Government Spending and Consumption in the Presence of Borrowing Constraints
(Job Market Paper)

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Abstract

Empirical estimates of the effect of government spending indicate crowding-in effect on aggregate output, consumption and labor supply, a positive co-movement between consumption of durables and non-durables and a cyclical crowding in-crowding out effect on investment. But most of the neo-classical real business cycle models fail to explain most of these empirical facts and frequently, all of them. I develop an RBC model where some agents face a binding borrowing constraint. The borrowing constraint is imposed in the form of a collateral constraint on these agents when they seek to borrow from the private debt market. I also propose an improved method for fitting the model to the data. Prior predictive analysis shows that the borrowing constraint increases the volatility of the model and helps to ensure a better fit. It also shows that the parameter space that is most consistent with the data is also consistent with the calibrated values used in the model. Finally, I show that once the model is properly calibrated, the impulse response functions of an unanticipated increase in government spending match all of their empirical counterparts.

Key Words: Crowding-in, crowding-out, borrowing constraint, collateral requirement, borrower-saver model, monte carlo simulation, stochastic non-singularity, impulse response function

JEL code: E13, E2, E62, C15, H3

1 Introduction

Does government spending crowd-in or crowd-out output, private consumption and investment? This has been one of the fundamental questions in macroeconomics. Empirical analysis of the effect of government spending(Blanchard and Perotti(2002), Fatas and Mihov(2001) and Burnside, Eigenbaum and Fisher(2004)) indicates that in case of an unanticipated temporary increase in government spending:

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a) there is an increase in output, aggregate consumption and employment; the crowding-in effect,
b) there is a positive co-movement and crowding-in effect on durable and non-durable consumption; the co-movement effect,
c) there is a crowding-in effect on investment followed by a crowding-out; the crowding-in crowding out effect.

These empirical results are, however, at odds with theoretical results derived from standard real business cycle (RBC) models. Under reasonable parametrization, a general real business cycle model (RBC) predicts that there would be a crowding-out effect on output, consumption, employment and investment (Baxter and King (1993), Fatas and Mihov (2001)). In this paper, I develop an RBC model where some consumers face a binding borrowing constraint and the borrowing constraint relates consumers' borrowing to their durable goods purchases. I show that my model can reconcile the tension between the theoretical literature and the empirical findings. I use a log-linearized version of my model to generate impulse response functions of the macroeconomic variables for an unanticipated and temporary increase in government spending. Under reasonable parametrization, the model predicts a crowding-in effect on aggregate consumption, output and labor supply, a crowding-in effect and a positive co-movement between durable and non-durable consumption and a cyclical crowding in-crowding out effect on investment. Simple statistical analysis conducted on this model and some of its variations indicates that the borrowing constraint could be important in explaining most of the variability in the data. It also provides evidence that the parameter values used for calibration purpose are quite consistent with the data. Robustness checks indicate that the dynamics of the model is very consistent across reasonable and accepted regions of the parameter space.

2 The Effect of Government Spending: Contacts with Literature

The effect of an unanticipated temporary increase in government spending on macroeconomic variables such as output, consumption and investment has been one of the most productive area of macroeconomic research for the last 30 years. Early contributions by Barro (1981) and Bailey (1971) only focused on the effect on output while Hall (1980) argued that such policy could have significant business cycle effect on other macroeconomic variables. These authors showed that empirical effect on output was consistent with their simple version of the RBC models. Baxter and king (1993) expanded the theoretical literature on this issue by looking at effects on other macroeconomic variables such as consumption, labor supply and private investment and also by considering alternative financing of the temporary increase in government spending in a standard RBC model. The authors found a crowding-out effect on consumption and investment and a positive impact on labor supply.

These results, however, have come under serious scrutiny as a result of a recent surge in empirical papers based on Vector Auto Regressions (VAR). Using a semi-structural VAR, Blanchard and Perotti (2002) found that an unanticipated, temporary one standard deviation orthogonal shock (increase) in government spending leads to a crowding-in in both output and consumption while crowding out private investment. Fatas and Mihov (2001) expanded the empirical analysis of Blanchard and Perotti (BP from now on, 2002) by looking at a larger set of macroeconomic variables. By employing the same identification strategy used by BP, they found that that an unanticipated increase in government spending leads to a) a persistent increase in GDP, b) a persistent increase in aggregate consumption, durable and non-durable consumption, c) an immediate crowding in of investment followed by a crowding out effect(a cyclical effect), d) a persistent increase in net tax revenue, e) a persistent increase in private employment and f) a crowding-in and clear positive co-movement between durable and non-durable consumption.
The above papers gave rise to several major puzzles in the macroeconomic effects of government spending. First, the discrepancy between theoretical and empirical analysis of aggregate consumption has given rise to the crowding in- crowding out puzzle. It appears that the prediction of the standard RBC model on the effect of consumption is inconsistent with empirical findings. A large literature using different versions of the RBC model has tried to address this puzzle. Most have failed. The only notable success is Linneman(2005) who used an unconventional preference structure in an RBC setup to solve puzzle. Following seminal work by Mankiw(2000), Gali, Salido and Valles(2007) used a sticky price model where a fraction of the consumers are rule-of-thumb consumers in the sense that they make no intertemporal decisions. The authors conducted both empirical (following BP) and theoretical analysis and showed that they are consistent. Coenen and Straub(2005) developed a "new synthesis" model by combining Gali, et al. (2007) and Smets and Wouter(2003) where they added several real frictions(such as external habit persistence and investment adjustment cost) and also included an extended structure for the stochastic process for their model. They used Bayesian technique to estimate their model for the Euro area. Their results indicate a crowding out of consumption and investment. Forni, Monforte and Sessa(2006) extended Coenen and Straub(2005) by including several real frictions(such as internal habit persistence and investment adjustment cost). They estimated their model by using Bayesian technique for the Euro area and show that both their model and their estimation result indicate a crowding in of consumption and increase in output. But the main assumption on the nature of the rule-of-thumb consumers used by the above three papers have been criticized by Yang(2007) and Rahman(2008) who argued that the assumption of the rule of the thumb consumers is quite strong and imposes considerable restriction on the theoretical results.

Second, there are concerns about the co-movement between durable and non-durable consumption in the face of an unanticipated fiscal policy experiment, the co-movement puzzle. The empirical analysis indicates that there should be a positive co-movement between the durable and non-durable goods purchase whenever there is an unanticipated increase in government spending. No paper including Fatas and Mihov(2001) has so far attempted to address this puzzle for fiscal policy in a theoretical setup. In monetary policy analysis, this co-movement problem has gained significant traction. Barsky et al. (2007) first pointed out the co-movement problem in case of a monetary policy experiment and concluded that standard sticky price model cannot account for this result. Monacelli(2008) showed that by using borrowing constraint in a new Keynesian model, one could reconcile the puzzle.

Finally, it has been difficult to show the cyclical response pattern of investment in the face of a government spending shock. None of the papers cited earlier was able to replicate this. The only notable success is Burnside, Eichenbaum and Fisher(2004). The authors used an alternative identification scheme in their VAR setup but found responses of output, consumption and labor supply similar to BP. In order to explain these results in a theoretical setup, the authors incorporated an investment adjustment cost friction and internal habit persistence in an otherwise standard RBC model. While the paper was able to match the cyclical pattern of investment, it failed to show crowding in effect on consumption.

In this paper, I address the above three puzzles. I develop a model which is a modified version of the standard two-sector RBC model incorporating elements such as investment adjustment cost and habit persistence taken from the existing literature. In my model, there is a fraction of consumers who face a binding borrowing constraint which relates private sector borrowing by the consumers to the level of durable good purchase which acts as collateral. I show that my model can account for all the above puzzles simultaneously.
The Importance of Collateral requirement in US economy: Theory and Empirics

In USA, almost all private sector borrowing is subject to collateral requirement. The mortgage market where the collateral requirement is determined has undergone significant structural change in the last 70 years. Prior to the Great Depression, typical home mortgage payments were only interest and homeowners refinanced their loans' principles every few years. Consumers were also provided with installment credit through retailers. The Federal Home Loan Bank Act of 1932 and the Homeowners' Loan Act of 1933 established a regulatory framework where mortgage markets were insulated from the fluctuations of other capital market by the federal government acting as the lender of last resort. Second, long-term amortized mortgages replaced the previous interest-only, periodically refinanced mortgages. But the volatile financial market of the 60's and 70's and the monetary policy of the Volker regime made the existing regulatory framework unsustainable. Faced with a rising amount of insolvent savings and loans, the Monetary Act of 1980 and the Garn-St.Germain Act of 1982 eliminated restrictions on mortgage lending and re-integrated it with other financial markets.

The structural change in the mortgage market had significant effect on the private sector borrowing and collateral requirements. First, there is a rising trend in the mortgaged debt and private debt accumulation. Figure 1 shows the trend in mortgaged and total household and non-profit organization’s private debt, most of which in the form of housing and automobile purchases. The data is taken from the Federal reserve flow of funds accounts. The amount of collateralized debt, as measured by the fraction of total household debt mortgaged, increased from 78% in the last quarter of 1951 to a staggering 91% in the first quarter of 2007. During the period of 1951-2007, we also see a significant increase in the share of private debt as a fraction of GDP. That ratio increased almost three fold, from 2% in 1951QIV to 6% in 2007QI. Secondly, there appears to be a dramatic change in the volatility of private debt. Figure 2 shows the trend in the HP filtered data on private debt for the period of 1970Q1 to 2007QIV. There appears to be a dramatic decline in the volatility of private debt after 1983.

The above mentioned changes in the mortgage market also has implications for the down payment requirements. Figure 3 recreates figure 1 of Campbell and Hercowitz(2004). It plots the ratio of mortgage debt to the value of owner-occupied housing and the ratio of household debt to the value of their durable goods stocks, which includes housing. These ratios decline from 1966 to the end of 1982, and then start a dramatic increase. As Campbell and Hercowitz(2004) points out, this surge reflects the emergence of the sub prime mortgage lending market and households’ greater use of home equity loans and mortgage refinancing to cash-out previously accumulated equity and unrealized capital gains. This reflects an increased importance of good credit history which eased the process of refinancing. Greater access to refinancing and home equity loans allowed homeowners to greatly delay repayment of effective mortgage principle, and access to additional sub prime mortgage reduced effective down payment requirements.

Greater access to the mortgage market and a reduction of down payment requirement appears to have significant macroeconomic effects. There appears to be a general decline in the macroeconomic volatility for US coinciding with the changes in the mortgage market. Table 1 reports several summary statistics for the US macroeconomy. Comparing the HP filtered and logged data between sub-sample of 1970Q1-1982Q4 and 1983Q1-2007Q4, there is clear decline in the standard deviations for all the major macroeconomic variables such as real GDP, consumption, investment, government spending, public and private debt. Furthermore, the strength of the co-movement between the variables have decline, as evident by a decline in the correlation coefficient. The co-movement between government spending and other variables, although negative, appears to have weakened significantly between the two sample period.

In summary, there appears to be an improved and easier access to the mortgage market which
seems to have coincided with a decline in macroeconomic volatility. Therefore, it appears that the mortgage market plays a very important role in the USA economy. The importance of the mortgage market has prompted a more thorough and accurate modeling of the borrowing constraints faced by agents in macroeconomic models. In a seminal paper, Kiyotaki and Moore (1997) used borrowing constraint to exhibit how credit constraints and collateral requirement could serve a powerful transmission mechanism by which effects of aggregate shocks could be amplified. In their heterogenous agent model of "farmers" and "gatherers", the former can borrow from the private credit market, which is subject to collateral requirements, defined by their land holding. If the farmer has a durable goods stock of $D_t$ at date $t$, then the borrowing constraint requires that he can borrow any amount $B_t$ as long as the repayment does not exceed the market value of the durable goods. Let $q_t$ and $R$ be the market price of durable goods and the risk free interest on debt, the borrowing constraint is specified as followed:

$$B_t R \leq q_{t+1} D_t$$ (a)

Monacelli (2006) modified the borrowing constraint by assuming that borrowing limit cannot exceed a certain fraction of the durable goods stock, which he defined to be the fraction of the durable goods that can be collateralized:

$$B_t \leq (1 - \pi) D_t$$ (b)

Here, $\pi$ is the fraction of durable goods that cannot be used as collateral. Monacelli (2008) and Iacoviello (2005) further modified the borrowing constraint by requiring that the repayment cannot exceed the value of the fraction of the durable goods that can be used as collateral:

$$B_t R_t \leq (1 - \pi) D_t$$ (c)

I will, however, use the borrowing constraint specified by Campbell and Hercowitz (2004). The authors developed a heterogeneous agent model consisting of two types of agents, borrowers and savers. They used a borrowing constraint to analyze the amplification mechanism of productivity shock in an otherwise standard RBC model to explain the reduction of macroeconomic volatility in USA. The authors made a significant modification to the borrowing constraint by making borrowing not only depend on collateral requirement but also on past credit history, as observed in the data. The household in their model faces three saving opportunities, invest in physical capital, buy government/public bond and serve as a lender in the private debt market. Household cannot short-sell one asset and buy another. Households also have the opportunity to borrow from the private market by issuing one period state contingent bond. The state-contingent claims are assumed to be unbacked and are unenforceable. As a result, the private credit market is incomplete. Private borrowing is subject to an endogenous limit. The collateralized value of the durable goods stock is generally less than its replacement cost. For a stock of durable good $D_{t+1}$, it is given by:

$$V_{t+1} = (1 - \pi) \sum_{j=1}^{\infty} (1 - \phi) \left[ D_{t-j+1} - (1 - \delta_D) D_{t-j} \right]$$ (1)

Here $\pi$ is the fraction of a new durable good that cannot serve as collateral. $\phi$ is the rate at which a good’s collateral value depreciates and $\delta_D$ is the depreciation rate of the durable good. Campbell and Hercowitz (2004) assumed that $\phi \geq \delta_D$, so that the good’s value to a creditor declines at least as rapidly as its value to its owner. Equation(1) can be written in the following recursive form:

$$V_{t+1} = (1 - \phi) V_t + (1 - \pi) \left[ D_t - (1 - \delta_D) D_{t-1} \right]$$ (1.a)

5
Collateral requirement limits household borrowing. That is:

\[ B_{t+1} \leq V_{t+1} \]  

(2)

Here \( B_{t+1} \) refers to the outstanding debts of the households at the end of period \( t \) and \( V_{t+1} \) are the collateralized value of their durable goods.

4 The Borrower-Saver Model

My model is an extended version of the two sector model developed by Leeper, Walker and Yang(2008) embedded in a Campbell and Hercowitz(2004) setup. The model combines heterogeneity across household’s rates of time preference with collateral constraint on borrowing. Household debt reflects intertemporal trade between an impatient borrower and a patient saver. In the model economy, durable goods collateralize all household debt. Without collateral constraints, the patient saver lends to the impatient borrower and the debt increases over time. Collateral constraints limit the borrower’s debt, so the economy possesses a unique steady state with positive consumption by both households. In general, the borrower’s collateral constraint may bind occasionally. However, I will show that it always binds in the steady state. So if my model remains close to the steady state, the borrowing constraint for the borrower will bind. Therefore, standard log-linearization techniques can characterize its equilibrium for small disturbances. This is the path that has been followed by Campbell and Hercowitz(2004) and Monacelli(2008). We will do the same thing.

Time is taken in discrete intervals, \( t = 1, 2, \ldots \). The economy is composed of a continuum of households in the interval of \((0,1)\). There are two types of households, named Borrowers and Savers, of measure of \((1-F)\) and \(F\). They only differ in their time preference. More specifically, I assume that they have different time preference rate, \( \beta_b \) and \( \beta_s \) where I assume \( \beta_s > \beta_b \) which makes the saver the patient agent and the borrower the impatient agent. Each household is endowed with one unit of time. Each household derives utility from three sources: consumption of durable goods \((C_{t,b,s})\), consumption of non-durable goods \((D_{t,b,s})\) and leisure \((1-L_{t,b,s})\) where the superscript \( s \) refers to savers and \( b \) refers to borrowers. They also incur disutility from working and making decisions on a durable goods purchase. The household faces three saving opportunities, as in Campbell and Hercowitz(2004). Using (1) and (2), the borrowing constraint can be written recursively as:

\[ B_{t+1}^{b,s} \leq (1-\phi)B_{t}^{b,s} + (1-\pi)D_{t+1}^{b,s} - (1-\delta_D)D_{t}^{b,s} \]  

(3)

When collateral limits a household’s borrowing, \( \pi \) is the required down-payment rate for the durable goods purchase and \( \phi \) is the rate at which the principal is repaid. Following the existing literature, I will assume these two parameters are exogenously determined by the regulatory environment.

4.1 Utility Maximization by the Borrower

Following Becker(1980), Campbell and Hercowitz(2004) and Monacelli(2008) , I will assume that the borrowers are impatience enough so that they face a borrowing constraint that always binds. Since they cannot short-sell, borrowers will not invest on physical capital or buy public debt. The within period utility function of the borrower look like:
function. The quadratic term-

\[ U(.) = \left( C_t^{*s} \right)^{1 - \frac{1}{\sigma}} + V^s (D_t^s)^{1 - \frac{1}{\sigma}} + \frac{\eta (D_t^s - D_{t-1}^s)}{D_t^s} + \frac{(1 - L_t^s)^{1 - \theta} - 1}{1 - \theta} \]

Here \( C_t^{*s} = C_t^s - bC_{t-1}^s \), with \( b \geq 0 \) indicating the degree of internal habit persistence. \( \tau \) and \( \sigma \) are the elasticities of inter-temporal and intratemporal substitution of consumption. \( \theta \) is the inverse elasticity of intertemporal substitution of leisure. \( \chi \) is the weight on leisure in the utility function. The quadratic term \( \frac{\eta (D_t^s - D_{t-1}^s)^2}{D_t^s} \) is interpreted as the deliberation cost where \( \eta \) captures the disutility of changing durable good stock. Household are also endowed with physical capital stocks, \( k_0^b \geq 0 \). Given their initial capital stock each household chooses a sequence of consumption of durable goods, non-durable goods, private debt and leisure; \( \{C_t^b, D_t^b, B_t^b, L_t^b\}_{t=1}^{\infty} \) to maximize his expected lifetime utility:

\[ E_0 \sum_{t=1}^{\infty} \beta_t^{t-1} U(C_t^b, D_t^b, L_t^b) \]

Subject to the budget constraint:

\[ C_t^b + D_t^b \leq (1 - \tau_t^b) W_t L_t^b + B_t^b - B_{t-1}^b R_{1t-1} + (1 - \delta_D) D_{t-1}^b + TR_t^b \]

and the borrowing constraint:

\[ B_t^b \leq (1 - \phi) B_{t-1}^b + (1 - \pi) [D_t^b - (1 - \delta_D) D_{t-1}^b] \]

4.2 Utility Maximization by the Saver

I will assume that the savers are patience enough so that their borrowing constraint never binds. The preference structure of the saver is very similar to the borrower:

\[ U(.) = \left( C_t^s \right)^{1 - \frac{1}{\sigma}} + V^s (D_t^s)^{1 - \frac{1}{\sigma}} + \frac{\eta (D_t^s - D_{t-1}^s)}{D_t^s} + \frac{(1 - L_t^s)^{1 - \theta} - 1}{1 - \theta} \]

Here all the parameters have the same interpretation as in the case of the borrower with \( C_t^{*s} = C_t^s - bC_{t-1}^s \). Given initial capital stock \( k_0^s \geq 0 \), the saver will therefore choose a sequence of non-durable goods, durable goods, private debt, labor supply, public debt, physical capital stock, investment in physical capital stock and capital utilization rate \( \{C_t^s, D_t^s, B_t^s, L_t^s, X_t^s, K_t^s, I_t^s, \mu_t\}_{t=1}^{\infty} \) to maximize

\[ E_0 \sum_{t=1}^{\infty} \beta_t^{t-1} U(C_t^s, D_t^s, L_t^s) \]

Subject to the budget constraint:

\[ C_t^s + D_t^s + X_t^s + I_t^s \leq (1 - \tau_t^s) W_t L_t^s + B_t^s - B_{t-1}^s R_{1t-1} + (1 - \delta_D) D_{t-1}^s + \delta \tau_t^s K_{t-1}^s + (1 - \tau_t^s) \mu_t K_{t-1}^s + X_{t-1}^s R_{2t-1} + TR_t^s \]
and the law of motion of capital stock:

$$K_t^s \leq \left\{ 1 - s \left( \frac{I_t^s}{I_{t-1}^s} \right) \right\} I_t^s + (1 - \delta_t)K_{t-1}^s$$  (11)

where:

$$S(1) = S'(1) = 0, S''(1) = \gamma > 0$$  (12)

Here $s \left( \frac{I_t^s}{I_{t-1}^s} \right)$ is defined as investment adjustment cost, taken from Burnside, Eichenbaum and Fisher(2004). Following Leeper, Walker and Yang(2008), I assume a constant depreciation rate for durables, but not for capital. As in Greenwood, Hercowitz and Huffman(1988), using capital more intensively makes capital depreciate at a faster rate. The depreciation rate of capital has the following form:

$$\delta_t = \delta \mu_t^\omega$$  (13)

where $0 < \delta < 1$ and $\omega > 0$. Leeper, Walker and Yang(2008) also pointed out that U.S. tax codes does not have depreciation allowances depending on the period-by-period capital utilization intensity; the depreciation allowance is based on a pre-determined statutory schedule. Similar to these authors, I will assume that the capital depreciation allowance($\tilde{\delta} \tau_t K_{t-1}$) is given according to the time-invariant steady state rate of capital depreciation, $\tilde{\delta} (= \delta \mu_t)$. Also, following Becker(1980), I will assume that in the initial period, $K^b_0 = I^b_0$. Finally, I assume that the utility function is strictly concave, twice differentiable and satisfies the inada condition.

### 4.3 Profit Maximization by the Firm

The production function used by the firm is defined as follows:

$$Y_t = f \left( L_t, K_t \right) = \{\mu_t K_{t-1}\}^\alpha \{L_t\}^{1-\alpha}$$  (14)

The Representative firm rents capital and labor from agents to maximize profit

\[
\text{Profit} = \{\mu_t K_{t-1}\}^\alpha \{L_t\}^{1-\alpha} - r_t \mu_t K_{t-1} - w_t L_t
\]

where $K_t$ and $L_t$ are aggregate capital stock and labor supply, to be defined later. I also assume that the production function is strictly concave, twice differentiable and satisfy the inada condition.

### 4.4 Government Budget Constraint

The government levies taxes on capital($\tau_t^K$) and labor income($\tau_t^L$) separately, sells one period government bond($X_t$) to the savers, issues a depreciation allowance for capital($\tilde{\delta} \tau_t^K \mu_t K_{t-1}$) and provide lump-sum transfers($TR_t$) to the consumers to balance the budget. The government budget constraint is:

$$G_t + X_{t-1} R_{2t-1} + \tilde{\delta} \tau_t^K \mu_t K_{t-1} + TR_t = T_t + X_t$$  (15)

where $T_t$ is the total tax collected defined as:

$$T_t = T_t^K + T_t^L$$  (16)

8
\[ T^l_t = F \tau^L_t w_t L^s_t + (1 - F) \tau^L_t w_t L^b_t \]  
\[ T^k_t = \tau^k_t \mu_t K_{t-1} \]  

Finally, the total transfer in the economy, \( TR_t \), is:

\[ TR_t = TR^s_t + TR^b_t \]  

Here, \( TR^s_t \) and \( TR^b_t \) are total transfers to the saver and borrower, to be defined later. In this paper, I assume that an increase in government spending could be financed in various ways. To study the implications of alternative financing, I will posit the simplest possible rules for fiscal policy instruments that are consistent with fiscal solvency. The fiscal instruments are chosen as a function of the state of government indebtedness, as measured by the debt-output ratio. The rules adopted here are abstractions designed to capture the practice of offsetting policy: when the fiscal budget deteriorates and debt rises, explicit fiscal actions are taken to improve the budget situations. Following Leeper and Yang (2006), the fiscal policy rules that the government uses are summarized as follows:

\[ \ln \left( \frac{s^TR^s_t}{s^{TR^s}} \right) = -q_{TR}^s * M * \ln \left( \frac{s^B_{t-1}}{s^B} \right) + \varepsilon^{TR^s}_t, q_{TR^s} \geq 0 \]  
\[ \ln \left( \frac{s^TR^b_t}{s^{TR^b}} \right) = -q_{TR}^b * N * \ln \left( \frac{s^B_{t-1}}{s^B} \right) + \varepsilon^{TR^b}_t, q_{TR^b} \geq 0 \]  
\[ \ln G_t = \rho_G \ln G_{t-1} + u^G_t \]  
\[ \ln \left( \frac{\tau^L_t}{\tau^L} \right) = q_L \ln \left( \frac{s^B_{t-1}}{s^B} \right) + \varepsilon^L_t, q_L \geq 0 \]  
\[ \ln \left( \frac{\tau^K_t}{\tau^K} \right) = q_K \left( \frac{s^B_{t-1}}{s^B} \right) + \varepsilon^K_t, q_K \geq 0 \]  

Here \( s^TR^h_t = \frac{TR^h_t}{TR} \), \( h = s, b \) and \( u^G_t \sim iid.N(0, \sigma^2_G) \). Variables without time subscript denote steady state values. The rules build in a one-year delay for the response of an offsetting policy\(^1\). The \( q \)'s in the rules 1-4(equations 20, 21, 23 and 24) are defined as "fiscal adjustment parameters". Sign restrictions on the \( q \)'s are also straightforward. When the debt-output ratio rises above the initial steady-state level, one of the future distorting taxes are raised or transfer-output is reduced to maintain fiscal solvency. To isolate the impact of each financing instruments, one of the \( q \)'s is non-zero in each experiment. For example, if transfer-output ratio adjusts, \( q_{TR} > 0 \) and \( q_L = q_K = 0 \). Furthermore, since there is income heterogeneity in this model, transfers are distributionally non-neutral by nature. This means that even if transfer-output ratio for both group of consumers adjust by the same rate, as measured by \( q_{TR} \), the actual magnitudes of the change are not equal. In order to achieve equal magnitude of adjustment, I introduce two new constants, \( M \) and \( N \) which are defined as follows:

\[ M = \frac{TR^s}{TR} \text{ if distributionally neutral transfer adjustment and 1 otherwise} \]

\(^1\)Leeper and Yang (2006) argued that longer delays such as five year do not change the results significantly.
\[ N = \frac{TR^b}{TR} \] if distributionally neutral transfer adjustment and 1 otherwise

The error terms in fiscal rules are all AR(1) process, defined as follows:

\[ \varepsilon^{TR^s}_t = \rho^{TR^s}_t \varepsilon^{TR^s}_{t-1} + u^{TR^s}_t, u^{TR^s}_t \sim iid.N(0, \sigma^2_{TR^s}) \]  

\[ \varepsilon^{TR^b}_t = \rho^{TR^b}_t \varepsilon^{TR^b}_{t-1} + u^{TR^b}_t, u^{TR^b}_t \sim iid.N(0, \sigma^2_{TR^b}) \]  

\[ \varepsilon^{rL}_t = \rho^{rL}_t \varepsilon^{rL}_{t-1} + u^{rL}_t, u^{rL}_t \sim iid.N(0, \sigma^2_{rL}) \]  

\[ \varepsilon^{rK}_t = \rho^{rK}_t \varepsilon^{rK}_{t-1} + u^{rK}_t, u^{rK}_t \sim iid.N(0, \sigma^2_{rK}) \]  

The government also has to maintain intertemporal fiscal solvency. First, any equilibrium must satisfy the transversality conditions:

\[ E_t \lim_{T \to \infty} \beta^{t+T}_h \lambda_{s,t+T}K^s = 0 \]  

\[ E_t \lim_{T \to \infty} \beta^{t+T}_h \lambda_{h,t+T}B^h = 0, h = b, s \]  

\[ E_t \lim_{T \to \infty} \beta^{t+T}_h \lambda_{s,t+T}X = 0 \]  

The TVC imply that in any optimum, the households do not over-accumulate government liabilities, or private debt or physical capital. Imposing the TVC on the flow budget constraint of the government, we derive the intertemporal budget constraint for the government:

\[ \frac{B_t}{Y_t} = s^B_t = \sum_{j=0}^{\infty} d_{t,t+j} \left[ (1 - \alpha) \tau^L_{t+j} \frac{F^L_{t+j}}{L_{t+j}} + (1 - \alpha) \tau^K_{t+j} \frac{(1-F)_{t+j}}{L_{t+j}} \right] \]  

Where \( d_{t,t+j} = \prod_{j=0}^{j-1} R^1_{2t+i} \frac{Y_{i+j}}{Y_{i+j+1}} \). In equilibrium, equation(30) determines the value of government debt. It also imposes restrictions on dynamic interaction between current debt and expected future policies. An increase in government spending raises \( \frac{B_t}{Y_t} \) which automatically requires some combination of fiscal variables and/or discount factors in the future to adjust. The above fiscal rules only a subset of expected sequences of fiscal policies that satisfy equation(30). Feasibility will be ensured by judicious choice of response magnitude parameters- the q’s in the rules. I will use the values used in Leeper and Yang(2006).

### 4.5 Aggregation and Market Clearing Conditions

I will aggregate the economy as follows:

\[ I_t = FI^s_t, X_t = FX^s_t, K_t = FK^s_t \]  

\[ B_t = FB^s_t + (1 - F)B^b_t = 0 \]  

\[ L_t = FL^s_t + (1 - F)L^b_t \]  

\[ C_t = FC^s_t + (1 - F)C^b_t \]
\[ D_t = FD_t^s + (1 - F)D_t^b \]  
\[ TR_t^s = S * tr_t^s, TR_t^b = (1 - S) * tr_t^b \]  

Finally, the goods market clearing condition or the aggregate resource constraint can be written as:

\[ C_t + I_t + G_t + D_t = Y_t + (1 - \delta_D)D_{t-1} \]  

For the purpose of comparing the variables from my model to the variables found in the National Income Accounting (NIPA) data I will define the flow of durable goods service as:

\[ \text{Durable Service} = D_{-S_t} = D_t - (1 - \delta_D)D_{t-1} \]  

Since the NIPA data reports the flow of durable goods, equation (38) will be used for calibration purpose. Also, the aggregate consumption in the economy is defined as follows:

\[ \text{Aggregate Consumption} = AD_{-C_t} = C_t + D_{-S_t} \]  

**Definition 1** A Rational Expectations Competitive Equilibrium is a pair of sequence of prices \( \{r_t, w_t\}_{t=1}^{\infty} \), a sequence of a set of consumers’ decisions \( \{C_t^h, D_t^h, B_t^h, K_t^h, \}_{t=1}^{\infty} \), a sequence of firm’s decisions \( \{K_t, L_t\}_{t=1}^{\infty} \), a sequence of policy variables, \( \{X_t^s, G_t, \tau_t^K, \}_{t=1}^{\infty} \), such that, given initial level of capital stock \( K_{t-1} \), private and public debt, the optimization for the agents and firm’s are solved; the goods, capital, labor and the debt markets clear; the transversality conditions for capital and debts hold; the government budget constraint and at least one of the policy rules and all the aggregate conditions are satisfied. Furthermore, we will only consider the ranges of the fiscal adjustment parameters- the q’s- that are consistent with the existence of a rational expectations competitive equilibrium.

Appendix A, section 1 shows the first order conditions of utility and profit maximization for this economy. It also shows the steady state conditions derived for this economy. Here, I will provide a simple proof that the borrowing constraint of the borrower binds in the steady state. Assume \( \lambda_b \) and \( \psi_b \) to be the Lagrangian multipliers associated with the budget constraint and borrowing constraint of the borrower. In steady state, the Kuhn-Tucker condition for \( B^b \) looks like:

\[ \lambda_b \left( 1 - \beta_b R_2 \right) - \psi_b \left( 1 - \beta_b \left( 1 - \phi \right) \right) \leq 0, B^b \geq 0 \text{ with } B^b \left[ \lambda_b \left( 1 - \beta_b R_2 \right) - \psi_b \left( 1 - \beta_b \left( 1 - \phi \right) \right) \right] = 0 \]

From the first part of the CS condition, we see:

\[ \psi_b \left( 1 - \beta_b \left( 1 - \phi \right) \right) \geq \lambda_b \left( 1 - \beta_b R_2 \right) \Rightarrow \psi_b > 0 \]

Therefore, the borrowing constraint for the borrower binds in the steady state.
4.6 Model Calibration

Table 2 reports the benchmark values of parameters and the steady state values of variables that will be used for calibrating the model to US data. The value of inter-temporal elasticity of substitution ($\tau$) is taken from Ogaki and Reinhart(1998) to be 0.447. The value of Intratemporal elasticity of substitution is taken to be 0.90 which is slightly below the value the authors reported. Later we will look at the implication of this value and provide robustness checks. The steady state share of total time devoted to production for both types of consumer ($L_s$ and $L_{st}$) is assumed to be 0.20, the average weekly hours of production workers to 144 hours ($24 \times 7$), reported by BLS. The variable capital depreciation parameter ($\omega$) is assumed to be 1.56, taken from the estimates of Burnside and Eichenbaum(1996) which implies a quarterly capital depreciation rate of 0.02. The investment adjustment cost parameter ($\gamma$) is assumed to be, taken from Coenen and Straub(2005). The habit persistence parameter ($\delta$) is assumed to be 0.80, taken from Burnside, Eichenbaum and Fisher(2004). The deliberation cost parameter ($\eta$) is taken from Leeper, Walker and Yang(2008). The steady state capital tax and labor tax rate are set at the historical average of U. S. data(1947Q1-2008Q4). The two tax rates follow the definition of Jones(2002). The weight to leisure ($\chi$) taken from Leeper and Yang(2006). The steady state capital depreciation rate ($\delta$) is taken from Leeper, Walker and Yang(2008) while the depreciation rate for durable goods ($\delta_D$) is taken from are taken from Campbell and Hercowitz(2004). The value of $q$'s are taken from Leeper and Yang(2006). The value of the AR(1) coefficients and the value of standard deviations of various shocks have been taken from Forni, Monforte and Sessa(2006) and Coenen and Straub(2005). The parameters related to the borrowing constraints are taken from Campbell and Hercowitz(2004). They reported the value of $\pi$ and $\phi$ to be 0.16 and 0.03 for the high collateral regime that corresponds to the period 1971-1982 and 0.11 and 0.01 for the low collateral regime that corresponds to 1983-2007. I will the use value of $\pi$ and $\phi$ to be 0.15 and 0.03 for the high regime. Finally, the value for the inverse elasticity of intertemporal substitution for leisure ($\theta$) is taken from Leeper and Yang(2006). The values of the betas; $\beta_s$ and $\beta_h$ are assumed to be 0.99 and 0.97, which are similar to the values used by Campbell and Hercowitz(2004) and Monacelli(2008). The value of the fraction of savers($F$) are taken from Rahman(2008) and Joint Committee of Taxation(2006). The fraction of transfers that goes to the savers ($S$) is taken from an earlier version of Yang(2007). Although I use specific values for calibrating my model, I will later conduct prior predictive analysis to understand the sensitivity of the above parameters.

In addition to the above parameter values, I will need the value of several ratios to solve the steady state values for the variables in the model. The government spending-output ratio ($SG$), investment-output ratio ($SI$), aggregate consumption-output ratio ($SC$), total transfer-output ratio ($STR$), public debt-output ratio ($SX$) are set at the historical average of U. S. data. The value of private debt-output ratio ($SB$) is also set at the historical average but for a different sample period. For the calibration purpose, I will also need the ratio of consumption expenditure on durables to consumption expenditures on non-durables ($\frac{D_s}{C}$) which is set to be 0.149 taken from the NIPA data.

4.7 Solution Method and Stability Conditions

An analytical solution of the model is not available; the equilibrium conditions are log-linearized around the original steady state and analyzed in terms of percentage deviations from that steady state. This means I will assume that the perturbation in the log-linearized model is small enough
so that the log-linear model exhibits the same dynamic behavior as the original model in the steady state. In the log-linear model, I will postulate that the borrowing constraint of the borrower binds all the time, although during the simulation exercise I will be constantly checking whether the Lagrangian multiplier associated with borrower’s borrowing constraint is positive or not. The model is solved using Sims’s (2001) algorithm. Also, the log-linearized version of the model is complicated enough that it prevents me from analyzing its stability conditions analytically. They can only be analyzed by using a computer. I will, however, provide a brief discussion of the technical aspects associated with evaluating the stability conditions for this model, following Novales, Dominguez, Perez and Ruiz (2003) and Uhlig (2006).

The log-linearized model is first cast in its canonical form:

\[ \Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi z_t + \Pi \eta_t \]

Here \( y_t \) is the vector of log-linearized variables, \( z_t \) is the vector of exogenous processes (5 shocks) and \( \eta_t \) is the vector of expectational error generated as result of inclusion of forward looking variables in the model dynamics. When \( \Gamma_0 \) is invertible, stability conditions are evaluated by computing the eigenvalues of \( \Gamma_0^{-1} \Gamma_1 \) and checking for signs and magnitudes. However, Novales et. al (2003) showed that in a typical RBC model when capital stock and investment shows up simultaneously, this creates a redundancy in the model because of their contemporaneous relationship and therefore, makes the \( \Gamma_0 \) matrix singular. The problem also arises in my model. As a result \( \Gamma_0 \) is not invertible and we cannot carry out standard eigenvector - eigenvalue decomposition. The alternative is to use the QZ-decomposition to derive generalized eigenvalues, as suggested by Sims (2001). According to this method, for any pair of square matrices like \( (\Gamma_0, \Gamma_1) \), there exist orthonormal matrices \( Q, Z \) \( (QQ' = ZZ' = I) \) and upper triangular matrices \( \Lambda \) and \( \Omega \) such that:

\[ \Gamma_0 = Q' \Lambda Z', \Gamma_1 = Q' \Omega Z' \]

Besides, \( Q \) and \( Z \) can be chosen so that all possible zeros of \( \Lambda \) occur in the lower right corner and such that the remaining ratios \( \frac{\lambda_{ii}}{\lambda_{ji}} \) of diagonal elements in \( \Lambda \) and \( \Omega \), are non-decreasing in absolute values as they move down the diagonal. These ratios are the generalized eigenvalue of the pair \( (\Gamma_0, \Gamma_1) \). The stability of the model depends on the magnitude of these eigenvalues. According, to Uhlig (2006), if the dimension of \( \Lambda \) is \( m \) and if there exists exactly \( m \) generalized eigenvalues smaller than unity in absolute value, the system is said to be saddle-path stable. In my model this condition is satisfied under baseline calibration. Hence my model is also saddle-path stable.3

5 Dynamic Impact of a Temporary Increase in Government Spending

This section reports the dynamic impact of an unanticipated temporary increase in government spending and show how those impact changes when there are a) c) changes in the nature of the borrowing constraints, b) changes in the financing schemes or fiscal rules, c) changes in the Intratemporal Elasticity of Substitution between the durable and non-durable good, d) changes in the modeling assumptions, and e) changes in the collateral regime. I will focus on which experiments derive the three main results highlighted in the introduction and in section 2.

5.1 Government Spending Shock under Alternative Borrowing Constraints

Figure 4 compares the baseline model with borrowing constraint following Campbell and Hercowitz (2004) with the one that uses borrowing constraint similar to Kiyotaki and Moore (1997),

\[ ^3 \text{A list of the set of generalized eigenvalues for my model is available upon request.} \]
which was defined in equation (a). In both case, baseline parameters defined in table 2 are used to calibrate the models and non-neutral transfers adjust. This means that the first and second fiscal rules (equation 20-21) are in effect with $q_{TR} = 0.341$, $q_{L} = q_{K} = 0$ and $M = N = 1$.

In the Campbell and Hercowitz (2004) case, we see a crowding-in in case of output, aggregate consumption and labor supply, and a positive co-movement between durable and non-dur able consumption and crowding-in. In case of investment, there is a crowding in-crowding-out effect. All these results are consistent with the empirical facts defined in the introduction and in section 2. We see that for the Kiyotaki and Moore (1997) case, the co-movement problem does not appear. There is, however, crowding out of aggregate consumption and its components. Both output and labor goes down upon impact and shows pattern contradicting the empirical findings. Investment shows inverse cyclical pattern.

Figure 5 compares the baseline model with borrowing constraint following Campbell and Hercowitz (2004) with the one that uses borrowing constraint similar to Monacelli (2008), which was defined in equation (c). In both case, we use baseline parameters and non-neutral transfers adjust. This time the co-movement problem arises again. There is crowding out of aggregate consumption and its components. Output and labor supply shows even more contractionary effect. Investment again shows the inverse cyclical pattern.

In summary, among alternative specifications of the borrowing constraint, only the baseline specification used in this paper taken from Campbell and Hercowitz (2004) can match all the empirical facts discussed in the introduction of the paper.

Table 3 reports the size of output, consumption and investment multipliers at different points on the transition path from the empirical works of BP (2002), Gali et al. (2007) and Fatas and Mihov (2001) and compare it with the multipliers derived from the baseline model with borrowing constraint similar to Campbell and Hercowitz (2004) where non-neutral transfers adjust. Numbers in the parentheses indicate an approximation of the one standard deviation confidence bands. Output and consumption multipliers appear to be similar to the values derived by Fatas and Mihov (2001). If we combine all the empirical results, we see that the value of the consumption and output multipliers are within one standard deviation of confidence bands of the empirical results. The size of the investment multiplier is larger than those derived in the empirical studies. But the qualitative nature and the pattern of the investment multiplier is similar to Fatas and Mihov (2001).

5.2 Government Spending Shock under Alternative Financing schemes

Figure 6-8 shows the effect of government spending under alternative financing schemes. For all the cases, we assume that durable and non-durable goods are Edgeworth complements; $\sigma = 0.90$. Figure 6 compares the effect between neutral and non-neutral transfer adjustment. This means that the first and second fiscal rules (equation 20-21) are in effect with $q_{TR} = 0.341$, $q_{L} = q_{K} = 0$. In case of non-neutral transfers, $M = N = 1$. In case of neutral transfer adjustment $M = \frac{TR^s}{TR}$ and $N = \frac{TR^b}{TR}$. In transfer adjustment case, The effects are quite similar. In case of the neutral transfer adjustment (solid line), the initial impact is identical to the non-neutral case (dotted line). During the transition path, the two groups face similar decline in transfer in the neutral case. In case of the non-neutral case, the borrowers face a larger reduction in their transfers while the savers face a smaller decline. This is reflected in the transition path as the dynamic response of the borrowers in non-neutral case trail their behavior in the neutral case. For the saver, we see opposite effect.

4Only BP (2002) reports the output multiplier for different points on the transition path. No other multiplier was reported in any of the cited paper. All the cited papers only reported the impulse response functions with one standard deviation confidence bands. Therefore, the size of the multipliers and the confidence bands are approximated values derived from visual inspection of the impulse response functions.
Figure 7 and 8 shows cases when government spending is adjusted by raising labor tax and capital tax. For the labor tax adjustment case, the third fiscal rule(equation 23) is in effect with \( q_L = 0.149, q_{TR} = q_K = 0 \). For the capital adjustment case, the fourth fiscal rule(equation 24) is in effect with \( q_K = 0.206, q_{TR} = q_L = 0 \). In case of labor tax adjustment, aggregate consumption crowds in but both output and labor supply goes down. The co-movement problem between durable and non-durable consumption goods also appear. In case of capital tax adjustment, even aggregate consumption crowds out along with other features mentioned in the labor tax adjustment case. Since savers are less effected by labor tax and more effected by capital tax, we see crowding-in in investment in the former case while crowding-out effect in the latter.

In summary, among alternative financing schemes, both neutral and non-neutral transfer adjustment can match all the empirical facts explained in the introduction of the paper. The other financing schemes cannot match most of the empirical facts.

5.3 Government Spending shock under different Intratemporal Elasticity of Substitution (InES)

Figure 9 shows the effect of government spending shock under two values of InES to highlight the importance of this parameter. In both case, we assume that non-neutral transfers adjust. This means that first and second fiscal rules(equation 20-21) are in effect with \( q_{TR} = 0.341, q_L = q_K = 0 \) and \( M = N = 1 \). The baseline case, the dotted line, is where durable and non-durable goods are Edgeworth complements, i.e. \( \sigma = 0.90 \). The solid line shows the case when they are Edgeworth substitutes, i.e. \( \sigma = 1.05 \). In the substitute case, we see the co-movement between durable and non-durable goods disappears; aggregate labor supply, output and aggregate consumption all crowds in after initial decline. Also investment appears to crowd-out. In the complementary case, as was discussed in the previous sub-section, we see positive co-movement and crowding-in effect between durable and non-durable goods; aggregate labor supply, output and aggregate consumption all exhibits immediate crowding in effect while investment displays cyclical response starting with immediate crowding in.

In order to explain the movement of our targeted macro-variables, we will decompose their response into five categories, although they all happen simultaneously. We will look at\(^5\) a) the debt effect, b) the collateral constraint effect, c) the interest rate effect, d) the wage effect and e) the habit persistence effect. An unanticipated increase in government spending creates a negative wealth effect because it takes away a larger fraction from consumption and investment. Upon impact, real interest rate increases(interest rate effect). This raises the service cost of debt, thereby increasing the shadow cost of borrowing(debt effect). At the same time, there is a reduction in wage(wage effect). A constrained borrower with habit persistence de-accumulates his debt by paying part of it off(constraint effect). Wage reduction, upon impact induces him to reduce labor supply. With a loss of wage income and diminishing borrowing opportunity, the borrower dramatically reduces consumption of durables(habit persistence effect). When durable and non-durables are weak substitutes, this induces substitution towards durables.

The saver, on the other hand, enjoys contrasting economic opportunities. A debt payment from the borrower and expected increased return from public bonds enables him to increase his consumption of non-durable sufficiently so that he can forgo consumption of durable, work less and even reduce investment in physical capital. Since the behavior of the borrower dominates in the aggregate, their joint behavior with the saver gives rise to co-movement problem, crowd-out aggregate consumption and investment, reduce output and labor supply upon impact.

During the transition path, as long as the persistent shock was in effect, a declining transfer

\(^5\)Monacelli(2008) looks at the first three effects in the context of monetary policy analysis. My analysis will differ from his in two aspects. I will look at real debt and real interest effect while Monacelli(2008) looked at nominal debt and nominal interest effect.
forces the borrower to increase labor supply, de-accumulate debt, and reduce both consumption of durable and non-durable. The savers on the other hand, faces a richer prospect during this time and their consumption of durable continues to increase while non-durable goes down further before increases back to their steady level. After the shock subsides, the weaker negative income effect from transfers alone enables the borrower to reduce his labor supply, increase non-durable consumption and reduce durables.

The dynamic responses look fundamentally different and more consistent with empirical findings when durables and non-durables are Edgeworth complements. For the borrower, this induces them to increase consumption of durable and non-durable, work harder and de-accumulate debt quite dramatically. The saver, upon impact, dramatically increases their labor supply in order to accommodate consumption smoothing. With debt payment and rising interest payment, they can now enjoy simultaneous increase in both kinds of consumption and still manage to investment in physical capital and even buy government bond. This process continues as long as the shock was in effect. After that, the economy converges back to the original steady state. During the entire transition path, we see a positive co-movement between the consumption of durable and non-durable.

In summary, when durable and non-durable goods are Edgeworth complements, the model with non-neutral adjustment can match all the empirical facts explained in the introduction of the paper.

5.4 Government Spending Shock under Alternative Modeling Assumptions

Figure 10 compares dynamic effect of the baseline model with habit persistence (dotted line) with one that does not have habit persistence (solid line). Here non-neutral transfers adjust. Without habit, borrowers behave quite differently. Similar to Monacelli (2008), an increased real interest rate forces the borrower to de-accumulate their debt. This relaxation of the borrowing constraint (although they do not engage any new borrowing) reduces the user cost of durables, which produces a substitution towards durable goods, out weighing the weak complementary effect between the durable and non-durable consumption. Their labor supply now dramatically increases. On the saver’s side, absence of habit increases the volatility of their consumption and labor decisions. Both consumption of durable and non-durable crowd out with a dramatic increase in investment. Labor supply, although more volatile, appears to show similar pattern as with the habit persistence case. Over all, the crowding of output, the response of aggregate labor supply and cyclical investment response are all retained. Although co-movement problem does not occur, crowding in effect on aggregate consumption is lost.

Figure 11 compares the baseline model with one that does not have borrowing constraint. Here also non-neutral transfers adjust. The borrower seeks to smooth consumption by changing his labor supply decisions, which appears to be remarkably volatile. Increased labor income can now enable them to afford more durable and non-durable goods. On the saver side, the effects are quite dramatic. They raise consumption of non-durable by reducing durable consumption and by dramatically reducing investment, indicating the habit persistence effect have dominated the weak complementarity effect. Expected increased income from government bonds also enable them to reduce labor supply which appears to be as volatile as the borrower’s. In the aggregate, crowding in effect in output and aggregate consumption, increase in aggregate labor supply are all lost and Co-movement problem again emerges.

During the transition path, we see some crowding-in in aggregate consumption, increase in output and labor supply. But Co-movement problem endures during the entire adjustment period. Although the investment exhibits cyclical pattern, the pattern is opposite to the baseline case, thereby contradicting the empirical findings.

Figure 12 compares the baseline model with a representative agent model with habit per-
sistence. Since representative agent cannot borrow, consumption of non-durable go down upon impact while durable goods remain unchanged throughout the transition path. Aggregate consumption crowds out but output increases. Labor supply increase on impact too. There appears to an asset swap where the agent dramatically increases purchase of government bonds and reduce investment. This creates a cyclical pattern in the investment response which again contradicts the empirical findings.

In summary, only the borrower-saver model with borrowing constraint and habit persistence can match all the empirical facts explained in the introduction of the paper.

5.5 Government Spending Shock under Alternative Collateral Regimes

Figure 13 compares the baseline model with two alternative specification of the collateral regime, following Campbell and Hercowitz (2004). The parameters for different regimes are summarized in table 2. In the high collateral regime (solid line), there is less de-accumulation of debt by the borrower with an greater increase in their consumption of durable and non-durables. The effect on labor supply is similar to the low regime. For the saver, however, a decline in their debt income forces them to reduce consumption of non-durable with mild increase in the durable goods consumption. In the aggregate, there is crowding out of consumption upon impact followed by a crowding in phase. The positive Co-movement between durable and non-durable is retained after the initial impact. Output shows similar effect while investment shows a dampened cyclical response compared to the low regime.

In summary, only the baseline borrower-saver model with a low regime specification for the borrowing constraint can match all the empirical facts explained in the introduction of the paper.

6 Prior Predictive Analysis

In this section, I will propose an improved method for analyzing the performance of my calibrated model. This prior predictive analysis will formalize the choice of parameters and the evaluation of the model and provide an efficient way to conduct sensitivity analysis for perturbations of the parameters within a reasonable way. It will also analyze the robustness of the parameter choice and the goodness of fit of the model by considering alternative modeling assumptions and alternative specification of the stochastic process that govern the dynamics of the model. I will use a modified version of an approach outlined in Canova (1994) and extend it to the study of both technological and policy shocks in my model. The basic idea of the method is to reproduce features of actual data, which is taken to be the realization of an unknown vector stochastic process. I am interested in reproducing the second order moments and functions of these quantities. Each model would be simulated using the Monte Carlo procedure without replacement which will randomize over both the exogenous stochastic process and the parameters. The validity of the model would be judged based on its ability to reproduce as many second order moments as possible. Most importantly, the metric used to evaluate the discrepancy of the model from the data would be probabilistic. I will construct the simulated distribution of the statistics of interest and, taking the actual realization of the statistic as a critical value, examine a) in what percentile of the simulated distribution the actual value lies and b) how much of the simulated distribution is within a $k\%$ region centered around the critical value. A smaller value of the first statistic and a larger value of the second would indicate a better fit of the model.

Formally, I assume that a researcher is faced with an $m \times 1$ vector of time series $\tilde{x}_t$, which are the realization of a vector stochastic process $\tilde{X}_t$ and that he is interested in reproducing features of $\tilde{X}_t$ using a dynamic general equilibrium model. I assume that dynamic economic theory gives us a model expressing endogenous variables $X_t$ as a function of exogenous variables $Z_t$ and of the
parameters, \( \beta; X_t = f(Z_t, \beta) \). \( f \) cannot be computed analytically and I will use a log-linearized approximation outlined in the previous section. I am interested in studying the behavior of the functions of the simulated data, denoted as \( \mu(X_t) \), evaluating objects like \( E(\mu(X_t)) \) by using Monte Carlo simulation as specified in Geweke(1989) and appealing to the strong law of large numbers to obtain:

\[
\frac{1}{N} \sum_{i=1}^{N} \mu_i(X_t) \xrightarrow{a.s.} E(\mu(X_t))
\]

Finally, let \( h(\bar{x}_t) \) be the corresponding vector of functions of the actual data, which would be used to create test statistic to evaluate the fit of the model, as discussed earlier.

My method, however, differs from Canova(1994)\(^6\) in four very important ways. First, unlike Canova(1994), I will use acceptance criterions for parameters and for generated moments that will be used later for statistical analysis. My acceptance region is based on three conditions: a) the random parameter pick has to satisfy parameter restriction imposed by the model (namely that \( \beta_s > \beta_b \)), b) the parameter pick must ensure existence and uniqueness of equilibrium in the model and c) the ratio of the simulated variance of output to the actual variance cannot exceed more than 1.05\(^7\). Therefore, all the statistical analysis in my model would be conditional on the acceptance region.

Second, unlike Canova(1994), I will assess the goodness of fit of the model based on a larger number of second order moments which includes variances of output, aggregate consumption, durables, non-durables, investment and government spending.

Third, unlike Canova(1994), I will consider three alternative specification of modeling assumptions: one will be the baseline model with borrowing constraint, one will be without borrowing constraint and third one will be the representative agent model from Leeper, Walker and Yang(2008). This will help me to see the robustness of the statistical analysis under modeling uncertainty and also will highlight the importance of the borrowing constraint in fitting the model to the data.

Fourth, unlike Canova(1994), I will consider three alternative specifications of the stochastic process. In the first case, the stochastic process will consist of 1 shock to the law of motion of government spending (equation 22)\(^8\). In the second case, the stochastic process would consist of

\(^6\) There are four widely popular alternative approaches for evaluating model performance. The first approach is the extensively used classical likelihood based approach, such as MLE in papers like Hansen and Sargent(1979) and Leeper and Sims(1994). Model estimation however requires a precise specification which variables are unobserved or measured with error. Second approach is the Generalized Method of Moments (GMM) pioneered by Hansen and Singleton(1993) and also extensively used in papers like Ogaki and Reinhart(1998), Burnside et al. (1993). The limitations of the this approach is well known. The third approach is the use of restricted and unrestricted VAR to analyze the relevance of model. The fourth approach is to use Bayesian technique. The first three approaches are beyond the scope of this paper. Canova(1994), however, argues that his model evaluation strategy has a clear Bayesian interpretation. For a more elaborate discussion of the relative merit of each of this alternative methods, please see Canova(1994) and the January 1990 special issue of the Journal of Business and Economic Statistics.

\(^7\) This means that we will not consider any parameter value or any simulation that generates variance of output which is 150% more than actual variance of output, which happens to 0.001 for the US GDP. This cutoff point is entirely arbitrary. But similar cutoff points have been used by Eichenbaum(1993) where the cutoff point was 300% to show that based on parametrization, a standard RBC model can explain anything from 1% to 300% of the variation of output. I believe that if the model simulates a variance of output which is 300% more than the actual output, this automatically becomes a bad fit.

\(^8\) For the Monte Carlo Simulation, the law of motion for government spending will be changed slightly, as follows:

\[
\ln(G_t) = -q_G \ln \left( \frac{s^G - 1}{s^G} \right) + \rho_G \ln(G_{t-1} - 1) + \varepsilon^G_G, q_G \geq 0
\]

This is done to allow for a more general dynamics of the model where government spending could also respond to debt-output ratio, just like other fiscal instruments.
5 shocks in my baseline model; one shock each for government spending, two kinds of transfers, labor tax and capital tax, according to equations 20-24. In the third case, the stochastic process would consist of 13 shocks where an additional 8 shocks would be added to the baseline model following Smets and Wouters(2003), Coenen and Straub(2005) and Forni, Monforte and Sessa(2006). While the first two specifications make the model stochastically singular (the model has 12 choice variables and only 5 shocks), the third specification ensures non-singularity. Appendix-A, section 2 explains the algorithm used for the Monte Carlo simulation. Appendix-A, section 3 shows how additional shocks are added to the baseline model.

Table 4 shows the prior distributions of the parameters used for the Monte Carlo simulation. These prior distributions are taken from Canova(1994), Coenen and Straub(2005) and Forni, Monforte and Sessa(2006). In case of a parameter whose distribution is unknown but the support is known from previous works, I have used uniform distribution. I will perform 20,000 simulations with time series length of 20,000 and burning period of 8,000. For each simulation, parameters are picked from the specified prior distributions, second order moments are generated and compared to the acceptance criterion.

Figure 14 shows smoothed versions of the simulated distributions of $\beta_b$ and $\beta_s$ from the acceptance region for each version of the model under alternative specification of the stochastic process. For $\beta_b$, the median value is 0.97 for all models under all specification of stochastic process. The distributions also appear to be very similar in each case and for each models (first column). For $\beta_s$, the median value is 1.00 for each model under baseline shock and single shock case and 0.99 for the non-singular shock case. Therefore, the value of $\beta_b$ is robust under both modeling and stochastic process specifications. The value of $\beta_s$ is robust under modeling assumptions but slightly changes with stochastic process specifications.

Figures 15, 16 and 17 show smoothed distribution of the 25 most important parameters of each of the models. There are several interesting features of these distributions. The support, or the range of accepted values for each of the parameters are remarkably stable under all modeling and stochastic specifications. The distributions appear to be identical for baseline model with and without borrowing constraint case and somewhat different for the representative model. The median values of most of the parameters are very close to the value used for the calibration, with the exception of $F$, $S$, $b$ and the $q$’s. The median value of $F$, the fraction of the savers in the economy, is roughly around 0.40 in all models for the single and baseline shock case but around 0.55 in case of the non-singular shocks. The value of $S$, the fraction of transfer that goes to the saver is about 0.40 in the former two case but is about 0.30 in case of the non-singular shocks. The median value of $b$, the habit persistence parameter also varies from 0.54 to 0.75 under different models and shocks. The median value of $q_G$ varies from 0.18 in the baseline model without borrowing constraint in case of non-singular shock to 0.52 in the representative agent model under single shock. The other $q$’s also vary in similar fashion and in similar dimensions.

Table 5 shows the order statistics for the ratio of the variance of the simulated macroeconomic variables to their empirical counterparts under the baseline model with and without borrowing constraint under alternative specification of the stochastic process. It reports the 25th, 50th and 75th quantile of the distribution of these ratios. For the model without borrowing constraint, explanatory power increases with number of shocks for output, as median value of the ratio increases from 0.39 to 0.455 to 1.0. The same is true for government spending. For aggregate consumption, it increases from single shock to baseline shocks but surprisingly decreases for the non-singular shocks. The same is true for non-durable consumption. For durable consumption, variance is significantly over-estimated. Variance of investment is over estimated in case of the baseline and non-singular shocks while single shock estimates the variance to be about 50% of the actual data. Similar trend is observed for the model with borrowing constraint.

Comparing the two models reveals some very interesting results. Output, non-durable and investment appear to be less volatile under the model with borrowing constraint in case of both
single and baseline shocks while aggregate consumption, durable and government spending appears to be more volatile. But in case of non-singular shock, all the macroeconomic variables appear to be more volatile under the model with borrowing constraint. Therefore, this result is consistent with Campbell and Hercowitz(2004) who argued that relaxing borrowing constraint reduces macroeconomic volatility. Figure 18 makes this point quite clear. It plots the smoothed version of the distribution of the ratio of variance of the simulated output to actual data. In case of the non-singular shocks, the entire distribution appears to be a rightward shifted version of the distribution found in case of single or baseline shocks.

Table 5 also reports Pr_1 which refers to the frequency of simulation for which the variance of simulated macroeconomic variables is in a classical 95% confidence region around the actual variance, conditional on being in the acceptance region. A larger value of this statistics indicates a better fit of the model to the data. Comparing the two models, this statistics has a smaller value for aggregate consumption, durable, non-durable and investment under the model with borrowing constraint in case of single shock while larger value for output and government spending. But in case of both baseline shock and non-singular shocks, baseline model with borrowing constraint has larger numbers for Pr_1 for all variables. Therefore, it provides a better fit.

Table 5 also reports Pr_2 which reports the percentile of the simulated distribution where the point estimate of the actual variance of the macroeconomic variables lie, conditional on being in the acceptance region. A smaller value of this statistics indicates a better fit of the model. This value is large for all variables under two models for all specification of the shock process. Comparing between the two models, we see that the value of Pr_2 is smaller under the model without borrowing constraint for all variables under all different shock specification (in some cases, they are equal). Therefore, the model with borrowing constraint provides a relatively better fit.

In summary, the prior predictive analysis shows that the parameter choice for calibration is quite consistent with the parameters that can also provide a better fit of the model to the data. This significantly increases our confidence over our calibration exercise in the previous section. Second, it appears that the use of borrowing constraint in our model increases the volatility of the model and also provides a better fit to the data.

7 Robustness of the Results

Figure 19 analyzes the robustness of the output and consumption multiplier as a function of the ratio of the discount factors at different points on the transition path of the impulse response function whenever there is an unanticipated increase in government spending. The reasonable parameter range for $\beta_1$ is from 0.96 to 0.984 and the parameter range for $\beta_2$ is from 0.97 to 0.994. In case of the output multiplier, the value of first period multiplier is quite stable as long as $\beta_1$ is smaller than $\beta_2$. When $\beta_1$ becomes larger than $\beta_2$ (a clear violation of our my modeling assumption) and the ratio crosses the value of 1, we see a jump in the value of the multiplier. The same is true for the 10th and 20th period multiplier. In case of the 50th period multiplier,

---

9 Campbell and Hercowitz(2004) showed that once the borrowing constraint in their model is relaxed by reducing effective downpayment requirement and reducing collateral requirement, this reduces the volatility of the output, hours worked and durable goods purchase. My baseline model without borrowing constraint could be thought as an extreme case of such experiment.

10 To calculate the actual variance of the data, I use the seasonally adjusted time series for real GDP, real consumption expenditures, real non-durable consumption expenditures(excluding services), real durable consumption expenditures, real investment expenditures and real government expenditure from the NIPA data series published by the BEA. The series is from 1947 Q1 to 2008 Q4. The variance for the HP filtered log data is calculated to be 0.001, 0.0009, 0.0008, 0.0035, 0.0071 and 0.002 respectively.

11 It is quite possible for a ratio to have both a higher value of Pr_1 and Pr_2. This means that most of the simulated values are below the critical value and most of the simulated values lie in the lower segment of the 95% confidence interval.

---
the value of the multiplier decreases as long as $\beta_b$ is below $\beta_s$. When they become very close, there is clear upward jump upward. The value of the multiplier continues to increase for cases when $\beta_b$ becomes larger than $\beta_s$. For consumption, the first period multiplier is a mirror image of the response of the first period output multiplier. For the 10th, 20th and 50th period multiplier, the value decreases as long as the ratio is below 1. When $\beta_b$ comes very close to $\beta_s$(the ratio becomes 0.99), there is a clear upward jump which continues to hold as long as $\beta_b$ is larger than $\beta_s$. Therefore, for the values of $\beta_b$ and $\beta_s$ that are consistent with my model(namely $\beta_b < \beta_s$), the sizes of the multipliers are quite robust to small changes in their values.

Figure 20 shows the robustness of the output multiplier at different points on the transition path for small changes in the values of 12 most critical parameters\textsuperscript{12}. The 10th period, 20th period and 50th period multipliers appear to quite robust under small changes in all of these parameters. However, the first period multiplier is a different story. This multiplier increases monotonically with values of $\chi$, $\gamma$ and decreases with the value of $\alpha, \theta, \pi, \phi$ and $S$, the fraction of transfer going to the savers. It also shows monotonically increasing trend for $F$, the fraction of savers in the economy as long as the value is below 0.6. The monotonicity breaks down for any value between 0.65 and 0.7. The value of $\sigma$, the elasticity of Intratemporal substitution also produces quite dramatic results. For values between 0.75 and 0.78, the value of the multiplier has no pattern, increasing for some values while decreasing for some other. Between 0.8 and 0.9, the range where my calibrated value lies, the relationship between the size of the multiplier appears to be stable and monotonic. Again at some intermediate values between 0.93 and 0.94, the multiplier increases while after that, goes down. For all the values of $b$, there appears to be a monotonic relationship. Between 0.2 and 0.6, the value of the multiplier increases. But between 0.6 and 0.9, the range where the calibrated value is, the size of the multiplier goes down slowly, but in a monotonic fashion. For $\omega$, the parameter for the time varying depreciation rate, the relationship between the parameter value and the size of the multiplier is non-monotonic for values less than 1.4. But for values between 1.4 and 3.0, the range for the calibrated values, the size of the multiplier is unchanged.

Figure 21 shows the robustness of the consumption multiplier at different points on the transition path for small changes in the values of 12 most critical parameters of the model. The multipliers at different points show a mirror image of output multipliers for changes in the parameters, with the exception of $\sigma$. As the value of $\sigma$ increases, we see a monotonically increasing first period consumption multiplier.

To summarize, the robustness exercise shows that the dynamics of the model is quite robust to small changes in the value of the most of the critical parameters of my model. While changes in the discount rates are important for dynamics at the latter part of the transition path, the values of other parameters mainly effect the initial impacts.

8 Conclusion

In this paper, I have analyzed the effect of unanticipated and temporary change in government spending on macroeconomic variables such as output, aggregate consumption, consumption of durable and non-durable goods, employment and investment. The objective of the analysis was to reconcile the difference between the results generally obtained in standard RBC models and those reported in empirical papers. Empirical results show a crowding-in effect on output, aggregate consumption and employment; a positive co-movement and crowding-in effect on durable and non-durable goods and finally, a crowding-in effect on investment followed by a crowding-out effect.

\textsuperscript{12}These 12 parameters were identified during our prior predictive analysis as the ones whose values changed somewhat under different modeling assumptions and different specification of the stochastic process. Therefore, an uni-dimensional robustness check is needed to perform on them to see their effects of the model dynamics.
Standard RBC models fail to explain most of the empirical facts and sometimes, all of them. I developed a model that combines heterogeneity across household’s rates of time preference with collateral constraint on borrowing in a standard two sector RBC model. The model also includes several other features which has recently being used with standard RBC models such as internal habit persistence, deliberation cost to durable goods’ consumption and investment adjustment cost. Once the model is properly calibrated, Impulse response functions for an unanticipated increase in government spending generated from this model seemed to match with those found in empirical literature. Therefore, the model has succeeded in meeting its objective. Simple statistical analysis showed that this model can provide a better fit to data compared to its simpler counterparts. Although statistical analysis conducted in the form of prior predictive analysis is sufficient within the scope of this paper, a full-fledged estimation is warranted to assess the goodness of fit of this model and analyze its implications.

References


[23] Joint Committee on Taxation (2006): "Background Information about the Dynamic Stochastic General Equilibrium Model Used by the Staff of the Joint Committee on Taxation in the Macroeconomic Analysis of Tax Policy". December 14, Washington, DC.


Appendix A

A.1: Model Solutions and Steady state Conditions

Assuming all the first order condition binds except for the first order for private bonds for the saver, the first order conditions look like:

\[
\begin{align*}
\left\{ C_t^h + D_t^h - (1 - \tau_t^L) W_t L_t^h - B_t^h + B_{t-1}^h R_{t-1} - (1 - \delta_D) D_{t-1}^h - TR_t^h \right\} &= 0, h = b, s \quad (A.1) \\
\left\{ B_t^h - (1 - \phi) B_{t-1}^h - (1 - \pi) \left[ D_t^h - (1 - \delta_D) D_{t-1}^h \right] \right\} &= 0, h = b, s \quad (A.2) \\
-\beta_t b E_t \left\{ \left( \frac{C_{t+1}^{s,b}}{\Delta} + V^b(D_{t+1}^b)^{1-\frac{1}{\sigma}} \right)^{\frac{\tau - \sigma}{\sigma - 1}} \left( \frac{C_{t+1}^{s,b}}{\Delta} \right)^{-\frac{1}{\sigma}} \right\} &= \lambda b_t \quad (A.3)
\end{align*}
\]
\[ \left[ (C_t^s)^{1-\frac{1}{\sigma}} + V^b(D_t^b)^{1-\frac{1}{\sigma}} \right] \frac{\tau_{t} \alpha}{(\tau - 1)} V^b(D_t^b)^{-\frac{1}{\sigma}} - \eta \frac{(D_t^b - D_{t-1}^b)}{D_t^b} \]

\[ -\lambda_{st} + \psi_{st}(1 - \pi) + \beta_{st} b E_t \left[ \left\{ \frac{\eta}{2} \left( \frac{D_{t+1}^b - D_t^b}{D_t^b} \right)^2 - 1 \right\} + \lambda_{st+1}(1 - \delta_D) - \psi_{st+1}(1 - \pi)(1 - \delta_D) \right] = 0 \]  

\[ \lambda_{st} - \psi_{st} - \beta_{st} E_t \{ \lambda_{st+1} R_{1t} - \psi_{st+1}(1 - \phi) \} = 0 \]  

\[ -\chi(1 - L_t^b)^{-\theta} + \lambda_{st} (1 - \tau_t^b) w_t = 0 \]  

The first order conditions for the saver looks like:

\[
\left\{ C_t^s + D_t^s + X_t^{sb} + I_t^s - (1 - \tau_t^s) W_t L_t^s - B_t^s + B_{t-1}^s R_{t-1}^s - (1 - \delta_D) D_{t-1}^s \right\} = 0 \tag{A.7}
\]

\[
\left\{ K_t^s - \left\{ 1 - s \left( I_t^s \right) \right\} I_t^s - (1 - \delta_t^s) K_{t-1}^s \right\} = 0 \tag{A.8}
\]

\[
\left[ (C_{t+1}^s)^{1-\frac{1}{\sigma}} + V^s(D_t^s)^{1-\frac{1}{\sigma}} \right] \frac{\tau_{t+1} \alpha}{(\tau - 1)} (C_{t+1}^s)^{-\frac{1}{\sigma}}
- \beta_{st} b E_t \left[ \left\{ (C_{t+1}^s)^{1-\frac{1}{\sigma}} + V^s(D_{t+1}^s)^{1-\frac{1}{\sigma}} \right\} \frac{\tau_{t+1} \alpha}{(\tau - 1)} (C_{t+1}^s)^{-\frac{1}{\sigma}} \right] - \lambda_{st} = 0 \tag{A.9}
\]

\[
\left[ (C_t^s)^{1-\frac{1}{\sigma}} + V^s(D_t^s)^{1-\frac{1}{\sigma}} \right] \frac{\tau_{t} \alpha}{(\tau - 1)} V^s(D_t^s)^{-\frac{1}{\sigma}} - \eta \frac{(D_t^s - D_{t-1}^s)}{D_t^s} \]
\[ -\lambda_{st} + \beta_{st} b E_t \left[ \left\{ \frac{\eta}{2} \left( \frac{D_{t+1}^s - D_t^s}{D_t^s} \right)^2 - 1 \right\} + \lambda_{st+1}(1 - \delta_D) \right] = 0 \tag{A.10}
\]

\[ \lambda_{st} - \beta_{st} E_t \{ \lambda_{st+1} R_{1t} \} = 0 \tag{A.11} \]

\[ \beta_{st} E_t \lambda_{st+1} \left\{ \delta^k_t + (1 - \tau^k_t - 1) \eta_{t+1}(1 - \delta(\mu_{t+1})^\omega) - \omega - \varphi_t \right\} = 0 \tag{A.12} \]

\[ -\lambda_{st} + \varphi_{st} \left\{ 1 - s \left( \frac{I_t^s}{I_{t-1}^s} \right) \right\} - \varphi_t \left\{ s \left( \frac{I_t^s}{I_{t-1}^s} \right) \frac{I_t^s}{I_{t-1}^s} \right\} - \beta_{st} E_t \varphi_{t+1} \left\{ s \left( \frac{I_{t+1}^s}{I_t^s} \right) \frac{I_{t+1}^s}{I_t^s} \right\} = 0 \tag{A.13} \]

\[ \lambda_{st}(1 - \tau^k_t - 1 - \varphi_t \delta(\mu_{t})^\omega - 1) = 0 \tag{A.14} \]

\[ -\lambda_{st} + \beta_{st} E_t \{ \lambda_{st+1} R_{2t} \} = 0 \tag{A.15} \]

\[ -\chi(1 - L_t^b)^{-\theta} + \lambda_{st} (1 - \tau_t^b) w_t = 0 \tag{A.16} \]
Also, the first order condition of profit maximization are as follows:

\[ r_t = \frac{\alpha Y_t}{\mu_t K_{t-1}} \]  \hspace{1cm} (A.17)

\[ w_t = \frac{(1 - \alpha)Y_t}{L_t} \]  \hspace{1cm} (A.18)

In steady state, A.13 implies

\[ \lambda_s = \varphi_s \]  \hspace{1cm} (A.19)

Using A.19, A.12 could be simplified as:

\[ r = \left[ \frac{\left\{ \frac{1 - \beta_s}{\beta_s (1 - \tau^K)} \right\} + \delta \mu^\omega}{\mu} \right] \]  \hspace{1cm} (A.20)

Combining A.20 with A.14 in steady state and simplifying, we can get:

\[ \mu = \left[ \frac{(1 - \tau^K) (1 - \beta_2)}{\beta_2 \{\omega - (1 - \tau^K)\delta\}} \right]^{\frac{1}{\omega}} \]  \hspace{1cm} (A.21)

If we substitute A.21 into A.20 and use the value of \( r \) into A.17, we get:

\[ K = \left[ \frac{\left\{ \frac{1 - \beta_2}{\beta_2 (1 - \tau^K)} \right\} + \delta \mu^\omega}{\alpha L^{1-\alpha} \mu^\omega} \right]^{\frac{1}{\omega - 1}} \]  \hspace{1cm} (A.22)

Given a value for \( L \), we can calculate the value \( K, Y \) and \( W \). Now from A.16, we get:

\[ \lambda_s = \varphi_s = \frac{\chi(1 - L^s)^{-\theta}}{(1 - \tau L)w} \]  \hspace{1cm} (A.23)

Similarly, we could get:

\[ \lambda_b = \frac{\chi(1 - L^b)^{-\theta}}{(1 - \tau L)w} \]  \hspace{1cm} (A.24)

Again, from A.15, we get: \( R_2 = \frac{1}{\rho_s} \). Combine this and A.5, we get:

\[ \varphi_b = \frac{\lambda_b \left(1 - \frac{\beta_b}{\rho_s}\right)}{1 - \beta_b (1 - \phi)} \]  \hspace{1cm} (A.25)

Now substituting A.3 into A.4 and also substitute the value of \( \lambda_b \) from A.24, we can simplify and get:

\[ V^b = \frac{(1 - b_b) \pi^b (1 - \beta_b b) \{1 - \beta_b (1 - \delta_D)\} \{\pi - \beta_b (1 - \phi - R_1)\}}{(1 - \beta_b (1 - \phi)) \left[ \frac{C_b}{D^b} \right]^\frac{1}{\beta}} \]  \hspace{1cm} (A.26)

Similarly, I could get:

\[ V^a = \left( \frac{D^a}{C_a} \right)^{\frac{1}{\beta}} (1 - \beta_a) \pi^a (1 - \beta_a b) \{1 - \beta_2 (1 - \delta_D)\} \]  \hspace{1cm} (A.27)

In the model, there are 6 unknown variables that needs to be solved simultaneously, \( C^s, C^b, D^a, D^b, V^b \) and \( V^a \). In order to solve them, we will use steady state version of A.3, A.9, A.26,
A.27, steady state version of the aggregate resource constraint (equation 37) and a ratio which we
deﬁned in section 4.6 and looks like:

\[
\frac{\delta D \{FD^s + (1 - F)D^b\}}{FC^s + (1 - F)C^b} = 0.149
\]  

(A.28)

Since there are now six equations and six unknowns, I can solve the system of equations. Thus I solve the entire steady state system of equations.

A.2: Algorithm for Monte Carlo Simulation for Prior Predictive Analysis

The algorithm for Monte Carlo simulation in this paper is a modiﬁed version of the method
used in Canova(1994) to account for model restriction, existence and uniqueness of equilibrium
and also for reasonable values of second order moments that could be accepted as plausible. The
algorithm for the Monte Carlo simulation without replacement is as follows:

Step 1: From a reasonable prior density, draw a vector of parameters.
   a. Check whether the parameter draw satisﬁes the model restriction.
   b. Check whether the parameter draw can ensure existence and uniqueness of equilibrium for
   the model

   If both conditions are satisﬁed, keep them and move to step 2. Otherwise draw another vector
   of parameters.

Step 2: For the accepted parameter draw:
   a. Generate a time series of observations for the variables of the model of length T=20,000.
   b. Drop the ﬁrst 8,000 observations as burning period.
   c. Generate variance for each of the variables of the model.
   d. Check whether the ratio of the simulated variance of output to actual variance of output
   is below accepted value. The critical value is set at 150%
   e. If the acceptance criterion is satisﬁed, keep the ratio of variances. Otherwise, go back to
   step 1.

Step 3: Repeat the two previous steps N=20,000 times.

Step 4: Construct frequency distributions for the ratio of variances, compute probabilities,
quantiles and other measures of interest.

A.3: Derivation of the Stochastically Non-singular Baseline Model

The baseline model of Borrower and saver has 12 choice variables and only ﬁve shocks. This
makes the model stochastically singular. In order to derive a stochastically non-singular model, I
need to include additional shocks, as suggested by previous papers such as Parkin(1988) McGrat-
tan(1989), Ingram, Kochelerkota and Savin() and Leeper and Sims(1994). I will however, follow
Smets and Wouters(2003), Forni, Monforte and Sessa(2006) and Coenen and Straub(2005) by
adding several preference shocks and one investment adjustment shock. The preference structure
for the non-singular model would be as follows; For the Borrower, the utility function looks like:

\[
U(.) = \frac{\varepsilon^b_t \left(C^b_t\right)^{1-\frac{1}{2}} + V^b_t \varepsilon^{D^b_t} \left(D^b_t\right)^{1-\frac{1}{2}}}{1 - \frac{1}{2}} - \frac{\eta (D^b_t - D^b_{t-1})^2}{2 D^b_{t-1}} + \chi^{I^b_t} \left(1 - L^{h^b_t}\right)^{1-\theta} - 1
\]

\[
\varepsilon^b_t \text{ is a preference shock to consumption of non-durable, } \varepsilon^{D^b_t} \text{ is a preference shock to the consumption of durables and } \varepsilon^{I^b_t} \text{ is a preference shock to leisure.}
\]

Similarly, for the Saver, the utility function looks like:
\[ U(\cdot) = \]
\[
\left[ \varepsilon_t^{C^a} (C^a)^{1-\frac{1}{2}} + V^s \varepsilon_t^{D^s} (D^s)^{1-\frac{1}{2}} \right]^{1-\frac{1}{2}} - \frac{\eta (D_t^s - D_{t-1}^s)^2}{2 D_t^{s^2} (1 - L_t^{s^2})^{1-\theta} - 1}
\]

Here the shocks have similar interpretations.

The law of motion for capital stock now looks like:

\[ K_t^s \leq \left( 1 - s \left( \frac{\varepsilon_t^s I_t^s}{I_{t-1}^s} \right) \right) I_t^s + (1 - \delta_t) K_{t-1}^s \]  

(A.31)

Here \( \varepsilon_t^s \) is a shock to investment adjustment cost. Finally, the production function now looks like:

\[ Y_t = A_t \left\{ \mu_t K_{t-1} \right\}^\alpha \left\{ L_t \right\}^{1-\alpha} \]  

(A.32)

Here \( A_t \) is the familiar productivity shock. The law of motion for the shock process are as follows:

\[ \varepsilon_t^{C^b} = \rho_{C^b} \varepsilon_{t-1}^{C^b} + u_t^{C^b}, u_t^{C^b} \sim iid.N(0, \sigma_{C^b}^2) \]  

(A.33)

\[ \varepsilon_t^{D^b} = \rho_{D^b} \varepsilon_{t-1}^{D^b} + u_t^{D^b}, u_t^{D^b} \sim iid.N(0, \sigma_{D^b}^2) \]  

(A.34)

\[ \varepsilon_t^{L^b} = \rho_{L^b} \varepsilon_{t-1}^{L^b} + u_t^{L^b}, u_t^{L^b} \sim iid.N(0, \sigma_{L^b}^2) \]  

(A.35)

\[ \varepsilon_t^{C^s} = \rho_{C^s} \varepsilon_{t-1}^{C^s} + u_t^{C^s}, u_t^{C^s} \sim iid.N(0, \sigma_{C^s}^2) \]  

(A.36)

\[ \varepsilon_t^{D^s} = \rho_{D^s} \varepsilon_{t-1}^{D^s} + u_t^{D^s}, u_t^{D^s} \sim iid.N(0, \sigma_{D^s}^2) \]  

(A.37)

\[ \varepsilon_t^{L^b} = \rho_{L^b} \varepsilon_{t-1}^{L^b} + u_t^{L^b}, u_t^{L^b} \sim iid.N(0, \sigma_{L^b}^2) \]  

(A.38)

\[ \varepsilon_t^I = \rho_I \varepsilon_{t-1}^I + u_t^I, u_t^I \sim iid.N(0, \sigma_I^2) \]  

(A.39)

\[ A_t = \rho_A A_{t-1} + u_t^A, u_t^A \sim iid.N(0, \sigma_A^2) \]  

(A.40)

With these 8 additional shocks, the number of shocks in the model are now 13 while the model has 12 choice variables. Hence the model is now stochastically non-singular.
Table 1: Standard Deviation and Correlation Coefficients for major macroeconomic variables. All variables are in real terms, logged and HP filtered at quarterly frequency. The data source for the GDP and its components are NIPA and Federal reserve flow of funds accounts, table B.100, Balance sheet of households and non-profit organizations for the private debt. Public debt is defined as the total government debt held my public, reported by NIPA.

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<td>0.34</td>
<td>0.09</td>
</tr>
<tr>
<td>Private Debt</td>
<td>0.50</td>
<td>0.59</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>Sample: 1983QI-2007QIV</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Y</td>
<td>C</td>
<td>I</td>
</tr>
<tr>
<td>Y</td>
<td>1.00</td>
<td>0.83</td>
<td>0.87</td>
</tr>
<tr>
<td>C</td>
<td>0.83</td>
<td>1.00</td>
<td>0.59</td>
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<tr>
<td>I</td>
<td>0.87</td>
<td>0.59</td>
<td>1.00</td>
</tr>
<tr>
<td>G</td>
<td>-0.16</td>
<td>-0.10</td>
<td>-0.36</td>
</tr>
<tr>
<td>Public Debt</td>
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<td>-0.22</td>
<td>0.18</td>
</tr>
<tr>
<td>Private Debt</td>
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<td>0.21</td>
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Table 2: Benchmark Parameter values used for model calibration

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<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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<td>$\alpha$</td>
<td>0.36</td>
<td>$S^T_R$</td>
<td>0.07</td>
<td>$\eta$</td>
<td>3.0</td>
<td>$\tau^L$</td>
<td>0.21</td>
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<tr>
<td>$\beta_s$</td>
<td>0.99</td>
<td>$S^B$</td>
<td>0.047</td>
<td>$b_s = b_s$</td>
<td>0.80</td>
<td>$\tau^K$</td>
<td>0.39</td>
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<td>$\beta_b$</td>
<td>0.97</td>
<td>$S$</td>
<td>0.4051</td>
<td>$\gamma$</td>
<td>0.80</td>
<td>$q_{TR^e} = q_{TR^e}$</td>
<td>0.341</td>
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<tr>
<td>$\phi$ for High Regime</td>
<td>0.03</td>
<td>$S^G$</td>
<td>0.2</td>
<td>$\theta$</td>
<td>1.0</td>
<td>$q^L$</td>
<td>0.149</td>
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<tr>
<td>$\phi$ for Low Regime</td>
<td>0.01</td>
<td>$S^C$</td>
<td>0.63</td>
<td>$\sigma$</td>
<td>0.90</td>
<td>$q^K$</td>
<td>0.206</td>
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<tr>
<td>$\pi$ for High Regime</td>
<td>0.15</td>
<td>$S^A$</td>
<td>0.18</td>
<td>$\omega$</td>
<td>1.56</td>
<td>$\rho_G$</td>
<td>0.80</td>
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<tr>
<td>$\pi$ for Low Regime</td>
<td>0.11</td>
<td>$\delta$</td>
<td>0.02</td>
<td>$F$</td>
<td>0.40</td>
<td>$\sigma_G$</td>
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<tr>
<td>$\tau$</td>
<td>0.447</td>
<td>$\delta_D$</td>
<td>0.0115</td>
<td>$L^b = L^b$</td>
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<td>$S^I$</td>
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<td>$\chi$</td>
<td>0.5</td>
<td>$\rho_{TR}$</td>
<td>0.80</td>
<td>$\sigma_{TR^e}$</td>
<td>0.02</td>
<td>$\sigma_{TR^b}$</td>
<td>0.02</td>
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<td>$\rho_{L_L}$</td>
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<td>$\sigma_{L_L}$</td>
<td>0.02</td>
<td>$\rho_{r_K}$</td>
<td>0.8</td>
<td>$\sigma_{r_K}$</td>
<td>0.02</td>
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</table>

Table 3: Comparing multipliers between Blanchard and Perotti(2002), Gali et al. (2007), Fatas and Mihov(2001) and the baseline model. For all the models, numbers in the parentheses indicate approximately one standard deviation confidence band.

<table>
<thead>
<tr>
<th>Model</th>
<th>Variable</th>
<th>1st Quarter</th>
<th>4 Quarter</th>
<th>12 Quarter</th>
<th>20 Quarter</th>
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<tr>
<td>Blanchard</td>
<td>Output</td>
<td>0.9</td>
<td>0.65</td>
<td>0.66</td>
<td>0.66</td>
</tr>
<tr>
<td>and</td>
<td></td>
<td>(0.55,1.3)</td>
<td>(-0.2,1.4)</td>
<td>(-0.8,2.0)</td>
<td>(-0.9,2.0)</td>
</tr>
<tr>
<td>Perotti</td>
<td>Consumption</td>
<td>0.33</td>
<td>0.34</td>
<td>0.43</td>
<td>0.44</td>
</tr>
<tr>
<td>(2002)</td>
<td>Investment</td>
<td>0.02</td>
<td>-0.74</td>
<td>-0.96</td>
<td>-0.95</td>
</tr>
<tr>
<td>Gali et al.</td>
<td>Output</td>
<td>0.1</td>
<td>0.2</td>
<td>0.27</td>
<td>0.24</td>
</tr>
<tr>
<td>(2007)</td>
<td></td>
<td>(0.05,0.15)</td>
<td>(0.35,0.04)</td>
<td>(0.15,0.49)</td>
<td>(0.0,0.48)</td>
</tr>
<tr>
<td></td>
<td>Consumption</td>
<td>0.12</td>
<td>0.22</td>
<td>0.28</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.06,0.18)</td>
<td>(0.15,0.35)</td>
<td>(0.15,0.37)</td>
<td>(0.10,0.36)</td>
</tr>
<tr>
<td></td>
<td>Investment</td>
<td>-0.2</td>
<td>-0.4</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.01,-0.3)</td>
<td>(-1.0,0.01)</td>
<td>(-0.46,0.7)</td>
<td>(-0.7,0.8)</td>
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<tr>
<td>Fatas</td>
<td>Output</td>
<td>0.00098</td>
<td>0.0023</td>
<td>0.003</td>
<td>0.0027</td>
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<tr>
<td>and</td>
<td></td>
<td>(-0.0008,0.001)</td>
<td>(0.001,0.0035)</td>
<td>(0.001,0.0052)</td>
<td>(0.001,0.0051)</td>
</tr>
<tr>
<td>Mihov</td>
<td>Consumption</td>
<td>0.0012</td>
<td>0.0022</td>
<td>0.004</td>
<td>0.0038</td>
</tr>
<tr>
<td>(2001)</td>
<td></td>
<td>(0.00098,0.0014)</td>
<td>(0.001,0.0032)</td>
<td>(0.002,0.006)</td>
<td>(0.0015,0.0066)</td>
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<tr>
<td></td>
<td>Investment</td>
<td>0</td>
<td>0.005</td>
<td>0.0056</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.005,0.003)</td>
<td>(-0.001,0.012)</td>
<td>(0.001,0.011)</td>
<td>(-0.003,0.007)</td>
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<tr>
<td>Baseline</td>
<td>Output</td>
<td>0</td>
<td>0.1114</td>
<td>0.0373</td>
<td>-0.0163</td>
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<tr>
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<td>Consumption</td>
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<td>0.086</td>
<td>0.023</td>
<td>-0.016</td>
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<td></td>
<td>Investment</td>
<td>0.2894</td>
<td>0.8383</td>
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<td>-0.7556</td>
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<td>Distribution</td>
<td>Property 1</td>
<td>Property 2</td>
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<tr>
<td>-----------</td>
<td>--------------</td>
<td>------------</td>
<td>------------</td>
<td></td>
<td></td>
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<tr>
<td>$F$</td>
<td>Uniform(a,b)</td>
<td>$a = 0.2$</td>
<td>$b = 0.60$</td>
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<tr>
<td>$S$</td>
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<td>$b = 0.60$</td>
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<td>$\phi$</td>
<td>Uniform(a,b)</td>
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<td>$b = 0.163$</td>
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<tr>
<td>$\delta$</td>
<td>Uniform(a,b)</td>
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<td>$b = 0.03$</td>
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<tr>
<td>$\delta_D$</td>
<td>Uniform(a,b)</td>
<td>$a = 0.0101$</td>
<td>$b = 0.0212$</td>
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<tr>
<td>$\sigma_G$</td>
<td>Uniform(a,b)</td>
<td>$a = 0.009$</td>
<td>$b = 0.0158$</td>
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<td>Uniform(a,b)</td>
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<td>$b = 0.0192$</td>
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<td>$b = 0.0202$</td>
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<td>$\sigma_C$</td>
<td>Uniform(a,b)**</td>
<td>$a = 0.00028$</td>
<td>$b = 0.0003$</td>
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<td>$a = 0.0001$</td>
<td>$b = 0.00015$</td>
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<tr>
<td>$\sigma_L$</td>
<td>Uniform(a,b)**</td>
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<td>$b = 0.00026$</td>
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<td>Uniform(a,b)</td>
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<td>$q_G$</td>
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<td>$b = 0.6$</td>
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<td>$q_K$</td>
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<td>$b = 0.6$</td>
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<tr>
<td>$q_L$</td>
<td>Uniform(a,b)</td>
<td>$a = 0.1$</td>
<td>$b = 0.6$</td>
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<tr>
<td>$q_Z$</td>
<td>Uniform(a,b)</td>
<td>$a = 0.1$</td>
<td>$b = 0.6$</td>
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<td>Uniform(a,b)</td>
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<td>$b = 1.7257$</td>
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<td>$\beta_b$</td>
<td>Truncated Normal(a,b)</td>
<td>$a = 0.95$</td>
<td>$b = 0.99$</td>
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<td>$\beta_s$</td>
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<td>$a = 0.99$</td>
<td>$b = 1.0$</td>
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<td>$a = 2.0$</td>
<td>$b = 5.0$</td>
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<tr>
<td>$\omega$</td>
<td>Truncated Normal(a,b)</td>
<td>$a = 1.0$</td>
<td>$b = 3.0$</td>
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<td>$\sigma_{GL}$</td>
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<td>$\sigma_A$</td>
<td>Uniform(a,b)**</td>
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<td>$b = 0.0003$</td>
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<tr>
<td>$\sigma_C^*$</td>
<td>Uniform(a,b)**</td>
<td>$a = 0.00016$</td>
<td>$b = 0.00019$</td>
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<td>Uniform(a,b)**</td>
<td>$a = 0.00025$</td>
<td>$b = 0.00027$</td>
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<td>Uniform(a,b)**</td>
<td>$a = 0.00028$</td>
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<td>$a = 0.009$</td>
<td>$b = 0.0202$</td>
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<tr>
<td>$\rho_C^*$</td>
<td>Uniform(a,b)**</td>
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<td>$b = 0.78$</td>
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<tr>
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<td>$a = 0.70$</td>
<td>$b = 0.71$</td>
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<td>$\rho_L^*$</td>
<td>Uniform(a,b)**</td>
<td>$a = 0.74$</td>
<td>$b = 0.76$</td>
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<tr>
<td>$\rho_I$</td>
<td>Uniform(a,b)**</td>
<td>$a = 0.73$</td>
<td>$b = 0.76$</td>
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</tbody>
</table>

Table 4: Prior Distributions of Parameters for Monte Carlo Simulation. The parameters with "**" are used only in the non-singular model.
<table>
<thead>
<tr>
<th>Model</th>
<th>Statistics</th>
<th>VAR(Y_M)</th>
<th>VAR(Y_D)</th>
<th>VAR(AgC_M)</th>
<th>VAR(AgC_D)</th>
<th>VAR(D_M)</th>
<th>VAR(D_D)</th>
<th>VAR(C_M)</th>
<th>VAR(C_D)</th>
<th>VAR(I_M)</th>
<th>VAR(I_D)</th>
<th>VAR(G_M)</th>
<th>VAR(G_D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Shock</td>
<td>Q_0.25</td>
<td>0.2</td>
<td>0.1</td>
<td>4.4</td>
<td>0.0</td>
<td>0.2</td>
<td>0.1</td>
<td>0.0</td>
<td>1.0</td>
<td>0.2</td>
<td>0.0</td>
<td>1.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Baseline Model</td>
<td>Q_0.50</td>
<td>0.4</td>
<td>0.2</td>
<td>14.7</td>
<td>0.0</td>
<td>1.0</td>
<td>0.1</td>
<td>11.0</td>
<td>0.2</td>
<td>0.2</td>
<td>0.0</td>
<td>1.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Without Borrowing</td>
<td>Q_0.75</td>
<td>0.7</td>
<td>0.6</td>
<td>45.1</td>
<td>0.1</td>
<td>11.0</td>
<td>0.2</td>
<td>0.2</td>
<td>0.0</td>
<td>1.0</td>
<td>0.0</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Constraint Pr_1</td>
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<td>0.0098</td>
<td>0.9994</td>
<td>0.9397</td>
<td>0.9972</td>
<td>0.9846</td>
<td>0.0003</td>
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<td></td>
<td>Pr_2</td>
<td>0.8796</td>
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<td>0.0762</td>
<td>0.9143</td>
<td>0.4982</td>
<td>0.9919</td>
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<tr>
<td>Single Shock</td>
<td>Q_0.25</td>
<td>0.1</td>
<td>0.1</td>
<td>7.4</td>
<td>0.0</td>
<td>0.2</td>
<td>0.1</td>
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<tr>
<td>Baseline Model</td>
<td>Q_0.50</td>
<td>0.4</td>
<td>0.3</td>
<td>25.9</td>
<td>0.0</td>
<td>0.4</td>
<td>0.2</td>
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<tr>
<td>With Borrowing</td>
<td>Q_0.75</td>
<td>0.8</td>
<td>0.8</td>
<td>70.2</td>
<td>0.1</td>
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Table 5: Order Statistics for Macroeconomic Variables derived from Monte Carlo Simulation.

*M* indicates model and *D* indicates data.
Figure 1: Trends in Debt: 1951Q4-2006Q4. Total debt equals total debt of the household and non-profit organization. Source of the data is the Federal reserve flow of funds accounts, table B.100, Balance sheet of households and non-profit organizations.
Figure 2: Trends in the volatility of Private debt held by household and non-profit organizations: 1970Q1-2007Q3. The data is HP filtered at quarterly frequency. Source of the data is the Federal reserve flow of funds accounts, table B.100, Balance sheet of households and non-profit organizations.

Figure 3: Ratio of Household’s Debts to their Tangible Assets. Taken from Campbell and Hercowitz(2004).
Figure 4: Effect of Government spending under different kinds of borrowing constraints when non-neutral transfer adjust. This means that $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Campbell and Hercowitz(2004)’s borrowing constraint(baseline): solid line; Kiyotaki and Moore(1997)’s borrowing constraint: dotted line.

Figure 5: Effect of Government spending under different kinds of borrowing constraints when non-neutral transfer adjust. This means that $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Campbell and Hercowitz(2004)’s borrowing Constraint(baseline): solid line; Monacelli(2008)’s borrowing constraint: dotted line.
Figure 6: Effect of Government spending under different Intratemporal elasticity of substitution when non-neutral transfers adjust. This means that $q_{fr} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Baseline is the low INES($\sigma = 0.90$): solid line; high INES($\sigma = 1.05$): dotted line.

Figure 7: Effect of Government spending when transfers adjust. Neutral Transfers adjustment: dotted line; Non-neutral adjustment: solid line.
Figure 8: Effect of Government spending when Labor Tax adjust. The Model is calibrated using baseline parameters, defined in table 2. This means that $q_{TR} = 0$, $q_L = 0.149$, $q_K = 0$.

Figure 9: Effect of Government spending when Capital Tax adjust. The Model is calibrated using baseline parameters, defined in table 2. This means that $q_{TR} = 0$, $q_L = 0$, $q_K = 0.206$
Figure 10: Effect of Government spending under different models when Non-neutral Transfer adjust. This means that $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Baseline model: solid line; Baseline model without habit persistence: dotted line.

Figure 11: Effect of Government spending under different models when Non-neutral Transfer adjust. This means that $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Baseline model: solid line; Baseline model without Borrowing Constraint: dotted line.
Figure 12: Effect of Government spending under different models when Non-neutral Transfer adjust. This means that $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Baseline model: solid line; Representative agent model: dotted line.

Figure 13: Effect of Government spending under different Collateral Regime when Non-neutral Transfer adjust. This means that $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Baseline model with low regime: solid line; Baseline model with high regime: dotted line.
Figure 14: Smoothed versions of the simulated distributions of $\beta_1$ and $\beta_2$ from the acceptance region for each version of the model under alternative specification of the stochastic process. Baseline line with borrowing constraint: solid line; Baseline line without borrowing constraint: dashed line; Representative agent model: dotted-dashed line.

Figure 15: Smoothed versions of the simulated distributions of 25 most important parameters from the acceptance region for each version of the model for single shock case. Baseline line with borrowing constraint: solid line; Baseline line without borrowing constraint: dashed line; Representative agent model: dotted-dashed line.
Figure 16: Smoothed versions of the simulated distributions of 25 most important parameters from the acceptance region for each version of the model for baseline shock(5) case. Baseline line with borrowing constraint: solid line; Baseline line without borrowing constraint: dashed line; Representative agent model: dotted-dashed line.

Figure 17: Smoothed versions of the simulated distributions of 25 most important parameters from the acceptance region for each version of the model for non-singular shocks case. Baseline line with borrowing constraint: solid line; Baseline line without borrowing constraint: dashed line; Representative agent model: dotted-dashed line.
Figure 18: Smoothed versions of the simulated distributions of ratio of variance for baseline model with and without borrowing constraint. Baseline line model with borrowing constraint: solid line; Baseline line model without borrowing constraint: dotted line.

Figure 19: Robustness of the $\beta$’s. The figure plots the value of Output and Consumption multiplier in four different points on the transition path for different values of the ratio $\frac{\beta_1}{\beta_2}$: 1st Period, 10th Period, 20th Period and 50th Period.
Figure 20: Robustness of output multiplier. The figure plots the value of Output multiplier in four different points on the transition path for different values of the 12 most critical parameters of the model. 1st Period multiplier: solid line; 10th period multiplier: dotted line; 20th Period multiplier:dotted-dashed line; 50th Period multiplier: dashed line.

Figure 21: Robustness of consumption multiplier. The figure plots the value of Consumption multiplier in four different points on the transition path for different values of the 12 most critical parameters of the model. 1st Period multiplier:solid line; 10th period multiplier:dotted line; 20th Period multiplier:dotted-dashed line; 50th Period multiplier:dashed line.