Spot-future Market Interaction and Price Co-movement: an Agent-based Modelling Method

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ABSTRACT
In this work, we build a multi-agent spot-futures market model based on continuous double auction mechanism. Traders in our model are heterogeneous, including fundamentalists, speculators, arbitragers, hedgers and noise traders. The spot price and futures price are endogenous and the spot market and futures market are linked through these traders’ investment behaviors. We conduct the simulation of our model and produce some stylized fact such as the co-movement between spot price and futures price, and the fat-tail distribution of basis. Furthermore, we introduce an exogenous shock into our market model and study the formation mechanism of backwardation in terms of arbitrage asymmetry. Based on the analysis of our simulation results, we find out that during the stock crash period, the investment behaviors of hedgers can pull the futures price down under the spot price and the arbitrage asymmetry due to the limit of short selling in spot market makes the backwardation persist for a while.

KEY WORD
Agent-based modelling, Continuous double auction, Arbitrage asymmetry, Spot-futures market
1. Introduction

Basis is the difference between stock index futures prices and spot index price, which will be affected by many factors, such as the cost of holding spot, the changes of the underlying index and reverse arbitrage forces. China’s CSI 300 stock index futures have been listed since April 16, 2010, and in the most of the trading days the stock index futures prices is higher than the spot index price, namely contango. On 12 June 2015, the Chinese stock market crash began with the popping of the stock market bubble. Within one month, the Shanghai index lost about a third of its value before rebounding. Major aftershocks occurred around the “Black Monday” on 27 July and 24 August 2015. Shorting stock indexes is considered to be the blasting fuse of this financial crash. Since the 2015 stock market crash, stock index futures price sharply became lower than spot index price, and the backwardation lasted for a long time. There are some possible causes, such as the overreaction in stock index futures market to the stock market crash, and the arbitrage asymmetry caused by the poor margin mechanism in spot market.

It is a widely accepted principle that futures price cannot exceed the spot price since, otherwise, if arbitrage opportunity exists, selling a futures contract, buying spot and storing to deliver would earn a riskless profit. But arbitrage opportunities can exist if short selling is difficult. Lamont (2012) discusses various impediments to short selling, and he also argues that impediments can become more severe when a stock becomes more overpriced, sometimes due to an action by a firm to deter shorting of its stock. Moreover, arbitrage asymmetry could be caused by the traders’ behavior. Hahn and Won (2011) argue that arbitrage opportunities can persist if the behaviors of individual traders regarding arbitrage opportunities overwhelms other traders who are against arbitrage opportunities, and underline the importance of combining arbitrage trading and the behavioral approach to explain arbitrage trading in the real world.

The financial market is a complex system. Using agent-based simulation method to research financial assets price dynamics is a hot topic in recent years. Agent-based modeling is a bottom-up approach based on modern computer technology, relying on established institutional arrangements and financial markets. This financial simulation method given the agent specific patterns of behavior and learning mechanism, emphasizing the microscopic interaction between the agent in order to reveal the macroscopic laws of the financial market. In agent-based models, traders can be heterogeneous and have bounded rationality, which is much closer to the real financial market. Compared to the theoretically oriented models, much more stylized facts, including volatility clustering and fat-tailed distribution of returns, can emerge through these computationally oriented agent-based models.

There are several studies on futures and spot markets using the agent-based model. Ohi et al. (2011) used agent simulation to reproduce the stylized facts such as fat-tail distribution of returns of prices, volatility clustering, and long memory properties of volatility. Raberto et al. (2001) introduced an agent-based artificial financial markets with heterogeneous agents, reproducing the leptokurtic shape of the probability density of log price returns and the clustering of volatility. Xu et al. (2014) built up an agent-
based computational model with spot-futures cross-market structure. Analyzing the key properties such as bid-ask spread distribution, spot futures basis distribution and volatility clustering, comparing with the real market data, showed the model can reproduce the Chinese financial markets’ features successfully. Chen and Liao (2005) examined the explanations for the presence of the relation between stock returns and trading volume from the perspective of the agent-based model of stock markets. Also agent-based modeling has been used to analyze the marketing rules such as price limit and tick size. Kobayashi and Hashimoto (2007) used an agent-based artificial market simulator called “U-Mart” to consider influence of the circuit breakers on a stock market, getting the conclusion that circuit breakers play a key role in the control of price fluctuations.

Inspired by the researches of Ohi et al. (2011) and Chiarella et al. (2012), this work analyzes the arbitrage asymmetry and backwardation in a framework with heterogeneous agents. This paper is organized as follows. Section 2 introduces the multi-agent market model. In Section 3, we provide an analysis of the simulation results of the model. Finally, in Section 4, we present our conclusion.

2. The model

We build a two-market model based on a double auction mechanism. In this model, there exists a spot market where one stock (marked as asset 1) is traded and a futures market where the stock index future (marked as asset 2) is traded. The spot market and the futures market are linked through the traders’ investment behaviors.

3.1. Types of agents

Traders are heterogeneous in our artificial market. According to the model design and the empirical analysis results, we classify the traders into five types - fundamentalists, speculators, arbitragers, hedgers and noise traders, depending on their investment strategies. Arbitragers and hedges are cross-market traders, namely they can submit orders both in spot market and futures market. On the contrary, fundamentalists, chartists and noise trader are single-market traders, which can trade only in spot market or futures market. At each trading day \( t \), each agent \( i \) has one chance to enter the market and submit orders. We will introduce different types of traders in details in the next installment.

Fundamentalist traders

*Spot fundamentalists*

Fundamentalists who only execute the trade in the spot market believe the stock has its own fundamental price which is given as follow

\[
p_{t+1}^* = p_{t+1}^* \exp(\sigma v_t)
\]

Where \( p_{t+1}^* \) is the spot fundamental price at day \( t \), \( \sigma \geq 0 \) is a given constant and \( v_t \sim N(0,1) \) subjects to the standard normal distribution. Fundamentalists consider the price is overestimated as the market price exceeds fundamental price, and they tend to
submit sell orders. We assume that fundamentalists will submit limit orders when the spread between fundamental price \( p_{i,t}^* \) and the current market price \( p_{1,t} \) at time \( \tau \) in day \( t \) is big enough and the quantity of stocks fundamentalists \( i \) want to trade at time \( \tau \) in day \( t \), namely \( \bar{q}_{i,\tau,t} \), is proportional to the spread of fundamental price and the current market price. So their trade strategies is described as follows:

\[
\begin{align*}
H_{1,\tau} &= 1, \bar{q}_{1,\tau} = \left[a\left(p_{1,\tau}^* - p_{1,\tau}^*\right)\right], l_{1,\tau} = U(p_{1,\tau}^*, p_{1,\tau}^*), & \text{ if } & p_{1,\tau} - p_{1,\tau}^* < -\delta p_{1,\tau}^* \\
H_{1,\tau} &= -1, \bar{q}_{1,\tau} = \left[a\left(p_{1,\tau}^* - p_{1,\tau}^*\right)\right], l_{1,\tau} = U(p_{1,\tau}^*, p_{1,\tau}^*), & \text{ if } & p_{1,\tau} - p_{1,\tau}^* > \delta p_{1,\tau}^* \\
H_{1,\tau} &= 0, \bar{q}_{1,\tau} = 0 & \text{ if } & \left|p_{1,\tau}^* - p_{1,\tau}\right| \leq \delta p_{1,\tau}^* \\
\end{align*}
\]

\( H_{1,\tau} \) is a sign function which determines the order type (\( H_{1,\tau} = +1 \) means buy, and \( H_{1,\tau} = -1 \) means sell), \( \delta > 0 \) is the threshold of submitting and \( a > 0 \) is a constant measuring the fundamentalist’s sensitivity to spread. The limit price \( l_{1,\tau} \) obeys uniform distribution.

**Futures fundamentalists**

The investment behaviors of fundamentalists in futures market is similar to these of spot fundamentalists as mentioned above. They believe the futures has its own fundamental price \( p_{2,j}^* \), too. We set the fundamental price of futures according to future parity theorem. Despite the storage cost, the dividends and convenience yield, the fundamental price of the stock index futures \( p_{2,j}^* \) is

\[
p_{2,j}^* = p_{1,j}^* \exp[r(T - t)]
\]

where \( T \) is the delivery day of the stock index futures and \( r \) is the risk-free rate of interest. The futures fundamentalists’ investment strategies are impressed as follows.

\[
\begin{align*}
H_{2,\tau} &= 1, \bar{q}_{2,\tau} = \left[a\left(p_{2,\tau}^* - p_{2,\tau}^*\right)\right], l_{2,\tau} = U(p_{2,\tau}^*, p_{2,\tau}^*), & \text{ if } & p_{2,\tau} - p_{2,\tau}^* < -\delta p_{2,\tau}^* \\
H_{2,\tau} &= -1, \bar{q}_{2,\tau} = \left[a\left(p_{2,\tau}^* - p_{2,\tau}^*\right)\right], l_{2,\tau} = U(p_{2,\tau}^*, p_{2,\tau}^*), & \text{ if } & p_{2,\tau} - p_{2,\tau}^* > \delta p_{2,\tau}^* \\
H_{2,\tau} &= 0, \bar{q}_{2,\tau} = 0 & \text{ if } & \left|p_{2,\tau}^* - p_{2,\tau}\right| \leq \delta p_{2,\tau}^* \\
\end{align*}
\]

**Speculators**

Speculators make decisions according to the moving average prices, namely, the average of last \( D_i \) days’ closing prices; thus

\[
m_{X,\tau} = \frac{\sum_{j=1}^{D_i} p_{X,i-1}}{D_i}, \quad X \in \{1,2\}
\]

\[
\]
For each agent the value of $D_i$ is different, which implies that the length of time window is individual.

**Spot speculators**

Speculators believe that if the market price exceed the moving average price, the market price will continue to rise, hence, they will trend to submit a buy order, and vice versa. Similarly, we assume that the quantity of stocks speculators want to trade is proportional to the spread between moving average price and the current market price and the limit price is close to the current market price. So, the order type and the desired order volume are determined as follows.

\[
H_{1,\text{trt}} = -1, \bar{q}_{1,\text{trt}} = \left[ b(m_{1,t} - p_{1,t}) \right], l_{1,\text{trt}} = p_{1,t}(1 - |\Delta z_{trt}|) \quad \text{if} \quad p_{1,t} - m_{1,t} < -\delta m_{1,t}
\]
\[
H_{1,\text{trt}} = 1, \bar{q}_{1,\text{trt}} = \left[ b(p_{1,t} - m_{1,t}) \right], l_{1,\text{trt}} = p_{1,t}(1 + |\Delta z_{trt}|) \quad \text{if} \quad p_{1,t} - m_{1,t} > \delta m_{1,t}
\]
\[
H_{1,\text{trt}} = 0, \bar{q}_{1,\text{trt}} = 0 \quad \text{if} \quad |p_{1,t} - m_{1,t}| \leq \delta m_{1,t}
\]

where $b > 0$ and $z_{trt} \sim N(0,1)$ is normal distribution and $\Delta > 0$ is a constant. A large $\Delta$ means chartists are aggressive and would like to submit buy (sell) limit prices much higher (lower) than the current market price.

**Futures speculators**

The speculators in futures market take chartists strategies similarly to spot speculators do.

\[
H_{2,\text{trt}} = -1, \bar{q}_{2,\text{trt}} = \left[ b(m_{2,t} - p_{2,t}) \right], l_{2,\text{trt}} = p_{2,t}(1 - |\Delta z_{trt}|) \quad \text{if} \quad p_{2,t} - m_{2,t} < -\delta m_{2,t}
\]
\[
H_{2,\text{trt}} = 1, \bar{q}_{2,\text{trt}} = \left[ b(p_{2,t} - m_{2,t}) \right], l_{2,\text{trt}} = p_{2,t}(1 + |\Delta z_{trt}|) \quad \text{if} \quad p_{2,t} - m_{2,t} > \delta m_{2,t}
\]
\[
H_{2,\text{trt}} = 0, \bar{q}_{2,\text{trt}} = 0 \quad \text{if} \quad |p_{2,t} - m_{2,t}| \leq \delta m_{2,t}
\]

**Arbitragers**

Arbitragers attempts to profit from the differences between actual futures price $p_{2,t}$ and theoretical futures prices $TP_{2,t} = p_{1,t}e^{r(T-t)}$. They get profit through buying (or selling) a stock index future while selling (or buying) the stock at the same time when the spread between theoretical price and actual price is big enough and closing the positions till the actual price is almost equal to the theoretical price. The investment decisions of arbitragers are expressed as follows.

\[
H_{1,\text{trt}} = 1, \bar{q}_{1,\text{trt}} = \left[ c(p_{2,t} - p_{1,t}e^{r(T-t)}) \right], l_{1,\text{trt}} = p_{2,t}(1 + |\Delta z_{trt}|) \quad \text{if} \quad p_{2,t} - p_{1,t}e^{r(T-t)} > \delta p_{1,t}e^{r(T-t)}
\]
\[
H_{2,\text{trt}} = -1, \bar{q}_{2,\text{trt}} = \bar{q}_{1,\text{trt}}, l_{2,\text{trt}} = p_{2,t}(1 - |\Delta z_{trt}|) \quad \text{if} \quad p_{2,t} - p_{1,t}e^{r(T-t)} < -\delta p_{1,t}e^{r(T-t)}
\]
\[
H_{1,\text{trt}} = -1, \bar{q}_{1,\text{trt}} = \left[ c(p_{1,t}e^{r(T-t)} - p_{2,t}) \right], l_{1,\text{trt}} = p_{1,t}(1 - |\Delta z_{trt}|) \quad \text{if} \quad p_{2,t} - p_{1,t}e^{r(T-t)} > \delta p_{1,t}e^{r(T-t)}
\]
\[
H_{2,\text{trt}} = 1, \bar{q}_{2,\text{trt}} = \bar{q}_{1,\text{trt}}, l_{2,\text{trt}} = p_{2,t}(1 + |\Delta z_{trt}|) \quad \text{if} \quad p_{2,t} - p_{1,t}e^{r(T-t)} < -\delta p_{1,t}e^{r(T-t)}
\]
\[
H_{1,\text{trt}} = 0, \bar{q}_{1,\text{trt}} = 0 \quad \text{if} \quad |p_{2,t} - p_{1,t}e^{r(T-t)}| \leq \delta p_{1,t}e^{r(T-t)}
\]
\[
H_{2,\text{trt}} = 0, \bar{q}_{2,\text{trt}} = 0
\]
Notice that when the price spread disappears, namely \( |p_{2,t} - p_{1,t} \cdot e^{(T-t)}| \leq \delta p_{1,t} \cdot e^{(T-t)} \), arbitragers will submit market orders and close their positions in spot market and futures market at the same time.

**Hedgers**

Hedgers are agents who own a fixed number of stock and sell stock index futures to hedge spot risk. We assume that the hedgers forecast the price of stock according to the moving average prices, which is similar to speculators’ prediction rule. If hedger consider the spot price will decrease he is willing to submit a sell limit order in futures market to hedge the loss of spot, otherwise he will submit a buy order to exit some short positions in futures market. The details are as follows.

\[
H_{2,_{ir}} = \begin{cases} 
-1, \bar{q}_{2,_{ir}} = \min \left\{ \left[ d \left( m_{i,t} - p_{2,t} \right) \right], S_{ir} + F_{ir}, l_{2,_{ir}} = p_t (1 - |\Delta z_i|) \right\} & \text{if } p_{1,t} - m_{i,t} < -\delta m_{i,t} \\
1, \bar{q}_{2,_{ir}} = \min \left\{ \left[ d \left( m_{i,t} - p_{2,t} \right) \right], -F_{ir}, l_{2,_{ir}} = p_t (1 + |\Delta z_i|) \right\} & \text{if } p_{1,t} - m_{i,t} > \delta m_{i,t} \\
0, \bar{q}_{2,_{ir}} = 0 & \text{if } |p_{1,t} - m_{i,t}| \leq \delta m_{i,t}
\end{cases}
\]

Despite the basis risk, taking the amount of short positions in futures market as same as the long position in spot market will provide a perfect hedge. Hence, the hedger will not sell futures more than the amount of \( S_{ir} + F_{ir} \), where \( S_{ir} > 0 \) is the stocks hedger \( i \) holds and \( F_{ir} < 0 \) is his short position in futures market.

**Noise traders**

We introduce some noise traders with zero intelligence both in spot market and future market. They submit the buy order and sell order with same probability. The limit volume \( q_{X,_{ir}} \ (X \in \{1,2\}) \) is random between 3 and 10, and the limit price is close to the current market price, namely \( l_{X,_{ir}} = p_{X,_{i}} (1 \pm |\Delta z_i|) (X \in \{1,2\}) \).

### 3.2. Order and price determination

In the spot market, short selling and buying on margin are forbidden, while in the futures market all transactions are executed by margin trading, hence, the actual quantity an agent can submit is

\[
q_{1,_{ir}} = \begin{cases} 
\min(\bar{q}_{1,_{ir}}, \frac{C_{ir} - \beta p_{2,_{ir}} \cdot F_{ir}}{l_{1,_{ir}}}), & (H_{1,_{ir}} = +1) \\
\min(\bar{q}_{1,_{ir}}, S_{ir}), & (H_{1,_{ir}} = -1)
\end{cases}
\]

\[
q_{2,_{ir}} = \begin{cases} 
\max\{0, \min\left[ C_{ir} / \beta l_{2,_{ir}} - F_{ir}, \bar{q}_{2,_{ir}} \right]\}, & (H_{2,_{ir}} = +1) \\
\max\{0, \min\left[ C_{ir} / \beta l_{2,_{ir}} + F_{ir}, \bar{q}_{2,_{ir}} \right]\}, & (H_{2,_{ir}} = -1)
\end{cases}
\]

where \( \beta > 0 \) is the margin ratio in futures market. \( C_{ir} > 0 \) and \( S_{ir} > 0 \) is the amount
of cashes and stocks that trader $i$ holds at time $t$ in day $t$, and $F_{it\tau}$ is trader’s position of stock index future.

The trading mechanism in our artificial market is continuous double auction (CDA), which has been widely used in real market. The limit orders are executed based on price-time priority. The market price changes at once when transaction happens.

3. Simulation results

We conducted simulation of the spot-futures market model and observed the agents’ behaviors and emergent phenomena at market level. Besides, we introduced an exogenous shock and studied the formation mechanism of backwardation in terms of arbitrage asymmetry.

There are 1000 agents in our spot-futures market, include 300 fundamentalists (150 in each market), 200 speculators (100 in each market), 200 hedgers, 200 arbitragers and 100 noise traders (50 in each market). The initial spot price and futures price both are 1000. We run the model for 550 periods but only analyze the data of last 300 periods to avoid transient effects. The other parameters are listed in Table 1.

Table 1. Parameters used in the simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$N$</td>
<td>1000</td>
<td>Number of agents</td>
</tr>
<tr>
<td>$S_{i0}$</td>
<td>1000 $S_{i0}$</td>
<td>Initial stock endowment</td>
</tr>
<tr>
<td>$C_{i0}$</td>
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<td>Volatility of fundamental value (daily)</td>
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<td>$r$</td>
<td>0</td>
<td>risk-free interest rate</td>
</tr>
<tr>
<td>$a$</td>
<td>1</td>
<td>Reaction coefficient for fundamentalists</td>
</tr>
<tr>
<td>$b$</td>
<td>1</td>
<td>Reaction coefficient for speculators</td>
</tr>
<tr>
<td>$c$</td>
<td>1</td>
<td>Reaction coefficient for arbitragers</td>
</tr>
<tr>
<td>$d$</td>
<td>1</td>
<td>Reaction coefficient for hedgers</td>
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<tr>
<td>$D_{i}$</td>
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<td>$\Delta$</td>
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<td>Aggressiveness parameter</td>
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<tr>
<td>$\delta$</td>
<td>0.001</td>
<td>Threshold to submit orders</td>
</tr>
</tbody>
</table>

3.1. Stylized facts

Some stylized facts, such as high co-movements between spot and futures prices and fat-tail distribution of basis, exist in spot and futures markets. These stylized facts are widely observed by empirical studies. The simulation results of our two market model can reproduce these price properties perfectly. We will show the simulation results in detail as follow.
Co-movement of prices

Fig. 1 shows the variations of the spot and futures daily prices in one of our model simulation. We can clearly see the co-movement between spot prices and futures prices from this line figure. Moreover, the correlation coefficient of the two prices is 0.8269, which further illustrates the high co-movement phenomenon.

Fat-tail distribution of basis

Fig. 2 shows the distribution of basis, namely the difference between spot and futures price. Comparing the distribution of basis to the normal distribution (green line), we can determine that the basis follow a fat-tail distribution.

The difference between spot prices and futures prices is not wide and the basis is close to 0 frequently, resulting from investment behaviors of arbitragers, who benefit from basis. However, investing activities of speculators and hedgers may cause significant discrepancies in prices of both and extreme basis values.

Fig. 1. Co-movement of spot (blue) and futures (red) prices

Fig. 2. Fat-Tail Distribution of Basis
3.2. Arbitrage asymmetry and backwardation

We give the market model an external shock through suddenly decreasing the fundamental value by 20%, and observe the agents’ behaviors. Fig. 3 shows the minutely prices’ variation in the three days after the shock. Once the fundamental value drops, the fundamentalists in both market firstly respond to the shock and submit sell orders. Numerous sell orders cause spot price and futures price both fell down. The market price sliding furtherly cause the speculators’ willingness to sell, which will aggravate the decline. Moreover, the hedgers sell futures in hopes of reduce the loss in spot market, resulting in the further decline of futures price and making futures prices fell down under the spot prices. The arbitragers then tend to sell shares while buying indexes because of the positive basis. But forbidden of short selling in spot market will limit the arbitragers’ sell investments in spot market. The arbitrage asymmetry makes the backwardation persist for a while.

![Fig. 3. The minutely prices of spot and futures under an external shock](image)

4. Conclusions

Inspired by the researches of Ohi et al. (2011) and Chiarella et al. (2012), we model a multi-agent spot-futures market. In our model, the spot market and futures market are linked through the investors’ behaviors. We conduct the simulation of our model and produce some stylized fact such as the co-movement between spot price and futures price, and the fat-tail distribution of basis. Our results indicate that the two-market model is effective for analyzing the relationship between spot price and futures price, and different types of investment behaviors, including arbitraging.

Furtherly, we introduce an exogenous shock into our market model and study the formation mechanism of backwardation in terms of arbitrage asymmetry. Based on the analysis of our simulation results, we find out that during the stock crash period, the investment behaviors of hedgers can pull the futures price down under the spot price and the arbitrage asymmetry due to the limit of short selling in spot market makes the backwardation persist for a while.

Generally, this model provides some meaningful results supported by many other literatures and can be extend to conduct more analyses in spot-futures market, such as “waterfall effect”, circuit breaker and raising limit.
Reference