The Impact of Fuel Taxation and Vehicle Production Subsidies in A Dynamic Two-Period Vintage Model*

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Abstract

We develop a dynamic general equilibrium two-period vintage model to study the endogenous determination of fuel efficiency and households' driving choices, under the policies of fuel taxes and fuel-efficiency production subsidies. The model also incorporates pollution and environmental quality to examine the policies' impact on social welfare. We analytically find that the ratio of gasoline consumption between new cars and old cars is greater than one which implies that new cars consume more gasoline. Households proportionally drive new cars more often than old cars. Numerical analyses show that the negatively sloped part on the tax revenue/tax rate relationship (Laffer Curve) doesn't exist if the tax revenue is fully used to subsidize the production of fuel efficiency. The increasing tax revenue improves the fuel economy status and reduces overall gasoline consumption. In the long run, the policy option also reduces pollution, enhances environmental quality and thus improves social welfare.

Keywords: Fuel Efficiency, Fuel Tax, Vehicle Production Subsidy, Dynamic General Equilibrium Model, Environmental Quality, Pollution

JEL Classification: D91, E20, H23, Q58, R48

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1 Introduction

Vehicles are the main culprit of urban air pollution, especially in developing countries (Small and Kazimi, 1995). The average emission levels of new domestic vehicles are 3-10 times higher in developing countries than that in developed countries due to lagging automotive manufacturing technology, poor fuel quality, poor vehicle exhaust control, and lenient laws limiting vehicle emissions (He et al., 2002). There has been heated discussion on addressing the pollution externality caused by vehicle driving. The most frequently discussed policy instruments are fuel tax and fuel efficiency standard.

Many developed countries have adopted fuel taxes. According to the data published by American Petroleum Institute, US average fuel tax is 49.44 cents per gallon for gasoline (2017).The corresponding figure for U.K. £2.19 per U.S. gallon according to HM Revenue Customs (2016).

Fuel efficiency standard has also been implemented in many countries. One example of fuel efficiency standard is Corporate Average Fuel Economy (CAFE) [1] which has been enacted in the United States in the wake of the 1973 oil crisis. Fuel efficiency standard targets at reducing gasoline consumption so as to reduce dependence on oil. This standard imposes a limit on the average fuel efficiency of the vehicles sold by a particular company in each year(Jacobsen, 2013).

While the empirical literature on estimating the impact of gasoline tax and fuel efficiency standard is vast[2], the theoretical literature is quite limited. Parry and Small

[1] The Corporate Average Fuel Economy (CAFE) achieved by a given fleet of vehicles in a given model year is the production-weighted harmonic mean fuel economy, expressed in miles per U.S. gallon (mpg), of a manufacturer's fleet of current model year passenger cars or light trucks with a gross vehicle weight rating of 8,500 pounds or less, produced for sale in the United States (EPA,2014). The standard for passenger vehicles in 2018 is 45.2 miles per gallon and 61.1 miles per gallon in 2025 (Administration et al., 2011).

[2] see Davis and Kilian (2011), Parry et al. (2007) and Austin and Dinan (2005)

(2005) sets up a structural static model to study the optimal gasoline tax solution where externalities caused by driving are included. They show that the current fuel tax rate is too low in the United States and too high in the United Kingdom. Wei (2003) overcomes the static limitation by constructing a dynamic general equilibrium model to analyse the transmission channels to reduce gasoline consumption under increasing gasoline tax and tightening fuel efficiency standards. With exogenous policy instruments, Wei (2003) shows that both policy instruments reduce gasoline consumption in the long run, but they are different in the transmission channel and may have different economic and environmental impact.

This paper aims to contribute to the literature with a new dynamic vintage model to endogenise the determination of fuel efficiency. To our knowledge, this is the first paper that employs a dynamic general equilibrium model to analyse the effect of gasoline taxes on fuel efficiency, environmental quality and welfare.

Our model is novel in several aspects. First, differently from Parry and Small (2005), we build a model where dynamic relationships are presented to capture the long-run nature of pollution and capital accumulation. Second, policy instruments are not imposed from outside the model, but instead are consistent derived from the government problem in a general equilibrium framework [3] which is different from Wei (2003). Also pollution and environmental quality dimension are incorporated in our model. A dynamic general equilibrium model is useful to interpret pollution issues as those are generally accumulate overtime and also affect environmental quality over time. Any policies addressing pollution issues will also have a long-run effects both on the environment and social welfare. Third, differently from Wei

[3] There has been heated discussion in the U.S. on whether fuel efficiency standard should be replaced by other policies. Lots of economists would like to replace CAFE standard with a gasoline tax, or at least pair them with policies that target congestion and accidents. Although policy makers have shown a consistent preference for fuel efficiency standard, Anderson and Sallee (2016) shows that this preference is costly and fuel tax can reduce pollution by the same amount at substantially lower social cost.

(2003) who adopts a vintage model with putty-clay technology, we introduce vintage vehicles using putty-putty technology. Solow et al. (1960) and Cooley et al. (1997) point out that the latest technology is only incorporated into the latest capital, while old capital still uses the technology from the time it was produced. Wei (2003) uses ”putty-clay” technology with Leontief production possibilities to model vehicle mileage of travel where the ratio of vehicle capital to energy consumption is fixed ex post production. Vehicle capital, however, is special in that it could generate mileage of travel given any amount of gasoline pumped in. Leontief production possibilities thus do not match with vehicle features. We therefore apply ”putty-clay” technology to model mileage of travel.

We summarize the results as follows. First, in the presence of fuel tax and government subsidy for fuel efficiency production, households purchase more gasoline for new cars than old cars. Households also prefer to use new cars more often than old cars. Second, under our benchmark calibration, the negatively sloped part on the tax revenue/tax rate relationship doesn't exist when tax revenue is fully used to subsidize the production of fuel efficiency. Raising gasoline tax rate also helps to improve fuel economy status and reduces gasoline consumption. It also enhances the environmental quality and thus improves social welfare.

The structure of the paper is as follows. Section 2 describes the model and its dynamics. Section 3 presents the steady-state analytical results. Section 4 presents the benchmark calibration. Section 5 measures the rebound effect under the model setting and section 6 examines the economic impact of increasing gasoline tax rates. Section 7 concludes.

2 The model

This section describes a decentralized economy where there are firms, households and government. 2.1 presents the production technology and the profit-maximizing problems for firms. Households' problems are discussed in 2.2 and we also explains how "putty-putty" technology is applied to model mileage of travel. We present government's budget and policy option in 2.3.

2.1 Firms

There are two sectors on the production side of the economy. Output from general production sector is used for consumption, capital accumulation and gasoline import. Vehicle production sector produces vehicle capital and fuel efficiency.

2.1.1 General production sector

At each period t, firms hire labour l_t^g and capital k_t^g at the rate of w_t^g and r_t^g from households to produce final output which can be used for consumption, capital accumulation and gasoline import. The profits generated π_t^g goes to households.

The final good, G, is produced with constant-return-to-scale technology:

$$G(k_t^g, l_t^g) = A_1 (k_t^g)^{\alpha_1} (l_t^g)^{1-\alpha_1} \tag{1}$$

and the resource constraint is:

$$G(k_t^g, l_t^g) = c_t + k_{t+1} - (1 - \epsilon_k) k_t + p_t (g_{t,1} + g_{t,2}) \tag{2}$$

where ϵ_k is capital depreciation rate and p_t is gasoline import price. k_t denotes the total capital at time period t.

Firms maximize profits π_t^g at each period with respect to the amount of capital k_t^g and labour l_t^g they hire.

$$\max_{k_t^a, l_t^a} = G(k_t^a, l_t^a) - w_t^a l_t^a - r_t^a k_t^a \tag{3}$$

The corresponding F.O.C are:

$$r_t^g = G_{k_t^g}, \qquad w_t^g = G_{l_t^g} \tag{4}$$

2.1.2 Vehicle production sector

Vehicle is composed of vehicle capital a and fuel efficiency δ. Vehicle capital can be considered as the metal parts of the car while fuel efficiency is the car engine. Vehicle capital and fuel efficiency are produced separately but must be sold as a combined product. Fuel efficiency is the crucial part in mitigating air pollution which will be further discussed.

Firms in this vehicle production sector hire labours l_t^a and capital k_t^a at the rate of w_t^a and r_t^a to produce vehicle capital a_t and fuel efficiency δ_t. Firms sell the final vehicle product $a_t\delta_t$ to households at the price q_t^a. Firms in this sector are also subsidized by government to produce more fuel-efficient vehicles (higher δ). The resource constraint reads:

$$F(k_t^a, l_t^a) = a_t + \mu\delta_t{}^{4}, \quad \mu > 0 \tag{5}$$

where μ measures the production preference for fuel efficiency.

The problem facing the firms in this sector is:

[4] The resource available for production in vehicle production sector 7 can be used to either produce more vehicle capital or more fuel efficiency. It shows the trade-off between quality and quantity. If firms decide to put more resource to produce vehicle capital a, then the fuel efficiency δ embedded in the vehicle will be low which means that vehicles are leff efficient in producing mileage of travel.

$$\max_{k_t^a, l_t^a, \delta_t} \pi_t^a = q_t^a(a_t\delta_t) - r_t^a k_t^a - w_t^a l_t^a + s_t\delta_t \tag{6}$$

and the production technology takes diminishing-return-to-scale form[5]:

$$F(k_t^a, l_t^a) = A_2 (k_t^a)^{\alpha_2} (l_t^a)^{\frac{1}{2}-\alpha_2} \tag{7}$$

The allocations of resources follow the governing rules:

$$r_t^a = q_t^a \delta_t F_{k_t^a}, \qquad w_t^a = q_t^a \delta_t F_{l_t^a}, \qquad q_t^a[F(k_t^a, l_t^a) - \mu\delta_t] - \mu q_t^a \delta_t + s_t = 0 \tag{8}$$

2.1.3 Equilibrium conditions in production

To clear the production side, we assume:

$$k_t^a + k_t^g = k_t \quad l_t^g + l_t^a = l_t \tag{9}$$

$$w_t^a = w_t^g = w_t \quad r_t^a = r_t^g = r_t \tag{10}$$

where l_t denotes the total labour at time period t.

[5]We specifically assume that the power adds up to $\frac{1}{2}$. When household purchase vehicle, they purchase the product of vehicle capital a and fuel efficiency δ,$a\delta$. If there is no externalities in the economy, the optimal solution will be zero tax rate on gasoline consumption and thus no subsidies on producing fuel efficiency. Equation 8 becomes $q_t^a(F - \mu\delta_t - \mu\delta_t)$. In order for the final production to be constant return to scale, we will have the profit for vehicle production sector $\pi_t^a = 0$. Thus, we will have $\frac{q_t^a}{\mu}(\frac{F}{2})^2 - r_t k_t^a - w_t l_t^a = 0$. Thus, we have to require $(\frac{F}{2})^2$ to be constant return to scale, and F has to be diminishing return to scale and the power has to sum up to $\frac{1}{2}$

2.2 Households

2.2.1 Preference

Representative agent lives infinitely in the economy. We assume log-preferences for consumption c_t, driving service M_t, leisure $1-l_t$ and environmental quality N_t.

$$U(c_t, M_t, 1-l_t, N_t) = \phi_1 \log c_t + \phi_2 \log M_t + (1-\phi_1-\phi_2)\log(1-l_t) + \phi_3 \log N_t \quad (11)$$

2.2.2 Production of driving services

There are two types of vehicles in the market: new cars and old cars. We follow Solow et al. (1960) and Cooley et al. (1997) to model vintage capital. The *exante* production technology of vehicle is assumed to be diminishing-return-to-scale (Eq.7). After production, the technology embedded in the vehicle will not change, which implies that the mileage of travel over one unit of fuel consumed is fixed for different vehicle vintages. Vehicles need one period of configuration and will be used by households for two periods before getting scraped. New cars are produced at time period $t-1$. Old cars are produced at time period $t-2$ and are also subject to depreciation $1-\rho$ from already being used for a time period. Miles of travel is linear in gasoline consumption. The mileage production technology is exogenous and is embedded in parameter γ.

Mileage of travel produced by new cars $m_{t,1}$ and old cars $m_{t,2}$ are:

$$m_{t,1} = (a_{t-1}\delta_{t-1})^{\gamma} g_{t,1} \quad (12)$$

$$m_{t,2} = (\rho a_{t-2}\delta_{t-2})^{\gamma} g_{t,2}, \qquad 0 < \rho < 1 \quad (13)$$

where $0 < \gamma < 1$.

Representative household owns both new cars and old cars. Driving service M_t at each period is composed of mileage of travel produced by new cars $m_{t,1}$ and old cars $m_{t,2}$:

$$M_t = (m_{t,1}^{\sigma} + m_{t,2}^{\sigma})^{\frac{1}{\sigma}}, \qquad 0 < \sigma < 1 \tag{14}$$

we set the preference for M_t based on Grossman and Helpman (1991) to guarantee that households exhibits preference for variety other quantity, which means that households always prefer to use both types of cars instead of just using new cars.

2.2.3 Environmental quality

Economic activities happen within environment, which is modelled as a type of renewable resource. The quality of the environment, N represents the stock of natural capital and accumulates based on the regenerating ability of nature while depreciates due to pollution P. N evolves over time according to the following function based on Bovenberg and Smulders (1995):

$$N_{t+1} - N_t = E(N_t) - P_t \tag{15}$$

where $E(N_t)$ represents the nature's absorption ability or ecological services produced by nature, that is the amount of pollution that can be assimilated without a change in environmental quality. We could also view Eq.15 as that environmental quality change and pollution are two rival users of ecological services. Nature's absorption ability $E(N_t)$ takes the function form:

$$E(N_t) = \phi - \epsilon N_t, \qquad 0 < \epsilon < 1$$

where ϕ denotes the original state and ϵ represents the nature's rate of absorbing pollutants.

The steady state of environmental quality reads:

$$\epsilon N_{ss} = \phi - P$$

and it means that for each level of pollution P, there exists a stable level of environmental quality which the absorbing capacity of nature could compensate the adverse effect of pollution so as to keep environmental quality unchanged over time. If the pollution level is too high, the equilibrium level of environmental quality will be lower. This corresponds to the description of steady-state environmental quality done by Smulders (2000).

Thus, substitute this back to Eq.15, we obtain the adjustment of environmental quality:

$$N_{t+1} - N_t = \epsilon(N_{ss} - N_t)$$

at each period, the pollution level decides the optimal level of environmental quality and the adjustment is made to keep the environmental quality unchanged over time.

2.2.4 Pollution

Our specification of pollution is based on Selden and Song (1995): pollution P_t is caused by the usage of gasoline but mitigated by vehicles' fuel-efficiency conditions, with $\partial P/\partial g > 0$, $\partial P/\partial \delta < 0$, $\partial^2 P/\partial g^2 = 0$ and $\partial^2 P/\partial \delta^2 > 0$.

$$P_t = \frac{g_{t,1}}{\delta_{t-1}^{\mu_1}} + \frac{g_{t,2}}{\delta_{t-2}^{\mu_2}}, \quad \mu_1 \geqslant \mu_2 \tag{16}$$

where parameters μ_1 and μ_2 measure the ability of mitigating pollution by different types of cars.

2.2.5 Household's problem

Each time period, the representative household supplies labour l_t and capital k_t to firms and receive all the profits generated in both sectors (π_t^g and π_t^a). Households purchase consumption goods, gasoline, new vehicles and make investment.

Household maximizes:

$$\max_{c_t, g_{t,1}, g_{t,2}, a_t \delta_t, l_t} \sum_{s=0}^{\infty} U(c_t, M_t, 1 - l_t, N_t) \tag{17}$$

under the restriction:

$$\pi_t^a + \pi_t^g + w_t l_t + r_t k_t = (p_t + \tau_t)(g_{t,1} + g_{t,2}) + k_{t+1} - (1 - \epsilon_k) k_t + c_t + q_t^a (a_t \delta_t) \tag{18}$$

taking environmental quality N_t as given. τ_t is the unit tax levied by government on the consumption of gasoline. q_t^a denotes the vehicle price.

The optimality conditions are derived in Appendix A, which will be discussed in later sections.

2.3 Government

Government levies tax τ_t on household's purchase of gasoline and uses the tax revenue to subsidize the production of more fuel-efficient vehicles in the vehicle production sector. As standard in public economics literature, we assume that the tax rate does not go over 100% [6] :

$$s_t \delta_t = \tau_t (g_{t,1} + g_{t,2}) \tag{19}$$

[6] we also experiment when tax rate goes beyond 100%, the result does not change.

3 Equilibrium

In this section, we characterize the steady-state solution of the model and derive long-run gasoline consumption and households' driving decisions in response to gasoline tax and constant fuel-efficiency improvement.

Proposition 1. *In the presence of fuel tax and government subsidy, the long-run ratio of gasoline consumption between new cars and old cars is given by:*

$$\frac{g_{1,ss}}{g_{2,ss}} = \rho^{\frac{\gamma\sigma}{\sigma-1}} \tag{20}$$

Here the subscript ss represents the steady state.

Proposition 1 characterizes the gasoline consumption ratio between new cars and old cars. The steady-state gasoline consumption ratio does not depend on the policy. It only depends on the depreciation rate of vehicle ρ, mileage production technology γ and driving service preference σ. In the long run, new cars in total will consume more gasoline compared to old cars even though new cars are more fuel-efficient.

Proof. The proof is contained in B □

Proposition 2. *Using the previous result, we can also obtain the mileage-of-travel ratio between new cars and old cars:*

$$\frac{m_{1,ss}}{m_{2,ss}} = \rho^{\frac{\gamma}{\sigma-1}} \tag{21}$$

Proof. The proof is contained in C □

Proposition 2 characterizes the equilibrium solution to the mileage ratio among two types of vehicles. In total, households prefer to use new cars more often than old cars given that new cars are more efficient in providing driving services.

In the next two section, we calibrate the model and use the analytical closed-form solutions to characterize the dynamic paths of the key endogenous variables responding to the increasing tax rates.

4 Calibration

This section describes the benchmark calibration for parameters. The values of parameter are based on the comprehensive reviews of relevant literature, like Wei (2003), Parry and Small (2005) and Chen et al. (2006). There are four categories of parameters: the first relates to driving service and gasoline usage. The second is about production technology and the third specifies the preference of household. The forth category is about environment quality and pollution. The details of calibration can be found in Appendix D.

Table 1: Benchmark Calibration

Category	**Parameters Description**	**Notation**	**Value**
Driving Service	Vehicle leftover rate	ρ	0.9
	Vehicle preference	σ	0.5
	Mileage production technology	γ	0.42
Production Technology	Capital depreciation rate	ϵ_k	0.1
	Capital share in production	α_1,α_2	0.33/0.42
	Productivity level	A_1, A_2	1
	Marginal Transformation rate	μ	1
	Capital rental price	r_t	0.1309
	Wage rate	w_t	1.0565
	Gasoline price	p_t	1.0872
Household Preference	Subjective discount rate	β	0.97
	Weight on consumption	ϕ_1	0.34
	Weight on driving	ϕ_2	0.05
	Weight on environmental quality	ϕ_3	1
Environmental Factor	The capacity of fuel efficiency	μ_1,μ_2	1
	Natural purifying capacity	ϵ	0.1

5 Rebound Effect

In this section, we use the calibrated model to investigate the effects of increasing gasoline prices and improving fuel economy of cars on miles of travel. Lots of research has been done on the economic reaction released by improving energy efficiency. As vehicles are more fuel efficient, less energy is needed for producing the same amount of mileage of travel. However, as it is ”cheaper” to travel the same amount of mileage, household tends to consume more driving services and the extra demand for driving services implies more fuel consumption. And the percentage of the energy conservation is called the rebound effect.

We base our measurement of rebound effect on Greene (1992). The key determinant of rebound effect is the elasticity of mileage travel to fuel cost per mile. We start from the elasticity of gasoline consumption with respect to a change in fuel efficiency $\eta(g)_e$. Given that in steady state, the mileages of new cars and old cars are of a constant ratio, we focus on the mileage of new cars only. Rearrange Eq.12, we get:

$$g_{1,ss} = \frac{m_{1,ss}}{e}$$

where e denotes the fuel efficiency of new cars: $e = (a\delta)^{\gamma}$. Thus, the elasticity of gasoline consumption to fuel efficiency $\eta(g)_e$ then reads:

$$\eta(g)_e = \frac{\partial m_{1,ss}}{\partial e}\frac{e}{m_{1,ss}} - 1 \tag{22}$$

We now introduce the realized cost per mile, defined as $cost = \frac{(p_t+\tau_t)}{e}$. Substitute this back to Eq.22, we get[7]:

[7] $\eta(g)_e$ measures the percentage change to gasoline consumption when fuel efficiency changes. Thus:

$$\eta(g)_e = \frac{\frac{\partial g_1}{g_1}}{\frac{\partial e}{e}} = \frac{\partial(\frac{m_1}{e})}{\partial e} = \frac{\partial m_1}{\partial e}\frac{e}{m} - 1$$

$$\eta(g)_e = -\eta(m)_c - 1$$

where $\eta(g)_c$ measures the elasticity of vehicle use to realized cost per mile.

Thus the elasticity of gasoline consumption to fuel efficiency equals to negative elasticity of vehicle mileage travel to fuel cost per mile minus 1. Using the calibrated model, we estimate $\eta(m)_c$ to -0.03. Thus the rebound effect is 3%, which corresponds to various empirical results. According to Greene (1992) and Berkhout et al. (2000), the magnitude of rebound effects has been quite small, between 5% to 15%, or less. This means that the rebound effect is negligible when considering the effect of improving fuel efficiency on energy consumption. Therefore, the policy works well if the government wants to control gasoline consumption whilst improving fuel efficiency.

6 Comparative statics of tax policy

In this section, we use the calibrated model to examine the efficacy of the policy tool to address pollution issues. The policy option is that government levies tax on gasoline consumption and uses the tax revenue to subsidize firms to produce more environmentally friendly vehicles. We first examine the tax revenue/tax rate relation to see whether the Laffer Curve exists or not. We then plot the long-run dynamic paths of endogenous variables responding to the increasing tax rates. Finally we examine whether the policy is efficient in coping with pollution problems.

Rearrange the expression and use some calculus:

$$\eta(g)_e = \frac{\partial m_1}{\partial cost}\frac{\partial cost}{\partial e}\frac{e}{m_1} - 1 = -\frac{\partial m_1}{\partial cost}(p_t + \tau_t)e^{-2}\frac{e}{m_1} - 1 = -\frac{m_1}{\partial cost}\frac{c}{m_1} - 1 = -\eta(g)_c - 1$$

6.1 Subsidy versus tax rates

One implication of Laffer Curve is that keeping increasing tax rates beyond a certain point will be counter-productive in raising tax revenue. Gahvari (1989) pointed out that if government expenditure takes the form of providing a good then the negatively-sloped part on the tax revenue/tax rate relationship will not exist. In our model, government does not directly transfer the revenue to households, instead, government subsidizes the production of fuel efficiency δ.

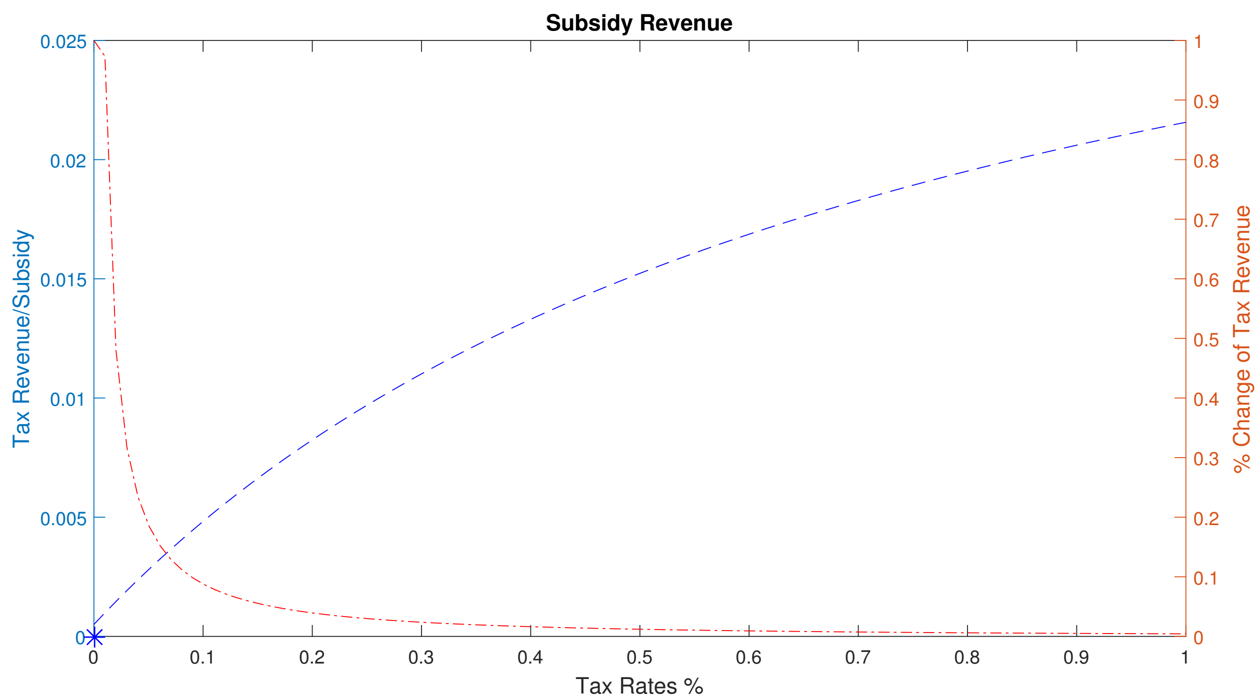

Figure 1: The change of subsidy over the increasing fuel tax and the percentage change of subsidy over the change of tax rate.

As shown in Figure 1, the subsidy keeps increasing as tax rates increases from zero percent to one-hundred percent. However, the subsidy is not strongly responsive to the increasing tax rates. The increasing rate of tax revenue plunges sharply as the tax rate increases from 0% to 10%.

6.2 The impact of increasing tax rate on economy

In this section, we plot the equilibrium paths of key endogenous variables responding to increasing gasoline prices.

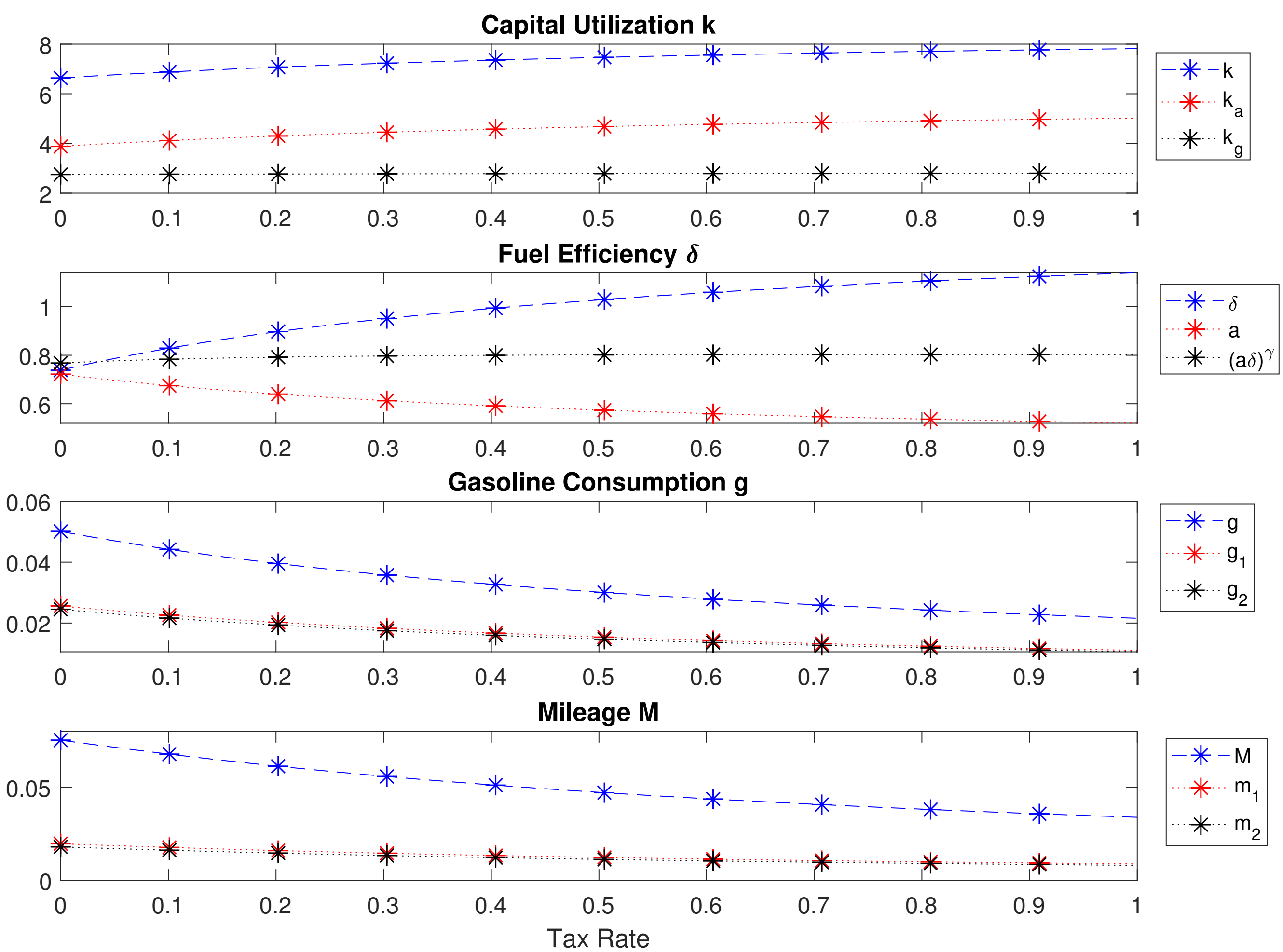

Figure 2: The effect of fuel tax rate increasing from zero to one-hundred percent on key endogenous variables.

As we can see from Figure 2, in the long run, the increasing fuel tax rate leads to the positive accumulation of capital. As illustrated in our model, vehicles are produced by vehicle capital k_a and labour l_a. As tax rate increases, the subsidy government provides to vehicle production firms increases which provides incentives for firms to expand production of more environmentally friendly vehicles. We therefore observe the accumulation of vehicle capital k_a. It is also shown in the figure that the general production capital k_g also slightly increases over the increasing gasoline price. By levying tax on gasoline, the increasing price makes driving service more expensive to obtain. Income effect makes households switch their demand towards consumption goods and leisure. However, based on previous researches like Brons et al. (2008), the demand for gasoline is not very price sensitive. This explains why the general production capital k_g increases with a relatively small magnitude. This also corresponds to the empirical results by Brons et al. (2008) that the impact of change in gasoline price is mainly driven by response in fuel efficiency.

As we mentioned above, the increasing tax rates have an impact on the vehicle production sector. As vehicle producing firms are directly subsidized, more resources are allocated to improve fuel efficiency δ. Thus proportionally, fuel efficiency δ takes heavier weight in the vehicle production sector. That explains the increment of δ and contraction of a. However, the productivity of the engine $(a\delta)^\gamma$ keeps rising, which shows that fuel efficiency δ is the driving force to improve the productivity of vehicles.

The direct impact of fuel tax is on gasoline consumption. Increasing tax rate makes it more expensive to purchase fuel so that gasoline consumption keeps decreasing. Eq.20 characterizes the constant gasoline consumption ratio between new cars and old cars. This is also observed in Fig.2. New cars are more fuel-efficient, thus g_1 is still higher than g_2 even though both of them are on a downward path.

We are also interested to see how household makes driving decisions based on two contradicting forces: 1) the improved fuel efficiency δ means that households can achieve more mileage using same amount of gasoline, 2) the increasing tax rates τ_t makes it more expensive to purchase fuel. Figure 2 shows that, the impact of tax on gasoline consumption overpowers the positive effect on improved fuel efficiency. Households eventually choose to drive less as gasoline tax rate increases.

6.3 The impact of increasing tax rate on environment and welfare

One major goals of levying tax and subsidizing the environmentally-friendly vehicles production is to address the pollution issues caused by vehicle driving so as to improve social welfare. In this section, we use the calibrated model to examine the impact of gasoline tax on environmental quality and welfare.

6.3.1 Environment

We start by looking at the efficacy of tax rate on pollution. Pollution is caused by vehicle driving and mitigated by fuel efficiency (Eq.16). In the long run, the increasing tax rate leads to decreasing gasoline consumption. Moreover, the tax revenue is used to produce more fuel-efficient vehicles which helps further in reducing pollution. We thus conduct sensitivity analysis to test the robustness.

Fig.3 presents two scenarios: when new cars and old cars are equally efficient in reducing pollution (μ_1=μ_2) and when new cars are more efficient in doing so ($\mu_1 > \mu_2$). As shown in the figure, the policy option of increasing tax and using the revenue to improve fuel efficiency works significantly in reducing pollution. The solid lines measures the pollution when new cars and old cars are equally efficient in reducing pollution while the dashed lines shows the pollution status when new cars are more efficient in reducing pollution. The benchmark calibration assumes that two types of vehicles are equally efficient in reducing pollution ($\mu = 1$). We also test the robustness by assigning different parameter values, which are in the vicinity of the benchmark. If new cars are more efficient in reducing pollution, the environment will be improved faster. We thus draw the conclusion that the policy works robustly in reducing pollution.

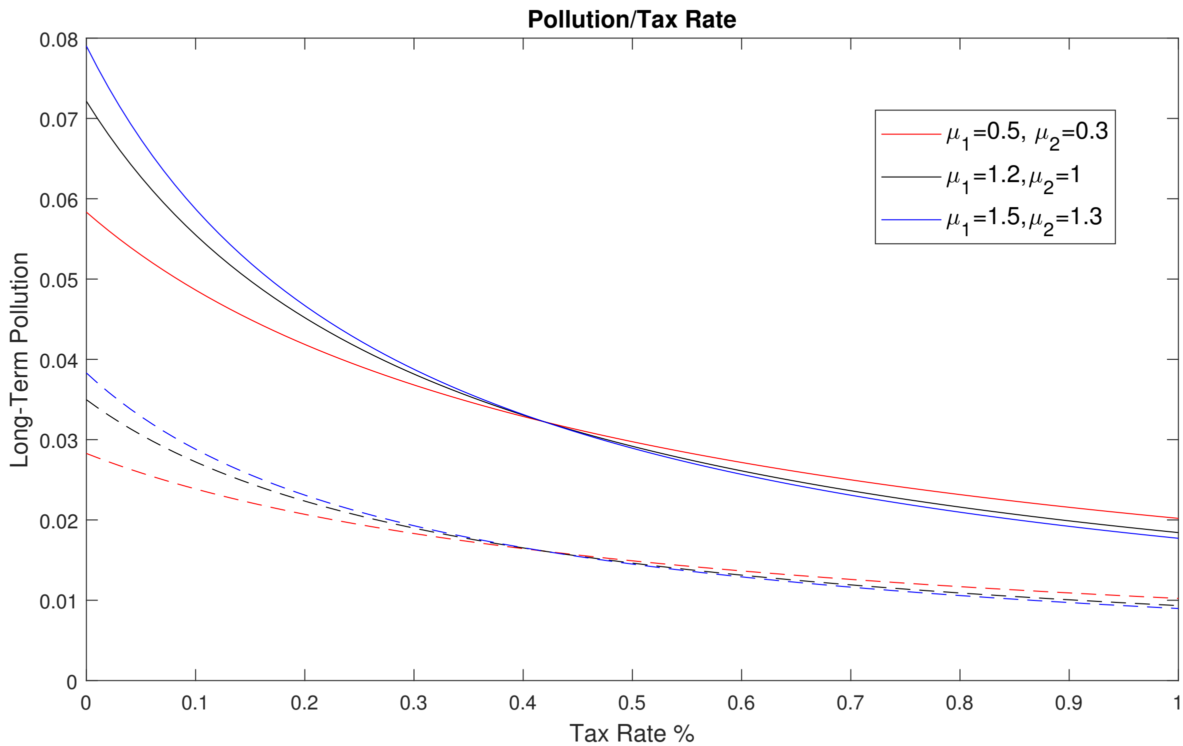

Figure 3: How efficient new cars and old cars are in mitigating pollution

We are also intrigued to see the long-run effect of the policy on environmental quality. Environmental quality N is a stock value and its change depends on two opposing factors: nature's assimilating capacity and pollution, as shown in Eq.15.

As shown in Fig.4, environmental quality improves with the increasing gasoline tax at a decreasing rate. In the long run, increasing tax rate leads to decreasing pollution. A lower level of pollution means that fewer ecological services are needed

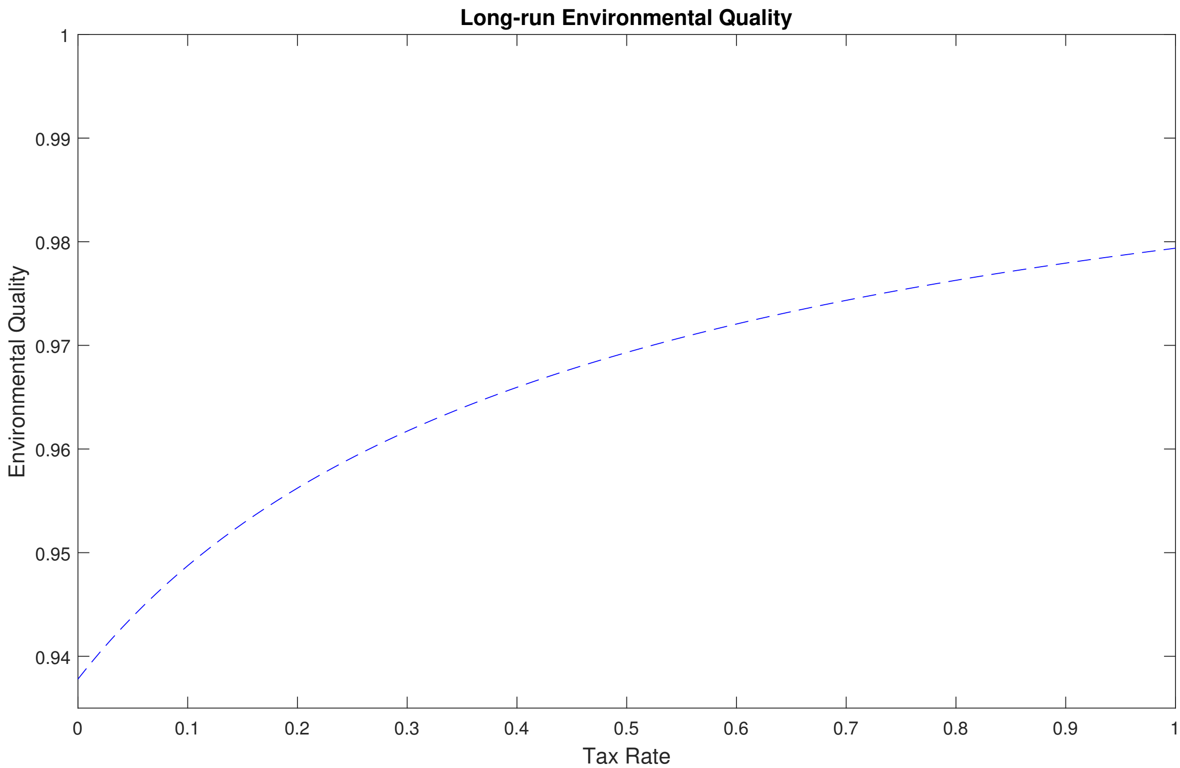

Figure 4: The impact of gasoline tax on environmental quality

to compensate on the adverse effect pollution has on environmental quality, which results in a higher level environmental quality condition. The concavity also reveals that environmental quality will not explode as the ecological services provided in nature is limited (Smulders, 2000).

6.3.2 Welfare

Households obtains utility from driving but also suffers from the pollution caused by vehicle driving. How much households value environmental quality depends on lots of factors. We are intrigued to see whether the policy works on improving social welfare given different willingness to pay for better environmental quality. According to Jackson (1983), place of residence affects household's preference for environmental quality: centre city has the benchmark preference value 1, suburb suffers more from pollution thus household living there put more weight on environmental quality, $\phi_3 = 1.4$. Rural areas has the least willingness to pay for better environmental quality, with $\phi_3 = 0.56$.

Fig.5 shows the impact of increasing gasoline tax rate on welfare. Three cases where people have different preference for environmental quality are shown in the figure. Overall, by increasing gasoline tax, the pollution level will be lower and the long run environmental quality will be improved. Households having higher preference for environmental quality will get higher welfare level from the improved environment. Households based in suburb area where pollution status is not severe get the least welfare improvement.

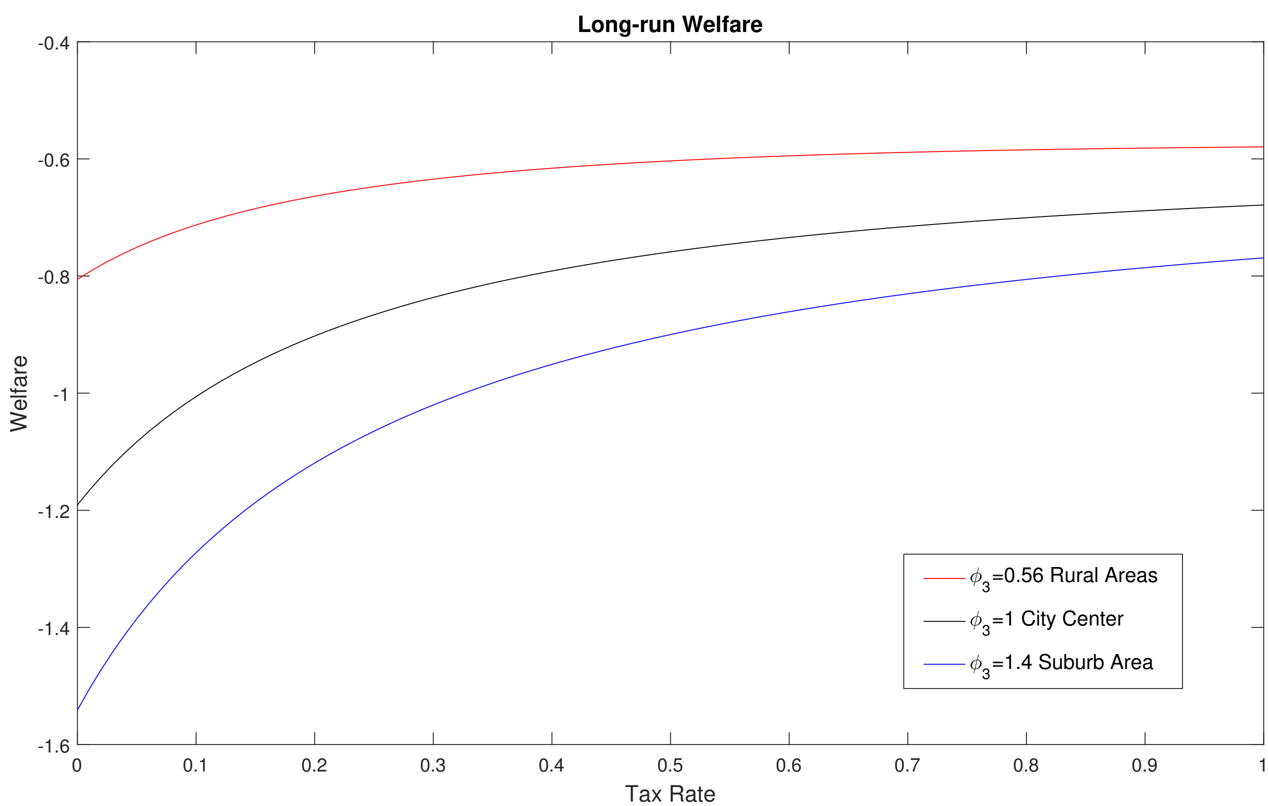

Figure 5: The impact of gasoline tax on welfare

7 Conclusion

This paper develops a dynamic general equilibrium model embedding vehicle heterogeneity using ”putty-putty” technology in modelling driving service production. Our aim is to analyse the endogenous determination of gasoline consumption, fuel efficiency and households’ driving choices among different vintage vehicles. Our analyses demonstrates that at the steady state, households prefer to use new cars more often than old cars and gasoline consumed by new cars is proportionally higher than gasoline consumed by old cars.

This paper also conducts numerical analysis on the impact of levying taxes on gasoline coupled with subsidies on the production of more fuel-efficient vehicles. By increasing fuel tax rate, governments can be assured to achieve gasoline consumption reduction and pollution control. Environmental quality and social welfare in the long run will also be improved without landing on the negative-sloped part of the Laffer Curve.

A Appendix: Optimality conditions derivation for household's problem

Household tries to maximize its discounted life time utility, as shown in Eq.17 subject to its budget constraint 18.

Thus, the Lagrangian reads:

$$\begin{aligned}\mathcal{L} = \sum_{s=0}^{\infty} \beta^{t+s} [U(c_t, M_t, 1 - l_t, Q_t) + \lambda_{t+s}(\pi_t^a + \pi_t^g + w_t l_t + r_t k_t - (p_t + \tau_t)(g_{t,1} + g_{t,2}) \\ -k_{t+1} + (1 - \epsilon_k)k_t - c_t - q_t^a(a_t \delta_t))]\end{aligned} \tag{A.1}$$

yielding the first-order conditions:

$$\frac{U_{c_t}}{U_{c_{t+1}}} = \beta(1 - \epsilon_k + r_{t+1}) \tag{A.2}$$

$$\frac{\partial U_t}{\partial M_t} \frac{\partial M_t}{\partial m_{t,i}} \frac{\partial m_{t,i}}{\partial g_{t,i}} = U_{c_t}(p_t + \tau_t) \qquad i = 1, 2 \tag{A.3}$$

$$\beta \left(\frac{\partial U_{t+1}}{\partial M_{t+1}} \frac{\partial M_{t+1}}{\partial m_{t+1,1}} \frac{\partial m_{t+1,1}}{\partial a_t \delta_t} \right) + \beta^2 \left(\frac{\partial U_{t+2}}{\partial M_{t+2}} \frac{\partial M_{t+2}}{\partial m_{t+2,2}} \frac{\partial m_{t+2,2}}{\partial a_t \delta_t} \right) = q_t^a U_{c_t} \tag{A.4}$$

$$U_{1-l_t} = w_t U_{c_t} \tag{A.5}$$

B Appendix: Proof for proposition 1

In steady state, Eq.A.3 becomes:

$$\frac{\partial U}{\partial M} M_{ss}^{1-\sigma} \frac{m_{2,ss}^{\sigma}}{g_{2,ss}} = \frac{\partial U}{\partial M} M_{ss}^{1-\sigma} \frac{m_{1,ss}^{\sigma}}{g_{1,ss}} \tag{B.1}$$

Eq.B.1 states that the in steady state, the marginal utility of gasoline consumption from two types of vehicles is the same. Simplify Eq. B.1 we can obtain Eq.20

C Appendix: Proof for proposition 2

In steady state, Eq.12 and 13 become:

$$m_{1,ss} = (a_{ss}\delta_{ss})g_{1,ss}, \quad m_{2,ss} = (\rho a_{ss}\delta_{ss})g_{2,ss} \tag{C.1}$$

Thus, using Eq.20 we could get the mileage ratio between two types of vehicles in equilibrium.

D Appendix: Calibration

D.1 Parameters related to driving service

The driving service is provided by two types of cars: old cars and new cars. ρ measures vehicle wear-out which will affect the fuel efficiency condition in the next period. We set ρ to be 0.9. σ measures preference over different types cars and we set it to 0.5. According to Chen et al. (2006), consumption output ratio is around 0.65 and Ferdous et al. (2010) states that personal cars spending over household expenditure ratio is 6%. According to the steady state equilibrium of the economy, equation 20 states the gasoline consumption ratio between new cars and old cars. According to U.S. Energy Information Administration (2010) data, the newly produced light-duty vehicles fuel consumption is 921 gallon per vehicle in 2010 and 882 gallon per vehicle in 2009. Thus, we use the ratio of two years to proxy the gasoline consumption ratio between two different types of cars. Using the ratio, we get:

$$\rho^{\frac{\gamma\sigma}{\sigma-1}} = 1.0452$$

In steady state, A.4 becomes:

$$\phi_1(1+\rho^{\frac{\gamma\sigma}{\sigma-1}})a\delta q^a = \beta\phi_2\gamma(\rho^{\frac{\gamma\sigma}{\sigma-1}}+\beta)g_2$$

Using the consumption out ratio (0.65) and personal cars spending over expenditure ratio (0.06) and substituting $\rho^{\frac{\gamma\sigma}{\sigma-1}=1.0452}$ in, we could get the value of γ to be 0.42.

D.2 Parameters related to production technology

The second category relates to production sector. Parameter ε_k denotes the depreciation rate of physical capital and we set that value to be 0.1. The mean value of the annual real gasoline price p_t is 1.0872 dollar. The aggregate productivity level of both sector, A_1 and A_2 are normalized to 1. The parameter α is considered as the share of total income paid to owners of capital and we set α_1 and α_2 to 0.33 and 0.42 based on the calibration results from Wei (2003). μ is the marginal transformation of vehicle capital and fuel efficiency and we set that to 1 for simplicity.

D.3 Parameters related to the preference of households

The third category of parameter describes the preferences of household. The subjective discount rate β is 0.97. In the model, we assume log-preference for consumption, driving service, leisure and environmental quality shown in 11, which implies that consumption and driving service are not perfect substitutes. We calibrate the parameters ϕ_1 and ϕ_2 to 0.34 and 0.05 respectively to match the fact that households allocate two thirds of their time in leisure.

Based on Ghez et al. (1975), households normally spent one third of their time to market activities(time not spent on sleeping or personal maintenance). The gasoline consumption output ratio (Eq.A.3) in steady state becomes:

$$\phi_2 c = \phi_1 (p_t + \tau_t)(g_1 + g_2)$$

Dividing each side by output and using the consumption output ratio and gasoline expenditure over income ratio, we calibrate the parameter ϕ_1 and ϕ_2 to 0.34 and 0.05.

Households benefit from good environmental quality. Jackson (1983) shows that although the household's willingness to pay for better environmental quality is sensitive to the model specification and other assumptions, the income elasticity is in the vicinity of 1. We thus set the benchmark value for ϕ_3 to 1.

D.4 Parameters related to environmental quality and pollution

The concept of environmental quality is depicted in Eq. 15. Environmental quality is a stock which is improved every period by natural purification capacity and damaged by the pollution. ϵ measures nature's purifying capacity and we set it to 0.1. Pollution, as expressed in Eq. 16, is positively related to gasoline consumption but mitigated by the fuel efficiency condition δ. Parameter μ_1 and μ_2 measures to which extent the fuel efficiency help in addressing pollution caused by gasoline consumption. For simplicity, we assume μ_1 and μ_2 to unity.

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