On the excessive entry in markets with moral hazard

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Abstract

We study an oligopoly market where, in addition to the business stealing externality, firms face a problem of moral hazard in hiring workers. The rent efficiency trade-off in oligopolistic markets adds additional considerations on the business stealing effect and can lead to reverse results about the excessive entry theorem.

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1 Introduction

In a principal-agent relationship, the rent-efficiency trade-off implies a loss for the society as a whole when the principal is a monopolist. In this paper, against this backdrop of the standard model, we study an oligopolistic market where several firms offer contracts to workers and a moral hazard problem emerges. Hence, we have an aggregative game where the result of a single match between workers and employer has an externality effect on competitors. This in turn implies that the rent extraction is also affected by the market's forces.

Previous works have modelled the effect on incetives in oligopolistic markets when the moral hazard enters because of cost reducing production from agent's effort; see for instance Hermalin (1994), Scharfstein (1988) and recently Etro (2011). We propose a simple framework where production is delegated. Despite the interest on the power of incentive, we can show under which conditions the excessive entry theorem does not hold.

2 Model

We consider a population of n firms each owned by an employer (principal). The cost of entry is F and the governance of each firm is delegated to a manager (agent) who provides with effort a quantity q.[1] From the employment, principals obtain a benefit from the delegated production q and

[1] Throughout the paper, the female pronoun is for the principal (firm) and the male pronoun is for the agent (manager).

design an incentive wage (scheme) W to incentivize the worker. Parties are risk-neutral.

Provided production implies for the agent a cost $C(e;D) = \frac{e^2}{2D}$ with D the ability; agents are identical in terms of abilities.[2] The agent's production generates for a principal i a benefit measured by a function $S(q_i, Q)$; where q_i is the quantity produced by the agent working for principal i and $Q = \sum_{i=1}^{n} q_i$ is the total quantity in the market.

For each unit of effort, manager provide a unit of a quantity. Hence, production is at constant return to scale, but it is affected by a random term; i.e., we assume $q_i = e_i + \epsilon_i$. Random terms across managers are *i.i.d.* with $E[\epsilon_i] = 0$. Moreover, without loss of generality we focus on the case where $\sum_{i=1}^{n} \epsilon_i = 0$: the random cancels out in the aggregate. Observe that since the random term affects the quantity in a linear manner, the remaining assumptions are w.l.o.g. and come with the advatage of excluding constant terms in the expressions without affecting any of the following results. Formally, profit and utility are:

$$\pi = S(q_i, Q) - W_i - F \tag{1}$$

$$U(w_i, e_i, D) = W_i - \frac{e^2}{2D} \tag{2}$$

The surplus for the principal (revenue) comes from a linear (inverse) demand: $p(Q) = a - bQ$. Hence, since it gives a closed form solution in case of asymmetric information (see e.g. Vives (2001)), we have the often used qudratic functional form:

$$S(q_i, Q) = aq_i - bQq_i \tag{3}$$

We focus on a linear incentive scheme $W = w + Bq$ where w is a fix wage and B is a piece rate. Outside options are zero for both parties. Since agents are equal in terms of abilities and profits are stricly concave we solve without loss of generality for symmetric equilibria. Moreover, to avoid trivial results, we assume that workers are protected by *limited liability* on the payments $(w, B) \geqslant 0$. For the purpose of this paper, it would be enough to have $w \geqslant -l$ with $l \geqslant 0$ and assuming that l is not too

[2] Assuming difference in the abilities would not affect the results.

large such that manager can buy the firm; i.e., we assume that agents are financially constrained.

3 Contractible efforts

As benchmark, we study the perfect information case where efforts are contractible. Each principal competes à la Cournot in the product market and then takes as given the quantities produced by the others. She solves the following PI problem in designing an incentive scheme W:

$$\underset{\{w,B\}}{Max}\, E[\pi \mid e] = E[S(q,Q) - w - Bq - F \mid e]$$

$$E[U(w,e,D) \mid e] \geqslant 0 \tag{PC}$$

$$w, B \geqslant 0 \tag{NNC}$$

Where the first contraint guarantees that the agent finds profitable to accept the contract. The second set of constraints limits contracts on non-negative payments.

Lemma 1. *If effort is contractible, in the oligopoly market principals' contracts extract all the rent. The incentive scheme consists of a zero piece rate and a wage which compensates the agent for the effort.*

Proof of Lemma 1. To maximize profit, principals set payments as small as possible. A piece rate equal to zero and a wage which compensates the agent for the effort satisfies all the constraints and leaves the agent with his outside option. ■

As corollary of lemma 1, principals solve the next relaxed problem:

$$\underset{\{q\}}{Max}\, E[\pi \mid e] = E[S(q,Q) - \frac{q^2}{2D} - F \mid e] \tag{RPI}$$

Which amounts to be a standard Cournot model with a quadratic cost function, but uncertainty.

Proposition 1. *If efforts are contractible, the excessive entry theorem* *holds. The average quantity of each firm and the aggregate quantity are:*

$$E[q^*] = \frac{aD}{bD(n+1)+1} \tag{4}$$

$$Q^* = \frac{aDn}{bD(n+1)+1} \tag{5}$$

Proof of Proposition 1. The entry decision is ex-ante and therefore principals consider expected profit with $E[\epsilon_i] = 0$. Since effort is contractible, the maximization of each principal consists in choosing effort instead of quantity in (RPI). By assumption, random terms are incorrelated and in the aggregate quantity the sum cancels out. Then, it is straightforward to see that 4 is the symmetric (Bayesian) Nash Equilibrium. The rest of the proof is standard. The social welfare is the sum of expected profits and consumer surplus. Hence:

$$W_{pi} = nE[\pi_{pi}] + CS_{pi} \tag{6}$$

We denote with n^*_{pi} the number of firms which maximizes the social welfare: $n^*_{pi} = \underset{n}{argmax}\ W_{pi}$. From expected profit equal to zero we have the total number of operating firms (say it n^e_{pi}). The f.o.c. to find the optimum number of firms is:

$$\frac{\partial W_{pi}}{\partial n} = 0 \Leftrightarrow E[\pi_{pi}] + n\frac{\partial E[\pi_{pi}]}{\partial n} + \frac{\partial CS_{pi}}{\partial n} = 0$$

Simplifying:

$$\frac{\partial W_{pi}}{\partial n} = 0 \Leftrightarrow E[\pi_{pi}] - \frac{a^2b^2D^3n}{(bD(n+1)+1)^3} = 0$$

and therefore $n^e_{pi} > n^*_{pi}$. ∎

4 The Moral hazard problem

We assume that agents' ability is observable, but neither the effort nor the realization of the random variable are observable and therefore a moral hazard problem arises. Payments can only be conditioned on observable quantities. The timing of our game is as follows:

1. Each firm decides to enter or not;
2. If a firm enters, she posts a contract $\langle w, B \rangle$;
3. An agent accepts or rejects the contract; if he rejects, then both parties obtain zero as outside option;
4. If an agent accepts, he exerts effort;
5. Firms compete à la Cournot in the product market;
6. Payments are made and profits realized.

Formally, each principal takes as given quantities produced by the others and maximizes the following MH problem:

$$\underset{\{w,B\}}{Max}\, E[\pi \mid e] = E[aq - bQq - w - Bq - F \mid e]$$

$$\hat{e} = \underset{e}{argmax}\, E[U(w, e, D) \mid e] \tag{IC}$$

$$E[w + Bq - \frac{e^2}{2D} \mid e] \geqslant 0 \tag{PC}$$

$$w, B \geqslant 0 \tag{NNC}$$

Where the first contraint ensures that the contract is incentive-compatible according to which the manager chooses his effort as to maximize his own expected utility for any given incentive. The second constrait guarantees that the agent finds profitable to accept the contract. The third set of constraints limits contracts on non-negative payments.

Equilibrium. Observe that $\hat{e} = BD$ and w must be equal to zero. We disregard the (PC) and (NNC) on B constraints and we check them at the optimum. Therefore, the reduced problem is:

$$\underset{B}{Max}\, E[\pi_i] = aBD - b\sum_{j \neq i} E[q_i]BD - bB^2D^2 - B^2D - F$$

The F.O.C. is:

$$aD - b\sum_{j \neq i} E[q_i]D - 2bBD^2 - 2BD \overset{!}{=} 0 \tag{7}$$

Since firms are symmetric it holds $\sum_{j\neq i} E[q_i] = (n-1)BD$ and therefore solving the F.O.C. for B:

$$B^m = \frac{a}{bD(n+1)+2} \tag{8}$$

Hence, at the optimum (NNC) are satisfied and (PC) is slack implying that agent obtains a rent equal to $\frac{(B^m)^2 D}{2}$. To conclude:

$$E[q^m] = \frac{aD}{bD(n+1)+2} \tag{9}$$

$$Q^m = \frac{aDn}{bD(n+1)+2} \tag{10}$$

We summurize our results in the following:

Lemma 2. *If effort is not contractible and agents are protected by limited liability, in the oligopoly market agents always obtain an informational rent. The incentive scheme consists of a wage as small as possible and a piece rate as in 8.*

The result of Lemma 2 about the informational rent are essentially in line with the literature about incentives where the limited liability implies a rent. However, we augment them studying the effect on the power of incentives when the principal is not a monopolist. Useful for subsequent results is the number of firms for which expected profits are equal to zero:

$$n^e = -1 - \frac{2}{D} + \frac{a}{b}\sqrt{\frac{1+bD}{DF}} \tag{11}$$

Hence, we define $\Upsilon \equiv \{n \in \mathbb{R} : 2 \leqslant n \leqslant n^e\}$ as the feasible interval over which we can study the behavior of fundamentals in a market equilibrium and for simplicity we treat the number of firms as a continuous variable. Next, we define $\phi(a, b, D, F) \equiv F - \frac{a^2 D(1+bD)}{4(2+bD)^2}$ wich involves the demand parameters, the worker's ability and the entry cost.

Corollary 1. *Whereas the individual rent is decreasing in the number of firms, the total rent in the market:*

a) for $\phi(a, b, D, F) < 0$ is an inverted U-shaped function of n in Υ;

b) for $\phi(a,b,D,F) \geqslant 0$ is an increasing function of n in Υ.

Proof of Corollary 1. The individual rent is $\frac{(B^m)^2 D}{2}$ and then

$$rent_i = \frac{(B^m)^2 D}{2} = \frac{a^2 D}{2(D(n+1)+2)^2} \tag{12}$$

Hence, it is decreasing in n. For the total rent $n \cdot rent_i$, even if it changes concavity in Υ the only global maximum is for $\overline{n} = 1 + \frac{2}{bD}$. Hence, if $\overline{n} > n^e$ it is always increasing in Υ. To conclude, observe that solving for F the equality $\overline{n} = n^e$ we have $F = \frac{a^2 D(1+bD)}{4(2+bD)^2}$ which implies $\phi(a,b,D,F) = 0$. ∎

Corollary 1 then shows first that an increase of operating firms in an oligopolistic market aggravates the (individual) problem of loss in efficiency aimed to decrease the paid rent (because $e^m = B^m D$). Second, from a total welfare point of view an initial increase in the number of firms is not necessarily detrimental. This means that the initial marginal reduction of the individual rent is compensated by the marginal increase for an additional employee.

4.1 Comparison: on the effect of the rent-efficiency trade-off in an oligopolistic market

Before claiming our main result, we observe that moral hazard implies a loss for the society as whole.

Lemma 3. *For n and other things being equal, it holds i) $Q_{pi} > Q_m$; ii) $q_{pi} > q_m$; iii) $welfare_{pi} > welfare_m$.*

However, we can show that the excessive entry theorem does not always hold. As for this, we define $\psi(a,b,D,F) \equiv F - \frac{a^2 D(1+2bD)^2}{4(1+bD)(2+bD)^2}$ wich involves the demand parameters, the worker's ability and the entry cost.

Proposition 2. *In an oligopolistic market with moral hazard as in this model, if the agent is protected by limited liability and $\psi(a,b,D,F) \geqslant 0$ then the excessive entry theorem does not hold.*

Proof of Preposition 2. The welfare is $W_m = nE[\pi_m] + n \cdot rent + CS_m$. Differentiating w.r.t. n:

$$\frac{\partial W_m}{\partial n} = E[\pi_m] + \underbrace{\left[n\frac{\partial E[\pi_m]}{\partial n} + rent + \frac{\partial rent}{\partial n} + \frac{\partial CS_m}{\partial n} \right]}_{\Omega(a,b,D,F;n)} \tag{13}$$

In $n = n^e$ expected profit is zero. Hence, we study the sign of the term in square brackets evaluated in n^e and we show that it is non-negative *iff* $\psi\left(a, b, D, F\right) \geqslant 0$. Simplifying it:

$$\Omega\left(\cdot; n = n^e\right) = F \left[\frac{(2 + bD)F}{a\sqrt{D(1 + bD)F}} - \frac{1 + 2bD}{2(1 + bD)} \right] \tag{14}$$

Solving $\Omega\left(\cdot; n = n^e\right) = 0$ for $F > 0$:

$$F = \frac{a^2 D(1 + 2bD)^2}{4(1 + bD)(2 + bD)^2} \Leftrightarrow \psi\left(a, b, D, F\right) = 0 \tag{15}$$

Therefore, $\Omega\left(\cdot; n = n^e\right) \gtreqless 0 \Leftrightarrow \psi\left(a, b, D, F\right) \gtreqless 0$ ∎

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