An opportunity egalitarian approach to the intertemporal evaluation of distributions

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September 4, 2019

Abstract

We propose an axiomatic approach to characterize normative criteria for the evaluation of lifetime income distributions based on the opportunity egalitarian perspective [28]. In a setting in which both individual incomes and predetermined circumstances are variable over time, we first aggregate over time, thereby characterizing measures of the individual income streams, and then aggregate the individual measures into an overall measure. In both stages of aggregation we adopt the opportunity egalitarian approach and in particular the opportunity gap methodology proposed by Fleurbaey and Schokkaert [17]. Our individual measure results to be a weighted average of individuals' opportunity gap experienced in each period. Our aggregate measure is a weighted average of a function of each individual's intertemporal opportunity gap and results to be decomposable into the amount of opportunity deprivation and opportunity advantage experienced by the society over time. We apply our framework to evaluate the Korean distribution of income from an intertemporal and opportunity egalitarian perspective.

Keywords: intertemporal distributions; inequality of opportunity; opportunity gap; time horizon.

JEL codes: D31, D63, I32, J62.

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1 Introduction

The equality of opportunity literature flourished in the field of normative economics and distributional analysis in the last two decades. This literature has developed concepts of fairness for the context in which individual achievements are partly the outcome of morally arbitrary circumstances (such as inherited endowments and social background) and partly the consequence of individual effort and responsibility (see 13, 30 for recent surveys).

Those concepts revolve around the idea that inequalities due to circumstances are unfair and should be eliminated as much as possible, while inequalities that result from unequal efforts should be considered acceptable. This literature has witnessed a rapidly growing number of theoretical modelizations for the measurement of inequality of opportunity and empirical applications interested in measuring the degree of inequality of opportunity (IOp) and in evaluating public policies in terms of equality of opportunity (EOp), mainly in the context of income distributions (see, among others, 1, 6, 9, 22, 29).

All of the contributions[1] - both in the theoretical and empirical literature - aimed at providing an evaluation of distributions according to the EOp perspective propose cross-sectional and unitemporal measures. Among the seminal philosophical contributions to the literature on EOp [31, 26, 3, 4, 11] there seems to be a common agreement on the idea that once the social planner is able to ensure the existence of EOp, the actual inequalities have to be considered fair. The focus of the EOp literature on uni-temporal analysis can be reconducted to this idea. However, it is unquestionable that the time dimension is crucial when making individual and social welfare evaluations, and it is common practice, in the standard unidimensional distributional analysis, to refer to the lifetime individual income. Both in Dworkin's [11] and Arneson's [3] seminal contributions, the moment from which individuals should be *left on their own feets* is not clear. At any moment of time there are many events that may occur, which are definitely out of individual control, but affect their income opportunities. Moreover, following Fleurbaey [14] human beings are constantly in evolution and defining a precise moment in time when they become fully responsible of their life is a complex issue.

We propose to measure EOp in each moment individual's life, we argue that the time dimension is relevant also in the opportunity egalitarian perspective and this paper aims at filling this gap: we want to incorporate the time dimension into the standard equality of opportunity framework. Hence, in this paper we provide distributional indicators framed into an intertemporal perspective and consistent with the EOp framework. We do this by adopting a three-step procedure.

[1] An exception to this is the paper by Aaberge et al. [1] that derives a framework for measuring IOp in the long run by making use of long term distributions of incomes.

In the first step we derive an optimal income distribution, i.e. a distribution that fully satisfies our principles of fairness. In the second step, we assess the extent of the penalty or premium experienced by each individual by comparing her income in the actual, observed, distribution with the one she would obtain in the optimal distribution. In particular, in each period we construct a measure of distance between each individual observed income and his optimal income and we propose an aggregation of this measures of distance over time that accounts for the opportunity history of individuals. In the third step, we aggregate the individual indicators into an aggregate measure of intertemporal opportunity gap.

An alternative way of conducting analysis of intertemporal EOp would consist in computing the permanent (i.e. life-time) income of each individual and comparing it with its optimal permanent income. An analysis based on permanent incomes, which considers circumstances at birth would be in line with the existing theoretical contribution in the literature on EOp. We favor our approach because it allows us to better characterize the impact of time on opportunity deprivation. Moreover, the strategy of computing an opportunity gap for each period seems to better represent our idea that individuals assess fairness in the exact moment in which they experiment it and they often lack resources, skills and willingness to look at the entire life-span before making such assessment. One can prefer an analysis based on permanent incomes because, in the spirit of a *permanent income hypothesis à la Milton Friedman,* individuals can rationally smooth income (and effort) over their life-span and, lacking enough information on effort, a permanent income would be an advisable solution to wash-out this distortion. Insufficient disposable information does not constitute a crucial limit for our approach, because we are able to define optimal distributions without referring to any effort measure. Moreover, the presence of borrowing constraints [10] (i.e market imperfections) and the relevant number of the so called *hand-to-mouth* consumers [20], make less attractive the advantage of using permanent incomes.

In the second step we define an individual measure of unfairness (the individual intertemporal opportunity gap), which turns out to be a weighted average of individuals' opportunity gap experienced in each period. The axiomatization of the individual measure is inspired by the contributions in the literature of intertemporal poverty measurement [18, 8]. From this step we obtain vectors of individual intertemporal opportunity gaps which, relying on the generalized Lorenz curve, can be used to rank society in terms of intertemporal equality of opportunity.

In the third step, we define an aggregate measure (the social opportunity gap) which is a weighted average of a function of each individual's intertemporal opportunity gap and results to be decomposable into the amount of opportunity deprivation and opportunity advantage experienced by the society over time. The social opportunity gap, while completing the Lorenz order, can be seen as the *social cost* of intertemporal inequality of opportunity.

Our setting allows for the possibility that both circumstances and income change over time. The assumption of time varying circumstances is quite uncommon in the literature, which is instead usually concerned with circumstances at birth, hence fixed over time. To appreciate the relevance of this assumption, take as an example the disability status: disability can affect an individual at birth or it can occur in any moment of his life and radically change his outcome possibilities.

Our contribution is especially related to Aaberge et al. [1], who also propose an evaluation of lifetime income distribution from an opportunity egalitarian perspective. However, we depart from Aaberge et al. [1] model in two respects: first, as said before, we allow for time varying circumstances, while they assume fixed circumstances; second, following Fleurbaey and Schokkaert [17], we employ a fairness gap approach for inequality of opportunity, which allows us to obtain individual, in addition to societal, measures of opportunity deprivation. Indeed, one might be interested in quantifying the pain inflicted to individuals or the premium benefited because of the existence of some kind of inequity. Our model is also related to Almås et al. [2], who propose distributional indicators that builds upon the individual fairness gap, but do not account for the time dimension.

Finally, we provide an empirical application of the measurement tools characterized in the paper by analyzing the Korean distribution of income from an intertemporal and opportunity egalitarian perspective.

The structure of the paper is as follows. Section 2 introduces some preliminary notation. Section 3 derives the optimal distributions according to the theory of equality of opportunity. Section 4 provides the individual and aggregate intertemporal measures of opportunity gap. Section 5 proposes an empirical application to Korean data. Section 6 concludes.

2 Preliminaries

We observe for $\tau \in \mathbb{N}_{++}$ periods a population of $n(>1) \in \mathbb{N}_{++}$ individuals that is represented by the set N. For each $i \in N$ and each $t \in \{1, \ldots, \tau\} = T$, the observed level of income is assumed to be a function of some observable circumstances $c \in \mathbb{R}^{\Psi}$, with $\Psi \subset \mathbb{N}_{++}$, and some responsibility variables $r \in \mathbb{R}^{\Theta}$, with $\Theta \subset \mathbb{N}_{++}$, that can be either observable or not. The set of all observable circumstance vectors is denoted by C and the set of all responsibility vectors is denoted by R . Following the existing literature**(author?)** [25, 24, 1, 13, 12], for any $i \in N$ and any $t \in T$, the observed income level is assumed to be function of circumstances and responsibility: $x_{it} = h(c_{it}, r_{it})$, with $c_{it} \in C$ and $r_{it} \in R$.

We denote the income distribution at time $t \in T$ with $X_t \in \mathbb{R}^n_{++}$. Given the observable circumstances and responsibilities, we can partition X_t in $\psi \in \mathbb{N}_{++}$ subgroups of individuals sharing the same circumstances that we call *types* and $\theta \in \mathbb{N}_{++}$ subgroups of individuals sharing the same vector of responsibility variables that we call *tranches*. At any moment $t \in T$, the income $x_{it} \in X_t$ of an agent $i \in N$ will be both in one of the types $c_i \in \{c_1, \ldots, c_\psi\} = C$ and one of the tranches $r_i \in \{r_1, \ldots, r_\theta\} = R$. Therefore, we can rewrite the vector X_t in matrix form as $\mathbf{X}_t \in \mathbb{R}_{++}^{\psi \times \theta}$ and indicate the income of the agent i, at time t, belonging to the type j and the tranche k with x_{it}^{jk}. We refer to this notation in the following section which introduces the concept of EOp and defines the characteristics of an optimal allocation.

3 Equality of opportunity

The concept of equality of opportunity is based on *compensation* and *reward* principles. The former claims that differences in achievements due to differences in factors out of individual control are unfair and have to be compensated. The latter claims that differences in achievements due to differences in effort or factors under individual control are fair, and have to be respected. In the literature [7, 12] we distinguish two interpretations of the *compensation principle* that lead to ex ante and ex post approaches. Based on the first one, inequality of opportunity will be removed if all agents face the same set of opportunities. Following the second approach, inequality of opportunity will be removed if agents in the same responsibility group (agents exerting the same level of effort) obtain the same results. There are also two prominent interpretations of *Reward principle* that lead to liberal and utilitarian approaches. Following the former, we should minimize the redistribution of differences in achievements due to differences in effort. According to the latter, we should focus only on the sum of the achievements obtained by each circumstance group, remaining neutral with respect to the way different levels of effort are remunerated within the circumstance groups.

To characterize the optimal distributions in terms of equality of opportunity, we first rewrite the vector X_t in matrix form ($\mathbf{X}_t$) and we assume that for each combination of circumstances and responsibility (c, r) there is one and only one individual[2]. Therefore, we indicate the income of an individual that at time t belonging to circumstances i and responsibility group k simply as x_t^{ik}.

The first principle we introduce is the compensation one, in its ex post interpretation.

- *Ex post compensation:* individuals exerting the same effort level (responsi-

[2]This is clearly an harmless simplification once we recall that the observed income is assumed to be function of circumstances and responsibility: $x_{it} = h(c_{it}, r_{it})$.

bility) achieve the same income level. Formally, for any income distribution $\mathbf{X}_t \in \mathbb{R}_+^{\psi\times\theta}$, the optimal distribution is $\tilde{\mathbf{X}}_t \in \mathbb{R}_+^{\psi\times\theta}$ such that $\tilde{x}_t^{ik} = \tilde{x}_t^{jk} = \cdots = \frac{1}{\psi}\sum_{c\in C}\tilde{x}_t^{ck}$, for all columns k.

Ex post compensation states that a poor-to-rich (i.e regressive) transfer between individuals belonging to the same responsibility group ($k \in R$), reduces the level of EOp. In other words, referring to the matrix representation of an income distribution, the incomes in each column have to be the same.

We now define the ex ante interpretation of the compensation principle.

- *Ex ante compensation:* the difference in circumstances do not have to influence the (value of) individual's opportunity sets. Formally, for any income distribution $\mathbf{X}_t \in \mathbb{R}_+^{\psi\times\theta}$, the optimal distribution is $\tilde{\mathbf{X}}_t \in \mathbb{R}_+^{\psi\times\theta}$ such that $\frac{1}{\theta}\sum_{r\in R}\tilde{x}_t^{ir} = \frac{1}{\theta}\sum_{r\in R}\tilde{x}_t^{jr}$, for all $i \neq j \in C$.

Ex ante compensation claims that a transfer from one individual in an opportunity advantaged type toward another one in an less advantaged type, constitutes a worsening in terms of EOp. To identify the advantage of a circumstance group with respect to the others, we refer to the expected income of individuals belonging to that type as in [25, 24, 32].

Given this two interpretations of the compensation principle, we deviate from two prominent views of the reward principle (utilitarian and liberal) introducing the following two principles.

- *Reward as agnosticism:* inequalities between individuals with the same circumstances are not taken into account. More precisely, the social evaluator takes no position with respect to such inequality.

- *Limited utilitarian reward*: inequalities between individuals with the same circumstances are fare if they are *non-extreme*. There are several ways to formalize it, one of these is the following. For any income distribution $\mathbf{X}_t \in \mathbb{R}_+^{\psi\times\theta}$, the optimal distribution is $\tilde{\mathbf{X}}_t \in \mathbb{R}_+^{\psi\times\theta}$ such that $\tilde{x}_t^{ik} \in [\alpha, \beta] \subset \mathbb{R}_{++}$.

Note that limited utilitarian reward takes a position (of limited neutrality) with respect to inequality within types, while reward as agnosticism takes no position at all. Moreover, limited utilitarian reward is simply a modification of the utilitarian reward principle in [25, 24, 12] which states *full* neutrality with respect to within type inequality.

We should note that the optimal distribution is *time-specific* so that, to apply our approach, one needs to compute τ optimal distributions. If this seems to slightly increase the complexity of our analysis, a time-specific optimal distribution is what allows us to include time varying circumstances.

In the literature (see [12, 7] for other examples) there are several alternative axioms one may use to define an optimal distribution. It is important to note that not all optimal distributions should be implemented in our framework; consider, for example, the combination of ex ante compensation and utilitarian reward. In this case, the optimal distribution is $\tilde{\mathbf{X}}_t \in \mathbb{R}^{\psi \times \theta}$ such that $\frac{1}{\theta} \sum_{k \in R} \tilde{x}_t^{ik} = \frac{1}{\theta} \sum_{k \in R} \tilde{x}_t^{jk}$ for all $i \neq j$. In other words, the condition for a distribution to satisfy ex ante compensation and utilitarian reward is to have type income distributions with the same expected value. This example is useful to underline the necessity of using well-defined optimal distributions, in order to limit the degree of subjectivity in choosing the optima for each individual. A well-defined optimal distribution is such that for each individuals there is only one possible optimal income, while in the previous example there are uncountably many alternative optimal incomes for each individual.[3] Again, there are several other axioms one may want to impose and therefore many other possible optimal distributions compatible with this paper's framework. Before proceeding with the characterization of the optimal distribution for the empirical exercise, we underline that one of the possible optima is the fully egalitarian distribution. This creates a link between the idea of inequality of opportunity and the more widespread idea of equality of outcomes. Moreover, it generalizes the measure we propose for other applications.

The following subsection describes two examples of well-defined optimal distributions we will refer to in the empirical exercise.

3.1 The optimal distributions

As mentioned above, a useful optimal distribution has to be such that each pair of actual and optimal incomes is well (uniquely) defined. In what follows we propose two alternative optimal distributions which assess equality of opportunity according to ex ante and ex post views. Note that, even if this is not fully in line with the principles of equality of opportunity, the fully egalitarian distribution can also be implemented as optimal distribution. This is because the social evaluation function we propose is coherent to any egalitarian principle that generates a well-defined optimal distribution. Moreover, in a *second-best* situation, lack of information on circumstances or effort and the quality of the data may force us to refer to such a distribution which one can use as a benchmark in the analysis.

In order to investigate equality of opportunity with an ex post view, we propose to define the optimal distribution as follows. First, we partition the population in ψ subgroups of individuals with identical circumstances (*types*). Second, applying

[3] Take type income vector $\mathbf{x} = (10, 50)$ and assume that the other types in the distribution have all a mean income of 30. Then, $\tilde{\mathbf{x}} = (10, 50)$, $\tilde{\mathbf{x}}' = (15, 45)$, $\tilde{\mathbf{x}}'' = (0, 60)$, $\tilde{\mathbf{x}}''' = (30, 30)$ are only some of the possible type income distribution coherent with ex ante compensation and utilitarian reward.

Roemer [27, 28] identification approach we partition each type's income CDF[4] in θ percentiles which identify comparable levels of effort across individuals. Then, the optimal income of each individual in the k-th percentile is the mean income across types of individuals in k-th percentile of their relative type income distribution. Formally, let $X_t \in \mathbb{R}^n_{++}$ be the income distribution and $\mathbf{X}_t \in \mathbb{R}^{\psi\times\theta}_{++}$ its matrix representation. Then, the optimal distribution $\tilde{\mathbf{X}}_t \in \mathbb{R}^{\psi\times\theta}_{++}$ is such that, for any agent in thranche k, and for all $k \in R$

$$\tilde{x}_t^{ik} = \frac{1}{\psi}\sum_{j\in C} x_t^{jk}$$

This optimal distribution satisfies a slightly different[5] version of ex post compensation (which we can rename *within tranches inequality aversion* as in Peragine [24]) while being agnostic to inequality between tranches (a form of reward principle used also in [24]).

In the previous section we showed the difficulties of implementing an ex ante approach. Indeed, the combination of ex ante compensation and utilitarian reward only imposes the type income distributions to have the same expected value without considering the inequality within types. We propose the following compromise solution to define the optimal distribution. First, we partition the population in ψ subgroups of individuals with identical circumstances (*types*). Second, we define an interval around the mean (μ) of the income distribution so that any income that belongs to such interval is considered optimal. The interval has the scope of laving room for the existence of a *fair* within type inequality in line with utilitarian reward while allowing us to obtain a well-defined optimal distribution. Moreover, since the interval is unique across types, our analysis would also take into account the volatility of incomes within types, which ex ante constitutes an element of risk that behind the veil of ignorance reduces the welfare of risk averse agents. In addition, one can set the interval in order to also take into account the minimum necessary income (which would constitute the lower bound) and assess the intertemporal distribution also in terms of poverty. Formally, given the interval $[\alpha\mu, \beta\mu]$, with $\alpha \in (0,1)$ and $\beta \in (1,\infty)$, the optimal distribution $\tilde{\mathbf{X}}_t \in \mathbb{R}^{\psi\times\theta}_{++}$ is such that for each $t \in T, i \in C, k \in R$,

$$\begin{cases} \tilde{x}_t^{ik} = \alpha\mu & x_t^{ik} < \alpha\mu \\ \tilde{x}_t^{ik} = \beta\mu & x_t^{ik} > \beta\mu \\ \tilde{x}_t^{ik} = x_t^{ik} & otherwise \end{cases}$$

[4] Cumulative distribution function.

[5] The difference is due to the fact that we do not directly refer to some responsibility variables to define the tranches. Moreover, in the practice, each percentile will contain more than one individual with heterogeneous income and actual responsibility variable.

The proposed optimal distribution satisfies ex ante compensation and a limited form of utilitarian reward which is neutral to inequality within types only if they are *non-extreme.* One may object that the proposed optimal distribution, being based on the actual realization of individual incomes, cannot be considered as following an ex ante approach. The answer to such a critic consists in noting that this optimal distribution looks at individual's ex ante income perspective "*with the benefit of insight about the ex post distribution*" (Fleurbaey, [15, p 158]) which allows us to identify individuals that, falling outside of the income interval, tend to be more or less opportunity advantaged and are considered so also because of an excessive (unfair) impact of effort on outcomes. An example of such concern for extreme outcomes of individual choices (which in our context are summarized with effort) is the Bert's story in Fleurbaey [14].

Starting from the existence of an optimal distribution for each period of observation, the following section axiomatizes indexes to assess the level of IOp from an intertemporal perspective.

4 A family of indexes of intertemporal opportunity gap

Given some of the egalitarian principles defined in the previous section, for each income distribution $X_t \in \mathbb{R}^n_{++}$ there exists an optimal distribution $\tilde{X}_t \in \mathbb{R}^n_{++}$ that assigns a unique optimal income to any individual in the population. For each agent i and each period t we define the *opportunity gap* (g_{it}) as a function $g : \mathbb{R}^2_{++} \to \mathbb{R}$ of the actual and the optimal income $g_{it} = g(x_{it}, \tilde{x}_{it})$.

The opportunity gap has to be such that $g_{it} > 0$ if the individual i is opportunity advantaged at time t (i.e. he has an income higher than the optimal one), $g_{it} < 0$ if he is opportunity disadvantaged and $g_{it} = 0$ otherwise. For the sake of uniformity in exposition, throughout the paper we refer to the following definition.

Definition 1. An individual $i \in N$ at time $t \in T$ is *opportunity deprived* (or disadvantaged) if and only if $g_{it} < 0$. The same individual is *opportunity advantaged* if and only if $g_{it} > 0$.

For a more precise characterization of the opportunity gap $g : \mathbb{R}^2_{++} \to \mathbb{R}$, we list some minimal desirable properties one may want to be satisfied.

- *Monotonicity* (Mg): For all $x_{it}, x_{jt} \in X_t$ and $\tilde{x}_{it}, \tilde{x}_{jt} \in \tilde{X}_t$, $g(x_{it}, \tilde{x}_{it}) \geq g(x_{jt}, \tilde{x}_{it})$ if and only if $x_{it} \geq x_{jt}$, and $g(x_{it}, \tilde{x}_{it}) \geq g(x_{it}, \tilde{x}_{jt})$ if and only if $\tilde{x}_{it} \leq \tilde{x}_{jt}$.

- *Scale invariance* (SIg): For all $x_{it} \in X_t$, $\tilde{x}_{it} \in \tilde{X}_t$ and $\lambda \in \mathbb{R}$, $g(\lambda x_{it}, \lambda \tilde{x}_{it}) = g(x_{it}, \tilde{x}_{it})$.

- *Additive invariance* (AIg): For all $x_{it} \in X_t$, $\tilde{x}_{it} \in \tilde{X}_t$ and $\lambda \in \mathbb{R}$, $g\left(x_{it}+\lambda, \tilde{x}_{it}+\lambda\right) = g\left(x_{it}, \tilde{x}_{it}\right)$.

Monotonicity claims that if two agents have the same optimal income, the one with higher actual income has a bigger gap. Vice versa, among two agents with the same actual income, the one with higher optimal income has a smaller (i.e. more negative) gap. Scale invariance requires the gap function to be invariant to multiplication of both its arguments for the same scalar. Additive invariance imposes the gap to remain unchanged if both its arguments are summed to a common scalar. These three properties define two families of opportunity gap measures [23]. The combination of Monotonicity with Scale invariance leads us to the following result.

Lemma 1. *For all $x_{it}, x_{jt} \in X_t$ and $\tilde{x}_{it}, \tilde{x}_{jt} \in \tilde{X}_t$, $g : \mathbb{R}^2_{++} \to \mathbb{R}$ satisfies Monotonicity (Mg) and Scale invariance (SIg) if and only if*

$$g\left(x_{it}, \tilde{x}_{it}\right) \geq g\left(x_{jt}, \tilde{x}_{jt}\right) \iff \frac{x_{it}}{\tilde{x}_{it}} \geq \frac{x_{jt}}{\tilde{x}_{jt}}$$

Proof. Appendix. □

The combination of Monotonicity with Additive invariance leads us to the following result.

Lemma 2. *For all $x_{it}, x_{jt} \in X_t$ and $\tilde{x}_{it}, \tilde{x}_{jt} \in \tilde{X}_t$, $g : \mathbb{R}^2_{++} \to \mathbb{R}$ satisfies Monotonicity (Mg) and Additive invariance (AIg) if and only if*

$$g\left(x_{it}, \tilde{x}_{it}\right) \geq g\left(x_{jt}, \tilde{x}_{jt}\right) \iff x_{it} - \tilde{x}_{it} \geq x_{jt} - \tilde{x}_{jt}$$

Proof. Appendix. □

Since some optimal distribution are reallocations of the total income in the society at time t, then $\sum_{i=1}^{n} g_{it} = 0$ for g linear. Therefore, to emphasize the presence of individuals penalized by the existence of inequality of opportunity in the society, we impose the following property.

- *Concavity* (Cg): For all $x_{it} \in X_t$, $\tilde{x}_{it} \in \tilde{X}_t$, $\frac{\partial^2 g}{\partial y^2} \leq 0$, for either $y = x_{it} - \tilde{x}_{it}$ or $y = x_{it}/\tilde{x}_{it}$.

Note that this axiom imposes $g : \mathbb{R}^2_{++} \to \mathbb{R}$ continuous, twice differenciable and non-convex.

Given the previous properties, we define two families of individual opportunity gap measures.

Definition 2. Following Lemma 1 and Cg, the opportunity gap of agent $i \in N$ in $t \in T$ is $g_{it} = g(x_{it}/\tilde{x}_{it})$, for $g : \mathbb{R}_{++} \rightarrow \mathbb{R}$ increasing, concave and such that Definition 1 holds.

This is a family of opportunity gap measures as in Definition 1 that is monotone, scale invariant and concave. A member of this family is, for example, $g(x_{it}/\tilde{x}_{it}) = \ln(x_{it}/\tilde{x}_{it})$.

Definition 3. Following Lemma 2 and Cg, the opportunity gap of agent $i \in N$ in $t \in T$ is $g_{it} = g(x_{it} - \tilde{x}_{it})$ for $g : \mathbb{R} \rightarrow \mathbb{R}$ increasing, concave and such that Definition 1 holds.

This is a family of opportunity gap measures as in Definition 1 that is monotone, additive invariant and concave. A member of this family is, for example, $g(x_{it} - \tilde{x}_{it}) = (x_{it} - \tilde{x}_{it}) - e^{(x_{it} - \tilde{x}_{it})} + 1$.

Since both families of measures satisfy Definition 1, in the following subsection we do not need to make a distinction between scale or additive invariant opportunity gaps. This is because independently of the invariance property it satisfies, the gap of an opportunity deprived individual is always negative, and vice versa for an opportunity advantaged. It is however implicit that once the evaluator defines a specific function g, all the gaps have to be computed according to such function.

Given a function g, for each $t \in T$, we have the opportunity gap distribution $\mathbf{g}_t \in \mathbb{R}^n$ generated by substituting the income of each individual with his opportunity gap in t. Moreover, since each index $i \in N$ is in a one to one relation with an individual, the values $g_{i1} \in \mathbf{g}_1$, ..., $g_{it} \in \mathbf{g}_t, \dots,$ $g_{i\tau} \in \mathbf{g}_\tau$ are the opportunity gaps experienced by the same i individual across time. We can therefore define the *intertemporal* opportunity gap distribution $G \in \mathbb{R}^{n \times \tau}$ where each row $\mathbf{g}_i \in \mathbb{R}^\tau \subseteq G$ is the agent i intertemporal opportunity gap distribution and each column $\mathbf{g}_t \in \mathbb{R}^n \subseteq G$ is the time t opportunity gaps distribution in the society. The intertemporal opportunity gap distributions, i. e. the rows of G, are the arguments of the individual intertemporal opportunity gap we axiomatize in the following section.

4.1 Individual intertemporal opportunity gap

In this section we axiomatically characterize an aggregator $\gamma : \mathbb{R}^\tau \rightarrow \mathbb{R}$ of the opportunity gaps of each individual across time which we label *individual intertemporal opportunity gap (or γ)* . We first propose two minimal axioms that γ has to satisfy, and that define its structure of our aggregator. Then, we introduce other axioms one would like to impose to model the effect of time on the individual intertemporal opportunity gap.

The first minimal axiom we impose is a criteria for normalizing our aggregator.

- *Normalization* (N) - For any $\mathbf{g}_i \in G$ such that $g_{it} = x$ for all $t \in T$, $\gamma(\mathbf{g}_i) = x$.

According to the Normalization axiom, if an individual's opportunity gaps are equal in all periods, then it is reasonable to say that this individual intertemporal opportunity gap can be appropriately represented by a snapshot opportunity gap. This axiom is similar to the normalization axiom used in the measurement of income inequality and creates a link between our intertemporal and the classical uni-temporal framework for the analysis of equality of opportunity. Moreover, it imposes γ to be increasing and concave in each $g_{it} \in \mathbf{g}_i \in G$.[6]

The second axiom we impose requires independence between opportunity gaps in different moments.

- *Independence* (I) - For any $\mathbf{g}_i \in G$, $\frac{\partial^2 \gamma(\mathbf{g}_i)}{\partial g_{it} \partial g_{is}} = 0$ for all $s \neq t \in T$.

With the independence axiom, we assume that the effect of an individual opportunity gap on γ is independent of the level of any other opportunity gap. It is important to recognize that this assumption effectively shuts down one possible channel for which the extent of opportunity deprivation in one period can have effects on the impact of opportunity deprivation in another period of life. It is questionable that individual opportunity gap in period t is independent from one in period s. If circumstances are constant over time then, the opportunity gaps in t and s should be correlated. It is here worthy to underline that our model allows for some transitory shocks in circumstances (temporary disabilities for example) that may weaken such correlation. Moreover, the independence axiom simplifies the analysis of the time distance between opportunity deprived periods. Indeed, the opportunity gap persistence axiom below considers one type of interaction between different gaps and does this specifically with regard to "how close" the gaps are. This approach [18] allows us to make more explicit the intertemporal interaction of opportunity gaps and emphasizes our concern for persistent opportunity deprivation.

The two axioms above characterize a family of additive individual intertemporal opportunity gaps as stated in the following proposition.

Proposition 1. *An individual intertemporal opportunity gap measure γ satisfies independence (I) and normalization (N) axioms if and only if it can be written as a weighted average of the opportunity gaps such that, for any $\mathbf{g}_i \in G$ with $i \in N$,*

$$\gamma(\mathbf{g}_i) = \sum_{t \in T} \alpha(t, T) g_{it} \tag{1}$$

where $0 < \alpha(t, T) < 1$ and $\sum_{t \in T} \alpha(t, T) = 1$.

[6] See proof of Proposition 1 in Appendix B.

Proof. Appendix. □

Eq. (1) constitutes the family of individual intertemporal opportunity gap measures we refer to throughout this paper. The remaining of this section focuses on how to model the time component of γ.

Many contributions (see for example [18]) point out that the earlier in life the individual experiences specific economic conditions, the stronger is their impact on his life. So, one might want to give more relevance on the opportunity gaps of early stages of life than those of later life periods. The following early gap axiom applies this idea in our framework.

- *Early period gap* (EG) - For any $\mathbf{g}_i \in G$ with $i \in N$, $\frac{\partial\gamma(\mathbf{g}_i)}{\partial g_{it}} \geq \frac{\partial\gamma(\mathbf{g}_i)}{\partial g_{is}}$ for all $t < s \in \{1, \ldots, \tau\}$.

This axiom specifies the functional form of the intertemporal weights we assign to each opportunity gap. Indeed, as claimed by the following corollary, the weights have to be decreasing in time.

Corollary 1. *An individual intertemporal opportunity gap measure γ satisfies early gap (EG) in addition to independence (I) and normalization (N) if and only if $\alpha(t, T)$ is decreasing in t*

From a subjective point of view, it might rather be the opposite, in the sense that recent gaps are more important. Consider, for example, the ability of people to adapt to event occurring in the distant past or the relevance of more recent events. The following axiom formalizes this idea.

- *Late period gap* (LG) - For any $\mathbf{g}_i \in G$ with $i \in N$, $\frac{\partial\gamma(\mathbf{g}_i)}{\partial g_{it}} \leq \frac{\partial\gamma(\mathbf{g}_i)}{\partial g_{is}}$ for all $t < s \in \{1, \ldots, \tau\}$.

The following corollary constitutes the counterpart of the previous one.

Corollary 2. *An individual intertemporal opportunity gap measure γ satisfies late gap (LG) in addition to independence (I) and normalization (N) if and only if $\alpha(t, T)$ is increasing in t*

The following axiom characterizes our concern for the persistence of opportunity deprivation. The main idea is that multiple opportunity deprived spells experienced consecutively are more harmful than if the same set of opportunity gaps were more spread out over time. We generalize this notion by requiring that any two negative opportunity gaps create less persistent opportunity deprivation the "more spread out" they are.

- *Opportunity gap persistence* (GP) - For any $\mathbf{g}_i, \mathbf{g}_j \in G$ such that $g_{is} = g_{iu} < 0$ with $g_{it} = 0$ for all $t \neq s, u \in T$, and $g_{jr} = g_{jv} < 0$ with $g_{jt} = 0$ for all $t \neq r, v \in T$. If $g_{is} = g_{jr}$, $g_{iu} = g_{jv}$, $1 \leq r < s < u < v \leq \tau$ and $s - r = v - u$ then, $\gamma(\mathbf{g}_i) \leq \gamma(\mathbf{g}_j)$.

It is important to note that the previous axiom focuses only on the negative opportunity gaps. However, as will be clear in Corollary 3, this axiom assumes that the compensation effect of a positive opportunity gap can be boosted by the presence of consecutive positive opportunity gaps. When combined with the first two axioms introduced (I and N), the opportunity gap persistence axiom leads to a simple implication.

Corollary 3. *An individual intertemporal opportunity gap measure γ satisfies opportunity gap persistence (GP) in addition to independence (I) and normalization (N) if and only if $\alpha(t, T)$ is concave in t.*

Proof. Appendix. □

Provided with a complete picture of the axiomatic structure we desire to impose, we can provide some more comments on the result in Proposition 1.

Independence axiom (I) imposed above surely is the most questionable one. It is a strong requirement that imposes the additive structure of γ which turns out to be extremely useful in isolating the *position* effect of an opportunity gap from its *intensity* effect. With a linear α one would capture only the latter and being able to isolate the two effects may reveal useful in the analysis of intertemporal EOp. The additive structure of γ makes it easier to interpret and model, and other more general aggregation procedure may conflict with (GP), (EG) or (LG). [7] We do not claim that an additive structure is the only possible way of aggregating gaps but we believe it is a simpler structure which maintain its effectiveness.

There are some circumstances that, besides affecting the actual income in a specific period, may have an impact on the shape of the income streams. Indeed, (I) imposes independency between the intensity of deprivation in different sub-periods but form (GP) the position of a gap matters. Hence, the effect of circumstances on income stream's shape is (at least partially) captured by (GP). Moreover, our opportunity gap approach in the intertemporal perspective we follow, is actually comparing the income path or each individual with some optimal income path so that the opportunity gaps can be seen as divergences between the two.

[7] Let us assume that we remove Independence axiom. By Normalization, $\gamma(x) = x = \gamma(\gamma(x), \gamma(x), \gamma(x, x))$. Set $x < 0$. Gap persistence requires $\gamma(0, x, x, 0) \leq \gamma(0, x, 0, x)$; by Normalization this inequality can be written as $\gamma(\gamma(0, x), \gamma(x, 0)) \leq \gamma(\gamma(0, x), \gamma(0, x))$ which implies $\gamma(x, 0) \leq \gamma(0, x)$. A contradiction to Late period gap. Following the same logic we can construct an example for Early period gap.

The implemented opportunity gap approach focuses on the effects of circumstances in a given period. However, there are circumstances that occurs at t but have effects in $t+1$ or after. The separability imposed by (I), and in general out approach, does not allow us to capture this effect. A solution to this drawback constitute in defining circumstances that capture the entire history of individuals. This, in a context where we consider only circumstances at birth does not arise concerns. However, if circumstances are variable, then the more the observation periods, the more will be the circumstance groups, the lower will be the population density of each type and the robustness of the resulting analysis [24].

Opportunity gap persistence imposes the weight function to be concave in time, this implies that the central part of the observed period tend to receive more emphasis in the aggregation process. Imposing (EG) or (LG) the emphasis (in terms of absolute value of the time-weight) is actually in the extremes of the observation period, and the relative relevance of the central part of lifespan with respect to one of the extreme seems to be a natural consequence of those two axioms more than (GP).

In the literature on EOp (1, 5, 12, 22, 24, 25, 29) we often refer to the Lorenz curve as a tool to define dominance condition between distributions. Almås et al. [2] define the *unfairness Lorenz curve* that ranks unitemporal gap distribution. It is easy to see how the distribution of the individual intertemporal opportunity gap (γ), which we call $\mathbf{s} \in \mathbb{R}^n$, is actually an intertemporal generalization of the unfairness distribution for which Almås et al. [2] define the *unfairness Lorenz curve*. Following the same logic, but noting that in our framewotk the sign and magnitude of the opportunity gaps are relevant, we can define the *intertemporal unfairness generalized Lorenz curve* as

$$GL\left(h, \mathbf{s}\right) = \sum_{i=1}^{nh} \gamma_i$$

for all ordered vectors $\mathbf{s} \in \mathbb{R}^m$ and $h \in [0,1]$. Moreover, following Almås et al. [2], if $GL\left(h, \mathbf{s}\right) \geq GL\left(h, \mathbf{s}'\right)$ for all $h \in [0,1]$ then the distribution $\mathbf{s}'$ shows more intertemporal inequality of opportunity.[8]

Although the inequality in the distribution of intertemporal gaps is *per se* an important criteria to assess EOp, this paper proposes aslo to consider the existence of IOp as a cost (or benefit) for some individual. According to such view one can also be interested in evaluating the *social cost* in terms of opportunity gap due to IOp. The following section axiomatically defines a social evaluation function

[8] Following the same intuition in Kakwani [19], one can also graph the ordered values, of the vector $\mathbf{s}$ to have a graphical instrument which represent the distribution of the individual intertemporal opportunity gaps.

according to which we can aggregate de individual intertemporal opportunity gaps in a society and assess the social cost of intertemporal IOp.

4.2 Social opportunity gap

In this section we define the *social opportunity gap* which can be seen as a social evaluation function. Given all the individual intertemporal opportunity gaps constructed as in the previous section, we can define the vector $\mathbf{s} \in \mathbb{R}^n$ which contains as elements all the $\gamma(\mathbf{g}_i)$ derived from G. To simplify the notation we indicate $\gamma(\mathbf{g}_i) = \gamma_i$. We axiomatically characterize the social opportunity gap $\Gamma : \mathbb{R}^n \to \mathbb{R}$ imposing the following axioms.

- *Monotonicity* (Ms) - For all $\mathbf{s}$ and $\mathbf{s}'$, $\Gamma(\mathbf{s}) \geq \Gamma(\mathbf{s}')$ if $\gamma_i \geq \gamma_i'$ for all $i \in N$.

Monotonicity is imposed because we want our social gap to mirror the individual social gaps.

- *Decomposability* (Ds*)* - For all $\mathbf{s}$, $\frac{\partial^2 \Gamma(\mathbf{s})}{\partial \gamma_i \partial \gamma_j} = 0$ for all $i \neq j \in N$.

Decomposability is a quite standard property imposed to aggregators of social phenomena. In our case, the importance of this property is emphasized by the eventual necessity of distinguishing opportunity advantaged and opportunity deprived individuals, for a deeper analysis of the phenomenon.

- *Population invariance* (Ps) - $\Gamma(\mathbf{s}, \ldots, \mathbf{s}) = \Gamma(\mathbf{s})$ for $\Gamma(\mathbf{s}, \ldots, \mathbf{s}) : \mathbb{R}^{nm} \to \mathbb{R}$ and $m \in \mathbb{N}_+$.

Population (replication) invariance is necessary for comparing populations with different sizes.

- *Lower gap concern*(LGs) - For all $\mathbf{s}$ and any $\epsilon^{\mathbf{i}} \in \mathbb{R}^n$ such that $\epsilon^i_j = 0$ for all $j \neq i \in N$ and $\epsilon^i_i \in \mathbb{R}_+$, $\Gamma\left(\mathbf{s} + \epsilon^{\mathbf{i}}\right) \geq \Gamma\left(\mathbf{s} + \epsilon^{\mathbf{j}}\right)$ if $\gamma_i \leq \gamma_j$.

Lower gap concern expresses aversion to opportunity inequality. Indeed, following LGs, a progressive transfer from an opportunity advantaged individual to an opportunity deprived one would increase the social gap. An higher value of our social evaluation function Γ has to be considered as an improvement in terms of intertemporal equality of opportunity.

The proposed axioms, characterize the social evaluation function (social opportunity gap) as in the following proposition.

Proposition 2. *A social opportunity gap measure* Γ *satisfies monotonicity (Ms), population invariance (Ps), decomposability (Ds) and lower gap concern (LGs) if and only if , for all* $\mathbf{s} \in \mathbb{R}$*,*

$$\Gamma(\mathbf{s}) = \frac{1}{n} \sum_{i=1}^{n} \omega(\gamma_i) \tag{2}$$

with γ defined as in Proposition 1 and $\omega : \mathbb{R} \rightarrow \mathbb{R}$ such that $\omega' \geq 0$ and $\omega'' \leq 0$.

Proof. Appendix. $\square$

The social opportunity gap is an average of the individual intertemporal opportunity gaps. Such measure aggregates the γs giving more weight to lower values, as all the inequality averse functions. Γ is a social evaluation function that constitutes our way of comparing and ordering different states of the world according to their respect for given EOp allocation criteria, which define the optimal distributions.

The empirical exercise will not perform comparison between countries. However, for countries in which we can apply the same criteria for defining circumstances and effort, a comparison based on our social evaluation function turns out to be meaningful thanks to the invariance properties of the functions g and to the fact that Γ takes into account the dimension of the population. Although one may find this normatively odd, to the extent that one accepts the fact that an optimal distribution is the goal of the society, and each society is allowed to have different goals, the fact that two countries have optimal distributions inspired to different interpretations of EOp principles, does not necessarily imply their non-comparability. We retain from taking a clear position on the issue of defining the extent to which two optimal distributions lead to comparable social opportunity gap. We believe that such considerations are quite subjective and have to be made case by case. Moreover, this does not influence the usefulness of our family of measure in making intertemporal comparisons for the same country, as in the empirical exercise.

One can also observe that Γ can be decomposed into two components: (1) intertemporal opportunity deprivation and (2) intertemporal opportunity advantage. Because of additive decomposability it is trivial to obtain that $\Gamma = \sum_{i \in I} \omega(\gamma_i^-) + \sum_{i \in I} \omega(\gamma_i^+)$. In fact, $\sum_{i \in I} \omega(\gamma_i^-)$ is an aggregate measure of intertemporal opportunity disadvantage whereas $\sum_{i \in I} \omega(\gamma_i^+)$ is an aggregate measure of intertemporal opportunity advantage.

Proposition 2 offers an instrument to complete the generalized Lorenz ranking criteria proposed in the previous section. One can also restrict the focus to the generalized Lorenz curve of intertemporal opportunity deprivate in order to perform a greater number of comparisons [33]. The most common solution to the incompleteness of the Lorenz ranking is the use of inequality indexes; for g, α and ω linear, Eq. (2) can be interpreted as an intertemporal complement measure to the Gini coefficient of fairness gaps as in Almås et al[2].

The proposed social evaluation function allows us to assess the improvement one country makes in terms of EOp. Suppose that each individual gets in the first period and in the consecutive periods exactly the same amount of income. In this specific case, Γ will correspond to the aggregate individual opportunity gap in the first period, that is: $\tilde{\Gamma} = \Gamma_1$ and can be interpreted as a benchmark situation of persistence in the status quo. Thus, every difference with respect to this benchmark can be used to evaluate how a society reacts over time in terms of opportunity gaps. In particular, for $\Gamma - \tilde{\Gamma} > 0$ we are in presence of a fairness improvement; instead, for $\Gamma - \tilde{\Gamma} < 0$ we are in presence of a fairness deterioration.

The following section provides some specifications for g, α, and ω.

4.3 Examples of social evaluation functions

In this section we formalize the social evaluation functions and the optimal distributions that will be implemented in the empirical exercise. As already mentioned, there are numerous optimal distributions satisfying the broad idea of equality of opportunity. Moreover, the functions g, γ and Γ can be defined in several ways. Therefore, given the uncountably many possible social evaluation functions, the followings constitute useful examples that cannot exhaust all the possible cases.

We propose two examples of concave opportunity gap functions (g):

1) $g_A = (x - \tilde{x}) - \sigma^{(x-\tilde{x})} + 1$;

2) $g_S = \log_\rho(\frac{x}{\tilde{x}})$.

The first opportunity gap function (g_A) is a monotone, additive invariant and concave function of $(x - \tilde{x})$. This particular functional form allows us to have an opportunity gap that is equal to zero if $x = \tilde{x}$, and emphasizes the negative opportunity gaps. Indeed, the value $\sigma \in [0, 1]$ can be interpreted as a measure of the emphasis one wants to assign to the negative opportunity gaps. The choice of the exact value of σ is also motivated by considerations on the standard of living in the analyzed countries and the type of income (monthly or yearly) considered. As an example, if the observed income is annual, one may assume that $x - \tilde{x} = -300$ does not constitute a relevant opportunity gap and set $\sigma = 0.997$ but if that difference is in terms of monthly incomes, then one may prefer to set $\sigma = 0.989$ and make g more concave. The second opportunity gap function (g_S) is a monotone, scale invariant and concave function of $(x/\tilde{x})$. The value $\rho \in [0, +\infty)$ approximates the slope of the function around the intersection with the horizontal axes so that the lower is ρ, the steeper is g_S. Again, the actual value of ρ has to be defined according to the income measure observed; as an example one can set $\rho = 2.5$ for monthly incomes and $\rho = 10$ for annual incomes.

To construct the individual intertemporal opportunity gap, we need to define the function $\alpha(t, T)$. Setting the first observed period as $t = 1$, we propose :

1) $\alpha_{LC}(t, T) = \sqrt[\nu]{\frac{t}{\tau}}$;

2) $\alpha_{EC}(t, T) = \sqrt[\nu]{\frac{\tau-t+1}{\tau}}$.

These are both concave functions of t which also satisfy alternatively early gap concern (α_{EC}) and late gap concern (α_{LC}). The exponent ν of the root is a measure of the concavity of the function α and the sensibility to the persinstency of gaps, in line with the opportunity gap persistence axiom.

The last function we need to define in order to characterize Γ is ω. Noting that $\gamma \in \mathbb{R}$, we cannot define ω as a log or a squared root because these functions are not defined for negative numbers. We can set $\omega(\gamma) = \gamma$ to consider the arithmetic mean of the individual intertemporal opportunity gaps as social evaluation function. Note that concavity of g implies that Γ is concave too.

After describing the data, the following section performs the empitical excercise.

5 An empirical application to Korea

5.1 Data

Our empirical analysis is based on the KLIPS (Korean Labor and Income Panel Study), which is a Korean census conducted every year on a sample of about 5,000 households. It started in 1998 and collects information at both household and individual level, in particular, it is one of the few panel surveys that contains information on individual socio-economic background. For our analysis we use 14 waves ranging from 2001 to 2014.

The unit of observation is the individual, in particular we consider all individuals aged between 20 and 65 and interviewed in each wave. The measure of living standards is equivalized disposable household income, which includes income after transfers and the deduction of income tax and social security contributions. Incomes are expressed in constant 2005 prices, using country and year-specific price indexes and are adjusted for differences in household size by dividing incomes by the square root of household size.[9] Individuals with zero sampling weights are excluded since our measures are calculated using sample weights designed to make the samples nationally representative. We also exclude individuals with non-positive income. Therefore, our sample size is made of 3,125 observation.

A fundamental step in the measurement of inequality of opportunity is the identification of the vector of observable circumstances. This is a normative choice, subject to the constraint of data availability. Our data contain information on a small set of basic circumstances, but nonetheless of prominent importance. For each wave, in fact, we can observe the following: birth place, parental education, parental occupation, parental support.

Birth place is categorized following the major administrative divisions of the country. The first category is represented by individuals born in the special city - namely Seoul. The second category is represented by individuals born

[9]Consumer Price Indexes are taken from Korea National Statistical Office KOSTAT.

in one of the metropolitan cities (self-governing cities that are not part of any province) - namely Busan, Daegu, Incheon, Gwangju, Daejeon, Ulsan - or in the autonomous metropolitan city - namely Sejong. The third categories is represented by individuals born in other provinces - namely Gyeonggi, Gangwon, North Chungcheong, South Chungcheong, North Jeolla, South Jeolla, North Gyeongsang, South Gyeongsang, Jeju - or outside South Korea. Parental education - measured by the highest educational attainment between mother and father - is also coded into 3 categories: individuals whose parents have elementary education or no education; individuals with at least one parents with middle/secondary education; individuals with at least one parent having attained tertiary education. Parental occupation - measured by father occupation - is again coded into 3 categories: Individuals whose father is either Legislator/Senior Officials or Managers or professional or Technicians and Associate Professionals; individuals whose father is either Clerks or Service/Shop/Market Sales Workers or Skilled Agricultural and Fishery Workers; individuals whose father is either Craft/Trades Workers or Plant and Machine Operators and Assemblers or Elementary Occupations. The last circumstance used is parental support that is a binary variable indicating whether or not the individual received any material/financial support received from the parent(s) during last year. Notice that differently from birth place, parental education and parental occupation, circumstances that are fixed over time, parental support might be quite variable.

As explained above, in order to compute our measure one needs to identify the optimal distribution. In the empirical exercise we refer to three optimal distributions:

1. Equality: obtained by equating the optimal income to the mean income in the population[10]

2. Ex ante optimal income belonging to an interval around the mean income in the population. To be more specific we use as the upper bound limit of our interval 1.5 times the mean income and as lower bound 0.5 times the mean income in the population

3. Ex post: optimal income equal to the mean income across types of individuals in the same percentile of the relative type income distribution. In particular, we partition types into 5 quantiles. This approach follows Roemer's [28] identification to approximate effort.

[10]Notice that, the use of such benchmark would allow to interpret our measure as an intertemporal generalization of relative deprivation measures.

5.2 Results

South Korea is among the most developed countries in the world especially in terms of per capita GDP and technological innovation. At the same time, suicide in South Korea is a serious and widespread problem and the country ranks poorly on world happiness reports for a high-income state. So it is of interest to shade light on the fairness aspect of this controversial country. We do this by applying our measurement framework.

Table 1 reports the results of our estimates for the whole sample and the whole 14 years-period considered. We compute six specifications of the family of indexes characterized in the previous section. In particular we compute the following:

- $OG_{a1} = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{1}{T}\sum_{t=1}^{T}(x_{it} - \tilde{x}_{it})\right]$
- $OG_{a2} = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{t=1}^{T}\sqrt{\frac{\tau-t+1}{\tau}}(x_{it} - \tilde{x}_{it})\right]$
- $OG_{a3} = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{t=1}^{T}\sqrt{\frac{t}{\tau}}(x_{it} - \tilde{x}_{it})\right]$
- $OG_{l1} = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{1}{T}\sum_{t=1}^{T}\log_{2.5}\left(\frac{x_{it}}{\tilde{x}_{it}}\right)\right]$
- $OG_{l2} = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{t=1}^{T}\sqrt{\frac{\tau-t+1}{\tau}}\log_{2.5}\left(\frac{x_{it}}{\tilde{x}_{it}}\right)\right]$
- $OG_{l3} = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{t=1}^{T}\sqrt{\frac{t}{\tau}}\log_{2.5}\left(\frac{x_{it}}{\tilde{x}_{it}}\right)\right]$
- $OG_{e1} = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{1}{T}\sum_{t=1}^{T}\left(x_{it} - \tilde{x}_{it} - 0.997^{(x_{it}-\tilde{x}_{it})+} + 1\right)\right]$
- $OG_{e2} = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{t=1}^{T}\sqrt{\frac{\tau-t+1}{\tau}}\left(x_{it} - \tilde{x}_{it} - 0.997^{(x_{it}-\tilde{x}_{it})} + 1\right)\right]$
- $OG_{e3} = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{t=1}^{T}\sqrt{\frac{t}{\tau}}\left(x_{it} - \tilde{x}_{it} - 0.997^{(x_{it}-\tilde{x}_{it})} + 1\right)\right]$

It immediately comes out that almost all entries of Table 1 have negative sign. This means that although South Korea is known as one of the most growing and progressive country, it still has some degree of unfairness. The only exception is represented by the case in which we look at the existence of EOp using an ex ante approach we measure intertemporal (un)fairness using additive invariant indexes. With respect to the other two optima, as specified above, the use of a linear g leads to a social evaluation function that is always equal to zero.

Table 1: Korea intertemporal fairness, 2001-14

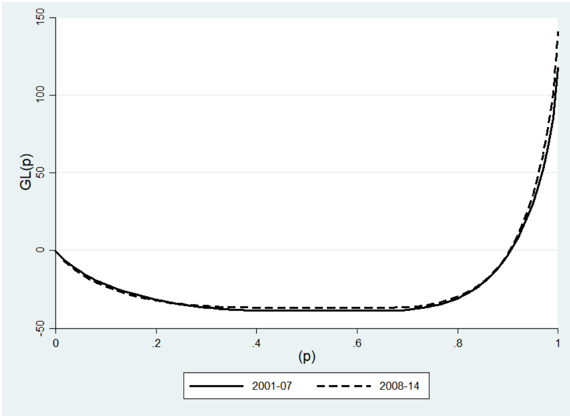
	Equality	Ex ante	Ex post
OG_{a1}	0.0000	129.39	0.0000
OG_{a2}	0.0002	1315.19	0.0000
OG_{a3}	0.0002	1227.87	0.0000
OG_{l1}	-0.2213	-0.0283	-0.0419
OG_{l2}	-2.1349	-0.2580	-0.4081
OG_{l3}	-2.2020	-0.3134	-0.4123
OG_{e1}	-3.7295	129.27	-1.6679
OG_{e2}	-39.7325	1314.03	-17.4188
OG_{e3}	-33.4308	1226.68	-15.5111

We then apply our indexes on the two sub-periods 2001-07 and 2008-14 to show how our framework can be used not only to evaluate single distributions but also to make comparisons across distributions. Table 2 shows the same signs for the different sub-periods. Comparing the numerical values of the two sub-periods, we can see that while in relative terms, i. e. using g scale invariant, 2008-14 tends to perform better in terms of fairness, looking at the absolute values of the opportunity gaps the ranking between the sub-period is inverted. Overall, the result of the comparison appears to be robust to the specific index and benchmark adopted. We can also use the intertemporal unfairness generalized Lorenz curves to deepen the investigation. Figures 1 and 2 show respectively OG_{e1} and OG_{l1}.As showed by the values in Table 2 the ranking between the sub-period is inverted according to the way we consider the gaps. Moreover, our index offers a way to solve the incompleteness of the ranking based on generalized Lorene curves in Figure 1(a).

South Korea has been characterized by a rapid economic growth process as well as by a rapid process of integration in the world economy. After the Korean war, end of 50s, per capita GDP was at the same level of some poorest African countries. One may argue that in our example we can distinguish two cohort of individuals: one which was involved in the initial phase of South Korean development, and one that was born in a more developed country. To investigate whether this has an impact on equality of opportunity, we divide the sample into two cohorts: the cohort of individuals aged between 20 and 40 in 2001 and the cohort of individuals older than 40. Table 3 shows that according to an ex ante perspective of intertemporal EOp, the old cohort performs better while ex post is the young generation the more advantaged by the existence of IOp. Therefore, while the

Figure 1: Intertemporal unfairness generalized Lorenz curves, 2001-07 vs 2008-14

(a) Ex ante, OG_{e1}



(b) Ex post, OG_{e1}

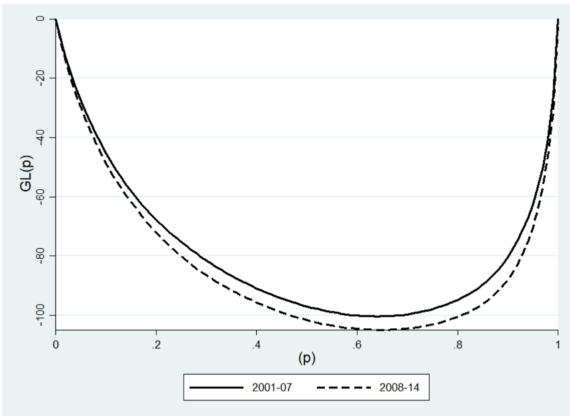
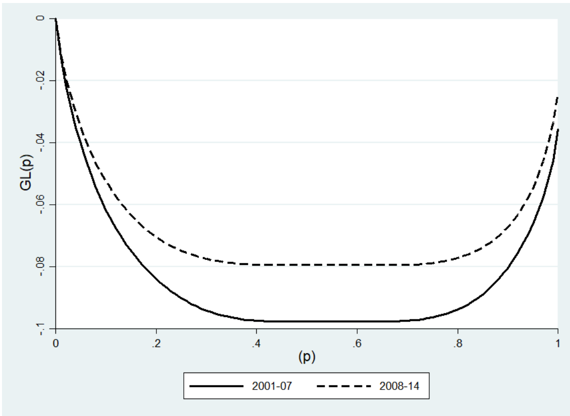


Figure 2: Intertemporal unfairness generalized Lorenz curves, 2001-07 vs 2008-14

(a) Ex ante, OG_{l1}



(b) Ex post,OG_{l1}

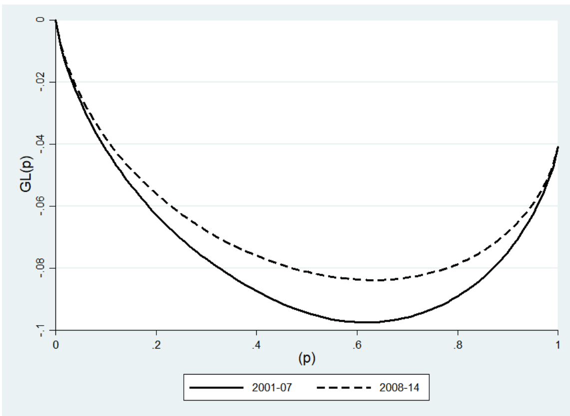


Table 2: Korea intertemporal fairness, 2001-07 vs 2008-14

2001-07	Equality	Ex ante	Ex post
OG_{a1}	0.0000	117.79	0.0000
OG_{a2}	0.0001	632.08	0.0000
OG_{a3}	0.0001	568.93	0.0000
OG_{I1}	-0.2307	-0.0359	-0.0410
OG_{I2}	-1.1776	-0.1701	-0.2078
OG_{I3}	-1.1739	-0.1951	-0.2102
OG_{e1}	-2.8910	117.67	-1.3921
OG_{e2}	-16.0219	631.42	-7.8237
OG_{e3}	-13.4133	568.31	-6.3642

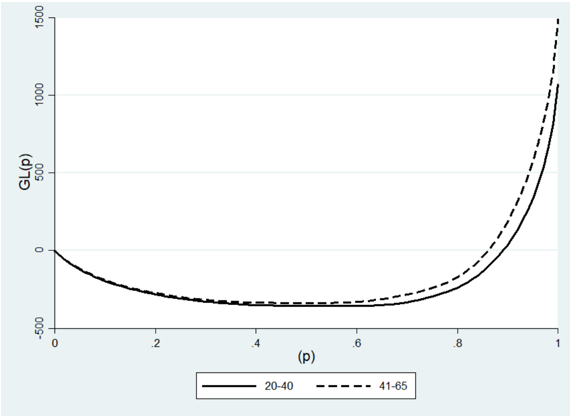
2008-14	Equality	Ex ante	Ex post
OG_{a1}	0.0000	140.99	0.0000
OG_{a2}	0.0001	710.92	0.0000
OG_{a3}	0.0001	725.03	0.0000
OG_{I1}	-0.2209	-0.0243	-0.0410
OG_{I2}	-1.1000	-0.1134	-0.2030
OG_{I3}	-1.1497	-0.1340	-0.2140
OG_{e1}	-4.5680	140.87	-1.9436
OG_{e2}	-23.6902	710.37	-9.5946
OG_{e3}	-22.7564	724.43	-10.1867

income of an individual in cohort 41-65 tends to be in a higher, his effort seems to be less remunerated. The use of generalized Lorenz curves highlight the difference between ex ante and ex post EOp a well known in the literature [16]. One should also note that Figures 3(b) and 4(b) do not show generalized Lorenz dominance, so that our measures complete this partial ranking.

Following [21] we investigate if the religion has an impact on the cost/benefit due to the existence of IOp. The increase in in the importance of the Christian religion in South Korea seems to coincide with the development and *westernization* of the country. Kim [21] argues that the difference between Buddhism and Christianity in the way they treat material achievement may push Christians to exert higher effort than Buddhists in economic activities. We investigate this hypothesis by dividing our sample according to declared religion. The ex post approach to

Figure 3: Intertemporal unfairness generalized Lorenz curves, cohorts 20-40 vs 41-65

(a) Ex ante, OG_{e3}.



(b) Ex post, OG_{e3}.

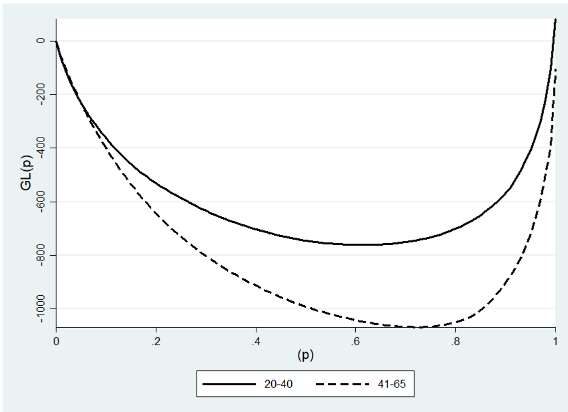
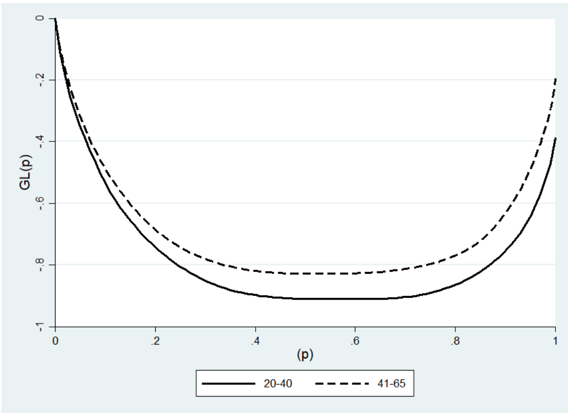


Figure 4: Intertemporal unfairness generalized Lorenz curves, cohorts 20-40 vs 41-65

(a) Ex ante, OG_{l3}.



(b) Ex post,OG_{l3}.

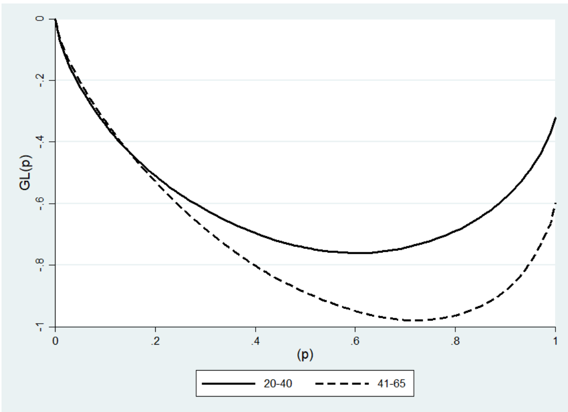


Table 3: Korea intertemporal fairness, 2001-14 by cohort

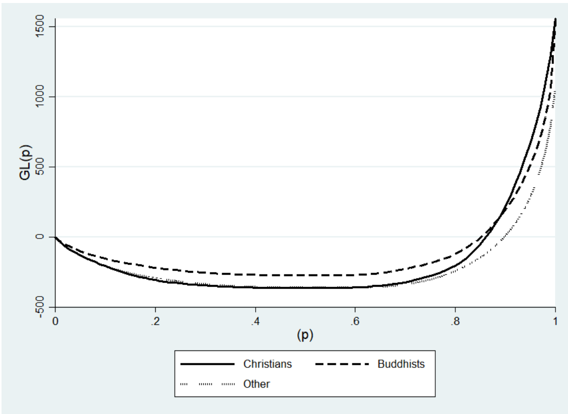
	Equality		Ex ante		Ex post	
	20/40	**41/65**	**20/40**	**41/65**	**20/40**	**41/65**
OG_{a1}	-38.4540	63.1834	108.89	163.07	5.7887	-6.2996
OG_{a2}	-461.97	759.06	1074.38	1710.84	31.6079	-17.9449
OG_{a3}	-289.92	476.36	1067.75	1490.96	83.2013	-104.91
OG_{I1}	-0.2487	-0.1778	-0.0380	-0.0154	-0.0335	-0.0591
OG_{I2}	-2.4542	-1.6266	-0.3560	-0.1021	-0.3364	-0.5554
OG_{I3}	-2.4210	-1.8533	-0.3875	-0.1954	-0.3206	-0.5987
OG_{e1}	-42.3318	59.6974	108.76	162.97	4.3189	-8.2861
OG_{e2}	-503.43	722.17	1073.09	1709.91	16.1430	-38.5013
OG_{e3}	-324.53	444.86	1066.46	1489.92	69.6373	-123.55

the analysis of EOp, which is the one that assumes to observe effort, is indeed in line with the hypothesis that Christians exert more effort. This is because, if we interpret each quantile as an effort interval, Christians tend to be in the upper part of this interval so that they have higher incomes. The opportunity advantage of Christians with respect to Buddhists disappear when we look at ex ante EOp, using a scale invariant gap function: a result that is coherent with the higher variability of the ex ate relative gaps for Christians (see Figure 6(a))

The results above show numerous aspect of the intertemporal inequality of opportunity we can explore using the proposed social evaluation functions. Our results are in line with the theory of EOp, showing the existence of differences in applying ex ante or ex post perspective, and analyzing the phenomena in absolute or relative terms. The social evaluation function is an instrument to complete the partial ordering given by the generalized Lorenz criteria which, moreover, has the advantage of distinguish between the existence of positive or negative effects of individual IOp.

Figure 5: Intertemporal unfairness generalized Lorenz curves by religion.

(a) Ex ante, OG_{e2}.



(b) Ex post, OG_{e2}.

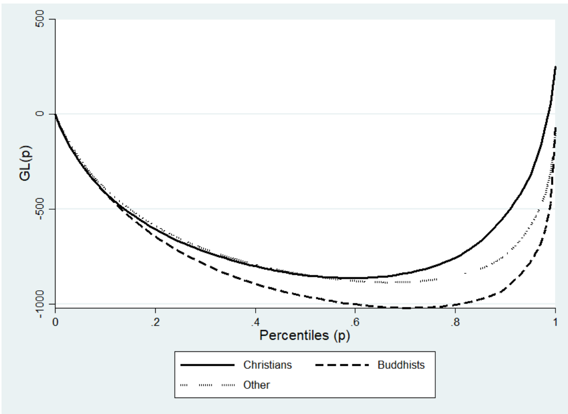
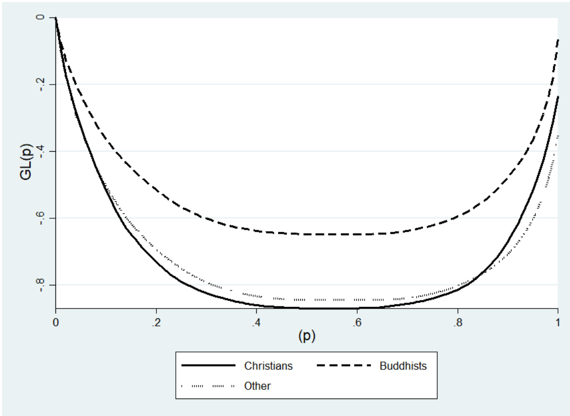


Figure 6: Intertemporal unfairness generalized Lorenz curves by religion.

(a) Ex ante, OG_{l2}.



(b) Ex post, OG_{l2}.

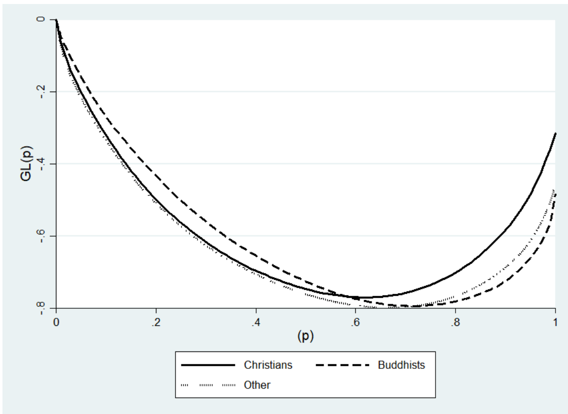


Table 4: Korea intertemporal fairness, 2001-14 by religion

Christians	Equality	Ex ante	Ex post
OG_{a1}	39.4501	152.85	23.4504
OG_{a2}	357.23	1559.37	249.94
OG_{a3}	432.04	1453.79	216.11
OG_{I1}	-0.2061	-0.0270	-0.0332
OG_{I2}	-2.0398	-0.2362	-0.3159
OG_{I3}	-1.9859	-0.2823	-0.3292
OG_{e1}	35.6625	152.73	21.7087
OG_{e2}	316.46	1558.21	231.70
OG_{e3}	398.61	1452.64	199.96

Buddhists	Equality	Ex ante	Ex post
OG_{a1}	38.8233	147.21	-8.6260
OG_{a2}	507.11	1538.92	-65.5352
OG_{a3}	238.65	1347.14	-102.88
OG_{I1}	-0.1911	-0.0130	-0.0517
OG_{I2}	-1.7413	-0.0658	-0.4833
OG_{I3}	-2.0126	-0.1868	-0.5271
OG_{e1}	35.6055	147.13	-10.5225
OG_{e2}	473.53	1538.29	-85.4193
OG_{e3}	209.02	1346.34	-120.39

Others/Atheist	Equality	Ex ante	Ex post
OG_{a1}	-39.5307	108.10	-7.6868
OG_{a2}	-423.56	1077.49	-93.2477
OG_{a3}	-354.26	1044.95	-59.3063
OG_{I1}	-0.2433	-0.0377	-0.0459
OG_{I2}	-2.3597	-0.3534	-0.4565
OG_{I3}	-2.4100	-0.3860	-0.4444
OG_{e1}	-43.4420	107.96	-9.2097
OG_{e2}	-465.28	1076.10	-109.0882
OG_{e3}	-389.29	1043.57	-73.5486

6 Conclusions

The theoretical and empirical literature on the measurement of inequality of opportunity has been very florid in the last decades. A vast variety of tools is available for researchers and policymakers to have information of the extent of (un)fairness of a society.

However, we have noticed that within this literature the temporal aspect has received less attention. In this paper we have tried to fill this gap by proposing a new measure of (un)fairness that is intertemporal. To obtain a societal measure of intertemporal (un)fairness we have followed a two-step procedure, that is, we have, first derived a measure of intertemporal unfairness at the individual level, we have then aggregated this measure into a measure of aggregate unfairness. In both stages of aggregation we adopt the opportunity egalitarian approach and in particular the opportunity gap methodology proposed by Fleurbaey and Schokkaert [17].

Last, we have applied our measurement tool using the KLIPS dataset, a rich source of data on the Korean population, which provides information not only on individuals' standard of livings but also on a different set circumstances and for a considerable number of years.

Notwithstanding its richness, scientific contributions exploring this data in the sphere of the distributional analysis are of a limited number. For this reason, we think that our paper provides both a theoretical as well as an empirical contribution to the literature focused on the analysis of social justice.

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Proofs

Lemma 1

Proof. Set $x = x_{it}$, $x' = x_{jt}$, $\tilde{x} = \tilde{x}_{it}$, $\tilde{x}' = \tilde{x}_{jt}$.

By scale invariance,

$$g(x, \tilde{x}) \geq g(x', \tilde{x}') \iff g\left(\frac{x}{\tilde{x}}, 1\right) \geq g\left(\frac{x'}{\tilde{x}'}, 1\right)$$

By monotonicity,

$$g\left(\frac{x}{\tilde{x}}, 1\right) \geq g\left(\frac{x'}{\tilde{x}'}, 1\right) \iff \frac{x}{\tilde{x}} \geq \frac{x'}{\tilde{x}'}$$

Q.E.D. □

Lemma 2

Proof. Set $x = x_{it}$, $x' = x_{jt}$, $\tilde{x} = \tilde{x}_{it}$, $\tilde{x}' = \tilde{x}_{jt}$.
By additive invariance,

$$g(x, \tilde{x}) \geq g(x', \tilde{x}') \iff g(x - (\tilde{x} - 1), 1) \geq g(x' - (\tilde{x}' - 1), 1)$$

By monotonicity

$$g(x - (\tilde{x} - 1), 1) \geq g(x' - (\tilde{x}' - 1), 1) \iff x - \tilde{x} \geq x' - \tilde{x}'$$

Q.E.D. □

Proposition 1

We begin this section by stating the following lemma proved in Hoy and Zheng [18].

Lemma 3. *For any twice differenciable function $f(\mathbf{x})$ with $\mathbf{x} \in \mathbb{R}^n$, the conditions $\frac{\partial f(\mathbf{x})}{\partial x_i \partial x_j} = 0$ for all $i \neq j \in \{1, \ldots, n\}$ are satisfied if and only if $f(\mathbf{x})$ can be written as*

$$f(\mathbf{x}) = f_1(x_1) + f_2(x_2) + \ldots + f_n(x_n)$$

for some twice differenciable f_is.

Proposition 1 is actually a corollary of the following proposition.

Proposition 3. *An individual intertemporal opportunity gap measure γ satisfies independence (I) and normalization (N) axioms if and only if there exist twice differenciable functions $\omega_1, \omega_2, \ldots, \omega_\tau$ such that, for any $\mathbf{g}_i \in G$,*

$$\gamma(\mathbf{g}_i) = \sum_{t \in T} \omega_t(t) g_{it} \tag{3}$$

where $\sum_{t \in T} \omega_t(t) = 1$.

Proof. By Lemma 3, independence (I) is satisfied if and only if γ is additively separable; i. e.

$$\gamma(\mathbf{g}_i) = \omega_1(g_{i1}) + \ldots + \omega_\tau(g_{i\tau}) \tag{4}$$

Independence axiom also impose twice differenciability of γ and ω.

We now want to show that the weights have to be independent on the value of the gaps.

Take $\mathbf{g}_i \in G$ such that $g_{i1} = \cdots = g_{i\tau} = x$. Normalization axiom (N) is satisfied if and only if $\gamma(\mathbf{g}_i) = x$, i.e.

$$\sum_{t \in T} \omega_t(g_{it}) = x \tag{5}$$

for some $x \in \mathbb{R}$.

From lemmas 1 and 2, we can express g_{it} as a function of some y_{it} that is equal to either $(x_{it} - \tilde{x}_{it})$ or $(x_{it}/\tilde{x}_{it})$. By totally differentiating the previous equation with respect to y_{it} we have:

$$\sum_{t \in T} \frac{\partial \omega_t}{\partial g_{it}} \cdot \frac{\partial g_{it}}{\partial y_{it}} = 0 \tag{6}$$

Set $\tau = 1$, then the previous equation becomes

$$\frac{\partial \omega_1}{\partial g_{i1}} \cdot \frac{\partial g_{i1}}{\partial y_{i1}} \geq 0 \tag{7}$$

This is because for $\tau = 1$, Normalization implies $\gamma(\mathbf{g}_i) = g_{i1}$, and by Definitions 2 and 3, we have $\frac{\partial g_{i1}}{\partial y_{i1}} \geq 0$ which implies $\frac{\partial \gamma}{\partial y_{i1}} \geq 0$.

Therefore, for EQ. (6) to hold, it must be $\frac{\partial \omega_t}{\partial g_{it}} = 0$; i. e. ω_t is independent from g_{it} and depends only on time.

Moreover, taking $\mathbf{g}_i \in G$ such that $g_{i1} = \cdots = g_{i\tau} = x$, from $\sum_{t \in T} \omega_t(t) g_{it} = \sum_{t \in T} \omega_t(t) x = x$ it is evident that $0 < \omega_t(t) < 1$ and $\sum_{t \in T} \omega_t(t) = 1$.[11] □

[11] This proof is inspired to the one of Proposition 2.1 in **(author?)** [18].

We now have to show that γ can be written as in Eq. (1). To see it clearly, note that

$$\sum_{t\in T}\omega_t\left(t\right)g_{it}=\sum_{t\in T}\omega_t\left(t\right)\cdot\sum_{t\in T}\frac{\omega_t\left(t\right)g_{it}}{\sum_{t\in T}\omega_t\left(t\right)}=\sum_{t\in T}\omega_t\left(t\right)\cdot\sum_{t\in T}\left(\frac{\omega_t\left(t\right)}{\sum_{t\in T}\omega_t\left(t\right)}\cdot g_{it}\right)$$

Since $\sum_{t\in T}\omega_t\left(t\right)=1$, we can rewrite Eq. (1) as

$$\gamma\left(\mathbf{g}_i\right)=\sum_{t\in T}\alpha(t,T)g_{it}$$

with $\alpha(t,T)=\frac{\omega_t(t)}{\sum_{t\in T}\omega_t(t)}\in(0,1)$. This conclude the proof of Proposition 1.

Corollary 3

Proof. Set $g_{is}=g_{iu}=x<0$. Opportunity gap persistence axiom requires

$$\alpha(s,T)x+\alpha(u,T)x\leq\alpha(r,T)x+\alpha(v,T)x$$

dividing by $x<0$ and rearranging

$$\alpha(u,T)-\alpha(v,T)\geq\alpha(r,T)-\alpha(s,T)$$

since $s-r=v-u=k$

$$\alpha(u,T)-\alpha(u+k,T)\geq\alpha(s-k,T)-\alpha(s,T)$$

with $u\geq s$, the inequality is satisfied if and only if $\alpha(t,T)$ is concave in t. □

Proposition 2

Proof. By Lemma 3, decomposability is satisfied if and only if Γ can be expressed as a function of the sum of the γ_i in the population. Decomposability and population invariance impose Γ to be an average of the γ_is that have to be transformed to satisfy Lower gap concern. Monotonicity is satisfied if and only if ω is increasing in γ_i. Lower gap concern is satisfied if and only if ω is concave. Indeed, take $\epsilon^{\mathbf{i}}\in\mathbb{R}^n$ such that $\epsilon^i_j=0$ for all $j\neq i\in N$ and $\epsilon^i_i=\epsilon\in\mathbb{R}_+$, $\Gamma\left(\mathbf{s}+\epsilon^{\mathbf{i}}\right)\geq\Gamma\left(\mathbf{s}+\epsilon^{\mathbf{j}}\right)$ if and only if $\Gamma\left(\mathbf{s}+\epsilon^{\mathbf{i}}\right)-\Gamma\left(\mathbf{s}+\epsilon^{\mathbf{j}}\right)\geq 0$. This inequality holds if and only if $\omega\left(\gamma_i+\epsilon\right)\geq\omega\left(\gamma_j+\epsilon\right)$. Given $\gamma_i\leq\gamma_j$, the inequality is satisfied if and only if ω is a concave function. □