Green fiscal policies in the presence of socially responsible investors

by T.I. Renström and Luca Spataro

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Abstract

We build a general equilibrium dynamic model in which individual investors are endowed with "warm-glow" preferences a la Andreoni (1990) so that they feel partly responsible for the pollution content of their portfolio. Through investors' portfolio choice, firms are induced to engage in costly abatement activities, given that higher pollution also implies a higher cost of capital. In this scenario, we characterize the equilibrium of the economy and investigate, through a fiscal reform analysis, the effects of such tax instruments on the equilibrium scale of the economy, per-capita consumption, pollution abatement and "pollution premium". We show that an increase of the pollution tax, while reducing pollution, also depresses consumption and the scale of the economy. On the contrary, an increase of subsidies on abatement activity increases the scale of the economy and, under mild assumptions on the abatement technology, can also decrease pollution.

JEL Classification: D21, D53, G11, H21, H23, M14, Q58.

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1. Introduction

In this paper, we analyse the effects of fiscal policies aimed at reducing pollution in the context of financial markets populated by socially responsible investors. More precisely, we build a general-equilibrium-dynamic model in which individual investors are endowed with "warm-glow" preferences a *la* Andreoni (1990), so that they feel partly responsible for the pollution content of their portfolio and, at the equilibrium, ask for a "pollution premium" in order to hold "dirty assets". Pollution emerges as a by-product of production activity of profit-maximizing firms, which are also endowed with a costly abatement technology. Finally, the Government levies a lump sum tax on firm's pollution flow and a subsidy on its abatement activity. In this scenario, we characterize the equilibrium of the economy and investigate, through a fiscal reform analysis, the effects of such fiscal instruments on the equilibrium scale of the economy, on pollution abatement, on per-capita consumption and on the "pollution premium".

The issue we investigate is relevant under several points of view. First of all, concern on natural resource depletion and environmental quality has been spreading throughout the world in recent decades. Several initiatives have been put in place by private organizations aimed at implementing common strategies to integrate social, environmental and governance (ESG) factors into both business reporting and traditional investment risk analysis (see OECD 2017).

Moreover, the mounting public debate on these issues promoted by shareholder activism, mobilization of NGOs and social media, the diffusion of new data from credit rating agencies, coupled with the effects of the recent financial crisis, have increased the demand for more transparent management from financial investors (DATA) and for more responsible behaviour from the productive sector. All these elements have contributed to the spreading of the well known phenomenon called "Social Responsible Investing" or "Sustainable investing".

According to Eurosif, SRI "is a long-term oriented investment approach, which integrates ESG [i.e. Environmental, Social, Governance] factors in the research, analysis and selection process of securities within an investment portfolio. It combines fundamental analysis and engagement with an evaluation of ESG factors in order to better capture long term returns for investors, and to benefit society by influencing the behaviour of companies." (Eurosif 2016, p. 9). Hence, SRI is a process of identifying and investing in companies that meet certain standards of Corporate Social Responsibility (CSR) [1] through such activities and strategies as positive or negative screening, shareholder advocacy, impact and community investing (for more details see GSIA, 2016). SRI has been argued to be a possible instrument to improve environmental quality through a market mechanism.

In this scenario, under the impulse of several international summits (from Kyoto in 1997 to Paris 2015), many public institutions have been implementing or proposing fiscal and regulatory policies aimed at contrasting the worsening of environmental quality and at increasing the ESG-practices (as for the recent initiatives, see OECD 2017 and, for the EU, see Eurosif 2018?)

For example, in 1995 the Dutch government has launched the Green Funds Scheme, a tax incentive scheme for investors into green initiatives. In the U.S. there are examples of tax-credit bonds, (bond investors receive tax credits instead of interest payments so issuers do not have to pay interest on their green bond issuances) or tax-exempt bonds.

In light of this recent trend and for its wide potential implications concerning, in particular, the design of pollution abatement policies and of fiscal incentives for green initiatives, in this paper, we aim at shedding new light on the effectiveness of fiscal policies in enhancing environmental quality and showing the conditions under which they can also improve the performance of the economy.

[1] For a recent review of economic literature on CSR, see Brekke and Pekovic (2018).

The paper is organized as follows: in section 2 we discuss the related literature, in section 3 we specify the model and characterize the equilibrium and its stability; in section 4 we carry out a tax-reform analysis and discuss the results. Section 5 concludes.

2. Related literature

Several scholars have analysed the issue of environmental quality and fiscal incentives from an economic perspective [2]. However, while the literature on pollution taxes has been flourishing, the economic literature on SRI [3] and on its consequences on taxation is still embryonic and results are mixed.

For example, Hainkel et al. (2001), adopt a one-period model to show that negative screening on polluting firms by fund managers can induce firms to adopt cleaner technologies in order to avoid higher costs of capital. The positive effects of financial markets on environmental quality are also stressed by Dam (2011), who argues that SRI creates a role for the stock market to deal with intergenerational environmental externalities. The author shows that, although socially responsible investors are short-lived, the forward-looking nature of stock prices, reflecting the warm-glow motive, can help to mitigate the conflict between current and future generations. Dam and Scholtens (2015) develop a model that links SRI and CSR, showing that responsible firms display higher returns on assets, although the overall effect on stock market returns depends on the relative strength of supply and demand side effects.

On the other hand, Barnea et al. (2005) argue that negative screening reduces the incentives of polluting firms to invest so that also the total level of investment in the economy decreases. Dam and Heijdra (2011) analyse the effects of SRI and public abatement on

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[2] The seminal work is Sandmo (1975). See also Cremer et al. (2001) and the survey by Bovenberg and Goulder (2002). More recent works on this subject are Bontems and Bourgeon (2005), Goulder and Parry (2008), Gahvari (2014), Jacobs and De Mooij (2015), Kampas and Horan (2016), Belfiori (2018), Pizer and Sexton (2019).

[3] For a survey on the topic, see Renneboog et al. (2008).

environmental quality in a growth model with socially responsible investors and show that SRI behaviour by households partially offsets the positive effects on environmental quality of public abatement policies.

Finally, Vanwalleghem (2017) argues that SRI may have a mixed effect on firms' incentives to remove negative externalities. In fact, whereas SRI screening incentivizes the removal of externalities (as predicted by Heinkel et al. 2001 and confirmed by the empirical work of Hong and Kacperczyk 2009), SRI trading can disincentivise it when traders disagree on the externality removal's cash flow effects.[4]

While providing interesting results, the above mentioned literature has not analysed the effects of fiscal instruments in presence of SRI.[5] To bridge this gap, in this paper we specify a continuous-time model, where pollution is a by-product of production, but firms can engage in abatement, reducing net pollution. We model investors' social-responsibility objective through a warm-glow mechanism as in Andreoni (1990) and Dam (2011)[6]. Through investors' portfolio choice, firms are induced to engage in socially responsible activities (abatement), given that higher pollution also implies a higher cost of capital on the capital markets. In this scenario, we carry out a tax-reform analysis to evaluate the effects of fiscal instruments (i.e. taxes on pollution and subsidies to abatement activities) on the economy scale of production, on pollution and on the "pollution premium".

[4] Graff Zivin and Small (2005) and Baron (2007) also focus on socially responsible firms and financial markets. However, these are partial-equilibrium and static models, in which social responsibility is concerned with charitable giving and not with abatement of externalities or public bads,

[5] In a recent paper, Renström et al. (2019) present a normative analysis of second-best taxes in presence of socially responsible investors, although disregarding subsidies to abatement and without providing positive analysis.

[6] While there is increasing evidence of the very existence of warm-glow preferences (see Andreoni et al. 2017), the exact shape is far from being clear. However, some recent works have produced axiomatizations of the warm glow that can help to characterize its shape (see Evren and Minardi 2017 and the literature therein). In this work, we follow Bernehim and Rangel (2005) when stating that "one can interpret it [the warm-glow] as a reduced form for a variety of mechanisms with starkly differing welfare implications" (p. 63).

Our results can be summarised as follows: an increase of the tax on pollution reduces pollution but also the installed capital of the economy and per-capita consumption.

On the other hand, a subsidy on firms' abatement activity increases consumption and the installed capital of the economy and can, under certain circumstances, reduce pollution.

3. The model setup

In this section, we present the benchmark model. The economy is populated by H identical households and J identical firms. We assume that in each period an infinitely lived consumer-investor is endowed with a unit of time that is exogenously allocated to work. Moreover, individuals are endowed with an instantaneous utility function $u(c(t), p(t))$, where $c(t)$, is consumption for that individual at period t, $p(t)$ is an index of the responsibility that the individual feels for the pollution caused by firms that it holds shares in (*warm-glow*). This utility is assumed to be increasing in $c(t)$, decreasing in $p(t)$ and strictly concave. Hence, in each period an individual chooses consumption and saving allocation.

As for firms, we assume perfectly competitive markets and constant return to scale technology both for production and abatement activity.

Finally, we assume the government finances an exogenous stream of per-capita expenditure g by issuing debt (which is the only clean asset in the market) and levying taxes on firm's pollution flow and subsidies on its abatement activity.

2.1. Households

The lifetime utility function of an individual household, at period 0, is:

$$U(0) = \int_0^\infty e^{-\rho t}\, u(c(t), p(t))dt \tag{1}$$

with $u_c > 0, u_p < 0, u_{cc}, u_{pp} < 0$ and $\rho > 0$ the intertemporal discount rate. Population size, H, is assumed to be constant. In line with Dam and Scholtens (2015), the warm-glow $p(t)$ is assumed to be a function of the individual's portfolio invested in polluting firms:

$$p(t) \equiv \sum_{j=1}^{J} \frac{e^j(t)}{\bar{E}^j} \bar{p}^j(t) \tag{2}$$

where $e^j(t)$ is the number of shares of firm j owned by the individual, $\bar{E}^j$ is number of total shares of firm j, assumed to be constant, $\bar{p}^j(t)$ is the "pollution content" of firm j as perceived by the individual. The idea is that the household feels responsible for the pollution associated with its share ownership, even though the pollution is not directly felt. For simplicity we assume that the instantaneous utility function is additive-separable in c and p, so that $u_{cp} = u_{pc} = 0$.

Following previous literature (e.g. Dam 2011 and Dam and Heijdra 2011), we assume that $\bar{p}^j$ is linear in x^j (as any non-linearity can be captured by u):

$$\bar{p}^j(t) = \gamma \cdot x^j(t) \tag{S.1}$$

where $x^j(t)$ is the flow of pollution produced by the jth firm and $X(t) = \sum_{j=1}^{J} x^j(t)$ is the aggregate flow of pollution. Notice that $x^j(t)$ is controlled by the firm j, i.e. each firm can affect its "rating" through its decision, while aggregate variables are taken as given by each firm. At each instant of time t, individual's wealth is

$$a(t) \equiv b(t) + \sum_{j=1}^{J} e^j(t)\, P_e^j(t) \tag{4}$$

where $b(t)$ is per-capita public debt, $P_e^j(t)$ the stock market price of shares. By defining

$\omega^j(t) \equiv \frac{e^j(t)P_e^j(t)}{a(t)}$ as the portfolio share invested in firm j, and $V^j(t) \equiv \bar{E}^j P_e^j(t)$ as the stock market value of firm j, we have:

$$p(t) \equiv \sum_{j=1}^{J} \frac{\omega^j(t)a(t)}{V^j(t)} \bar{p}^j(t) \qquad (5)$$

and the individual budget constraint reads as[7]:

$$\dot{a}(t) = \sum_{j=1}^{J} \omega^j(t) r_e^j(t)\, a(t) + \left[1 - \sum_{j=1}^{J} \omega^j(t)\right] r(t)a(t) + w(t) - c(t) - z(t) \qquad (6)$$

where $r_e^j(t)$ is the return on share j, $r(t)$ is net-of-tax interest rate on public debt, $w(t)$ is the wage, $z(t)$ a lump sum tax used to balance the Government budget. Returns on shares of firm j are:

$$r_e^j(t) \equiv \frac{\dot{V}^j(t)}{V^j(t)} e^j(t) + \frac{d^j(t)}{V^j(t)} \qquad (7)$$

where $\frac{d^j(t)}{V^j(t)}$ is the dividend pay-out ratio and $d^j(t)$ total dividend payments by firm j.

The individual's problem is to maximize (1) w.r.t. $c(t), l(t), \omega^j(t)$ subject to (4) and (6). The associated current value Hamiltonian is:

$$\Lambda(t) = u(t) + q(t)\dot{a}(t) \qquad (8)$$

with $q(t)$ the shadow price of wealth. FOCs yield:

[7] We follow Merton (1971).

$$u_c(t) - q(t) = 0 \quad (9)$$

$$u_p(t)\frac{\bar{p}^j(t)}{V^j(t)}a(t) + q(t)a(t)[r_e^j(t) - r(t)] = 0 \quad (11)$$

$$q(t)\left[\sum_{j=1}^{J}\omega^j(t)r_e^j + \left(1 - \sum_{j=1}^{J}\omega^j(t)\right)r(t)\right] + u_p(t)\sum_{j=1}^{J}\frac{\omega^j(t)\bar{p}^j(t)}{V^j(t)} = \rho q(t) - \dot{q}(t) \quad (12)$$

Note that eq. (9) and eq. (11) provide:

$$\frac{u_p(t)}{u_c(t)}\bar{p}^j(t) + V^j(t)[r_e^j(t) - r(t)] = 0 \quad (13)$$

Equations (9)-(10) provide the usual optimality conditions for consumption and labour supply; (13) is the optimal portfolio-choice condition. Notice that the return on assets in production is greater than the return on government bonds and the difference is proportional to the pollution content by the firm, thus there is a pollution premium, a compensation required by the household for holding "dirty assets". Exploiting (7), (13) becomes:

$$\frac{u_p(t)}{u_c(t)}\bar{p}^j(t) + \dot{V}^j(t) + d^j(t) - r(t)V^j(t) = 0 \quad (14)$$

Finally, pre-multiplying (11) by $\omega^j(t)$ and summing from j=1 to J and using (12) we have:

$$q(t)r(t) = \rho q(t) - \dot{q}(t). \quad (15)$$

and, by time-differentiating eq. (9) and exploiting (15), we get the usual Euler's equation for consumption:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma(t)}\left(r(t) - \rho\right). \tag{15'}$$

with $\sigma(c) \equiv -\frac{u_{cc}(t)c(t)}{u_c(t)}$.

2.2. Firms

We assume that each firm runs its business in a perfectly competitive market, endowed with constant-returns-to-scale production technology that uses capital and labour inputs to produce a homogenous good. We shall also assume that each firm's technologies are the same. Hence, it will be possible to aggregate the firms to obtain a representative firm. The production function for firm j is:

$$y^j(t) = f^j\left(k^j(t), l^j(t)\right) \tag{16}$$

with $k^j(t)$ physical capital input and $l^j(t)$ labour input, respectively. We follow Brock and Taylor (2005) by assuming that, at any time t, every unit of output generates ε units of pollution as a joint product of output and that pollution can be reduced by abatement activity by the firm, $\alpha(t)$. The latter is supposed to be carried out through a CRS technology that is an increasing function of the total scale of firm activity $f(t)$ and of the firm's efforts at abatement, $f^\alpha(t)$. If abatement at level $\alpha(t)$ removes $\varepsilon \cdot \alpha(t)$ units of pollution, we have that total emissions (pollution) $x(t)$ by firm j is equal to:

$$x^j(t) = \varepsilon \cdot f^j(t) - \varepsilon \cdot \alpha\left(f^j(t), f^{\alpha j}(t)\right) \tag{17}$$

Defining $\psi^j(t) \equiv \frac{f^{\alpha j}(t)}{f^j(t)}$ as the fraction of output devoted to abatement activity and exploiting CRS, we get:

$$\frac{x^j(t)}{f^j(t)} = \varepsilon \cdot \left[1 - \alpha\left(1, \psi^j(t)\right)\right] = \varepsilon \cdot \left[1 - \alpha\left(\psi^j(t)\right)\right] \tag{18}$$

with α increasing in ψ^j and, thus, eq. (18) gives $\psi^j(t) = \Psi\left(\frac{x^j(t}{f^j(t)}\right)$ [8], with $\Psi' < 0$, $\Psi'' > 0$.

For example, by assuming an abatement technology of the form $\alpha(f, f^\alpha) = f^{(1-\xi)} f^{\alpha^\xi} = \psi^\xi f$, then $\frac{x}{f} = \varepsilon \cdot \left[1 - \psi^\xi\right]$ and $\psi = \left(1 - \frac{x}{f}\frac{1}{\varepsilon}\right)^{\frac{1}{\xi}}$.

Assuming that the government levies taxes on pollution and subsidies on abatement activities, gross operating profits of firm j are:

$$\pi^j(t) \equiv \left[1 - (1 - s(t))\Psi\left(\frac{x^j(t}{f^j(t)}\right)\right] f^j\left(k^j(t), l^j(t)\right) - w(t)l^j(t) - \tau^x(t)x^j(t) \tag{19}$$

where $\tau^x(t)$ is the tax on pollution at time t and $s(t)\Psi f^j = S(t)$ is the public subsidy on firms' abatement activity. Given that we assume that the number of shares remains constant and that we abstract from corporate bonds[9], new investments, $i^j(t)$, can only by financed via retained earnings, $Re(t)$, i.e. $\pi^j(t) = d^j(t) + Re^j(t)$. That is, by exploiting the capital accumulation identity:

$$\dot{k}^j(t) = i^j(t) - \delta k^j(t) \tag{20}$$

[8] From (18), $\alpha(\psi^j(t)) = 1 - \frac{1}{\varepsilon} \cdot \frac{x^j(t}{f^j(t)}$ gives $\psi^j(t) = \alpha^{-1}\left(1 - \frac{1}{\varepsilon} \cdot \frac{x^j(t}{f^j(t)}\right) \equiv \Psi\left(\frac{x^j(t}{f^j(t)}\right)$.

[9] Corporate bonds would be equivalent to shares in our model, as they would also carry the same pollution premium as shares.

with δ the (constant) instantaneous depreciation rate, we get:

$$\dot{k}^j(t) = \pi^j(t) - d^j(t) - \delta k^j(t) \tag{21}$$

and, exploiting (19), (21) becomes:

$$\dot{k}^j(t) = \left[1 - (1 - s(t))\Psi\left(\frac{x^j(t}{f^j(t)}\right)\right] f^j\big(k^j(t), l^j(t)\big) - w(t)l^j(t) - \tau^x(t)x^j(t) - d^j(t) - \delta k^j(t) \tag{22}$$

Now, integrating (14) we get:

$$V^j(0) = \int_0^\infty e^{-\int_0^t r(s)ds}\left[d^j(t) + \frac{u_p(t)}{u_c(t)}\bar{p}^j(t)\right]dt \tag{23}$$

which provides the value of the firm at time 0. Substituting for $d^j(t)$ from (22), (23) reads as:

$$V^j(0) = \int_0^\infty e^{-\int_0^t r(s)ds}\left\{\left[1 - (1 - s(t))\Psi\left(\frac{x^j(t}{f^j(t)}\right)\right] f^j(k^j(t), l^j(t)) - w(t)l^j(t) - \tau^x(t)x^j(t) - \delta k^j(t) + \frac{u_p(t)}{u_c(t)}\bar{p}^j(t) - \dot{k}^j(t)\right\}dt \tag{24}$$

Given the assumption of perfect competition, the firm hires labour, $l^j(t)$, on the spot market and remunerates it according to its marginal productivity. In fact, FOCs on (24) w.r.t. $l^j(t)$ and $x^j(t)$ yield, respectively:

$$\left[1 - (1 - s(t))\Psi\left(\frac{x^j(t}{f^j(t)}\right) + (1 - s(t))\Psi'\left(\frac{x^j(t}{f^j(t)}\right)\frac{x^j(t)}{f^j(t)}\right] f_l^j(t) - w(t) = 0 \tag{25}$$

$$\frac{u_p(t)}{u_c(t)}\frac{\partial \bar{p}^j(t)}{\partial x^j(t)} - (1-s(t))\Psi'\left(\frac{x^j(t}{f^j(t)}\right) - \tau^x(t) = 0 \tag{26}$$

The optimality condition for $k^j(t)$, $\frac{dV^j(0)}{dk^j(t)} = \frac{d}{dt}\frac{dV^j(0)}{d\dot{k}^j(t)}$, classical calculus of variation, gives:

$$\int_0^\infty e^{-\int_0^t r(s)ds}\left\{\left[1-(1-s(t))\Psi\left(\frac{x^j(t}{f^j(t)}\right) + (1-s(t))\Psi'\left(\frac{x^j(t}{f^j(t)}\right)\frac{x^j(t)}{f^j(t)}\right] f_k^j(t) - \delta\right\} dt =$$

$$\frac{d}{dt}\int_0^\infty e^{-\int_0^t r(s)ds}\, dt \Rightarrow$$

$$\left[1-(1-s(t))\Psi\left(\frac{x^j(t}{f^j(t)}\right) + (1-s(t))\Psi'\left(\frac{x^j(t}{f^j(t)}\right)\frac{x^j(t)}{f^j(t)}\right] f_k^j(t) - \delta = r(t) \tag{27}$$

It can be shown that, by plugging (25)-(27) into (24) and exploiting CRS in $f^j(t)$, then $maxV^j(0) \equiv \bar{E}^j {P_e}^j(0) = k^j(0)$.

3. Equilibrium and stability

We now characterise the equilibrium of the economy. At each date t, given that all firms are equal, plugging eq. (27) into (15)' yields:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma(c)}\left\{\left[1-(1-s(t))\Psi\left(\frac{x(t)}{f(t)}\right) + (1-s(t))\Psi'\left(\frac{x(t)}{f(t)}\right)\frac{x(t)}{f(t)}\right] F_K(t) - \delta - \rho\right\}. \tag{28}$$

Moreover, under the assumption that firms are equal, the feasibility constraint stating that private plus investment be equal to aggregate output (recall that we assume a balanced Government budget), reads as.[10]

[10] In fact aggregating over firms we get:

$$\dot{K}(t) = \left[1 - \Psi\left(\frac{X(t)}{F(t)}\right)\right] F(K(t), H) - c(t)H - \delta K(t) \tag{29}$$

Notice that in equilibrium $\omega^j(t) \equiv \frac{V^j(t)}{H \cdot a(t)}$; then, by (4), the warm-glow function is:

$$p(t) \equiv \sum_{j=1}^{J} \frac{\omega^j(t)a(t)}{V^j(t)} \bar{p}^j(t) = \sum_{j=1}^{J} \frac{\bar{p}^j\left(x^j(t)\right)}{H} = \frac{\bar{p}\left(X(t)\right)}{H} = \gamma \cdot \frac{X(t)}{H}$$

where the second equality follows from $\bar{p}^j(t)$ being linear in $x^j(t)$, and, hence, $\frac{\partial \bar{p}}{\partial X} = \gamma$. Finally, at the equilibrium, eq. (26) becomes:

$$\frac{u_p(t)}{u_c(t)} \frac{\partial \bar{p}(t)}{\partial X(t)} - (1 - s(t))\Psi'\left(\frac{X(t)}{F(t)}\right) - \tau^x(t) = 0 \tag{30}$$

Eqs. (28), (29) and (30) characterise the equilibrium at each date for $(k(t), c(t), x(t))$. From (30) we obtain:

$$X(t) = X(c(t), s(t), \tau^x(t), K(t)) \tag{31}$$

Total differentiation of (31) yields:

$$-\left(R\eta + T\frac{F}{K}\right)\frac{dX}{X} + \left(T\frac{F}{K}\theta\right)\frac{dK}{K} - (R\sigma)\frac{dc}{c} - \left(\frac{X}{K}\right)d\tau^x + \left(\Psi'\frac{X}{K}\right)ds = 0 \tag{31\#}$$

where

$\sum_{j=1}^{J}\left[1 - (1 - s(t))\Psi^j\left(\frac{x^j(t}{f^j(t)}\right)\right] f^j(k^j(t), l^j(t)) = \sum_{j=1}^{J}\left[1 - (1 - s(t))\Psi\left(\frac{x(t)}{f(t)}\right)\right] f(k(t), l(t)) = \left[1 - (1 - s(t))\Psi\left(\frac{X(t)}{F(t)}\right)\right] F(K(t), L(t))$, with $L(t) = H$.

$$T \equiv \left(\frac{X}{F}\right)^2 (1-s)\Psi'' > 0, R \equiv \frac{u_p}{u_c}\frac{\partial \bar{p}}{\partial X}\frac{X}{K} > 0, \eta \equiv \frac{u_{pp}}{u_p} p > 0, \theta \equiv \frac{F_K}{F} K > 0$$

Hence, from (31#) we get:

$$\frac{\partial X}{\partial K} = \frac{X}{K}\frac{T\frac{F}{K}\theta}{R\eta + T\frac{F}{K}} > 0, \frac{\partial X}{\partial c} = -\frac{X}{c}\frac{R\sigma}{R\eta + T\frac{F}{K}} < 0, \frac{\partial X}{\partial s} = \frac{\Psi'\frac{X}{K}X}{R\eta + T\frac{F}{K}} < 0, \frac{\partial X}{\partial \tau^x} = -\frac{\frac{X}{K}X}{R\eta + T\frac{F}{K}} < 0 \qquad (32\#)$$

By substituting (31) into (28) and (29), the dynamic system describing the equilibrium path of the economy boils down to the following two equations in $[c(t), K(t)]$:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma(c)}\left\{\left[1 - (1 - s(t))\Psi\left(\frac{X(t)}{F(t)}\right) + (1 - s(t))\Psi'\left(\frac{X(t)}{F(t)}\right)\frac{X(t)}{F(t)}\right] F_K(t) - \delta - \rho\right\}. \qquad (32)$$

$$\dot{K}(t) = \left[1 - \Psi\left(\frac{X(t)}{F(t)}\right)\right] F(K(t), H) - c(t)H - \delta K(t). \qquad (33)$$

By recognizing that

$$\frac{d\left(\frac{X}{F}\right)}{dK} = \frac{1}{F}\frac{dX}{dK} - \frac{X}{F}\frac{F_K}{F} = \frac{X}{K}\frac{T\theta\frac{1}{K}}{R\eta + T\frac{F}{K}} - \frac{X}{F}\frac{\theta}{K} = \frac{X}{F}\frac{\theta}{K}\left(\frac{R\eta}{R\eta + T\frac{F}{K}}\right)$$

$$\frac{d\left(\frac{X}{F}\right)}{dc} = \frac{1}{F}\frac{dX}{dc} = -\frac{X}{F}\frac{1}{c}\left(\frac{R\sigma}{R\eta + T\frac{F}{K}}\right)$$

the Jacobian matrix of system (32)-(33) can be written as:

$$J = \begin{bmatrix} -\frac{F_K T R}{R\eta + T\frac{F}{K}} & -\frac{c}{\sigma}\frac{F_K}{K}\left[T\theta\frac{R\eta}{R\eta + T\frac{F}{K}} + \left(1 - (1-s)\Psi + (1-s)\Psi'\frac{X}{F}\right)\beta\right] \\ \Psi'\frac{X}{c}\frac{R\sigma}{R\eta + T\frac{F}{K}} - H & \Psi'\theta\frac{X}{K}\frac{R\eta}{R\eta + T\frac{F}{K}} + (1-\Psi)F_K - \delta \end{bmatrix}$$

where $\beta \equiv -\frac{F_{KK}}{F_K}K$.

The following Proposition contains sufficient conditions on the tax instruments for our economy to display saddle-path stability.

Proposition 1: *Sufficient for saddle path stability of the economic system is:*

Either $\delta = 0$ *or*

$\delta > 0$

and $\tau^x \geq 0, \frac{1-\left(\Psi-\Psi'\frac{X}{F}\right)}{\Psi-\Psi'\frac{X}{F}}\frac{\rho}{\delta} \geq s \geq 0$

Proof: We recall that saddle-path stability requires $Tr(J) > 0$ and $Det(J) < 0$.

The trace of the Jacobian matrix can be written as:

$$Tr(J) = F_K\left[-\frac{T}{R\eta + T\frac{F}{K}}\left(R + \Psi'\frac{X}{K}\right) + \left(1 - \Psi - \Psi'\frac{X}{F}\right)\right] - \delta$$

Notice that by (30), $R + \Psi'\frac{X}{K} = -\frac{X}{K}(-s\Psi' + \tau^X)$, which is non-positive if $s, \tau^X \geq 0$. Under these assumptions, it follows that $Tr(J) \geq F_K\left(1 - \Psi - \Psi'\frac{X}{F}\right) - \delta$. Notice that, if $\delta = 0$, given that $F_K\left(1 - \Psi - \Psi'\frac{X}{F}\right) > 0$, then $Tr(J) > 0$. If $\delta > 0$, as standard in Ramsey-Cass-Koopmans models, dynamic efficiency requires $F_K\left(1 - \Psi - \Psi'\frac{X}{F}\right) - \delta > 0$. By (27) and (32) we get that, at the steady state, the inequality $F_K\left(1 - \Psi - \Psi'\frac{X}{F}\right) > \delta$ can be written as $\rho > s\left(\Psi - \Psi'\frac{X}{F}\right)F_K$, or $\frac{\rho}{\rho+\delta} > \frac{s\left(\Psi-\Psi'\frac{X}{F}\right)F_K}{\rho+\delta}$, that is $\frac{\rho}{\rho+\delta} > \frac{s\left(\Psi-\Psi'\frac{X}{F}\right)F_K}{\left(1-\Psi-\Psi'\frac{X}{F}\right)+s\left(\Psi-\Psi'\frac{X}{F}\right)}$ and, finally, $\frac{1-\left(\Psi-\Psi'\frac{X}{F}\right)}{\Psi-\Psi'\frac{X}{F}}\frac{\rho}{\delta} > s$. The latter inequality implies $Tr(J) > F_K\left(1 - \Psi - \Psi'\frac{X}{F}\right) - \delta > 0$.

Next, given that $Tr(J) = J_{11} + J_{22}$ and that $J_{11} < 0, Tr(J) > 0$ implies $J_{22} > 0$. Moreover, given that $Det(J) = J_{11}J_{22} - J_{12}J_{21}$ and that $J_{12} < 0, J_{21} < 0$ and $J_{22} > 0$, then the result $Det(J) < 0$ follows.

4. Tax reforms

In this section we carry out a comparative statics analysis to verify the effects of the tax instruments on the endogenous variables of the model, i.e. the scale of the economy, per-capita consumption, pollution and the pollution premium. In this exercise we assume that the reforms are carried out by keeping the public budget balanced, i.e. any tax change is financed by a corresponding change in individuals' lump sum tax z(t).

Total differentiation of system (31#), (32) and (33) provides:

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} dc \\ dK \end{bmatrix} = \begin{bmatrix} -\frac{c}{\sigma} F_K \left[\Psi - \Psi' \frac{X}{F} \frac{R\eta}{R\eta + T\frac{F}{K}} \right] ds + \frac{c}{\sigma} F_K \frac{X}{K} \frac{T}{R\eta + T\frac{F}{K}} d\tau^X \\ \frac{(\Psi')^2}{K} \frac{X^2}{R\eta + T\frac{F}{K}} ds - \frac{\Psi'}{K} \frac{X^2}{R\eta + T\frac{F}{K}} d\tau^X \end{bmatrix}$$

Hence, using Cramer's rule we get the following results:

Proposition 2: An increase of the tax on pollution reduces the capital installed in the economy, while an increase of the subsidy on abatement activity increases the capital installed.

Proof: As for the effect of the tax on pollution, by defining $A \equiv \frac{c}{\sigma} F_K \frac{X}{K} \frac{T}{R\eta + T\frac{F}{K}} > 0$ and $B \equiv -\frac{\Psi'}{K} \frac{X^2}{R\eta + T\frac{F}{K}} > 0$, and given that $J_{11} < 0$ and $J_{21} < 0$, Cramer's rule provides the following result:

$$|J| \frac{dK}{d\tau^X} = \begin{vmatrix} J_{11} & A \\ J_{21} & B \end{vmatrix} = J_{11}B - AJ_{21} > 0$$

Given that $|J| < 0$, it follows that $\frac{dK}{d\tau^X} < 0$.

As for the effect of the subsidy on abatement activity, by defining $a \equiv -\frac{c}{\sigma} F_K \left[\Psi - \Psi' \frac{X}{F} \frac{R\eta}{R\eta + T\frac{F}{K}} \right] < 0$ and $b \equiv \frac{(\Psi')^2}{K} \frac{X^2}{R\eta + T\frac{F}{K}} > 0$, and given that $J_{11} < 0$ and $J_{21} < 0$, Cramer's rule provides the following:

$$|J|\frac{dK}{ds}=\begin{vmatrix}J_{11} & a\\ J_{21} & b\end{vmatrix}=J_{11}b-aJ_{21}<0$$

Given that $|J|<0$, it follows that $\frac{dK}{ds}>0$.

The economic rationale behind the results is the following

The tax on pollution corresponds to a tax on profits, so that it reduces the marginal productivity of capital, so that firms will reduce the capital invested. On the contrary, the subsidy on abatement reduces the production costs, so that will provide an incentive to increase the scale of the firm.

As for the effects on pollution, we can summarise the results in the following proposition:

Proposition 3: An increase of the tax on pollution reduces pollution, while an increase of the subsidy on abatement activity can either increase or reduce pollution. However, when $\tau^X=0$, sufficient for $\frac{dX}{ds}<0$ is $-\frac{\frac{\Psi'' X}{\Psi' F}\Psi}{1-\Psi+\Psi'\frac{X}{F}}\leq\frac{\beta}{\theta}$.

Proof:

As for the effect of τ^X on pollution, by eq. (31#) it follows that

$$\frac{dX}{d\tau^X}=\frac{X}{K}\frac{1}{R\eta+T\frac{F}{K}}\left(T\frac{F}{K}\theta\frac{dK}{d\tau^X}-R\frac{\sigma}{c}K\frac{dc}{d\tau^X}-X\right)$$

and, by Cramer's rule

$$|J|\frac{dX}{d\tau^X}=\frac{X}{K}\frac{1}{R\eta+T\frac{F}{K}}\left[T\frac{F}{K}\theta(J_{11}B-AJ_{21})-R\frac{\sigma}{c}K(J_{22}A-BJ_{12})-X(J_{11}J_{22}-J_{12}J_{21})\right]$$

By expanding J_{11}, J_{21}, A, B and J_{22} and collecting terms we get:

$$|J|\frac{dX}{d\tau^X}=\frac{X}{K}\frac{1}{R\eta+T\frac{F}{K}}\left[T\frac{F}{K}\theta H\frac{\sigma}{c}F_K\frac{X}{K}\frac{T}{R\eta+T\frac{F}{K}}-XHJ_{12}\right]>0$$

so that $\frac{dX}{d\tau^X}<0$.

As for the effect of s on pollution, by exploiting eq. (31) it follows that

$$|J|\frac{dX}{ds}=|J|\frac{\partial X}{\partial K}\frac{dK}{ds}+|J|\frac{\partial X}{\partial c}\frac{dc}{ds}+|J|\frac{\partial X}{\partial s}$$

Applying (32#), the expression above can be written as:

$$\left(R\eta+T\frac{F}{K}\right)|J|\frac{dX}{ds}=T\frac{X}{K}\frac{F\theta}{K}|J|\frac{dK}{ds}-\frac{X}{c}R\sigma|J|\frac{dc}{ds}+\Psi'\frac{X^2}{K}|J|$$

Using Cramer's rule for $|J|\frac{dK}{ds}$ and $|J|\frac{dc}{ds}$, we obtain:

$$\left(R\eta+T\frac{F}{K}\right)\frac{|J|}{X}\frac{dX}{ds}=\frac{T}{K}\frac{F\theta}{K}(J_{11}b-aJ_{21})-\frac{R\sigma}{c}(aJ_{22}-bJ_{12})+\Psi'\frac{X}{K}(J_{11}J_{22}-J_{12}J_{21})$$

After some manipulation and collecting terms the above equation can be written as:

$$\left(R\eta+T\frac{F}{K}\right)\frac{|J|}{X}\frac{dX}{ds}=\frac{T}{K}\frac{F\theta}{K}(J_{11}b-aJ_{21})+J_{22}RF_K\left(\Psi-\Psi'\frac{X}{F}\right)+H\Psi'\frac{X}{K}J_{12}$$

The term $(J_{11}b-aJ_{21})$ turns out to be equal to $F_K\frac{c}{\sigma}\left(\Psi-\Psi'\frac{X}{F}\right)\Psi'\frac{X}{c}\frac{R\sigma}{R+T\frac{F}{K}}-HF_K\frac{c}{\sigma}\left(\Psi-\Psi'\frac{X}{F}\frac{R\eta}{R+T\frac{F}{K}}\right)$. Morover, recognizing that, by eqs. (27) and (28), $J_{12}=T\theta\frac{R\eta}{R\eta+T\frac{F}{K}}+\left(1-(1-s)\Psi+(1-s)\Psi'\frac{X}{F}\right)\beta=T\theta\frac{R\eta}{R\eta+T\frac{F}{K}}+\frac{\rho+\delta}{F_K}\beta$ the equation above is:

$$\left(R\eta+T\frac{F}{K}\right)\frac{|J|}{X}\frac{dX}{ds}=J_{22}RF_K\left(\Psi-\Psi'\frac{X}{F}\right)-\frac{c}{\sigma}HF_K\Psi'\frac{X}{K}\frac{F_K}{K}\frac{\rho+\delta}{F_K}\beta+\left[\left(\Psi-\Psi'\frac{X}{F}\right)\Psi'\frac{X}{c}\frac{R\sigma}{R\eta+T\frac{F}{K}}-H\Psi\right]\frac{T}{K}\frac{F\theta F_K}{K}\frac{c}{\sigma}$$

Given that $J_{22}RF_K\left(\Psi - \Psi'\frac{X}{F}\right) > 0$, $-\frac{c}{\sigma}HF_K\Psi'\frac{X}{K}\frac{F_K}{K}\frac{\rho+\delta}{F_K}\beta > 0$ and $\left[\left(\Psi - \Psi'\frac{X}{F}\right)\Psi'\frac{X}{c}\frac{R\sigma}{R\eta+T\frac{F}{K}} - H\Psi\right]\frac{T}{K}\frac{F\theta F_K}{K}\frac{c}{\sigma} < 0$, sufficient for the RHS of the above equation to be positive is $T < \frac{J_{22}R\left(\Psi-\Psi'\frac{X}{F}\right) - \frac{c}{\sigma}HX\Psi'\frac{X}{K^2}\frac{\rho+\delta}{F_K}\beta}{\left[-\left(\Psi-\Psi'\frac{X}{F}\right)c\Psi'\frac{R}{R\eta+T\frac{F}{K}}+\frac{cH\Psi}{\sigma}\right]\frac{F_K}{K}}$.

Recognizing that $J_{22} = \Psi'\theta\frac{X}{K}\frac{R\eta}{R\eta+T\frac{F}{K}} + (1-\Psi)F_K - \delta$ and after some manipulation, the above inequality can be written as:

$$T < \frac{\sigma}{\Psi}\frac{K}{F_K}R\left(\Psi - \Psi'\frac{X}{F}\right)\frac{[(1-\Psi)F_K-\delta]}{Hc} - \frac{\Psi'}{\Psi}\frac{X}{F}\frac{\rho+\delta}{F_K}\frac{\beta}{\theta}.$$

Finally, exploiting the steady state relations $cH = (1-\Psi)F - \delta K = F\left(1 - \Psi - \frac{\theta}{F_K}\delta\right)$, $R = -\frac{X}{K}(-s\Psi' + \tau^X) - \Psi'\frac{X}{K}$, $\frac{\rho+\delta}{F_K} = 1 - \Psi + \Psi'\frac{X}{F} + s(\Psi - \Psi'\frac{X}{F})$ and setting $\tau^X = 0$, the above inequality has the following form:

$$T < -\frac{\Psi'}{\Psi}\frac{X}{F}\left\{\sigma(1-s)\left(\Psi - \Psi'\frac{X}{F}\right)\frac{1-\Psi+\Psi'\frac{X}{F}-\frac{\delta}{F_K}}{1-\Psi-\frac{\theta}{F_K}\delta} + \left[1 - \Psi + \Psi'\frac{X}{F} + s\left(\Psi - \Psi'\frac{X}{F}\right)\right]\frac{\beta}{\theta}\right\}.$$

Given that $\sigma(1-s)\left(\Psi - \Psi'\frac{X}{F}\right)\frac{1-\Psi+\Psi'\frac{X}{F}-\frac{\delta}{F_K}}{1-\Psi-\frac{\theta}{F_K}\delta} > 0$, sufficient for the above inequality is $T \equiv \left(\frac{X}{F}\right)^2(1-s)\Psi'' < -\frac{\Psi'}{\Psi}\frac{X}{F}\left[1 - \Psi + \Psi'\frac{X}{F} + s\left(\Psi - \Psi'\frac{X}{F}\right)\right]\frac{\beta}{\theta}$

that is,

$$\frac{\beta}{\theta} \geq (1-s)\left[\frac{\beta}{\theta}\left(1 - \Psi + \Psi'\frac{X}{F}\right) - \frac{\Psi''}{\Psi'}\frac{X}{F}\Psi\right]$$

or $\frac{\beta}{\theta} \geq \frac{-\frac{\Psi''}{\Psi'}\frac{X}{F}\Psi}{1-\Psi+\Psi'\frac{X}{F}}$.

The economic rationale of the results is the following: as for the pollution tax, as expected, an increase of the latter will reduce pollution. As for the subsidy, the latter exerts two opposite effects on pollution: on one hand, by increasing the installed capital, it will also increase production and, thus, pollution (which is a by-product of production); on the other hand, the it tends to increase the resources that the firms devote to abatement, thus reducing pollution. The final result depends on the relative strength of each effect.

Proposition 4: An increase of the tax on pollution reduces per-capita consumption, while an increase of the subsidy on abatement activity can either increase or reduce per-capita consumption.

Proof: As for the effect of the tax on pollution, given that $A, B, J_{22}, > 0$ and $J_{12} < 0$, Cramer's rule yields:

$$|J|\frac{dc}{d\tau^X} = \begin{vmatrix} A & J_{12} \\ B & J_{22} \end{vmatrix} = AJ_{22} - BJ_{12} > 0$$

Given that $|J| < 0$, it follows that $\frac{dc}{d\tau^X} < 0$.

As for the effect of the subsidy on abatement activity, by defining given that $J_{11} < 0$ and $J_{21} < 0$, Cramer's rule provides the following:

$$|J|\frac{dc}{ds} = \begin{vmatrix} a & J_{12} \\ b & J_{22} \end{vmatrix} = aJ_{22} - bJ_{12}$$

whose sign is ambiguous.

We can summarise the economic rationale of the results as follows: recall that steady state consumption, by eq. (33) is proportional to net-of-abatement product. Given that an increase of the tax on pollution reduces net-of-abatement production, it follows that also consumption decreases. On the other hand, the subsidy can either increase or decrease net-of-production, given that, it exerts two opposite effects: on one hand, by increasing the installed capital, it will also increase production and, thus, consumption; on the other hand, the it tends to increase the resources that the firms devote to abatement, thus net production and, thus, consumption. The final result depends on the relative strength of each effect.

We now turn to the effects of the fiscal instruments on the pollution premium, which, by eq. (13), at the steady state, is:

$$R \equiv [r_e^j - r] = \frac{u_p}{u_c}\gamma\frac{X}{K} \tag{34}$$

The following Proposition summarises the results:

Proposition 4: An increase of the tax on pollution reduces the pollution premium, while an increase of the subsidy on abatement activity can increase or decrease the pollution premium. when $\tau^X = 0$, sufficient to have a negative effect is $-\frac{\Psi''}{\Psi'}\frac{X}{F} \leq 1$.

Proof: As for the effect of the tax on pollution, by total differentiation of logs of (34) we get:

$$\frac{dR}{R} = p\ \frac{u_{pp}}{u_p}\frac{dp}{p} - \frac{u_{cc}}{u_c}c\frac{dc}{c} + \frac{dX}{X} - \frac{dK}{K} = (1+\eta)\frac{dX}{X} + \sigma\frac{dc}{c} - \frac{dK}{K} \tag{35}$$

Taking (31#) and collecting terms we get:

$$\frac{\left(R\eta + T\frac{F}{K}\right)}{(1+\eta)}\frac{dR}{R} = \left(T\frac{F}{K}\frac{\theta}{K}\right)dK + \left(T\frac{F}{K} - R\right)\frac{\sigma}{c(1+\eta)}\ dc - \left(\frac{X}{K}\right)d\tau^x + \left(\Psi'\frac{X}{K}\right)ds = 0 \tag{36}$$

Focusing on τ^x, exploiting the results of previous Propositions, we can write:

$$|J|\frac{\left(R\eta+T\frac{F}{K}\right)}{(1+\eta)R}\frac{dR}{d\tau^{x}}=\left(T\frac{F}{K}\frac{\theta}{K}\right)(J_{11}B-AJ_{21})+\left(T\frac{F}{K}-R\right)\frac{\sigma}{c(1+\eta)}(AJ_{22}-BJ_{12})-\frac{X}{K}(J_{11}J_{22}-J_{12}J_{21})$$

After some manipulation and collecting terms we can write

$$|J|\frac{\left(R\eta+T\frac{F}{K}\right)}{(1+\eta)R}\frac{dR}{d\tau^{x}}=\left(T\frac{F}{K}\frac{\theta}{K}\right)H\frac{c}{\sigma}F_{K}\frac{X}{K}\frac{T}{R\eta+T\frac{F}{K}}+\frac{T}{1+\eta}\frac{X}{K}F_{K}J_{22}+\frac{\Psi^{'}X}{(1+\eta)}\frac{\sigma}{c}\frac{X}{K}J_{12}-\frac{X}{K}HJ_{12}>0$$

so that $\frac{dR}{d\tau^{x}}<0$.

As for the subsidy on pollution abatement, from (35) and (31#) we know that

$$\frac{dR}{R}=(1+\eta)\frac{dX}{X}+\sigma\frac{dc}{c}-\frac{dK}{K}=\left[\frac{(1+\eta)}{X}\frac{\partial X}{\partial K}-\frac{1}{K}\right]dK+\left[\frac{(1+\eta)}{X}\frac{\partial X}{\partial c}+\frac{\sigma}{c}\right]dc+\frac{(1+\eta)}{X}\frac{\partial X}{\partial s}ds \qquad (35)$$

Substituting from (32#) and rearranging terms we get

$$\frac{1}{R}\frac{dR}{ds}=\frac{[(1+\eta)\theta-1]T\frac{F}{K}-R\eta}{R\eta+T\frac{F}{K}}\frac{dK}{ds}+\frac{\sigma}{c}\frac{T\frac{F}{K}-R\eta}{R\eta+T\frac{F}{K}}\frac{dc}{ds}+\frac{(1+\eta)\Psi'}{R\eta+T\frac{F}{K}}\frac{X}{K}$$

By exploiting previous results on $|J|\frac{dK}{ds}$ and $|J|\frac{dc}{ds}$ we can write the following

$$|J|\frac{\left(R\eta+T\frac{F}{K}\right)}{R}\frac{dR}{ds}=\left\{[(1+\eta)\theta-1]T\frac{F}{K}-R\eta\right\}(J_{11}b-Aa)+\frac{\sigma}{c}\left(T\frac{F}{K}-R\eta\right)(aJ_{22}-bJ_{12})+(1+\eta)\Psi'\frac{X}{K}(J_{11}J_{22}-J_{12}J_{21})$$

which can also be written as:

$$|J|\frac{\left(R\eta+T\frac{F}{K}\right)}{R}\frac{dR}{ds}=\left\{[(1+\eta)\theta-1]T\frac{F}{K}-R\eta\right\}(J_{11}b-Aa)+\left[a\frac{\sigma}{c}T\frac{F}{K}-\frac{\sigma}{c}Ra+(1+\eta)\Psi'\frac{X}{K}J_{11}\right]J_{22}$$
$$+\left[-\frac{\sigma}{c}\left(T\frac{F}{K}-R\eta\right)b-(1+\eta)\Psi'\frac{X}{K}J_{21}\right]J_{12}$$

Recognizing that $\Psi'\frac{X}{K}J_{11}-\frac{\sigma}{c}Ra=RF_{K}\left(\Psi-\Psi^{'}\frac{X}{F}\right)$ and that $\frac{\sigma}{c}b=\Psi'\frac{X}{K}J_{21}+\frac{H}{R}\Psi'\frac{X}{K}$, the above equation becomes:

$$|J|\frac{\left(R\eta+T\frac{F}{K}\right)}{R}\frac{dR}{ds}=\left\{[(1+\eta)\theta-1]T\frac{F}{K}-R\eta\right\}(J_{11}b-Aa)+\left(T\frac{F}{K}+R\eta\right)\frac{\Psi'}{R}\frac{X}{K}J_{11}J_{22}-\left(T\frac{F}{K}+R\eta\right)\frac{\Psi'}{R}\frac{X}{K}J_{21}J_{12}$$
$$-\left(T\frac{F}{K}-R\right)F_{K}\left(\Psi-\Psi'\frac{X}{F}\right)J_{22}-\frac{\Psi'}{R}\frac{X}{K}H\left(T\frac{F}{K}-R\right)J_{12}$$

that is

$$|J|\frac{\left(R\eta+T\frac{F}{K}\right)}{R}\frac{dR}{ds}=\left\{[(1+\eta)\theta-1]T\frac{F}{K}-R\eta\right\}(J_{11}b-Aa)+\left(T\frac{F}{K}+R\eta\right)\frac{\Psi'}{R}\frac{X}{K}|J|-\left(T\frac{F}{K}-R\right)\left[\frac{\Psi'}{R}\frac{X}{K}HJ_{12}+F_K\left(\Psi-\Psi'\frac{X}{F}\right)J_{22}\right]$$

Hence, given that $(J_{11}b-Aa)<0$, sufficient for the RHS to be positive (i.e. $\frac{dR}{ds}<0$) is $T\frac{F}{K}-R\leq 0$ and $[(1+\eta)\theta-1]T\frac{F}{K}-R\eta\leq 0$. However, if $T\frac{F}{K}-R\leq 0$, then also $R\eta+T\frac{F}{K}\leq(1+\eta)T\frac{F}{K}\leq\theta(1+\eta)T\frac{F}{K}$, given that $\theta<1$, so that also the inequality $[(1+\eta)\theta-1]T\frac{F}{K}-R\eta\leq 0$ holds true. Hence, sufficient for $\frac{dR}{ds}<0$ is $T\frac{F}{K}-R\geq 0$, which, by eq. (30), with $\tau^X=0$, reads as $-\frac{\Psi''}{\Psi'}\frac{X}{F}\leq 1$.

5. Conclusions

In this paper we analysed the effects that fiscal instruments, aimed at reducing pollution, can exert on the scale of the economy, on pollution and on the pollution premium. In particular, we compared two different instruments: a tax on pollution and a subsidy on abatement activity.

We found that the former, besides reducing pollution, depresses also per capita consumption and the capital installed in the economy. As for the subsidy, rather interestingly, we found that it increases the capital installed and may, under certain conditions concerning the shape of the abatement technology, decrease pollution and increase per-capita consumption.

Some policy implications follow: in an economy populated by socially responsible investors, pollution abatement, a goal which is on the political agenda of most developed countries and international organizations, is not necessarily at odds with economic performance, albeit the adoption of the fiscal instruments to be used in order to avoid the possible trade-off between environmental quality and the scale of the economy is conditioned on the characteristics of the abatement technology, which is an empirical matter.

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