1 Dancing with the Populist. New parties, electoral rules and Italian Municipal elections

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2 Theoretical framework

In this section we set up a theoretical framework to provide hindsight about the effect of the different electoral rules on populist strategies and behavior. We will use the results of the model to interpret our data, deriving empirically testable predictions.

2.1 The model

We begin by setting up a very simple version of the model, where we deliberately avoid many complications in order to sharpen results and intuition. Some potential extensions are discussed in the next section.

2.1.1 Voters

We assume that the utility of voters depend on two elements: their preferences along a "left to right" policy dimension, and their trust on the traditional political system. The first element captures the traditional cleavage among voters, between who is in favor of more government intervention (say, more taxes and redistribution) and who is against, perhaps as a result of voters' different position on the labor market (say, dependent workers versus self-employers and entrepreneurs). Traditional policy and party systems have evolved along this dimension in all Western democracies in the last two centuries. However, the recent surge of populist parties has been predicated on a different conflict, the one between the "elité" and the "people", where the elité is depicted as inevitably corrupt and the people as fundamentally sound, honest and blessed with some natural wisdom (CAS, 2018). Populists portrait themselves as the defenders of the people against the self-serving elité. Traditional parties, both on the right and the left of the political spectrum, just for having been in charge in the past or being represented in Parliament, are naturally part of the elité and are therefore accused by populists to be self serving and corrupt. This is believed by voters with little trust in the traditional political system, regardless their position on the first policy dimension.

We capture these ideas here by assuming that a voter' utility function can be written as:

(1) $U^i = B - (q^i - q)^2 - \alpha^i \gamma E$

where q^i captures voter i's preference on the left to right ideological dimension and the parameter α^i her trust in the established political system. More specifically, we assume that the ideological political dimension can be represented by an interval of the real line, ranging from -1 (say, extreme left) to $+1$ (extreme right); q^i is the bliss point of voter i on this line, while q is the policy chosen by elected politicians trough the working of the electoral system (to be described below). Notice that the formulation in eq.(1) implies that the voter loses utility at an increasing rate as the selected policy is farther from her bliss point.

The parameter α^i instead ranges from $\underline{\alpha}$ to $\overline{\alpha}$, where $\overline{\alpha} > 0 > \underline{\alpha}$ and $E = \{0, 1\}$ captures the fact whether the party that proposes (and if elected, implements) the policy q is an established party ($E = 1$) or a new, populist, one ($E = 0$). Note that in line with the above discussion, we model the trust dimension α^i as separate and independent from voter i's ideological position on the policy dimension. Further notice that for voters such that $\alpha^i < 0$ the fact that a policy is proposed and implemented by an established party is actually a plus; these voters feel comforted by the fact that the policy is chosen by a reliable party that has already been in charge in the past. On the contrary, for voters with $\alpha^i > 0$, the opposite is true and regardless of the policy itself, they dislike the fact that this is chosen and implemented by an established party, perhaps because they believe that traditional politicians are corrupted and will end up cashing some of the public money devoted to policy. $0 \leq \gamma \leq 1$ is a parameter that captures how strong are these "elité-anti elité" preferences among voters. For concreteness, we assume $\overline{\alpha} \geq 1$ and $\underline{\alpha} \geq -\frac{1}{2}$, so that, depending on the specific form of the distribution of α^i among voters, an increase in γ would increase on average the populist, anti-establishment preferences in the population. In general (as in our empirical applications below) one can think to an increase in γ as a shock to preferences induced by the fact that the traditional political system has shown to be unable to address the consequences of some deep crises hitting society (say, globalization, increased immigration, or an economic crisis).

Finally, $B \geq 2$ is just some parameter whose only role is to guarantee interior solutions in the analytical developments to follow.

We assume that concerning their ideological position, there are only two types of voters, "leftish" voters, with $q^i = -1/2$ and "rightish" voters, with $q^i = 1/2$. We index the former voters with l and the latter with r, so that $i = l, r$. This description of the electorate as sharply divided in two distant and polarized groups on ideological grounds might look overly simplistic, but as we will discuss in section XX, is not very far the mark as a representation of the political competition in our testing grounds, municipal elections in Italy. However, we will discuss in section XX the effect of adding other types of voters.

The parameter α^i instead is a continuous variable distributed across the population, according to the distribution function $g(\alpha^i)$. For simplicity and to ease computations, in what follows we assume $g(\alpha^i)$ to be the uniform distribution. In order to exploit symmetry, we also assume that there is (in expected terms) the same (large) number of rightish and leftish voters, N. This number is fix, but

the day of the election there is some small shock ϵ that switch some voters from the leftish to the rightist side of the political spectrum[1]. The distribution of ϵ, $F(\epsilon)$, is symmetric around zero in the interval $[-\bar{\epsilon}; \bar{\epsilon}]$ and $E(\epsilon) = 0$. The only role of ϵ is to introduce some ex ante uncertainty in the results of the election; ϵ can be thought of as a last minute participation shock affecting differently the size of the two types of voters[2].

Voters decide which political party (or candidate)[3] to support on the basis of the utility they would gain if this party were in charge, taking into account both the policy platform of the party and the fact whether the party proposing the policy is an established or a new party. However, support for a party does not necessarily translate in a vote for this party, as voters are also subject to a participation cost on the day of the election equal to $c^i > 0$. Voters will then only vote if the perceived benefit from voting (the so called "expressive" component) is larger than this cost; they will abstain otherwise. However, we also assume that for each voter this perceived benefit from voting is proportional to the utility that the favored party would guarantee her if it were to be elected. This seems reasonable; for the same level of cost in participating at the elections, a voter that strongly likes a candidate might be more willing to reach the ballot box and vote rather than another voter who only mildly supports the same candidate. With this formulation, voters just vote sincereously for the candidate they like the most, but more motivated voters are also more likely to vote. Finally, for analytical simplicity, we assume that c^i is distributed with a uniform distribution in the interval $[0, 2]$. α^i, c^i and ϵ are all i.d.d. random variables.

2.1.2 Parties

There are two established or traditional parties, R and L with policy preferences coinciding with those of the two groups of voters that they represent. The two parties cannot change their policy positions; because of history and tradition, a new policy position proposed by a traditional party would not be believed by voters. This also seems reasonable (but see section XX for an alternative formulation). Thus, if we let q^P be the policy proposed by party P, $q^R = 1/2$ and $q^L = -1/2$. If one of the two traditional parties wins the elections, it also earns political rents equal to $V > 0$.

There is also a potential entrant, a not established party, M. This party, being new, has not pre-determined policy position and can freely decide where to position itself on the policy line. It also gains V if it wins the elections, but differently from the established parties (that have paid it already), if M wants to participate at the electoral competition it needs to pay some organizational costs. Let this cost be fix and equal to $K > 0$. We assume $V > K$, so that

[1] Analysis of electoral flows in Italy typically shows that there is very little shift from one side to the opposite side of the political spectrum, usually less than 5%. See Itanes, 2018.

[2] As an alternative explanation, one might think that parties are ex ante slightly uncertain about the specific dimension of each group of voters; this is only revealed ex post at the ballot, when ϵ is realized.

[3] In what follows we do not distinguish between the party and its candidate and use the two words indifferently.

M would certainly enter the competition if it had a large enough probability to win the elections.

2.1.3 Electoral systems

We compare two different electoral systems, a single ballot mechanism (SB) and a dual ballot (DB) mechanism (or runoff). According to the first system, a candidate (party) obtaining the highest number of votes is directly elected to guide the executive; according to the second, there are two rounds of election, and if no candidate (party) obtains at least 50%+1 of votes at the first round, the two candidates who have gained more votes at the first round run again at a second ballot. We focus on these two electoral systems because the Italian municipalities composing our data set vote for the Mayor and the municipality legislative body (the Council) according to one of these two mechanisms. More specifically, as discussed more in detail in section XX, below a given threshold of population (15,000 inhabitants), Mayors are elected through a first pass the post system; the candidate obtaining more votes becomes Mayor and the political party or parties supporting its candidature obtain the majority in the municipality legislative body (the Council). In municipalities above 15,000 inhabitants there is instead a runoff between the two most voted candidates at the first round. Again, who wins the second round is elected and the party or parties supporting him obtain the majority in the Council.

2.1.4 Sequence of events

We postulate the following sequence of events.

1. At the first stage, parties decide if running and on which political platform. Specifically, established parties always run (as they have already paid the organizational cost) and by assumption they run on predetermined policy positions, so they have in fact no decision to make. Party M must instead decide if entering and paying the fix cost K or not entering and saving the cost. Moreover, conditional on the entry decision, he must also decide which policy platform to propose. Notice that when making these decisions, parties know the distribution of a^i, c^i among voters and the distribution of ϵ, but not their realization.

2. At the second stage, each voter i observes the policy proposals of the different political parties and the realization of a^i. On this basis, she decides which party she would potentially support at the elections.

3. At the election day, each voter observes the realization of her participation cost, c^i. On the basis of the utility offered by the most favored party, she then decides whether to vote for this party or to abstain.

4. Citizens who decide to turn out at the ballot box, vote for the different parties. At this point, ϵ is also realized. If the electoral system is SB, the party (candidate) that gain more votes wins and implements the proposed policy. If the electoral system is DB, the two most voted parties run again at the second

ballot to determine the winner. Who wins, again implements the proposed policy.

Of interest here, as potential predictions to guide the empirical analysis, are:

1) the conditions under which party M decides to run and the policy platform it chooses under the two electoral systems;

2) the turnout of voters in all possible cases, under SB or DB.

The model is solved by backwards induction.

2.1.5 Single ballot

Consider first the case where M decides not to run, so that only the two traditional parties are competing at the election. Consider a rightish voter. In deciding which party he prefers between R and L, he compare his utility under the two opposing policy platforms:

(2) $U^r(q^R) = B - \alpha^i\gamma$; $U^r(q^L) = B - 1 - \alpha^i\gamma$;

Inspections of (2) immediately reveals that the rightish voter will always prefer R to L for any realization of α^i. The reason is simple; as both traditional parties are established parties ($E = 1$) the trust component has the same effect on the utility of voters for any realization of α^i, while on the policy dimension rightish voters clearly prefer the right party. By symmetry, the same is true for the leftist voter that would always prefer L to R.

At the time of the elections, however, voter i is also subject to a participation cost equal to c^i; on the basis of our assumptions, she will then vote only if the utility promised by the favored party is larger than this cost, that is if

(3) $B - \alpha^i\gamma \geq c^i$

She will abstain otherwise. Exploiting the properties of the distribution function for α^i, the expected utility of the N rightish voters is then $(B - \gamma\alpha)$ where $\alpha = (\overline{\alpha} + \underline{\alpha})/2 > 0$. The same is true for the N leftish voters. Considering the participation shock c^i and its distribution, this implies that the expected votes for each party are equal to:

(4) $NE(B - \alpha\gamma) = N\frac{1}{2}\int_o^{(B-\gamma\alpha)} dc^i = \frac{N}{2}(B - \gamma\alpha)$.

Ex post, however, the shock ϵ will shift some voters from the two parties, so that $\frac{N}{2}(1 + \epsilon)(B - \gamma\alpha)$ will vote for R and $\frac{N}{2}(1 - \epsilon)(B - \gamma\alpha)$ will vote for party L; by symmetry of $F(\epsilon)$ around zero, and as $E(\epsilon) = 0$, each of the two traditional parties, when running again another traditional party, then wins with probability 1/2. Once elected, the party then implements its promised platform. Notice also, that as the shock ϵ is perfectly symmetric, the global effect of ϵ on total turnout cancels out and the total participation rate will be $N(B - \gamma\alpha)/2N = \min\left[(\frac{1}{2}B - \gamma\frac{\alpha}{2}); 1\right]$ [4]. Notice that electoral turnout is

[4] Given our assumptions on parameters, partecipation will always be positive as $B \geq 2$ and $\gamma\alpha \leq 1$. For $B = 2$ it will be however smaller than 100% and will decrease as γ and α increases.

decreasing in γ and α; the larger the share of voters who do not trust the established political system, the lower the participation rate at the elections when only the two traditional parties run. Let us summarize our results so far in the following proposition:

Proposition 1 *Suppose that only the two established parties participate at the elections. Then, under the Single Ballot rule each party wins with probability 1/2, with equal probability policy is either* $q = 1/2$ *or* $q = -1/2$, *and the turnout rate of voters is equal to* $\min\left[\frac{1}{2}(B - \gamma\alpha); 1\right]$.

Suppose now instead that the populist party decides to participate at the elections, running on the platform q^M. Consider again the rightish voter. For the reasons discussed above, this voter will never support the Left party; thus, the only relevant utility comparison to be made by this voter is between the utility offered by R and by M. Specifically, the rightish voter will support M rather than R if:

(5) $B - (1/2 - q^M)^2 \geq B - \alpha^i\gamma$

That is, she will support the M party if $\alpha^i \geq \frac{(1/2-q^M)^2}{\gamma} \equiv \alpha^*$. Assuming $\alpha^* < \overline{\alpha}$ (to be verified below), in expected terms, a share $\frac{\alpha^*-\underline{\alpha}}{\overline{\alpha}-\underline{\alpha}}$of the N rightish voters will then support the R party and a share $\frac{\overline{\alpha}-\alpha^*}{\overline{\alpha}-\underline{\alpha}}$ will support the M party. Exploiting the properties of the distribution function for a^i, the expected utility for the rightish voters voting for M and for R respectively can then be computed as

(6) $E(B - \gamma\alpha^*) = \int_{\alpha^*}^{\overline{\alpha}} (B - \gamma\alpha^*) f(\alpha^i) d\alpha^i = (B - \gamma\alpha^*)$

and

(7) $E(B - \alpha^i\gamma) = \int_{\underline{\alpha}}^{\alpha^*} (B - \alpha^i\gamma) f(\alpha^i) d\alpha^i = (B - \gamma\frac{\alpha^*+\underline{\alpha}}{2})$

Adding the participation shock c^i, this implies that the total number of (expected) rightish votes for M and for R respectively are:

(8) $N\frac{\overline{\alpha}-\alpha^*}{\overline{\alpha}-\underline{\alpha}}E(B - \gamma\alpha^*) = N\frac{\overline{\alpha}-\alpha^*}{\overline{\alpha}-\underline{\alpha}}\frac{1}{2}\int_{o}^{(B-\gamma\alpha^*)} dc^i = \frac{N}{2}\frac{\overline{\alpha}-\alpha^*}{\overline{\alpha}-\underline{\alpha}}(B - \gamma\alpha^*)$

and

(9) $N\frac{\alpha^*-\underline{\alpha}}{\overline{\alpha}-\underline{\alpha}}E(B - \gamma\frac{\alpha^*+\underline{\alpha}}{2}) = N\frac{\alpha^*-\underline{\alpha}}{\overline{\alpha}-\underline{\alpha}}\frac{1}{2}\int_{o}^{(B-\gamma\frac{\alpha^*+\underline{\alpha}}{2})} dc^i = \frac{N}{2}\frac{\alpha^*-\underline{\alpha}}{\overline{\alpha}-\underline{\alpha}}(B - \gamma\frac{\alpha^*+\underline{\alpha}}{2})$

Clearly, by choosing appropriately q^M , M can influence the number of rightish voters who will vote for him. In particular, by letting q^M get closer to

less than 100% for any $\gamma > 1/3$.

1/2 (that is, by reducing α^*) he can increase the potential number of rightish votes that he can get. However, choosing a platform closer to the bliss point of the rightish voters means losing votes on the leftish side. In fact, by invoking symmetry, the (expected) number of leftish voters willing to vote for the M party is given by $\frac{N}{2}(\frac{\overline{\alpha}-\alpha^o}{\overline{\alpha}-\underline{\alpha}})(B-\gamma\alpha^o)$ where $\alpha^o=(1/2+q^M)^2/\gamma$. The optimal policy platform for M clearly is the one that maximizes the expected number of votes from both the rightish and the leftish side. Intuitively, by symmetry, this should occur at the middle point of the two bliss points, that is at $q^M=0$. Given our parametric assumptions, this is indeed confirmed by the proposition below.

Proposition 2 *The optimal policy platform for M is $q^M=0$.*
Proof. *Summing over the expected number of votes for party M we get* ■

$F(q^M)=\frac{N}{2}((\frac{\overline{\alpha}-\alpha^o}{\overline{\alpha}-\underline{\alpha}})(B-\gamma\alpha^o)+(\frac{\overline{\alpha}-\alpha^*}{\overline{\alpha}-\underline{\alpha}})(B-\gamma\alpha^*))$

substituting for $\alpha^o=(1/2+q^M)^2/\gamma$ and $\alpha^*=(1/2-q^M)^2/\gamma$ and differentiating, $F'(q^M=0)=0$ and

$F''(q^M=0)\leq 0$ for $B\geq\frac{3}{2}$. *Hence $q^M=0$ is a local maximum. QED*

To strenghten the intuition behind the proposition, consider the opposite strategy where the M party chooses instead an extreme policy position. Clearly any choice of q^M outside the interval $(-1/2,1/2)$ would be self-defeating for party M; it would alienate some of the ideologically closer voters while also alienating the voters of the further group. Suppose then that party M chooses the extreme $q^M=1/2$ (or $q^M=-1/2$), that is, it chooses the political position of the R party (L party). This would bring him all rightish (leftish) voters with $a^i>0$, a share $\overline{\alpha}/(\overline{\alpha}-\underline{\alpha})\geq\frac{2}{3}$ of these voters, meaning that the M party would surely beat the R party (L party) at the elections. However, if $\gamma\overline{\alpha}\leq 1$, M will also lose all leftish (rightish) voters to the L party (R party). And, as can be easily checked, even under the most optimistic forecasts on the result of the shock ϵ, M could never possibly win the elections if $\overline{\epsilon}<\frac{1}{5}$.

Thus, given our parametric restrictions, if the party M decides to enter, it will do so by proposing the policy platform $q^M=0$. Substituting in (8) and (9) and manipulating, the total number of (expected) votes for M and for any of the other traditional party are:

(10) $\frac{N}{(\overline{\alpha}-\underline{\alpha})\gamma}(\gamma\overline{\alpha}-\frac{1}{4})(B-\frac{1}{4})$

(11) $\frac{N}{(\overline{\alpha}-\underline{\alpha})\gamma}(\frac{B}{2}(\frac{1}{4}-\gamma\underline{\alpha})-(\frac{1}{4})^3+\frac{1}{4}(\gamma\underline{\alpha})^2)$

Clearly, however, as it has to pay the entry cost K, party M will not enter unless it has some positive probability of winning. We now prove:

Proposition 3 *There exists a value of γ, $\gamma^*<1$ such that if M enters it ranks at the second place at the elections. There exists a level of γ, $1>\gamma^{**}>\gamma^*$ such that M enters and wins with probability $\frac{K}{V}$.*

Proof. To prove the proposition, note first that because ϵ is symmetric across N the expected numbers of votes for M do not depend on the realization of this shock, while the votes of its opponents do. This also means that if the expected number of votes in (11) and (10) are equal, M will certainly arrive second, as one of the other two traditional parties is bound to get a negative shock ϵ. Inspection of (10) and (11) shows that (10) is increasing in $\overline{\alpha}$ and in B and (11) is increasing in $\underline{\alpha}$. To prove the proposition, let us then take the smallest admissible values for $\overline{\alpha}$ and in B (that is, $\overline{\alpha} = 1$ and $B = 2$) and the largest admissible value for $\underline{\alpha}$ ($\underline{\alpha} = -\frac{1}{2}$). Substituting in (10) and (11), then γ^* is then defined as the minimum level of γ such that: A.1 $(\gamma - \frac{1}{4})(2 - \frac{1}{4}) \geq (\frac{1}{4} + \frac{1}{2}\gamma) - (\frac{1}{4})^3 + (\frac{1}{4})^2\gamma^2$ ■

Solving the equation, it is easy to see that for $\gamma > \frac{1}{2}$ the inequality in A.1. is satisfied. At fortiori, this means that for other admissible values of the above parameters there always exists a $\gamma^* < 1$ such that the values in (10) and (11) coincide. This proves the first part of the proposition. For the second, let us first note that in order for M to win it must the case that

A.2 $\frac{N}{(\overline{\alpha}-\underline{\alpha})\gamma}(\gamma\overline{\alpha} - \frac{1}{4})(B - \frac{1}{4}) \equiv NS \geq N(1+\epsilon)T = \frac{N(1+\epsilon)}{(\overline{\alpha}-\underline{\alpha})\gamma}(\frac{B}{2}(\frac{1}{4} - \gamma\underline{\alpha}) - (\frac{1}{4})^3 + \frac{1}{4}(\gamma\underline{\alpha})^2)$

where S and T are just shorthands for the two coefficients on the left and right side of A.2. This means that in order to win, the realization of the shock must be such that:

A.3 $$\frac{S}{T} - 1 \geq \epsilon$$

For $\gamma = \gamma^*$, $\frac{S}{T}$ is equal to 1 and therefore the inequality in A.3 is certainly unsatisfied. But note that using the above values for the parameters and evaluating A.1 at $\gamma = 1$, it turns out that M would certainly wins if $\epsilon < \frac{33}{51}$. But a realization of $\epsilon > \frac{33}{51}$ is certainly beyond the admissible interval of the shock ϵ (it would imply that about 2/5 of voters shift from an ideological extreme to the other). A fortiori, this then implies that for any admissible values of the parameters M would certainly win the elections if $\gamma = 1$. As $\frac{S}{T}$ is increasing in γ, this then implies that there exists a $1 > \gamma^{**} > \gamma^*$ such that M would enter the political competition. Implicitly, γ^{**} is defined by the equation

$$VF(\frac{S}{T}(\gamma^{**}) - 1) = K$$

Where $F(\frac{S}{T}(\gamma^{**}) - 1)$, the probability that the realization of the shock is lower than $(\frac{S}{T} - 1)$ is equal to the probability that M will win the elections, $\frac{K}{V}$. QED.

Proposition X then shows that M will then enter if the range of unsatisfied voters γ is large enough and will not enter otherwise. An interesting question is what happens to electoral turnout when M enters. Does it increase?

Lemma 4 *Upon M entry, electoral participation will increase if the following condition is satisfied:* $(\overline{\alpha}\gamma)(\overline{\alpha}\gamma - \frac{1}{2}) > 1/16$.

Proof. *Invoking (10) and (11) the participation rate when M enters it is equal to* ■

A.2 $\frac{1}{(\overline{\alpha}-\underline{\alpha})\gamma}\frac{1}{2}((\gamma\overline{\alpha}-\frac{1}{4})(B-\frac{1}{4})+(B(\frac{1}{4}-\gamma\underline{\alpha})-2(\frac{1}{4})^3+\frac{1}{2}(\gamma\underline{\alpha})^2)$

Manipulating and simplifying terms A.2 can be rewritten as

A.3 $\frac{B}{2}+\frac{1}{2}\frac{1}{(\overline{\alpha}-\underline{\alpha})\gamma}(+\frac{1}{2}(\gamma\underline{\alpha})^2-\frac{1}{32}-\frac{1}{4}\gamma\overline{\alpha})$

Invoking (4), the participation rate when only the two traditional parties run is

A.4 $\frac{1}{2}(B-\gamma\ (\ \overline{\alpha}+\underline{\alpha})/2)$

It follows that the participation increases if

A.5 $\frac{1}{(\overline{\alpha}-\underline{\alpha})\gamma}(+\frac{1}{2}(\gamma\underline{\alpha})^2-\frac{1}{32}-\frac{1}{4}\gamma\overline{\alpha})+\gamma\ \frac{(\overline{\alpha}+\underline{\alpha})}{2}>0$

Solving, we get the condition in the statement of the Lemma. QED

Clearly, as $\overline{\alpha}\geq 1$, for γ close to 1, the condition of the Lemma will be certainly satisfied. But if γ is close to 1, by the previous proposition, M will also certainly enter. We can then safely conclude that in general, entry by M will increase the turnout rate to the elections. Intuitively, voters strongly unsatisfied by the traditional political system and that therefore on average abstained when only the two traditional parties competed, would return to the ballot box when the populist party also runs. Summing up:

Proposition 5 *If the share of unsatisfied voters is large enough, the populist party will enter in the political arena, winning with a positive probability. Depending on who is the winner, policy will either be one of the platforms desired by the traditional parties or a more centrist policy. Turnout at the elections will increase as a result of the entrance in the political arena of the populist party.*

2.2 Dual ballot

How do these results change when we consider the dual ballot system? To address this question and easy the comparison, let us start by assuming that nothing else expect the electoral rule changes with respect to the previous section. Specifically, let us assume that the realization of α^i, c^i and ϵ all occur at the first round and that their values just remain unchanged when moving to the second ballot. Under these conditions, it becomes easy to analyse the different cases.

Suppose then that the two traditional parties reach the second ballot, either because the populist party has not run at the first ballot or because it has been defeated at the first round. In this case, results at the second round do not change with respect to the first round. In particular, if the populist party did not run at the first ballot, than no voter when it reaches the second ballot has an incentive to revise its turnout and voting decision. Leftish voters who voted for the left party at first round will just confirm their vote at the second round and so will do the rightist voters. As participation is the same and the shock ϵ is permanent, whoever won at the first round will also win at the second round and be elected. The electoral system therefore does not make any difference in terms of result; the winner would be the same under both electoral systems.

Notice that the same will happen even if the populist party did run at the first round[5]. If the populist party run at the first round, but it was defeated, finishing at the third place in the electoral competition, some voters will change their voting behavior at the second round. Specifically, some of the rightish voters who had voted for the populist party at the first round, since the populist party is not running anymore, will revise their choice and vote for the R party at the second round. And so will do some of the leftish voters who had voted for the populist at the first round, voting for the L party at the second round. But the important point is that given the symmetry of the game -and in particular the fact that as we proved in Proposition the populist party at the first round ran on a centrist policy to attract voters from both sides –these "repented" traditional voters on both side of the political spectrum will have exactly the same size. Thus, if the shock ϵ is permanent, whoever of the two traditional parties won at the first round will also win at the second round and be elected. Again, the electoral system does not make any difference in terms of the final outcome.

When the electoral system can make a difference, it is in the case where the populist and a traditional party candidate reach the second round. To see this, suppose that the R party did not make at the first round, so that at the second round voters can only vote for either the L party or the populist party M. Let us also suppose that L was the more voted party at the first round, so that it leads the competition on M at the second round. In terms of our model, this could happen because party L was hit by a particular positive shock ϵ at the first round. Again, let us assume that this shock is permanent, so that L keeps benefiting from this positive shock even at the second round.

Now consider the incentives of different type of voters at the second ballot. Clearly, whoever voted for M or for L at the first round has no incentive to change her voting behavior at the second round; given our hypothesis of permanence of all shocks, each one of these voters will just confirm her vote at the second ballot. Consider then the rightish voters who had preferred and then voted for R at the first ballot. According to our analysis in the previous section, these are the voters for whom $\gamma\alpha^i < (1/2 - q^M)^2 = \frac{1}{4}$, and their size is $\frac{\frac{1}{4\gamma}-\underline{\alpha}}{\overline{\alpha}-\underline{\alpha}}$of the $N(1-\epsilon)$ realized ex post rightish voters. At the second ballot, these voters face only the option to either choose M or L. Their utility under the two options is given by:

(12) $U^r(q^M) = B - \frac{1}{4}$; $U^r(q^L) = B - 1 - \alpha^i\gamma$;

Given our assumptions above on the distribution of α^i, it is clear that all these rightish voters will certainly prefer M to L. The utility of all these voters under this choice will be given by $(B - 1/4)$ leading to a participation rate of $\frac{1}{2}(B - 1/4)$ of the potential $\frac{\frac{1}{4\gamma}-\underline{\alpha}}{\overline{\alpha}-\underline{\alpha}}N(1-\epsilon)$ voters.

[5] Strictly speaking, this is the only possible case given the assumptions of the model. With only two candidates running at the first round, and given the symmetry of the game, one of the candidates necessariky gets more than 50% of votes at the first ballot. He is then elected directly, without the need of setting up a second ballot.

The conclusion is while confirming all its first round votes, the M party will also gain a considerable amount of votes from the rightish voters at the second round, so making it much more likely that M will win at the final ballot, as these extra votes largely overweight (in the contest of our model) the first round advantage of the L party. In this case, the electoral system does make a difference. If the electoral system were SB the L candidate would win the elections, while with the DB, keeping everything constant, the M candidate prevails.

Also note that in the contest of our model the opposite cannot happen; that is, it cannot happen that a M candidate leading at the first ballot on a traditional party candidate, will be beaten by the latter at the second round. The reason is again the same. While the M party can attract some voters from the traditional party which did not make at the second round, this is not possible for the other candidate, that is then bound to lose the election. In sum, our model suggests that the populist party will be a much more likely winner if it manages to reach the second round.

A potential objection to the result above is that we kept the policy choices of parties unchanged when comparing the first to the second round, while a party, knowing that it might have to run for a second round, might make different policy choices at the first round. While this argument is correct in general, it has no bite in this particular context. The traditional parties, by assumption, have no choice to make in either the single or the second ballot electoral system. For the M party, we already provided conditions that imply that it would prefer to run on a centrist platform under a single ballot mechanism. This is *a fortiori* even truer for the second ballot mechanism, as a centrist platform is needed to attract the ideological voters who had lost their representative party at the second ballot.

Finally, there are two other observations that are worth stressing, as they might provide testable implications of the model.

First, as the M party is more likely to win under the DB electoral system, we should expect the populist party, ceteris paribus, to enter more often in contexts that adopt this system than in others that adopt a SB mechanism. Specifically, under a DB system, what the populist party only needs to do it is to reach the second position at the first round as this will be enough to make him an almost sure winner at the second round. Going back to Proposition XX of the previous section, while γ needs to be at least as large as γ^{**} for the populist party to enter in the political arena under the SB electoral rule, under the DB he will enter even if $\gamma \geq \gamma^{*}$, where $\gamma^{**} > \gamma^{*}$.

Second, suppose we compare electoral turnout at the *second round* in two different cases, when the two candidates who compete at this round are from the two traditional parties, and when a populist candidate and traditional party candidate compete at this round. Then turnout in the latter case should be larger. Specifically, we already computed in the previous section turnout when only the traditional parties run: $(\frac{1}{2}B - \gamma\frac{\alpha}{2})$. Notice that under our assumption of permanence of all shocks between rounds, the same expression characterize turnout at the second round, irrespective on whether the populist ran (and was

defeated) or did not run at the first round [6]. On the contrary, when a populist and a traditional party run at the second ballot, turnout will be given by:

(13) $\frac{1}{2(\overline{\alpha}-\underline{\alpha})\gamma}((\gamma\overline{\alpha}-\frac{1}{4})(B-\frac{1}{4})+(1+\epsilon)(\frac{B}{2}(\frac{1}{4}-\gamma\underline{\alpha})-(\frac{1}{4})^3+\frac{1}{4}(\gamma\underline{\alpha})^2)+(1-\epsilon)(\frac{1}{4}-\underline{\alpha}\gamma)(\frac{B}{2}-\frac{1}{8}))$

This expression in (13) is very similar to the expression that we have obtained for total participation at first ballot in the previous section and we have already proved that for $\gamma\overline{\alpha}$ sufficiently large (which is the case when the populist enters) (13) implies a larger turnout than in the case when only the two traditional parties compete. The same happens to turnout at the second round of a SB system. Intuitively, when two candidates from the two traditional parties run at the second ballot, only the voters of these two parties participate at the elections. But when one populist and a traditional candidate run at the elections, not only the voters of the traditional parties participate (the members of one group to vote for its own traditional party, the members of the other to vote for the populist) but also the unsatisfied voters of the populist party (that would have otherwise abstained) participate. Hence, turnout is larger.

Let us summarize the results of this section in the following proposition.

Proposition 6 *Under the DB, populists have an advantage with respect to the traditional parties. If they reach the second round they are more likely to win because they can attract the voters of the excluded party. This also implies that 1) ceteris paribus, populists are more likely to run in localities that adopts a DB electoral system 2) turnout at the second round is larger when one of the two contenders is a populist.*

2.3 Empirical predictions

The model discussed so far it is clearly very stylized; in section xx we will briefly address some potential extensions. But as the basic intuition behind the stylized model will be shown to survive these extensions, it is worth pausing here and summarize the basic implications that derive from the model as a guide for empirical research.

1) First, we have shown that in order to be worth for the populist party to enter in the political arena, paying the unavoidable organizational cost, there has to be a large number of voters unsatisfied with the traditional political system. As this lack of satisfaction is likely to be related with a lack of capacity of the traditional political class to solve some serious past problems (say, an immigration or an economic crisis), we should expect to find more populists running in localities hit more severely by unsolved crises.

2) The above is true under any electoral system. However, as the dual ballot advantages the populist party, we should observe, ceteris paribus, more populist

[6] Since nothing changes between rounds, the result is obviuos when only the two traditional parties ran at the first round. But even if the populist ran and was defeated at the first round result do not change. This is because as the populist party is no longer running at the second round, all traditional voters that at the first round had voted for the populist, just vote for their traditional parties at the second ballot.

running in jurisdictions adopting this system rather than the SB.

3) The DB should help the populists, in the sense that a populist arriving second at the first round is more likely to win the second ballot than a traditional party candidate arriving second at the first ballot.

4) The presence of a populist party at the elections should increase turnout, as unsatisfied voters unwilling to vote for the traditional parties and therefore abstaining, would be willing to turn out and vote for the populist.

5) At the second round of a DB, participation at the elections should be larger when a populist is running. The reason is that the populist, besides keeping his own votes, should also attract the votes of the excluded traditional candidate, while this would not happen if only traditional parties compete at the elections.

6) One unpalatable further prediction of our model is that the populist party should also run on a more moderate (centrist) platform than either traditional party. This is somewhat harder to believe (and also more difficult to test). But what the populists really like to announce is not so much that they run on a somewhat more moderate platform, intermediate between those of the more extreme left and right parties; this is what a normal centrist party would like to do. Rather, the populist like to claim that their political platform is "beyond" the traditional distinction between left and righ policies, and so it can be supported by voters belonging to both ideological positions. Our model cannot capture this claim and it is not even obvious how one could do it, as the expression "beyond" has no clear meaning. In practise, one would have to see which specific policies the populists implement once they are elected, and check if they are truly more centrist than the ones advocated by the traditional parties.

7) This last observation lead us directly to the last (implicit) prediction of our model. What happens after that a populist has seized power? Our model does not directly address this issue. However, it implicitly suggests that the specific advantage of the populist on traditional parties should not last. This, for two reasons. First, because once it has been in power for a sufficient period of time, the populist also becomes part of the "elité". If problems are not solved, the lack of satisfaction and trust of voters will also extend to the populist party, eliminating its advantage with respect to the traditional parties. Second, because the populist once elected will have to take decisions and it is not obvious that its proposed platform, "beyond" the traditional distinction between left and right policies, can be put in practise.

3 Potential extensions to the model (perhaps an appendix)

- what if traditional parties could react and change their policy position to block M entry? Hint. It should not change much. Even if traditional parties converge at the center, if α^i is large enough they should lose elections.

The populist party is not a centrist party.

- what if there are more parties? what if the bigger parties are extreme? Hint. Ceteris paribus it should not make a difference.
- what if we divide the final shock in two parts (first round, second round) with some possibility of reversal? Again, don't think would change anything.
- Strategic behavior might have a bite here?