Portfolio Optimization based on Higher Moment Coherent Risk Measures

Zheng Chengli
Huazhong Normal University
School of Economics and Business Administration
Wuhan China
Email: zhengchengli168@163.com

Yao Yinhong
Huazhong Normal University
School of Economics and Business Administration
Wuhan China
Email: yyh0418@126.com

Abstract: This paper utilizes the axiomatic foundation of coherent risk measures to introduce MV(Mean-Variance), VaR (Value at Risk), ES (Expected Shortfall) and HMCR (Higher Moment Coherent Risk Measure). And based on the analysis of relationship between HMCR and stochastic dominance, it shows the HMCR measures are consistent with the second order stochastic dominance. Then, four kinds of optimization models are used to constituent stocks portfolio selection in Chinese A300 index. According to the different performance indicators, we get the cumulative expected return rate of the optimal portfolio. The results turn out that all the performance indicators of optimal portfolios by HMCR (p=3) are the best, which reveals that it has the highest ability of risk identification, and the higher cumulative expected return rate confirms the validity of this measure.

Keywords: risk management; higher moment coherent risk measure; portfolio optimization; consistence with stochastic dominance

I. Introduction

How to measure risk and how to choose the optimal portfolio are the problems that the risk managers and investors must concern about. In 1952, the theory of investment portfolio was proposed by Markowitz[1], based on Mean-Variance (or standard deviation) analysis, defining the risk as the rate for the expected return volatility, aimed at resolving the problem of minimizing the risk under a certain expected income level or achieving the optimal portfolio in the premise of risk. Introducing the quantitative analysis method to the financial field laid the foundation of modern finance. However, the higher standard deviation does not necessarily imply higher risk. That is an important defect of standard deviation as the risk measure, because it has non-monotonicity.

At the end of 1980's, Philippe Jorion[2] proposed the VaR (Value at Risk) model to cover the uncertainty and loss, that is the expected maximum loss under a certain confidence level and a certain goal. Soon, VaR became the benchmark risk measure, see the deep research of Duffie(1997) et al[3]. However, the VaR method also has serious defects as the risk measure. Artzner (1999)[4] pointed out that this method of risk measurement did not satisfy subadditivity. Cherny (2006) [5] thought that VaR only considered the information of a single point, it only cared about the probability of loss and ignored the size of loss. More defects about MV and VaR see Artzner (1997)[6].

ES (Expect Shortfall) is one kind of coherent risk measures. It is also called CVaR (conditional value at risk) or TVaR (tailed value at risk). We can understand it by probability threshold. It is a convex function of the combination of asset position and satisfies all the coherent axioms. Acerbi (2002) studied the differences between ES and VaR, and he pointed out the effectiveness of ES. However, ES still has defects: it only depends on the tail information, resulted it to be 0-1 risk measure, and it is not smooth. Cherny (2006) pointed out that Weighted VaR is more smoother than TVaR (also named ES).

Dellbaen et al. extended the definition of coherent measures of risk to the entire probability space. They explained how to define risk measure in the random variable space, and proved that all the coherent risk measures are equivalent to risk measurement under the Fatou property, where P is absolutely continuous probability measure set. They gave the list of some coherent risk measures that satisfied Fatou lemma. P-norm is one in the list. Cheridito and Li (2009), Dentcheva et al. (2010) introduced the theory of high order tail risk measure. Krokhmal proposed the concept of higher moment coherent risk measures(HMCR). He found that ES is a special case of HMCR (p=1), and proved that HMCR has the property of second order stochastic dominance consistency and match the utility theory. So it can describe the portfolio risk better. Chen and Wang (2007) gave the proof and derivation of p-norm (equal to HMCR).

Based on these literatures, we will analyze the effectiveness of the MV, VaR, ES, HMCR from the view of the consistency axiom via the concept of stochastic dominance. We focus on the HMCR. Then, we optimize constituent stocks portfolio of the Chinese A300 stock Index based on HMCR(p=1,2,3). Four different performance indicators are applied to test the performance of the optimal portfolio.

The rest of the paper is arranged as follows: section 2 introduces four kinds of risk measures, section 3 analyzes the characteristics of HMCR, section 4 applies the portfolio selection models and gives the empirical results based on four risk measures, and section 5 concludes.

II. Analysis of risk measure theory

In this section, We analyze the several important risk measures based on the coherent axioms.

A. Coherent axioms

Denote that \(X, Y\) are random variables on the space of \((\Omega, F, P)\), which are risk assets or assets portfolio. If the map \(\rho(\cdot)\) satisfies:

1. Monotonicity: if \(X \leq Y\), then \(\rho(X) \geq \rho(Y)\);
2. Translation Invariance: if any \(c \in R\), then \(\rho(X + c) = \rho(X) - c\);
3. Positive Homogeneity: if \(\lambda \geq 0\), then \(\rho(\lambda X) = \lambda \rho(X)\);
Subadditivity: \[ \rho(X + Y) \leq \rho(X) + \rho(Y) \]

Or Convexity: \[ \rho(\lambda X + (1-\lambda)Y) \leq \lambda \rho(X) + (1-\lambda) \rho(Y), \quad 0 \leq \lambda \leq 1. \]

then \( \rho(\cdot) \) is coherent risk measure.

Here, \( \rho_\alpha(X) = \inf \{ m \in \mathbb{R} | m + X \in A \} \), \( \rho(\cdot) \) is the corresponding set of acceptable risk measure.

It can be explained that the minimum of money in order to let the risk asset \( X \) become acceptable (we only consider the discounted situation). The financial implication of monotonicity is very clear. Translation invariance here means that when we increase the amount of risk-free cash, the economic capital of new portfolio will reduce the same number. Positive homogeneity means that the risk is proportional to the size of the risk assets. Sub-additivity is very important, it requires that the risk by combinations of the entire portfolio does not exceed or even lower than the sum of the risk of all sub-portfolios. Convexity implies that diversification do not increase the risk, it is equivalent to the sub-additivity plus positive homogeneity.

**B. Several kinds of risk measures**

The model of Markowitz maximizes the expected income of portfolio under the same risk level, or equivalently, it minimizes the risk of portfolio under the same return level. However, this method usually established with the assumption of elliptical distributions of risk assets, while the financial data has a fat tail, and the high standard deviation does not necessarily mean high risk.

Here we introduce three kinds of quantile-based risk measures: VaR(Value at Risk), ES(Expected Shortfall), and HMCR(Higher Moment Coherent Risk), and we put more attention to the last one.

For the random variable \( X \in L^1(\Omega,F,P) \), the cumulative distribution function and the higher-order cumulative distribution function is expressed as:

\[
F_X(\eta) = P[X \leq \eta] = F_X^1(\eta); F_X^{(k)}(\eta) = \int_{-\infty}^{\eta} F_X^{(k-1)}(\alpha) d\alpha \quad \text{for } \eta \in \mathbb{R}, \quad k \geq 2
\]

In which \( \eta \) is the expected return value.

The left-continuous inverse of the cumulative distribution function is defined as follows:

\[
F_X^{-1}(\alpha) = \inf \{ \eta : F_X(\eta) \geq \alpha \} \quad \text{for } 0 < \alpha < 1
\]

\( F_X^{(-2)}(\cdot) \) is named Lorenz function in \( R \rightarrow \overline{R} \), is defined as the cumulative quantile:

\[
F_X^{(-2)}(\alpha) = \int_0^\alpha F_X^{-1}(t) dt \quad \text{for } 0 < \alpha \leq 1
\]

We define that \( F_X^{(-2)}(0) = 0 \), and then find \( F_X^{(-2)}(1) = E[X] \).

At the level of \( \alpha \in (0,1) \), VaR is defined as:

\[
\text{VaR}(X) = -F_X^{-1}(\alpha) = \inf \{ \eta : F_X(\eta) \geq \alpha \}
\]

We can observe that VaR is based on quantile, but VaR model can only consider one quantile
without considering other information, and it only consider the loss probability and ignore the size of loss\cite{3}. It satisfies monotonicity, translation invariance and homogeneous, but does not meet the sub-additivity, so it isn’t a convex risk measure. While the convexity is very important when we deal with financial data, because it means that a combination of diversified investment strategy will not increase risk. And a non convex optimization problem is very difficult to get the optimal solution.

At the level \( \alpha \in (0,1] \), ES is defined as:

\[
ES_{\alpha} (X) = -\frac{1}{\alpha} F_{X}^{-1} (\alpha) = \frac{1}{\alpha} \int_0^\alpha Var_{\alpha} (X) dt
\] (5)

ES is a coherent risk measure, it satisfies monotonicity, positive homogeneity and translation invariance, sub-additivity, and it is the law invariant coherent risk measure that dominates VaR\cite{23}. The definition of this section can also refer to the Matmoura and Penev(2013)\cite{24}.

According to Matmoura and Penev (2013)\cite{24}, HMCR can be defined as:

\[
\rho_{c,p} (X) = \inf_{q \in \mathbb{R}} \left\{ c \left\| \eta - X \right\|, \left| \eta - \eta \right| \right\} = \inf_{q \in \mathbb{R}} \left\{ \frac{1}{\alpha} \left( E \left[ \left( \eta - X \right)^{\alpha} \right] \right) \right\}^{1/p} - \eta
\] (6)

In which, \( p > 1, c > 1, \alpha \in (0,1), c > 1 \).

The kusuoka\cite{25} representation of HMCR is:

\[
g_{c,p} (X) = \sup_{\mu \in \mathcal{M}_c} \int_0^1 AVaR_{\alpha} (X) \mu (d\alpha)
\] (7)

In which, \( p \in (1,\infty), q : 1/p + 1/q = 1, c \geq 1 \).

ES is the special case of HMCR (\( p=1 \)):

\[
ES_{\alpha} (X) = \inf_{q \in \mathbb{R}} \left\{ \frac{1}{\alpha} E \left[ \left( \eta - X \right)^{\alpha} \right] \right\}
\] (8)

HMCR satisfies four axioms, it is the coherent risk measure, now consider the relations between VaR, ES and HMCR at the same confidence level. That is:

\[
HMCR_{\alpha} (X) \geq ES_{\alpha} (X) \geq VaR_{\alpha} (X)
\] (9)

We can observe that \( ES_{\alpha} (X) \geq VaR_{\alpha} (X) \) from the definition, next ES is the special case of HMCR (\( p=1 \), and HMCR is a non-decreasing function\cite{22}, so \( HMCR_{\alpha} (X) \geq ES_{\alpha} (X) \).

III. Higher moment coherent risk measure

This part also based on the coherent axioms to analyze the property of HMCR. In addition, analyzing the stochastic dominance of HMCR based on the stochastic dominance theory.

A. HMCR is coherent risk measure

Under the condition of \( p > 1, \alpha \in (0,1) \), HMCR is coherent risk measure. Firstly, we define the optimal choice of HMCR named \( \eta_{c,p}^* \)\cite{22}:
\[ \eta^*_n, p = \arg \inf_{\eta, p} \left\{ \frac{1}{\alpha} \left[ E \left[ \left( \left( \eta - X \right)_+ \right]^p \right] \right]^{1/p} - \eta \right\}. \]  

(10)

(1) Monotonicity

For any \( X, Y \in L^r(\Omega, F, P) \) with \( X \leq Y \), and then \( (\eta - X)^+ \geq (\eta - Y)^+ \), for \( \eta \in R \).

We can derived the following two inequalities from the definition of \( \rho_{n, p} \):

\[ \frac{1}{\alpha} \left[ E \left[ \left( \left( \eta^*_n, p - Y \right)_+ \right)^p \right] \right]^{1/p} - \eta^*_n, p \leq \frac{1}{\alpha} \left[ E \left[ \left( \left( \eta^*_n, p - X \right)_+ \right)^p \right] \right]^{1/p} - \eta^*_n, p = \rho_{n, p}(X) \]  

(11)

\[ \rho_{n, p}(Y) = \frac{1}{\alpha} \left[ E \left[ \left( \left( \eta^*_n, p - Y \right)_+ \right)^p \right] \right]^{1/p} - \eta^*_n, p \leq \frac{1}{\alpha} \left[ E \left[ \left( \left( \eta^*_n, p - Y \right)_+ \right)^p \right] \right]^{1/p} - \eta^*_n, p \]  

(12)

Which means that \( \rho_{n, p}(Y) \leq \rho_{n, p}(X) \).

(2) Translation invariance

For any arbitrary \( c \in R \), we have:

\[ \rho_{n, p}(X + c) = \inf_{\eta, p} \left\{ \frac{1}{\alpha} \left[ E \left[ \left( \left( \eta - (X + c) \right)_+ \right)^p \right] \right]^{1/p} - \eta \right\} \]

\[ = \inf_{\eta, p} \left\{ \frac{1}{\alpha} \left[ E \left[ \left( \left( \eta - (X + c) \right)_+ \right)^p \right] \right]^{1/p} - \eta + c - c \right\} \]

\[ = \inf_{\eta, p} \left\{ \frac{1}{\alpha} \left[ E \left[ \left( \left( \eta^* - X \right)_+ \right)^p \right] \right]^{1/p} - \eta^* - c \right\} = \rho_{n, p}(X) - c \]  

(13)

(3) translation invariance

For any \( h > 0 \), from the formula of HMCR, we have:

\[ \rho_{n, p}(hX) = \inf_{\eta, p} \left\{ \frac{1}{\alpha} \left[ E \left[ \left( \left( \eta - hX \right)_+ \right)^p \right] \right]^{1/p} - \eta \right\} \]

\[ = \inf_{\eta, p} \left\{ \frac{1}{\alpha} \left[ E \left[ \left( \left( \eta - hX \right)_+ \right)^p \right] \right]^{1/p} - \eta / h \right\} \]

\[ = \inf_{\eta, p} \left\{ \frac{1}{\alpha} \left[ E \left[ \left( \left( \eta^* - X \right)_+ \right)^p \right] \right]^{1/p} - \eta^* \right\} = h \rho_{n, p}(X) \]  

(14)

When the function is continuous, it can also be established when \( p=0 \).

(4) sub-additivity

For any tow random variable \( X, Y \in L^r(\Omega, F, P) \), from the formula of HMCR, we have:

\[ \rho_{n, p}(X) + \rho_{n, p}(Y) = \frac{1}{\alpha} \left[ E \left[ \left( \left( \eta^*_n, p - X \right)_+ \right)^p \right] \right]^{1/p} + \frac{1}{\alpha} \left[ E \left[ \left( \left( \eta^*_n, p - Y \right)_+ \right)^p \right] \right]^{1/p} - (\eta^*_n, p + \eta^*_n, p) \]  

(15)

based on the Minkovski inequality and \( (x + y)^+ \leq x^+ + y^+ \) (\( \forall x, y \in R \)), the following formula can be established:

\[ \left[ E \left[ \left( \left( \eta^*_n, p - X \right)_+ \right)^p \right] \right]^{1/p} + \left[ E \left[ \left( \left( \eta^*_n, p - Y \right)_+ \right)^p \right] \right]^{1/p} \geq \left[ E \left[ \left( \left( \eta^*_n, p - X + \eta^*_n, p - Y \right)_+ \right)^p \right] \right]^{1/p} \]  

(16)
Therefore,
\[
\rho_{n,p}(X) + \rho_{n,p}(Y) \geq \frac{1}{\alpha} \left[ E \left[ \left( (\eta_{n,p}^* + \eta_{n,p}^{**}) - (X + Y) \right)^+ \right] \right]^{1/p} - (\eta_{n,p}^* + \eta_{n,p}^{**}) \geq \inf_{\eta \in \mathbb{R}} \left\{ \frac{1}{\alpha} \left[ E \left[ \left( (X + Y - \eta)^+ \right)^+ \right] \right]^{1/p} - \eta \right\} = \rho_{n,p}(X + Y)
\]
(17)

If and only if \( \eta_{n,p} = \eta_{n,p}^* + \eta_{n,p}^{**} \), and
\[
\eta_{n,p} \in \arg \inf_{\eta \in \mathbb{R}} \left\{ \frac{1}{\alpha} \left[ E \left[ \left( (X + Y - \eta)^+ \right)^+ \right] \right]^{1/p} - \eta \right\}
\]
(18)

The proof of this section can also refer to Chen and Wang\(^{[22]}\).

B. HMCR and stochastic dominance

(1) The concept of stochastic dominance

Stochastic dominance and the expected utility, rational choice theory have inevitable connection, it maybe the most important criteria to test risk sorting or the ability of recognition. The relationship between risk measure and stochastic dominance determines the ability of risk identification. If two risk assets \( X \) and \( Y \) have the relationship of \( X \overset{\text{nSD}}{\geq} Y \Rightarrow \rho_n(X) \leq \rho_n(Y), \forall \alpha \in (0,1], \) we think this risk measure method \( \rho \) is consistent with \( n \) order stochastic dominance. If two risk assets don’t have a corresponding order stochastic dominance, the risk measure does not recognize the ranking.

(2) Risk measure and stochastic dominance

VaR is consistent with the first order stochastic dominance. For any random variable \( X, Y \), the following statements are equivalent.

1) \( X \overset{\text{FSD}}{\geq} Y \);

2) \( \text{VaR}_\alpha(X) \leq \text{VaR}_\alpha(Y), \forall \alpha \in (0,1] \);

3) \( E\left(U(X)\right) \geq E\left(U(Y)\right), \) for all bounded and increasing utility functions \( U \in C(\mathbb{R}) \).

That means if two risky assets are consistent with first-order stochastic dominance, the VaR can be given sorting and identification, which has the consistent results with the investors whose utility function is increasing.

ES is consistent with the first order stochastic dominance. For any absolute integrable random variable \( X, Y \), the following established:
\[
X \overset{\text{FSD}}{\geq} Y \Rightarrow ES_n(X) \leq ES_n(Y), \forall \alpha \in (0,1]
\]
(19)

Further, ES is consistent with second order stochastic dominance. That is for all \( X, Y \) that have continuum supports and are atom less and absolute integrable, the following conditions are equivalent:

1) \( X \overset{\text{SSD}}{\geq} Y \);

2) \( ES_n(X) \leq ES_n(Y), \forall \alpha \in (0,1] \);
3) \( E(U(X)) \geq E(U(Y)) \), for all increasing and concave utility function \( U \in C^1(R) \) with bounded first-order derivatives.

That means if two risky assets are consistent with second-order stochastic dominance, ES can sort and identify the risk, ES does not have the higher order stochastic dominance. Ma and Wong\((2010)^{[26\, 27]}\) has the detailed proof.

HMCR are consistent with the first order stochastic dominance. For any absolute integrable random variable \( X, Y \), the following established:

\[
X \overset{PDO}{\succeq} Y \Rightarrow \rho_{\alpha}(X) \leq \rho_{\alpha}(Y), \quad \forall \alpha \in (0,1]
\]

(20)

By the definition of HMCR, HMCR is increasing by \( X \), this formula was clearly established.

Next, we will proof that HMCR are consistent with second-order stochastic dominance. Define \( u(\eta) = \left( (\eta^+)^p \right)^\alpha \) is a convex and non decreasing function, there are:

\[
\left( E[u(X)] \right)^{1/p} \leq \left( E[u(Y)] \right)^{1/p}
\]

(21)

for any \( X \succeq Y \). So HMCR are consistent with second order stochastic dominance.

More detailed proof see Krokhmal (2007)^{[22]}.

IV. Portfolio optimization based on HMCR

A. Portfolio optimization model

Suppose there are \( n \) stocks, each stock has the return rate of \( r_i \), we structure combination of \( P \), and the weight of each stock investment is \( w_i \), \( i = 1, 2, \cdots, n \), the return rate of those combination is \( r_p = \sum w_ir_i \). Under the condition of the expected rate of \( E[r_p] = \mu \), we doesn’t consider short selling, the portfolio selection problems in a confidence level of \( \alpha \) is as follows:

\[
\begin{align*}
\min & \quad \rho_{\alpha}(r_p) \\
\text{s.t.} & \quad E[r_p] = \mu \\
& \quad \sum w_i = 1 \\
& \quad w_i \geq 0
\end{align*}
\]

(22)

Let MV, VaR\(_n\)\((r_p)\), ES\(_n\)\((r_p)\), \(\rho_{\alpha}(r_p)\) instead the optimization objective function of formula (22).

Using the historical data method to estimate those risk measures, calculated by the frequency of data.

In the traditional standard deviation as the risk measure case, information ratio \( \frac{\mu}{\sigma} \) is an usual performance indicator, which means the ratio of average return and the standard deviation during the holding time. According to the different methods of risk measurement, given different performance indicators, that are \( \mu/\sigma, \mu/VaR_n, \mu/ES_n, \mu/\rho_{\alpha} \). In the same performance indicators, we compare the performance of different portfolio optimization during the holding period, if a combination is better than other combinations in all performance indicators, which means this combination has the strongest ability of stock selection.

B. Data description
We choose the constituent stocks in Chinese A300 index between 2009.01.05 and 2015.03.27, totally 1511 trading days, eliminating 79 stocks that missing data, remaining 221 stocks. We take the daily logarithmic return rate (not considering outliers). Divided the total data in two intervals of in sample and out of sample, the in sample interval between January 2009.01.06 and 2013.02.07, totally 1000 trading days as the historical sample for calculating optimization coefficient. The out of sample between 2013.02.08 and 2015.02.27, totally 510 trading days, as a follow-up period, investigating the actual performance of pre optimized combination.

C. The result of portfolio optimization

We analyze the result of portfolio optimization under the different expected return rate. The expected rate of return based on the average income of 221 stocks, taking equidistant 100 copies between the maximum and minimum average of return rate, getting 101 points of $E_\mu = \mu$. Under the confidence level of 95%, optimizing the calculation of MV, VaR, ES, HMCR. According to the portfolio frontier method of Markowitz, we structure the relationship between the optimal risk measures and portfolio return rate. The effective frontier is obtained as figure 1:

Analyzing the efficient frontier of different risk measures, we can observe that the classic Markowitz portfolio frontier located on the left side, the efficient frontier of VaR is not convex. HMCR(p=1) and CVaR are almost completely overlap, HMCR(p=2,3) are located on the right side. When $p > 3$, the outcome is close to $p = 3$, so we analyze the HMCR model of p=1,2,3.

D. The performance of portfolio optimization during the holding time

According to the optimization coefficient of part 4.3, we will calculate the combined performance of the four kinds of risk measures. Because the VaR model is non-convex, the optimal combination does not have reference significance, here we put more attention to those risk measures of MV, ES and HMCR(p=1,2,3). From the left of figure 2, the ordinate represents a given expected return rate of $\mu$, corresponding to the 99 (removal of the maximum and minimum) points, in which $h_i$ refer to
\( HMCR_i(p = i), i = 1, 2, 3, \) the abscissa represents the performance indicator, that is the corresponding information ratio, we only list the result of \( \mu / \sigma \) in order to save space, other risk measures have the similar results. In the portfolio frontier, those that below the global minimum points are abnormal, which can be remove because nobody would like to choice them. We can get the effective area like the right of figure 2.

**FIGURE 2 DIFFERENT OPTIMIZATION PERFORMANCE OF SEVERAL RISK MEASURE OUT-OF-SAMPLE**

Here we list the efficient frontier of \( \mu / h \), other performance indicators are similar to this. From right of the figure 2, we can observe that the performance indicators of \( \rho_a(p = 2, 3) \) are better than MV, ES (or \( \rho_a(p = 1) \)). It proved that HMCR is very suitable to measure portfolio optimization, showing its strongest risk measure identification, illustrating it is appropriate to judge risk measure by stochastic dominance.

Select 15 optimal combinations in effective area, calculating the average cumulative expected return, determining the ability of combination optimization of several risk measures in general. Figure 3 reports the value of five portfolios of MV, CVaR, HMCR (p=1,2,3). We can observe that the clear winner is the portfolio based on the \( HMCR_i(p = 3) \), the result of \( HMCR_i(p = 2) \) is similar to \( HMCR_i(p = 3) \).
We prove that HMCR are coherent risk measures based on the coherent axioms, and they are consistent with second stochastic dominance according to the concept of stochastic dominance. Then we build a portfolio optimization of constituent stocks in Chinese A300 index based on MV, VaR, ES and HMCR \((p=1,2,3)\), and testing stock selecting ability of those risk measures. Finally, we get the cumulative expected return rate of optimal portfolio. The results turn out that all the performance indicators of optimal portfolios by HMCR \((p=1)\) and ES are very similar. The performance indicators of HMCR \((p=2,3)\) are winner than others, the cumulative expected return rates of HMCR \((p=3)\) and HMCR \((p=2)\) are similar. HMCR \((p=3)\) is the best in general, which reveals that it has the highest ability of risk identification, and the higher cumulative expected return rate confirms the validity of this measure. The high order stochastic dominance of HMCR is worthy of further study.

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