Prices, schedules, and passenger welfare in multi-service transportation systems

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Abstract

We consider a multi-service transportation system in which passengers are heterogeneous along two dimensions, namely ideal departure time and value of time, leading to both horizontal and vertical differentiation. We investigate the behaviour of passengers, and assess how service pricing and scheduling affect their travel choices and welfare. We show that this depends, first, on whether passengers are uninformed or informed about the timetable of services, supplied at different prices, upon arrival at the station. Besides, given the information passengers hold, it also depends on their (individual-specific) value of time. The market segmentation results accordingly, and is found to be finer, in general, when passengers are informed. Our analysis offers policy-makers a scientifically founded tool to make sensible decisions, based on the exact identification of those who would gain and those who would lose from policy changes. The analysis further highlights the potential benefits of information, and points to the importance of facilitating information accessibility to passengers.

Keywords: Travel demand; service scheduling; market segmentation; targeted policy-making; impact of information

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1 Introduction

Travel behaviour is complex and multi-faceted. Most of travel demand has a derived nature and its determinants are many, with price being only one of those. Whereas some of the determinants are unrelated to transport decision-making, others result directly from policy-makers' and transport operators' choices. When appraising the performance of a (passenger) transportation system, it is of primary importance to look beyond the sole prices and also consider these other determinants.

Since the first attempts to measure travel demand, which we owe to McFadden [19], it has been recognized that two major determinants are (on-vehicle) travel time and service scheduling. Whereas travel time is essentially related to network characteristics and technological choices, and can thus be regarded as a given attribute, at least to some extent, service scheduling (hence, frequency) reflects operators' and/or policy-makers' decisions nearly in the same fashion as price. Therefore, it looks natural to examine the impact of *both* price and scheduling on passenger choices and welfare.

This task is complicated by the fact that, in a number of instances, passengers are not bound to purchase services from a single provider, and can rather swing across operators to their best convenience. One obvious reason for this is that many transportation sectors are (imperfectly) competitive markets. In any case, the choice of a given service depends, first, on how the prices differ over time and, possibly, across operators; second, on how the services, possibly supplied at different prices, are scheduled over time.

The goal of our paper is to investigate the behaviour of passengers in multi-service transportation systems, and assess how price and schedule (or frequency) of the available services affect their choices and welfare. This analysis further permits to draw policy insights, particularly in light of distributional considerations.

Given that the sequence of differently priced services matters for travel decisions, the exact time at which passengers reach the station/stop of departures is relevant as well. This, in turn, depends on the information passengers hold about the schedule of available services. To account for this, we rely on a comprehensive analytical framework, which enables us to represent two types of passengers, namely uninformed and informed. Uninformed passengers do not know the schedule of services before reaching the station. More precisely, they know neither at what time services will depart nor whether each of them is cheap or expensive. Hence, once they are in the station, they can only choose between taking the next departing service and waiting for a subsequent service. This latter option is attractive only if the subsequent service happens to be sufficiently cheap that the price saving will be worth the waiting time. By contrast, informed passengers know both how services are scheduled and at what price each of them is supplied, and decide in advance what exact service they will take. Accordingly, they adjust their arrival time at the station and avoid waiting. In other words, information grants flexibility to travel choices. However, even the most suitable service, among the available ones, may not depart at the their ideal time. A shift in departure time harms the concerned passengers. In either case, individual choices will reflect a trade-off between willingness to pay and willingness to shift departure, which in turn depends on the individual value of time.

Far from adding gratuitous complexity to the model, the introduction of an individual-specific value of time, on top of an individual-specific ideal departure time, is paramount to the understanding of passenger behaviour and the derivation of policy insights. First, as already pinpointed by Small and Yan [21], if passengers were all identical, then the consequences of multiple services being supplied, which enables people to indulge their varying preferences, would remain concealed. Second, it is now well established in economics that the value of time is positively related to the individual income. To illustrate, using data from a natural experiment, in which motorists are required to choose between waiting in a queue for purchasing low-price gasoline and purchasing at a higher price without waiting, Deacon and Sonstelie [10] obtain estimates of the value of time which, in most cases, are similar to individuals' after-tax wages. Accordingly, the value of time in a transportation market is a reasonably accurate indicator of passenger earnings. In addition, it is also an important determinant of passenger choices, as it reflects the opportunity cost of waiting or, more generally, shifting departure.[1]

Being based on this description, one might believe that making an assessment of the performance of a multi-service transportation system, as here considered, is a simple exercise. In fact, it is a complex problem, for two critical reasons. First, whereas our model is purposely silent about the specific industry structure, the demand for transportation services is made endogenous. Indeed, it is taken to depend on the characteristics of the supply of services, particularly prices and schedules. Second, the assessment is not based on a set of pre-defined criteria, as is the case, for instance, when it is assumed that an ideal system is one in which the difference between actual and ideal departure time is minimized over the whole population of passengers. Rather, in this work, the assessment is made through the very lens of those who use the transportation system, with the acknowledgement that the associated benefits do not need be alike across users. More specifically, we account for the fact that (*i*) the ideal departure time varies across users, in general; (*ii*) not only monetary costs but also time costs are relevant; and (*iii*) the trade-off between monetary costs and time costs, which is ordinarily rooted in the individual earnings, is all the more heterogeneous that income inequalities are becoming more pronounced in the population.

We find that low-value-of-time (low-income) passengers tend to privilege relatively

[1]See Wardman [23] for a report on research about the evaluations of values of time in the use of public transport.

cheap services. By contrast, high-value-of-time (high-income) passengers tend to attach more importance to the timetable of services than to their prices. Although this is a natural result, there are a few important implications coming to the forefront of our study. First, changes in price and/or schedule have a different impact on the various passengers in the population, depending on their value of time, which ultimately mirrors their economic conditions. Even budget-neutral changes, like an increase in service frequency, as coupled with an increase in price that compensates for the resulting cost increase, may appear to be welfare-enhancing for some individuals and welfare-degrading for others. Second, not only does the value of time represent a key to identify 'winners' and 'losers' of specific policies. It also provides a tool to appraise their differentiated impact. Third, a more accurate appraisal can be made as passengers are better informed on the available travel options. In substance, by delivering a fine segmentation of the population of passengers, which permits to discern winners from losers, our detailed model of travel demand in a multi-service transportation system offers policy-makers a scientifically founded tool to make sensible decisions.

There is also a parallel lesson, concerning the value of information, to be retained from our analysis. By allowing for more flexible transport choices, information enhances the matching between heterogeneous passengers and differently priced services. In addition, by permitting a more accurate appraisal of the impact of policy decisions on passenger welfare, information can also help public decision-makers fine-tune service supply and targeted redistribution strategies. These conclusions have implications for the design and implementation of devices which could facilitate information acquisition by passengers.

1.1 Related literature

Our paper is first related to the domain of literature on modal and/or service choice in transportation systems. To investigate how individuals accrue to services, we inspire ourselves to the approach developed in the studies on traffic flow predictions (such as Leurent [18]). According to that approach, passengers choose the option minimising the generalised cost of the trip, which depends on the monetary price and some individual characteristics (value of time and ideal departure time, in our setting).[2] A similar approach is followed by Yang *et al.* [24] to study the effect of time value distributions on price and frequency competition among three transportation modes (low-quality bus, high-quality bus, and private car). Our work diverges from theirs in that rather than representing vertically differentiated modes, we consider horizontally differentiated services, which are tantamount to varieties of a single product ranked by passengers according to their ideal departure times, whereas vertical differentiation is rooted in the heterogeneous

[2] A more general theory of consumer behaviour, as designed to address economic problems with relevant time dimensions, is provided by DeSerpa [11].

value of time in our model. In addition to determining the allocation of passengers across services for given prices and schedule, we also study how changes in price and frequency of the available services affect passenger surplus, mirroring the individual trade-off between willingness to pay and willingness to depart at the favourite time. In so doing, we highlight how the distribution of demand over the course of the reference time interval (say, the day) responds to both explicit price changes and the price changes implicit in the variation of departure. This approach brings our model somewhat closer to queuing models, also applied to other sectors such as telecommunications.

The impact of service (re)scheduling is also considered by Jiang *et al.* [16]. Particularly, they focus on how to schedule passenger trains in a highly congested railway in such a way as to meet the increased demand for transportation services as new users add up. We follow a somewhat different approach and wash out any 'volume' effects, abstracting from any change in the number of passengers that comes along with a change in the time interval between subsequent services. This enables us to restrict attention to how those passengers, who are already using the services, react to policy changes in terms of both allocation across services and quantity of demanded travels.

Cantos-Sànchez and Moner-Colonques [7] model a mixed duopoly in which two transportation modes (bus and train) are offered, each characterized by both a quality attribute and departure frequency. In that setting, in addition to quality determining a unanimous ranking of modes in the population of users in the same vein as in Yang *et al.* [24], frequency also introduces an element of horizontal differentiation, capturing the multiplicity of supplied products. This strikes a more pronounced similarity with our model. As an additional ingredient, we also consider that passengers are either uninformed or informed, and highlight the impact of information on the resulting market segmentation. In particular, we show the consequences associated with the possibility for informed passengers of anticipating departure (in addition to postponing it), relative to their ideal departure time, in order to take advantage of a lower price. The possibility of passengers anticipating actions related to their trips is also considered by Koster *et al.* [17] with regards to air transportation. However, what these authors exactly account for is the choice of passengers to anticipate departure from their place in order to reach the airport in due time. Whereas this focus is functional to their goal, which is to assess the cost of access travel time variability for air passengers through the analysis of the determinants of the preferred arrival times at airports, we are concerned with access to transportation services themselves.

In transport economics the dichotomy between uninformed and informed passengers has long been associated with the following two situations: passengers prefer to go to urban transport stops without consulting the timetable of service departures; they prefer to plan long-distance journeys according to the timetable. After being explored separately in earlier studies, these two situations are first analysed in a unified framework by Jansson [15], who concludes that the choice of passengers to acquire or not information depends on service frequency. That is, passengers do not acquire information on the timetable in systems with frequent services, such as urban transport; they do so, instead, in systems with infrequent services, such as long-distance transport.[3] Although we also offer a comprehensive model, which permits to represent uninformed and informed passengers within the same analytical framework, we rather attempt to pinpoint the effects of the (lack of) information on passenger behaviour, being suggestive of the role that devices for information diffusion could play to enhance the performance of multi-service transportation systems. In this respect, our work is in line with recent studies assessing the effects of information disclosure on market outcomes. Among those, being based on a field experiment on the wholesale market for used cars, Tadelis and Zettelmeyer [22] conclude that information disclosure leads to a better matching of vertically differentiated cars with heterogeneous buyers. Whereas information helps to solve a classical lemon market problem in that study, the finding that it improves on the commodity-customer matching is akin to our work.

There are a number of studies on welfare effects in multi-service transportation systems in which price and scheduling are the relevant policy dimensions.[4] In general, such studies rest on specific hypotheses about the industry structure. This is the case of many of the works previously referred to, but also of several other papers, such as Yang and Zhang [25], who focus on competition between air transport and high-speed rail. Diverging from that approach, we avoid imposing assumptions on the supply side of the market, and only require the transportation system to include several services offered at possibly different prices. This rests on the implicit idea that a multiplicity of differently priced alternatives (services), among which customers can choose, is consistent with two or more operators being active in the market. To the extent that the richness of the set of customer choices proxies competition in supply, the analysis can be developed through the lens of customers only, with an important benefit. The effects of policy choices on passenger welfare so identified, and the distributional considerations drawn thereof, are not restricted to hold within a given industry structure, and might rather be indicative of what a desirable industry structure would look like.

Of course, as most of the papers recalled so far, also our study is related to the wide literature on product differentiation. The origins of that literature are rather distant in the past. Horizontal (or spatial) differentiation is first studied by Hotelling [14]; in a

[3]Informed passengers are identified as those engaging in long-distance journeys also in a more recent work by Abrantes and Wardman [1].

[4]More generally, there is now a rich literature on the optimal pricing and frequencies of transportation services. In addition to the aforementioned paper by Jansson [15], this includes, for instance, Börjesson *et al.* [4] who derive the optimal pricing and frequencies for buses in Stockholm, showing how they depend on the congestion charges levied on the corridors leading into the city.

later stage, D'Aspremont *et al.* [9] provide clarifications on basic theoretical aspects. The equilibria of the basic vertical differentiation model are characterized by Gabszewicz and Thisse [12] and Shaked and Sutton [20]. In the specification adopted in these studies, and in several papers thereafter, the product varieties (or quality attributes) are bound to two. By contrast, in our model the service schedule includes more than two departures. In this respect, our work is more akin to Barigozzi and Ma [3], who model vertical differentiation with an arbitrary number of quality attributes.[5] Besides, in many models of product differentiation it is assumed that customers have only one unit of consumption to allocate so that the individual problem boils down to selecting one variety/quality. We also admit a quantity dimension, letting passengers both pick the variety (service) to be used and express demand for the number of travels to be made. This enables us to identify the effects of policy changes on the distribution of passengers across services and, in addition, on the amount of demanded travels for each service.

Lastly, there is a flourishing strand of studies on the distributional effects of transport policies and their use for (re)distribution purposes, including poverty alleviation.[6] Among those studies, Bureau and Glachant [5] use data in the Paris Region from the Global Transport Survey 2001-2002 to conclude that low-income individuals benefit more from a reduction in fares than from an increase in the speed of urban public transport. Rather than focusing on how variations in fare and speed affect the well-being of the various income groups, we explore the welfare effects of variations in service price and frequency, looking at the market segmentation that results from time value heterogeneity.

1.2 Outline

The remainder of the paper is organized as follows. In section 2 we describe the problem to be analysed, presenting the key ingredients of the model and the methodology to be followed. In section 3 and 4 we respectively investigate how uninformed and informed passengers decide what service to take, characterizing the resulting market segmentation for a given price pattern. We further explore the impact on passenger welfare of marginal changes in price and in schedule. In section 5 we discuss how one can make use of our

[5] The juxtaposition of our model, which is interpretable as one of both horizontal and vertical differentiation, and the model of pure vertical differentiation proposed by Barigozzi and Ma [3] does not look inconsistent, if it is considered that the horizontal differentiation model *à la* Hotelling is a special case of the vertical differentiation model, as shown by Cremer and Thisse [8]. It must nonetheless be acknowledged that there may be differences between the two kinds of models, due to the specification of the strategy sets available to firms (more generally, to decision-makers). In particular, just as firms' positions in location models, service scheduling in transportation models like ours may face more restrictions than do quality attributes in vertically differentiated models.

[6] For an analysis of transportation policies as a tool for poverty alleviation see Gannon and Liu [13], for instance. Less related to ours, yet worth mentioning, are the studies on direct redistribution policies in transport sectors. For instance, Adler and Cetin [2] discuss redistribution through toll collection from drivers on a more desirable route and subsidization of drivers on a less desirable route.

methodology to derive policy insights. In section 6 we are based on our results to provide a clue on the value of information in multi-service transportation systems. Section 7 briefly concludes. Lengthy calculations, including for alternative price patterns, are relegated to an appendix.

2 Description of the problem

Transportation system We consider an origin-destination pair over which several transportation services are available. A given service $s \in S$ differs from the others in its departure time $t \in H$ and, possibly, in its price $p \in \mathbb{R}_+$. The set H of departure times is a time interval within the day; transportation operations are restricted to take place within that interval. We assume that there is a discontinuity in services from one day to the next one, say, for maintenance reasons, due to legal restrictions on the working time, or simply because demand is very low at night. Under this assumption, it is legitimate to focus on the scheduling problem within a single day.

Passengers Passengers are heterogeneous along two dimensions. The first is the ideal departure time $t^* \in \hat{H}$, which we take to be uniformly distributed with a density of $n > 0$. The second is a marginal disutility of $\tau \in \mathbb{R}_+$, which is incurred if the actual departure time $t \in H$ differs from t^*, and has cumulative distribution function $G(\tau)$ such that $(dG(\tau)/d\tau) = g(\tau)$. The set $\hat{H}$ of ideal departure times does *not* need coincide with the set H of actual departure times. We nonetheless assume that all passengers with ideal departure time $t^* \in \hat{H}$ end up using a service scheduled at some time $t \in H$ (alternatively, they do not travel at all).[7] Therefore, there will be no leftover demand at the end of the day.

Choice of service Each passenger decides what service to use given the supply S, taking into account, on the one hand, her ideal departure time and, on the other, the actual departure time and the price of the available services. Formally, the choice is made in such a way as to maximize the individual net surplus

$$b(x, \tilde{p}) = u(x) - \tilde{p}x. \tag{1}$$

[7]The fact that passengers care about the time at which they begin - rather than complete - their trips is plausible in various instances. To illustrate, one can think of the trips made by commuters and shoppers in the evening to return home. Moreover, the fact that the travel time is not represented reflects the (implicit) assumption that all services take the same time to reach the destination point, the in-vehicle time being essentially related to network and technological features. In light of previous empirical findings, this restriction does not seem to be a severe one. For instance, using stated preference models to estimate the value that commuters are willing to pay to save on travel time, Calfee and Winston [6] find that even high-income commuters attach a very low value to travel time savings, and rather adjust other choices, such as the departure time.

In (1) $u(x)$ denotes the gross utility derived from a total of x travels, which is increasing ($u'(x) > 0$) and concave ($u''(x) < 0$) in its argument. Furthermore, $\tilde{p} = p + \tau \|t^* - t\|$ denotes the so-called *generalised price*, including both the monetary price p, which is the same for all passengers using a given service, and the disutility $\tau \|t^* - t\|$, which is individual specific instead. In fact, the optimal service choice is the one yielding the *lowest* generalised price. Notice that different choices across individuals do not reflect heterogeneity in preferences, provided the function $u(\cdot)$ is the same for all of them. They rather reflect heterogeneity in wealth and travel needs, as proxied by the parameters τ and t^*, which affect the individual generalised price. Moreover, the individual demand $x(\tilde{p})$ depends on the information the passenger holds about the schedule of departure times. This is because the generalised price $\tilde{p}$ includes the wedge $\|t^* - t\|$ between the ideal and the actual departure time, and the latter depends on the individual choice, hence on the individual information set. Henceforth, we let $v(\tilde{p}) = b(x(\tilde{p}), \tilde{p})$ be the individual surplus when the generalised price is $\tilde{p}$.

Information Passengers are either uninformed or informed. *Uninformed* passengers do not know how differently priced services are scheduled until after they reach the station of departures. Once they learn the schedule of departure times, either they take the first available service or, alternatively, they wait for a *later* (but cheaper) service, if available. Thus, for these individuals the difference between the true departure time and the ideal one measures the *waiting time* (henceforth, WT) until departure. By contrast, *informed* passengers learn how differently priced services are scheduled before reaching the station. They can thus decide to take services departing both *earlier* and *later* than their ideal departure time, planning their arrival at the station accordingly. For these individuals the divergence of the true departure time from the ideal one measures the *departure time shift* (henceforth, DTS).

In what follows, we investigate the choice of services and its impact on the market outcome first for uninformed passengers, next for informed passengers. In either case, the problem also admits an interpretation along the lines of models with product differentiation. Specifically, it can be viewed as a case of both *horizontal* (or *spatial*) and *vertical* differentiation. Suppose for a moment that the value of time is alike for all passengers, whereas the ideal departure time is not. Then, the available services represent different varieties (or store locations) of the same commodity, which each passenger orders according to the distance from her ideal departure time. The ranking of services is thus individual specific, and we are in a standard setting of horizontal differentiation. Next suppose that passengers would like to depart all at the same time, whereas they are not equally hindered by a DTS. Then, the available services represent product versions of different quality levels, with the service scheduled at (or closest to) the ideal time representing the best version and more distant services representing lower-quality versions. The ranking of services is thus univocal in the population of passengers, and we are in a standard setting of vertical differentiation. In our comprehensive framework, these two features are blended together, with a quantity dimension being nested on top of that. Actually, in addition to picking a service, each individual expresses a demand of travels for that service. There is an intuitive reading key of the quantity dimension too, as regarded at the individual level. Depending on the individual value of time, the passenger may be keen on shifting departure to take advantage of a cheap service, if not available at (or close to) her ideal departure time; or she may not be prone to that and rather prefer to forego any price savings. Provided the individual value of time is persistent, the favourite option to the passenger will of course be the same in *any* travel occasion. Accordingly, the optimal choice will be replicated across travels, and the individual demand for the service will result.

3 Uninformed passengers

Recall that uninformed passengers can either take the first available service upon arrival at the station, or they can opt for one of the subsequent services. Taking a subsequent service requires waiting the time interval until its departure time, denoted $\Delta\mathcal{T}$. This is not convenient unless a price saving, denoted Δp, is obtained. Accordingly, only sufficiently patient passengers will decide to wait for a subsequent service, namely passengers with

$$\tau \leq \frac{\triangle p}{\Delta\mathcal{T}}. \tag{2}$$

One can interpret the threshold $\triangle p/\Delta\mathcal{T}$ as being the value of time of an individual whose willingness to pay for transportation services is exactly balanced by the willingness to wait.

The attractiveness of a given service depends on how its price compares with those of the other services. A 'cheap' service can attract (patient) passengers even if they reached the station before the departure of previous services. An 'expensive' service can lose some of its potential clients, who may prefer to wait for a subsequent service, if that choice permits to pay less. Overall, the demand for the various services depends on both the pattern of prices and the schedule of departures.

For readability, we will consider a setting where service departures are evenly distributed over time, and there are only two levels of price, high (p_h) and low (p_l). Hence, the time interval ΔT between any two subsequent departures and the price saving $\Delta p = p_h - p_l$ are both constant. Furthermore, $\Delta\mathcal{T} = \Delta T$ and $\Delta\mathcal{T} = 2\Delta T$ for passengers who postpone departure, respectively, by one and two services. Accordingly, the cut-off value of time in (2) specifies as $\hat{\tau} \equiv \Delta p/\Delta T$ in the former case, and as $\hat{\tau}/2$ in the latter case. Without loss of generality, we will take a service s_1, departing at t_1 and offered at a price of p_1, to be the ‘reference’ service. We will characterize the distribution of passengers with ideal departure time $t^* \in [t_1 - 2\Delta T, t_1]$ across this service, the previous service s_0, which is offered at a price of p_0, and the subsequent service s_2, which is offered at a price of p_2.

Depending on the exact level of p_0, p_1 and p_2, there are four possible allocations of passengers across services. Hereafter, rather than plunging into a long description of the various cases, we only present a case where prices are such that $p_1 = p_h > p_l = p_2$, which admits a simple and realistic interpretation. That is, t_1 may represent a pick hour of the day, in which the service is priced more, whereas t_0 and t_2 are off-pick hours, in which the service is priced less. For this case, to be denoted U.1 to ease reference, we will explore the impact of a marginal change in price and in schedule on passenger welfare. The other cases (U.2 to U.4) are reported in Appendix A.

Before turning to present case U.1, it is useful to briefly mention the limit case where the price is uniform across services ($p_0 = p_1 = p_2$). As no saving is available in that case, there is obviously no point to wait for later services. Each passenger will thus take the first service next to her ideal departure time, regardless of how (im)patient she is.

3.1 Case U.1: $p_1 = p_h > p_l = p_2$

When service s_1 is at least as expensive as the previous service s_0 and more expensive than the later service s_2 it attracts fewer passengers than any of the other two services. To see this, we need to explore the behaviour of passengers with different ideal departure times within the range $[t_1 - 2\Delta T, t_1]$.

Let us first consider passengers with $t^* \in [t_{-1}, t_0]$, where $t_{-1} = t_1 - 2\Delta T$ is the departure time of a hypothetical service s_{-1} preceding s_0. Since $p_1 \geq p_0$, for these passengers there is no point to wait until t_1, and they all take s_0. Passengers with $t^* \in [t_0, t_1]$ choose s_1, if they are impatient, namely $\tau > \widehat{\tau}$; they wait for s_2 otherwise. In substance, neglecting passengers with $t^* > t_1$, who would take s_2 in any case, all those who choose s_2 are actually passengers who could have taken s_1 but tolerate a longer WT to pay less. On the other hand, there is no passenger with $t^* < t_0$ who waits until t_2. Accordingly, the partial and total demand for s_0, s_1 and s_2, are respectively given by

$$\begin{aligned} X_0(\tau) &= \int_{t_{-1}}^{t_0} x\left(p_0 + \tau \left\|t_0 - t^*\right\|\right) n dt^* \\ \tilde{X}_0 &= \int_0^{+\infty} X_0(\tau)\, dG(\tau), \end{aligned}$$

$$
\begin{aligned}
X_1(\tau) &= \int_{t_0}^{t_1} x\left(p_1 + \tau \left\|t_1 - t^*\right\|\right) n dt^* \\
\bar{X}_1 &= \int_{\widehat{\tau}}^{+\infty} X_1(\tau)\, dG(\tau),
\end{aligned}
$$

and

$$
\begin{aligned}
X_{1 \rhd 2}(\tau) &= \int_{t_0}^{t_1} x\left(p_2 + \tau \left\|t_2 - t^*\right\|\right) n dt^* \\
\underline{X}_2 &= \int_0^{\widehat{\tau}} X_2(\tau)\, dG(\tau),
\end{aligned}
$$

where the subscript $1 \rhd 2$ refers to passengers who would prefer to depart at t_1 but wait until t_2. This market segmentation is represented in the graphs on the right in Figure 1, whereas the graphs on the left represent the market segmentation in a case where the price is uniform across services ($p_0 = p_1 = p_2 = p_l$). Contrasting the two pairs of graphs, it is evident that when s_1 is expensive s_2 attracts some (patient) passengers with earlier ideal departure time, whereas this is not the case when services are all cheap. Using the capital letter V to refer to aggregate surplus, together with further notation along the previous lines, we can also write the surplus derived by the use of the three services, namely

$$
\begin{aligned}
\widetilde{V}_0 &= \int_0^{+\infty} \left(\int_{t_{-1}}^{t_0} v\left(p_0 + \tau \left\|t_0 - t^*\right\|\right) n dt^* \right) dG(\tau) \\
\overline{V}_1 &= \int_{\widehat{\tau}}^{+\infty} \left(\int_{t_0}^{t_1} v\left(p_1 + \tau \left\|t_1 - t^*\right\|\right) n dt^* \right) dG(\tau) \\
\underline{V}_2 &= \int_0^{\widehat{\tau}} \left(\int_{t_0}^{t_1} v\left(p_2 + \tau \left\|t_2 - t^*\right\|\right) n dt^* \right) dG(\tau).
\end{aligned}
$$

The total surplus is $V = \widetilde{V}_0 + \overline{V}_1 + \underline{V}_2$.

3.1.1 A change in price

We now turn to assess the effect induced on passenger surplus by a marginal change in some price p_i, where $i \in \{0, 1, 2\}$, which does not alter the overall ordering of prices across services. Of course, the aggregate impact cannot be understood without first considering the individual level. Recalling that (1) is true for any given passenger, one straightforwardly obtains the well-known result that an infinitesimal increase in the monetary price

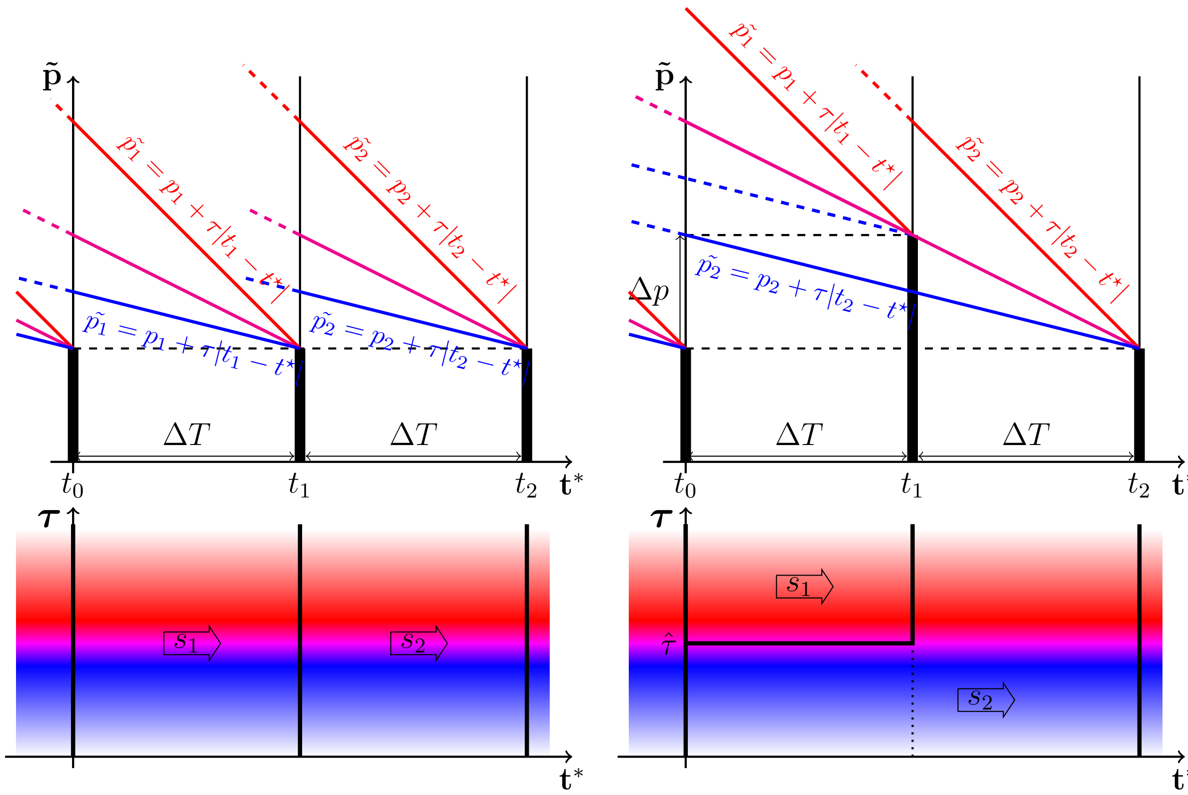

Figure 1: **Uninformed passengers - Case U.1: $\mathbf{p_1 = p_h > p_l = p_2}$ (right)**
Top graphs: The generalised price is plotted against the ideal departure time when the price is uniform (left) and when it is not (right). The blue line represents the generalised price of a patient passenger ($\tau < \hat{\tau}$), the red line that of an impatient passenger ($\tau > \hat{\tau}$), the magenta line that of a passenger with $\tau = \hat{\tau}$. The individual generalised price decreases to the monetary price (the thick black vertical line placed in the service locations) as the departure time approaches the ideal one, t^*.
Bottom graphs: Passenger distribution over the two heterogeneity dimensions (t^*, τ) and across services with uniform price (left) and in case U.1 (right). Colors refer to the value of time τ; the intensity is associated with the number of passengers displaying the characteristics (t^*, τ). For this representation, we assume it independent of t^*. When s_1 is more expensive than the other services, s_2 attracts some patient passengers with earlier ideal departure time.

p triggers a reduction in individual surplus equal to the demanded quantity:

$$\begin{aligned} \frac{dv(\tilde{p})}{dp} &= \left(u'\left(x\left(\tilde{p}\right)\right) - \tilde{p}\right)\frac{dx\left(\tilde{p}\right)}{dp} - x\left(\tilde{p}\right) \\ &= x\left(\tilde{p}\right). \end{aligned} \tag{3}$$

At the aggregate level, two effects are at work, respectively on the *extensive* and *intensive* margin. First, if there is an infinitesimal increase in the price saving Δp, then this induces an increase of $1/\Delta T$ in $\widehat{\tau}$, the cut-off value of the marginal disutility of waiting in this case. The distribution of passengers across services is thereby altered, with some passengers moving away from the service which is now more expensive. Second, there is a reduction in the individual demand expressed by the passengers who stick to that service. The impact on passenger surplus results from the combination of these two effects.

With the prices being such that $p_0 \leq p_1 = p_h$, the passenger distribution is not altered by a change in p_0. It is altered, instead, by both a change in p_1 and a change in p_2, each of which triggers a variation in the cut-off value of time $\widehat{\tau}$. However, because this boils down to some passengers switching between s_1 and s_2, the effect on the extensive margin is offset in the aggregate, and only the effect on the intensive margin matters. Formally, following an infinitesimal increase in some price p_i, where $i \in \{0, 1, 2\}$, surplus is reduced by the exact size of the demand for service s_i, namely (see Appendix A.1 for details)

$$\frac{dV}{dp_0} = -\tilde{X}_0; \quad \frac{dV}{dp_1} = -\bar{X}_1; \quad \frac{dV}{dp_2} = -\underline{X}_2.$$

Although this is a familiar result in microeconomics, there are a few insights to be retained in this context, using the individual value of time as a proxy for individual earnings. First, an increase in the price of an expensive service operated in a peak hour would hinder high$-\tau$ (high-earning) passengers only. Second, the overall loss of surplus associated with that policy would be greater, if a later but cheaper service (s_2) were not available, given that this service attracts some of the passengers who were using the expensive service prior to the price change. Third, a raise in the lower price (p_2) would essentially penalize low$-\tau$ (low-earnings) passengers, who are ready to wait in order to pay less. Thus, in a transportation system with the characteristics here considered, a policy-maker could actually favour passengers in difficult economic conditions by decreasing the price of a late but cheap service.

3.1.2 A change in schedule

Of course, any variation in the schedule of services will affect passenger surplus as well. For any given passenger with ideal departure time $t^* \leq \inf\{t, t+dt\}$, a delay of dt in the actual departure time induces a decrease in surplus equal to the marginal disutility of waiting for each of the demanded travels, namely

$$\begin{aligned}\frac{dv(\tilde{p})}{dt} &= \left[\left(u'(x(\tilde{p}))-\tilde{p}\right)\frac{dx(\tilde{p})}{dp}-x(\tilde{p})\right]\frac{d\tilde{p}}{dt} \\ &= -\tau x(\tilde{p}). \end{aligned} \tag{4}$$

Again this result is in line with product differentiation models, in which it is found that the reduction induced in the individual surplus, as quality is curtailed, equals the benefit attached to quality for each consumed product unit. It is thus not surprising that, as can be seen by taking (4) together with (3), the marginal rate of substitution between time and money is equal to the marginal value of time, which measures the loss of utility from not consuming the ideal commodity (not departing at the ideal time):

$$\frac{dv(\tilde{p})/dt}{dv(\tilde{p})/dp} = \tau.$$

More insights can be derived at the aggregate level. Following a variation in schedule, which raises the time interval ΔT between any two subsequent services, some patient individuals renounce to wait for a cheaper service as the cost of delaying departure becomes too high for them. Formally, this is captured by a decrease in the cut-off value of time $\widehat{\tau}$. Letting dT be the increase in ΔT, one can verify that $(d\widehat{\tau}/\widehat{\tau}) = -(dT/\Delta T)$, *i.e.*, the elasticity of $\widehat{\tau}$ with respect to ΔT is constant and equal to -1.

To examine the effects on the aggregate surplus, we proceed as follows. We consider an increase in the time interval between the departure time of service s_0 and the departure times of the two services which are respectively scheduled before and after s_0. This is done by fixing t_0, and letting t_{-1} decrease by dT, t_1 increase by dT, and t_2 increase by $2dT$. To avoid dealing with pure 'volume' effects associated with an increased number of passengers, we cut out of the surplus variation the terms which are due to the $2ndT$ additional passengers whose ideal departure time coincides with any of the two extremes of the time interval, namely, those with $t^* \in [t_{-1} - dT; t_{-1}]$ and with $t^* \in [t_1; t_1 + dT]$.

With the departure time t_0 being fixed, the change in schedule does not concern any of the passengers who were already using s_0. The only effect on the surplus associated with the use of s_0 would pertain to the passengers who come to take this service following the change, which we neglect, as explained. As the departure of s_1 is postponed, passengers who were already using (and stick to) s_1 all wait longer. As the departure of s_2 is postponed even more, passengers with $\tau = \hat{\tau}$ shift to the previous service, those who stick to s_2 in order to pay less wait even longer. Omitting the specific expressions that capture the impact on surplus associated with the single services, the overall impact (net of any volume effects) is formalised as follows (see Appendix A.1 for details):

$$\frac{dV}{dT} = -\left(\bar{\tau}_1 \bar{X}_1 + 2\underline{\tau}_2 \underline{X}_2\right), \tag{5}$$

where

$$\bar{\tau}_1 = \int_{\hat{\tau}}^{+\infty} \tau \frac{X_1(\tau)}{\bar{X}_1} dG(\tau) \quad \text{and} \quad \underline{\tau}_2 = \int_0^{\hat{\tau}} \tau \frac{X_{1\triangleright 2}(\tau)}{\underline{X}_2} dG(\tau)$$

provide an average measure of the value of time of passengers respectively using s_1 and s_2. From (5), it is apparent that the impact on surplus through service s_2 accounts doubly, which is precisely because, as ΔT is increased keeping t_0 fixed and postponing t_1 by dT, t_2 is postponed by $2dT$. One useful property of our approach is precisely its capability of highlighting these effects. Whereas (5) provides an aggregate measure of the impact of changes in service scheduling on the welfare of the overall population of passengers, it also pins down a segmentation of that population based on an individual characteristic, namely the value of time, which is a reliable indicator of the passenger economic conditions and preferences. In the situation here considered, if it is decided to increase the service frequency, then the benefit to patient individuals has a higher weight in aggregate surplus enhancement than the benefit to impatient individuals. Note however that, whereas the marginal impact of a change in schedule is proportional to the value of *all* passengers, the cost of WT may differ sensibly across them. As high-income passengers typically face a higher cost of WT, it might be the case that they are favoured to a greater extent by a decision to make services more frequent.

4 Informed passengers

Unlike uninformed passengers, who can only decide to *wait*, informed passengers may alternatively decide to use one of the services which are scheduled *before* their ideal departure time. For any given pair of services $\{s_i, s_j\} \in S$, an individual with $t^* \in [t_i, t_j]$ prefers s_i to s_j, if the generalised price associated with the former service is lower than that associated with the latter, namely

$$p_i + \tau(t^* - t_i) < p_j + \tau(t_j - t^*),$$

where now the parameter τ expresses the value attached to the DTS. For any given value of τ, there exists a critical departure time

$$t_c^*(\tau) \equiv \frac{t_i + t_j}{2} + \frac{p_j - p_i}{2\tau}$$

splitting passengers into two groups: those who would like to leave before $t_c^*(\tau)$ take s_i, the others opt for s_j. Notice that passengers with a high (marginal) disutility of DTS prefer the service which is closer to their ideal departure time, overlooking price differences. Passengers with a low (marginal) disutility of DTS choose the cheaper service, caring little about the departure time.

Focusing on passengers with $t^* \in [t_0, t_2]$, we now turn to characterize their distribution across the three subsequent services s_0, s_1 and s_2. We will next investigate how their surplus is affected by changes in price and in schedule. Again, for readability, we focus on situations in which service departures are evenly distributed over time, and there are only two levels of price, high (p_h) and low (p_l), so that the time interval ΔT and the price saving Δp are both constant.

As with the uninformed passengers, there are again four cases to consider, depending on the pattern of prices across services, but we only discuss the case where prices are such that $p_1 = p_h > p_l = p_0 = p_2$, to be referred to as I.1, for clarity. The other cases (I.2 to I.4) are analysed in Appendix B.

Before turning to present case I.1, we briefly return to the limit case where the price is uniform across services so that no saving is available. When so, any given service collects all passengers whose ideal departure time lies within *half* the time interval which separates that service from the previous and the subsequent service. Unlike with uninformed passengers, this includes passengers who would like to depart both *earlier* and *later* than the service is actually scheduled.

4.1 Case I.1: $p_1 = p_h > p_l = p_0 = p_2$

When s_1 is the only expensive service passengers with a low value of time all accrue to other services to save on price. Actually, these are passengers with $\tau < \hat{\tau}$, highlighting that the cut-off value of time is *just the same as for uninformed passengers.* Individuals with $t^* \in [t_0; t_1]$ opt for s_0; those with $t^* \in [t_1; t_2]$ prefer to wait and take s_2. Passengers with a high value of time, namely $\tau \geq \hat{\tau}$, stick to s_1 in the event that t_1 is sufficiently close to their ideal departure time, and choose other services otherwise. Specifically, these passengers respectively use s_0, s_1 and s_2 when

$$t^* \in \left[t_0; \frac{t_0 + t_1}{2} + \frac{\Delta p}{2\tau}\right]$$
$$t^* \in \left[\frac{t_0 + t_1}{2} + \frac{\Delta p}{2\tau}; \frac{t_1 + t_2}{2} - \frac{\Delta p}{2\tau}\right]$$
$$t^* \in \left[\frac{t_1 + t_2}{2} - \frac{\Delta p}{2\tau}; t_2\right].$$

Accordingly, we can respectively write the demand for service s_0, s_1 and s_2 as

$$\begin{aligned}
\widetilde{X}_0 &= \underline{X}_0 + \overline{X}_0 \\
&= \int_0^{\hat{\tau}} X_{(1]\rhd 0}(\tau)\, dG(\tau) + \int_{\hat{\tau}}^{+\infty} X_{1\rhd 0}(\tau)\, dG(\tau) \\
\overline{X}_1 &= \int_{\widehat{\tau}}^{+\infty} X_1(\tau)\, dG(\tau) \\
\widetilde{X}_2 &= \underline{X}_2 + \overline{X}_2 \\
&= \int_0^{\hat{\tau}} X_{(1]\rhd 2}(\tau)\, dG(\tau) + \int_{\widehat{\tau}}^{+\infty} X_{1\rhd 2}(\tau)\, dG(\tau)
\end{aligned}$$

where

$$\begin{aligned}
X_{(1]\rhd 0}(\tau) &= \int_{t_0}^{t_1} x\left(p_0 + \tau \left\|t^* - t_0\right\|\right) n dt^* \\
X_{1\rhd 0}(\tau) &= \int_{t_0}^{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}} x\left(p_0 + \tau \left\|t^* - t_0\right\|\right) n dt^*,
\end{aligned}$$

together with

$$X_1(\tau) = \int_{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}} x\left(p_1 + \tau \left\|t^* - t_1\right\|\right) n dt^*$$

and with

$$\begin{aligned}
X_{(1]\rhd 2}(\tau) &= \int_{t_1}^{t_2} x\left(p_2 + \tau \left\|t^* - t_2\right\|\right) n dt^* \\
X_{1\rhd 2}(\tau) &= \int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2 + \tau \left\|t^* - t_2\right\|\right) n dt^*,
\end{aligned}$$

where the subscripts $(1] \rhd 0$ and $1 \rhd 0$ respectively indicate passengers with $t^* \leq t_1$ and with $t^* \leq \frac{t_0+t_1}{2} + \frac{\Delta p}{2\tau} < t_1$, who would take s_1 rather than s_0, if they were to abstract from any price consideration; and similarly for $(1] \rhd 2$ and $1 \rhd 2$. A representation of this market segmentation is found in the graphs on the right in Figure 2, whereas the graphs on the left represent the market segmentation with a (nearly) uniform price. We can also write the surplus respectively derived from the use of s_0, s_1 and s_2 as

$$\begin{aligned}
\widetilde{V}_0 &= \int_0^{\hat{\tau}} \left(\int_{t_0}^{t_1} v\left(p_0 + \tau \left\| t^* - t_0 \right\|\right) ndt^* \right) dG(\tau) \\
&\quad + \int_{\hat{\tau}}^{+\infty} \left(\int_{t_0}^{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}} v\left(p_0 + \tau \left\| t^* - t_0 \right\|\right) ndt^* \right) dG(\tau) \\
\overline{V}_1 &= \int_{\hat{\tau}}^{+\infty} \left(\int_{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}} v\left(p_1 + \tau \left\| t^* - t_1 \right\|\right) ndt^* \right) dG(\tau) \\
\widetilde{V}_2 &= \int_0^{\hat{\tau}} \left(\int_{t_1}^{t_2} v\left(p_2 + \tau \left\| t^* - t_2 \right\|\right) ndt^* \right) dG(\tau) \\
&\quad + \int_{\hat{\tau}}^{+\infty} \left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} v\left(p_2 + \tau \left\| t^* - t_2 \right\|\right) ndt^* \right) dG(\tau),
\end{aligned}$$

with total surplus $V = \widetilde{V}_0 + \overline{V}_1 + \widetilde{V}_2$.

4.1.1 A change in price

Also in this case, an infinitesimal change in price p_i, $i = 0, 1, 2$, has a double impact. First, it affects all passengers who take s_i both before and after the change (the *intensive* margin). Second, it affects the distribution of passengers across services (the *extensive* margin). Here, the sole additional twist is that the latter effect works not only through the cut-off value of time $\hat{\tau}$ but also through the critical departure time $t_c^*(\tau)$. Overall, we find the standard result that an increase in the price of a given service triggers a reduction in surplus equal to the demand for that service, namely (see Appendix B.1 for details):

$$\frac{dV}{dp_0} = -\widetilde{X}_0; \quad \frac{dV}{dp_1} = -\overline{X}_1; \quad \frac{dV}{dp_2} = -\widetilde{X}_2.$$

4.1.2 A change in schedule

We next consider an infinitesimal change in the time interval ΔT that separates two subsequent services. Our strategy is to look at an increase in the time interval around the departure time of service s_1, with the number of passengers being held constant. Formally, we fix t_1 and let t_0 decrease by dT and t_2 increase by dT, cutting out of the surplus variation the terms which are due to the $2ndT$ additional passengers whose ideal departure time coincides with either t_0 or t_2. This approach is tantamount to that followed to investigate uninformed behaviour, once it is considered that informed passengers can not only delay but also anticipate departure.

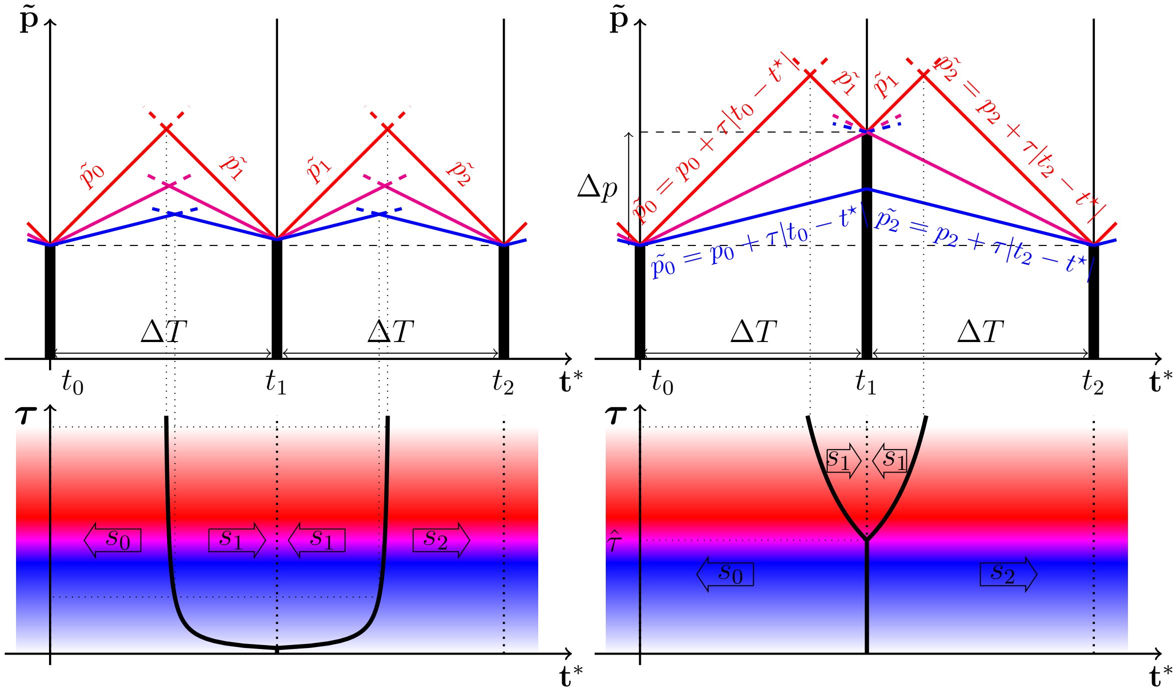

Figure 2: **Informed passengers - Case I.1:** $\mathbf{p_1 = p_h > p_l = p_0 = p_2}$ **(right)**
Top graphs: The generalised price is plotted against the ideal departure time when the price is (nearly) uniform (left) and when it is not (right). The blue line represents the generalised price of a patient passenger ($\tau < \hat{\tau}$), the red line that of an impatient passenger ($\tau > \hat{\tau}$), the magenta line that of a passenger with $\tau = \hat{\tau}$. The individual generalised price decreases to the monetary price (the thick black vertical line placed in the service locations) as the departure time approaches the ideal one, t^*.
Bottom graphs: Passenger distribution over the two heterogeneity dimensions (t^*, τ) and across services with uniform price (left) and in case I.1. Colors refer to the value of time τ; the intensity is associated with the number of passengers displaying the characteristics (t^*, τ). For this representation, we assume it independent of t^*. When s_1 is more expensive than the other services, it only attracts passengers with high value of time ($\tau > \hat{\tau}$) who would like to depart immediately before and after t_1.

Let

$$\underline{\tau}_0 = \int_0^{\hat{\tau}} \tau \frac{X_{(1]\rhd 0}(\tau)}{\underline{X}_0} dG(\tau) \quad \text{and} \quad \overline{\tau}_0 = \int_{\hat{\tau}}^{+\infty} \tau \frac{X_{1\rhd 0}(\tau)}{\overline{X}_0} dG(\tau)$$
$$\underline{\tau}_2 = \int_0^{\hat{\tau}} \tau \frac{X_{(1]\rhd 2}(\tau)}{\underline{X}_2} dG(\tau) \quad \text{and} \quad \overline{\tau}_2 = \int_{\hat{\tau}}^{+\infty} \tau \frac{X_{1\rhd 2}(\tau)}{\overline{X}_2} dG(\tau)$$

measure the value of time of more to less patient passengers respectively using s_0 and s_2. Omitting again the specific expressions that capture the impact of an increase of dT in the infra-service time interval ΔT on surplus derived by the use of the single services, the overall impact (net of any volume effects) amounts to

$$\frac{dV}{dT} = -\left(\underline{\tau}_0 \underline{X}_0 + \overline{\tau}_0 \overline{X}_0 + \underline{\tau}_2 \underline{X}_2 + \overline{\tau}_2 \overline{X}_2\right). \tag{6}$$

Essentially, this expression mirrors two consequences to transportation services becoming less frequent. First, passengers using s_1 are not concerned, provided this service still departs at t_1. Second, as the DTS increases for patrons of s_0 and s_2, welfare reduces by an aggregate measure of their value of time, given their travel demand, according to whether that value is low or high. In this context, a decision to make services more frequent around the expensive service (s_1) would benefit individuals with both low and high value of time, hence individuals in any economic conditions. Although it would not benefit all passengers in the system, it would leave out those with a high value of time, who are plausibly wealthier.

5 Assessing policy choices: winners and losers

So far we have investigated travel choices, looking at the impact on passenger welfare of *marginal* variations in price and in schedule, which alter neither the pattern of prices across services nor the number of available services. To illustrate how our analysis can be made fully operational in practice, and help decision-makers assess and fine-tune targeted policies, we now turn to consider *discrete* variations in price and in frequency, which do lead to a different pattern of prices and number of services. We identify winners and losers in the population of passengers.

5.1 A change in price

To illustrate the impact of a change in price leading to a different pattern of prices across services, we compare a situation in which prices are such that $p_1 = p_h > p_l = p_0 = p_2$, as in cases U.1 and I.1, with a situation in which prices are such that $p_0 = p_1 = p_h > p_2 = p_l$, as in cases U.4 and I.4, which are respectively developed in Appendix A and B.

Clearly, the only difference between the two situations is that service s_0 is expensive in the former and cheap in the latter.

Assessing the consequences for *uninformed* passengers requires comparing case U.1 with case U.4. A graphical representation is found in Figure 3. Although the top graph on the left refers to case U.1 and the top graph to the right refers to case U.4, they look nearly the same. This is because, as p_0 is raised to p_h, nothing changes for passengers with $t^* \in [t_0, t_2]$, namely, those whose behaviour is apparent from the graphs. The passengers who are actually concerned by the variation in p_0 are those with $t^* \in [t_{-1}, t_0]$ instead. Specifically, those with $\tau \leq \frac{\hat{\tau}}{2}$ are induced to switch from s_0 to s_2, that is, they decide to wait much longer to still be able to take a cheaper service. Hence, in fact, s_2 receives more passengers in the graph on the right than in the graph on the left.

There are essentially two insights which a policy-maker could retain by looking at these effects in a transportation system with uninformed passengers. First, making a *cheap* service available before an *expensive* service (*i.e.*, switching from case U.4 to case U.1) would allow for travelling to be not only less expensive but also more timely in the overall system. Second, within the whole population, this policy would be beneficial to some of the passengers who have low earnings, in different ways. The poorest would still travel for cheap but face a shorter WT. The others would still wait little but spend less.

We now turn to consider *informed* passengers, and compare case I.1 with case I.4. This is made graphically in Figure 3, in which the bottom graph on the left represents case I.1 and the bottom graph to the right represents case I.4. These graphs show that passengers with $t^* \in [t_0, t_1]$ are now concerned as p_0 is increased to p_h. Among those, passengers with $\tau \leq \frac{\hat{\tau}}{2}$ and some with $\tau \in \left(\frac{\hat{\tau}}{2}, \hat{\tau}\right]$ are induced to switch from s_0 to s_2, that is, they decide to wait long, rather than anticipating departure slightly, to still be able to use a cheap service. Passengers with $\tau > \hat{\tau}$, whose ideal departure time is closer to t_1 than t_0, switch from s_0 to s_1 to wait less than they were anticipating departure prior to the change, but spend more to travel.

The insights previously drawn on the possibility of making a cheap service available before an expensive service (here, a switch from case I.4 to case I.1), carry over only partially when passengers are informed. First, although it is still true that travelling would become overall less expensive, it is not clear that it would also be overall more timely, since some passengers decide to anticipate departure when s_0 is cheap more then they wait when s_0 is expensive. Second, the price cut would produce more extensive effects in that, in addition to low-earning individuals, it would benefit high-earning individuals as well. Thus, it would be less powerful a pro-poor tool in this context.

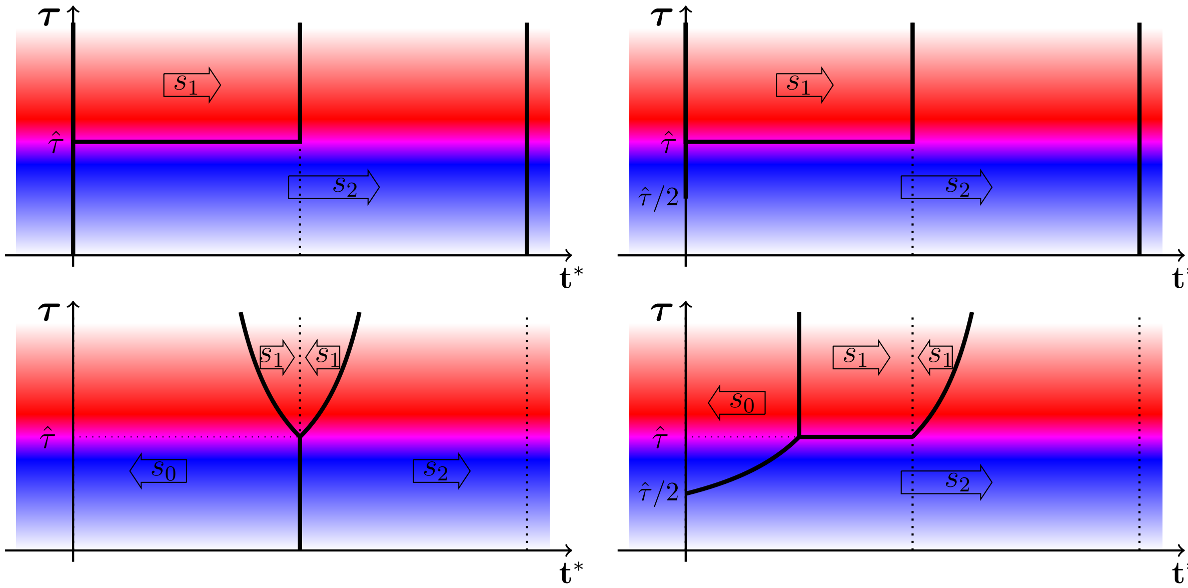

Figure 3: **Change in price: p_1 is increased from p_l (left) to p_h (right)**
Top graphs: Distribution of uninformed passengers. As p_1 is increased, s_2 attracts very patient passengers from s_0.
Bottom graphs: Distribution of informed passengers. As p_1 is increased, passengers with very low value of time and some passengers with intermediate value of time switch from s_0 to s_2. Passengers with high value of time switch from s_0 to s_1 only if their ideal departure time is very closer to t_1.

5.2 Introduction (or cancellation) of a service

We now turn to consider a change in the number of services due to the introduction or cancellation of a service. To that end, we compare cases U.1 and I.1, in which service s_1 is offered at a price of p_h, with a situation in which, whereas the other services are still both cheap, s_1 is infinitely expensive instead ($p_1 \to \infty$). A switch from the latter to the former situation is tantamount to the service frequency being doubled since, after the switch, the first truly affordable service after s_0 is scheduled at t_1 rather than at t_2. This makes it plane that scheduling considerations cannot be disjoint from price considerations. More precisely, it is not possible for a decision-maker to assess the consequences of fewer/more services being provided in the transportation system, without taking into account the *price* of the services which are eliminated or newly introduced and, hence, the resulting price pattern.

The effects for the *uninformed* passengers with $t^* \in [t_0, t_2]$ are represented in Figure 4. The graphs on the left refer to the case where s_1 is unavailable; those on the right refer to the case where s_1 is offered at a price of p_h. From the graphs it appears that the switch from the former case to the latter concerns passengers who would like to depart after t_0 but no later than t_1, and dislike waiting ($\tau > \hat{\tau}$). They all switch from s_2 to s_1 as s_1 becomes affordable (or available). Although they spend more in the presence of

s_1, provided s_1 is more expensive than s_2, this does not make them worse off as they all sustain a lower opportunity cost of waiting.

In the *informed* regime, as represented in Figure 5 with similar interpretation to Figure 4, concerned passengers appear to be those who would like to depart immediately before and after t_1, and dislike shifting departure away from their ideal time ($\tau > \hat{\tau}$). They respectively switch from s_0 and s_2 to s_1 when this latter service is available. Again, they all spend more since both s_0 and s_2 are cheaper than s_1; yet, they all benefit from a decrease in the DTS.

There is a clear insight to be retained, which applies to both uninformed and informed passengers. The introduction of an *expensive* service between two *cheap* services (*i.e.*, a switch from a regime without s_1 to case U.1 / I.1) represents a suitable tool, if a decision-maker chases to favour high-earnings (high$-\tau$) passengers, who are prone to pay more in order to depart at a more desirable time, without penalizing low-earnings (low$-\tau$) passengers, who could still travel for cheap as if the schedule of services were unaltered.

6 On the impact of information on passenger welfare

We complete our analysis with a few considerations on the impact that the possibility for passengers of making informed travel decisions (potentially) has in multi-service transportation systems as here explored. Since the choices, which are available to passengers when they lack information on services, are all equally available when they hold information, it is clear that, if information were irrelevant, then informed passengers would simply behave in exactly the same manner as uninformed passengers. Our previous analysis shows that this is not the case of (at least) some passengers, and it is straightforward to conclude that information is relevant indeed.

One effect of information is precisely that, for an equal supply of services, the set of services which are actually accessible to passengers is richer. Suppose that $S = \{s_0, s_1, s_2\}$, where $p_0 = p_1 = p_2$ so as to abstract from any price difference, and restrict attention to passengers with $t^* \in [t_0, t_2]$. If uninformed, then these passengers take either s_1 or s_2, whereas s_0 is scheduled too early for their use. If informed, then s_0 becomes accessible and some passengers do switch to it, namely, those with $t^* \in \left[t_0, t_0 + \frac{\Delta T}{2}\right]$. In good substance, given the supply S, informed passengers have one more service to use, if they wish, which is instead inaccessible to uninformed passengers, even with equal ideal departure time and value of time. Thereby, with a uniform price, the DTS will be halved, in aggregate, if passengers are informed.

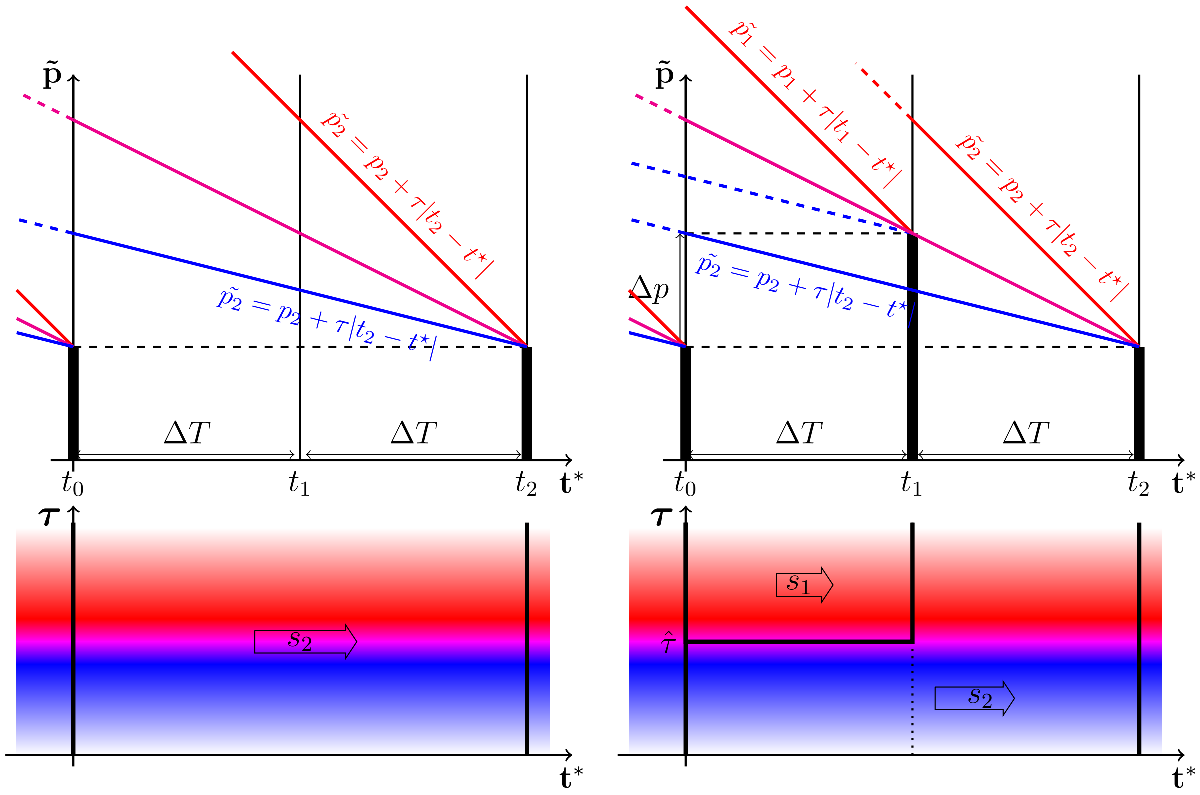

Figure 4: **Introduction of a service with uninformed passengers: s_1 is unavailable (left); s_1 is available (right)**
Top graphs: The generalised price is plotted against the ideal departure time, with similar interpretation to previous figures.
Bottom graphs: Passenger distribution across services. When s_1 is introduced (right), it attracts some passengers from s_2. These are impatient passengers who would like to depart after t_0 but no later than t_1.

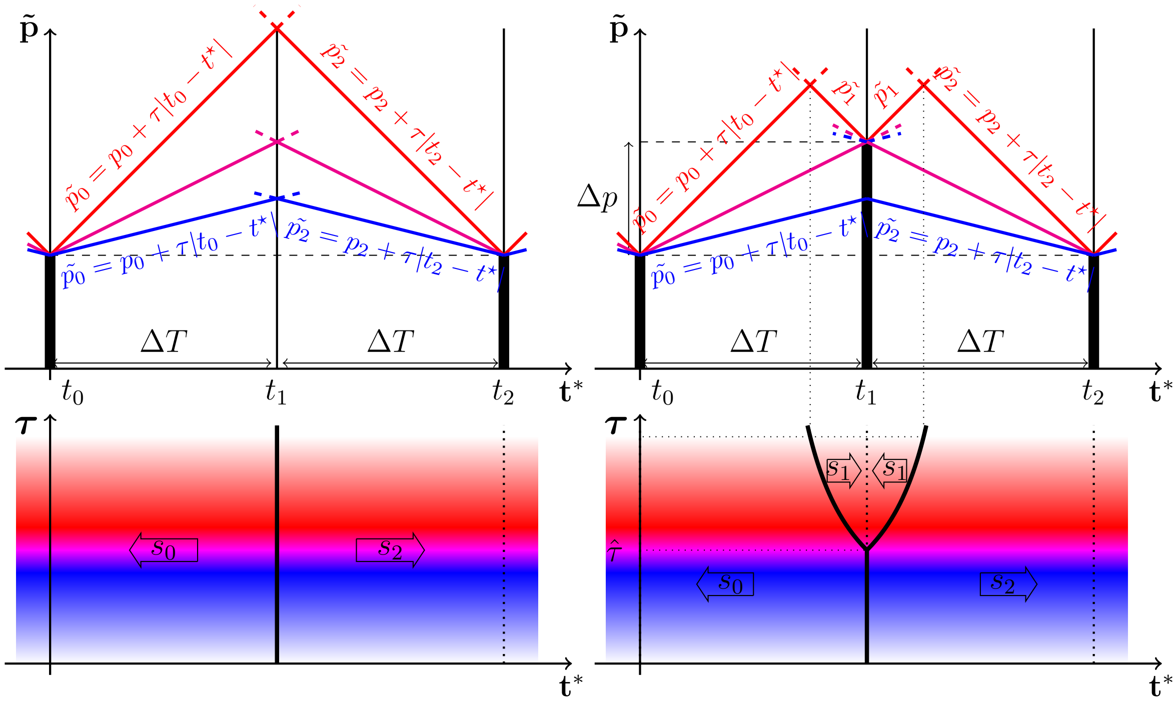

Figure 5: **Introduction of a service with informed passengers:** s_1 **is unavailable (left);** s_1 **is available (right)**

Top graphs: The generalised price is plotted against the ideal departure time, with similar interpretation to previous figures.

Bottom graphs: Passenger distribution across services. When s_1 is introduced (right), it attracts some passengers from both s_0 and s_2. The former are passengers with high value of time who would like to depart no much earlier than t_1. The latter are passengers with high value of time who would like to depart no much later than t_1.

Of course, looking at the impact of information with the monetary dimension being sterilized is a somewhat simplistic approach. One should also consider possible differences in the monetary price to assess the impact of information on passengers welfare. In some situations, information affects passengers in the same direction along both the monetary and the non-monetary dimension. This is the case, for instance, when prices are such that $p_1 = p_h > p_l = p_0 = p_2$. Actually, by comparing again U.1 and I.1, it is easy to verify that informed passengers face both a *lower monetary price* and a *lower non-monetary cost*, in aggregate. Thus, clearly, information reduces the generalised price for the whole population of passengers.

This is not a general rule though, and conclusions may be different with other price patterns. To illustrate, let us take $p_0 = p_1 = p_h > p_2 = p_l$, as in cases U.4 and I.4 analysed in Appendix B.4. With this price pattern, passengers still benefit from a *lower non-monetary cost*, if informed; yet, the *monetary price* is now *higher*, in aggregate (the mathematical proof is found in Appendix C). Intuitively, those who really care about early departure will have to pay more to take advantage of s_0, which is now expensive. As a result, monetary costs are higher, overall. Of course, it remains that the monetary penalty must be worth the non-monetary benefit for those passengers to be willing to modify their behaviour when aware of the service scheduling. This conclusion brings us back to considerations previously made about the possibility of introducing an expensive service in a system with informed passengers. In that case, we concluded that the additional service favours those with high earnings without, yet, damaging those who stick to cheaper services. In a similar fashion, information permits passengers with a high willingness to pay to depart more timely, whereas the others can still choose to spend less, accepting to face a bigger DTS.

In spite of a general rule not being identifiable, an interesting lesson seems to emerge. Although information might lead to higher monetary payments for the population of passengers, it nonetheless reduces aggregate time costs. This lesson has emerged by taking the supply of services to be the same, regardless of passenger information. Plausibly enough, in a transportation system the supply of services will be adjusted to account for passenger behaviour, which obviously depends on the held information. If this is considered, then it is clear that making information accessible to passengers is also an indirect way of promoting efficiency gains in service frequency and scheduling.

One last point is worth making. As information does not need affect all passengers, one may be interested in disentangling those who take advantage of information from those who do not. To avoid delivering additional calculations, which would add little to the main message of the analysis, we content ourselves with a graphical representation, which we provide in Figure 6. In the top graphs, the distribution of uninformed passengers is compared with that of informed passengers when the price pattern is $p_1 = p_h > p_l = p_0 = p_2$

(cases U.1 and I.1). First, with information, the early service s_0 attracts some passengers with ideal departure time beyond t_0, who would rather take s_1 and s_2 if uninformed. The former are passengers with high value of time, who welcome the possibility of facing a smaller DTS by anticipating departure, and all the more that s_0 is cheaper than s_1. The latter are passengers with low value of time, who welcome the possibility of travelling for cheaper by anticipating departure slightly rather than waiting until t_2. Also service s_1 attracts some passengers who would take s_2 if uninformed. These are passengers with high value of time, who are prone to spend more money in order to face a smaller DTS. In good substance, to the extent that the individual value of time is indicative of the individual earnings, information is beneficial to both passengers with low and high earnings, albeit the latter seem to take more advantage of it, in that they adjust their behaviour through the double channel of a switch to s_0 and s_1. The bottom graphs in Figure 6 compare the distributions of uninformed and informed passengers when prices are $p_1 = p_0 = p_h > p_l = p_2$ (cases U.4 and I.4 in Appendix B.4). One can read and use them along the same lines, as a support to draw useful policy insights.

7 Conclusion

We constructed a micro-founded travel demand model which allows for two dimensions of heterogeneity. Passengers differ both in their ideal departure time (horizontal differentiation) and in their value of time (vertical differentiation). We showed that either aspect is relevant in terms of both their travel choices and the impact of changes in price and/or schedule on welfare. If policy-makers are to account for these effects, then our model shall be precious at enabling them to make informed decisions.

Besides, the model sheds light on the role and the importance of information for the performance of multi-service transportation systems. *Ceteris paribus*, information on prices and service schedules enhances passenger welfare. Again, the impact of information varies across passengers. Some are unaffected, others may derive significant benefits from it - and their behaviour may be altered radically, if this information is made available (or easily accessible) to them.

Whereas our paper still falls short of providing a full analysis of the impact of information on passenger welfare, we believe it paves the way for that. Making further progress in that direction is on our research agenda.

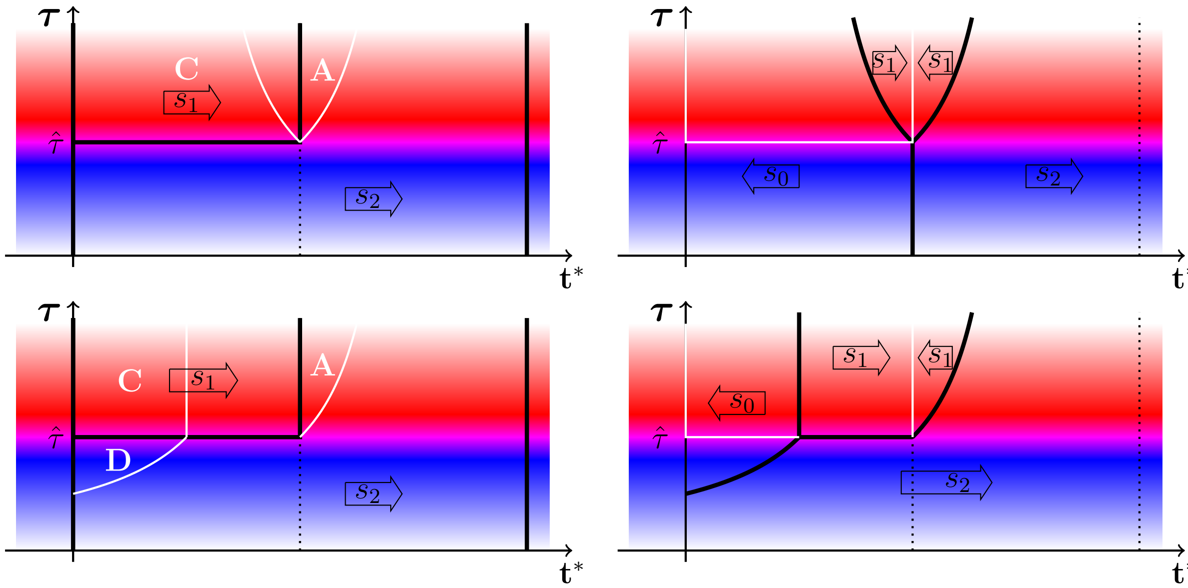

Figure 6: **Impact of information**

Top graphs: Passengers distribution with prices $\mathbf{p_1 = p_h > p_l = p_0 = p_2}$

On the left, uninformed passengers wait until t_2 if their value of time is low ($\tau < \hat{\tau}$); they take s_1 otherwise. With information, some passengers, rather than waiting for s_2, anticipate departure slightly and take s_1 (area **A**), in spite of the higher price ($p_1 > p_2$). Others, who patronize s_1 without information, anticipate departure even more and take s_0 (area **C**) in order to take advantage of the lower price $p_0 < p_1$.

On the right, informed passengers pay a higher price to use s_1 only if it is scheduled immediately before or after their ideal departure time, and they find it costly to wait ($\tau > \hat{\tau}$).

Bottom graphs: Passengers distribution with prices $\mathbf{p_0 = p_1 = p_h > p_l = p_2}$

On the left, uninformed passengers wait until t_1 and even until t_2, if they are sufficiently patient. With information, those with not-too-low value of time who would like to depart early enough, switch to s_0 (areas **C** and **D**); those with high value of time who would like to depart slightly later than t_1 (area **A**), anticipate departure and take s_1.

On the right, informed passengers pay a higher price to use s_1 only if the service is scheduled sufficiently close to their ideal departure time, and they dislike shifting departure ($\tau > \hat{\tau}$).

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A Uninformed passengers

A.1 Case U.1: $p_1 = p_h > p_2 = p_l$

A.1.1 The impact of a change in price

As similar results are obtained in all cases, we provide detailed calculations of the impact of a price change on surplus only for this case, omitting them for cases U.2 to U.4, to be reported below.

A marginal change in p_0 We compute

$$\begin{aligned}\frac{d\widetilde{V}_0}{dp_0} &= -\int_0^{+\infty}\left(\int_{t_{-1}}^{t_0} x\left(p_0+\tau\left\|t_0-t^*\right\|\right)ndt^*\right)dG\left(\tau\right)\\ &= -\tilde{X}_0,\end{aligned}$$

whereas $\left(d\overline{V}_1/dp_0\right)=0$ and $(d\underline{V}_2/dp_0)=0$. Hence,

$$\frac{dV}{dp_0} = -\tilde{X}_0.$$

A marginal change in p_1 We compute

$$\begin{aligned}\frac{d\overline{V}_1}{dp_1} &= -\frac{d\hat{\tau}}{dp_1}\left(\int_{t_0}^{t_1} v\left(p_1+\hat{\tau}\left\|t_1-t^*\right\|\right)ndt^*\right)g\left(\hat{\tau}\right)\\ &\quad -\int_{\hat{\tau}}^{+\infty}\left(\int_{t_0}^{t_1} x\left(p_1+\tau\left\|t_1-t^*\right\|\right)ndt^*\right)dG\left(\tau\right)\\ &= -\frac{d\hat{\tau}}{dp_1}\left(\int_{t_0}^{t_1} v\left(p_1+\hat{\tau}\left(t_1-t^*\right)\right)ndt^*\right)g\left(\hat{\tau}\right)-\bar{X}_1\end{aligned}$$

and

$$\frac{d\underline{V}_2}{dp_1} = \frac{d\hat{\tau}}{dp_1}\left(\int_{t_0}^{t_1} v\left(p_2+\hat{\tau}\left(t_2-t^*\right)\right)ndt^*\right)g\left(\hat{\tau}\right),$$

whereas $\left(d\widetilde{V}_0/dp_1\right) = 0$. By the definition of $\hat{\tau}$ we see that $p_1+\hat{\tau}\left(t_1 - t^*\right) = p_2+\hat{\tau}\left(t_2 - t^*\right)$ so that

$$\begin{aligned}\frac{dV}{dp_1} &= -\bar{X}_1 - \frac{d\hat{\tau}}{dp_1}\left(\int_{t_0}^{t_1} v\left(p_1 + \hat{\tau}\left(t_1 - t^*\right)\right) ndt^*\right) g\left(\hat{\tau}\right) \\ &+ \frac{d\hat{\tau}}{dp_1}\left(\int_{t_0}^{t_1} v\left(p_2 + \hat{\tau}\left(t_2 - t^*\right)\right) ndt^*\right) g\left(\hat{\tau}\right) \\ &= -\bar{X}_1.\end{aligned}$$

A marginal change in p_2 We compute

$$\frac{d\overline{V}_1}{dp_2} = -\frac{d\hat{\tau}}{dp_2}\left(\int_{t_0}^{t_1} v\left(p_1 + \hat{\tau}\left(t_1 - t^*\right)\right) ndt^*\right) g\left(\hat{\tau}\right)$$

and

$$\begin{aligned}\frac{d\underline{V}_2}{dp_2} &= \frac{d\hat{\tau}}{dp_2}\left(\int_{t_0}^{t_1} v\left(p_2 + \tau\left\|t_2 - t^*\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\ &- \int_0^{\hat{\tau}}\left(\int_{t_0}^{t_1} x\left(p_2 + \tau\left\|t_2 - t^*\right\|\right) ndt^*\right) dG\left(\tau\right) \\ &=\frac{d\hat{\tau}}{dp_2}\left(\int_{t_0}^{t_1} v\left(p_2 + \tau\left(t_2 - t^*\right)\right) ndt^*\right) g\left(\hat{\tau}\right) - \underline{X}_2,\end{aligned}$$

whereas $\left(d\widetilde{V}_0/dp_2\right) = 0$. Summing up and using again the equality $p_1 + \hat{\tau}\left(t_1 - t^*\right) = p_2 + \tau\left(t_2 - t^*\right)$, we obtain

$$\begin{aligned}\frac{dV}{dp_2} &= -\frac{d\hat{\tau}}{dp_2}\left(\int_{t_0}^{t_1} v\left(p_1 + \hat{\tau}\left(t_1 - t^*\right)\right) ndt^*\right) g\left(\hat{\tau}\right) \\ &+ \frac{d\hat{\tau}}{dp_2}\left(\int_{t_0}^{t_1} v\left(p_2 + \tau\left(t_2 - t^*\right)\right) ndt^*\right) g\left(\hat{\tau}\right) - \underline{X}_2 \\ &= -\underline{X}_2.\end{aligned}$$

A.1.2 The impact of a change in schedule

We compute

$$\begin{aligned}\frac{1}{n}\frac{d\widetilde{V}_0}{dT} &= 0 + \int_0^{+\infty}\left(\frac{dt_0}{dT}v\left(p_0\right) - \frac{dt_{-1}}{dT}v\left(p_0 + \tau\Delta T\right)\right) dG\left(\tau\right) \\ &- \int_0^{+\infty}\left(\int_{t_{-1}}^{t_0} \frac{dt_0}{dT}\tau x\left(p_0 + \tau\left(t^* - t_0\right)\right) dt^*\right) dG\left(\tau\right).\end{aligned}$$

With t_0 the reference point, $(dt_0/dT) = 0$ and $(dt_{-1}/dT) = -1$ and we have

$$\frac{d\widetilde{V}_0}{dT} = \int_0^{+\infty} v\left(p_0 + \tau\Delta T\right) ndG\left(\tau\right).$$

We next compute

$$\begin{aligned}\frac{1}{n}\frac{d\overline{V}_1}{dT} = & -\frac{d\hat{\tau}}{dT}\left(\int_{t_0}^{t_1} v\left(p_1 + \widehat{\tau}\left(t_1 - t^*\right)\right) dt^*\right) g\left(\widehat{\tau}\right) \\ & + \int_{\widehat{\tau}}^{+\infty}\left(\frac{dt_1}{dT}v\left(p_1\right) - \frac{dt_0}{dT}v\left(p_1 + \tau\Delta T\right)\right) dG\left(\tau\right) \\ & - \int_{\widehat{\tau}}^{+\infty}\left(\int_{t_0}^{t_1}\frac{dt_1}{dT}\tau x\left(p_1 + \tau\left(t_1 - t^*\right)\right) dt^*\right) dG\left(\tau\right).\end{aligned}$$

Being based on the definitions of $\bar{X}_1$ and $\bar{\tau}_1$, and considering again that t_0 is the reference point, we can further write

$$\frac{d\overline{V}_1}{dT} = -\frac{d\hat{\tau}}{dT}\left(\int_{t_0}^{t_1} v\left(p_1 + \widehat{\tau}\left(t_1 - t^*\right)\right) ndt^*\right) g\left(\widehat{\tau}\right) - \bar{\tau}_1\bar{X}_1 + n\left(1 - G\left(\widehat{\tau}\right)\right) v\left(p_1\right).$$

We also compute

$$\begin{aligned}\frac{1}{n}\frac{d\underline{V}_2}{dT} = & \frac{d\hat{\tau}}{dT}\left(\int_{t_0}^{t_1} v\left(p_2 + \widehat{\tau}\left(t_2 - t^*\right)\right) dt^*\right) g\left(\widehat{\tau}\right) \\ & + \int_0^{\widehat{\tau}}\left(\frac{dt_1}{dT}v\left(p_2 + \tau\Delta T\right) - \frac{dt_0}{dT}v\left(p_2 + 2\tau\Delta T\right)\right) dG\left(\tau\right) \\ & - \int_0^{\widehat{\tau}}\left(\int_{t_0}^{t_1}\frac{dt_2}{dT}\tau x\left(p_2 + \tau\left(t_2 - t^*\right)\right) dt^*\right) dG\left(\tau\right).\end{aligned}$$

With t_0 the reference point, $(dt_1/dT) = 1$ and $(dt_2/dT) = 2$ and we have

$$\frac{d\underline{V}_2}{dT} = \frac{d\hat{\tau}}{dT}\left(\int_{t_0}^{t_1} v\left(p_2 + \widehat{\tau}\left(t_2 - t^*\right)\right) ndt^*\right) g\left(\widehat{\tau}\right) - 2\underline{\tau}_2\underline{X}_2 + \int_0^{\widehat{\tau}} v\left(p_2 + \tau\Delta T\right) dG\left(\tau\right).$$

Overall:

$$\begin{aligned}\frac{dV}{dT} & = \frac{d\widetilde{V}_0}{dT} + \frac{d\overline{V}_1}{dT} + \frac{d\underline{V}_2}{dT} \\ & = -\bar{\tau}_1\bar{X}_1 - 2\underline{\tau}_2\underline{X}_2 \\ & \quad + \frac{d\hat{\tau}}{dT}\left(\int_{t_0}^{t_1}\left[v\left(p_2 + \widehat{\tau}\left(t_2 - t^*\right)\right) - v\left(p_1 + \widehat{\tau}\left(t_1 - t^*\right)\right)\right] ndt^*\right) g\left(\widehat{\tau}\right) \\ & \quad + \int_0^{+\infty} v\left(p_0 + \tau\Delta T\right) ndG\left(\tau\right) + n\left(1 - G\left(\widehat{\tau}\right)\right) v\left(p_1\right) \\ & \quad + \int_0^{\widehat{\tau}} v\left(p_2 + \tau\Delta T\right) ndG\left(\tau\right).\end{aligned}$$

By definition, $p_2 + \widehat{\tau}(t_2 - t^*) = p_2 + \widehat{\tau}\Delta T + \widehat{\tau}(t_1 - t^*) = p_1 + \widehat{\tau}(t_1 - t^*)$, and we can rewrite

$$\begin{aligned}\frac{dV}{dT} &= -\bar{\tau}_1 \bar{X}_1 - 2\underline{\tau}_2 \underline{X}_2 + n\left(1 - G\left(\widehat{\tau}\right)\right) v\left(p_1\right) \\ &+ \int_0^{+\infty} v\left(p_0 + \tau\Delta T\right) ndG\left(\tau\right) + \int_0^{\widehat{\tau}} v\left(p_2 + \tau\Delta T\right) ndG\left(\tau\right).\end{aligned}$$

Neglecting the last three terms, which capture volume effects, yields (5).

A.2 Case U.2: $p_1 = p_l < p_h = p_0$

Passengers with $t^* \in [t_{-1}, t_0]$ would all take s_0, if they were to choose the service considering the sole departure time. Because price matters as well, only those who are impatient (with $\tau > \widehat{\tau}$) are not attracted by the price saving available with s_1, and do take s_0. Accordingly, the partial and total demand for s_0, and the average value of time of its patrons are given by

$$\begin{aligned}X_0\left(\tau\right) &= \int_{t_{-1}}^{t_0} x\left(p_0 + \tau\left\|t_0 - t^*\right\|\right) ndt^* \\ \overline{X}_0 &= \int_{\widehat{\tau}}^{+\infty} X_0\left(\tau\right) dG\left(\tau\right) \\ \overline{\tau}_0 &= \int_{\widehat{\tau}}^{+\infty} \tau \frac{X_0\left(\tau\right)}{\overline{X}_0} dG\left(\tau\right).\end{aligned}$$

By contrast, patient passengers (with $\tau \leq \widehat{\tau}$) prefer to wait for s_1 and save some money. The partial and total demand for s_1, and the average value of time of its patrons are given by

$$\begin{aligned}X_{0\rhd 1}\left(\tau\right) &= \int_{t_{-1}}^{t_0} x\left(p_1 + \tau\left\|t_1 - t^*\right\|\right) ndt^* \\ \underline{X}_1 &= \int_0^{\widehat{\tau}} \underline{X}_1\left(\tau\right) dG\left(\tau\right) \\ \underline{\tau}_1 &= \int_0^{\widehat{\tau}} \tau \frac{X_{0\rhd 1}\left(\tau\right)}{\underline{X}_1} dG\left(\tau\right),\end{aligned}$$

where the subscript $0 \rhd 1$ is used to indicate passengers who would prefer to depart at t_0 and, yet, take s_1.

For passengers with $t^* \in [t_0, t_1]$, s_1 is the best matching in terms of departure time. Provided $p_1 = p_l \leq p_2$, there is no benefit to postponing departure, and they all stick to s_1. Thus, whereas there is no demand for s_2, the partial and total demand for s_1 and the value of time are given by

$$\begin{aligned}X_1(\tau) &= \int_{t_0}^{t_1} x(p_1+\tau\|t_1-t^*\|)\, ndt^* \\ \tilde{X}_1 &= \int_0^{+\infty} \tilde{X}_1(\tau)\, dG(\tau) \\ \tilde{\tau}_1 &= \int_0^{+\infty} \tau \frac{X_1(\tau)}{\tilde{X}_1} dG(\tau).\end{aligned}$$

In definitive, $\overline{X}_0$, $\underline{X}_1$, and $\tilde{X}_1$ are respectively the aggregate demand of passengers who choose to take s_0, prefer to wait for s_1 although they could have taken s_0, and take s_1 having no incentive to wait for a later and equally expensive service. A graphical representation is provided in Figure 7. We also write the surplus respectively associated with s_0 and s_1 as follows:

$$\overline{V}_0 = \int_{\widehat{\tau}}^{+\infty} \left(\int_{t_{-1}}^{t_0} v(p_0+\tau\|t_0-t^*\|)\, ndt^* \right) dG(\tau)$$

and

$$\begin{aligned}\underline{V}_1 + \widetilde{V}_1 &= \int_0^{\widehat{\tau}} \left(\int_{t_{-1}}^{t_0} v(p_1+\tau\|t_1-t^*\|)\, ndt^* \right) dG(\tau) \\ &\quad + \int_0^{+\infty} \left(\int_{t_0}^{t_1} v(p_1+\tau\|t_1-t^*\|)\, ndt^* \right) dG(\tau).\end{aligned}$$

A.2.1 The impact of a change in price

As expected, we have

$$\begin{aligned}\frac{dV}{dp_0} &= \frac{d\overline{V}_0}{dp_0} \\ &= -\bar{X}_0.\end{aligned}$$

An increase in the price of the early but expensive service would hinder high$-\tau$ (high-income) passengers only. The overall loss of surplus would be greater, if the cheaper service s_1 were not available, provided this service attracts some of the passengers who were using s_0 prior to the price increase. Similarly, we have

$$\begin{aligned}\frac{dV}{dp_1} &= \frac{d\underline{V}_1}{dp_0} + \frac{d\widetilde{V}_1}{dp_0} \\ &= -\left(\underline{X}_1 + \tilde{X}_1\right),\end{aligned}$$

showing that a variation in the price of the late but cheap service would have more pervasive effects. In addition to high-income passengers, it would concern also (and above all) low-income (low$-\tau$) passengers. A policy-maker, who were to decrease the price of the late but cheap service, at the aim of favouring little wealthy passengers, would end

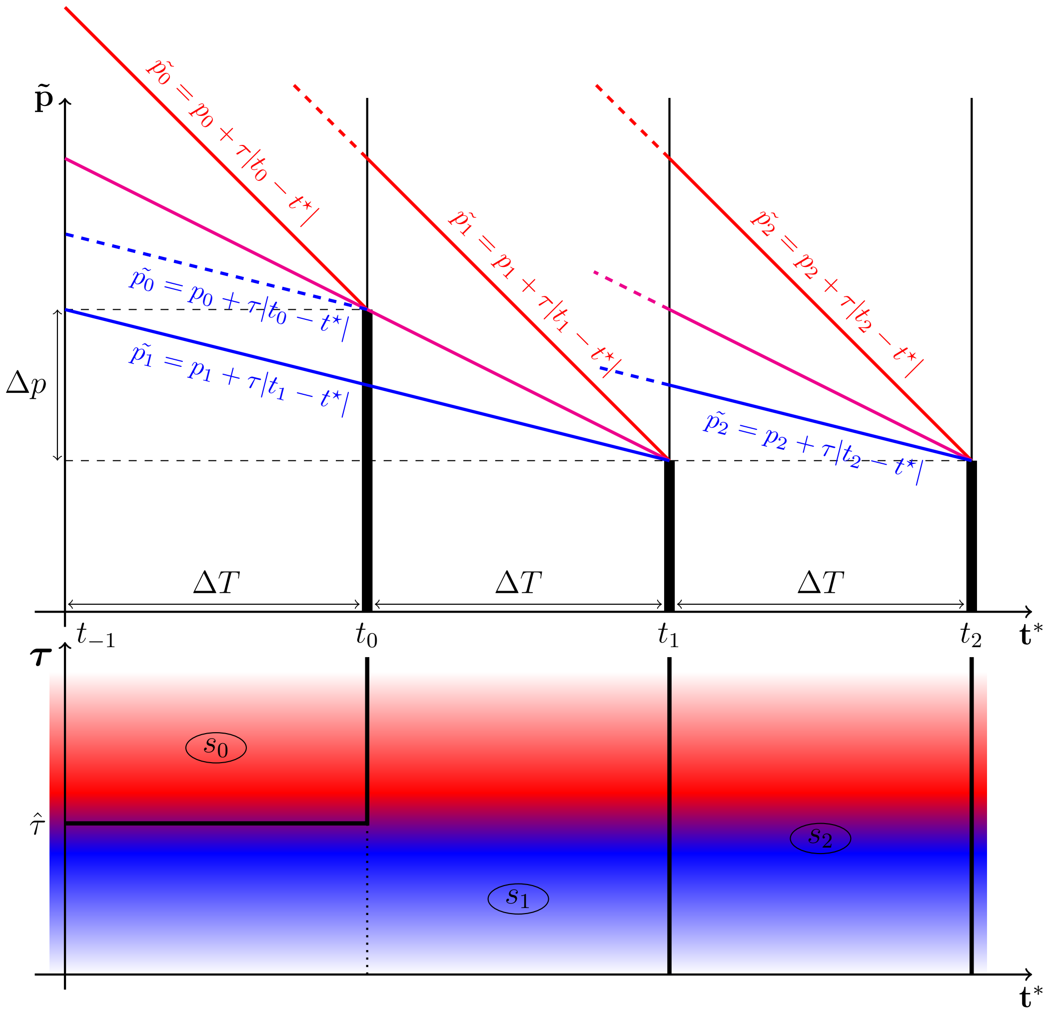

Figure 7: **Uninformed passengers - Case U.2:** $\mathbf{p_1 = p_l < p_h = p_0}$

Top graph: The generalised price is plotted against the ideal departure time. The blue line represents the generalised price of a patient passenger ($\tau < \hat{\tau}$), the red line that of an impatient passenger ($\tau > \hat{\tau}$), the magenta line that of a passenger with $\tau = \hat{\tau}$. The individual generalised price decreases to the monetary price (the thick black vertical line placed in the service locations) as the departure time approaches the ideal one, t^*.

Bottom graph: Passenger distribution over the two heterogeneity dimensions (t^*, τ) and across services. Colors refer to the value of time τ; the intensity is associated with the number of passengers displaying the characteristics (t^*, τ). For this representation, we assume it independent of t^*. Patient passengers, who would use s_0 if it were cheap, wait until t_1 to save on the price.

up benefiting also passengers with high earnings through that same policy.

A.2.2 The impact of a change in schedule

We compute

$$\frac{1}{n}\frac{d\overline{V}_0}{dT} = -\frac{d\widehat{\tau}}{dT}\left(\int_{t_{-1}}^{t_0} v\left(p_0+\widehat{\tau}\left\|t_0-t^*\right\|\right)dt^*\right)g\left(\widehat{\tau}\right) + \int_{\widehat{\tau}}^{+\infty}\left(\frac{dt_0}{dT}v\left(p_0\right)-\frac{dt_{-1}}{dT}v\left(p_0+\tau\Delta T\right)\right)dG\left(\tau\right) - \int_{\widehat{\tau}}^{+\infty}\left(\int_{t_{-1}}^{t_0}\frac{dt_0}{dT}\tau x\left(p_0+\tau\left\|t_0-t^*\right\|\right)dt^*\right)dG\left(\tau\right).$$

With t_0 the reference point, t_{-1} decreases by dT, t_1 increases by dT (and t_2 by $2dT$) so that

$$\frac{d\overline{V}_0}{dT} = \frac{\widehat{\tau}}{\Delta T}\left(\int_{t_{-1}}^{t_0} v\left(p_0+\widehat{\tau}\left\|t_0-t^*\right\|\right)ndt^*\right)g\left(\widehat{\tau}\right) + \int_{\widehat{\tau}}^{+\infty} v\left(p_0+\tau\Delta T\right)ndG\left(\tau\right),$$

where the last term is associated with the additional passengers using s_0 as the time interval around s_0 is extended. As t_0 is fixed, the change in schedule does not concern any of the passengers already using s_0. The only effect associated with s_0 pertains to the passengers who come to take this service following the change. Similarly, we compute

$$\frac{1}{n}\left(\frac{d\underline{V}_1}{dT}+\frac{d\widetilde{V}_1}{dT}\right) = \frac{d\widehat{\tau}}{dT}\left(\int_{t_{-1}}^{t_0} v\left(p_1+\widehat{\tau}\left\|t_1-t^*\right\|\right)dt^*\right)g\left(\widehat{\tau}\right) + \int_0^{\widehat{\tau}}\left(\frac{dt_0}{dT}v\left(p_1+\tau\Delta T\right)-\frac{dt_{-1}}{dT}v\left(p_1+2\tau\Delta T\right)\right)dG\left(\tau\right) - \int_0^{\widehat{\tau}}\left(\int_{t_{-1}}^{t_0}\frac{dt_1}{dT}\tau x\left(p_1+\tau\left\|t_1-t^*\right\|\right)dt^*\right)dG\left(\tau\right) + 0 + \int_0^{+\infty}\left(\frac{dt_1}{dT}v\left(p_1\right)-\frac{dt_0}{dT}v\left(p_1+\tau\Delta T\right)\right)dG\left(\tau\right) - \int_0^{+\infty}\left(\int_{t_0}^{t_1}\frac{dt_1}{dT}\tau x\left(p_1+\tau\left\|t_1-t^*\right\|\right)dt^*\right)dG\left(\tau\right),$$

which is rewritten as

$$\frac{d\underline{V}_1}{dT}+\frac{d\widetilde{V}_1}{dT} = -\underline{\tau}_1\underline{X}_1-\tilde{\tau}_1\tilde{X}_1-\frac{\widehat{\tau}}{\Delta T}\left(\int_{t_{-1}}^{t_0} v\left(p_1+\widehat{\tau}\left\|t_1-t^*\right\|\right)ndt^*\right)g\left(\widehat{\tau}\right) + nv\left(p_1\right)+\int_0^{\widehat{\tau}} v\left(p_1+2\tau\Delta T\right)ndG\left(\tau\right),$$

where again the last two terms capture volume effects. As the departure of s_1 is postponed, there is a double reduction in the surplus derived from the use of s_1: passengers with a value of time of $\widehat{\tau}$ shift to the previous service; those who stick to s_1 all wait longer. The total effect on surplus amounts to

$$\begin{aligned}
\frac{dV}{dT} &= \frac{d\overline{V}_0}{dT} + \frac{d\underline{V}_1}{dT} + \frac{d\widetilde{V}_1}{dT} \\
&= -\underline{\tau}_1 \underline{X}_1 - \tilde{\tau}_1 \tilde{X}_1 \\
&+ \frac{\widehat{\tau}}{\Delta T} \left(\int_{t_{-1}}^{t_0} v\left(p_0 + \widehat{\tau} \left\| t_0 - t^* \right\|\right) ndt^* \right) g\left(\widehat{\tau}\right) \\
&- \frac{\widehat{\tau}}{\Delta T} \left(\int_{t_{-1}}^{t_0} v\left(p_1 + \widehat{\tau} \left\| t_1 - t^* \right\|\right) ndt^* \right) g\left(\widehat{\tau}\right) \\
&+ \int_{\widehat{\tau}}^{+\infty} v\left(p_0 + \tau \Delta T\right) ndG\left(\tau\right) + \int_0^{\widehat{\tau}} v\left(p_1 + 2\tau \Delta T\right) ndG\left(\tau\right) + nv\left(p_1\right).
\end{aligned}$$

Using $p_0 + \widehat{\tau} \left\| t_0 - t^* \right\| = p_1 + \widehat{\tau} \left\| t_1 - t^* \right\|$ and neglecting the last three terms (to rule out any volume effects), the aggregate impact of a variation in schedule boils down to

$$\begin{aligned}
\frac{dV}{dT} &= \frac{dV_0}{dT} + \frac{d\underline{V}_1}{dT} + \frac{d\widetilde{V}_1}{dT} \\
&= -\left(\underline{\tau}_1 \underline{X}_1 + \tilde{\tau}_1 \tilde{X}_1 \right). \qquad (7)
\end{aligned}$$

Since s_1 is cheaper than s_0, the postponement of s_1 affects not only passengers with $t^* \in [t_0; t_1]$ but also passengers with $t^* \in [t_{-1}; t_0]$ and $\tau \leq \hat{\tau}$.

A.3 Case U.3: $p_0 \leq p_1 \leq p_2$

Under this price ordering, there is no point for any passengers to wait for the subsequent service. They will all take advantage of the first available departure. With similar notation to the previous case, we let the partial and total demand for service $i = 0, 1$, and the value of time of its patrons respectively be

$$\begin{aligned}
X_i\left(\tau\right) &= \int_{t_{i-1}}^{t_i} x\left(p_i + \tau \left\| t_i - t^* \right\|\right) ndt^* \\
\tilde{X}_i &= \int_0^{+\infty} X_i\left(\tau\right) dG\left(\tau\right) \\
\tilde{\tau}_i &= \int_0^{+\infty} \tau \frac{X_i\left(\tau\right)}{\tilde{X}_i} dG\left(\tau\right).
\end{aligned}$$

No demand for s_2 is expressed by passengers with $t^* \in \left[t_1 - 2\Delta T, t_1\right]$. A graphical representation is provided in Figure 8. The surplus associated with s_i, $i = 0, 1$, is given by

$$\widetilde{V}_i = \int_0^{+\infty} \left(\int_{t_{i-1}}^{t_i} v\left(p_i + \tau \left\| t_i - t^* \right\|\right) ndt^* \right) dG\left(\tau\right).$$

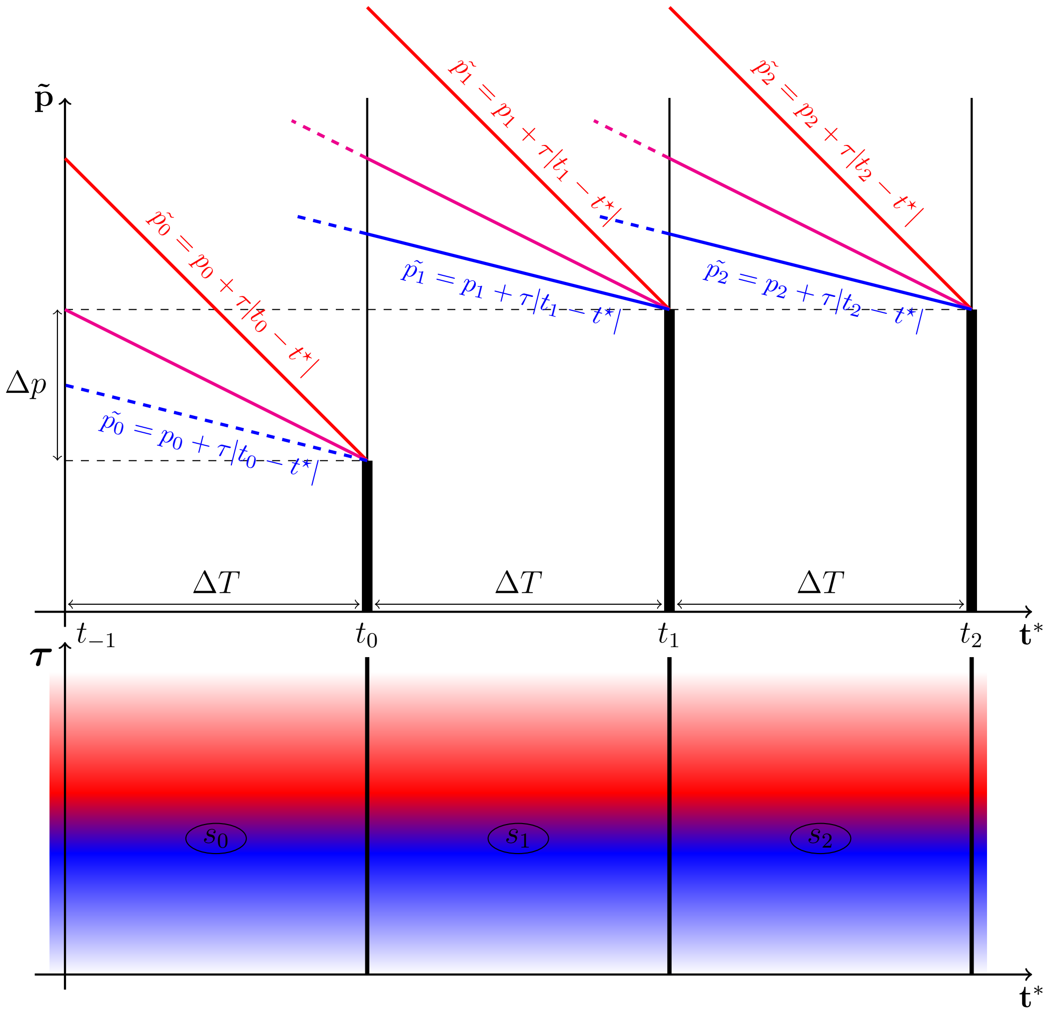

Figure 8: **Uninformed passengers - Case U.3: $\mathbf{p_0 \leq p_1 \leq p_2}$**
Top graph: The generalised price is plotted against the ideal departure time. The blue lines are associated with lower values of time than the magenta lines; in turn, these are associated with lower values of time than the red lines. The individual generalised price decreases to the monetary price (the thick black vertical line placed in the service locations) as the departure time approaches the ideal one, t^*.
Bottom graph: Passenger distribution over the two heterogeneity dimensions (t^*, τ) and across services. Colors refer to the value of time τ; the intensity is associated with the number of passengers displaying the characteristics (t^*, τ). For this representation, we assume it independent of t^*. No passenger has any interest in waiting for later services since s_1 and s_2 are both more expensive than s_0.

A.3.1 The impact of a change in price

In this context, a change in price p_i alters neither the distribution of passengers across services nor their WT. It only affects the surplus of those who take s_i. We thus find the following usual result:

$$\frac{dV}{dp_i} = -\tilde{X}_i.$$

A.3.2 The impact of a change in schedule

With t_0 the reference departure time, an infinitesimal increase in the time interval ΔT around t_0 involves that s_1 is re-scheduled at $t_1 + dT$, whereas the service preceding s_0 is re-scheduled at $t_{-1} - dT$. First take $i = 0$ and compute

$$\begin{aligned}\frac{1}{n}\frac{d\widetilde{V}_0}{dT} = 0 + \int_0^{+\infty}\left(\frac{dt_0}{dT}v\left(p_0\right) - \frac{dt_{-1}}{dT}v\left(p_0 + \tau\Delta T\right)\right)dG\left(\tau\right) \\ - \int_0^{+\infty}\left(\int_{t_{-1}}^{t_0}\frac{dt_0}{dT}\tau x\left(p_0 + \tau\left(t^* - t_0\right)\right)dt^*\right)dG\left(\tau\right).\end{aligned}$$

Using $(dt_0/dT) = 0$ and $(dt_{-1}/dT) = -1$, this reduces to

$$\frac{d\widetilde{V}_0}{dT} = \int_0^{+\infty} v\left(p_0 + \tau\Delta T\right)ndG\left(\tau\right).$$

Next take $i = 1$ and compute

$$\begin{aligned}\frac{1}{n}\frac{d\widetilde{V}_1}{dT} = 0 + \int_0^{+\infty}\left(\frac{dt_1}{dT}v\left(p_1\right) - \frac{dt_0}{dT}v\left(p_1 + \tau\Delta T\right)\right)dG\left(\tau\right) \\ - \int_0^{+\infty}\left(\int_{t_0}^{t_1}\frac{dt_1}{dT}\tau x\left(p_1 + \tau\left(t_1 - t^*\right)\right)dt^*\right)dG\left(\tau\right),\end{aligned}$$

which further yields

$$\frac{d\widetilde{V}_1}{dT} = nv\left(p_1\right) - \widetilde{\tau}_1\widetilde{X}_1.$$

Overall:

$$\begin{aligned}\frac{dV}{dT} &= \frac{d\widetilde{V}_0}{dT} + \frac{d\widetilde{V}_1}{dT} \\ &= -\widetilde{\tau}_1\widetilde{X}_1 + \int_0^{+\infty} v\left(p_0 + \tau\Delta T\right)dG\left(\tau\right) + nv\left(p_1\right).\end{aligned}$$

Neglecting the last two terms, we end up with

$$\frac{dV}{dT} = -\tilde{\tau}_1\tilde{X}_1. \tag{8}$$

A.4 Case U.4: $p_0 = p_1 = p_h > p_l = p_2$

Given that s_1 is as expensive as s_0, for passengers who can take s_0 there is no point to wait for s_1. These are passengers with $t^* \in [t_{-1}, t_0]$, who reach the station before t_0 and choose between s_0 and s_2. They prefer the latter if and only if $\tau \leq \hat{\tau}/2$: only very patient passengers are prone to wait until t_2 to benefit from the price saving. Passengers with $t^* \in [t_0, t_1]$ choose between s_1, which departs earlier, and s_2, which is cheaper. The latter is preferred if and only if $\tau \leq \hat{\tau}$: only sufficiently patient passengers are available to wait until t_2 to pay less. Because two cut-off values of time are relevant, the distribution of passengers across services depends on whether τ takes low, intermediate, or high values. The partial and total demand for s_0 are respectively given by

$$X_0(\tau) = \int_{t_{-1}}^{t_0} x\left(p_0 + \tau \left\|t_0 - t^*\right\|\right) ndt^*$$
$$X_0 = \int_{\frac{\hat{\tau}}{2}}^{\infty} X_0(\tau)\, dG(\tau),$$

those for s_1 are given by

$$X_1(\tau) = \int_{t_0}^{t_1} x\left(p_1 + \tau \left\|t_1 - t^*\right\|\right) ndt^*$$
$$X_1 = \int_{\hat{\tau}}^{\infty} X_1(\tau)\, dG(\tau).$$

For s_2 one has

$$X_{0\triangleright 2}(\tau) = \int_{t_{-1}}^{t_0} x\left(p_2 + \tau \left\|t_2 - t^*\right\|\right) ndt^*$$
$$X_{1\triangleright 2}(\tau) = \int_{t_0}^{t_1} x\left(p_2 + \tau \left\|t_2 - t^*\right\|\right) ndt^*$$
$$X_2 = \underline{\underline{X}}_2 + \underline{X}_2,$$

where

$$\underline{\underline{X}}_2 = \int_0^{\frac{\hat{\tau}}{2}} X_{0\triangleright 2}(\tau) dG(\tau) \quad \text{and} \quad \underline{X}_2 = \int_0^{\hat{\tau}} X_{1\triangleright 2}(\tau) dG(\tau),$$

and the subscripts $0 \triangleright 2$ and $1 \triangleright 2$ respectively refer to passengers who would depart at t_0 and t_1, if s_2 were not cheaper than s_0 and s_1. A graphical representation is provided in Figure 9, which evidences that s_2 attracts some passengers from both s_0 and s_1. The total surplus is given by

$$V = V_0 + V_1 + \underline{\underline{V}}_2 + \underline{V}_2,$$

where

$$
\begin{aligned}
V_0 &= \int_{\frac{\widehat{\tau}}{2}}^{\infty} \left(\int_{t_{-1}}^{t_0} v\left(p_0 + \tau \left\| t_0 - t^* \right\|\right) n dt^* \right) dG(\tau) \\
V_1 &= \int_{\widehat{\tau}}^{\infty} \left(\int_{t_0}^{t_1} v\left(p_1 + \tau \left\| t_1 - t^* \right\|\right) n dt^* \right) dG(\tau) \\
\underline{\underline{V}}_2 &= \int_{0}^{\frac{\widehat{\tau}}{2}} \left(\int_{t_{-1}}^{t_0} v\left(p_2 + \tau \left\| t_2 - t^* \right\|\right) n dt^* \right) dG(\tau) \\
\underline{V}_2 &= \int_{0}^{\widehat{\tau}} \left(\int_{t_0}^{t_1} v\left(p_2 + \tau \left\| t_2 - t^* \right\|\right) n dt^* \right) dG(\tau).
\end{aligned}
$$

A.4.1 The impact of a change in price

As in previous cases, a change in any of the prices determines a 'standard' reduction in total surplus:

$$\frac{dV}{dp_i} = -X_i, \ \forall i \in \{0,1,2\}.$$

A.4.2 The impact of a change in schedule

We compute

$$
\begin{aligned}
\frac{1}{n}\frac{dV_0}{dT} &= -\frac{1}{2}\frac{d\widehat{\tau}}{dT}\left(\int_{t_{-1}}^{t_0} v\left(p_0 + \frac{\widehat{\tau}}{2}\left\| t_0 - t^* \right\|\right) dt^* \right) g\left(\frac{\widehat{\tau}}{2}\right) \\
&\quad - \frac{dt_{-1}}{dT}\int_{\frac{\widehat{\tau}}{2}}^{\infty} v\left(p_0 + \tau \Delta T\right) dG(\tau) + \frac{dt_0}{dT}\int_{\frac{\widehat{\tau}}{2}}^{\infty} v\left(p_0\right) dG(\tau) \\
&\quad - \int_{\frac{\widehat{\tau}}{2}}^{\infty} \tau \left(\int_{t_{-1}}^{t_0} \frac{dt_0}{dT} x\left(p_0 + \tau \left\| t_0 - t^* \right\|\right) dt^* \right) dG(\tau) \\
&= -\frac{1}{2}\frac{d\widehat{\tau}}{dT}\left(\int_{t_{-1}}^{t_0} v\left(p_0 + \frac{\widehat{\tau}}{2}\left\| t_0 - t^* \right\|\right) dt^* \right) g\left(\frac{\widehat{\tau}}{2}\right) \\
&\quad - \frac{dt_{-1}}{dT}\int_{\frac{\widehat{\tau}}{2}}^{\infty} v\left(p_0 + \tau \Delta T\right) dG(\tau) + 0 - 0
\end{aligned}
$$

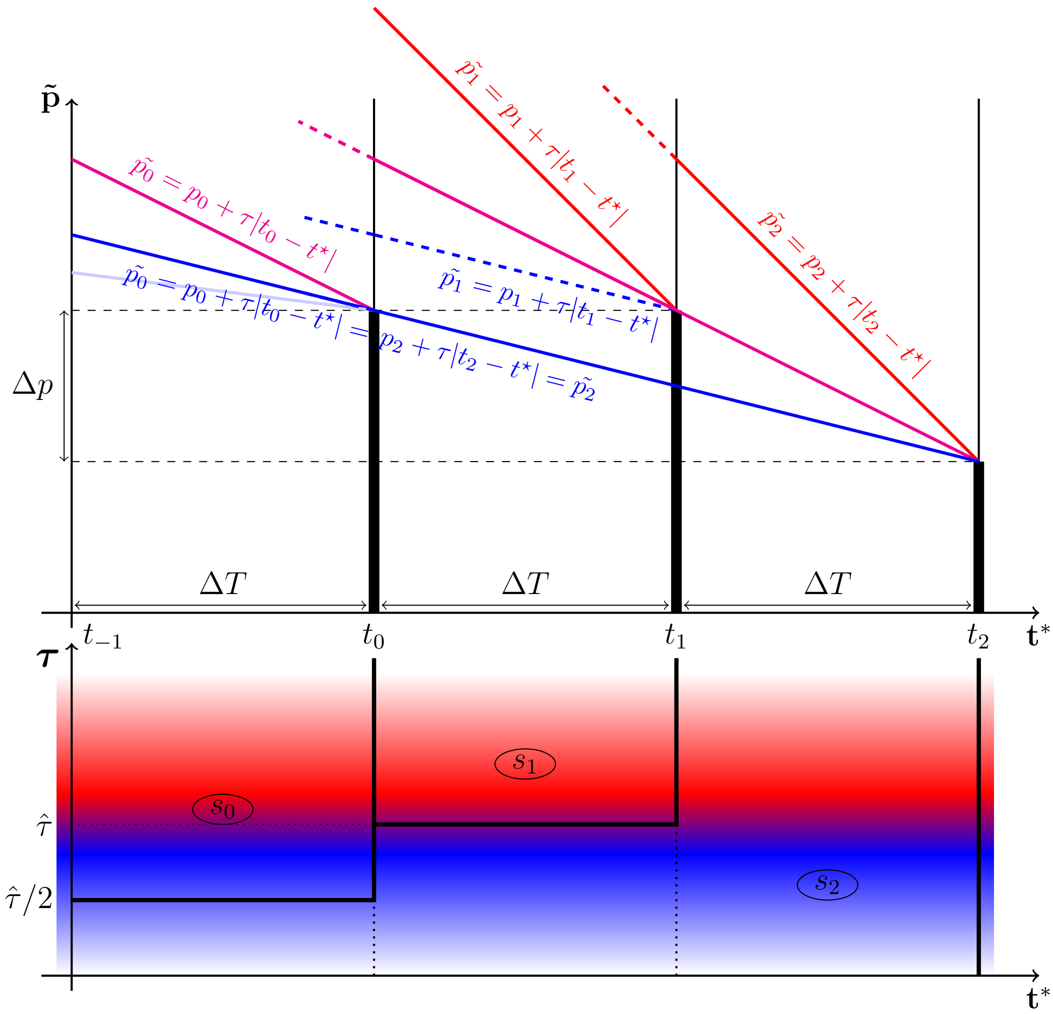

Figure 9: **Uninformed passengers - Case U.4:** $\mathbf{p_0 \geq p_1 = p_h > p_l = p_2}$
Top graph: The generalised price is plotted against the ideal departure time. The red line represents the generalised price of an impatient passenger ($\tau > \hat{\tau}$), the magenta line that of a passenger with $\tau = \hat{\tau}$. The blue line represents the generalised price of a passenger with $\tau = \hat{\tau}/2$. The light blue line shows that for passengers with a strictly value of time below (above) $\hat{\tau}/2$, it is (not) worth waiting $2\Delta T$ in order to face a price cut of Δp. The individual generalised price decreases to the monetary price (the thick black vertical line placed in the service locations) as the departure time approaches the ideal one, t^*.
Bottom graph: Passenger distribution over the two heterogeneity dimensions (t^*, τ) and across services. Colors refer to the value of time τ; the intensity is associated with the number of passengers displaying the characteristics (t^*, τ). For this representation, we assume it independent of t^*. Very patient passengers, who would use s_0 if it were cheap, and not too impatient passengers, who would use s_1 if it were cheap, all wait until t_2 to take advantage of a cheaper service.

and

$$\begin{aligned}
\frac{1}{n}\frac{dV_1}{dT} &= v\left(p_1\right)\int_{\widehat{\tau}}^{\infty}\frac{dt_1}{dT}dG\left(\tau\right) \\
&\quad -\int_{\widehat{\tau}}^{\infty}\tau\left(\int_{t_0}^{t_1}\frac{dt_1}{dT}x\left(p_1+\tau\left\|t_1-t^*\right\|\right)dt^*\right)dG\left(\tau\right) \\
&\quad -\int_{t_0}^{t_1}\frac{d\widehat{\tau}}{dT}\left(v\left(p_1+\widehat{\tau}\left\|t_1-t^*\right\|\right)dt^*\right)g\left(\widehat{\tau}\right) \\
&\quad -\int_{\widehat{\tau}}^{\infty}\frac{dt_0}{dT}v\left(p_1+\tau\Delta T\right)dG\left(\tau\right) \\
&= v\left(p_1\right)\int_{\widehat{\tau}}^{\infty}\frac{dt_1}{dT}dG\left(\tau\right)-\int_{\widehat{\tau}}^{\infty}\tau\left(\int_{t_0}^{t_1}\frac{dt_1}{dT}x\left(p_1+\tau\left\|t_1-t^*\right\|\right)dt^*\right)dG\left(\tau\right) \\
&\quad -\int_{t_0}^{t_1}\frac{d\widehat{\tau}}{dT}\left(v\left(p_1+\widehat{\tau}\left\|t_1-t^*\right\|\right)dt^*\right)g\left(\widehat{\tau}\right)-0
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{n}\frac{d\underline{\underline{V}}_2}{dT} &= \frac{1}{2}\frac{d\widehat{\tau}}{dT}\left(\int_{t_{-1}}^{t_0}v\left(p_2+\frac{\widehat{\tau}}{2}\left\|t_2-t^*\right\|\right)dt^*\right)g\left(\frac{\widehat{\tau}}{2}\right) \\
&\quad -\int_0^{\frac{\widehat{\tau}}{2}}\frac{dt_{-1}}{dT}v\left(p_2+3\tau\Delta T\right)dG\left(\tau\right) \\
&\quad -\int_0^{\frac{\widehat{\tau}}{2}}\left(\int_{t_{-1}}^{t_0}\frac{dt_2}{dT}\tau x\left(p_2+\tau\left\|t_2-t^*\right\|\right)dt^*\right)dG\left(\tau\right) \\
&\quad +\int_0^{\frac{\widehat{\tau}}{2}}\frac{dt_0}{dT}v\left(p_2+2\tau\Delta T\right)dG\left(\tau\right) \\
&= \frac{1}{2}\frac{d\widehat{\tau}}{dT}\left(\int_{t_{-1}}^{t_0}v\left(p_2+\frac{\widehat{\tau}}{2}\left\|t_2-t^*\right\|\right)dt^*\right)g\left(\frac{\widehat{\tau}}{2}\right) \\
&\quad -\int_0^{\frac{\widehat{\tau}}{2}}\frac{dt_{-1}}{dT}v\left(p_2+3\tau\Delta T\right)dG\left(\tau\right) \\
&\quad -\int_0^{\frac{\widehat{\tau}}{2}}\left(\int_{t_{-1}}^{t_0}\frac{dt_2}{dT}\tau x\left(p_2+\tau\left\|t_2-t^*\right\|\right)dt^*\right)dG\left(\tau\right)+0
\end{aligned}$$

and

$$
\begin{aligned}
\frac{1}{n}\frac{d\underline{V}_2}{dT} &= \int_0^{\widehat{\tau}} \frac{dt_1}{dT} v\left(p_2+\tau\Delta T\right) dG\left(\tau\right) \\
&+\left(\int_{t_0}^{t_1} \frac{d\widehat{\tau}}{dT} v\left(p_2+\widehat{\tau}\left\|t_2-t^*\right\|\right) dt^*\right) g\left(\widehat{\tau}\right) \\
&-\int_0^{\widehat{\tau}} \tau\left(\int_{t_0}^{t_1} \frac{dt_2}{dT} x\left(p_2+\tau\left\|t_2-t^*\right\|\right) dt^*\right) dG\left(\tau\right) \\
&-\int_0^{\widehat{\tau}} \frac{dt_0}{dT} v\left(p_2+2\tau\Delta T\right) dG\left(\tau\right) \\
&= \int_0^{\widehat{\tau}} \frac{dt_1}{dT} v\left(p_2+\tau\Delta T\right) dG\left(\tau\right)+\left(\int_{t_0}^{t_1} \frac{d\widehat{\tau}}{dT} v\left(p_2+\widehat{\tau}\left\|t_2-t^*\right\|\right) dt^*\right) g\left(\widehat{\tau}\right) \\
&-\int_0^{\widehat{\tau}} \tau\left(\int_{t_0}^{t_1} \frac{dt_2}{dT} x\left(p_2+\tau\left\|t_2-t^*\right\|\right) dt^*\right) dG\left(\tau\right)-0.
\end{aligned}
$$

Overall:

$$
\begin{aligned}
\frac{1}{n}\frac{dV}{dT} &= -\frac{1}{2}\frac{d\widehat{\tau}}{dT}\left(\int_{t_{-1}}^{t_0} v\left(p_0+\frac{\widehat{\tau}}{2}\left\|t_0-t^*\right\|\right) dt^*\right) g\left(\frac{\widehat{\tau}}{2}\right) \\
&-\frac{dt_{-1}}{dT}\int_{\frac{\widehat{\tau}}{2}}^{\infty} v\left(p_0+\tau\Delta T\right) dG\left(\tau\right)+v\left(p_1\right)\int_{\widehat{\tau}}^{\infty} \frac{dt_1}{dT} dG\left(\tau\right) \\
&-\int_{\widehat{\tau}}^{\infty} \tau\left(\int_{t_0}^{t_1} \frac{dt_1}{dT} x\left(p_1+\tau\left\|t_1-t^*\right\|\right) dt^*\right) dG\left(\tau\right) \\
&-\int_{t_0}^{t_1} \frac{d\widehat{\tau}}{dT}\left(v\left(p_1+\widehat{\tau}\left\|t_1-t^*\right\|\right) dt^*\right) g\left(\widehat{\tau}\right) \\
&+\frac{1}{2}\frac{d\widehat{\tau}}{dT}\left(\int_{t_{-1}}^{t_0} v\left(p_2+\frac{\widehat{\tau}}{2}\left\|t_2-t^*\right\|\right) dt^*\right) g\left(\frac{\widehat{\tau}}{2}\right) \\
&-\int_0^{\frac{\widehat{\tau}}{2}} \frac{dt_{-1}}{dT} v\left(p_2+3\tau\Delta T\right) dG\left(\tau\right) \\
&-\int_0^{\frac{\widehat{\tau}}{2}}\left(\int_{t_{-1}}^{t_0} \frac{dt_2}{dT}\tau x\left(p_2+\tau\left\|t_2-t^*\right\|\right) dt^*\right) dG\left(\tau\right) \\
&+\int_0^{\widehat{\tau}} \frac{dt_1}{dT} v\left(p_2+\tau\Delta T\right) dG\left(\tau\right) \\
&+\left(\int_{t_0}^{t_1} \frac{d\widehat{\tau}}{dT} v\left(p_2+\widehat{\tau}\left\|t_2-t^*\right\|\right) dt^*\right) g\left(\widehat{\tau}\right) \\
&-\int_0^{\widehat{\tau}} \tau\left(\int_{t_0}^{t_1} \frac{dt_2}{dT} x\left(p_2+\tau\left\|t_2-t^*\right\|\right) dt^*\right) dG\left(\tau\right).
\end{aligned}
$$

We rewrite

$$\begin{aligned}
\frac{1}{n}\frac{dV}{dT} &= -\frac{1}{2}\frac{d\widehat{\tau}}{dT}\left(\int_{t_{-1}}^{t_0} v\left(p_0+\frac{\widehat{\tau}}{2}\left\|t_0-t^*\right\|\right)dt^*\right)g\left(\frac{\widehat{\tau}}{2}\right)\\
&+\frac{1}{2}\frac{d\widehat{\tau}}{dT}\left(\int_{t_{-1}}^{t_0} v\left(p_2+\frac{\widehat{\tau}}{2}\left\|t_2-t^*\right\|\right)dt^*\right)g\left(\frac{\widehat{\tau}}{2}\right)\\
&-\int_{t_0}^{t_1}\frac{d\widehat{\tau}}{dT}\left(v\left(p_1+\widehat{\tau}\left\|t_1-t^*\right\|\right)dt^*\right)g\left(\widehat{\tau}\right)\\
&+\int_{t_0}^{t_1}\frac{d\widehat{\tau}}{dT}v\left(p_2+\widehat{\tau}\left\|t_2-t^*\right\|\right)dt^* g\left(\widehat{\tau}\right)\\
&-\int_0^{\frac{\widehat{\tau}}{2}}\frac{dt_{-1}}{dT}v\left(p_2+3\tau\Delta T\right)dG\left(\tau\right)-\int_{\frac{\widehat{\tau}}{2}}^{\infty}\frac{dt_{-1}}{dT}v\left(p_0+\tau\Delta T\right)dG\left(\tau\right)\\
&-\int_{\widehat{\tau}}^{\infty}\tau\left(\int_{t_0}^{t_1}\frac{dt_1}{dT}x\left(p_1+\tau\left\|t_1-t^*\right\|\right)dt^*\right)dG\left(\tau\right)\\
&-\int_0^{\frac{\widehat{\tau}}{2}}\tau\left(\int_{t_{-1}}^{t_0}\frac{dt_2}{dT}x\left(p_2+\tau\left\|t_2-t^*\right\|\right)dt^*\right)dG\left(\tau\right)\\
&+\int_0^{\widehat{\tau}}\frac{dt_1}{dT}v\left(p_2+\tau\Delta T\right)dG\left(\tau\right)+v\left(p_1\right)\int_{\widehat{\tau}}^{\infty}\frac{dt_1}{dT}dG\left(\tau\right)\\
&-\int_0^{\widehat{\tau}}\tau\left(\int_{t_0}^{t_1}\frac{dt_2}{dT}x\left(p_2+\tau\left\|t_2-t^*\right\|\right)dt^*\right)dG\left(\tau\right).
\end{aligned}$$

We have $p_0+\frac{\widehat{\tau}}{2}\left\|t_0-t^*\right\| = p_2+\frac{\widehat{\tau}}{2}\left\|t_2-t^*\right\|$. Indeed, using the definition of $\widehat{\tau}$, this is rewritten as

$$p_0+\frac{\Delta p}{2\Delta T}\left\|t_0-t^*\right\| = p_2+\frac{\Delta p}{2\Delta T}\left\|t_2-t^*\right\| \Leftrightarrow p_0=p_2+\Delta p,$$

which is true. We also have $p_1+\widehat{\tau}\left\|t_1-t^*\right\| = p_2+\widehat{\tau}\left\|t_2-t^*\right\|$. Indeed, this is equivalent to $p_1=p_2+\Delta p$, which is true. We end up with

$$\begin{aligned}
\frac{1}{n}\frac{dV}{dT} &= -\int_{\widehat{\tau}}^{\infty}\tau\left(\int_{t_0}^{t_1}\frac{dt_1}{dT}x\left(p_1+\tau\left\|t_1-t^*\right\|\right)dt^*\right)dG\left(\tau\right)\\
&-\int_0^{\frac{\widehat{\tau}}{2}}\tau\left(\int_{t_{-1}}^{t_0}\frac{dt_2}{dT}x\left(p_2+\tau\left\|t_2-t^*\right\|\right)dt^*\right)dG\left(\tau\right)\\
&-\int_0^{\widehat{\tau}}\tau\left(\int_{t_0}^{t_1}\frac{dt_2}{dT}x\left(p_2+\tau\left\|t_2-t^*\right\|\right)dt^*\right)dG\left(\tau\right)\\
&-\int_0^{\frac{\widehat{\tau}}{2}}\frac{dt_{-1}}{dT}v\left(p_2+3\tau\Delta T\right)dG\left(\tau\right)-\int_{\frac{\widehat{\tau}}{2}}^{\infty}\frac{dt_{-1}}{dT}v\left(p_0+\tau\Delta T\right)dG\left(\tau\right)\\
&+\int_0^{\widehat{\tau}}\frac{dt_1}{dT}v\left(p_2+\tau\Delta T\right)dG\left(\tau\right)+v\left(p_1\right)\int_{\widehat{\tau}}^{\infty}\frac{dt_1}{dT}dG\left(\tau\right).
\end{aligned}$$

Using $(dt_{-1}/dT) = -1$, $(dt_1/dT) = 1$, and $(dt_2/dT) = 2$, we further rewrite

$$\begin{aligned}
\frac{1}{n}\frac{dV}{dT} &= -\int_{\widehat{\tau}}^{\infty} \tau \left(\int_{t_0}^{t_1} x\left(p_1 + \tau \left\| t_1 - t^* \right\|\right) dt^* \right) dG\left(\tau\right) \\
&\quad -2\int_{0}^{\frac{\widehat{\tau}}{2}} \tau \left(\int_{t_{-1}}^{t_0} x\left(p_2 + \tau \left\| t_2 - t^* \right\|\right) dt^* \right) dG\left(\tau\right) \\
&\quad -2\int_{0}^{\widehat{\tau}} \tau \left(\int_{t_0}^{t_1} x\left(p_2 + \tau \left\| t_2 - t^* \right\|\right) dt^* \right) dG\left(\tau\right) \\
&\quad +\int_{0}^{\frac{\widehat{\tau}}{2}} v\left(p_2 + 3\tau\Delta T\right) dG\left(\tau\right) + \int_{\frac{\widehat{\tau}}{2}}^{\infty} v\left(p_0 + \tau\Delta T\right) dG\left(\tau\right) \\
&\quad +\int_{0}^{\widehat{\tau}} v\left(p_2 + \tau\Delta T\right) dG\left(\tau\right) + v\left(p_1\right)\int_{\widehat{\tau}}^{\infty} dG\left(\tau\right).
\end{aligned}$$

Letting

$$\tau_1 = \int_{\hat{\tau}}^{\infty} \tau \frac{X\left(\tau\right)}{X_1} dG\left(\tau\right), \quad \underline{\underline{\tau}}_2 = \int_{0}^{\frac{\hat{\tau}}{2}} \tau \frac{X_{0\triangleright 2}(\tau)}{\underline{\underline{X}}_2} dG(\tau) \quad \text{and} \quad \underline{\tau}_2 = \int_{0}^{\hat{\tau}} \tau \frac{X_{1\triangleright 2}(\tau)}{\underline{X}_2} dG(\tau),$$

and grouping terms, we ultimately obtain

$$\begin{aligned}
\frac{dV}{dT} &= -\tau_1 X_1 - 2(\underline{\underline{\tau}}_2 \underline{\underline{X}}_2 + \underline{\tau}_2 \underline{X}_2) \\
&\quad +\int_{\frac{\hat{\tau}}{2}}^{\infty} nv\left(p_0 + \tau\Delta T\right) dG\left(\tau\right) + n\left(1 - G\left(\hat{\tau}\right)\right) v\left(p_1\right) \\
&\quad +\int_{0}^{\frac{\widehat{\tau}}{2}} nv\left(p_2 + 3\tau\Delta T\right) dG\left(\tau\right) + \int_{0}^{\widehat{\tau}} nv\left(p_2 + \tau\Delta T\right) dG\left(\tau\right).
\end{aligned}$$

The terms in the second and third line all identify volume effects, to be neglected. Thus, the overall impact on surplus, working through $s_1$and s_2, amounts to

$$\frac{dV}{dT} = -\left[\tau_1 X_1 + 2\left(\underline{\underline{\tau}}_2 \underline{\underline{X}}_2 + \underline{\tau}_2 \underline{X}_2\right)\right], \tag{9}$$

The impact on surplus through s_2 accounts doubly, and includes the effect on very patient passengers and that on sufficiently patient passengers.

B Informed passengers

B.1 Case I.1: $p_1 = p_h > p_l = p_0 = p_2$

B.1.1 The impact of a change in price

Again, as similar results are obtained in all cases, we provide detailed calculations of the impact of a price change on surplus only for this case, omitting them for cases I.2 to I.4, to be reported below.

A marginal change in p_0 We first compute

$$\begin{aligned}
\frac{d\widetilde{V}_0}{dp_0} &= \frac{d\hat{\tau}}{dp_0}\left(\int_{t_0}^{t_1} v\left(p_0+\hat{\tau}\left\|t^*-t_0\right\|\right)ndt^*\right)g\left(\hat{\tau}\right) \\
&- \int_0^{\hat{\tau}}\left(\int_{t_0}^{t_1} x\left(p_0+\tau\left\|t^*-t_0\right\|\right)ndt^*\right)dG\left(\tau\right) \\
&- \frac{d\hat{\tau}}{dp_0}\left(\int_{t_0}^{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}} v\left(p_0+\hat{\tau}\left\|t^*-t_0\right\|\right)ndt^*\right)g\left(\hat{\tau}\right) \\
&- \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(v\left(p_0+\tau\left\|\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}-t_0\right\|\right)n\right)dG\left(\tau\right) \\
&- \int_{\hat{\tau}}^{+\infty}\left(\int_{t_0}^{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}} x\left(p_0+\tau\left\|t^*-t_0\right\|\right)ndt^*\right)dG\left(\tau\right)
\end{aligned}$$

Using $\left(d\hat{\tau}/dp_0\right)=-\left(1/\Delta T\right)$ and

$$p_0+\tau\left(\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}-t_0\right)=p_0+\left(\tau+\hat{\tau}\right)\frac{\Delta T}{2},$$

we rewrite

$$\begin{aligned}
\frac{d\widetilde{V}_0}{dp_0} &= \frac{-1}{\varDelta T}\left(\int_{t_0}^{t_1} v\left(p_0+\hat{\tau}\left\|t^*-t_0\right\|\right)ndt^*\right)g\left(\hat{\tau}\right) \\
&+ \frac{1}{\varDelta T}\left(\int_{t_0}^{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}} v\left(p_0+\hat{\tau}\left\|t^*-t_0\right\|\right)ndt^*\right)g\left(\hat{\tau}\right) \\
&- \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(v\left(p_0+\left(\tau+\hat{\tau}\right)\frac{\Delta T}{2}\right)n\right)dG\left(\tau\right)-\widetilde{X}_0.
\end{aligned}$$

We also compute

$$\begin{aligned}
\frac{d\overline{V}_1}{dp_0} &= \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(v\left(p_1+\tau\left\|\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}-t_1\right\|\right)n\right)dG\left(\tau\right) \\
&+ \frac{1}{\varDelta T}\left(\int_{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\hat{\tau}}} v\left(p_1+\hat{\tau}\left\|t^*-t_1\right\|\right)ndt^*\right)g\left(\hat{\tau}\right)
\end{aligned}$$

Using

$$p_1+\tau\left(t_1-\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}\right)=p_1+\left(\tau-\hat{\tau}\right)\frac{\Delta T}{2}$$

we rewrite

$$
\begin{aligned}
\frac{d\overline{V}_1}{dp_0} &= \frac{1}{2\tau}\int_{\widehat{\tau}}^{+\infty}\left(v\left(p_1+(\tau-\hat{\tau})\frac{\Delta T}{2}\right)n\right)dG(\tau)\\
&+\frac{1}{\mathit{\Delta T}}\left(\int_{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\hat{\tau}}} v\left(p_1+\hat{\tau}\left\|t^*-t_1\right\|\right)ndt^*\right)g(\hat{\tau}).
\end{aligned}
$$

Since $\left(d\widetilde{V}_2/dp_0\right)=0$, overall we have

$$
\begin{aligned}
\frac{dV}{dp_0} &= \frac{-1}{\mathit{\Delta T}}\left(\int_{t_0}^{t_1} v\left(p_0+\hat{\tau}\left\|t^*-t_0\right\|\right)ndt^*\right)g(\hat{\tau})\\
&+\frac{1}{\mathit{\Delta T}}\left(\int_{t_0}^{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}} v\left(p_0+\hat{\tau}\left\|t^*-t_0\right\|\right)ndt^*\right)g(\hat{\tau})\\
&+\frac{1}{\mathit{\Delta T}}\left(\int_{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\hat{\tau}}} v\left(p_1+\hat{\tau}\left\|t^*-t_1\right\|\right)ndt^*\right)g(\hat{\tau})\\
&-\frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(v\left(p_0+(\tau+\hat{\tau})\frac{\Delta T}{2}\right)n\right)dG(\tau)\\
&\frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(v\left(p_1+(\tau-\hat{\tau})\frac{\Delta T}{2}\right)n\right)dG(\tau)\\
&-\widetilde{X}_0.
\end{aligned}
$$

Considering that $p_1=p_h>p_l=p_0=p_2$, we can write

$$
\begin{aligned}
p_0+(\tau+\widehat{\tau})\frac{\Delta T}{2} &= p_1-\Delta p+\frac{\Delta T}{2}(\tau+\widehat{\tau})\\
&= p_1-\widehat{\tau}\Delta T+\frac{\Delta T}{2}(\tau+\widehat{\tau})\\
&= p_1+(\tau-\widehat{\tau})\frac{\Delta T}{2}.
\end{aligned}
$$

Moreover,

$$
\begin{aligned}
p_1+\hat{\tau}\left\|t^*-t_1\right\| &= p_0+\Delta p+\hat{\tau}\left\|t^*-t_1\right\|\\
&= p_0+\hat{\tau}\left\|t^*-t_0\right\|+\Delta p-\hat{\tau}\Delta T\\
&= p_0+\hat{\tau}\left\|t^*-t_0\right\|+\left(\frac{\Delta p}{\Delta T}-\hat{\tau}\right)\Delta T\\
&= p_0+\hat{\tau}\left\|t^*-t_0\right\|.
\end{aligned}
$$

Hence, we end up with

$$\begin{aligned}
\frac{dV}{dp_0} &= \frac{-1}{\Delta T}\left(\int_{t_0}^{t_1} v\left(p_0+\hat{\tau}\left\|t^*-t_0\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
&+ \frac{1}{\Delta T}\left(\int_{t_0}^{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}} v\left(p_0+\hat{\tau}\left\|t^*-t_0\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
&+ \frac{1}{\Delta T}\left(\int_{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\hat{\tau}}} v\left(p_0+\hat{\tau}\left\|t^*-t_0\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
&- \widetilde{X}_0.
\end{aligned}$$

Summing the second and the third integral and rearranging, we further obtain

$$\begin{aligned}
\frac{dV}{dp_0} &= -\widetilde{X}_0 - \frac{1}{\Delta T}\left(\int_{t_0}^{t_1} v\left(p_0+\hat{\tau}\left\|t^*-t_0\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
&+ \frac{1}{\Delta T}\left(\int_{t_0}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\hat{\tau}}} v\left(p_0+\hat{\tau}\left\|t^*-t_0\right\|\right) ndt^*\right) g\left(\hat{\tau}\right).
\end{aligned}$$

Using $\frac{t_1+t_2}{2}-\frac{\Delta p}{2\hat{\tau}} = t_1+\frac{\Delta T}{2}-\frac{\Delta T}{2} = t_1$, we ultimately obtain

$$\frac{dV}{dp_0} = -\widetilde{X}_0.$$

A marginal change in p_1 We compute

$$\begin{aligned}
\frac{d\widetilde{V}_0}{dp_1} &= \frac{d\hat{\tau}}{dp_1}\left(\int_{t_0}^{t_1} v\left(p_0+\hat{\tau}\left\|t^*-t_0\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
&- \frac{d\hat{\tau}}{dp_1}\left(\int_{t_0}^{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}} v\left(p_0+\hat{\tau}\left\|t^*-t_0\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
&+ \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_0+\tau\left\|\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}-t_0\right\|\right)\right) dG\left(\tau\right).
\end{aligned}$$

Using $(d\hat{\tau}/dp_1) = (1/\Delta T)$ together with

$$\begin{aligned}
p_0+\tau\left(\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}-t_0\right) &= p_0+\tau\left(\frac{t_0+t_1-2t_0}{2}+\frac{\Delta p}{2\tau}\right) \\
&= p_0+\left(\hat{\tau}+\tau\right)\frac{\Delta T}{2}
\end{aligned}$$

and with $\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}} = t_1-\frac{\Delta T}{2}+\frac{\Delta T}{2} = t_1$, we end up with

$$\frac{d\widetilde{V}_0}{dp_1} = \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_0+\left(\hat{\tau}+\tau\right)\frac{\Delta T}{2}\right)\right) dG\left(\tau\right).$$

We also compute

$$
\begin{aligned}
\frac{d\overline{V}_1}{dp_1} = & -\frac{d\hat{\tau}}{dp_1}\left(\int_{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\hat{\tau}}} v\left(p_1+\hat{\tau}\left\|t^*-t_1\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
& -\frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_1+\tau\left\|\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}-t_1\right\|\right)\right) dG\left(\tau\right) \\
& -\frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_1+\tau\left\|\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}-t_1\right\|\right)\right) dG\left(\tau\right) \\
& -\int_{\hat{\tau}}^{+\infty}\left(\int_{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}} x\left(p_1+\tau\left\|t^*-t_1\right\|\right) ndt^*\right) dG\left(\tau\right) \\
= & -\overline{X}_1-\frac{1}{\Delta T}\left(\int_{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\hat{\tau}}} v\left(p_1+\hat{\tau}\left\|t^*-t_1\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
& -\frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_1+\tau\left(\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}-t_1\right)\right)\right) dG\left(\tau\right) \\
& -\frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_1+\tau\left\|\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}-t_1\right\|\right)\right) dG\left(\tau\right).
\end{aligned}
$$

We lastly compute

$$
\begin{aligned}
\frac{d\widetilde{V}_2}{dp_1} = & \frac{d\hat{\tau}}{dp_1}\left(\int_{t_1}^{t_2} v\left(p_2+\hat{\tau}\left\|t^*-t_2\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
& -\frac{d\hat{\tau}}{dp_1}\left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} v\left(p_2+\hat{\tau}\left\|t^*-t_2\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
& +\frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_2+\tau\left\|\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}-t_2\right\|\right)\right) dG\left(\tau\right) \\
= & \frac{1}{\Delta T}\left(\int_{t_1}^{t_2} v\left(p_2+\hat{\tau}\left\|t^*-t_2\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
& -\frac{1}{\Delta T}\left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} v\left(p_2+\hat{\tau}\left\|t^*-t_2\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
& +\frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_2+\tau\left\|\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}-t_2\right\|\right)\right) dG\left(\tau\right).
\end{aligned}
$$

Overall we have

$$
\begin{aligned}
\frac{dV}{dp_1} &= -\overline{X}_1 + \frac{1}{\Delta T}\left(\int_{t_1}^{t_2} v\left(p_2 + \hat{\tau}\left\|t^* - t_2\right\|\right) n dt^*\right) g\left(\hat{\tau}\right) \\
&- \frac{1}{\Delta T}\left(\int_{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\hat{\tau}}} v\left(p_1 + \hat{\tau}\left\|t^* - t_1\right\|\right) n dt^*\right) g\left(\hat{\tau}\right) \\
&- \frac{1}{\Delta T}\left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} v\left(p_2 + \hat{\tau}\left\|t^* - t_2\right\|\right) n dt^*\right) g\left(\hat{\tau}\right) \\
&+ \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_0 + \left(\hat{\tau}+\tau\right)\frac{\Delta T}{2}\right)\right) dG\left(\tau\right) \\
&- \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_1 + \tau\left(\frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau} - t_1\right)\right)\right) dG\left(\tau\right) \\
&- \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_1 + \tau\left\|\frac{t_0+t_1}{2} + \frac{\Delta p}{2\tau} - t_1\right\|\right)\right) dG\left(\tau\right) \\
&+ \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_2 + \tau\left\|\frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau} - t_2\right\|\right)\right) dG\left(\tau\right).
\end{aligned}
$$

Using $\frac{t_0+t_1}{2} + \frac{\Delta p}{2\hat{\tau}} = t_1 = \frac{t_1+t_2}{2} - \frac{\Delta p}{2\hat{\tau}}$, as previously computed, we obtain

$$
\begin{aligned}
\frac{dV}{dp_1} &= -\overline{X}_1 \\
&+ \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_0 + \left(\hat{\tau}+\tau\right)\frac{\Delta T}{2}\right)\right) dG\left(\tau\right) \\
&- \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_1 + \tau\left(\frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau} - t_1\right)\right)\right) dG\left(\tau\right) \\
&- \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_1 + \tau\left(t_1 - \frac{t_0+t_1}{2} - \frac{\Delta p}{2\tau}\right)\right)\right) dG\left(\tau\right) \\
&+ \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_2 + \tau\left(t_2 - \frac{t_1+t_2}{2} + \frac{\Delta p}{2\tau}\right)\right)\right) dG\left(\tau\right).
\end{aligned}
$$

Further using

$$
\begin{aligned}
p_1 + \tau\left(\frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau} - t_1\right) &= p_0 + \Delta p + \tau\left(\frac{t_2-t_1}{2} - \frac{\Delta p}{2\tau}\right) \\
&= p_0 + \left(\tau + \hat{\tau}\right)\frac{\Delta T}{2}
\end{aligned}
$$

and noticing that $p_1 + \tau\left(t_1 - \frac{t_0+t_1}{2} - \frac{\Delta p}{2\tau}\right) = p_2 + \tau\left(t_2 - \frac{t_1+t_2}{2} + \frac{\Delta p}{2\tau}\right)$, we end up with

$$
\frac{dV}{dp_1} = -\overline{X}_1.
$$

A marginal change in p_2 We compute

$$\frac{d\overline{V}_1}{dp_2} = -\frac{d\hat{\tau}}{dp_2}\left(\int_{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\hat{\tau}}} v\left(p_1+\hat{\tau}\left\|t^*-t_1\right\|\right)ndt^*\right)g\left(\hat{\tau}\right)$$
$$+\frac{1}{2\tau}\int_{\widehat{\tau}}^{+\infty}\left(nv\left(p_1+\tau\left(\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}-t_1\right)\right)\right)dG\left(\tau\right)$$
$$=\frac{1}{\Delta T}\left(\int_{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\hat{\tau}}} v\left(p_1+\hat{\tau}\left\|t^*-t_1\right\|\right)ndt^*\right)g\left(\hat{\tau}\right)$$
$$+\frac{1}{2\tau}\int_{\widehat{\tau}}^{+\infty}\left(nv\left(p_1+\left(\tau-\hat{\tau}\right)\frac{\Delta T}{2}\right)\right)dG\left(\tau\right)$$

together with

$$\frac{d\widetilde{V}_2}{dp_2} = -\int_0^{\hat{\tau}}\left(\int_{t_1}^{t_2} x\left(p_2+\tau\left\|t^*-t_2\right\|\right)ndt^*\right)dG\left(\tau\right)$$
$$+\frac{d\hat{\tau}}{dp_2}\left(\int_{t_1}^{t_2} v\left(p_2+\hat{\tau}\left\|t^*-t_2\right\|\right)ndt^*\right)g\left(\hat{\tau}\right)$$
$$-\frac{d\hat{\tau}}{dp_2}\left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} v\left(p_2+\hat{\tau}\left\|t^*-t_2\right\|\right)ndt^*\right)g\left(\hat{\tau}\right)$$
$$-\frac{1}{2\tau}\int_{\widehat{\tau}}^{+\infty}\left(nv\left(p_2+\tau\left(\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}-t_2\right)\right)\right)dG\left(\tau\right)$$
$$-\int_{\widehat{\tau}}^{+\infty}\left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2+\tau\left\|t^*-t_2\right\|\right)ndt^*\right)dG\left(\tau\right)$$
$$=-\widetilde{X}_2-\frac{1}{\Delta T}\left(\int_{t_1}^{t_2} v\left(p_2+\hat{\tau}\left\|t^*-t_2\right\|\right)ndt^*\right)g\left(\hat{\tau}\right)$$
$$+\frac{1}{\Delta T}\left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} v\left(p_2+\hat{\tau}\left\|t^*-t_2\right\|\right)ndt^*\right)g\left(\hat{\tau}\right)$$
$$-\frac{1}{2\tau}\int_{\widehat{\tau}}^{+\infty}\left(nv\left(p_2+\left(\tau+\hat{\tau}\right)\frac{\Delta T}{2}\right)\right)dG\left(\tau\right).$$

Since $\left(d\widetilde{V}_0/dp_2\right) = 0$, overall we have

$$\begin{aligned}
\frac{dV}{dp_2} &= -\widetilde{X}_2 + \frac{1}{\Delta T}\left(\int_{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\hat{\tau}}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\hat{\tau}}} v\left(p_1 + \hat{\tau}\left\|t^* - t_1\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
&- \frac{1}{\Delta T}\left(\int_{t_1}^{t_2} v\left(p_2 + \hat{\tau}\left\|t^* - t_2\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
&+ \frac{1}{\Delta T}\left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} v\left(p_2 + \hat{\tau}\left\|t^* - t_2\right\|\right) ndt^*\right) g\left(\hat{\tau}\right) \\
&+ \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_1 + \left(\tau - \hat{\tau}\right)\frac{\Delta T}{2}\right)\right) dG\left(\tau\right) \\
&- \frac{1}{2\tau}\int_{\hat{\tau}}^{+\infty}\left(nv\left(p_2 + \left(\tau + \hat{\tau}\right)\frac{\Delta T}{2}\right)\right) dG\left(\tau\right).
\end{aligned}$$

Using again $\frac{t_0+t_1}{2} + \frac{\Delta p}{2\hat{\tau}} = t_1 = \frac{t_1+t_2}{2} - \frac{\Delta p}{2\hat{\tau}}$ and $p_1 + \left(\tau - \hat{\tau}\right)\frac{\Delta T}{2} = p_2 + \left(\tau + \hat{\tau}\right)\frac{\Delta T}{2}$, this reduces to

$$\frac{dV}{dp_2} = -\widetilde{X}_2.$$

B.1.2 The impact of a change in schedule

We begin by computing

$$\begin{aligned}
\frac{1}{n}\frac{d\widetilde{V}_0}{dT} &= \frac{d\widehat{\tau}}{dT}\left(\int_{t_0}^{t_1} v\left(p_0 + \widehat{\tau}\left\|t^* - t_0\right\|\right) dt^*\right) g\left(\widehat{\tau}\right) \\
&+ \int_0^{\hat{\tau}} v\left(p_0\right) dG\left(\tau\right) - \int_0^{\hat{\tau}} \tau\left(\int_{t_0}^{t_1} x\left(p_0 + \tau\left(t^* - t_0\right)\right) dt^*\right) dG\left(\tau\right) \\
&- \frac{d\widehat{\tau}}{dT}\left(\int_{t_0}^{t_1} v\left(p_0 + \widehat{\tau}\left\|t^* - t_0\right\|\right) dt^*\right) g\left(\widehat{\tau}\right) \\
&+ \int_{\hat{\tau}}^{+\infty}\left(-\frac{1}{2}v\left(p_0 + \frac{1}{2}\left(\tau + \widehat{\tau}\right)\Delta T\right) + v\left(p_0\right)\right) dG\left(\tau\right) \\
&- \int_{\hat{\tau}}^{+\infty} \tau\left(\int_{t_0}^{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}} x\left(p_0 + \tau\left(t^* - t_0\right)\right) dt^*\right) dG\left(\tau\right),
\end{aligned}$$

which is rearranged as

$$\begin{aligned}
\frac{1}{n}\frac{d\widetilde{V}_0}{dT} &= v\left(p_0\right) - \frac{1}{2}\int_{\hat{\tau}}^{+\infty} v\left(p_0 + \frac{1}{2}\left(\tau + \widehat{\tau}\right)\Delta T\right) dG\left(\tau\right) \\
&- \int_0^{\hat{\tau}} \tau\left(\int_{t_0}^{t_1} x\left(p_0 + \tau\left(t^* - t_0\right)\right) dt^*\right) dG\left(\tau\right) \\
&- \int_{\hat{\tau}}^{+\infty} \tau\left(\int_{t_0}^{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}} x\left(p_0 + \tau\left(t^* - t_0\right)\right) dt^*\right) dG\left(\tau\right).
\end{aligned}$$

We next compute

$$\begin{aligned}\frac{1}{n}\frac{d\overline{V}_1}{dT} &= -\frac{d\widehat{\tau}}{dT}\left(\int_{t_1}^{t_1} v\left(p_1+\widehat{\tau}\left\|t^*-t_1\right\|\right)dt^*\right)g\left(\widehat{\tau}\right)\\ &+\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_1+\frac{\Delta T}{2}\left(\tau-\widehat{\tau}\right)\right)dG\left(\tau\right)\\ &+\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_1+\frac{\Delta T}{2}\left\|\widehat{\tau}-\tau\right\|\right)dt^* dG\left(\tau\right)\end{aligned}$$

which yields

$$\frac{1}{n}\frac{d\overline{V}_1}{dT}=\int_{\widehat{\tau}}^{+\infty} v\left(p_1+\frac{\Delta T}{2}\left(\tau-\widehat{\tau}\right)\right)dG\left(\tau\right).$$

We lastly compute

$$\begin{aligned}\frac{1}{n}\frac{d\widetilde{V}_2}{dT} &= \frac{d\widehat{\tau}}{dT}\left(\int_{t_1}^{t_2} v\left(p_2+\widehat{\tau}\left\|t^*-t_2\right\|\right)dt^*\right)g\left(\widehat{\tau}\right)\\ &+\int_0^{\hat{\tau}} v\left(p_2\right)dG\left(\tau\right)-\int_0^{\hat{\tau}}\tau\left(\int_{t_1}^{t_2} x\left(p_2+\tau\left(t_2-t^*\right)\right)dt^*\right)dG\left(\tau\right)\\ &-\frac{d\widehat{\tau}}{dT}\left(\int_{t_1}^{t_2} v\left(p_2+\widehat{\tau}\left\|t^*-t_2\right\|\right)dt^*\right)g\left(\widehat{\tau}\right)\\ &+\int_{\widehat{\tau}}^{+\infty}\left(v\left(p_2\right)-\frac{1}{2}v\left(p_2+\left(\widehat{\tau}+\tau\right)\frac{\Delta T}{2}\right)\right)dG\left(\tau\right)\\ &-\int_{\widehat{\tau}}^{+\infty}\tau\left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2+\tau\left(t_2-t^*\right)\right)dt^*\right)dG\left(\tau\right),\end{aligned}$$

which is rearranged to obtain

$$\begin{aligned}\frac{1}{n}\frac{d\widetilde{V}_2}{dT} &= v\left(p_2\right)-\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_2+\left(\widehat{\tau}+\tau\right)\frac{\Delta T}{2}\right)dG\left(\tau\right)\\ &-\int_0^{\hat{\tau}}\tau\left(\int_{t_1}^{t_2} x\left(p_2+\tau\left(t_2-t^*\right)\right)dt^*\right)dG\left(\tau\right)\\ &-\int_{\widehat{\tau}}^{+\infty}\tau\left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2+\tau\left(t_2-t^*\right)\right)dt^*\right)dG\left(\tau\right).\end{aligned}$$

Summing up yields

$$
\begin{aligned}
\frac{1}{n}\frac{dV}{dT} &= \frac{1}{n}\left(\frac{d\widetilde{V}_0}{dT}+\frac{d\overline{V}_1}{dT}+\frac{d\widetilde{V}_2}{dT}\right) \\
&= v\left(p_0\right)-\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_0+\frac{\Delta T}{2}\left(\tau+\widehat{\tau}\right)\right) dG\left(\tau\right) \\
&\quad +\int_{\widehat{\tau}}^{+\infty} v\left(p_1+\frac{\Delta T}{2}\left(\tau-\widehat{\tau}\right)\right) dG\left(\tau\right) \\
&\quad +v\left(p_2\right)-\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_2+\frac{\Delta T}{2}\left(\tau+\widehat{\tau}\right)\right) dG\left(\tau\right) \\
&\quad -\int_0^{\hat{\tau}} \tau\left(\int_{t_0}^{t_1} x\left(p_0+\tau\left(t^*-t_0\right)\right) dt^*\right) dG\left(\tau\right) \\
&\quad -\int_{\widehat{\tau}}^{+\infty} \tau\left(\int_{t_0}^{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}} x\left(p_0+\tau\left(t^*-t_0\right)\right) dt^*\right) dG\left(\tau\right) \\
&\quad -\int_0^{\hat{\tau}} \tau\left(\int_{t_1}^{t_2} x\left(p_2+\tau\left(t_2-t^*\right)\right) dt^*\right) dG\left(\tau\right) \\
&\quad -\int_{\widehat{\tau}}^{+\infty} \tau\left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2+\tau\left(t_2-t^*\right)\right) dt^*\right) dG\left(\tau\right).
\end{aligned}
$$

Recalling that $p_0+\left(\tau+\widehat{\tau}\right)\frac{\Delta T}{2}=p_1+\left(\tau-\widehat{\tau}\right)\frac{\Delta T}{2}$, we reformulate the above expression to obtain

$$
\begin{aligned}
\frac{dV}{dT} &= n\left(v\left(p_0\right)+v\left(p_2\right)\right) \\
&\quad -\int_0^{\hat{\tau}} \tau\left(\int_{t_0}^{t_1} x\left(p_0+\tau\left(t^*-t_0\right)\right) ndt^*\right) dG\left(\tau\right) \\
&\quad -\int_{\widehat{\tau}}^{+\infty} \tau\left(\int_{t_0}^{\frac{t_0+t_1}{2}+\frac{\Delta p}{2\tau}} x\left(p_0+\tau\left(t^*-t_0\right)\right) ndt^*\right) dG\left(\tau\right) \\
&\quad -\int_0^{\hat{\tau}} \tau\left(\int_{t_1}^{t_2} x\left(p_2+\tau\left(t_2-t^*\right)\right) ndt^*\right) dG\left(\tau\right) \\
&\quad -\int_{\widehat{\tau}}^{+\infty} \tau\left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2+\tau\left(t_2-t^*\right)\right) ndt^*\right) dG\left(\tau\right).
\end{aligned}
$$

Making use of the notation introduced in the text, we further rewrite

$$
\frac{dV}{dT}=-\underline{\tau}_0\underline{X}_0-\overline{\tau}_0\overline{X}_0-\underline{\tau}_2\underline{X}_2-\overline{\tau}_2\overline{X}_2+n\left[v\left(p_0\right)+v\left(p_2\right)\right].
$$

Neglecting the bracketed term, which captures volume effects, (6) is derived.

B.2 Case I.2: $p_1 = p_l < p_h = p_0 = p_2$

All passengers with $\tau < \hat{\tau}$ are eager to increase their DTS by more than ΔT in order to benefit from a price saving of Δp. Hence, provided s_1 is cheaper than any other service, passengers with $t^* \in [t_0, t_2]$ and $\tau < \hat{\tau}$ all take s_1. Passengers with $\tau \geq \hat{\tau}$ respectively use s_0, s_1 or s_2 when

$$\begin{aligned}
t^* &\in \left[t_0; \frac{t_0+t_1}{2} - \frac{\Delta p}{2\tau}\right] \\
t^* &\in \left[\frac{t_0+t_1}{2} - \frac{\Delta p}{2\tau}; \frac{t_1+t_2}{2} + \frac{\Delta p}{2\tau}\right] \\
t^* &\in \left[\frac{t_1+t_2}{2} + \frac{\Delta p}{2\tau}; t_2\right].
\end{aligned}$$

Accordingly, the total demand for s_0, s_1 and s_2 is respectively written as

$$\begin{aligned}
\overline{X}_0 &= \int_{\widehat{\tau}}^{+\infty} X_0(\tau)\, dG(\tau) \\
\widetilde{X}_1 &= \underline{X}_1 + \overline{X}_1 \\
&= \int_0^{\hat{\tau}} X_{[0,2]\triangleright 1}(\tau)\, dG(\tau) + \int_{\widehat{\tau}}^{+\infty} X_{0,2\triangleright 1}(\tau)\, dG(\tau) \\
\overline{X}_2 &= \int_{\widehat{\tau}}^{+\infty} X_2(\tau)\, dG(\tau),
\end{aligned}$$

where

$$\begin{aligned}
X_0(\tau) &= \int_{t_0}^{\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}} x\left(p_0 + \tau \left\|t^* - t_0\right\|\right) ndt^* \\
X_{[0,2]\triangleright 1}(\tau) &= \int_{t_0}^{t_2} x\left(p_1 + \tau \left\|t^* - t_1\right\|\right) ndt^* \\
X_{0,2\triangleright 1}(\tau) &= \int_{\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}}^{\frac{t_1+t_2}{2}+\frac{\Delta p}{2\tau}} x\left(p_1 + \tau \left\|t^* - t_1\right\|\right) ndt^* \\
X_2(\tau) &= \int_{\frac{t_1+t_2}{2}+\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2 + \tau \left\|t^* - t_2\right\|\right) ndt^*,
\end{aligned}$$

and the subscript $[0,2] \triangleright 1$ (resp., $0,2 \triangleright 1$) indicates that, in addition to passengers whose ideal departure time is t_1, s_1 also attracts all (resp., some) passengers who would have taken s_0 and s_2 absent the price saving. This is all graphically represented in Figure

10. The surplus respectively associated with s_0, s_1 and s_2 is written as

$$\begin{aligned}
\overline{V}_0 &= \int_{\widehat{\tau}}^{+\infty}\left(\int_{t_0}^{\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}} v\left(p_0+\tau\left\|t^*-t_0\right\|\right)ndt^*\right)dG(\tau),\\
\widetilde{V}_1 &= \underline{V}_1+\overline{V}_1=\int_0^{\hat{\tau}}\left(\int_{t_0}^{t_2} v\left(p_1+\tau\left\|t^*-t_1\right\|\right)ndt^*\right)dG(\tau)\\
&\quad+\int_{\widehat{\tau}}^{+\infty}\left(\int_{\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}}^{\frac{t_1+t_2}{2}+\frac{\Delta p}{2\tau}} v\left(p_1+\tau\left\|t^*-t_1\right\|\right)ndt^*\right)dG(\tau),\\
\overline{V}_2 &= \int_{\widehat{\tau}}^{+\infty}\left(\int_{\frac{t_1+t_2}{2}+\frac{\Delta p}{2\tau}}^{t_2} v\left(p_2+\tau\left\|t^*-t_2\right\|\right)ndt^*\right)dG(\tau).
\end{aligned}$$

B.2.1 The impact of a change in price

As standard, a price increase of dp triggers a surplus reduction equal to the aggregate demand for service s_i, where $i\in\{1,1,2\}$:

$$\frac{dV}{dp_0}=-\overline{X}_0;\quad \frac{dV}{dp_1}=-\widetilde{X}_1;\quad \frac{dV}{dp_2}=-\overline{X}_2.$$

B.2.2 The impact of a change in schedule

Define

$$\bar{\tau}_0=\int_{\widehat{\tau}}^{+\infty}\tau\frac{X_0(\tau)}{\overline{X}_0}dG(\tau)\quad\text{and}\quad \bar{\tau}_2=\int_{\widehat{\tau}}^{+\infty}\tau\frac{X_2(\tau)}{\overline{X}_2}dG(\tau),$$

the value of time of the passengers respectively using s_0 and s_2. We begin by computing

$$\begin{aligned}
\frac{1}{n}\frac{d\overline{V}_0}{dT} &= -\frac{d\widehat{\tau}}{dT}\left(\int_{t_0}^{\frac{t_0+t_1-\Delta T}{2}} v\left(p_0+\widehat{\tau}\left\|t^*-t_0\right\|\right)dt^*\right)\\
&\quad-\int_{\widehat{\tau}}^{+\infty}\left(\int_{t_0}^{\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}}\tau\frac{d\left\|t^*-t_0\right\|}{dT}x\left(p_0+\tau\left\|t^*-t_0\right\|\right)dt^*\right)dG(\tau)\\
&\quad+\int_{\widehat{\tau}}^{+\infty}\frac{1}{2}\frac{dt_0}{dT}v\left(p_0+\tau\left\|\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}-t_0\right\|\right)dG(\tau)\\
&\quad-\int_{\widehat{\tau}}^{+\infty}\frac{dt_0}{dT}v(p_0)\,dG(\tau),
\end{aligned}$$

which is rewritten as

$$\begin{aligned}
\frac{1}{n}\frac{d\overline{V}_0}{dT} &= -\int_{\widehat{\tau}}^{+\infty}\tau\left(\int_{t_0}^{\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}} x\left(p_0+\tau\left\|t^*-t_0\right\|\right)dt^*\right)dG(\tau)\\
&\quad-\frac{1}{2}\int_{\widehat{\tau}}^{+\infty}v\left(p_0+(\tau-\widehat{\tau})\frac{\Delta T}{2}\right)dG(\tau)+(1-G(\widehat{\tau}))\,v(p_0),
\end{aligned}$$

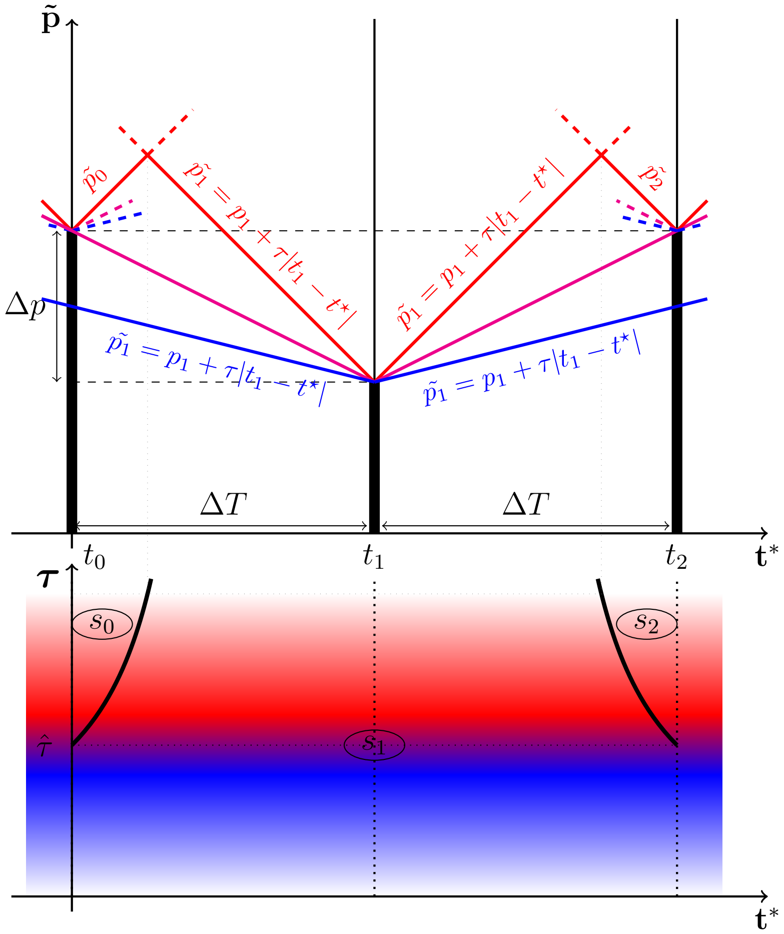

Figure 10: **Informed passengers - Case I.2: $\mathbf{p_1 = p_l < p_h = p_0 = p_2}$**
Top graph: The generalised price is plotted against the ideal departure time. The blue line represents the generalised price of a patient passenger ($\tau < \hat{\tau}$), the red line that of an impatient passenger ($\tau > \hat{\tau}$), the magenta line that of a passenger with $\tau = \hat{\tau}$. The individual generalised price decreases to the monetary price (the thick black vertical line placed in the service locations) as the departure time approaches the ideal one, t^*.
Bottom graph: Passenger distribution over the two heterogeneity dimensions (t^*, τ) and across services. Colors refer to the value of time τ; the intensity is associated with the number of passengers displaying the characteristics (t^*, τ). For this representation, we assume it independent of t^*. The two expensive services s_0 and s_2 are only used by passengers with high value of time and whose ideal departure time is very close to t_0 and t_2 respectively. The cheap service s_1 attracts all the other passengers.

hence as

$$\frac{d\overline{V}_0}{dT} = -\bar{\tau}_0\overline{X}_0 - \frac{n}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_0 + (\tau - \widehat{\tau})\frac{\Delta T}{2}\right) dG(\tau) + n\left(1 - G(\hat{\tau})\right) v(p_0). \tag{10}$$

We next compute

$$\begin{aligned}
\frac{1}{n}\frac{d\widetilde{V}_1}{dT} &= \frac{d\widehat{\tau}}{dT}\int_{t_0}^{t_2} v\left(p_1 + \hat{\tau}\left\|t^* - t_1\right\|\right) dt^* - \frac{d\widehat{\tau}}{dT}\int_{\frac{t_0+t_1-\Delta T}{2}}^{\frac{t_1+t_2+\Delta T}{2}} v\left(p_1 + \hat{\tau}\left\|t^* - t_1\right\|\right) dt^* \\
&- \int_0^{\hat{\tau}}\left(\int_{t_0}^{t_2} \tau\frac{d\left\|t^* - t_1\right\|}{dT} x\left(p_1 + \tau\left\|t^* - t_1\right\|\right) dt^*\right) dG(\tau) \\
&- \int_{\widehat{\tau}}^{+\infty}\left(\int_{\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}}^{\frac{t_1+t_2}{2}+\frac{\Delta p}{2\tau}} \tau\frac{d\left\|t^* - t_1\right\|}{dT} x\left(p_1 + \tau\left\|t^* - t_1\right\|\right) dt^*\right) dG(\tau) \\
&+ \int_0^{\hat{\tau}}\left[v\left(p_1 + \tau\left\|t_2 - t_1\right\|\right) + v\left(p_1 + \tau\left\|t_0 - t_1\right\|\right)\right] dG(\tau) \\
&+ \frac{1}{2}\int_{\widehat{\tau}}^{+\infty} \frac{dt_2}{dT} v\left(p_1 + \tau\left\|\frac{t_1 + t_2}{2} + \frac{\Delta p}{2\tau} - t_1\right\|\right) dG(\tau) \\
&- \frac{1}{2}\int_{\widehat{\tau}}^{+\infty} \frac{dt_0}{dT} v\left(p_1 + \tau\left\|\frac{t_0 + t_1}{2} - \frac{\Delta p}{2\tau} - t_1\right\|\right) dG(\tau).
\end{aligned}$$

This is rearranged to obtain

$$\begin{aligned}
\frac{d\widetilde{V}_1}{dT} &= n\int_0^{\hat{\tau}}\left[v\left(p_1 + \tau\left\|t_2 - t_1\right\|\right) + v\left(p_1 + \tau\left\|t_0 - t_1\right\|\right)\right] dG(\tau) \\
&+ n\int_{\widehat{\tau}}^{+\infty} v\left(p_1 + \tau\left(1 + \frac{\hat{\tau}}{\tau}\right)\frac{\Delta T}{2}\right) dG(\tau)
\end{aligned} \tag{11}$$

We lastly compute

$$\begin{aligned}
\frac{1}{n}\frac{d\overline{V}_2}{dT} &= -\frac{d\widehat{\tau}}{dT}\left(\int_{\frac{t_1+t_2}{2}+\frac{\Delta p}{2\widehat{\tau}}}^{t_2} v\left(p_2 + \widehat{\tau}\left\|t^* - t_2\right\|\right) dt^*\right) \\
&+ \int_{\widehat{\tau}}^{+\infty}\left(\int_{\frac{t_1+t_2}{2}+\frac{\Delta p}{2\tau}}^{t_2} \tau\frac{d\left\|t^* - t_2\right\|}{dT} x\left(p_2 + \tau\left\|t^* - t_2\right\|\right) dt^*\right) dG(\tau) \\
&- \int_{\widehat{\tau}}^{+\infty} \frac{1}{2}\frac{dt_2}{dT} v\left(p_2 + \tau\left\|\frac{t_1 + t_2}{2} + \frac{\Delta p}{2\tau} - t_2\right\|\right) dG(\tau) \\
&+ \int_{\widehat{\tau}}^{+\infty} \frac{dt_2}{dT} v(p_2)\, dG(\tau) \\
&= 0 - \int_{\widehat{\tau}}^{+\infty} \tau\left(\int_{\frac{t_1+t_2}{2}+\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2 + \tau\left\|t^* - t_2\right\|\right) dt^*\right) dG(\tau) \\
&- \frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_2 + (\tau - \widehat{\tau})\frac{\Delta T}{2}\right) dG(\tau) + \left[1 - G(\hat{\tau})\right] v(p_2),
\end{aligned}$$

which further becomes

$$\frac{d\overline{V}_2}{dT} = -\bar{\tau}_2\overline{X}_2 - \frac{n}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_2 + (\tau - \widehat{\tau})\frac{\Delta T}{2}\right) dG(\tau) \qquad (12)$$
$$+ n(1 - G(\hat{\tau}))\, v(p_2).$$

Summing up, we obtain

$$\begin{aligned}
\frac{dV}{dT} &= \frac{d\overline{V}_0}{dT} + \frac{d\widetilde{V}_1}{dT} + \frac{d\overline{V}_2}{dT} \\
&= -\bar{\tau}_0\overline{X}_0 - \bar{\tau}_2\overline{X}_2 - \frac{n}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_0 + (\tau - \widehat{\tau})\frac{\Delta T}{2}\right) dG(\tau) \\
&\quad + n\int_{\widehat{\tau}}^{+\infty} v\left(p_1 + (\tau + \hat{\tau})\frac{\Delta T}{2}\right) dG(\tau) - \frac{n}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_2 + (\tau - \widehat{\tau})\frac{\Delta T}{2}\right) dG(\tau) \\
&\quad + 2n\int_0^{\hat{\tau}} (v(p_1 + \tau\Delta T))\, dG(\tau) + n(1 - G(\hat{\tau}))\left[v(p_0) + v(p_2)\right].
\end{aligned}$$

By the definition of $\widehat{\tau}$, we have

$$\begin{aligned}
p_0 + (\tau - \widehat{\tau})\frac{\Delta T}{2} &= p_0 - \frac{\Delta p}{2} + \tau\frac{\Delta T}{2} \\
&= p_1 + \tau\frac{\Delta T}{2} + \frac{\Delta p}{2} \\
&= p_1 + (\tau + \hat{\tau})\frac{\Delta T}{2}
\end{aligned}$$

and

$$\begin{aligned}
p_2 + (\tau - \widehat{\tau})\frac{\Delta T}{2} &= p_2 - \frac{\Delta p}{2} + \tau\frac{\Delta T}{2} \\
&= p_1 + \tau\frac{\Delta T}{2} + \frac{\Delta p}{2} \\
&= p_1 + (\tau + \hat{\tau})\frac{\Delta T}{2}
\end{aligned}$$

so that

$$\begin{aligned}
v\left(p_0 + (\tau - \widehat{\tau})\frac{\Delta T}{2}\right) &= v\left(p_1 + (\tau + \hat{\tau})\frac{\Delta T}{2}\right) \\
v\left(p_2 + (\tau - \widehat{\tau})\frac{\Delta T}{2}\right) &= v\left(p_1 + (\tau + \hat{\tau})\frac{\Delta T}{2}\right),
\end{aligned}$$

and we can rewrite

$$\frac{dV}{dT} = -\bar{\tau}_0\overline{X}_0 - \bar{\tau}_2\overline{X}_2 \\ + 2n\int_0^{\hat{\tau}} \left(v\left(p_1 + \tau\Delta T\right)\right) dG\left(\tau\right) + n\left(1 - G\left(\hat{\tau}\right)\right)\left[v\left(p_0\right) + v\left(p_2\right)\right].$$

Once the volume effects (the terms in the second line) are net out, this yields

$$\frac{dV}{dT} = -\left(\bar{\tau}_0\overline{X}_0 + \bar{\tau}_2\overline{X}_2\right). \tag{13}$$

As the service frequency is reduced, and the DTS increases for patrons of s_0 and s_2, there is a reduction in welfare equal to an aggregate measure of their value of time. Passengers using s_1 are not concerned, provided this service still departs at t_1. A policy choice to make services more frequent around the cheap service s_1 would exclusively benefit wealthy passengers, who are keen to spend more in order to contain departure shifting.

B.3 Case I.3: $p_0 = p_h > p_l = p_2 = p_1$

Services s_1 and s_2 are now both cheaper than s_0.[8] First consider passengers with $\tau < \hat{\tau}$. Again, they are eager to increase their DTS by more than ΔT to take advantage of the price saving. Thus, all such passengers who also have $t^* \in [t_0, t_1]$ use s_1. Next consider passengers with $\tau \geq \hat{\tau}$. Those with $t^* \in \left[t_0, \frac{t_0+t_1}{2} - \frac{\Delta p}{2\tau}\right]$ use s_0; those with $t^* \in \left[\frac{t_0+t_1}{2} - \frac{\Delta p}{2\tau}, t_1\right]$ opt for s_1. Clearly, regardless of the value of τ, all passengers with $t^* \in \left[t_1, \frac{t_1+t_2}{2}\right]$ choose s_1; those with $t^* \in \left[\frac{t_1+t_2}{2}, t_2\right]$ prefer s_2. Accordingly, the total demand for s_0, s_1 and s_2 is respectively written as

$$\begin{aligned}
\overline{X}_0 &= \int_{\hat{\tau}}^{+\infty} X_0\left(\tau\right) dG\left(\tau\right) \\
\widetilde{X}_1 &= \underline{X}_1 + \overline{X}_1 \\
&= \int_0^{\hat{\tau}} X_{[0)\triangleright 1}\left(\tau\right) dG\left(\tau\right) + \int_{\hat{\tau}}^{+\infty} X_{0\triangleright 1}\left(\tau\right) dG\left(\tau\right) \\
\widetilde{X}_2 &= \int_0^{+\infty} X_2\left(\tau\right) dG\left(\tau\right),
\end{aligned}$$

[8] This is one of two possible cases where s_1 is one of the *cheap* services. In the other such case, s_1 and s_0 would both be cheaper than s_2, namely $p_2 = p_h > p_l = p_0 = p_1$. The analysis of this latter case would be analogous, *mutatis mutandis*, and is thus omitted to avoid redundancy.

where

$$\begin{aligned}
X_0(\tau) &= \int_{t_0}^{\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}} x\left(p_0+\tau\left\|t^*-t_0\right\|\right) ndt^* \\
X_{[0)\rhd 1}(\tau) &= \int_{t_0}^{\frac{t_1+t_2}{2}} x\left(p_1+\tau\left\|t^*-t_1\right\|\right) ndt^* \\
X_{0\rhd 1}(\tau) &= \int_{\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}}^{\frac{t_1+t_2}{2}} x\left(p_1+\tau\left\|t^*-t_1\right\|\right) ndt^* \\
X_2(\tau) &= \int_{\frac{t_1+t_2}{2}}^{t_2} x\left(p_2+\tau\left\|t^*-t_2\right\|\right) ndt^*
\end{aligned}$$

and the subscript $[0) \rhd 1$ (resp., $0 \rhd 1$) is used to indicate that s_1 also attracts all (resp., some) passengers who would have departed at t_0 absent the price saving. A graphical representation is provided in Figure 11. The surplus respectively associated with s_0, s_1 and s_2 is given by

$$\begin{aligned}
\overline{V}_0 &= \int_{\widehat{\tau}}^{+\infty}\left(\int_{t_0}^{\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}} v\left(p_0+\tau\left\|t^*-t_0\right\|\right) ndt^*\right) dG(\tau) \\
\widetilde{V}_1 &= \underline{V}_1+\overline{V}_1 \\
&= \int_0^{\hat{\tau}}\left(\int_{t_0}^{\frac{t_1+t_2}{2}} v\left(p_1+\tau\left\|t^*-t_1\right\|\right) ndt^*\right) dG(\tau) \\
&\quad +\int_{\widehat{\tau}}^{+\infty}\left(\int_{\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}}^{\frac{t_1+t_2}{2}} v\left(p_1+\tau\left\|t^*-t_1\right\|\right) ndt^*\right) dG(\tau) \\
\widetilde{V}_2 &= \int_0^{+\infty}\left(\int_{\frac{t_1+t_2}{2}}^{t_2} v\left(p_2+\tau\left\|t^*-t_2\right\|\right) ndt^*\right) dG(\tau).
\end{aligned}$$

B.3.1 The impact of a change in price

The impact on surplus of a price increase is analogous to the other cases, namely

$$\frac{dV}{dp_0}=-\overline{X}_0;\quad \frac{dV}{dp_1}=-\widetilde{X}_1;\quad \frac{dV}{dp_2}=-\widetilde{X}_2.$$

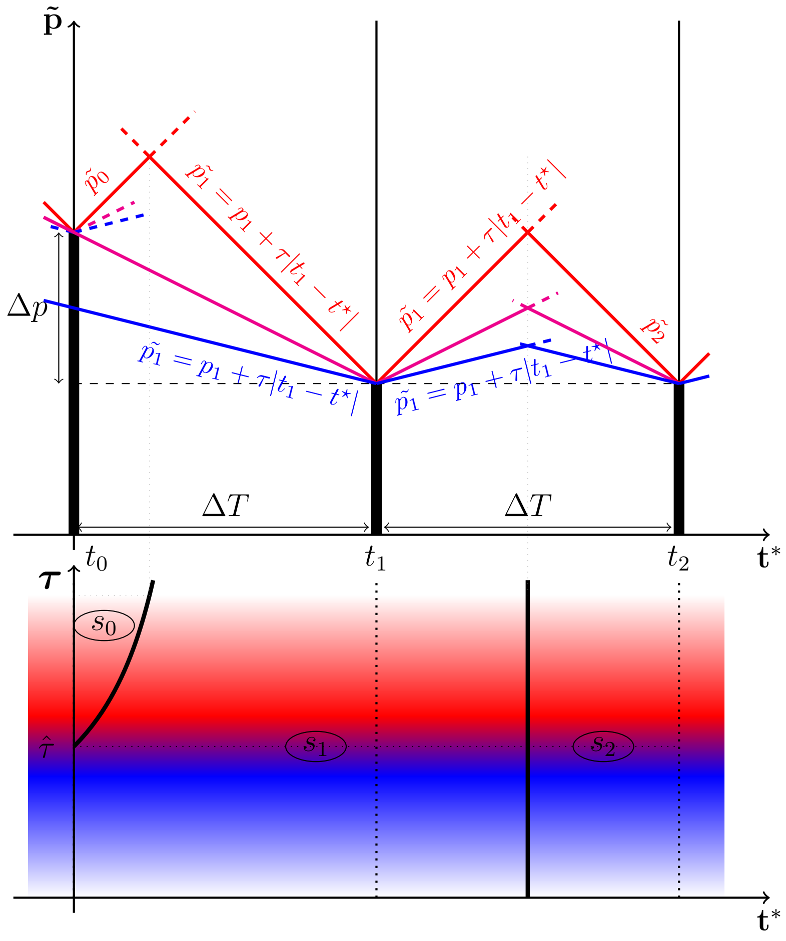

Figure 11: **Informed passengers - Case I.3:** $\mathbf{p_0 = p_h > p_l = p_2 = p_1}$
Top graph: The generalised price is plotted against the ideal departure time. The blue line represents the generalised price of a patient passenger ($\tau < \hat{\tau}$), the red line that of an impatient passenger ($\tau > \hat{\tau}$), the magenta line that of a passenger with $\tau = \hat{\tau}$. The individual generalised price decreases to the monetary price (the thick black vertical line placed in the service locations) as the departure time approaches the ideal one, t^*.
Bottom graph: Passenger distribution over the two heterogeneity dimensions (t^*, τ) and across services. Colors refer to the value of time τ; the intensity is associated with the number of passengers displaying the characteristics (t^*, τ). For this representation, we assume it independent of t^*. The expensive service s_0 is only used by passengers with high value of time and ideal departure time very close to t_0. The cheap service s_1 attracts all the other passengers who would otherwise take s_0. Instead, it does not attract passengers from s_2, which is equally cheap.

B.3.2 The impact of a change in schedule

We first compute

$$\begin{aligned}\frac{1}{n}\frac{d\overline{V}_0}{dT} &= -\frac{d\widehat{\tau}}{dT}\left(\int_{t_0}^{t_0} v\left(p_0+\widehat{\tau}\left\|t^*-t_0\right\|\right)dt^*\right)g\left(\widehat{\tau}\right)\\ &-\int_{\widehat{\tau}}^{+\infty}\tau\left(\int_{t_1-\Delta T}^{t_1-\frac{\Delta T}{2}\left(1+\frac{\widehat{\tau}}{\tau}\right)} x\left(p_0+\tau\left\|t^*-t_0\right\|\right)dt^*\right)dG\left(\tau\right)\\ &+\left[1-G\left(\hat{\tau}\right)\right]v\left(p_0\right)-\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_0+\left(\tau-\widehat{\tau}\right)\frac{\Delta T}{2}\right)dG\left(\tau\right),\end{aligned}$$

from which we obtain

$$\begin{aligned}\frac{d\overline{V}_0}{dT} &= -\int_{\widehat{\tau}}^{+\infty}\tau\left(\int_{t_1-\Delta T}^{t_1-\frac{\Delta T}{2}\left(1+\frac{\widehat{\tau}}{\tau}\right)} x\left(p_0+\tau\left\|t^*-t_0\right\|\right)ndt^*\right)dG\left(\tau\right)\\ &+n\left(1-G\left(\hat{\tau}\right)\right)v\left(p_0\right)-\frac{n}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_0+\left(\tau-\widehat{\tau}\right)\frac{\Delta T}{2}\right)dG\left(\tau\right).\end{aligned}$$

We next compute

$$\begin{aligned}\frac{1}{n}\frac{d\widetilde{V}_1}{dT} &= \frac{d\widehat{\tau}}{dT}\left(\int_{t_0}^{\frac{t_1+t_2}{2}} v\left(p_1+\hat{\tau}\left\|t^*-t_1\right\|\right)dt^*\right)g\left(\hat{\tau}\right)\\ &-\frac{d\widehat{\tau}}{dT}\left(\int_{\frac{t_0+t_1}{2}-\frac{\Delta p}{2\widehat{\tau}}}^{\frac{t_1+t_2}{2}} v\left(p_1+\hat{\tau}\left\|t^*-t_1\right\|\right)dt^*\right)g\left(\hat{\tau}\right)\\ &+\frac{1}{2}\int_0^{\hat{\tau}} v\left(p_1+\tau\left\|\frac{t_1+t_2}{2}-t_1\right\|\right)dG\left(\tau\right)+\int_0^{\hat{\tau}} v\left(p_1+\tau\left\|t_0-t_1\right\|\right)dG\left(\tau\right)\\ &+\int_0^{\hat{\tau}}\left(\int_{t_0}^{\frac{t_1+t_2}{2}}\tau\frac{d\left\|t^*-t_1\right\|}{dT}v'\left(p_1+\tau\left\|t^*-t_1\right\|\right)dt^*\right)dG\left(\tau\right)\\ &+\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_1+\tau\frac{\Delta T}{2}\right)dG\left(\tau\right)+\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_1+\tau\left\|\frac{\Delta T}{2}\left(1+\frac{\widehat{\tau}}{\tau}\right)\right\|\right)dG\left(\tau\right)\\ &+n\int_{\widehat{\tau}}^{+\infty}\left(\int_{\frac{t_0+t_1}{2}-\frac{\Delta p}{2\tau}}^{\frac{t_1+t_2}{2}}\tau\frac{d\left\|t^*-t_1\right\|}{dT}v'\left(p_1+\tau\left\|t^*-t_1\right\|\right)dt^*\right)dG\left(\tau\right),\end{aligned}$$

which is rearranged as

$$\begin{aligned}\frac{1}{n}\frac{d\widetilde{V}_1}{dT} &= -\frac{\widehat{\tau}}{\Delta T}\left(\int_{t_0}^{\frac{t_1+t_2}{2}} v\left(p_1+\hat{\tau}\left\|t^*-t_1\right\|\right)dt^*\right)g\left(\hat{\tau}\right)\\ &+\frac{\widehat{\tau}}{\Delta T}\left(\int_{t_0}^{\frac{t_1+t_2}{2}} v\left(p_1+\hat{\tau}\left\|t^*-t_1\right\|\right)dt^*\right)g\left(\hat{\tau}\right)\\ &+\frac{1}{2}\int_0^{\hat{\tau}} v\left(p_1+\tau\frac{\Delta T}{2}\right)dG\left(\tau\right)+\int_0^{\hat{\tau}} v\left(p_1+\tau\Delta T\right)dG\left(\tau\right)\\ &+\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_1+\tau\frac{\Delta T}{2}\right)dG\left(\tau\right)+\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_1+\tau\left(1+\frac{\widehat{\tau}}{\tau}\right)\frac{\Delta T}{2}\right)dG\left(\tau\right).\end{aligned}$$

This ultimately yields

$$\begin{aligned}\frac{d\widetilde{V}_1}{dT} &= \frac{n}{2}\int_0^{+\infty} v\left(p_1+\tau\frac{\Delta T}{2}\right)dG\left(\tau\right)+\int_0^{\hat{\tau}} v\left(p_1+\tau\Delta T\right)ndG\left(\tau\right)\\ &\quad+\frac{n}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_1+\tau\left(1+\frac{\widehat{\tau}}{\tau}\right)\frac{\Delta T}{2}\right)dG\left(\tau\right).\end{aligned}$$

We also compute

$$\begin{aligned}\frac{1}{n}\frac{d\widetilde{V}_2}{dT} &= \int_0^{+\infty} v\left(p_2\right)dG\left(\tau\right)\\ &-\frac{1}{2}\int_0^{+\infty} v\left(p_2+\tau\left\|\frac{t_1+t_2}{2}-t_2\right\|\right)dG\left(\tau\right)\\ &+\int_0^{+\infty}\left(\int_{\frac{t_1+t_2}{2}}^{t_2}\tau\frac{d\left\|t^*-t_2\right\|}{dT}v'\left(p_2+\tau\left\|t^*-t_2\right\|\right)dt^*\right)dG\left(\tau\right).\end{aligned}$$

Using $t_2=t_1+\Delta T$ and $\frac{t_1+t_2}{2}=t_1+\frac{\Delta T}{2}$, this is rewritten as

$$\begin{aligned}\frac{d\widetilde{V}_2}{dT} &= -\int_0^{+\infty}\tau\left(\int_{t_1+\frac{\Delta T}{2}}^{t_1+\Delta T} x\left(p_2+\tau\left\|t^*-t_2\right\|\right)ndt^*\right)dG\left(\tau\right)\\ &\quad+nv\left(p_2\right)-\frac{n}{2}\int_0^{+\infty} v\left(p_2+\tau\frac{\Delta T}{2}\right)dG\left(\tau\right).\end{aligned}$$

As a result

$$\begin{aligned}
\frac{dV}{dT} &= \frac{d\overline{V}_0}{dT} + \frac{d\widetilde{V}_1}{dT} + \frac{d\widetilde{V}_2}{dT} \\
&= -\int_{\widehat{\tau}}^{+\infty} \tau \left(\int_{t_1-\Delta T}^{t_1 - \frac{\Delta T}{2}\left(1+\frac{\widehat{\tau}}{\tau}\right)} x\left(p_0 + \tau \left\|t^* - t_0\right\|\right) n dt^* \right) dG\left(\tau\right) \\
&- \frac{n}{2} \int_{\widehat{\tau}}^{+\infty} v\left(p_0 + \left(\tau - \widehat{\tau}\right) \frac{\Delta T}{2}\right) dG\left(\tau\right) + n\left[1 - G\left(\hat{\tau}\right)\right] v\left(p_0\right) \\
&+ \frac{n}{2} \int_0^{+\infty} v\left(p_1 + \tau \frac{\Delta T}{2}\right) dG\left(\tau\right) + n \int_0^{\hat{\tau}} v\left(p_1 + \tau \Delta T\right) dG\left(\tau\right) \\
&+ \frac{n}{2} \int_{\widehat{\tau}}^{+\infty} v\left(p_1 + \tau\left(1 + \frac{\widehat{\tau}}{\tau}\right) \frac{\Delta T}{2}\right) dG\left(\tau\right) \\
&+ nv\left(p_2\right) - \frac{n}{2} \int_0^{+\infty} v\left(p_2 + \tau \frac{\Delta T}{2}\right) dG\left(\tau\right) \\
&- \int_0^{+\infty} \tau \left(\int_{t_1 + \frac{\Delta T}{2}}^{t_1+\Delta T} x\left(p_2 + \tau \left\|t^* - t_2\right\|\right) n dt^* \right) dG\left(\tau\right),
\end{aligned}$$

from which we further obtain

$$\begin{aligned}
\frac{dV}{dT} = &-\int_{\widehat{\tau}}^{+\infty} \tau \left(\int_{t_1-\Delta T}^{t_1 - \frac{\Delta T}{2}\left(1+\frac{\widehat{\tau}}{\tau}\right)} x\left(p_0 + \tau \left\|t^* - t_0\right\|\right) n dt^* \right) dG\left(\tau\right) \\
&- \int_0^{+\infty} \tau \left(\int_{t_1 + \frac{\Delta T}{2}}^{t_1+\Delta T} x\left(p_2 + \tau \left\|t^* - t_2\right\|\right) n dt^* \right) dG\left(\tau\right) \\
&- \frac{n}{2} \int_{\widehat{\tau}}^{+\infty} v\left(p_0 + \left(\tau - \widehat{\tau}\right) \frac{\Delta T}{2}\right) dG\left(\tau\right) + n \int_0^{\hat{\tau}} v\left(p_1 + \tau \Delta T\right) dG\left(\tau\right) \\
&+ \frac{n}{2} \int_{\widehat{\tau}}^{+\infty} v\left(p_1 + \left(\tau + \widehat{\tau}\right) \frac{\Delta T}{2}\right) dG\left(\tau\right) + n\left[\left(1 - G\left(\hat{\tau}\right)\right) v\left(p_0\right) + v\left(p_2\right)\right].
\end{aligned}$$

Recall that when $t^* = \frac{t_0+t_1}{2} - \frac{\Delta p}{2\tau}$ we have $p_0 + \tau \left\|t^* - t_0\right\| = p_1 + \tau \left\|t^* - t_1\right\|$ so that

$$p_0 + \tau \left\| \frac{\Delta T}{2} - \frac{\Delta p}{2\tau} \right\| = p_1 + \tau \left\| \frac{\Delta T}{2} + \frac{\Delta p}{2\tau} \right\|,$$

which further yields

$$p_0 + \left(\tau - \widehat{\tau}\right) \frac{\Delta T}{2} = p_1 + \left(\tau + \hat{\tau}\right) \frac{\Delta T}{2}.$$

Observing also that $p_0 = p_1 + \hat{\tau} \Delta T$, we end up with

$$v\left(p_0 + \left(\tau - \widehat{\tau}\right) \frac{\Delta T}{2}\right) = v\left(p_1 + \left(\tau + \hat{\tau}\right) \frac{\Delta T}{2}\right) = v\left(p_2 + \left(\tau + \widehat{\tau}\right) \frac{\Delta T}{2}\right)$$

and we can write

$$\frac{dV}{dT} = -\int_{\hat{\tau}}^{+\infty} \tau \left(\int_{t_1-\Delta T}^{t_1-\frac{\Delta T}{2}\left(1+\frac{\hat{\tau}}{\tau}\right)} x\left(p_0+\tau\left\|t^*-t_0\right\|\right) ndt^* \right) dG(\tau)$$
$$-\int_{0}^{+\infty} \tau \left(\int_{t_1+\frac{\Delta T}{2}}^{t_1+\Delta T} x\left(p_2+\tau\left\|t^*-t_2\right\|\right) ndt^* \right) dG(\tau)$$
$$+n\left[\left(1-G(\hat{\tau})\right)v(p_0)+v(p_2)\right]+n\int_0^{\hat{\tau}} v\left(p_1+\tau\Delta T\right)dG(\tau).$$

Using $t_0 = t_1 - \Delta T$, $\frac{t_0+t_1}{2} - \frac{\Delta p}{2\tau} = t_1 - \frac{\Delta T}{2}\left(1+\frac{\hat{\tau}}{\tau}\right)$, and $\frac{t_1+t_2}{2} = t_1 + \frac{\Delta T}{2}$, and defining

$$\bar{\tau}_0 = \int_{\hat{\tau}}^{+\infty} \tau \frac{X_0(\tau)}{\overline{X}_0} dG(\tau) \quad \text{and} \quad \widetilde{\tau}_2 = \int_0^{+\infty} \tau \frac{X_2(\tau)}{\widetilde{X}_2} dG(\tau),$$

we can further express

$$\begin{aligned} \frac{dV}{dT} &= -\bar{\tau}_0\overline{X}_0 - \widetilde{\tau}_2\widetilde{X}_2 \\ &\quad +n\left[\left(1-G(\hat{\tau})\right)v(p_0)+v(p_2)\right]+n\int_0^{\hat{\tau}} v\left(p_1+\tau\Delta T\right)dG(\tau). \end{aligned}$$

Neglecting the volume effect (the terms in the second line), we obtain

$$\frac{dV}{dT} = -\left(\bar{\tau}_0\overline{X}_0 + \widetilde{\tau}_2\widetilde{X}_2\right), \tag{14}$$

which evidences that the reduction in surplus works entirely through s_0 and s_2.

B.4 Case I.4: $p_1 = p_0 = p_h > p_l = p_2$

We conclude with a case where s_0 and s_1 are both more expensive than s_2.[9] Given that $p_0 = p_1$, all passengers with $t^* \in \left[t_0, \frac{t_0+t_1}{2}\right]$ are better off if they take s_0 instead of s_1. However, it is more advantageous to take s_2, if

$$p_2 + \tau(t_2 - t^*) \leq p_0 + \tau(t^* - t_0) \Leftrightarrow t^* \geq t_0 + \Delta T - \frac{\Delta p}{2\tau}. \tag{15}$$

Here, in addition to $\hat{\tau}$, a second cut-off value of time is found to be relevant, namely $\hat{\tau}/2$, just *as in the uninformed case* (recall U.4). For $\tau \leq \hat{\tau}/2$, (15) holds true for all passengers with $t^* \in [t_0, t_2]$, hence they all prefer to use s_2. These passengers are so patient that they find it worth taking the cheapest service, although it is the latest to depart. For $\tau > \hat{\tau}/2$, there exists $t_c \in [t_0, t_2]$ such that all passengers with $t^* \in [t_0, t_c]$ prefer s_0 to s_2

[9] This is one of two possible cases where s_1 is one of the *expensive* services. In the other such case, s_1 and s_2 would both be more expensive than s_0, namely $p_1 = p_2 = p_h > p_l = p_0$. Developing this alternative case would bring no additional insight, hence it is omitted.

instead. By definition

$$t_c = t_0 + \Delta T - \frac{\Delta p}{2\tau} = t_1 - \frac{\Delta p}{2\tau},$$

hence these are passengers with $t^* \in \left[t_0, t_1 - \frac{\Delta p}{2\tau}\right]$. When $t_c \in \left[t_0, \frac{t_0+t_1}{2}\right]$, namely

$$t_1 - \frac{\Delta p}{2\tau} < \frac{t_0 + t_1}{2} \Leftrightarrow \frac{\hat{\tau}}{2} < \tau < \hat{\tau},$$

passengers with $t^* \in [t_0, t_c]$ also prefer s_0 to s_1, and choose s_0. By contrast, passengers with $t^* \in \left[t_c, \frac{t_0+t_1}{2}\right]$ prefer s_2 to s_0 and s_0 to s_1; hence, they use s_2. For passengers with $t^* \geq \frac{t_0+t_1}{2}$, s_0 is never convenient. Those with $t^* \in \left[\frac{t_0+t_1}{2}, t_1\right]$ find s_2 more convenient than s_1 if

$$p_l + \tau\left(t_2 - t^*\right) \leq p_h + \tau\left(t_1 - t^*\right) \Leftrightarrow \tau \leq \hat{\tau}.$$

In turn, passengers with $t^* \in [t_1, t_2]$ find s_2 more convenient than s_1 if

$$p_l + \tau\left(t_2 - t^*\right) \leq p_h + \tau\left(t^* - t_1\right) \Leftrightarrow t^* \geq t_1 + \frac{1}{2}\left(\Delta T - \frac{\Delta p}{\tau}\right),$$

which is again the case for $\tau \leq \hat{\tau}$. Thus, none of these passenger uses s_1, which is equally expensive but departs later than s_0. For $\tau > \hat{\tau}$, all passengers with $t^* \in \left[t_1, \frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau}\right]$ find s_1 more convenient, whereas those with $t^* \in \left[\frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau}, t_2\right]$ find s_2 more convenient. Moreover, all passengers with $t^* \in \left[\frac{t_0+t_1}{2}, t_1\right]$ prefer s_1 to s_2. As they also prefer s_1 to s_0, they take s_1. There is, thus, a discontinuity at $\tau = \hat{\tau}$: whereas for $\tau < \hat{\tau}$ no passenger uses s_1, regardless of t^*, for $\tau > \hat{\tau}$ there is a set of passengers of strictly positive measure using s_1. As seen, this set must contain all passengers with $t^* \in \left[\frac{t_0+t_1}{2}, t_1\right]$. In definitive, for $\tau > \hat{\tau}$, passengers with $t^* \in \left[t_0, \frac{t_0+t_1}{2}\right]$ choose s_0, passengers with $t^* \in \left[\frac{t_0+t_1}{2}, \frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau}\right] \supset \left[\frac{t_0+t_1}{2}, t_1\right]$ choose s_1, and passengers with $t^* \in \left[\frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau}, t_2\right]$ choose s_2. Taking this all into account, we can respectively write the demand for service s_0, s_1 and s_2 as

$$\begin{aligned}
X_0 &= \underline{X}_0 + \overline{X}_0 \\
&= \int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} X_0(\tau)\, dG(\tau) + \int_{\hat{\tau}}^{+\infty} X_{[0]}(\tau)\, dG(\tau) \\
\overline{X}_1 &= \int_{\hat{\tau}}^{+\infty} X_1(\tau)\, dG(\tau) \\
X_2 &= \underline{\underline{X}}_2 + \underline{X}_2 + \bar{X}_2 \\
&= \int_0^{\frac{\hat{\tau}}{2}} X_{[0,1]\rhd 2}(\tau)\, dG(\tau) + \int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} X_{0,1\rhd 2}(\tau)\, dG(\tau) + \int_{\hat{\tau}}^{+\infty} X_{1\rhd 2}(\tau)\, dG(\tau),
\end{aligned}$$

where now

$$\begin{aligned}
X_0(\tau) &= \int_{t_0}^{t_1 - \frac{\Delta p}{2\tau}} x\left(p_0 + \tau\left\|t^* - t_0\right\|\right) n dt^* \\
X_{[0]}(\tau) &= \int_{t_0}^{\frac{t_0+t_1}{2}} x\left(p_0 + \tau\left\|t^* - t_0\right\|\right) n dt^*,
\end{aligned}$$

together with

$$X_1(\tau) = \int_{\frac{t_0+t_1}{2}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}} x\left(p_1 + \tau\left\|t^* - t_1\right\|\right) n dt^*$$

and with

$$\begin{aligned}
X_{[0,1]\triangleright 2}(\tau) &= \int_{t_0}^{t_2} x\left(p_2 + \tau\left\|t^* - t_2\right\|\right) n dt^* \\
X_{0,1\triangleright 2}(\tau) &= \int_{t_1-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2 + \tau\left\|t^* - t_2\right\|\right) n dt^* \\
X_{1\triangleright 2}(\tau) &= \int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2 + \tau\left\|t^* - t_2\right\|\right) n dt^*.
\end{aligned}$$

This market segmentation is represented in Figure 12. The surplus respectively associated with s_0, s_1 and s_2 is given by

$$\begin{aligned}
V_0 &= \underline{V}_0 + \overline{V}_0 \\
& \int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \left(\int_{t_0}^{t_1-\frac{\Delta p}{2\tau}} v\left(p_0 + \tau\left\|t^* - t_0\right\|\right) n dt^*\right) dG(\tau) \\
& + \int_{\hat{\tau}}^{+\infty} \left(\int_{t_0}^{\frac{t_0+t_1}{2}} v\left(p_0 + \tau\left\|t^* - t_0\right\|\right) n dt^*\right) dG(\tau) \\
\overline{V}_1 &= \int_{\widehat{\tau}}^{+\infty} \left(\int_{\frac{t_0+t_1}{2}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}} v\left(p_1 + \tau\left\|t^* - t_1\right\|\right) n dt^*\right) dG(\tau) \\
\widetilde{V}_2 &= \underline{\underline{V}}_2 + \underline{V}_2 + \overline{V}_2 \\
&= \int_0^{\frac{\hat{\tau}}{2}} \left(\int_{t_0}^{t_2} v\left(p_2 + \tau\left\|t^* - t_2\right\|\right) n dt^*\right) dG(\tau) \\
& + \int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \left(\int_{t_1-\frac{\Delta p}{2\tau}}^{t_2} v\left(p_2 + \tau\left\|t^* - t_2\right\|\right) n dt^*\right) dG(\tau) \\
& + \int_{\widehat{\tau}}^{+\infty} \left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} v\left(p_2 + \tau\left\|t^* - t_2\right\|\right) n dt^*\right) dG(\tau).
\end{aligned}$$

B.4.1 The impact of a change in price

As usual, an infinitesimal increase in price p_i, where $i = 0, 1, 2$, will determine a reduction in surplus equal to the aggregate demand for service i.

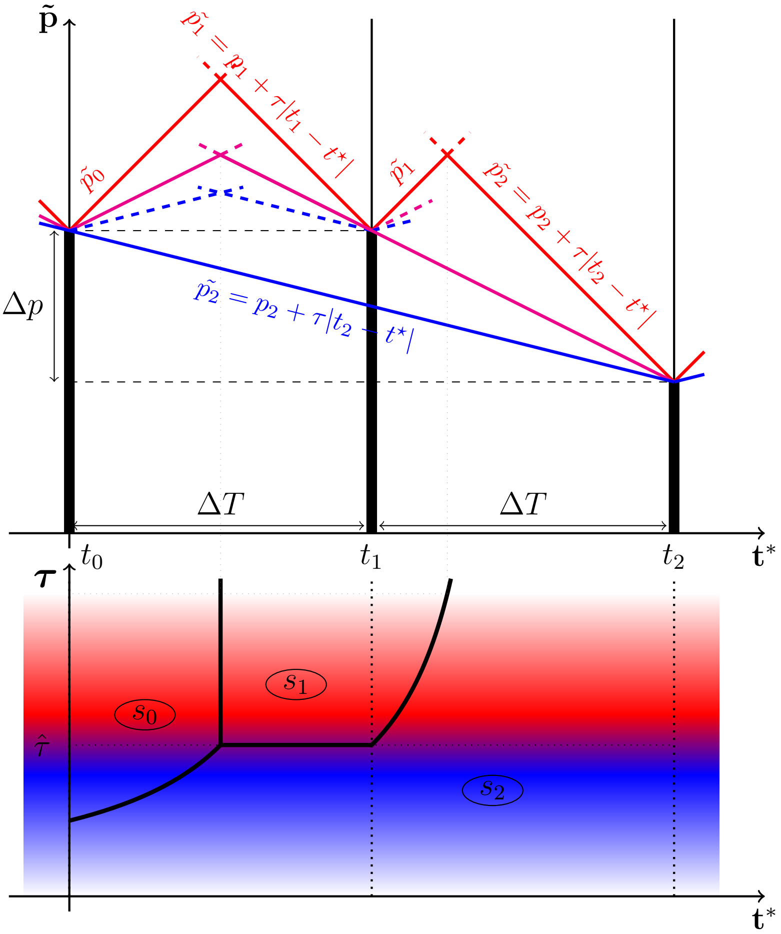

Figure 12: **Informed passengers - Case I.4:** $p_0 = p_1 = p_h > p_l = p_2$

Top graph: The generalised price is plotted against the ideal departure time. The blue line represents the generalised price of a patient passenger ($\tau < \hat{\tau}$), the red line that of an impatient passenger ($\tau > \hat{\tau}$), the magenta line that of a passenger with $\tau = \hat{\tau}$. The individual generalised price decreases to the monetary price (the thick black vertical line placed in the service locations) as the departure time approaches the ideal one, t^*.

Bottom graph: Passenger distribution over the two heterogeneity dimensions (t^*, τ) and across services. Colors refer to the value of time τ; the intensity is associated with the number of passengers displaying the characteristics (t^*, τ). For this representation, we assume it independent of t^*. The cheap service s_2 attracts passengers with low value of time from s_0, and passengers with intermediate value of time from s_1. From s_1, s_2 further attracts some passengers with high value of time who would like to depart slightly after t_1, but are available to wait until t_2 to pay less.

B.4.2 The impact of a change in schedule

We first compute

$$
\begin{aligned}
\frac{1}{n}\frac{dV_0}{dT} = {} & \frac{d\widehat{\tau}}{dT}\left(\int_{t_0}^{t_1-\frac{\Delta p}{2\widehat{\tau}}} v\left(p_0+\widehat{\tau}\left\|t^*-t_0\right\|\right)dt^*\right)g\left(\widehat{\tau}\right) \\
& -\frac{1}{2}\frac{d\widehat{\tau}}{dT}\left(\int_{t_0}^{t_1-\frac{\Delta p}{\widehat{\tau}}} v\left(p_0+\frac{\widehat{\tau}}{2}\left\|t^*-t_0\right\|\right)dt^*\right)g\left(\frac{\widehat{\tau}}{2}\right) \\
& +\int_{\hat{\tau}/2}^{\hat{\tau}} v\left(p_0\right)dG\left(\tau\right)-\int_{\hat{\tau}/2}^{\hat{\tau}} \tau\left(\int_{t_0}^{t_1-\frac{\Delta p}{2\tau}} x\left(p_0+\tau\left(t^*-t_0\right)\right)dt^*\right)dG\left(\tau\right) \\
& -\frac{d\widehat{\tau}}{dT}\left(\int_{t_0}^{\frac{t_0+t_1}{2}} v\left(p_0+\widehat{\tau}\left\|t^*-t_0\right\|\right)dt^*\right)g\left(\widehat{\tau}\right) \\
& +\int_{\hat{\tau}}^{+\infty}\left(-\frac{1}{2}v\left(p_0+\tau\left\|\frac{t_0+t_1}{2}-t_0\right\|\right)+v\left(p_0\right)\right)dG\left(\tau\right) \\
& -\int_{\hat{\tau}}^{+\infty}\tau\left(\int_{t_0}^{\frac{t_0+t_1}{2}} x\left(p_0+\tau\left(t^*-t_0\right)\right)dt^*\right)dG\left(\tau\right),
\end{aligned}
$$

from which we then obtain

$$
\begin{aligned}
\frac{1}{n}\frac{dV_0}{dT} = {} & -\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \tau\left(\int_{t_0}^{t_1-\frac{\Delta p}{2\tau}} x\left(p_0+\tau\left(t^*-t_0\right)\right)dt^*\right)dG\left(\tau\right) \\
& -\int_{\hat{\tau}}^{+\infty}\tau\left(\int_{t_0}^{\frac{t_0+t_1}{2}} x\left(p_0+\tau\left(t^*-t_0\right)\right)dt^*\right)dG\left(\tau\right) \\
& +\int_{\frac{\hat{\tau}}{2}}^{+\infty} v\left(p_0\right)dG\left(\tau\right)-\frac{1}{2}\int_{\hat{\tau}}^{+\infty} v\left(p_0+\tau\left\|\frac{t_0+t_1}{2}-t_0\right\|\right)dG\left(\tau\right).
\end{aligned}
$$

We next compute

$$
\begin{aligned}
\frac{1}{n}\frac{d\overline{V}_1}{dT} = {} & -\frac{d\widehat{\tau}}{dT}\left(\int_{\frac{t_0+t_1}{2}}^{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\widehat{\tau}}} v\left(p_1+\widehat{\tau}\left\|t^*-t_1\right\|\right)dt^*\right)g\left(\widehat{\tau}\right) \\
& +\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_1+\tau\left\|\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}-t_1\right\|\right)dG\left(\tau\right) \\
& +\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_1+\tau\left\|\frac{t_0+t_1}{2}-t_1\right\|\right)dG\left(\tau\right),
\end{aligned}
$$

which is rearranged as

$$\begin{aligned}\frac{1}{n}\frac{d\overline{V}_1}{dT} &= -\frac{d\widehat{\tau}}{dT}\left(\int_{\frac{t_0+t_1}{2}}^{t_1} v\left(p_1+\widehat{\tau}\left\|t^*-t_1\right\|\right)dt^*\right)g\left(\widehat{\tau}\right)\\ &\quad +\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_1+\frac{\Delta T}{2}\left(\tau-\widehat{\tau}\right)\right)dG\left(\tau\right)\\ &\quad +\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_1+\frac{1}{2}\tau\Delta T\right)dG\left(\tau\right).\end{aligned}$$

We also compute

$$\begin{aligned}\frac{1}{n}\frac{d\widetilde{V}_2}{dT} &= \frac{1}{2}\frac{d\widehat{\tau}}{dT}\left(\int_{t_0}^{t_2} v\left(p_2+\frac{\widehat{\tau}}{2}\left\|t^*-t_2\right\|\right)dt^*\right)g\left(\frac{\widehat{\tau}}{2}\right)\\ &+\int_0^{\frac{\hat{\tau}}{2}}\left(v\left(p_2\right)+v\left(p_2+\tau\left\|t_0-t_2\right\|\right)\right)dG\left(\tau\right)\\ &-\int_0^{\frac{\hat{\tau}}{2}}\left(\tau\int_{t_0}^{t_2} x\left(p_2+\tau\left\|t^*-t_2\right\|\right)dt^*\right)dG\left(\tau\right)\\ &+\frac{d\widehat{\tau}}{dT}\left(\int_{t_1-\frac{\Delta p}{2\hat{\tau}}}^{t_2} v\left(p_2+\hat{\tau}\left\|t^*-t_2\right\|\right)dt^*\right)g\left(\widehat{\tau}\right)\\ &-\frac{1}{2}\frac{d\widehat{\tau}}{dT}\left(\int_{t_1-\frac{\Delta p}{\hat{\tau}}}^{t_2} v\left(p_2+\frac{\widehat{\tau}}{2}\left\|t^*-t_2\right\|\right)dt^*\right)g\left(\frac{\widehat{\tau}}{2}\right)\\ &+\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} v\left(p_2\right)dG\left(\tau\right)-\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\tau\int_{t_1-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2+\tau\left\|t^*-t_2\right\|\right)dt^*\right)dG\left(\tau\right)\\ &-\frac{d\widehat{\tau}}{dT}\left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\hat{\tau}}}^{t_2} v\left(p_2+\widehat{\tau}\left\|t^*-t_2\right\|\right)dt^*\right)g\left(\widehat{\tau}\right)\\ &+\int_{\widehat{\tau}}^{+\infty}\left(v\left(p_2\right)-\frac{1}{2}v\left(p_2+\tau\left\|\frac{\Delta p}{2\tau}+\frac{t_2-t_1}{2}\right\|\right)\right)dG\left(\tau\right)\\ &-\int_{\widehat{\tau}}^{+\infty}\left(\tau\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2+\tau\left\|t^*-t_2\right\|\right)dt^*\right)dG\left(\tau\right).\end{aligned}$$

This simplifies to

$$
\begin{aligned}
\frac{1}{n}\frac{dV_2}{dT} &= \frac{d\widehat{\tau}}{dT}\left(\int_{\frac{t_0+t_1}{2}}^{t_1} v\left(p_2+\hat{\tau}\left\|t^*-t_2\right\|\right)dt^*\right)g\left(\widehat{\tau}\right) \\
&+ v\left(p_2\right) + \int_0^{\frac{\hat{\tau}}{2}} v\left(p_2+\tau\left\|t_0-t_2\right\|\right)dG\left(\tau\right) \\
&- \frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_2+\frac{1}{2}\Delta T\left(\hat{\tau}+\tau\right)\right)dG\left(\tau\right) \\
&- \int_0^{\frac{\hat{\tau}}{2}}\left(\tau\int_{t_0}^{t_2} x\left(p_2+\tau\left\|t^*-t_2\right\|\right)dt^*\right)dG\left(\tau\right) \\
&- \int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\tau\int_{t_1-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2+\tau\left\|t^*-t_2\right\|\right)dt^*\right)dG\left(\tau\right) \\
&- \int_{\widehat{\tau}}^{+\infty}\left(\tau\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2+\tau\left\|t^*-t_2\right\|\right)dt^*\right)dG\left(\tau\right).
\end{aligned}
$$

As a result

$$
\begin{aligned}
\frac{1}{n}\frac{dV}{dT} &= \frac{1}{n}\left(\frac{dV_0}{dT}+\frac{d\overline{V}_1}{dT}+\frac{d\widetilde{V}_2}{dT}\right) \\
&= -\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \tau\left(\int_{t_0}^{t_1-\frac{\Delta p}{2\tau}} x\left(p_0+\tau\left(t^*-t_0\right)\right)dt^*\right)dG\left(\tau\right) \\
&\quad -\int_{\hat{\tau}}^{+\infty} \tau\left(\int_{t_0}^{\frac{t_0+t_1}{2}} x\left(p_0+\tau\left(t^*-t_0\right)\right)dt^*\right)dG\left(\tau\right) \\
&\quad +\int_{\frac{\hat{\tau}}{2}}^{+\infty} v\left(p_0\right)dG\left(\tau\right)-\frac{1}{2}\int_{\hat{\tau}}^{+\infty}\left(v\left(p_0+\tau\left\|\frac{t_0+t_1}{2}-t_0\right\|\right)\right)dG\left(\tau\right) \\
&\quad -\frac{d\widehat{\tau}}{dT}\left(\int_{\frac{t_0+t_1}{2}}^{t_1} v\left(p_1+\widehat{\tau}\left\|t^*-t_1\right\|\right)dt^*\right)g\left(\widehat{\tau}\right) \\
&\quad +\frac{1}{2}\int_{\widehat{\tau}}^{+\infty}\left(v\left(p_1+\frac{1}{2}\Delta T\left(\tau-\widehat{\tau}\right)\right)+v\left(p_1+\frac{1}{2}\tau\Delta T\right)\right)dG\left(\tau\right) \\
&\quad +\frac{d\widehat{\tau}}{dT}\left(\int_{\frac{t_0+t_1}{2}}^{t_1} v\left(p_2+\hat{\tau}\left\|t^*-t_2\right\|\right)dt^*\right)g\left(\widehat{\tau}\right) \\
&\quad +v\left(p_2\right)+\int_0^{\hat{\tau}/2} v\left(p_2+\tau\left\|t_0-t_2\right\|\right)dG\left(\tau\right) \\
&\quad -\frac{1}{2}\int_{\widehat{\tau}}^{+\infty} v\left(p_2+\frac{1}{2}\Delta T\left(\hat{\tau}+\tau\right)\right)dG\left(\tau\right) \\
&\quad -\int_0^{\frac{\hat{\tau}}{2}}\left(\tau\int_{t_0}^{t_2} x\left(p_2+\tau\left\|t^*-t_2\right\|\right)dt^*\right)dG\left(\tau\right) \\
&\quad -\int_{\hat{\tau}/2}^{\hat{\tau}}\left(\tau\int_{t_1-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2+\tau\left\|t^*-t_2\right\|\right)dt^*\right)dG\left(\tau\right) \\
&\quad -\int_{\widehat{\tau}}^{+\infty}\left(\tau\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} x\left(p_2+\tau\left\|t^*-t_2\right\|\right)dt^*\right)dG\left(\tau\right).
\end{aligned}
$$

Given that $p_1 = p_0 = p_h > p_l = p_2$, we have $p_1 = p_2 + \Delta p$, hence $p_1 = p_2 + \hat{\tau}\Delta T$. We can thus write $p_1 + \widehat{\tau}t_1 = p_2 + \hat{\tau}\left(\Delta T + t_1\right) = p_2 + \hat{\tau}t_2$ and so

$$
p_1 - \frac{1}{2}\widehat{\tau}\Delta T = p_2 + \frac{1}{2}\widehat{\tau}\Delta T.
$$

Using this equality in the previous computation, we further obtain

$$
\begin{aligned}
\frac{1}{n}\frac{dV}{dT} = & \left(1 - G\left(\frac{\hat{\tau}}{2}\right)\right) v\left(p_0\right) + v\left(p_2\right) + \int_0^{\frac{\hat{\tau}}{2}} v\left(p_2 + \tau 2\Delta T\right) dG\left(\tau\right) \\
& - \int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \tau \left(\int_{t_0}^{t_1 - \frac{\Delta p}{2\tau}} x\left(p_0 + \tau\left(t^* - t_0\right)\right) dt^*\right) dG\left(\tau\right) \\
& - \int_{\hat{\tau}}^{+\infty} \tau \left(\int_{t_0}^{\frac{t_0 + t_1}{2}} x\left(p_0 + \tau\left(t^* - t_0\right)\right) dt^*\right) dG\left(\tau\right) \\
& - \int_0^{\frac{\hat{\tau}}{2}} \tau \left(\int_{t_0}^{t_2} x\left(p_2 + \tau\left\|t^* - t_2\right\|\right) dt^*\right) dG\left(\tau\right) \\
& - \int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \tau \left(\int_{t_1 - \frac{\Delta p}{2\tau}}^{t_2} x\left(p_2 + \tau\left\|t^* - t_2\right\|\right) dt^*\right) dG\left(\tau\right) \\
& - \int_{\hat{\tau}}^{+\infty} \tau \left(\int_{\frac{t_1 + t_2}{2} - \frac{\Delta p}{2\tau}}^{t_2} x\left(p_2 + \tau\left(t_2 - t^*\right)\right) dt^*\right) dG\left(\tau\right).
\end{aligned}
$$

Denoting

$$
\begin{aligned}
& \underline{\tau}_0 = \int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \tau \frac{X_0\left(\tau\right)}{\underline{X}_0} dG\left(\tau\right),\ \bar{\tau}_0 = \int_{\hat{\tau}}^{+\infty} \tau \frac{X_{[0]}\left(\tau\right)}{\bar{X}_0} dG\left(\tau\right) \\
& \underline{\underline{\tau}}_2 = \int_0^{\frac{\hat{\tau}}{2}} \tau \frac{X_{[0,1]\triangleright 2}\left(\tau\right)}{\underline{\underline{X}}_2} dG\left(\tau\right),\ \underline{\tau}_2 = \int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \tau \frac{X_{0,1\triangleright 2}\left(\tau\right)}{\underline{X}_2} dG\left(\tau\right) \text{ and } \bar{\tau}_2 = \int_{\hat{\tau}}^{+\infty} \tau \frac{X_{1\triangleright 2}\left(\tau\right)}{\bar{X}_2} dG\left(\tau\right),
\end{aligned}
$$

this is rewritten as

$$
\begin{aligned}
\frac{dV}{dT} = & -\left(\underline{\tau}_0 \underline{X}_0 + \bar{\tau}_0 \bar{X}_0 + \underline{\underline{\tau}}_2 \underline{\underline{X}}_2 + \underline{\tau}_2 \underline{X}_2 + \bar{\tau}_2 \bar{X}_2\right) \\
& + n\left[\left(1 - G\left(\frac{\hat{\tau}}{2}\right)\right) v\left(p_0\right) + v\left(p_2\right)\right] + n \int_0^{\frac{\hat{\tau}}{2}} v\left(p_2 + 2\tau\Delta T\right) dG\left(\tau\right).
\end{aligned}
$$

Neglecting volume effects (the terms in the second line), we obtain

$$
\frac{dV}{dT} = -\left(\underline{\tau}_0 \underline{X}_0 + \bar{\tau}_0 \bar{X}_0 + \underline{\underline{\tau}}_2 \underline{\underline{X}}_2 + \underline{\tau}_2 \underline{X}_2 + \bar{\tau}_2 \bar{X}_2\right). \tag{16}
$$

In line with previous findings, as the service becomes less frequent and the DTS increases for patrons of s_0 and s_2, welfare is reduced by an aggregate measure of their value of time. This includes the impact on passengers using s_0 with low and high value of time, and that on passengers using s_2 with low, intermediate, and high value of time. Although it is clear that an increase in frequency around the expensive service would benefit passengers with any level of income, the market segmentation expressed by (16) would be extremely useful to appraise the *extent* of the benefit to passengers with different levels of income.

C Aggregate monetary and non-monetary price

Consider the price pattern $p_0 = p_1 = p_h > p_2 = p_l$ and passengers with $t^* \in [t_0, t_2]$. We will compare first the aggregate monetary price and next the aggregate non-monetary cost in cases U.4 and I.4. In all computations we will omit n, for shortness; hence, all findings are to be intended up to a scaling factor of n.

C.1 Aggregate monetary price

C.1.1 Case U.4

The aggregate monetary price of passengers using $s_1$is given by

$$\begin{aligned}\int_{\hat{\tau}}^{\infty}\left(\int_{t_0}^{t_1} p_1 dt^*\right) dG(\tau) &= p_1 \int_{\hat{\tau}}^{\infty}\left(\int_{t_0}^{t_1} dt^*\right) dG(\tau)\\ &= p_1 \Delta T \int_{\hat{\tau}}^{\infty} dG(\tau)\\ &= p_1 (1 - G(\hat{\tau}))\Delta T.\end{aligned}$$

For passengers using s_2 it is given by

$$\begin{aligned}&\int_{0}^{\hat{\tau}}\left(\int_{t_0}^{t_1} p_2 dt^*\right) dG(\tau) + \int_{0}^{\infty}\left(\int_{t_1}^{t_2} p_2 dt^*\right) dG(\tau)\\ &= p_2\left(\int_{0}^{\hat{\tau}} dG(\tau) + \int_{0}^{\infty} dG(\tau)\right)\Delta T\\ &= p_2 (1 + G(\hat{\tau}))\Delta T.\end{aligned}$$

Overall, the aggregate monetary price for uninformed passengers is given by

$$p_1 (1 - G(\hat{\tau}))\Delta T + p_2 (1 + G(\hat{\tau}))\Delta T = 2p_l \Delta T + \Delta p (1 - G(\hat{\tau}))\Delta T.$$

C.1.2 Case I.4

The aggregate monetary price of passengers using s_0 is computed as follows:

$$\begin{aligned}&\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\int_{t_0}^{t_1 - \frac{\Delta p}{2\tau}} p_0 dt^*\right) dG(\tau) + \int_{\hat{\tau}}^{+\infty}\left(\int_{t_0}^{\frac{t_0+t_1}{2}} p_0 dt^*\right) dG(\tau)\\ &= p_0\left[\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(t_1 - \frac{\Delta p}{2\tau} - t_0\right) dG(\tau) + \frac{\Delta T}{2}\int_{\hat{\tau}}^{+\infty} dG(\tau)\right]\\ &= p_0\left[\left(G(\hat{\tau}) - G\left(\frac{\hat{\tau}}{2}\right)\right)\Delta T - \frac{\Delta p}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\frac{dG(\tau)}{\tau} + (1 - G(\hat{\tau}))\frac{\Delta T}{2}\right]\\ &= p_h\left[G(\hat{\tau}) - G\left(\frac{\hat{\tau}}{2}\right) - \frac{\hat{\tau}}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\frac{dG(\tau)}{\tau} + \frac{1}{2}(1 - G(\hat{\tau}))\right]\Delta T.\end{aligned}$$

For passengers using s_1 it is given by

$$\begin{aligned}
&\int_{\hat{\tau}}^{+\infty}\left(\int_{\frac{t_0+t_1}{2}}^{t_1+\frac{\Delta T}{2}\left(1-\frac{\hat{\tau}}{\tau}\right)} p_1 dt^*\right) dG(\tau) \\
&= p_1 \int_{\hat{\tau}}^{+\infty}\left[t_1+\frac{\Delta T}{2}\left(1-\frac{\hat{\tau}}{\tau}\right)-\frac{t_0+t_1}{2}\right] dG(\tau) \\
&= p_1 \Delta T \int_{\hat{\tau}}^{+\infty}\left(1-\frac{\hat{\tau}}{2\tau}\right) dG(\tau) \\
&= p_h\left(1-G(\hat{\tau})-\frac{\hat{\tau}}{2}\int_{\hat{\tau}}^{+\infty}\frac{dG(\tau)}{\tau}\right)\Delta T.
\end{aligned}$$

For passengers using s_2 it is given by

$$\begin{aligned}
&\int_0^{\frac{\hat{\tau}}{2}}\left(\int_{t_0}^{t_2} p_2 dt^*\right) dG(\tau)+\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\int_{t_1-\frac{\Delta p}{2\tau}}^{t_2} p_2 dt^*\right) dG(\tau) \\
&+\int_{\hat{\tau}}^{+\infty}\left(\int_{\frac{t_1+t_2}{2}-\frac{\Delta p}{2\tau}}^{t_2} p_2 dt^*\right) dG(\tau) \\
&= p_2\left[2\int_0^{\frac{\hat{\tau}}{2}} dG(\tau)+\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\Delta T+\frac{\Delta p}{2\tau}\right) dG(\tau)+\int_{\hat{\tau}}^{+\infty}\left(\frac{\Delta T}{2}+\frac{\Delta p}{2\tau}\right) dG(\tau)\right]\Delta T \\
&= p_2\left[2G\left(\frac{\hat{\tau}}{2}\right)+G(\hat{\tau})-G\left(\frac{\hat{\tau}}{2}\right)+\frac{1}{2}\left(1-G(\hat{\tau})\right)+\frac{\hat{\tau}}{2}\int_{\frac{\hat{\tau}}{2}}^{+\infty}\frac{dG(\tau)}{\tau}\right]\Delta T \\
&= p_l\left[1+2G\left(\frac{\hat{\tau}}{2}\right)+G(\hat{\tau})+\hat{\tau}\int_{\frac{\hat{\tau}}{2}}^{+\infty}\frac{dG(\tau)}{\tau}\right]\frac{\Delta T}{2}.
\end{aligned}$$

Overall, the aggregate monetary price of informed passengers amounts to

$$\begin{aligned}
& p_h\left[G(\hat{\tau})-G\left(\frac{\hat{\tau}}{2}\right)-\frac{\hat{\tau}}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\frac{dG(\tau)}{\tau}+\frac{1}{2}\left(1-G(\hat{\tau})\right)\right]\Delta T \\
&+p_h\left(1-G(\hat{\tau})-\frac{\hat{\tau}}{2}\int_{\hat{\tau}}^{+\infty}\frac{dG(\tau)}{\tau}\right)\Delta T \\
&+p_l\left[1+2G\left(\frac{\hat{\tau}}{2}\right)+G(\hat{\tau})+\hat{\tau}\int_{\frac{\hat{\tau}}{2}}^{+\infty}\frac{dG(\tau)}{\tau}\right]\frac{\Delta T}{2} \\
&= p_h\left(3-2G\left(\frac{\hat{\tau}}{2}\right)-G(\hat{\tau})-\hat{\tau}\int_{\frac{\hat{\tau}}{2}}^{+\infty}\frac{dG(\tau)}{\tau}\right)\frac{\Delta T}{2} \\
&+p_l\left(1+2G\left(\frac{\hat{\tau}}{2}\right)+G(\hat{\tau})+\hat{\tau}\int_{\frac{\hat{\tau}}{2}}^{+\infty}\frac{dG(\tau)}{\tau}\right)\frac{\Delta T}{2}.
\end{aligned}$$

Using $p_h = p_l + \Delta p$, this further becomes

$$
\begin{aligned}
&(p_l + \Delta p)\left(3 - 2G\left(\frac{\hat{\tau}}{2}\right) - G(\hat{\tau}) - \hat{\tau}\int_{\frac{\hat{\tau}}{2}}^{+\infty} \frac{dG(\tau)}{\tau}\right)\frac{\Delta T}{2} \\
&+ p_l\left(1 + 2G\left(\frac{\hat{\tau}}{2}\right) + G(\hat{\tau}) + \hat{\tau}\int_{\frac{\hat{\tau}}{2}}^{+\infty} \frac{dG(\tau)}{\tau}\right)\frac{\Delta T}{2} \\
&= 2p_l\Delta T + \Delta p\left(3 - 2G\left(\frac{\hat{\tau}}{2}\right) - G(\hat{\tau}) - \hat{\tau}\int_{\frac{\hat{\tau}}{2}}^{+\infty} \frac{dG(\tau)}{\tau}\right)\frac{\Delta T}{2}.
\end{aligned}
$$

C.1.3 Comparison between U.4 and I.4

The aggregate monetary price is at least as high in case U.4 as in case I.4 if and only if

$$
\begin{aligned}
&2p_l\Delta T + \Delta p\,(1 - G(\hat{\tau}))\,\Delta T \\
&\geq 2p_l\Delta T + \Delta p\left(3 - 2G\left(\frac{\hat{\tau}}{2}\right) - G(\hat{\tau}) - \hat{\tau}\int_{\frac{\hat{\tau}}{2}}^{+\infty} \frac{dG(\tau)}{\tau}\right)\frac{\Delta T}{2},
\end{aligned}
$$

which is equivalent to

$$
3 - 2G\left(\frac{\hat{\tau}}{2}\right) - G(\hat{\tau}) - \hat{\tau}\int_{\frac{\hat{\tau}}{2}}^{+\infty} \frac{dG(\tau)}{\tau} \geq 2\,(1 - G(\hat{\tau}))
$$

and so to

$$
1 + G(\hat{\tau}) - 2G\left(\frac{\hat{\tau}}{2}\right) \geq \hat{\tau}\int_{\frac{\hat{\tau}}{2}}^{+\infty} \frac{dG(\tau)}{\tau}. \tag{17}
$$

Observing that

$$
\begin{aligned}
\hat{\tau}\int_{\frac{\hat{\tau}}{2}}^{+\infty} \frac{dG(\tau)}{\tau} &= \hat{\tau}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \frac{dG(\tau)}{\tau} + \hat{\tau}\int_{\hat{\tau}}^{+\infty} \frac{dG(\tau)}{\tau} \\
&= 2\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \frac{dG(\tau)}{\tau/\frac{\hat{\tau}}{2}} + \int_{\hat{\tau}}^{+\infty} \frac{dG(\tau)}{\tau/\hat{\tau}} \\
&\leq 2\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} dG(\tau) + \int_{\hat{\tau}}^{+\infty} dG(\tau) \\
&= 2\left(G(\hat{\tau}) - G\left(\frac{\hat{\tau}}{2}\right)\right) + 1 - G(\hat{\tau}) \\
&= 1 + G(\hat{\tau}) - 2G\left(\frac{\hat{\tau}}{2}\right),
\end{aligned}
$$

it is clear that (17) is satisfied.

C.2 Aggregate non-monetary cost

C.2.1 Case U.4

The aggregate non-monetary cost of passengers using s_1 is given by

$$\int_{\widehat{\tau}}^{\infty} \tau \left(\int_{t_0}^{t_1} (t_1 - t^*) \, dt^* \right) dG(\tau) = \int_{\widehat{\tau}}^{\infty} \tau \left(t_1 \Delta T - \int_{t_0}^{t_1} t^* dt^* \right) dG(\tau) .$$

Computing

$$\begin{aligned} \int_{t_0}^{t_1} t^* dt^* &= \frac{t_1^2 - t_0^2}{2} \\ &= \frac{t_1 + t_0}{2} \Delta T \\ &= \left(t_0 + \frac{\Delta T}{2} \right) \Delta T, \end{aligned}$$

we can rewrite

$$\begin{aligned} &\int_{\widehat{\tau}}^{\infty} \tau \left(t_1 \Delta T - \int_{t_0}^{t_1} t^* dt^* \right) dG(\tau) \\ &= \Delta T \int_{\widehat{\tau}}^{\infty} \tau \left(t_1 - t_0 - \frac{\Delta T}{2} \right) dG(\tau) \\ &= \frac{(\Delta T)^2}{2} \int_{\widehat{\tau}}^{\infty} \tau dG(\tau) . \end{aligned}$$

For passengers using s_2 the aggregate non-monetary cost amounts to

$$\int_{0}^{\widehat{\tau}} \tau \left(\int_{t_0}^{t_1} (t_2 - t^*) \, dt^* \right) dG(\tau) + \int_{0}^{\infty} \tau \left(\int_{t_1}^{t_2} (t_2 - t^*) \, dt^* \right) dG(\tau)$$

Computing

$$\begin{aligned} \int_{t_0}^{t_1} (t_2 - t^*) \, dt^* &= t_2 \Delta T - \frac{t_1^2 - t_0^2}{2} \\ &= \frac{3}{2} (\Delta T)^2 \end{aligned}$$

and

$$\begin{aligned} \int_{t_1}^{t_2} (t_2 - t^*) \, dt^* &= (t_1 + \Delta T) \Delta T - \left(t_1 + \frac{\Delta T}{2} \right) \Delta T \\ &= \frac{(\Delta T)^2}{2}, \end{aligned}$$

we can rewrite

$$\frac{3}{2}(\Delta T)^2\int_0^{\widehat{\tau}}\tau dG(\tau)+\frac{(\Delta T)^2}{2}\int_0^{\infty}\tau dG(\tau).$$

Overall, the aggregate non-monetary cost of uninformed passengers is given by

$$\begin{aligned}&\frac{(\Delta T)^2}{2}\int_{\widehat{\tau}}^{\infty}\tau dG(\tau)+\frac{3}{2}(\Delta T)^2\int_0^{\widehat{\tau}}\tau dG(\tau)+\frac{(\Delta T)^2}{2}\int_0^{\infty}\tau dG(\tau)\\&=\left(\frac{3}{2}(\Delta T)^2+\frac{(\Delta T)^2}{2}\right)\int_0^{\widehat{\tau}}\tau dG(\tau)+\left(\frac{(\Delta T)^2}{2}+\frac{(\Delta T)^2}{2}\right)\int_{\widehat{\tau}}^{\infty}\tau dG(\tau)\\&=2(\Delta T)^2\int_0^{\widehat{\tau}}\tau dG(\tau)+(\Delta T)^2\int_{\widehat{\tau}}^{\infty}\tau dG(\tau).\end{aligned}$$

C.2.2 Case I.4

The aggregate non-monetary cost of passengers using s_0 is given by

$$\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\tau\left(\int_{t_0}^{t_1-\frac{\Delta p}{2\tau}}\|t^*-t_0\|\,dt^*\right)dG(\tau)+\int_{\hat{\tau}}^{+\infty}\tau\left(\int_{t_0}^{\frac{t_0+t_1}{2}}\|t^*-t_0\|\,dt^*\right)dG(\tau)$$

Computing

$$\begin{aligned}&\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\tau\left(\int_{t_0}^{t_1-\frac{\Delta p}{2\tau}}\|t^*-t_0\|\,dt^*\right)dG(\tau)\\&=\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\tau\left(\int_{t_0}^{t_1-\frac{\Delta p}{2\tau}}(t^*-t_0)\,dt^*\right)dG(\tau)\\&=\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\tau\left[\frac{1}{2}\left(\left(t_1-\frac{\Delta p}{2\tau}\right)^2-t_0^2\right)-t_0\left(\Delta T-\frac{\Delta p}{2\tau}\right)\right]dG(\tau)\\&=\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\tau\left[\frac{1}{2}\left((\Delta T)^2-\frac{\Delta p}{\tau}\Delta T+\frac{(\Delta p)^2}{4\tau^2}\right)\right]dG(\tau)\\&=\frac{1}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\tau\left(\Delta T-\frac{\Delta p}{2\tau}\right)^2dG(\tau)\end{aligned}$$

and

$$\begin{aligned}&\int_{\hat{\tau}}^{+\infty}\tau\left(\int_{t_0}^{\frac{t_0+t_1}{2}}\|t^*-t_0\|\,dt^*\right)dG(\tau)\\&=\left[\int_{t_0}^{\frac{t_0+t_1}{2}}t^*dt^*-t_0\left(\frac{t_0+t_1}{2}-t_0\right)\right]\int_{\hat{\tau}}^{+\infty}\tau dG(\tau)\\&=\frac{(\Delta T)^2}{8}\int_{\hat{\tau}}^{+\infty}\tau dG(\tau),\end{aligned}$$

we can rewrite

$$\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \tau \left(\int_{t_0}^{t_1 - \frac{\Delta p}{2\tau}} \|t^* - t_0\| \, dt^* \right) dG(\tau) + \int_{\hat{\tau}}^{+\infty} \tau \left(\int_{t_0}^{\frac{t_0+t_1}{2}} \|t^* - t_0\| \, dt^* \right) dG(\tau)$$
$$= \frac{1}{2} \int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \left(\Delta T - \frac{\Delta p}{2\tau} \right)^2 \tau dG(\tau) + \frac{(\Delta T)^2}{8} \int_{\hat{\tau}}^{+\infty} \tau dG(\tau).$$

For passengers using s_1 the aggregate non-monetary cost is given by

$$\int_{\widehat{\tau}}^{+\infty} \left(\int_{\frac{t_0+t_1}{2}}^{\frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau}} \tau \|t^* - t_1\| \, dt^* \right) dG(\tau)$$
$$= \int_{\widehat{\tau}}^{+\infty} \tau \left(\int_{\frac{t_0+t_1}{2}}^{t_1} (t_1 - t^*) \, dt^* \right) dG(\tau) + \int_{\widehat{\tau}}^{+\infty} \tau \left(\int_{t_1}^{\frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau}} (t^* - t_1) \, dt^* \right) dG(\tau).$$

Computing

$$\int_{\widehat{\tau}}^{+\infty} \tau \left(\int_{\frac{t_0+t_1}{2}}^{t_1} (t_1 - t^*) \, dt^* \right) dG(\tau) = \frac{(\Delta T)^2}{8} \int_{\widehat{\tau}}^{+\infty} \tau dG(\tau)$$

and

$$\int_{\widehat{\tau}}^{+\infty} \tau \left(\int_{t_1}^{\frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau}} (t^* - t_1) \, dt^* \right) dG(\tau)$$
$$= \frac{1}{2} \int_{\widehat{\tau}}^{+\infty} \tau \left[\left(\frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau} \right)^2 - t_1^2 - t_1 \left(\Delta T - \frac{\Delta p}{\tau} \right) \right] dG(\tau)$$
$$= \frac{1}{2} \int_{\widehat{\tau}}^{+\infty} \tau \left[\frac{1}{4} \left(\Delta T - \frac{\Delta p}{\tau} \right)^2 + t_1 \left(\Delta T - \frac{\Delta p}{\tau} \right) - t_1 \left(\Delta T - \frac{\Delta p}{\tau} \right) \right] dG(\tau)$$
$$= \frac{1}{8} \int_{\widehat{\tau}}^{+\infty} \tau \left(\Delta T - \frac{\Delta p}{\tau} \right)^2 dG(\tau),$$

we can rewrite

$$\frac{1}{8} \left[(\Delta T)^2 \int_{\widehat{\tau}}^{+\infty} \tau dG(\tau) + \int_{\widehat{\tau}}^{+\infty} \tau \left(\Delta T - \frac{\Delta p}{\tau} \right)^2 dG(\tau) \right].$$

For passengers using s_2 the aggregate non-monetary cost is given by

$$\int_0^{\frac{\hat{\tau}}{2}} \left(\int_{t_0}^{t_2} \tau \|t^* - t_2\| \, dt^* \right) dG(\tau) + \int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \left(\int_{t_1 - \frac{\Delta p}{2\tau}}^{t_2} \tau \|t^* - t_2\| \, dt^* \right) dG(\tau)$$
$$+ \int_{\hat{\tau}}^{+\infty} \left(\int_{\frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau}}^{t_2} \tau \|t^* - t_2\| \, dt^* \right) dG(\tau)$$
$$= 2(\Delta T)^2 \int_0^{\frac{\hat{\tau}}{2}} \tau dG(\tau) + \int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \tau \left(\int_{t_1 - \frac{\Delta p}{2\tau}}^{t_2} (t_2 - t^*) \, dt^* \right) dG(\tau)$$
$$+ \int_{\hat{\tau}}^{+\infty} \tau \left(\int_{\frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau}}^{t_2} (t_2 - t^*) \, dt^* \right) dG(\tau).$$

Computing

$$\int_{t_1 - \frac{\Delta p}{2\tau}}^{t_2} (t_2 - t^*) \, dt^*$$
$$= t_2 \left(\Delta T + \frac{\Delta p}{2\tau} \right) - \frac{1}{2} t_2^2 + \frac{1}{2} \left(t_1 - \frac{\Delta p}{2\tau} \right)^2$$
$$= \frac{1}{2} \left(\frac{\Delta p}{\tau} \Delta T + \Delta T^2 + \frac{\Delta p^2}{4\tau^2} \right)$$
$$= \frac{1}{2} \left(\Delta T + \frac{\Delta p}{2\tau} \right)^2$$

and

$$\int_{\frac{t_1+t_2}{2} - \frac{\Delta p}{2\tau}}^{t_2} (t_2 - t^*) \, dt^*$$
$$= \frac{t_2}{2} \left(\Delta T + \frac{\Delta p}{\tau} \right) - \frac{1}{2} t_2^2 + \frac{1}{2} \left(\frac{t_1 + t_2}{2} - \frac{\Delta p}{2\tau} \right)^2$$
$$= \frac{1}{2} \left(\frac{\Delta p}{2\tau} \Delta T + \frac{\Delta T^2}{4} + \frac{\Delta p^2}{4\tau^2} \right)$$
$$= \frac{1}{2} \left(\frac{\Delta p}{2\tau} \Delta T + \frac{\Delta T^2}{4} + \frac{\Delta p^2}{4\tau^2} \right)$$
$$= \frac{1}{8} \left(\Delta T + \frac{\Delta p}{\tau} \right)^2,$$

we can rewrite

$$2(\Delta T)^2 \int_0^{\frac{\hat{\tau}}{2}} \tau dG(\tau) + \frac{1}{2} \int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}} \tau \left(\Delta T + \frac{\Delta p}{2\tau} \right)^2 dG(\tau) + \frac{1}{8} \int_{\hat{\tau}}^{+\infty} \tau \left(\Delta T + \frac{\Delta p}{\tau} \right)^2 dG(\tau).$$

Overall, the aggregate non-monetary cost of informed passengers amounts to

$$
\begin{aligned}
&\frac{1}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\Delta T-\frac{\Delta p}{2\tau}\right)^{2}\tau dG\left(\tau\right)+\frac{\left(\Delta T\right)^{2}}{8}\int_{\hat{\tau}}^{+\infty}\tau dG\left(\tau\right)\\
&+\frac{\left(\Delta T\right)^{2}}{8}\int_{\hat{\tau}}^{+\infty}\tau dG\left(\tau\right)+\frac{1}{8}\int_{\hat{\tau}}^{+\infty}\tau\left(\Delta T-\frac{\Delta p}{\tau}\right)^{2}dG\left(\tau\right)\\
&+2\left(\Delta T\right)^{2}\int_{0}^{\frac{\hat{\tau}}{2}}\tau dG\left(\tau\right)+\frac{1}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\tau\left(\Delta T+\frac{\Delta p}{2\tau}\right)^{2}dG\left(\tau\right)\\
&+\frac{1}{8}\int_{\hat{\tau}}^{+\infty}\tau\left(\Delta T+\frac{\Delta p}{\tau}\right)^{2}dG\left(\tau\right)\\
&=2\left(\Delta T\right)^{2}\int_{0}^{\frac{\hat{\tau}}{2}}\tau dG\left(\tau\right)+\frac{\left(\Delta T\right)^{2}}{4}\int_{\hat{\tau}}^{+\infty}\tau dG\left(\tau\right)\\
&+\frac{1}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\Delta T-\frac{\Delta p}{2\tau}\right)^{2}\tau dG\left(\tau\right)+\frac{1}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\Delta T+\frac{\Delta p}{2\tau}\right)^{2}\tau dG\left(\tau\right)\\
&+\frac{1}{8}\int_{\hat{\tau}}^{+\infty}\left(\Delta T-\frac{\Delta p}{\tau}\right)^{2}\tau dG\left(\tau\right)+\frac{1}{8}\int_{\hat{\tau}}^{+\infty}\left(\Delta T+\frac{\Delta p}{\tau}\right)^{2}\tau dG\left(\tau\right).
\end{aligned}
$$

C.2.3 Comparison between U.4 and I.4

The aggregate non-monetary cost is at least as high in case I.4 as in case U.4 if and only if

$$
\begin{aligned}
&2\left(\Delta T\right)^{2}\int_{0}^{\hat{\tau}}\tau dG\left(\tau\right)+\left(\Delta T\right)^{2}\int_{\hat{\tau}}^{\infty}\tau dG\left(\tau\right)\\
&\geq 2\left(\Delta T\right)^{2}\int_{0}^{\frac{\hat{\tau}}{2}}\tau dG\left(\tau\right)+\frac{\left(\Delta T\right)^{2}}{4}\int_{\hat{\tau}}^{+\infty}\tau dG\left(\tau\right)\\
&+\frac{1}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\Delta T-\frac{\Delta p}{2\tau}\right)^{2}\tau dG\left(\tau\right)+\frac{1}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\Delta T+\frac{\Delta p}{2\tau}\right)^{2}\tau dG\left(\tau\right)\\
&+\frac{1}{8}\int_{\hat{\tau}}^{+\infty}\left(\Delta T-\frac{\Delta p}{\tau}\right)^{2}\tau dG\left(\tau\right)+\frac{1}{8}\int_{\hat{\tau}}^{+\infty}\left(\Delta T+\frac{\Delta p}{\tau}\right)^{2}\tau dG\left(\tau\right).
\end{aligned}
$$

Let us first rewrite this inequality as

$$
\begin{aligned}
&2\left(\Delta T\right)^{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\tau dG\left(\tau\right)+\frac{3}{4}\left(\Delta T\right)^{2}\int_{\hat{\tau}}^{+\infty}\tau dG\left(\tau\right)\\
&\geq\frac{1}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\Delta T-\frac{\Delta p}{2\tau}\right)^{2}\tau dG\left(\tau\right)+\frac{1}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\Delta T+\frac{\Delta p}{2\tau}\right)^{2}\tau dG\left(\tau\right)\\
&+\frac{1}{8}\int_{\hat{\tau}}^{+\infty}\left(\Delta T-\frac{\Delta p}{\tau}\right)^{2}\tau dG\left(\tau\right)+\frac{1}{8}\int_{\hat{\tau}}^{+\infty}\left(\Delta T+\frac{\Delta p}{\tau}\right)^{2}\tau dG\left(\tau\right).
\end{aligned}
$$

Computing

$$\begin{aligned}
&\frac{1}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\Delta T-\frac{\Delta p}{2\tau}\right)^{2}\tau dG\left(\tau\right)\\
&=\frac{1}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\left(\Delta T\right)^{2}+\frac{\left(\Delta p\right)^{2}}{4\tau^{2}}-\frac{\Delta p}{\tau}\Delta T\right)\tau dG\left(\tau\right)\\
&=\frac{1}{2}\left[\left(\Delta T\right)^{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\tau dG\left(\tau\right)+\Delta p\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\frac{\Delta p}{4\tau}-\Delta T\right)dG\left(\tau\right)\right]
\end{aligned}$$

and

$$\begin{aligned}
&\frac{1}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\Delta T+\frac{\Delta p}{2\tau}\right)^{2}\tau dG\left(\tau\right)\\
&=\frac{1}{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\left(\Delta T\right)^{2}+\frac{\left(\Delta p\right)^{2}}{4\tau^{2}}+\frac{\Delta p}{\tau}\Delta T\right)\tau dG\left(\tau\right)\\
&=\frac{1}{2}\left[\left(\Delta T\right)^{2}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\tau dG\left(\tau\right)+\Delta p\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\left(\frac{\Delta p}{4\tau}+\Delta T\right)dG\left(\tau\right)\right],
\end{aligned}$$

we can rewrite

$$\begin{aligned}
&\left(\Delta T\right)^{2}\int_{\frac{\hat{\tau}}{2}}^{\widehat{\tau}}\tau dG\left(\tau\right)+\frac{3}{4}\left(\Delta T\right)^{2}\int_{\widehat{\tau}}^{\infty}\tau dG\left(\tau\right)\\
&\geq\frac{\left(\Delta p\right)^{2}}{4}\int_{\frac{\hat{\tau}}{2}}^{\hat{\tau}}\frac{dG\left(\tau\right)}{\tau}\\
&+\frac{1}{8}\int_{\widehat{\tau}}^{+\infty}\left(\Delta T-\frac{\Delta p}{\tau}\right)^{2}\tau dG\left(\tau\right)+\frac{1}{8}\int_{\widehat{\tau}}^{+\infty}\left(\Delta T+\frac{\Delta p}{\tau}\right)^{2}\tau dG\left(\tau\right).
\end{aligned}$$

Further computing

$$\begin{aligned}
&\frac{1}{8}\int_{\widehat{\tau}}^{+\infty}\left(\Delta T-\frac{\Delta p}{\tau}\right)^{2}\tau dG\left(\tau\right)\\
&=\frac{1}{8}\int_{\widehat{\tau}}^{+\infty}\left(\left(\Delta T\right)^{2}+\frac{\left(\Delta p\right)^{2}}{\tau^{2}}-2\frac{\Delta p}{\tau}\Delta T\right)\tau dG\left(\tau\right)\\
&=\frac{1}{8}\left[\left(\Delta T\right)^{2}\int_{\widehat{\tau}}^{+\infty}\tau dG\left(\tau\right)+\Delta p\int_{\widehat{\tau}}^{+\infty}\left(\frac{\Delta p}{\tau}-2\Delta T\right)dG\left(\tau\right)\right]
\end{aligned}$$

and

$$\begin{aligned}
&\frac{1}{8}\int_{\widehat{\tau}}^{+\infty}\left(\Delta T+\frac{\Delta p}{\tau}\right)^{2}\tau dG\left(\tau\right)\\
&=\frac{1}{8}\int_{\widehat{\tau}}^{+\infty}\left(\left(\Delta T\right)^{2}+\frac{\left(\Delta p\right)^{2}}{\tau^{2}}+2\frac{\Delta p}{\tau}\Delta T\right)\tau dG\left(\tau\right)\\
&=\frac{1}{8}\left[\left(\Delta T\right)^{2}\int_{\widehat{\tau}}^{+\infty}\tau dG\left(\tau\right)+\Delta p\int_{\widehat{\tau}}^{+\infty}\left(\frac{\Delta p}{\tau}+2\Delta T\right)dG\left(\tau\right)\right],
\end{aligned}$$

with some manipulation we further rewrite

$$2\int_{\frac{\hat{\tau}}{2}}^{\widehat{\tau}}\frac{\tau}{\hat{\tau}}dG\left(\tau\right)+\int_{\hat{\tau}}^{\infty}\frac{\tau}{\hat{\tau}}dG\left(\tau\right)\geq\int_{\frac{\hat{\tau}}{2}}^{\infty}\frac{dG\left(\tau\right)}{\tau/\frac{\hat{\tau}}{2}}. \tag{18}$$

We see that

$$\begin{aligned}
\int_{\frac{\hat{\tau}}{2}}^{\infty}\frac{dG\left(\tau\right)}{\tau/\frac{\hat{\tau}}{2}}&\leq\int_{\frac{\hat{\tau}}{2}}^{\infty}dG\left(\tau\right)\\
&=1-G\left(\frac{\hat{\tau}}{2}\right)
\end{aligned}$$

and that

$$\begin{aligned}
\int_{\hat{\tau}}^{\infty}\frac{\tau}{\hat{\tau}}dG\left(\tau\right)&\geq\int_{\hat{\tau}}^{\infty}dG\left(\tau\right)\\
&=1-G\left(\hat{\tau}\right).
\end{aligned}$$

We also see that

$$\begin{aligned}
2\int_{\frac{\hat{\tau}}{2}}^{\widehat{\tau}}\frac{\tau}{\hat{\tau}}dG\left(\tau\right)&=\int_{\frac{\hat{\tau}}{2}}^{\widehat{\tau}}2\frac{\tau}{\hat{\tau}}dG\left(\tau\right)\\
&\geq\int_{\frac{\hat{\tau}}{2}}^{\widehat{\tau}}dG\left(\tau\right)\\
&=G\left(\hat{\tau}\right)-G\left(\frac{\hat{\tau}}{2}\right)
\end{aligned}$$

because $2\frac{\tau}{\hat{\tau}}\in[1,2]$, hence $2\frac{\tau}{\hat{\tau}}\geq 1$ for $\tau\in\left[\frac{\hat{\tau}}{2},\hat{\tau}\right]$. Since

$$G\left(\hat{\tau}\right)-G\left(\frac{\hat{\tau}}{2}\right)+1-G\left(\hat{\tau}\right)\geq 1-G\left(\frac{\hat{\tau}}{2}\right)\Leftrightarrow 0\geq 0,$$

which clearly holds as an equality, (18) is satisfied.