & Time-Varying Trade Costs

Let us define the log real exchange rate as $q_t = s_t - p_t + p_t^*$, where $s_t$ is the logarithm of the spot exchange rate (the domestic price of foreign currency), $p_t$ is the logarithm of the domestic price level and $p_t^*$ the logarithm of the foreign price level.
3.1 The ESTAR Model

A widely employed nonlinear econometric model that can capture the behavior of the real exchange rate in the presence of constant trade costs is the Exponential STAR (ESTAR) model advocated by Teräsvirta (1994). The appealing feature of the ESTAR model is that it allows transitions between a continuum of regimes to occur smoothly and symmetrically. In this setting, the speed of mean reversion is an increasing function of the size of absolute deviation from equilibrium. This property is suggested by the analysis of Dumas (1992) and demonstrated by Berka (2005). In addition, Teräsvirta (1994) argues that if an aggregated process is observed, regime changes may be smooth rather than discrete as long as heterogeneous agents do not act simultaneously even if they individually make dichotomous decisions. All the above favor the use of ESTAR models over Threshold Autoregressive (TAR) models, in which changes of persistence occur abruptly.

A STAR model for the process \( \{q_t\} \) may be written as

\[
q_t - \mu = \sum_{p=1}^{\bar{p}} \phi_p (q_{t-p} - \mu) G_j(\cdot) + \epsilon_t, \tag{5}
\]

where \( \mu \) is a constant representing the long run equilibrium, \( \epsilon_t \) is a white noise process with mean 0 and variance \( \sigma_\epsilon \), and \( G_j(\cdot) \) is the transition function. For a given AR structure, \( \sum_{p=1}^{\bar{p}} \phi_p \), the transition function, \( G_j(\cdot) \), specifies the degree of persistence of the real exchange rate at each point in time. In the absence of trade costs, the transition function for the ESTAR model is given by

\[
G_1(q_{t-d}) = \exp \left( -\gamma^2 (q_{t-d} - \mu)^2 \right). \tag{6}
\]

where \( q_{t-d} \) is the transition variable and \( \gamma \) is the smoothness (or transition) parameter. The exponential transition function \( G_1(\cdot) \) is particularly applicable because it implies symmetric adjustment for positive and negative deviations from the equilibrium. Furthermore, the speed of adjustment is increasing with the smoothness parameter \( \gamma \) and the absolute value of the past deviation from the equilibrium. For expositional reasons, we assume that \( \sum_{p=1}^{\bar{p}} \phi_p = 1 \) throughout this section. In this case, at the equilibrium \( G_1(\cdot) = 1 \) and the real exchange rate behaves as a unit root process, \( q_t = \sum_{p=1}^{\bar{p}} \phi_p (q_{t-p} - \mu) + \epsilon_t \). Whilst, for nonzero deviations \( G_j(\cdot) \in [0, 1) \) and the process becomes mean reverting. Finally, if \( |q_{t-d} - \mu| \to \infty \) the function value approaches zero and the process is white noise, \( q_t = \epsilon_t \). The speed of transition between regimes is specified by the smoothness parameter \( \gamma \). If \( \gamma \) is equal to zero the real exchange rate behaves as a linear unit root process irrespective of the regime. Whilst, if \( \gamma \to \infty \) the process becomes white noise. Intermediate values

\[\text{Note also that the incorporation of trade costs in TAR models is not straightforward.}\]
of $\gamma$ imply smooth adjustment of the real exchange rate.

Let us consider two deviations from PPP which have the same size but occur at different time periods, $|q_{t_1-d} - \mu| = |q_{t_2-d} - \mu| \neq 0$ with $t_1 < t_2$. The fact that $\gamma$ is constant in the typical ESTAR model implies that the real exchange rate will exhibit the same degree of persistence at time $t_1$ and $t_2$. Conditional on the assumption of constant trade costs this is an attractive property. However, if trade costs vary in time so will the speed of adjustment. An increase (decline) in trade costs, $\tau$, during the two time periods, $\tau_{t_1-d} \neq \tau_{t_2-d}$, will induce higher (lower) persistence of the real exchange rate and, therefore, a decrease (increase) of the $\gamma$ parameter. Hence, time varying trade costs can be incorporated into Equation (6) by allowing $\gamma$ to change over time depending on $\tau_{t-d}$. By assuming a linear relationship between the value of the smoothness parameter and trade costs, the transition function for the Time Varying Trade Costs ESTAR (TVTC-ESTAR) is given by

$$G_2(q_{t-d}, \tau_{t-d}) = \exp\left(- (\gamma - \gamma_\tau \tau_{t-d})^2 (q_{t-d} - \mu)^2 \right),$$

where the coefficient, $\gamma_\tau$, on trade costs is greater than zero. The above equation allows both the degree of trade restrictiveness and the size of the deviation from the equilibrium to determine the speed of adjustment of the real exchange rate at a particular point in time (see Figure 3).

### 3.2 The QLSTAR Model

An alternative model to the ESTAR that captures the theoretical insights of the authors above and allows us to parsimoniously encompass the influence of fixed and proportional time-varying trade costs is the Quadratic Logistic Smooth Transition Autoregressive (QLSTAR) model of Jansen and Teräsvirta (1996). The main advantage of the QLSTAR model over the widely used ESTAR model is that it can approximate a trade costs band more closely and, at the same time, it can preserve all the other properties of the ESTAR model.

Jansen and Teräsvirta (1996) propose the following quadratic logistic function

$$G_3^*(q_{t-d}) = 1 - \left(1 + \exp\left(-\gamma^2 (q_{t-d} + c_1)(q_{t-d} + c_2)\right)\right)^{-1},$$

where $c_1 = \mu - c$ and $c_2 = \mu + c$ with $c > 0$ are the band coefficients. The quadratic logistic transition function $G_3^*(\cdot)$ is particularly applicable because it, as the exponential function, implies symmetric adjustment for positive and negative deviations from the equilibrium. Further, the QLSTAR model specified by Equation (8) can approximate ESTAR models but also nests three regime Threshold Autoregressive (TAR) models and linear AR models. In contrast to TAR and ESTAR models, the QLSTAR allows the type of adjustment (smooth or discrete) between regimes to be
specified by the data and, at the same time, can approximate narrow and wide “bands of inaction”. Hence, the model allows for both fixed and proportional costs. Overall, the model is particularly applicable when one is agnostic about the range of the “band of inaction” and the type of transition.

Suppose that regime changes occur abruptly rather than gradually (see Sercu et al., 1995), which favors the use of TAR over ESTAR models. If \( \gamma \rightarrow \infty \) and \( q_{t-d} < c_1 \) or \( q_{t-d} > c_2 \) the transition function value equals zero and \( q_t \) becomes white noise. Whilst, inside the “band of inaction”, \( c_1 < q_{t-d} < c_2 \), \( G_3^*(\cdot) \) equals one and \( q_t \) behaves as a unit root process. Note that an increase in trade costs will widen the “band of inaction” and, therefore, result in higher absolute values of the band coefficients, \( c_1 \) and \( c_2 \). At the other extreme, when \( \gamma = 0 \) the model becomes linear. For moderate values of \( \gamma \), the QLSTAR model can approximate both ESTAR and TAR models. The speed of mean reversion increases with the deviation from the equilibrium \( q_{t-d} - \mu \).

If \( q_{t-d} - \mu \rightarrow \infty \) the process approaches the white noise regime (outer regime). Whilst, in the inner regime, \( q_{t-d} - \mu = 0 \), the degree of persistence is given by the maximum value of the transition function \( G_3^* \)

\[
G_3^*(\mu) = 1 - \left( 1 + \exp(\gamma^2 c^2) \right)^{-1},
\]

which is determined by the transition parameter \( \gamma \) and the coefficient \( c \). Consequently, changes in \( \gamma \) or \( c \) due to movements in trade costs lead to different degrees of persistence at the equilibrium. Due to the fact that there is no a priori reason why changes in trade costs should alter the degree of persistence in the inner regime, we modify Equation (8) as follows

\[
G_3(q_{t-d}) = 1 - \left( 1 + \exp\left( -\frac{\gamma^2}{c^2} (q_{t-d} + c_1)(q_{t-d} + c_2) \right) \right)^{-1}.
\]

The maximum value of \( G_3(\cdot) \), which again occurs at the equilibrium rate, is

\[
G_3(\mu) = 1 - (1 + \exp(\gamma^2))^{-1},
\]

and is independent of the value of the band coefficient. Note that dividing the smoothness parameter \( \gamma^2 \) by \( c^2 \) also implies that changes in the persistence of the process become more abrupt as \( c \) decreases (see Figure 4). This behavior is in line with the presence of both fixed and proportional costs which move together in time (O’Connell and Wei, 2002). The above modification enables the incorporation of time-varying trade costs in the QLSTAR model in a straightforward manner.
The transition function for the Time-Varying Trade Costs QLSTAR (TVTC-QLSTAR) is given by

$$G_4(q_{t-d}, \tau_{t-d}) = \left[ 1 - \left( 1 + \exp \left( -\frac{\gamma^2}{(c + c_\tau \tau_{t-d})^2} (q_{t-d} + c_3)(q_{t-d} + c_4) \right) \right)^{-1} \right] + \epsilon_t, \quad (12)$$

where $c_3 = \mu - c - c_\tau \tau_{t-d}$ and $c_4 = \mu + c + c_\tau \tau_{t-d}$ with $c_3 < c_4$ are the time-varying band coefficients, $c$ is a positive constant, $c_\tau$ is the coefficient on trade costs $\tau$. Controlling for $\gamma$, the speed of mean reversion decreases with the absolute value of the band coefficients $c_1$ and $c_2$, and increases with the past deviation from the equilibrium rate. In this setting, the null hypothesis that changes in trade costs have no effect on the degree of persistence of PPP deviations is $H_0 : c_\tau = 0$. While the alternative hypothesis is $H_0 : c_\tau > 0$. We examine the impact of trade costs on the speed of mean reversion of the real exchange rate in the next section.

4 Empirical Results

Our data set consists of annual observations for the dollar-sterling real exchange rate and the corresponding trade costs index from 1830 to 2005. For the construction of the real exchange rate we use the International Financial Statistics database to update the nominal exchange rate and the price indexes analyzed in Lothian and Taylor (1996). International trade data are obtained by Mitchell (2008b,a) and GDP series for the United States and the United Kingdom are taken from Officer (2008) and Johnston and Williamson (2008), respectively.

Figures 1 and 2 show the demeaned real exchange rate and the trade costs series, respectively. In line with Jacks et al. (2008), the latter exhibits significant fluctuations throughout the period. Specifically, until the beginning of the 20th century trade costs were relatively low. Subsequently, the war and interwar periods were associated with a remarkable increase of bilateral trade costs with respect to intranational domestic costs. During this time interval the series displays two peaks, the first in 1935 following the Great Depression, and the second in 1946 at the end of the second World War and the establishment of the Bretton Woods system. A gradual decline has occurred since then.

After running a battery of linearity tests on the real exchange rate series, which indicate the presence of smooth transition nonlinearity, we examine whether trade costs are an important constituent of the nonlinear adjustment mechanism of the real exchange rate. Specifically, we employ the testing procedures proposed by Ter"asvirta (1994), Harvey and Leybourne (2007), and...
nonlinear models with constant and time-varying trade costs are reported in Table 1. Overall, all models provide a parsimonious fit to the real exchange rate. However, the incorporation of time-varying trade costs leads to a radically different adjustment process. The statistical significance of the coefficient $\gamma_\tau$ and the band coefficient $\hat{c}_\tau$ of the TVTC-ESTAR and TVTC-QLSTAR models, respectively, indicates that movements in trade costs can help explain changes in the level of persistence of the real exchange rate. An increase in trade costs widens the “band of inaction” and reduces the speed of mean reversion for a given PPP deviation.

Figures 3 and 4 display the transition functions of the time-varying trade costs models for three representative time periods, namely 1900, 1950 and 2000, which correspond to relatively low, large and moderate levels of trade costs, respectively. At those time periods, for the TVTC-ESTAR model, a PPP deviation of 0.4—which is roughly the maximum realized deviation—would suggest that the real exchange rate behaves similar to an AR process with coefficient around 0.3, a near unit root and an AR process with coefficient around 0.5. For the TVTC-QLSTAR model, the same PPP deviation would suggest that the real exchange rate behaves similar to a white noise, a near unit root and an AR process with coefficient around 0.2. On the other hand, according to the ESTAR and QLSTAR models with constant trade costs the real exchange rate would behave as an AR process with coefficient of about 0.7 and 0.5, respectively, at all points in time. These findings also show that the inability of the ESTAR model to approximate a wide “band of inaction” results in higher persistence than that implied by the QLSTAR model for large deviations.

A natural question that arises in the nonlinear framework under examination is how fast does...
the process adjust to such a large deviation. We employ the fitted models so as to simulate data and compute the half-life of a shock which drives the process from its equilibrium value to the maximum realized PPP deviation. Starting with the standard ESTAR and QLSTAR models, the real exchange rate process would absorb half of the shock in four years. Turning to the time-varying trade costs models, we consider three scenarios. As above, we set trade costs equal to their 1900, 1950 and 2000 levels. In the former and latter cases, both the TVTC-ESTAR and TVTC-QLSTAR models suggest that the time required for the process to absorb half of the maximum PPP deviation is only two years, which is half of that corresponding to constant trade costs. Obviously, large deviations of the real exchange rate appear to mean revert much faster (than that implied by the ESTAR and QLSTAR models) during the beginning of the 20th century and the recent floating period. On the contrary, the high level of trade costs around the middle of the 20th century leads to an increase in the half-life of the shock with respect to the constant trade costs benchmark. In particular, the TVTC-QLSTAR and TVTC-ESTAR models imply a half-life of five and twenty years, respectively. As above, the large discrepancy between the results of the two models can be attributed to the inability of the ESTAR model to capture the effect of wide “bands of inaction”. When the level of trade costs is high the transition parameter in the ESTAR model approaches unity (infinite band width) falsely suggesting that the real exchange rate series is a unit root process.

In order to examine which model is superior in terms of capturing the effect of time-varying trade costs, we conduct two bootstrap experiments. For each experiment, we employ either the estimated TVTC-QLSTAR or the TVTC-ESTAR model (reported in Table 1), the original trade costs series and the corresponding estimated residuals so as to generate B artificial samples of size 176. In turn, we fit the alternative model to each artificial sample and compute the t-statistic for the coefficient on trade costs, $\tilde{t}_b$. This provides the empirical distributions for the t-statistics for $\hat{\gamma}_\tau$ and $\hat{c}_\tau$ under the null that the true DGP is given by the alternative model. The probability of

9Half-lifes for the nonlinear models are computed by using a procedure similar to the Generalized Impulse Response Function proposed by [Koop et al. (1996)]. Specifically, we employ the estimated coefficients and bootstrap from the residuals so as to generate one thousand artificial real exchange rate series. The initial values of these series are set equal to zero and the size of the shock equal to the maximum PPP deviation of the model under consideration. Next, we average the generated series at each horizon and compute the horizon at which the resultant series crosses the threshold of half the shock.

10To further illustrate this point we consider the maximum level of trade costs, which occurs in 1946. The corresponding half-lives are twelve and fifty seven years for the TVTC-QLSTAR and TVTC-ESTAR models, respectively.

11We set the number of generated samples B equal to 1000 and initialize the bootstrap DGP by using the first observations of the original real exchange rate series.
obtaining a $t$-statistic as extreme as the original is

$$p_b = \frac{1}{B} \sum_{b=1}^{B} I(\tilde{t} \leq \tilde{t}_b),$$

where $I(A)$ is the indicator function, which takes the value of 1 if event $A$ occurs and 0 otherwise, and $\tilde{t}$ is the original $t$-statistic. When the DGP is the TVTC-ESTAR model, the probability of the $t$-statistic for $\hat{c}_\tau$ exceeding 4.488 is only 13.8 percent. Whilst, when the DGP is given by the fitted TVTC-QLSTAR, there is a 52.1 percent probability that the value of the $t$-statistic for $\hat{\gamma}_\tau$ is greater than 3.145. Hence, it is very likely to obtain a $t$-statistic for the coefficient on trade costs in the TVTC-ESTAR model as extreme as the original when the DGP is given by the estimated TVTC-QLSTAR model. However, the opposite is not true. In this respect, the TVTC-QLSTAR model outperforms the TVTC-ESTAR model in capturing the rich dynamics of the real exchange rate.

Clearly, the assumption of constant trade costs can result in severe overestimation / underestimation of persistence. Figures 5 and 6 depict the difference between the degrees of persistence estimated by the time-varying and constant trade costs models. For the ESTAR model, a broad conclusion that emerges is that the exclusion of time-varying trade costs results more frequently in overestimating rather than underestimating the degree of persistence of the process. Hence, it appears that most of the time the dollar-sterling real exchange rate series is less persistent than what standard ESTAR models suggest. On the contrary, the QLS TAR model with constant trade costs appears to underestimate the degree of persistence in most periods. Overestimation occurs on rare occasions, such as 1986, which are usually associated with substantially larger differences in the speed of mean reversion. This implies higher speed of adjustment of the real exchange rate.

## 5 Conclusion

The insights of Jacks et al. (2008), motivated by the recent gravity literature, show how to compute a long span trade costs series. The authors illustrate that trade costs related to the exchange of goods across countries have exhibited substantial and non-monotonic changes from 1870 to 2000. In empirical work on the dynamic behavior of the real exchange rates trade costs have typically been assumed constant even though trade costs play a key role in theoretical models in determining real exchange rate dynamics. Essentially, arbitrage will commence, *ceteris paribus*, when it

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12The degree of persistence in this case is measured by the value of the transition function of the corresponding model.
is profitable and PPP deviations are outside the transactions band. We develop and estimate a nonlinear model for the real exchange rate which incorporates time-varying trade costs and is a generalization of the ESTAR nonlinear model employed in previous analysis, where trade costs were assumed constant. Our empirical approach is supported by a battery of statistical tests and simulation methods. Our results provide strong evidence in favor of a time-varying “band of inaction”, which widens with the level of trade costs. The persistence of the real exchange rate is found to depend on both the magnitude of trade frictions and the size of the deviation from PPP. For instance, a given shock to the real exchange rate would be absorbed at significantly different speeds in 1950 and 2000 due to the existence of different trade costs levels. Although trade costs appear to have declined substantially since the second World War, their magnitude is still significant. Consequently, our empirical results are also consistent with the documented high persistence of real exchange rates in the post Bretton Woods era.
References


## Table 1: Estimated Nonlinear Models

### Panel A, ESTAR

\[
q_t + 0.016 = (q_{t-1} + 0.016) \exp(-1.505^2 (q_{t-1} + 0.016)^2).
\]

\[
s = 0.064; \; Q_1 = 0.140 \; [0.062]; \; Q_5 = -0.127 \; [0.227]; \; \text{ARCH}_1 = 0.557 \; [0.456]; \; \text{ARCH}_5 = 0.802 \; [0.550].
\]

### Panel B, TVTC-ESTAR

\[
q_t - 0.066 = (q_{t-1} - 0.066) \exp\left(-\left(3.552 - 5.324 \tau_{t-2}\right)^2 \left(q_{t-2} - 0.066\right)^2\right).
\]

\[
s = 0.063; \; Q_1 = 0.035 \; [0.642]; \; Q_5 = -0.161 \; [0.374]; \; \text{ARCH}_1 = 1.538 \; [0.217]; \; \text{ARCH}_5 = 0.538 \; [0.747].
\]

### Panel C, QLSTAR

\[
q_t + 0.014 = (q_{t-1} + 0.014) \left[1 - \left(1 + \exp\left(-1.829^2 / (6.700)(q_{t-1} - 0.387)(q_{t-1} + 0.416)\right)\right)^{-1}\right].
\]

\[
s = 0.064; \; Q_1 = 0.141 \; [0.061]; \; Q_5 = -0.126 \; [0.219]; \; \text{ARCH}_1 = 0.535 \; [0.465]; \; \text{ARCH}_5 = 0.786 \; [0.561].
\]

### Panel D, TVTC-QLSTAR

\[
q_t - 0.059 = (q_{t-1} - 0.059) \left[1 - \left(1 + \exp\left(-2.146^2 / (7.811)(q_{t-1} - 0.172)(q_{t-1} + 0.587)\right)^2 \right) \right.
\times \left(q_{t-2} - 0.231 - 0.587 \tau_{t-2}\right)(q_{t-2} + 0.1128 + 0.587 \tau_{t-2})\right]^{-1}\]

\[
s = 0.063; \; Q_1 = 0.020 \; [0.787]; \; Q_5 = -0.154 \; [0.426]; \; \text{ARCH}_1 = 0.667 \; [0.411]; \; \text{ARCH}_5 = 0.344 \; [0.886].
\]
Notes: Figures in parentheses and square brackets denote absolute $t$-statistics and $p$-values, respectively. The $p$-values for the coefficients on trade costs $\hat{\gamma}_\tau$ and $\hat{c}_\tau$ are obtained through a simulation exercise, where the bootstrap DGPs are the fitted ESTAR and QLSTAR models, respectively. For illustration purposes, we report the summation of the long run equilibrium estimate and the constant part of the band coefficients $\hat{\mu} \pm \hat{c}$. $s$ is the standard error of the regression. $Q_1$ and $Q_5$ denote the Ljung-Box $Q$-statistic for serial correlation up to order 1 and 5, respectively. ARCH$_1$ and ARCH$_5$ denote the LM test statistic for conditional heteroskedasticity up to order 1 and 5, respectively.

Figure 1: The dollar-sterling real exchange rate (demeaned).
Figure 2: The United States-United Kingdom trade costs index.

Figure 3: The quadratic logistic function for the estimated TVTC-ESTAR model at 1900, 1950 and 2000.
Figure 4: The quadratic logistic function for the estimated TVTC-QLSTAR model at 1900, 1950 and 2000.

Figure 5: Difference in the degree of persistence between the TVTC-ESTAR and ESTAR models.
Figure 6: Difference in the degree of persistence between the TVTC-QLSTAR and QLSTAR models.