Efficient University Funding Formulas

Pier-André Bouchard St-Amant*

This version: August 2019

Abstract

The set of rules through which public appropriations are transferred to universities is dubbed a university funding formula. A prevalent formula is based on student enrollment and unconditional transfers. We provide the characterization of an efficient allocation of research and graduates and show that it can be implemented in a decentralized, enrollment based formula. The efficient formula is a function of the private value of graduates, the accrued production of public good through accrued research, the elasticity of a university system at producing graduates and the elasticity of the same system at producing research. Using a stark change in the funding formula that occurred in Québec, we perform a difference in difference to provide an empirical estimation of these statistics. The most important finding is that enrollment patterns changed starkly in programs rewarded by the formula. Practical policy guidelines as how to implement an efficient formula based on observed statistics is also discussed.

Keywords: Universities, Public Subsidies, Funding Formulas, Incentives.
JEL Codes: H74, H41, H42.

1 Introduction

Plenty of jurisdictions use publics funds to finance universities. For instance, the states of California, Texas, Florida and New York provide public subsidies through a public university system [57, 54, 55, 47, 59]. In Canada, the three largest provinces, Ontario, Québec and British Columbia publicly fund all of their universities ([38, 23, 20, 42]). In Europe, France, England, Sweden and Spain (to name a few) also provide public funds to their universities [14, 7, 53, 1, 26, 17]. The rule through which public funds are transferred to universities is dubbed a **funding formula**.

A popular scheme, as in Texas, Sweden, Quebec, or France is based on enrollment. Despite the prevalence of this scheme, there is very little theoretical work done on how it should be characterized. This paper seeks to fill this gap. It first presents a theoretical model with three "layers" of economic agents: individuals who must decide if they enroll to a university program, universities who decide where they allocate their funds, and the government, who determines the level of funding and the formula parameters. When the government changes any of the funding parameters, it changes the incentives given to universities and thus, may change enrollment decisions and research outputs. A funding formula is thus an indirect steering mechanism that must account for the non-trivial interactions between these layers.

This paper first clarifies the impacts that funding formulas have on universities. It is shown that a robust result of an enrollment based formula is that an increase of the amount of money per student should increase both enrollment and the research output. However, specific funds devoted to either research or teaching quality may decrease or increase, depending on the nature of universities and what they find "prestigious". Research oriented universities will favor spending on research funds while teaching universities will prefer the converse. These behaviors and the efficient parameters of an efficient formula is also characterized. The formulas are presented in terms of sufficient statistics, which provides a practical rules for its implementation.

*Professor of Public Finance, École nationale d'administration publique (ÉNAP), 4750 Ave. Henri-Julien, 5e étage, Montréal, Québec, Canada, H2T 3E5, pier-andre.bouchardst-amant@enap.ca. Empirical results remain preliminary. I am thankful to Hugo Morin for his invaluable research assistance, Jean-Denis Garon, John Bowden, Chad Doerksen, Lisa Ransom, Martin Maltais, Nicolas Bérubé, Vincent Larivière as well as the participants to the Societé canadienne de sciences économiques. All errors remain my own.

From a theoretical standpoint, this paper extends the model provided by De Fraja and Valbonesi [9] and provide five key theoretical contributions. First, the wage premium of graduates is endogenized and is a function of the teaching quality. From a welfare perspective, this changes the behavior of the government, as it then faces incentives to increase enrollment, then increasing the fiscal base through higher wages, which at the same time generates inequalities. The optimal funding of universities thus weights the trade-off between these effects. Second, the research output is characterized as a public good. It is shown that as long as the government has two university specific funding components, it can implement the Samuelson condition and provide an efficient level of research. Third, on a conceptual standpoint, this paper generalizes what universities value. They are modeled as choosing their allocation of funds between teaching and research based on what they find “prestigious”. Fourth, a characterization of an efficient formula is presented in terms of observable sufficient statistics. This approach thus provides a formula that can be directly implemented based on the response of a university system to public subsidies. Fifth, the paper introduces a connection between government spending and taxation. Depending on how universities charge in tuition fees, it is shown that the government can set negative tax rates (a tax credit) for students. The opposite, akin to a graduate tax, can also happen.

From an empirical standpoint, the paper contributes to the literature by showing how a shift in a funding policy changed the behavior of a system of universities. The results relies on a natural experiment that occurred in the province of Québec, Canada, to estimate the impact of shifting the funding policy from lump-sum transfers to an enrollment based formula. The most important result is that the policy shift changes enrollment patterns by family of programs: business, engineering and health related programs see a stark increase in enrollment while humanities and education see a decrease. These change in enrollment follow where the funding per student is the highest. This extends the works of Umbricht, Fernandez, & Ortagus [56], who studied policy shifts from an enrollment based formula to a performance based formula. In particular, they show that this shift leads to a decline in enrollment. Based on the results of this paper, this result may be the outcome of abandoning the first policy rather than embracing the second one.

An additional section with very little mathematics is also presented, where some practical policy guidelines are given based on the results of this paper. This additional section, although more prescriptive than inquisitive, aims to provide a practical sense to non-economists of how the theory can be applied.[1]

1.1 Related Literature

From a theoretical perspective, the analysis presented in this paper is a direct extension of De Fraja and Valbonesi [9]. They provide a characterization of a university system in terms of its industrial structure (concentration of research, specialization, etc) and how it should be funded. Using the revelation principle in a mechanism design approach, they provide a “menu of policies”, comprised of both unconditional transfers and a funding amount per student, such that universities will choose the combination that will maximize social efficiency. This menu of policies constitutes a funding formula. In their paper, the universities solely care about research, the market premium that graduates acquire through education is constant and they assume that jurisdictions use lump-sum tax rates. As they focus on the concentration of research and the “efficient design” of the university system, little welfare analysis is produced as wether an “efficient” amount of teaching or research is produced. We broaden the analysis by letting universities care about research and the teaching quality by maximizing a “prestige function”, in line with the works of [8], [46] and [16]. Second, we let the market premium of graduates be a function of teaching quality. This provides a richer environment in which we can discuss the impact of efficient policies. We show that as long as universities are funded through a combination of lump-sum transfers and an amount per student, an efficient allocation can be implemented on the basis of observable statistics.

From a methodological standpoint, the theoritical analysis borrows from the optimal taxation literature, where the focus is on the maximisation of indirect utility [44]. The revelation principle is rather framed as a choice made by agents, once the policy tools have been set. The characterization of efficient formulas is characterized in terms of elasticities and other observable “sufficient statistics” [5, 27]. As such, elasticities of the university system provide an equivalent characterization of the efficiency of the university system, as in [9], but are rather observed through statistics than induced by a revelation principle.

[1] It can also be removed if out of scope.

Our theoretical work also relates to the enquiry of Del Rey [10]. She used a game theoretical approach to analyze universities' responses to funding formulas comprised of a mix of lump-sum transfers and enrolment-based transfers. She showed that changing the mix would change the equilibrium, either comprised of teaching only strategies, research only strategies and combinations thereof. While her paper links directly the incentives of the funding rules to universities' practices, it does not examine the welfare implications. De Fraja and Ioassa [8] studied the competition between public universities in the context of decentralized budget decisions. In their paper, universities maximize their prestige by choosing where to allocate funds between research and teaching. They then show that universities adopt different practices depending on the degree of mobility costs (competition) between students. When mobility costs are high, universities adopt similar practices in terms of teaching and research. When they are low, competition leads to the differentiation of objectives (pure teaching vs pure research institutions). Although it is not directly related to funding formulas, the paper goes to show that universities respond to their environment.

Some other papers studied funding formulas indirectly. Poyago-Theotoky and Tampieri [45] studied the competition between local and international universities. Their focus is on investment decisions by the international university, but one chapter studies a funding rule tied strictly to the graduates' incomes. Lien [30, 31] also studied Higher Education and focuses the local impact of a market entry by an international universiy. Trannoy and Gary-Bobo [17, 18, 19] also discussed the impacts of the French funding formula (based on enrollment), but mainly as a support for a funding model focused on higher tuition and an income contingent loan (or graduate tax scheme).

Our work also connects to some empirical studies linking the impact of funding formulas and universities' behaviour. Plenty of american states embraced a performance-based model in the 90s, went back to enrolment-based models and then came back to performance-based ones in the years before 2010 [13, 22, 40].[2] These variations in policies were used as a basis for difference in difference studies. Hillman, Tandberg and Gross [24] used this identification strategy to study the shift that occurred in the state of Washington. Using several control groups and robustness checks, they show that the policy had little short-run impacts on completion and retention rates. They however find an increase in the output of short-term certificates. Umbricht, Fernandez and Ortagus [56] also used a difference in difference to evaluate the same policy shift in Indiana. Using also various control groups, they find that the policy has little impact on graduation rates, but induces a significant decline in enrollment, suggesting that abandoning an enrollment based formula may have more impact than embracing a performance based one. Combined with our empirical results, this suggests that the decline in enrollment may be more related to abandoning an enrollment based formula than embracing a performance based one. A critical review of empirical studies can be found in [12] and [56].

The rest of this paper is organised as follows. In section 2.1, we characterize the economic environment for individuals. In the following section (2.2), we model how universities behave and provide some clear results for "undergraduate universities" (teaching oriented) and "graduate universities" (research oriented). Robust results regarding enrollment and research are also provided for universities doing both. In section 3.1, we model the government's objective, characterize an efficient allocation of research and teaching and show that it can be decentralized through an enrollment based funding formulas. In section 4, we describe the policy shift that occurred in Québec and estimate its impact both on research output and enrollment intake. Some policy advice regarding implementation are also discussed in section 5.

2 A Model of Student and University's Behaviour

2.1 Individuals and Student Demand

We focus first on a regional market with a single university. Individuals in the region must decide how much to consume, how much labour to supply over the course of their lifetime and if they if they should undertake university studies. Individuals who decide to enroll are designated as **students** while those who do not are referenced as **workers**. Each individual has a talent for studies denoted ϕ. For simplicity, this talent is expressed in units of utility, meaning that individuals find that attending university is enjoyable according

[2] See [15] for a classification of various types of funding formulas.

to ϕ. The cumulative distribution of individuals is noted by $F(\phi)$ with support $[\phi_{\min}, \phi_{\max}]$ and it is assumed that there exists at least one subset of $[\phi_{\min}, \phi_{\max}]$ such hat $F_{\phi\phi} > 0$.[3][4]

A student must choose how much time to devote to consumption (c) and leisure ($1-L$) over his lifetime according to a strictly concave utility function $V(c, 1-L, G)$.[5] Thus, L designates hours worked and G designates the total provision of public good. Over her lifetime, a student earns a wage rate premium $p(t)$ above the standard wage rate ($\bar{w}$) and so her wage rate is $(p(t)+\bar{w})$. The wage premium is an increasing and concave function of t, the teaching quality ($p_t > 0, p_{tt} < 0$). Once enrolled, the maximization problem of a student is thus:

$$\max_{c,L} V(c, 1-L, G) + \phi \text{ s.t. } (1-\tau)(p(t)+\bar{w})L - f + a \geq c, \tag{1}$$

where f are the tuition fees paid, τ is the tax rate and a is a lump-sum transfer. Denote $V^{e*}(t, f, G, \tau, a)$ the indirect utility function resulting from such problem.[6] We assume the condition:

$$-V^{e*}_{Lc} < (1-\tau)(p(t)+\bar{w})V^{e*}_{cc} \tag{2}$$

which amounts to assuming that leisure is a normal good and hours worked decrease with tax rates. This guarantees a trade-off between funding universities (increasing tax rates) and providing incentives to work (decreasing tax rates).

A worker faces the same allocation problem with the same utility function, but does not pay tuition fees, nor he enjoys the utility of being a student ϕ. The associated maximization problem is thus:

$$\max_{c,l} V(c, 1-l, G) \text{ s.t. } (1-\tau)\bar{w}l + a \geq c. \tag{3}$$

With assumption (2), the worker's labor supply also decreases with the tax rate ($l^*_\tau < 0$). Denote the indirect utility functions of the worker $V^*(G, \tau, a)$.

An individual decides to enroll to the university if $V^{e*}(f, t, \tau, G) + \phi > V^*(\tau, G)$, which defines an implicit talent level above which every individual enrolls:

$$\phi^e(f, t, \tau, a, G) \equiv V^*(\tau, a, G) - V^{e*}(f, t, \tau, a, G). \tag{4}$$

Enrollment is thus given by:

$$e(f, t, \tau, a, G) \equiv F(\phi_{\max}) - F(\phi^e(f, t, \tau, a, G)), \tag{5}$$

and it is completely characterized in the following proposition.

Proposition 1 (Student Enrollment)**.**

$$e_f < 0, \qquad e_{ff} < 0 \qquad e_t > 0, \qquad e_{tf} > 0, \tag{6}$$

$$e_{tt} < 0 \Leftrightarrow \underbrace{\frac{\mu^{e*}_t}{\mu^{e*}} - \frac{F_{\phi\phi}}{(F_\phi)^2}}_{-} + \frac{p_{tt}}{p_t} + \frac{L^*_t}{L^*} < 0, \tag{7}$$

$$e_G > 0 \Leftrightarrow \phi_G = V^{e*}_G - V^*_G > 0, \tag{8}$$

$$e_{GG} < 0 \Leftrightarrow \phi_G > 0, \phi_{GG} > 0, \tag{9}$$

$$e_a < 0 \Leftrightarrow \frac{\mu^*}{\mu^{e*}} > 1, \tag{10}$$

$$e_\tau < 0 \Leftrightarrow \frac{\mu^*}{\mu^{e*}} < \frac{(p(t)+\bar{w})L^*}{\bar{w}l^*}. \tag{11}$$

where μ^{e*}, μ^* are the Lagrange multipliers of respectively the students' and the workers' problem.

[3]Notation: subscripts will be used to denote the partial derivative of a function (e.g.: $f_x \equiv \frac{\partial f}{\partial x}, f_{xy} \equiv \frac{\partial^2 f}{\partial x \partial y}$). Throughout, a variable with a star as a superscript ($x^*, y^*, \ldots$) denotes the value of such variable at the optimum of the related maximization problem.

[4]Alternatively, one could make the stronger assumption of a strictly decreasing hazard rate. As long as F is convex on at least one subset of $[\phi_{\min}, \phi_{\max}]$, the function can be convexified and the optimum problem based on F will be well defined. See [3] for a discussion.

[5]For simplicity, total hours are normalized to one.

[6]All formal derivations are left out in the appendix.

Equations on line 6 provides an intuitive characterization of enrollment: it declines with tuition and increases with teaching quality. It is furthermore a concave function of tuition and the cross-derivative of teaching quality with respect to tuition is positive. It means that an increase of one dollar in tuition that is devoted to teaching quality increases enrollment. Equation (7) shows that a sufficient condition for e to be concave in t is when the marginal wage premium increases enough to compensate the increase in the labour supply. Equation (8) shows that enrollment increases with the public good solely if it is a complement to consumption. If the public good is separable in utility, then enrollment does not depend on the amount of public good. If it is a substitute, then enrollment decreases. In such case, the intuition is that a higher provision of public good reduces the attractiveness of a wage premium and thus to enroll in a university. Equation (9) states the technical condition under which enrollment is concave with the provision of the public good. It depends on the third derivatives of the utility function (see appendix) and it is assumed throughout.

Equation (10) shows that the impact on lump-sum transfers is a function of the difference in Lagrange multipliers. Consider a pair $(p(t), f)$ such that the marginal utility derived from income is identical for both types of workers (i.e. $\mu^{e*} = \mu^*$). Intuitively, this is the case when tuition fees absorb all benefits from the market premium and (10) is then equal to zero. In such case, lump-sum transfers move both the workers' and the students' marginal utility equally, therefore leaving incentives to study unchanged. If the pair $(p(t), f)$ is such that students derive more marginal utility from consumption and leisure ($\mu^{e*} > \mu^*(\tau, a)$), then enrollment declines with lump-sum transfers. The converse is also true.

Equation (11) states that enrollment decreases with the tax rate only if the ratio of earned income is lower than the ratio marginal utility. Again, if a pair $(f, p(t))$ equalizes marginal utility from income, which means that increasing tax rates decreases enrollment. However, if the pair $(p(t), f)$ is such that students have an higher marginal utility of income ($\mu^{e*} < \mu^*$), then enrollment increases with the tax rate. This is so because the marginal impact of taxation is higher on the marginal utility of workers than on the marginal utility of students. Thus, there is a displacement of individuals towards university.

2.2 Universities

There is one regional university that produces graduates and research. It can choose how much funds to allocate to research (R), the wage to pay a single professor (t) and tuition fees (f). In the discussions below, the pay given to the professor is also used as a proxy for the quality of teaching, the assumption being that a higher wage rate allows for a more productive hire.

Student enrollment is given by (5). Research is the outcome of a strictly concave function $g(R, t)$ that depends on research funds and the teaching quality. The research output of the university, as well as other sources of research, contributes to the provision of the public good. Hence, the research output of the university $g(R, t)$ is a fraction of G. It is assumed that research and teaching funds are complement for producing research ($g_{Rt} > 0$).

Each university receives appropriations F from the public legislature according to the following funding formula:

$$F = T + \alpha e, \tag{12}$$

where T is an unconditional transfer and α is a funding amount per student. Since the university can charge tuition fees, the total revenues are given by $F = T + (\alpha + f)e$. The total costs of the university are the sum of the salary and funds devoted to research ($R + t$). The budget constraint of the university is thus:

$$T + (\alpha + f)e \geq t + R. \tag{13}$$

Following the works of [8], [46] and [16], we assume that the university maximises a strictly increasing and concave "prestige function", noted $\tilde{\Pi}(g, t)$, that depends on the research output and the teaching quality.[7] The prestige function reflects the university's perspective on its role in society. Albeit this perspective might change drastically by field or position within the university, it is assumed that the ideals of the governing

[7]It is worth mentioning that if a university had prestige dependent on enrollment, the analysis performed below would also work. In this paper, prestige is a function of R and t: $\tilde{\Pi}(g(R, t), t) \equiv \Pi(R, t)$. Since enrollment is also a function of R and t, the conclusion follows.

body are adequately represented by such function. From a prestige standpoint, research and teaching are not substitutes ($\tilde{\Pi}_{gt} > 0$). The university is limited by its available funds, which yields the following prestige maximization problem:

$$\max_{R,t,f} \tilde{\Pi}(g(R,t),t) \text{ s.t. } T - R + (\alpha + f)e(t,f,G) - t. \tag{14}$$

Denote $R^*(\alpha,T), t^*(\alpha,T), f^*(\alpha,T)$ the resulting allocation of research funds, teaching funds and tuition fees. This allocation satisfies the following first-order conditions:

$$\tilde{\Pi}_g g_R + \lambda\left((\alpha + f)e_G g_R - 1\right) = 0, \tag{15}$$

$$\tilde{\Pi}_g g_t + \tilde{\Pi}_t + \lambda\left((\alpha + f)e_t - 1\right) = 0, \tag{16}$$

$$\lambda\left(e + (\alpha + f)e_f\right) = 0, \tag{17}$$

$$(T - R) + (\alpha + f)e - t = 0, \tag{18}$$

where λ is the Lagrange multiplier.

The budget constraint of the university (18) is the concave function shown in Figure 1a. Because of concavity, it is first possible to increase t while raising revenues, thus increasing R at the same time. However, as funds devoted to teaching increase, the marginal increase in enrollment (e_t) becomes smaller than the unit cost of funds and past the point where both effects are equal ($(\alpha + f)e_t - 1 = 0$), there is a trade-off between allocating funds to research and allocating funds to teaching and the slope of the budget constraint is negative. Equation (16) shows that the efficient solution is necessarily on that second part of the budget constraint. The first two terms of the equation ($\tilde{\Pi}_g g_t + \tilde{\Pi}_t$) are positive. Since λ is positive, it must be that $(\alpha + f)e_t - 1$ is negative, which happens only on that side of the slope. This means that the university spends more on teaching funds than what is profit maximizing (the point where $(\alpha + f)e_t = 1$).

The current setup can also be interpreted in terms of the usual marginal rates of substitution (the convex iso-curve in Figure 1a). When (15) and (16) are combined, one finds:

$$\frac{\tilde{\Pi}^*_g g^*_t + \tilde{\Pi}^*_t}{\tilde{\Pi}^*_g g^*_R} = \frac{1 - (\alpha + f^*)e^*_t}{1 - (\alpha + f^*)e^*_g g^*_R}. \tag{19}$$

Equation (19) states the familiar result that the efficient solution (R^*, t^*) is where the ratio of marginal benefits must equal the ratio of marginal costs.

Equation (17) reveals that the university will set tuition so as to maximize revenues (see Figure 1b). Denoting the elasticity of student demand with respect to tuition by $\epsilon^e_f \equiv \frac{f^*}{e^*}e^*_f$, rearranging (17) yields:

$$\frac{f^*}{\alpha + f^*} = -\epsilon^e_f. \tag{20}$$

Tuition fees are chosen so that the magnitude of the elasticity of demand equals the percentage of fees in the total revenues per student. If the subsidy per student were to be equal to zero ($\alpha = 0$), the result would then yield a first order condition typical of monopoly pricing. When there is a subsidy for enrollment ($\alpha > 0$), the fees are then chosen so as to account for their impact on the fraction of revenues per student they do generate. Hence, the fees are smaller when α is positive. This result is not general and stems from having a single university in the regional market (no competition between universities). A study of the competition between universities is provided in [8].

[Figures 1a and 1b about here.]

2.2.1 Comparative Statics: The Undergraduate University

If the production of public good has little impact on enrollment, the term e_g can be assumed close to zero and (15) becomes:

$$\tilde{\Pi}_g g_R - \lambda \approx 0. \tag{21}$$

This is exact if the utility function of both workers and students is separable in the public good, or it is a good approximation if the production of public good is high enough so that $V_G^{e*} \approx V_G^*$. This section uses the interpretation that e_G is close to zero because undergraduatestudents may have little interest in research. Hence, this section focuses on the analysis of an "undergraduate university" (the "graduate university" is analyzed in section 2.2.2). When (21) is true, funds devoted to teaching are higher than those to research. Since $\tilde{\Pi}_g g_t + \tilde{\Pi}_t > \tilde{\Pi}_R$ and that both functions g and $\tilde{\Pi}$ are concave, it must be that $t^* > R^*$ for the first order conditions to hold. In an undergraduate university, more funds are devoted to teaching quality than on research.

Proposition 2 (Undergraduate university). *Consider (14) and assume that $e_G = 0$. Then:*

$$t_\alpha^* > 0, \qquad R_\alpha^* > 0 \Leftrightarrow \frac{e}{1-(\alpha+f)e_t} > t_\alpha^*, \qquad f_\alpha^* > 0 \Leftrightarrow \frac{-e_f}{e_t + (\alpha+f)e_{tf}} < t_\alpha^*, \tag{22}$$

$$t_T^* \in (0,1), \qquad R_T^* \in (0,1) \qquad f_T^* > 0, \tag{23}$$

$$e_\alpha^* > 0 \qquad e_T^* > 0 \qquad e_\alpha^* \neq e_T^*, \tag{24}$$

$$g_\alpha^* > 0. \qquad g_T^* > 0, \qquad g_\alpha^* \neq g_T^* \tag{25}$$

The defining characteristic of an undergraduate university is that teaching funds increase when the per-student component of the funding formula increases ($t_\alpha^* > 0$). Its solution is proportional to:

$$t_\alpha^* \propto \underbrace{(2e_f + ((\alpha+f)e_{ff})}_{-} \underbrace{\left(\frac{\Pi_{RR}}{\Pi_R} - \frac{\Pi_{Rt}}{\Pi_t}\right)}_{-1\times\text{prestige effect (-)}} - e_t \frac{\Pi_R}{\Pi_t} \underbrace{\left(\frac{e_{tf}}{e_t} - \frac{e_{ff}}{e_f}\right)}_{-1\times\text{income effect (-)}} > 0, \tag{26}$$

where $\Pi(R,t) \equiv \tilde{\Pi}(g(R,t),t)$ is used to shorten the notation (see (106) for the details). The structure of t_α^* can be understood as a (weighted) sum of the of the prestige effect and the cost effect. Since teaching quality is both prestige and income enhancing, both curvatures push t_α^* upwards, thus leaving no ambiguity on its sign.

The same cannot be said of the derivative of research funds (R_α^*): it can either decrease or increase, depending on the impact of teaching funds on the budget maneuver. The second equation of (22) presents necessary and sufficient conditions for an increase in research funds. The numerator of the condition measures the marginal revenues induced by the increase in α. The numerator measures the marginal costs induced by the increase in teaching funds. The ratio is thus a measure of the marginal budget maneuver. If the marginal spending on teaching funds is lower than this ratio, then research funds can increase as well.

There is also a necessary and sufficient condition for tuition fees to increase with α, as shown in the third equation of (22). The left-hand side of the condition compares the ratio of marginal revenues when tuition is simultaneously increased and spend on teaching quality. The numerator measures the direct effect of tuition on enrollment while the denominator measures the compensation effect by increasing teaching quality. If the marginal change in teaching quality is lower than the ratio, then the impact of tuition is more important on enrollment than the increase in equality. In such case, tuition decrease (the converse is also true).

An increase in unconditional transfers (T) yields simpler comparative statics: teaching funds, research funds and tuition fees necessarily increase. This happens because unconditional transfers do not affect enrollment while allowing for an increase in both research and teaching funds. The increase in teaching funds attracts more students, thus allowing the university to increase fees to maximize revenues.

Equations on lines (30) and (31) show that enrollment and research output increase with any of the funding formula parameters. The increase in teaching funds always has a larger impact on research output that an eventual decrease in research funds. The last statements of each line are useful building blocks for the next stage of the model, as they state that the variables α and T fully span the (g, e) space. In other words, for any pair of desired research output and enrollment (g, e), there exists a funding formula with a pair (α, T) that will steer the university into reaching the desired point.

2.2.2 Comparative Statics: The Graduate University

This section assumes that students are primarily interested in enrolling to a university because of its impact on research. The quality of teaching is assumed to have little effect on enrollment ($e_t \approx 0$). This assumes either that the wage premium is constant ($p(t) = p$) regardless of teaching quality (as in [9]), or that a marginal increase in the premium has little effect on the student utility. From the standpoint of student demand, it also requires the assumption that the public good is complementary to consumption. In such case, (16) becomes:

$$\tilde{\Pi}_g g_t + \tilde{\Pi}_t - \lambda \approx 0. \tag{27}$$

Since the marginal cost of research funds is lower than one ($1 - (\alpha + f^*)e^*_G g_{R^*} < 1$), the prestige maximizing allocation of the graduate university has more funds dedicated to research than to teaching. The rest of the first-order conditions remains unchanged.

We let this setup characterize a university focused on graduate programs and of course, research. Compared to the undergraduate university, the following proposition shows that the change in the first order conditions implies reflexive comparative statics: substituting t for R in the last proposition yields the behavior of the graduate university.

Proposition 3 (Graduate University)**.** *Consider (14), assume $e_t = 0$ and ($V^*_G - V^{*e}_G < 0$). Then:*

$$R^*_\alpha > 0, \qquad t^*_\alpha > 0 \Leftrightarrow \frac{e}{1 - (\alpha + f)e_R} > R^*_\alpha, \qquad f^*_\alpha > 0 \Leftrightarrow \frac{-e_f}{e_R + (\alpha + f)e_{Rf}} < R^*_\alpha, \tag{28}$$

$$R^*_T \in (0, 1), \qquad t^*_T \in (0, 1), \qquad f^*_T > 0, \tag{29}$$

$$e^*_\alpha > 0 \qquad e^*_T > 0 \qquad e^*_\alpha \neq e^*_T, \tag{30}$$

$$g^*_\alpha > 0. \qquad g^*_T > 0, \qquad g^*_\alpha \neq g^*_T \tag{31}$$

When the public good is a complement to consumption, research funds increase with α. Increasing research funds is both prestige and income increasing. Whether t^*_α increases or not is fully characterized by the budget dynamics: if the marginal income (e) induced by α is higher than the marginal cost of accrued research funds ($1 - (\alpha + f)e_R$), there is then some (marginal) budget maneuver. If it is greater than the additional marginal spending on research (R^*_α), then the university can use some of the maneuver to increase teaching funds. The converse is also true.

The dynamics for tuition fees is similar: the numerator of the condition evaluates the marginal loss of income if tuition were increased by one dollar (e_f). The denominator evaluates how much marginal costs increase due to research funds, accounting for the additional dollar of tuition spent on research. The ratio is thus a measure of how tuition affects the budget, considering that research funds increase. If the spending in research funds (R^*_α) is greater, its an indication that the increase in research funds has more impact on demand than on supply, meaning that the university can increase tuition. The converse is also true.

As for the undergraduate university the increase in unconditional transfers (T) by one dollar translates into an increase in research funds by less than one dollar, an increase in teaching funds by less than one dollar and an increase in tuition. The increase in tuition can be higher than one dollar if the increase in demand is important. Similar to the undergraduate university, an increase in any of the two components of the funding formula leads to an increase of research output and enrollment. This proposition also shows that the parameters α and T can be used to steer graduate universities in a particular direction.

2.3 Comparative Statics: the General Case

If universities both face enrollment that responds to the research output and the teaching quality, then the robust results can be characterized in the following proposition:

Proposition 4. *Consider the university problem (14) and assume $V_G - V^e_G < 0$, then:*

1. *Enrollment increases with α and with T ($e_\alpha > 0, e_T > 0, e_\alpha \neq e_T$);*
2. *The research output increases with α and with T ($g_\alpha > 0, g_T > 0, g_\alpha \neq g_T$);*
3. *If α is increased and tuition fees increase, then either research funds or teaching funds increase (or both);*
4. *If either α or T increases, then either research funds increase, teaching quality increase, or both.*

This proposition conveys what is robust to any prestige maximizing university. Both enrollment and research output increase with changes in each of the components. Also, the two channels through which funding can be channeled (T and α) are enough to span the full space of outcomes (g and e).

2.4 A Critique of A Formula Based on Observed Costs

These results can be used to show the fallacy of setting α on the basis of costs per students, as it was analyzed in some north american jurisdictions such as in California (USA), Texas (USA) or Québec (CA) [28, 58, 60]. For the sake of the discussion, assume that universities complain they are not funded enough to cover teaching costs per student, defined as:

$$C \equiv \frac{t^*}{e^*}. \tag{32}$$

In response, assume that the the funding body yields to the complaints and corrects α to cover C. If ϵ_α^e and ϵ_α^t are used to respectively denote the elasticity of enrollment and the elasticity of teaching funds with respect to α, then the change in costs per student is given by:

$$C_\alpha = \frac{C}{\alpha}\left[\epsilon_\alpha^t - \epsilon_\alpha^e\right]. \tag{33}$$

If the elasticity of enrollment with respect to teaching quality is lower than one, then the term between brackets is positive ($\epsilon_\alpha^e < \epsilon_\alpha^t$). This means that costs will further *increase* with the increase in α. An attempt to fund the universities appropriately leads to another funding deficit, this time induced by the policy change. This goes to show that accounting for the response of universities to the funding formula must be considered when designing such formula. The converse is also true if $\epsilon_\alpha^e > \epsilon_\alpha^t$, that is when an increase in the funding per student will lead to a decrease in costs. In such case, the public legislature covers more than the costs. It is only when both elasticities are equal that a "static view" turns out to be accurate.

3 Efficient Centralized and Decentralized Policies

3.1 Centralized Policy

If the government could implement itself the production of research and students, how would the policy be set? This question is examined here by letting the government own the technology to produce market premiums (through students) and public goods (through research). We also let the lump-sum transfers vary by type of individuals and denote them a^e, a. The government thus seeks to maximize welfare while being constrained by the technology available to universities. Using (4), its problem is given by:[8]

$$W \equiv \max \underbrace{-e\phi^e(\tau, a, a^e, f, t, g(R,t))}_{\text{Additional welfare of students}} + \underbrace{\int_{\phi^e}^{\phi_{\max}} \phi dF(\phi)}_{\text{enjoyment of students}} + \underbrace{F(\phi_{\max})V^*(\tau, a, g(R,t))}_{\text{Basic welfare of everyone}} \text{ s.t :} \tag{34}$$

$$0 \leq \underbrace{(\tau(p(t)+\bar{w})L^* - a^e - (\tau\bar{w}l^* - a))}_{\equiv \Delta I} e + \underbrace{(\tau\bar{w}l^* - a)}_{\equiv I} F(\phi_{\max}) + fe - t - R,$$

[8]Note that for this problem to be concave, it must be that (9) holds.

Denote ψ the Lagrange multiplier. The first-order conditions of this problem can be shown equivalent to (see B.1):

$$\frac{(\mu^{*e}-\psi)}{\psi} = \frac{1}{e}\left[\Delta I e_f + e\Delta I_f + f e_f\right], \tag{35}$$

$$-\frac{(\mu^{e*}-\psi)}{\psi} = \frac{1}{e}\left[\Delta I e_{a^e} + e\Delta I_{a^e} + f e_{a^e}\right], \tag{36}$$

$$-\frac{\mu^{*}-\psi}{\psi} = \frac{1}{F(\phi_{\max})}\left[\Delta I e_a + I_a F(\phi_{\max}) + e + f e_a\right], \tag{37}$$

$$-\frac{eV_G^{e*} + (F(\phi_{\max})-e)V_G^{*}}{\psi} = \left[\Delta I_G e + \Delta I e_G + I_G F(\phi_{\max}) + f e_G - \frac{1}{g_R}\right], \tag{38}$$

$$-\frac{(1-\tau)\mu^{*e} + \tau\psi}{\psi} = \frac{1}{e p_t L^*}\left[\left[\tau(p(t)+\bar{w})L_t^*\right]e + \Delta I e_t + f e_t + \frac{g_t - g_R}{g_R g_t}\right], \tag{39}$$

$$\frac{e\mu^{e*}(p(t)+\bar{w})L^* + F(\phi_{\max})\mu^*\bar{w}l^*}{\psi} = \left[\Delta I_\tau e + \Delta I e_\tau + I_\tau F(\phi_{\max}) + f e_\tau\right]. \tag{40}$$

Equation (38) has a negative left-hand side and for the exception of $-\frac{1}{g_R}$, all terms of the right-hand side are positive. This means that g_R must be small enough to balance the equation and thus, $R^{eff} > 0$. Similarly, (39) implies that t^{eff} must be "high enough" so that $g_t < g_R$. It should be noted that the ratio of (38) and (39) is nothing but the specialized version of the Samuelson condition. Here, part of the value of the public good production influences tax revenues, as its production encourages enrollment. Hence, the trade-off must account for the gain in the production of the "private good" in terms of marginal utility, but also its marginal impact on the tax base.

The first-order conditions show that a^e and f are equivalent for the planner. Indeed, (35) and (36) are identical. By design, there is a mathematical equivalence between a lump-sum tax on students a^e and tuition fees f: the government can either charge tuition or use a "graduate tax". This equivalence stems from the absence of liquidity constraints and of asymmetry of information. For a discussion on a graduate tax that distinguishes both instruments, see ([17]).

It should be noticed than when $\tau = 0$, (35) can be re-organized to:

$$e + \frac{\psi}{\mu^{e*}-\psi}(a - a^e + f)e_f = 0, \tag{41}$$

which has the same structure as (17) (although generally not equal). The central planner can thus be thought of (and usually is) a monopoly planner with a different marginal cost. If students have more income than the government ($\mu^{e*} < \psi$), then the net transfers to students $a^e - f$ are smaller than the lump-sum transfers to workers a. It means that when students are better off, an efficient allocation compensates workers more than students. The converse is also true.

These first-order conditions leads to the following proposition:

Proposition 5 (Efficient Centralized Policy)**.** *Consider (34) and assume $V_G^* - V_G^* \leq 0$. Then there exists an efficient solution with $(f + a^e)^{eff} > 0$, $t^{eff} > 0, R^{eff} > 0$ and $\tau^{eff} = 0$.*

It should be noted that the proposition makes no assumption on the student demand. Hence, the proposition holds for any type of university.

3.2 Decentralized Policy

In a decentralized setup, the question is whether the government can choose a funding policy (α and T) to induce a given university to implement the efficient allocation. Below, proposition 6 tells us that it is the case. We thus focus on the decentralized problem where the parameters are designed for a specific university. It thus shows the basis of an efficient family of funding formulas, similar in spirit to Di Fraja and Valbonesi ([9]). As in the previous section, we assume individual specific lump-sum transfers and analyse first the case

$\tau^* = 0$. The problem of the government is given by:

$$\max_{a^e, a, \tau, \alpha, T} - e\phi^e + \int_{\phi^e}^{\phi_{\max}} \phi dF(\phi) + F(\phi_{\max})V^* \text{ s.t.:} \tag{42}$$
$$0 \leq \Delta I e - \alpha e + IF(\phi_{\max}) - T,$$
$$G = g(T, \alpha),$$

which leads to the following proposition:

Proposition 6 (Decentralized Policy)**.** *Consider (42) and assume $V_G^* - V_G^{*e} \leq 0$. Then there exists a solution $(\alpha^*, T^*, a^{e*}, a^*)$ that implements the allocation of (34). Such solution is given by the following implicit equations:*

$$t^{eff} = t^*(\alpha^*, T^*), \tag{43}$$
$$R^{eff} = R^*(\alpha^*, T^*), \tag{44}$$
$$(f + a^e)^{eff} = f^*(\alpha^*, T^*) + a^{e*}, \tag{45}$$
$$a^{eff} = a^* \tag{46}$$

The proof for such proposition is trivial: since α and T span the full space for t and R, there exists values α^*, T^* such that the efficient values are met. This generates a (perhaps inefficient) level of tuition fees, which is then compensated by the lump sum tax a^{e*}. Because the efficient levels of teaching funds and research funds are chosen, the production level $g(t, R)$ is efficient. Furthermore, because both students and workers face the same trade-offs as in the efficient solution, they make the same decisions. Finally, because the efficient parameters are mimicked, the budget constraint is also verified. Thus, as long as there exists a pair α, T for a given university, the government can steer that university towards the efficient allocation. Obviously, a policy with only two parameters falls short if there are more than one universities with different response functions.

The immediate corollary of the last proposition is that for as long as there are two distinct lump-sum transfers, a second best decentralized allocation can be reached if the government implements a second-best tax scheme. In other words, an enrollment based university formula that is university specific will not worsen welfare. Depending on the level of tuition fees, the lump-sum transfer for students can either be negative (graduate tax) or positive (tuition tax credit).

3.2.1 Efficient Funding Per Student in Terms of Sufficient Statistics

If proposition 6 establishes that an efficient decentralization scheme exists, it does not reveal much on the nature of efficient parameters. We thus focus on problem (42) and express the first-order conditions for α^* as a function of total elasticities or elasticities that account for all effects except for research. The formal definitions are contained in Table 2. The first-order conditions for a and α are:

$$0 = -e\phi_a^e + F(\phi_{\max})V_c^* + \psi\left[\Delta I_a e + \Delta I e_a + F(\phi_{\max})I_a\right], \tag{47}$$
$$0 = -e\phi_{\alpha/G}^e + \psi\left[\Delta I_{\alpha/G} e + \Delta I e_{\alpha/G} - \alpha e_{\alpha/G} - e + (\alpha - f^{eff})e_G g_\alpha + \frac{g_\alpha}{g_R}\right]\dots$$
$$\dots + \underbrace{(eV_G^{e*} + (F(\phi_{\max}) - e)V_G^*)g_\alpha + \psi\left[\Delta I_G e + \Delta I e_G + f^{eff} e_G + I_G F(\phi_{\max}) - \frac{1}{g_R}\right] g_\alpha}_{=0 \text{ (by (38))}} \tag{48}$$
$$. \tag{49}$$

Using the fact $V_c^* = \phi_a^e = -\frac{e_a}{F_\phi}$ and the definition of total elasticities, (47) yields:

$$\psi\frac{F_\phi}{e} = \frac{\xi_a^e(F(\phi_{\max}) - e)}{[\Delta I e(\xi_a^{\Delta I} + \xi_a^e) + I\xi_a^I F(\phi_{\max})]} \tag{50}$$

[Table 2 about here.]

The numerator expresses the change in welfare that is induced by increasing the lump-sum transfers of the workers. The denominator measures the marginal impact on the government's budget. The part first part of the denominator accounts for the change in the taxable market premium while the second one accounts for the change in the taxable base income. With a similar approach, (48) yields:

$$-\frac{F_\phi}{e}\psi = \frac{\xi^e_{\alpha/G}}{\Delta I(\xi^{\Delta I}_{\alpha/G} + \xi^e_{\alpha/G}) - (1 + \xi^e_{\alpha/G} - \epsilon^e_G \xi^G_\alpha)\alpha + \frac{R}{e}\frac{\xi^G_\alpha}{\epsilon^G_R} - f^{eff}\epsilon^e_G \xi^G_\alpha}. \tag{51}$$

Combining (51) with (50) yields:

$$\alpha^* = \underbrace{\frac{1}{1 + \xi^e_{\alpha/G} - \epsilon^e_G \xi^G_\alpha}}_{\text{Marg. cost of }\alpha} \left[\underbrace{\Delta I \left(\xi^{\Delta I}_{\alpha/G} + \frac{\xi^e_{\alpha/G}}{\xi^e_a} \frac{e}{F - e} \xi^{\Delta I}_a \right)}_{\text{Marg. increase market premium}} + \underbrace{\left(\frac{R}{e} \frac{1}{\epsilon^G_R} - f^{eff}\epsilon^e_G \right) \xi^G_\alpha}_{\text{Marg. increase in research}} + \underbrace{\frac{\xi^e_{\alpha/G}}{\xi^e_a} IF \xi^I_a}_{\text{Marg. change in base income}} \right] \tag{52}$$

Equation (52) shows how α^* is set to balance the impact of reducing a by one dollar and to use the resulting funds to increase α. The first term within the brackets measures the net impact of increasing α and of reducing a on the market premium of a student. Since reducing the lump-sum transfer increases both enrollment and the market premium, this term is positive. The second term measures the net impact per student at increasing the public good. The last term measures how changing a changes the gains in marginal income. All these benefits must be divided by the increase in costs induced by the accrued funding the university.

It is also worth analyzing (48) in terms of wealth redistribution. It can be reorganized to find:

$$\frac{e}{\psi}\frac{e_\alpha}{F_\phi} = e\left(\tau p_t t_\alpha L^* + (p(t) + \bar{w})L^*_G - \bar{w} l^*_G - 1\right) + \dots$$
$$\dots + (\Delta I - \alpha)\left(\frac{\mu^{e*}}{\psi}((1-\tau)p_t t^*_\alpha L^* - f^*_\alpha) - \frac{\mu^*}{\psi} 0 \right) + \frac{eV^{e*}_G + (F(\phi_{\max}) - e)V^*_G}{\psi} G_\alpha, \tag{53}$$

which shows explicitly how α changes the premium of students increases in comparison to workers. The second term:

$$\left(\frac{\mu^{e*}}{\psi}((1-\tau)p_t t^*_\alpha L^* - f^*_\alpha) - \frac{\mu^*}{\psi} 0 \right) \tag{54}$$

shows the difference in welfare-weighted effect of α on each persons' income. Because the income of workers is lower, their marginal utility is higher and thus, they have an higher weight ($\mu^* > \mu^{e*}$). A university produces a higher wage for students increasing the wage spread, which increases inequalities.Thus, the efficient subsidy per student is dampened to account for accrued inequalities.

4 From Unconditional Transfers to Enrollment Based: An Empirical Evaluation

An efficient funding formula can be expressed in terms of elasticities of both enrollment and research output. If one assumes that the public good is separable in utility ($V^{e*}_G = V^*_G$) and changes from α have a negligible on Individual's tax base ($\xi^I_\alpha \approx \xi^{\Delta I}_{\alpha/G} \approx \xi^I_a \approx 0$), then (52) reduces to:

$$\alpha^* \approx \frac{1}{1 + \xi^e_\alpha}\left[\Delta I \xi^e_\alpha \frac{F}{F - e} + \frac{R}{e}\frac{\xi^G_\alpha}{\epsilon^G_R} \right] \tag{55}$$

A crucial question is thus the magnitude of ξ^e_α and $\frac{\epsilon^G_\alpha}{\epsilon^G_R}$ and this section turns to an estimation of these parameters.

Prior to year 2000, universities in Québec were funded on unconditional transfers [35]. The public funding of an institution in a given year was based on what has been allocated in the year before, with some marginal adjustments. In the year 2000, the Department of Higher Education changed its funding rule to an enrollment based formula. We use this policy shift to how Quebec universities changed their behavior in comparison with the provinces of British Columbia and Manitoba, two provinces whose funding formula are, in essence, unconditional transfers [11, 42, 32, 33].

The Québec policy shift introduced three new funding channels, which are transfers based on full-time equivalent students (hereafter FTEs), transfers based on weighted FTEs, and transfers based on the size of buildings and related metrics (like the cost of energy) [35]. To give a sense of proportion, 70% of appropriations were transferred through the enrollment based component in the year 2017 [37]. The remainder was distributed through unconditional transfers (11%) and the remaining percentage (9%) in the building related component (id.).

Québec introduced a classification of eleven (11) families of programs combined with three (3) tiers (undergraduate, masters, phd), thus yielding 33 families of FTEs for weighting purpose [36]. Higher weights were given to "heavy/hard sciences" disciplines (engineering, medicine, computer science, etc) while administration, social sciences and arts and letters were given lower weights (id.). If we denote by FTE_{ik} the FTEs of university i in the weighing family k and approximate the building related component to a fixed transfer, the funding given to university i follows the following formula:

$$F^i = T^i + \alpha \sum_{k=1}^{n} (1 + w_k) f(FTE_{ik}), \tag{56}$$

where w_k are the funding weights, α is the base amount per student and $f(\cdot)$ is a strictly increasing function. Thus, $\alpha(1 + w_k)$ is the weighted funding per FTE in the funding family k. During the academic years 2000-2001 to 2004-2005, f was the identify function ($f(FTE) = FTE$) and there were 33 funding categories ($n = 33$). For academic years from 2005-2006 up to 2016-2017, the formula increased the number of families to twenty-four (24), thus increasing the number of families for funding purpose to seventy-two ($n = 24$) [37]. The new weights provided a refinement on categories of programs for funding purpose. Furthermore, a smoothing mechanism was introduced to limit the impact of sharp declines in enrollment. The enrollment statistics used for funding are now either the last years' FTEs or the average FTEs over the last three years, whichever is the highest $f(FTE) = \max\left[FTE_{t-1}, \frac{1}{3}(FTE_{t-1} + FTE_{t-2} + FTE_{t-3})\right]$.

In British Columbia, the funding formula gives yearly unconditional transfers to universities. Each of them has a "budget letter" that specifies how much funds they are to receive in the current year. The basic operating grant is based on a targeted number of FTEs [11]. The key difference, however, is that funds do not evolve with true FTEs, as in the provinces of Québec or Ontario ([39, 38]). As the core operating grant changes from year to year, the funding formula has characteristics that are closer to historical (or unconditional) funding. In Manitoba, the funding formula consists mainly of unconditional transfers adjusted yearly, with some additional funding for specific projects [42, 32, 33].

In terms of university finances, there are three systematic differences between the province of Québec and British Columbia. First, tuition fees have evolved differently [25]. In British Columbia, tuition fees were frozen from 1995 to 2001, increased sharply between the 2001 and up to year 2005, and were then capped to inflation afterwards (see Figure 2) [21]. In Québec, tuition fees were frozen from 1989 to 2007 and have been indexed at roughly 3% since then. Second, Québec requires a two-year college degree in preparation for university before being eligible for a university program. Thus, most undergraduate university degrees in Québec last three years. Third, Quebec shows a historical lag in its post-secondary graduation rates, although the creation of a public postgraduate system in the sixties has mitigated that lag [43, 29]. The last two issues can be seen as systematic differences that have not changed during the period of study. They are thus being absorbed by the constant (or fixed effects) of a regression design. We however add tuition as a control in some of our specifications to account for the potentiel effect of having different tuition levels.

[Figure 2 about here.]

4.1 Identification Strategy for Enrollment and Research

We use a Difference in differences (DID) as our main estimation technique [62]. In its simplest form, a DID compares the rate of increase of the outcome variable between the test group and the control group. In each group, a first difference in the outcome variable is done before and after the policy shift. The main assumption of a DID is that no significant event, other than the policy shift, occurred at the same time in the test group. If true, then the sole explanation for the difference in the rate of increase is the change in policy [61, 2].

Regression wise, let $\mathbb{1}_a$ be a binary variable equal to zero if an observation is before the policy change and equal to one if it is after, $\mathbb{1}_g$ be a binary variable be equal to one if the observation is within the test group and zero if it is in the control group. For FTEs, the estimation model of a DID is then given by:

$$FTE_{kt} = \beta_0 + \beta_1 \mathbb{1}_g + \beta_2 \mathbb{1}_a + \beta_3 [\mathbb{1}_g \mathbb{1}_a] + \sum_j [x_j]_{kt} \Gamma_j + \mu_k + \epsilon_{kt}, \quad (57)$$

where μ_k are program fixed-effects, ϵ_{kt} is the estimation error, $[x_j]_{gt}$ are other control variables and Γ_j other coefficients.

We further perform an estimation of the impact of the policy shift on the number of research articles published by universities in both provinces. If we denote by n_{it} the number of research articles published by university i at time t, the DID equation then has the following form:

$$n_{it} = \phi_0 + \phi_1 \mathbb{1}_g + \phi_2 \mathbb{1}_a + \phi_3 [\mathbb{1}_g \mathbb{1}_a] + \sum_j [x_j]_{it} \Lambda_j + \mu_i + \epsilon_{it}, \quad (58)$$

where ϕ_j and Λ_j are the associated estimated coefficients.

Provided the assumption for identification is true, then testing if the third coefficient β_3, ϕ_3 is equal to zero ($H_0 : \beta_3 = 0$, or $H_0 : \phi_3 = 0$) tests if the policy has any impact on the outcome variable. If the null hypothesis is rejected, its magnitude reveals how strong universities react to the change in the formula. In both specifications, we enrich the baseline model to include the unemployment rate, the 16-25 population and year effect variables as controls. We further bootstrap our error terms 400 times.

4.1.1 Dataset and Descriptive Statistics

We combine a variety of datasets to implement our estimation strategy. We use the Postsecondary Information System (PSIS) survey from the national statistics agency (Statistic Canada) [48]. The survey details enrollment in Canadian universities from 1992 to 2016 and is desegregated by province and field of study (k). For the number of research articles, we use data from Clarivate Analytics [6] that was gathered and cleaned by Université de Montréal's Observatory on Science and Technology. The survey details the number of publications by university and year of publication. We combine these two datasets with time series of tuition fees, unemployment and population counts from Statistics Canada [51, 49, 50] . We provide graphs of total enrollment in Figure 3 below, as well as descriptive statistics in Table **??**.

[Figure 3 about here.]

[Table **??** about here.]

4.2 Results

Table 4 presents the baseline results on enrollment. Each column present the result of (57), where the dependent variable is the FTE enrollment, with the successive addition of a covariate. The coefficient of interest is presented on the first line. It represents the aggregate effect of the policy shift across programs. The first column shows an average effect 668 FTE, although not significant. The next two columns, were tuition fees and the 18-25 year-old population is added, show a coefficient of respectively 2862 FTE and 2135 FTE (nearly significant at the 10% level). The last column shows a positive, but non-significant effect on enrollment. Combined, these results suggest an aggregate increase in enrollment, although it is not significant. In the paragraphs below, it is shown that this aggregate effect hides significant impacts that varies by families of programs.

[Table 4 about here.]

The change in the funding formula introduced important differences in the funding per student. For instance, "hard sciences" were given a much higher weight than "soft sciences". Because weights multiply the base amount per student (α), the resulting amount per student varies by funding family. We thus performed multiple regressions, each one restricted to a category of enrollment. A single coefficient thus isolates the effect of the change in the formula on this category of enrollment. The results are presented in Table 6, 7 and 8 in Appendix. Each table presents the regression results per program, each time adding more covariates. Because the results are robust and do not significantly differ from one table to another, only Table 7 is presented in the main text.

The tables present significant increases in enrollment in programs that have heavier weights in the funding formula, with the sole exception of "Business, management and public administration", who is a lightweight program but still faces a significant increase.[9] Besides business, the most important effects are found in the categories "Architecture and Engineering" and "Health and related studies". The categories "Mathematics, computer and information science", "Education" and "Humanities" show a negative and significant effect. The last two categories are those with the lowest weights in the funding formula. Combined together, this goes to suggest that enrollment weights can be used to steer categories of enrollment.

[Table 7 about here.]

The estimation results of (58) are presented in table (5). The first column suggests that the policy shift increased the number of published articles by roughly 3200 articles. The addition of unemployment variables and of tuition fees - the two inner columns - however suggest a non-significant impact. The last column however suggest a positive impact of roughly 3000 additional research articles. As the data provided has no field identifier, the analysis of impact by field is not possible.

[Table 5 about here.]

4.3 Back of the Enveloppe Estimation

The estimates found in this paper can be used to estimate the value of α with the use of equation (55). Taking an increase of 2800 FTEs as the average impact of the change in formula and using the the average figure of 174 315 FTE in Québec, this implies a 1.6% increase in FTEs. Given that the base amount per student increased from 0 to 2 742\$, the use of the "midpoint method" [34] yields a 200% increase. Combined together, this implies that the elasticity $\xi^e_\alpha \approx 0.008$. Using an increase of 3100 research articles as the impact of the policy shift and an average of 132 598 research articles published in Québec over the timespan, one can find an estimate for the elasticity of the research output $\frac{\xi^G_\alpha}{\epsilon^G_R} \approx 0.012$. Combined with the related statistics found in Table 1, this suggests an approximative value of α of:

$$\alpha^* \approx \frac{1}{1.08}\left[1050000 \cdot 1.49 \cdot 0.008 + \frac{1752240000}{238738}0.012\right] = 11\ 670\$ \tag{59}$$

5 Practical Policy Guidelines

This section provides practical advice based on the theory and the results developed above. It is intended for policy analysts in charge of monitoring or elaborating a funding formula. Very little mathematical background is required to understand this section. It aims to answer the following question: how should a funding formula be designed in practical terms? An answer is provided below through four broad statements.

[9] We hypothesize that this is were there is significant demand and a high compressibility of costs.

Table 1: Parameters For A Back of the Enveloppe Estimation of α

Parameter	Value	Source
$\frac{F}{F-e}$	1.49	Statistics Canada [52]
e	238 738	Statistics Canada [52] and calculations
R	1 752 240 000 ($)	CAUBO [4]
ΔI	1 050 000	Demers [?]

Statistics gather for the 2013-2014 academic year. Enrollment (e) is calculated by assuming that three part-time students equal one FTE.

Universities Respond to Incentives

Universities respond to incentives. They will adapt their behavior, albeit slowly, to make the maximum of the funds provided. Hence, a policy analyst must try to figure out how universities will respond to a funding formula, both qualitatively and quantitatively. This implies in turn that a formula whose components are determined on a costing analysis over a given period of time may induce a response from universities such that the original costs will no longer be observed. This analysis is outlined in the discussion around (33). Depending on how important is the increase in enrollment due to an increase in the quality of teaching, costs per student may either decrease or increase with additional funding. Hence, if the elaboration of a formula is somewhat based on a "static view" of the costs, it is at best a starting point that neglects how the system will respond.

A Funding Formula is A Steering Mechanism

If a funding formula changes decisions made by universities, the policy maker must spend time thinking the proper design of the formula to reach desired social objectives. Increasing the funding per student will alter the fund allocation between research and teaching. The same can be said of unconditional transfers (T). Thus, changing these tools should be done in a way that the response of universities is in the direction of some established qualitative goals. The components of a funding formula should thus be seen as a signaling or steering device much more than an attempt to replicate some desired costs. In order to find how important the changes in production will be, a policy maker requires a measure of universities' responses to changes in funding formulas, which is the next broad statement.

An Efficient Funding Formula Requires Measurements

In order to establish the proper value of the components of a funding formula, one requires plausible empirical values of how universities respond to the said components. Hence, an empirical evaluation of the response to an increase/decrease of funding through a given component should be known and used to devise an efficient one. Building this knowledge requires time and of course, changes in funding formulas. Natural experiments, such as the the ones reported in section 1.1, should be used as initial guidelines to infer some practical starting points. Then, the induced changes in the university system becomes useful feedbacks that should be used to update the components at a later stage.

An Efficient Funding Formula Evolves With Measurements

As the last sentence suggest, an efficient formula requires updates based on the measured feedbacks. Each new update should lead to changes to the formula and bring the university system closer to the desired goals, provided those goals do not change over time.

6 Conclusion

This paper examines how public appropriations to Higher Education should be transferred to a system of universities, that is the design of an efficient funding formula. In broad terms, we show that an enrollment based formula is socially efficient. It should be based on two key observable factors. First, the social value of what university produces, namely the private value bestowed to graduates through higher wages, but also the public value of research. Second, how universities respond to accrued funding. The more responsive they are at increasing research or graduates when funding is increased, the more funding should be provided.

References

[1] Adnot, P., Dupont, J.-L., Gaudin, C., Lagauche, S., Longuet, G., & Richerd, P. (2008). Le financement des universités: pour un SYstème de répartition des Moyens à l'Activité et à la Performance (SYMPA). Rapport du Sénat. Tech. rep., Le Sénat de France.
URL https://www.senat.fr/rap/r07-382/r07-382.html

[2] B Angrist, J., & Pischke, J.-S. (2008). *Mostly Harmless Econometrics*. Princeton University Press.

[3] Bergin, J. (2005). *Microeconomic Theory: A Concise Course*. Oxford University Press UK.

[4] Canadian Association of University Business Officers (2015). Financial Information of Universities and Colleges 2013-2014. Tech. rep., Canadian Association of University Business Officers.
URL www.caubo.ca

[5] Chetty, R. (2009). Sufficient Statistics for Welfare Analysis: A Bridge Between Structural and Reduced-Form Methods. *Annual Review of Economics*, *1*(1), 451–488.
URL http://www.annualreviews.org/doi/10.1146/annurev.economics.050708.142910

[6] Clarivate Analytics, & sur la science et la technologie, O. (2019). Dataset on Published Articles By University. Tech. rep., Université de Montréal.

[7] Cretan, G. C., & Gherghina, R. (2012). Education Funding Methods in European States. *Journal of Knowledge Management, Economics and Information Technology*, *II*(5).
URL www.scientificpapers.org

[8] De Fraja, G., & Iossa, E. (2002). Competition Among Universities and the Emergence of the Elite Institution. *Bulletin of Economic Research*, *54*(3), 275–293.
URL http://doi.wiley.com/10.1111/1467-8586.00153

[9] De Fraja, G., & Valbonesi, P. (2012). The design of the university system. *Journal of Public Economics*, *96*(3-4), 317–330.
URL https://www.sciencedirect.com/science/article/pii/S0047272711001630

[10] Del Rey, E. (2001). Teaching Versus Research: A Model of State University Competition. *Journal of Urban Economics*, *49*(2), 356–373.

[11] Department of Higher Education of British Columbia (2019). Budget Letters for Post-secondary Institutions - Province of British Columbia.
URL https://www2.gov.bc.ca/gov/content/education-training/post-secondary-education/institution-resources-administration/budget-letters

[12] Dougherty, K., & Reddy, V. (2011). The Impacts of State Performance Funding Systems on Higher Education Institutions: Research Literature Review and Policy Recommendations.
URL https://ccrc.tc.columbia.edu/publications/impacts-state-performance-funding.html

[13] Dougherty, K. J., Jones, S. M., Lahr, H., Natow, R. S., Pheatt, L., & Reddy, V. (2014). Performance Funding for Higher Education. *The ANNALS of the American Academy of Political and Social Science*, *655*(1).

[14] Erina, J., & Erins, I. (2015). Assessment of Higher Education Financing Models in the CEE Countries. *Procedia - Social and Behavioral Sciences*, *177*, 186–189.
URL https://www.sciencedirect.com/science/article/pii/S1877042815017334

[15] Frølich, N., Schmidt, E. K., & Rosa, M. J. (2010). Funding systems for higher education and their impacts on institutional strategies and academia: A comparative perspective. *International Journal of Educational Management*, *24*(1), 7–21.
URL https://doi.org/10.1108/09513541011013015

[16] Garvin, D. A. (1980). *The economics of university behavior*. Academic Press.
URL https://eric.ed.gov/?id=ED231251

[17] Gary-Bobo, R., & Trannoy, A. (1998). L'économique politique simplifiée du "mammouth". *Revue française d'économie*, *13*(3), 85–126.
URL http://www.persee.fr/web/revues/home/prescript/article/rfeco{_}0769-0479{_}1998{_}num{_}13{_}3{_}1062

[18] Gary-Bobo, R., & Trannoy, A. (2008). Efficient tuition fees, examinations, and subsidies. *Journal of the European Economic Association*, *6*(6), 1211–1243.
URL http://papers.ssrn.com/sol3/papers.cfm?abstract{_}id=551424

[19] Gary-Bobo, R. J., & Trannoy, A. (2015). Optimal student loans and graduate tax under moral hazard and adverse selection. *RAND Journal of Economics*, *46*(3), 546–576.
URL http://onlinelibrary.wiley.com/store/10.1111/1756-2171.12097/asset/rand12097.pdf?v=1{&}t=j6zg0egn{&}s=27ca7f6a2718549ff1ed2749edf191705720fc35

[20] Government of British Columbia (2019). Budget Letters for Post-secondary Institutions.

[21] Government of British Columbia (2019). Tuition Limit Policy.
URL https://www2.gov.bc.ca/gov/content/education-training/post-secondary-education/institution-resources-administration/tuition-limit-policyhttps://www2.gov.bc.ca/gov/content/health/health-drug-coverage/pharmacare-for-bc-residents/pharmanet

[22] Hearn, J. C., Corcoran, K., Gandara, D., Ness, E., Snyder, M., Stanley, J., & Tierney, S. (2015). Outcomes-Based Funding in Historical and Comparative Context. Tech. rep., Lumina foundation.
URL https://www.luminafoundation.org/files/resources/hearn-obf-full.pdf

[23] Higher Education Quality Council of Ontario (2015). The Ontario University Funding Model in Context. Tech. rep., Higher Education Quality Council of Ontario.
URL http://www.heqco.ca/SiteCollectionDocuments/ContextualBackgroundtotheOntarioUniversityFundingFo
pdf

[24] Hillman, N. W., Tandberg, D. A., & Gross, J. P. K. (2014). Performance Funding in Higher Education: Do Financial Incentives Impact College Completions? *The Journal of Higher Education*, *85*(6), 826–857.
URL https://www.tandfonline.com/doi/full/10.1080/00221546.2014.11777349

[25] Ivanova, I. (2012). Paid in Full Update: Who Pays for University Education in BC? Tech. rep., Canadian Center for Policy Alternatives.
URL www.policyalternatives.ca

[26] Jongbloed, B. (2010). FUNDING HIGHER EDUCATION : A VIEW ACROSS EUROPE. Tech. rep., European Centre for Strategic Management of Universities (EMSU).
URL http://www.utwente.nl/mb/cheps/publications/Publications2010/MODERN{_}Funding{_}Report.pdf

[27] Lawson, N. (2017). Liquidity Constraints, Fiscal Externalities, and Optimal Tuition Subsidies. *American Economic Journal: Economic Policy*, *0*(0).
URL https://www.aeaweb.org/articles?id=10.1257/pol.20150079{&}{&}from=f

[28] Legislative Budget Board Staff (2011). Financing Higher Education In Texas. Tech. rep., Government of Texas.

[29] Lewis, M. J. (2003). *Three essays on labor and urban economics*. Ph.D. thesis, Massachussets Institute of Technology.

[30] Lien, D. (2006). Borderless Education and Domestic Programs. *Education Economics*, *14*(3), 297–308.
URL http://www.tandfonline.com/doi/abs/10.1080/09645290600777519

[31] Lien, D. (2008). Economic analysis of transnational education. *Education Economics*, *16*(2), 149–166.
URL http://www.tandfonline.com/doi/abs/10.1080/09645290701273475

[32] Manitoba Council on Post-Secondary Education (2006). Annual Report 2004-2005. Tech. rep., Manitoba Council on Post-Secondary Education.
URL https://www.edu.gov.mb.ca/ald/pdf/copse{_}ar{_}0405.pdf

[33] Manitoba Council on Post-Secondary Education (2014). Annual Report 2013-2014. Tech. rep., Manitoba Council on Post-Secondary Education.
URL https://www.edu.gov.mb.ca/ald/pdf/copse{_}ar{_}1314.pdf

[34] Mas-Colell, A., Whinston, M. D., & Green, J. R. (1995). *Microeconomic theory*. Oxford University Press.

[35] Ministère de l'Éducation et de l'Enseignement Supérieur du Québec (2000). Règles budgétaires et calcul des subventions de fonctionnement aux universités du Québec. Tech. rep., Ministère de l'Éducation et de l'Enseignement Supérieur du Québec.
URL http://www.education.gouv.qc.ca/references/tx-solrtyperecherchepublicationtx-solrpublicationnou
resultats-de-la-recherche/detail/article/subventions-de-fonctionnement-aux-universites-du-quebec/

[36] Ministère de l'Éducation et de l'Enseignement Supérieur du Québec (2005). Règles budgétaires et calcul des subventions de fonctionnement aux universités du Québec. Tech. rep., Ministère de l'Éducation et de l'Enseignement Supérieur du Québec.
URL http://www.education.gouv.qc.ca/references/tx-solrtyperecherchepublicationtx-solrpublicationnou
resultats-de-la-recherche/detail/article/subventions-de-fonctionnement-aux-universites-du-quebec/

[37] Ministère de l'Éducation et de l'Enseignement Supérieur du Québec (2017). Règles budgétaires et calcul des subventions de fonctionnement aux universités du Québec. Tech. rep., Ministère de l'Éducation du Québec.

[38] Ministère de l'Éducation et de l'Enseignement Supérieur du Québec (2018). Règles budgétaires et calcul des subventions de fonctionnement aux universités du Québec. Tech. rep., Ministère de l'Éducation et de l'Enseignement Supérieur du Québec.

[39] Ministry of Training, C., & Universities (2009). The ontario operating funds distribution manual 2009-2010. Tech. Rep. October, The ontario operating funds distribution manual 2009-2010.
URL http://www.uoguelph.ca/analysis{_}planning/images/pdfs/2009-10-Operating-Manual-Sept09.pdf

[40] Moore, A., & Russ-Eft, D. (2016). Outcomes-Based Funding: Origins and Implications for the SEM Practitioner. *Strategic Enrollment Management Quarterly*, *3*(4), 261–281.
URL http://doi.wiley.com/10.1002/sem3.20074

[41] Neill, C. (2009). Tuition fees and the demand for university places. *Economics of Education Review*, *28*(5), 561–570.
URL http://linkinghub.elsevier.com/retrieve/pii/S0272775709000211

[42] Pakravan, P. (2006). The Future Is Not What It Used to Be: Re-Examining Provincial Postsecondary Funding Mechanisms in Canada. Tech. Rep. 227, C.D.-Howe Institute.
URL http://www.cdhowe.org/pdf/commentary{_}227.pdf

[43] Parent, D. (2014). Institutions et scolarité: le cas des francophones au Québec Vs les américains d'origine canadienne-française. *L'actualité économique*, *90*(3), 147–189.
URL http://expertise.hec.ca/actualiteeconomique/wp-content/uploads/2015/06/90{_}3{_}parent{_}147{_}189.pdf

[44] Piketty, T., & Saez, E. (2013). Optimal labor income taxation. In *Handbook of Public Economics*. Elsevier.

[45] Poyago-Theotoky, J., & Tampieri, A. (2016). University Competition and Transnational Education: The Choice of Branch Campus. *The B.E. Journal of Theoretical Economics*, *16*(2), 739–766.
URL http://www.degruyter.com/view/j/bejte.2016.16.issue-2/bejte-2015-0052/bejte-2015-0052.xml

[46] Raines, J. P., & Leathers, C. G. (2003). *The Economic Institutions of Higher Education*. Edward Elgar Publishing.
URL https://econpapers.repec.org/RePEc:elg:eebook:2721

[47] State University System of Florida, B. o. G. (2019). About the State University System of Florida.

[48] Statistics Canada (2019). Postsecondary Student Information System.
URL https://www.statcan.gc.ca/eng/survey/business/5017

[49] Statistics Canada (2019). Table 14-10-0287-03 Labour force characteristics by province, monthly, seasonally adjusted. Tech. rep., Statistics Canada.
URL https://doi.org/10.25318/1410028701-eng

[50] Statistics Canada (2019). Table 17-10-0005-01 Population estimates on July 1st, by age and sex. Tech. rep., Statistics Canada.

[51] Statistics Canada (2019). Table 37-10-0003-01 Canadian undergraduate tuition fees by field of study. Tech. rep., Statistics Canada.
URL https://doi.org/10.25318/3710000301-eng

[52] Statistics Canada (2019). Table 37-10-0103-01 Participation rate in education, population aged 18 to 34, by age group and type of institution attended. Tech. rep., Statistics Canada.
URL https://doi.org/10.25318/3710010301-eng

[53] Swedish National Agency for Higher Education (2011). Swedish Universities & University Colleges. Tech. rep., Swedish National Agency for Higher Education.
URL http://www.hsv.se/download/18.7dac986013389229f6e800011035/Engelsk+{_}2010.pdf

[54] The State University of New York (2019). What is SUNY?

[55] The University of Texas System (2019). About The University of Texas System.

[56] Umbricht, M. R., Fernandez, F., & Ortagus, J. C. (2017). An Examination of the (Un)Intended Consequences of Performance Funding in Higher Education. *Educational Policy*, *31*(5), 643–673.
URL http://journals.sagepub.com/doi/10.1177/0895904815614398

[57] University of California (2019). The UC System.
URL universityofcalifornia.edu/uc-system

[58] University of California Rebenching Budget Comittee (2012). University of California Rebenching Budget Comittee. Tech. rep., University of California.

[59] University of Hawaii (2007). Funding Formula Use In Higher Education. Tech. rep., University of Hawaii.
URL http://www.hawaii.edu/act188/docs/MGTAmer-funding{_}forumula{_}use.pdf

[60] Venne, J.-F. (2018). Major changes to the funding formula for Quebec universities. *University Affairs*.

[61] Wooldridge, J. M. (2001). *Econometric Analysis of Cross Section and Panel Data*. MIT Press.
URL http://mitpress.mit.edu/catalog/item/default.asp?tid=8632{&}ttype=2

[62] Wooldridge, J. M. (2002). *Econometric analysis of cross section and panel data*. MIT Press.
URL https://books.google.ca/books/about/Econometric{_}Analysis{_}of{_}Cross{_}Section{_}an.html?id=cdBPOJUP4VsC{&}source=kp{_}cover{&}redir{_}esc=y

A Appendix

A.1 Individual's Problem

Denote μ the Lagrange multiplier of (1). The first-order conditions are:

$$0 = V_c - \mu \qquad 0 = -V_L + \mu(1-\tau)(p(t)+\bar{w}), \qquad 0 = (1-\tau)(p(t)+\bar{w})L - c - f + a. \tag{60}$$

The system of derivatives with respect to $x \in \{\tau, a, f, t, G\}$ is given by:

$$\underbrace{\begin{bmatrix} V_{LL} & -V_{Lc} & (1-\tau)(p(t)+\bar{w}) \\ -V_{Lc} & V_{cc} & -1 \\ (1-\tau)(p(t)+\bar{w}) & -1 & 0 \end{bmatrix}}_{\equiv H_1} \begin{bmatrix} L_x \\ c_x \\ \mu_x \end{bmatrix} = \begin{bmatrix} a_x \\ b_x \\ c_x \end{bmatrix}, \tag{61}$$

where a_x, b_x, c_x are the direct effects of variable x on each equation of first order conditions. It can be solved to find:

$$L^*_x = -\frac{(1-\tau)(p(t)+\bar{w})b_x + a_x + c_x((1-\tau)(p(t)+\bar{w})V_{cc} - V_{Lc})}{|H_1|},$$
$$c^*_x = \frac{-(1-\tau)(p(t)+\bar{w})(a_x + (1-\tau)(p(t)+\bar{w})b_x) + (V_{LL} - (1-\tau)(p(t)+\bar{w})V_{Lc})c_x}{|H_1|},$$
$$\mu^*_x = \frac{(V_{LL}V_{cc} - V_{Lc}^2)c_x - ((1-\tau)(p(t)+\bar{w})V_{Lc} - V_{LL})b_x - ((1-\tau)(p(t)+\bar{w})V_{cc} - V_{Lc})a_x}{|H_1|},$$

where $|H_1| \equiv -\left(V_{LL} - 2V_{Lc}(1-\tau)(p(t)+\bar{w}) + (1-\tau)^2(p(t)+\bar{w})^2 V_{cc}\right) > 0$.
Comparative statics for τ:
Solving for τ means that $[a_x, b_x, c_x] = [\mu(p(t)+\bar{w}), 0, (p(t)+\bar{w})L]$ and the system resolves to:

$$L^*_\tau = -(p(t)+\bar{w})\frac{\mu^* + ((1-\tau)(p(t)+\bar{w})V_{cc} - V_{Lc})L^*}{|H_1|} < 0, \tag{62}$$
$$c^*_\tau = (p(t)+\bar{w})\frac{-(1-\tau)(p(t)+\bar{w})\mu^* + (V_{LL} - (1-\tau)(p(t)+\bar{w})V_{Lc})L^*}{|H_1|}, \tag{63}$$
$$\mu^*_\tau = (p(t)+\bar{w})\frac{\mu(V_{LL}V_{cc} - V_{Lc}^2) + L(V_{Lc} - V_{cc}(1-\tau)(p(t)+\bar{w}))}{|H_1|} > 0, \tag{64}$$

With the condition $V_{Lc} < (1-\tau)(p(t)+\bar{w})V_{cc}$ the first derivative is negative and the third one is positive.
Comparative statics for f and a:
Solving for f yields $[a_x, b_x, c_x] = [0, 0, 1]$ and for a the same vector yields $[0, 0, -1]$. Thus, (61) has for solution:

$$L^*_f = \frac{V_{Lc} - (1-\tau)(p(t)+\bar{w})V_{cc}}{|H_1|} > 0, \qquad c^*_f = \frac{V_{LL} - V_{Lc}(1-\tau)(p(t)+\bar{w})}{|H_1|}, \qquad \mu^*_f = \frac{V_{LL}V_{CC} - V_{Lc}^2}{|H_1|} > 0, \tag{65}$$
$$L^*_a = -\frac{V_{Lc} - (1-\tau)(p(t)+\bar{w})V_{cc}}{|H_1|} < 0, \quad c^*_a = -\frac{V_{LL} - V_{Lc}(1-\tau)(p(t)+\bar{w})}{|H_1|}, \quad \mu^*_a = -\frac{V_{LL}V_{CC} - V_{Lc}^2}{|H_1|} < 0. \tag{66}$$

Comparative statics for t: Solving for t yields $[a_x, b_x, c_x] = [-\mu(1-\tau)p_t, 0, -(1-\tau)Lp_t]$ and (61) has for solution:

$$L^*_t = -(1-\tau)p_t\frac{((1-\tau)(p(t)+\bar{w})V_{cc} - V_{Lc})L^* + \mu}{|H_1|} > 0 \tag{67}$$
$$c^*_t = -(1-\tau)p_t\frac{(V_{LL} - (1-\tau)(p(t)+\bar{w})V_{Lc})L^* - (1-\tau)(p(t)+\bar{w})\mu}{|H_1|}, \tag{68}$$
$$\mu^*_t = -(1-\tau)p_t\frac{L(V_{LL}V_{cc} - V_{Lc}^2) - \mu(V_{cc}(1-\tau)(p(t)+\bar{w}) - V_{Lc})}{|H_1|} < 0. \tag{69}$$

Comparative statics for G:
Solving for G yields $[a_x, b_x, c_x] = [-V_{cG}, -V_{LG}, 0]$ and (61) has for solution:

$$L^*_G = \frac{(1-\tau)(p(t)+\bar{w})V_{LG} + V_{cG}}{|H_1|}, \tag{70}$$

$$c^*_G = (1-\tau)(p(t)+\bar{w})L^*_G, \tag{71}$$

$$\mu^*_G = \frac{-V_{LG}(V_{Lc} - (1-\tau)(p(t)+\bar{w})V_{LL}) - V_{cG}(1-\tau)(V_{Lc} - (p(t)+\bar{w})V_{cc})}{|H_1|} < 0. \tag{72}$$

An individual who does not decide to enroll faces the same utility function, but does not pay tuition fees, nor he/she enjoys the additional utility ϕ. The associated maximization problem is thus:

$$\max_{c,l} V(c, 1-l, G) \text{ s.t. } (1-\tau)\bar{w}l \geq c, \tag{73}$$

With the assumption that $V_{Lc} < (1-\tau)\bar{w}V_{cc}$, the labour supply is also negative with the tax rate ($l^*_\tau < 0$). Beside tuition and teaching quality that has no effect, the behavior of the worker is similar to the behavior of the student in every other aspect.

A.1.1 First and Second Order Derivatives of Enrollment

Proof of Proposition 1.
The partial derivative of (5) with respect to f is given by:

$$\begin{aligned}
e_f &= \frac{\partial}{\partial f}\left[F(\phi_{\max}) - F(V(c^*, l^*, G) - V^e(c^{e*}, L^{e*}, G))\right], \\
&= -F_\phi \frac{\partial}{\partial f}\left[V(c^*, l^*, G) - V^e(c^{e*}, L^{e*}, G)\right], \\
&= F_\phi \left[\underbrace{V^e_c(1-\tau)(p(t)+\bar{w})L^{e*}_f - V^e_L L^{e*}_f}_{=0} - \underbrace{V^{*e}_c}_{=\mu}\right], \\
&= -F_\phi \mu^{e*} < 0.
\end{aligned} \tag{74}$$

Similarly, one can find:

$$e_a = -F_\phi\left(\mu^* - \mu^{e*}\right) \tag{75}$$

$$e_t = F_\phi \mu^{e*}(1-\tau)p_t L^* > 0, \tag{76}$$

$$e_G = -F_\phi\left[V^*_G - V^{e*}_G\right], \tag{77}$$

$$e_R = e_G g_R, \tag{78}$$

$$e_\tau = -F_\phi\left[\mu^{e*}(p(t)+\bar{w})L^* - \mu^*\bar{w}l^*\right] \tag{79}$$

Differentiating (74) and (75-78) again yields:

$$e_{ff} = e_f \left(\frac{\mu_f^e}{\mu^e} - \frac{F_{\phi\phi}}{(F_\phi)^2} e_f \right) < 0 \tag{80}$$

$$e_{tf} = e_f \left(\frac{\mu_t^e}{\mu^e} - \frac{F_{\phi\phi}}{(F_\phi)^2} e_t \right) > 0 \tag{81}$$

$$e_{tt} = e_t \left(\frac{\mu_t^e}{\mu^e} - \frac{F_{\phi\phi}}{(F_\phi)^2} e_t + \left(\frac{p_{tt}}{p_t} + \frac{L_t^*}{L^*} \right) \right) \tag{82}$$

$$e_{Gf} = e_f \left(\frac{\mu_G^e}{\mu^e} - \frac{F_{\phi\phi}}{(F_\phi)^2} e_G \right) > 0, \tag{83}$$

$$e_{Rf} = e_f g_R \left(\frac{\mu_G^e}{\mu^e} - \frac{F_{\phi\phi}}{(F_\phi)^2} e_G \right) > 0, \tag{84}$$

$$e_{\tau f} = e_\tau \left((p(t) + \bar{w}) \frac{((1-\tau)(p(t)+\bar{w})V_{cc} - V_{Lc})L^* L_f^* + V_c^{e*} L_f^*}{\mu^{e*}(p(t)+\bar{w})L^* - \mu^* \bar{w} l^*} - \frac{F_{\phi\phi}}{(F_\phi)^2} e_f \right), \tag{85}$$

$$e_{\tau t} = e_\tau \left((p(t) + \bar{w}) \frac{((1-\tau)(p(t)+\bar{w})V_{cc} - V_{Lc})L^* L_t^* + V_c^{e*} L_t^* + V^{e*} L^* p_t}{\mu^{e*}(p(t)+\bar{w})L^* - \mu^* \bar{w} l^*} - \frac{F_{\phi\phi}}{(F_\phi)^2} \frac{e_t}{1-\tau} \right), \tag{86}$$

$$e_{GG} = e_G \left(\frac{\phi_{GG}}{\phi_G} - \frac{F_{\phi\phi}}{(F_\phi)^2} e_G \right), \tag{87}$$

$$e_{RR} = e_R \left(\frac{\phi_{GG}}{\phi_G} - \frac{F_{\phi\phi}}{(F_\phi)^2} e_G + \frac{g_{RR}}{g_R} \right), \tag{88}$$

$$e_{\tau R} = -\frac{F_{\phi\phi}}{F_\phi^2} e_\tau e_R - g_R F_\phi \left[\bar{w} V_{Gc}^* \left((1-\tau)\frac{\partial l^*}{\partial \tau} - l^* \right) - V_{Gl}^* \frac{\partial l^*}{\partial \tau} - (p(t)+\bar{w}) V_{Gc}^{e*} \left((1-\tau) \frac{\partial L^*}{\partial \tau} - L^* \right) + V_{GL}^{e*} \frac{\partial L^*}{\partial \tau} \right], \tag{89}$$

$$e_{aR} = -\frac{F_{\phi\phi}}{F_\phi^2} e_a e_R - g_R F_\phi \left[\bar{w} V_{Gc}^* \left((1-\tau)\frac{\partial l^*}{\partial a} - l^* \right) - V_{Gl}^* \frac{\partial l^*}{\partial a} - (p(t)+\bar{w}) V_{Gc}^{e*} \left((1-\tau) \frac{\partial L^*}{\partial a} - L^* \right) + V_{GL}^{e*} \frac{\partial L^*}{\partial a} \right]. \tag{90}$$

With the assumptions on $F_{\phi\phi}$, (80) is negative and (81) is positive. Equation (82) also requires that $-\frac{p_{tt}}{p_t} > \frac{L_\tau^*}{L^*} + \frac{\mu_t}{\mu} - \frac{F_{\phi\phi}}{(F_\phi)^2} e_t$ to be negative. The signs of (83) and (84) depend on the sign of e_G. If the public good is a complement to consumption, which means that $e_G < 0$, then both cross-derivatives are negative.

Sufficient conditions on the sign of (85) and (86) are that (11) is met. In such case, (85) is negative and (86) is positive. Otherwise, the sign is undetermined and depends on if tax rates have more impact than either tuition or teaching quality.

The sign of (87) depends on the sign of:

$$\phi_{GG} = (V_{Gc}(1-\tau)\bar{w} - V_{GL}^*) l_G^* - (V_{Gc}^e (1-\tau)(p(t)+\bar{w}) - V_{GL}^e) L_G^* + (V_{GG}^e - V_{GG}^{e*}). \tag{91}$$

Its sign requires assumptions on the third (cross-)derivatives of the utility function or assumptions of homogeneity on V. If $G\frac{V_{GL}}{V_L}$, $G\frac{V_{cG}}{V_c}$ are constants and $V_{GG}^* < V_{GG}^{e*}$, then e_{GG} is negative. If the utility function is separable *$V(c, 1-l, G) = u(c, 1-l)G^\alpha$), then e_{GG} is negative. In general terms, if $\phi_{GG} < 0$ or if $\frac{\phi_{GG}}{\phi_G} - \frac{F_{\phi\phi}}{(F_\phi)^2} e_G$, then e_{GG} is negative. These conditions are also sufficient to sign (88) as negative, but they are not necessary.

The signs of (89-90) requires assumptions on the third derivative of V. □

These equations below turn-out useful for algebraic manipulations in the next section:

$$e_t e_{ff} - e_f e_{tf} = \frac{e_f e_t}{L^*} \frac{\mu_f^e}{\mu^*} < 0, \tag{92}$$

$$e_f e_{tt} - e_t e_{ft} = e_f e_t \left(\frac{p_{tt}}{p_t} + \frac{L_t^*}{L^*} \right). \tag{93}$$

If $-\frac{p_{tt}}{p_t} < \frac{L_t^*}{L^*}$, then (93) is negative.

A.2 The University Problem

The following definitions are useful to lighten the notation:

$$\Pi(R,t) \equiv \tilde{\Pi}(g(R,t),t), \tag{94}$$

$$N_1 \equiv (2e_f^* + (\alpha+f^*)e_{ff}^*) < 0 \qquad \text{and} \qquad P_1 \equiv (e_t^* + (\alpha+f^*)e_{tf}^*) > 0, \tag{95}$$

The Lagrangean of each university is given by:

$$\mathcal{L} \equiv \Pi(R,t) + \lambda\left(T + (\alpha+f)e - t - R\right) \tag{96}$$

where λ is the Lagrange multiplier.

A.2.1 Comparative Statics For the Undergraduate University

Proof of Proposition 2. The first order conditions are given by:

$$\Pi_R - \lambda = 0, \tag{97}$$

$$\Pi_t + \lambda\left((\alpha+f)e_t - 1\right) = 0, \tag{98}$$

$$\lambda\left(e + (\alpha+f)e_f\right) = 0, \tag{99}$$

$$(T-R) + (\alpha+f)e - t = 0 \tag{100}$$

Because Π_R is positive, λ is positive. For a change in variable $x \in \{\alpha, T, \tau, a\}$, the system yields the following equations:

$$\underbrace{\begin{bmatrix} \Pi_{RR} & \Pi_{Rt} & 0 & -1 \\ \Pi_{Rt} & \Pi_{tt} + \lambda(\alpha+f)e_{tt} & \lambda P_1 & -\frac{\Pi_t}{\lambda} \\ 0 & \lambda P_1 & \lambda N_1 & 0 \\ -1 & -\frac{\Pi_t}{\lambda} & 0 & 0 \end{bmatrix}}_{\equiv H} \begin{bmatrix} R_x \\ t_x \\ f_x \\ \lambda_x \end{bmatrix} = \begin{bmatrix} 0 \\ a_x \\ b_x \\ c_x \end{bmatrix}. \tag{101}$$

where a_x, b_x, c_x are each equation's direct effect of variable x. Solving for (101), yields:

$$R_x^* = -c_x - \frac{\Pi_t}{\Pi_R} \underbrace{\frac{\Pi_R\Pi_t}{|H|}\left[c_x N_1\left(\frac{\Pi_{RR}}{\Pi_R} - \frac{\Pi_{Rt}}{\Pi_t}\right) + \frac{1}{\Pi_t}\left(b_x P_1 - a_x N_1\right)\right]}_{=t_x^*}, \tag{102}$$

$$t_x^* = \frac{\Pi_R\Pi_t}{|H|}\left[c_x N_1\left(\frac{\Pi_{RR}}{\Pi_R} - \frac{\Pi_{Rt}}{\Pi_t}\right) + \frac{1}{\Pi_t}\left(b_x P_1 - a_x N_1\right)\right], \tag{103}$$

$$f_x^* = \frac{b_x}{\Pi_R}\frac{1}{N_1} - \frac{P_1}{N_1}\underbrace{\frac{\Pi_t\Pi_R}{|H|}\left[c_x N_1\left(\frac{\Pi_{RR}}{\Pi_R} - \frac{\Pi_{Rt}}{\Pi_t}\right) + \frac{1}{\Pi_t}(b_x P_1 - a_x N_1)\right]}_{t_x^*}, \tag{104}$$

$$|H| = \Pi_R\Pi_t\left(N_1 \underbrace{\left(\left(\frac{\Pi_{Rt}}{\Pi_R} - \frac{\Pi_{tt}}{\Pi_t}\right) + \left(\frac{\Pi_{Rt}}{\Pi_t} - \frac{\Pi_{RR}}{\Pi_R}\right)\frac{\Pi_t}{\Pi_R}\right)}_{-1\times(\text{Net})\text{ curvature of the prestige function}} + \frac{\Pi_R}{\Pi_t}\underbrace{\left(P_1^2 - (\alpha+f)e_{tt}N_1\right)}_{-1\times\text{Curvature of the cost function}}\right) < 0. \tag{105}$$

Comparative statics for α

When $x = \alpha$, the right-hand side of (101) equals $[0, -\lambda e_t, -\lambda e_f, -e]'$ and the system resolves to:

$$t_\alpha^* = \frac{\Pi_R\Pi_t}{|H|}\left[-N_1 e\left(\frac{\Pi_{RR}}{\Pi_R} - \frac{\Pi_{Rt}}{\Pi_t}\right) - (\alpha+f)\frac{\Pi_R}{\Pi_t}\left(e_{tf}e_f - e_t e_{ff}\right)\right] > 0, \tag{106}$$

$$R_\alpha^* = e - (1 - (\alpha+f)e_t)t_\alpha^* > 0 \Leftrightarrow \frac{e}{(1-(\alpha+f)e_t)} > t_\alpha^*, \tag{107}$$

$$f_\alpha^* = -\frac{1}{N_1}\left(e_f + P_1 t_\alpha^*\right) > 0 \Leftrightarrow \frac{-e_f}{e_t + (\alpha+f)e_{tf}} < t_\alpha^*. \tag{108}$$

Comparative statics for T
When $x = T$, the right-hand side of (101) is $[0, 0, 0, -1]'$, leading to:

$$t_T^* = \frac{\Pi_R \Pi_t}{-|H|} N_1 \left(\frac{\Pi_{RR}}{\Pi_R} - \frac{\Pi_{Rt}}{\Pi_t} \right) \in (0, 1), \tag{109}$$

$$R_T^* = 1 - \frac{\Pi_t}{\Pi_R} t_T^* \in (0, 1), \tag{110}$$

$$f_T^* = -\frac{P_1}{N_1} t_T^* > 0. \tag{111}$$

Comparative statics for τ
When $x = \tau$, the right-hand side of (101) is :

$$[0, -\lambda(\alpha + f)e_{t\tau}, -\lambda(e_\tau + (\alpha + f)e_{f\tau}), -(\alpha + f)e_\tau]', \tag{112}$$

and the solution is:

$$R_\tau^* = (\alpha + f)e_\tau - \frac{\Pi_t}{\Pi_R} t_\tau^* > 0 \Leftrightarrow (\alpha + f)e_\tau \frac{\Pi_R}{\Pi_t} > t_\tau^* \tag{113}$$

$$t_\tau^* = \frac{\Pi_R \Pi_t}{-|H|} \left[(\alpha + f)e_\tau N_1 \left(\frac{\Pi_{RR}}{\Pi_R} - \frac{\Pi_{Rt}}{\Pi_t} \right) + \frac{\Pi_R}{\Pi_t} \left((e_\tau + (\alpha + f)e_{\tau f})P_1 - (\alpha + f)e_{t\tau} N_1 \right) \right], \tag{114}$$

$$f_\tau^* = -\frac{(e_\tau + (\alpha + f)e_{\tau f})}{N_1} - \frac{P_1}{N_1} t_\tau^* > 0 \Leftrightarrow -\frac{e_\tau + (\alpha + f)e_{\tau f}}{e_t + (\alpha + f)e_{tf}} > t_\tau^*. \tag{115}$$

The sign of t_τ^* depends on the sign of e_τ and $e_{\tau f}$. If both are negative, then taxation decreases teaching quality. Necessary conditions for which this occurs is given by (11). Accordingly, the total derivative of enrollment ($\frac{de}{d\tau} = e_\tau^* + e_f^* f_\tau^* + e_t^* t_\tau^*$) and for research output $\frac{dg}{d\tau} = g_R R_\tau + g_t t_\tau$ are both negative.
Enrollment increases with α
If $f_\alpha^* < 0$, then $e_t t_\alpha + e_f f_\alpha > 0$. So assume that $f_\alpha > 0$, then, from the third equation of (101) and using (80) and (81), one finds:

$$\underbrace{\left[1 - \frac{F_{\phi\phi}}{F_\phi^2 e_f} \right]}_{+} e_\alpha^* + \underbrace{\frac{(\alpha + f)}{\mu} [\mu_t t_\alpha - \mu_f f_\alpha]}_{-} = \underbrace{e - e_f f_\alpha}_{+}, \tag{116}$$

which implies that e_α is positive.
Research output increases with α
Notice first that $1 > 1 - (\alpha + f)e_t > 0$ and $(\alpha + f)e_t > 0$. Direct calculations show that $\Pi_\alpha^* > 0$, which means $\tilde{\Pi}_g g_R R_\alpha^* + \tilde{\Pi}_g g_t t_\alpha^* + \tilde{\Pi}_t t_\alpha^* > 0$. Also, (98) can be re-written as $\tilde{\Pi}_t = \lambda(1 - (\alpha + f)e_t) - \tilde{\Pi}_g g_t$, which means, by substitution of (97), that $\tilde{\Pi}_t = \tilde{\Pi}_g (g_R(1 - (\alpha + f)e_t) - g_t)$. Since both $\tilde{\Pi}_g$ and $\tilde{\Pi}_t$ are positive, one can deduce that $g_R > \frac{g_t}{(1-(\alpha+f)e_t)}$. Finally, consider:

$$\begin{aligned} g_\alpha^* &= g_R^* R_\alpha^* + g_t^* t_\alpha^*, \\ &> g_t^* \left(\frac{R_\alpha^*}{1 - (\alpha + f^*)e_t^*} + t_\alpha^* \right). \end{aligned} \tag{117}$$

From (101), we know that the term within parentheses on the right-hand side is equal to $\frac{e^*}{1-(\alpha+f)e_t^*} > 0$. Thus:

$$g_\alpha^* > \frac{g_t^* e^*}{1 - (\alpha + f^*)e_t^*} > 0. \tag{118}$$

(The proof for g_T^* is the same.)
Enrollment increases with T

Direct calculation of the derivatives gives:

$$e_T^* = \frac{\Pi_R \Pi_{Rt} - \Pi_t \Pi_{RR}}{|H|}(\alpha + f)\underbrace{(e_{ff}e_t - e_f e_{tf})}_{-} > 0. \tag{119}$$

Derivatives are not equal
Consider that the solution to comparative statics for an increase in α and an increase in T are respectively given by:

$$\begin{bmatrix} R_\alpha^* \\ t_\alpha^* \\ f_\alpha^* \\ \lambda_\alpha^* \end{bmatrix} = H^{-1}\begin{bmatrix} 0 \\ -\lambda e_t \\ -\lambda e_f \\ -e \end{bmatrix} \quad \begin{bmatrix} R_T^* \\ t_T^* \\ f_T^* \\ \lambda_T^* \end{bmatrix} = H^{-1}\begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} R_\alpha^* \\ t_\alpha^* \\ f_\alpha^* \\ \lambda_\alpha^* \end{bmatrix} - \begin{bmatrix} R_T^* \\ t_T^* \\ f_T^* \\ \lambda_T^* \end{bmatrix} = H^{-1}\begin{bmatrix} 0 \\ -\lambda e_t \\ -\lambda e_f \\ -e+1 \end{bmatrix}, \tag{120}$$

which implies that teaching funds and tuition do not evolve similarly (research, as similar dynamics, regardless of the policy tool). This implies that $e_\alpha^* = e_t^* t_\alpha^* + e_f^* f_\alpha^* \neq e_t^* t_T^* + e_f^* f_T^* = e_T^*$. Similarly, the research output will evolve differently since $g_\alpha^* = g_R^* R_\alpha^* + g_t^* t_\alpha^* \neq g_R^* R_T^* + g_t^* t_T^* = g_T^*$. □

A.2.2 Comparative Statics For the Graduate University

Proof of Proposition 3. With the assumption $e_t = 0$, the first order conditions of the problem are:

$$\Pi_R + \lambda\left((\alpha + f)e_G g_R - 1\right) = 0, \tag{121}$$

$$\Pi_t - \lambda = 0, \tag{122}$$

$$\lambda\left(e + (\alpha + f)e_f\right) = 0, \tag{123}$$

$$(T - R) + (\alpha + f)e - t = 0. \tag{124}$$

A change in variable $x \in \{\alpha, T, \tau, a\}$ leads to the following system:

$$\underbrace{\begin{bmatrix} \Pi_{RR} + \lambda((\alpha+f)e_{RR}) & \Pi_{Rt} & \lambda P_3 & \frac{-\Pi_R}{\lambda} \\ \Pi_{Rt} & \Pi_{tt} & 0 & -1 \\ \lambda P_3 & 0 & \lambda N_1 & 0 \\ -\frac{\Pi_R}{\lambda} & -1 & 0 & 0 \end{bmatrix}}_{\equiv H_2} \begin{bmatrix} R_x \\ t_x \\ f_x \\ \lambda_x \end{bmatrix} = \begin{bmatrix} a_x \\ 0 \\ c_x \\ d_x \end{bmatrix}. \tag{125}$$

with $P_3 \equiv (e_R + (\alpha + f)e_{Rf})$. Its solution is given by:

$$R_x^* = \frac{\Pi_t \Pi_R}{|H_2|}\left(d_x N_1\left(\frac{\Pi_{tt}}{\Pi_t} - \frac{\Pi_{Rt}}{\Pi_R}\right) + (P_3 c_x - N_1 a_x)\frac{1}{\Pi_R}\right), \tag{126}$$

$$t_x^* = -d_x - \frac{\Pi_R}{\Pi_t}\underbrace{\frac{\Pi_R \Pi_t}{|H_2|}\left(d_x N_1\left(\frac{\Pi_{tt}}{\Pi_t} + \frac{\Pi_{Rt}}{\Pi_R}\right) + (P_3 c_x - N_1 a_x)\frac{1}{\Pi_R}\right)}_{=R_x^*}, \tag{127}$$

$$f_x^* = \frac{c_x}{N_1 \Pi_t} - \frac{P_3}{N_1}\underbrace{\frac{\Pi_t \Pi_R}{|H_2|}\left(d_x N_1\left(\frac{\Pi_{tt}}{\Pi_t} - \frac{\Pi_{Rt}}{\Pi_R}\right) + (P_3 c_x - N_1 a_x)\frac{1}{\Pi_R}\right)}_{=R_x^*} \tag{128}$$

where $|H_2| = -\Pi_R \Pi_t\left(N_1 \frac{\Pi_R}{\Pi_t}\left(\frac{\Pi_{tt}}{\Pi_t} - \frac{\Pi_{Rt}}{\Pi_R}\right) + N_1\left(\frac{\Pi_{RR}}{\Pi_R} - \frac{\Pi_{Rt}}{\Pi_t}\right) + \frac{\Pi_t}{\Pi_R}((\alpha+f)e_{RR}N_1 - P_3^2)\right) < 0$.

Comparative statics for α
For $x = \alpha$, $[a_x, 0, c_x, d_x] = [-\lambda e_R, 0, -\lambda e_f, -e]$ and (125) solves to:

$$R_\alpha^* = \frac{\Pi_t \Pi_R}{-|H_2|}\left(eN_1\left(\frac{\Pi_{tt}}{\Pi_t} - \frac{\Pi_{Rt}}{\Pi_R}\right) - e_f e_R\left(1 + (\alpha + f)\left(\frac{e_{ff}}{e_f} - \frac{e_{Rf}}{e_R}\right)\right)\frac{\Pi_t}{\Pi_R}\right), > 0 \tag{129}$$

$$t_\alpha^* = e - (1 - (\alpha + f)e_G g_R)R_\alpha^* > 0 \Leftrightarrow \frac{e}{1 - (\alpha + f)e_R} > R_\alpha^*, \tag{130}$$

$$f_\alpha^* = -\frac{1}{N_1}\left(e_f + P_3 R_\alpha^*\right) > 0 \Leftrightarrow \frac{-e_f}{e_R + (\alpha + f)e_{Rf}} < R_\alpha^*. \tag{131}$$

With the assumptions on the public good, the term $\frac{e_{ff}}{e_f} - \frac{e_{Rf}}{e_R}$ is positive and (129) is greater than zero.

Comparative statics for T

For $x = T$, $[a_x, 0, c_x, d_x] = [0, 0, 0, -1]$ and (125) solves to:

$$R_T^* = \frac{\Pi_t \Pi_R}{-|H_2|}\left(N_1\left(\frac{\Pi_{tt}}{\Pi_t} - \frac{\Pi_{Rt}}{\Pi_R}\right)\right) \in (0,1), \tag{132}$$

$$t_T^* = 1 - \frac{\Pi_R}{\Pi_t} R_T^* \in (0,1), \tag{133}$$

$$f_T^* = P_3 \frac{\Pi_t \Pi_R}{|H_2|}\left(\frac{\Pi_{tt}}{\Pi_t} - \frac{\Pi_{Rt}}{\Pi_R}\right) > 0. \tag{134}$$

Comparative statics for τ

For $x = \tau$, $[a_x, 0, c_x, d_x] = [-\lambda(\alpha+f)e_{R\tau}, 0, -\lambda(e_\tau + (\alpha+f)e_{f\tau}), -(\alpha+f)e_\tau]$ and (125) solves to:

$$R_\tau^* = \frac{\Pi_t \Pi_R}{-|H_2|}\left((\alpha+f)e_\tau N_1\left(\frac{\Pi_{tt}}{\Pi_t} - \frac{\Pi_{Rt}}{\Pi_R}\right) + (P_3(e_\tau + (\alpha+f)e_{f\tau}) - N_1(\alpha+f)e_{R\tau})\frac{\Pi_t}{\Pi_R}\right), \tag{135}$$

$$t_\tau^* = (\alpha+f)e_\tau - \frac{\Pi_R}{\Pi_t} R_\tau^*, \tag{136}$$

$$f_\tau^* = \frac{-(e_\tau + (\alpha+f)e_{f\tau})}{N_1} - \frac{P_3}{N_1} R_\tau^*. \tag{137}$$

If condition (11) is met and if $e_{R\tau} > 0$, then (135) is negative.

Comparative statics for a

For $x = a$, $[a_x, 0, c_x, d_x] = [-\lambda(\alpha+f)e_{Ra}, 0, -\lambda(e_a + (\alpha+f)e_{fa}, -(\alpha+f)e_a]$ and (125) solves to:

$$R_a^* = \frac{\Pi_t \Pi_R}{-|H_2|}\left((\alpha+f)e_a N_1\left(\frac{\Pi_{tt}}{\Pi_t} - \frac{\Pi_{Rt}}{\Pi_R}\right) + (P_3(e_a + (\alpha+f)e_{fa}) - N_1(\alpha+f)e_{Ra})\frac{\Pi_t}{\Pi_R}\right), \tag{138}$$

$$t_a^* = (\alpha+f)e_a - \frac{\Pi_R}{\Pi_t} R_a^*, \tag{139}$$

$$f_a^* = \frac{-(e_a + (\alpha+f)e_{f\tau})}{N_1} - \frac{P_3}{N_1} R_a^*. \tag{140}$$

If condition $(\mu^* - \mu^{*e} < 0)$ and if $e_{Ra} > 0$, then (138) is negative.

Enrollment increases with T:

A direct calculation of enrollment shows that:

$$e_T^* = \frac{\Pi_R \Pi_T}{|H_2|}\left(\frac{\Pi_{Rt}}{\Pi_R} - \frac{\Pi_{tt}}{\Pi_t}\right)\frac{e g_R^* F_\phi}{\mu^*}\left([V_G^* - V_G^{e*}]\mu_f^* - \mu^* \mu_G^*\right). \tag{141}$$

Such expression is positive with the assumptions that :$V_G^* - V_G^{e*} < 0$.

Enrollment increases with α:

If $f_\alpha^* \geq 0$, then a direct calculation shows that $e_\alpha^* > 0$. So assume that $f_\alpha^* > 0$, we can thus deduce:

$$R_\alpha^* > -\frac{N_1}{P_3} f_\alpha^* > 0. \tag{142}$$

Using the fact that $e_\alpha^* = e_R^* R_\alpha^* + e_f f_\alpha^*$ and the previous inequality, one can find:

$$e_\alpha^* > \left(-e_R^* \frac{N_1}{P_3} + e_f^*\right) f_\alpha^*. \tag{143}$$

The expression between brackets is equivalent to $1 + (\alpha+f)\left(\frac{e_{ff}}{e_f} - \frac{e_{Rf}}{e_R}\right)$, which is positive only if $V_G^* - V_G^{e*} < 0$.

Research Output increases with T:

The proof follows by direct calculation.
Research Output increases with α:
If $t^*_\alpha \geq 0$, then, direct calculation shows that $g^*_\alpha = g_R R^*_\alpha + g_t t^*_\alpha > 0$. So assume that $t^*_\alpha < 0$. Expanding and simplifying the last equation in (125) yields:

$$\begin{aligned} \tilde{\Pi}^*_g g^*_R R^*_\alpha + \tilde{\Pi}^*_g g^*_t t^*_\alpha + \tilde{\Pi}_t t^*_\alpha &= \lambda e, \\ \Leftrightarrow \tilde{\Pi}^*_g g^*_\alpha + \tilde{\Pi}_t t^*_\alpha &= \lambda e. \\ \Rightarrow g^*_\alpha &> 0. \end{aligned} \tag{144}$$

The derivatives of T and α are different:
The proof is identical to the case of the undergraduate university.

□

A.2.3 Proof of Statements For the General University

Proof. The system of comparative statics equations is given by:

$$\begin{bmatrix} \Pi_{RR} + \lambda(\alpha + f)e_{RR} & \Pi_{Rt} & \lambda P_3 & -1 \\ \Pi_{Rt} & \Pi_{tt} + \lambda(\alpha + f)e_{tt} & \lambda P_1 & -\frac{\Pi_t}{\lambda} \\ \lambda P_3 & \lambda P_1 & \lambda N_1 & 0 \\ -1 & -\frac{\Pi_t}{\lambda} & 0 & 0 \end{bmatrix} \begin{bmatrix} R_x \\ t_x \\ f_x \\ \lambda_x \end{bmatrix} = \begin{bmatrix} a_x \\ b_x \\ c_x \\ d_x \end{bmatrix}. \tag{145}$$

The proof relies on the previous results:

1. Research output: it is sufficient to notice that it already increases for the undergraduate university. Adding an additional enrollment incentive is income enhancing and can thus only strengthen the allocation to research funds.

2. Enrollment: it already increases for the graduate university. Adding an additional enrollment incentive makes it income enhancing to enroll students. It can thus only be increasing enrollment.

3. The proof for the difference in the derivatives with respect to the instrument (T or α) is similar to the two previous cases: the left-hand side of the bordered hessian system (the "direct effects") differs. Thus, the derivatives are different.

4. The equation for f_α is given by:

$$P_3 R_\alpha + P_1 t_\alpha + N_1 f_\alpha = -e_f \tag{146}$$

   which means that if tuition increase with α, then it must be that either research funds or teaching funds decrease.

5. The last equation yields of the system yields:

$$R_x + \frac{\Pi_t}{\lambda} T_x = d_x > 0, x \in \{\alpha, T\} \tag{147}$$

   At least one of the two must increase with any of the two variables.

□

B Government's Problem

B.1 Centralized Problem

The first-order condition with respect to variable f is given by:

$$0 = e_f(V^{*e} - V^*) - eV^{*e}_c + \phi F_\phi \phi^e_f + \psi \left[\Delta I e_f + \Delta I_f e + f e_f + e\right] \tag{148}$$

It should be understood that since $-F_\phi \phi^e_x = e_x$ (see (5)), the first and the third term of (148) cancel. This means that marginal utility can be expressed in terms of marginal enrollment. The interpretation is that any loss of utility that is induced by a marginal worker becoming a student (first term) is balanced by the marginal increase in total utility of enjoying studies (third term). Further recall that $V^{*e}_c = \mu^{*e}$, one thus obtains:

$$\frac{(\mu^{*e} - \psi)}{\psi} = \frac{1}{e}\left[\Delta I e_f + e\Delta I_f + f e_f\right] \tag{149}$$

Similarly, with a^e:

$$-\frac{(\mu^{e*} - \psi)}{\psi} = \frac{1}{e}\left[\Delta I e_{a^e} + e\Delta I_{a^e} + f e_{a^e}\right] \tag{150}$$

Since $e_{a^e} = -e_f$, these two FOCs are identical. This is so because f and a^e are substitutes from the student's perspective. They can thus be used interchangeably. The other first-order conditions are given by:

$$-\frac{\mu^* - \psi}{\psi} = \frac{1}{F(\phi_{\max})}\left[\Delta I e_a + I_a F(\phi_{\max}) + e + f e_a\right], \tag{151}$$

$$0 = e(V^{*e}_G - V^*_G) + F(\phi_{\max})V^*_G + \psi\left[\Delta I_G e + \Delta I e_G + I_G F(\phi_{\max} + f e_G - \frac{1}{g_R}\right]. \tag{152}$$

$$\begin{aligned} 0 &= eV^*_c(1-\tau)p_t L^* + e(V^{*e}_G - V^*_G)g_t + F(\phi_{\max})V^*_G g_t + \dots \\ &\dots + \psi\left[\Delta I_t e + \Delta I_G g_t e + \Delta I(e_t + e_G g_t) + (I_G g_t)F(\phi_{\max}) + f(e_t + e_G g_t) - 1\right]. \end{aligned} \tag{153}$$

If one multiplies (152) by g_t and subtracts the result from (153), one obtains:

$$0 = e p_t L^* \frac{(1-\tau)\mu^{*e} + \tau\psi}{\psi} + \left[\left[\tau(p(t) + \bar{w})L^*_t\right]e + \Delta I e_t + f e_t + \frac{g_t - g_R}{g_R}\right]. \tag{154}$$

The first-order condition with respect to τ is:

$$0 = -e\mu^{e*}(p(t) + \bar{w})L^* - F(\phi_{\max})\mu^* \bar{w} l^* + \psi\left[\Delta I_\tau e + \Delta I e_\tau + I_\tau F(\phi_{\max}) + f e_\tau\right]. \tag{155}$$

Proof of proposition 5. If τ is set to zero and one removes one equation because of the equivalence between a^e and f, the system reduces to:

$$\frac{\mu^{*e}}{\psi} = (a^e - a + f)\frac{e_f}{e} + 1, \tag{156}$$

$$-\frac{\mu^*}{\psi} = (a^e + f - a)\frac{e_a}{F(\phi_{\max})} + \frac{e}{F(\phi_{\max})}, \tag{157}$$

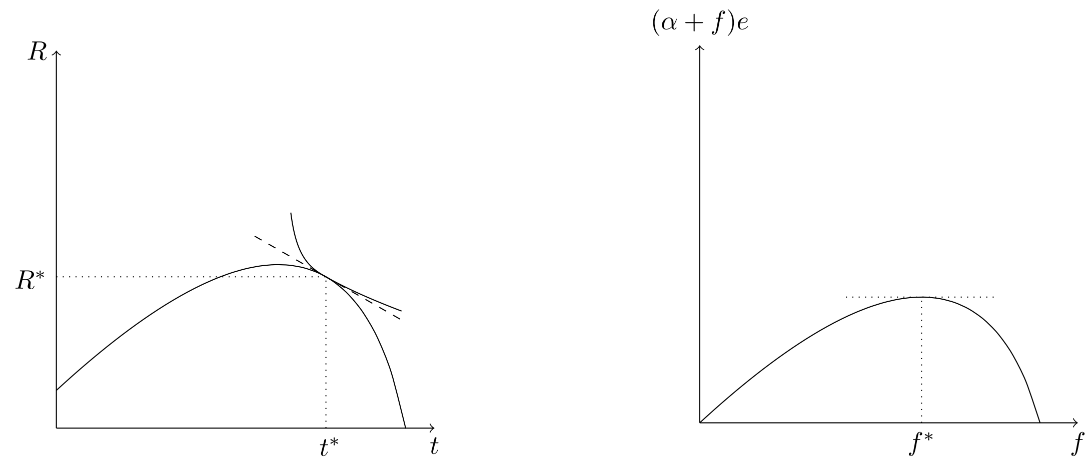
$$-\frac{eV^{*e}_G + (F(\phi_{\max}) - e)V^*_G}{\psi} = \left[(a^e - a + f)e_G - \frac{1}{g_R}\right], \tag{158}$$

$$-\frac{\mu^{*e}}{\psi} = \frac{1}{e p_t L^*}\left[(a^e - a + f)e_t + \frac{g_t - g_R}{g_R}\right], \tag{159}$$

$$t + R = fe + a(F(\phi_{\max} - e), \tag{160}$$

$$0 = 0. \tag{161}$$

This is a system of four equations with five unknowns, with two of them interchangeable. It has a solution and it respects the original first-order conditions of the problem. It is thus a solution to the maximization problem. From (157), one can deduce that $(a^{e*} + f^* - a^*)$ is positive. From (158), one can deduce that R^* is also positive. From (159), the right hand side is negative, which implies that $t > R$. □



(a) Efficient Teaching and Research Funds

(b) Tuition Maximizing Revenues

Figure 1: First-Order Conditions of the University Problem

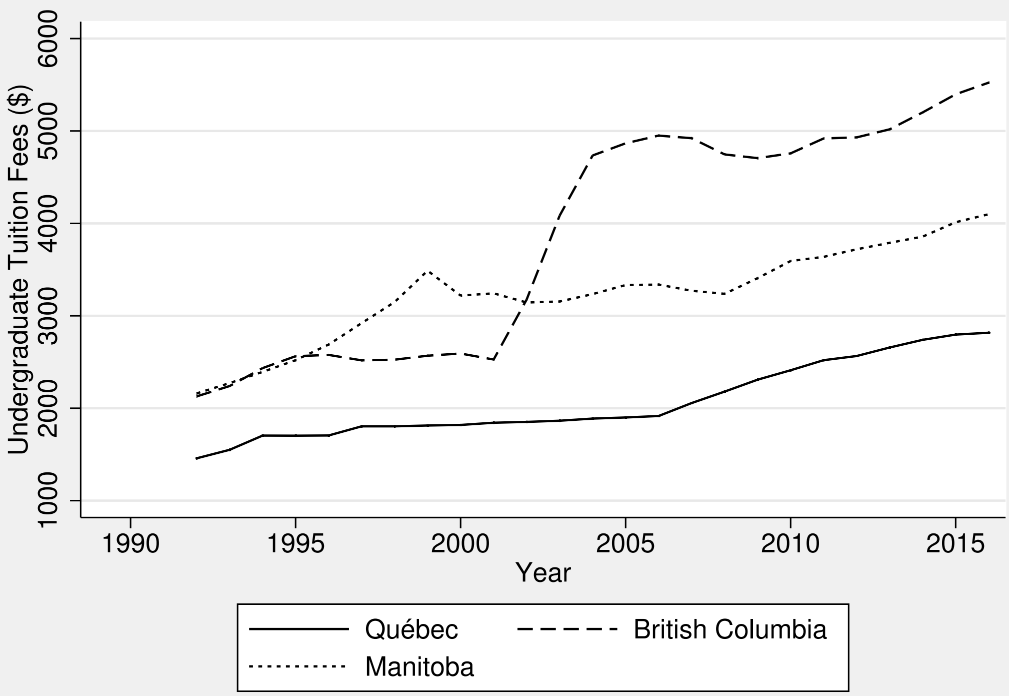
C Tables and Figures

Table 2: Definitions of Elasticities

Symbol	Meaning	Definition
ϵ_x^y	partial elasticity	$\frac{x}{y}\frac{\partial y}{\partial x}$
ξ_y^G	total elasticity of research	$\sum_i \left(g_R R \epsilon_y^R + g_t t \epsilon_y^t\right)$
$\xi_{y/G}^e$	private elasticity of enrollment	$e_f \epsilon_y^f + e_t \epsilon_y^p$
$\xi_{y/G}^{\Delta I}$	private elasticity of the market premium	$p(t)L^* \epsilon_y^p + (p(t) + \bar{w})L^* \left(\epsilon_f^L \epsilon_y^f + \epsilon_t^L \epsilon_y^t\right)$
ξ_y^I	total elasticity of the workers' fiscal base	$\bar{w} l^* (\xi_y^G + \epsilon_y^L)$

D Empirical Tables and Figures

Figure 2: Undergraduate Tuition Fees by Province and Year



Source: Statistics Canada (2019).

Figure 3: Total FTE by Province (Policy Shift in Maroon).

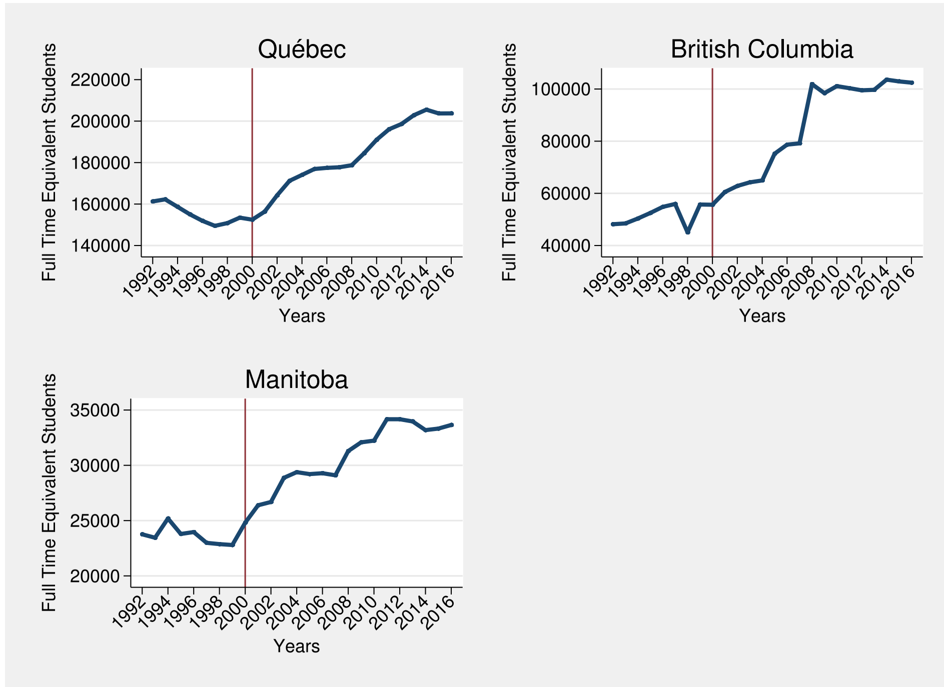


Table 3: Mean FTEs Per Program Type (Québec and British Columbia with Manitoba)

Table 4: Estimation Results for Enrollment

	(1) FTE	(2) FTE	(3) FTE	(4) FTE
Enrolment × Post-reform	668.26	2862.28**	2134.82	1401.93
	(0.608)	(0.033)	(0.107)	(0.286)
Enrolment	10715.01***	12886.63***	17308.13***	16249.54***
	(0.000)	(0.000)	(0.000)	(0.000)
Post-reform	1867.34***	-1687.10***	-103.63	230.23
	(0.000)	(0.001)	(0.853)	(0.618)
Tuition, undergraduates, Canadian students		2.47***	1.63***	1.72***
		(0.000)	(0.000)	(0.000)
18 to 25 y/o population			-0.01***	-0.01***
			(0.000)	(0.000)
Unemployment, 2 lag				211.85
				(0.362)
Observations	825	825	825	825
R^2	0.351	0.379	0.396	0.397
Adjusted R^2	0.349	0.376	0.393	0.392

p-values in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
The *FTE FD* dependant variable is a one year difference in FTE. Standard Errors are bootstraped with 400 replications.
Source: PSIS, Statistics Canada and author calculations. Standard Errors are bootstraped with 400 replications.

Table 5: Estimation Results for the Number of Articles

	(1) Papers	(2) Papers	(3) Papers	(4) Papers
Enrolment × Post-reform	3213.00***	-1920.11	820.84	3026.37**
	(0.002)	(0.134)	(0.573)	(0.016)
Enrolment	6305.44***	3093.08***	5570.82***	11456.11***
	(0.000)	(0.000)	(0.000)	(0.000)
Post-reform	5460.67**	9233.41***	2797.55	4211.20*
	(0.025)	(0.000)	(0.258)	(0.057)
Tuition, undergraduates, Canadian students			2.36***	1.00*
			(0.000)	(0.053)
18 to 25 y/o population				-0.01***
				(0.000)
Unemployment, 2 lag		1337.32***	1168.95***	285.47
		(0.000)	(0.000)	(0.248)
Observations	75	75	75	75
R^2	0.838	0.895	0.940	0.955
Adjusted R^2	0.750	0.834	0.904	0.926

p-values in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
The *FTE FD* dependant variable is a one year difference in FTE. Standard Errors are bootstraped with 400 replications.
Source: Clarivate Analytics, Statistics Canada and author calculations. Standard Errors are bootstraped with 400 replications.

Table 6: Impact on FTEs Per Enrollment Family (Baseline)

	(1) FTE	(2) FTE	(3) FTE	(4) FTE	(5) FTE	(6) FTE	(7) FTE	(8) FTE	(9) FTE	(10) FTE	(11) FTE
Enrolment × Post-reform	4.96	3456.16***	10929.91***	-1292.07***	4575.97***	-7651.38***	-2884.32***	-2757.70***	-1742.86*	4386.66***	325.54
	(0.976)	(0.000)	(0.000)	(0.004)	(0.001)	(0.000)	(0.000)	(0.000)	(0.095)	(0.002)	(0.416)
Enrolment	890.12***	12701.81***	25597.00***	17236.12***	11757.06***	7089.44***	5494.50***	8419.87***	4948.06***	19215.31***	4515.75***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Post-reform	113.74	1014.98**	3145.66***	332.34	2539.28***	6478.40***	503.26*	219.64	2111.83**	2884.48**	1197.11***
	(0.453)	(0.029)	(0.002)	(0.310)	(0.000)	(0.000)	(0.087)	(0.540)	(0.034)	(0.010)	(0.001)
Observations	75	75	75	75	75	75	75	75	75	75	75
R^2	0.465	0.933	0.917	0.981	0.844	0.297	0.784	0.835	0.257	0.870	0.756
Adjusted R^2	0.442	0.930	0.914	0.980	0.838	0.268	0.775	0.829	0.226	0.864	0.745

p-values in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
Each column is a program. Categories are 1) Agriculture, natural resources and conservation 2) Architecture, engineering and related technologies 3) Business, management and public administration 4) Education 5) Health and related studies 6) Humanities 7) Mathematics, computer and information science 8) Other instructional programs 9) Physical and life science and technologies 10) Social and behavioural science and law 11) Visual and performing arts and communication technologies.
Source: PSIS, Statistics Canada and author calculations. Standard Errors are bootstraped with 400 replications.

Table 7: Impact on FTEs Per Enrollment Family (Added Covariates)

	(1) FTE	(2) FTE	(3) FTE	(4) FTE	(5) FTE	(6) FTE	(7) FTE	(8) FTE	(9) FTE	(10) FTE	(11) FTE
Enrolment × Post-reform	310.01	4989.24***	14497.27***	-1216.78***	5362.44***	-3735.76	-2449.28***	-1777.52	72.06	5681.53***	1288.26
	(0.181)	(0.000)	(0.000)	(0.001)	(0.000)	(0.219)	(0.000)	(0.253)	(0.942)	(0.006)	(0.182)
Enrolment	2803.86***	19698.46***	39550.90***	20534.99***	18236.53***	24236.84***	9032.74***	9943.57***	18424.24***	32398.01***	8214.47***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Post-reform	666.77**	1673.62	1852.48	-268.04	6026.99***	1804.32	542.20	-2064.46	1006.63	2132.88	-958.12
	(0.015)	(0.314)	(0.656)	(0.495)	(0.010)	(0.579)	(0.268)	(0.286)	(0.412)	(0.420)	(0.363)
Unemployment, 2 lag	-69.16	-321.35	-580.31	70.34	-77.88	-762.84	-42.33	-54.93	-208.11	70.96	-11.22
	(0.255)	(0.137)	(0.357)	(0.494)	(0.789)	(0.151)	(0.707)	(0.895)	(0.361)	(0.864)	(0.966)
Tuitions, undergraduates, Canadian students	0.04	0.31	1.46	0.37***	0.52	1.04	0.29	0.86	1.09**	1.71**	1.02**
	(0.709)	(0.475)	(0.229)	(0.004)	(0.412)	(0.366)	(0.106)	(0.212)	(0.012)	(0.027)	(0.029)
18 to 25 y/o population	-0.00***	-0.01***	-0.02***	-0.01***	-0.01***	-0.03***	-0.01***	-0.00	-0.02***	-0.02***	-0.01***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.725)	(0.000)	(0.000)	(0.003)
Observations	75	75	75	75	75	75	75	75	75	75	75
R^2	0.949	0.989	0.980	0.999	0.971	0.913	0.980	0.927	0.974	0.988	0.936
Adjusted R^2	0.916	0.983	0.967	0.998	0.952	0.857	0.968	0.881	0.957	0.980	0.895
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

p-values in parentheses, $* \ p < 0.1$, $** \ p < 0.05$, $*** \ p < 0.01$

Each column is a program. Categories are 1) Agriculture, natural resources and conservation 2) Architecture, engineering and related technologies 3) Business, management and public administration 4) Education 5) Health and related studies 6) Humanities 7) Mathematics, computer and information science 8) Other instructional programs 9) Physical and life science and technologies 10) Social and behavioural science and law 11) Visual and performing arts and communication technologies.

Source: PSIS, Statistics Canada and author calculations. Standard Errors are bootstraped with 400 replications.

Table 8: Impact on FTEs Per Enrollment Family (Covariates and Fixed Effects)

	(1) FTE	(2) FTE	(3) FTE	(4) FTE	(5) FTE	(6) FTE	(7) FTE	(8) FTE	(9) FTE	(10) FTE	(11) FTE
Enrolment × Post-reform	464.67***	5452.81***	15084.01**	-1271.06**	6005.70*	-3907.91***	-2431.42***	-1237.91	539.01	7125.14***	1820.19
	(0.000)	(0.000)	(0.011)	(0.019)	(0.079)	(0.000)	(0.000)	(0.361)	(0.627)	(0.001)	(0.175)
Enrolment	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(.)	(.)	(.)	(.)	(.)	(.)	(.)	(.)	(.)	(.)	(.)
Post-reform	1264.89***	3466.39	4121.58	-477.96	8514.68	1138.54	611.30	22.37	2812.47***	7715.77	1098.99
	(0.000)	(0.378)	(0.803)	(0.234)	(0.301)	(0.796)	(0.559)	(0.990)	(0.000)	(0.368)	(0.187)
Unemployment, 2 lag	-143.57**	-544.40**	-862.62	96.46**	-387.39	-680.01	-50.93	-314.56	-432.78	-623.64	-267.16*
	(0.011)	(0.013)	(0.433)	(0.018)	(0.415)	(0.107)	(0.417)	(0.398)	(0.107)	(0.247)	(0.088)
Tuitions, undergraduates, Canadian students	-0.13**	-0.19	0.82	0.43***	-0.17	1.23	0.27	0.28	0.58	0.15	0.45
	(0.013)	(0.905)	(0.913)	(0.010)	(0.970)	(0.550)	(0.820)	(0.657)	(0.548)	(0.971)	(0.312)
18 to 25 y/o population	-0.01**	-0.04*	-0.05	-0.00	-0.04	-0.02	-0.01	-0.03	-0.05	-0.10***	-0.03
	(0.016)	(0.092)	(0.545)	(0.717)	(0.385)	(0.418)	(0.539)	(0.310)	(0.177)	(0.000)	(0.278)
Observations	75	75	75	75	75	75	75	75	75	75	75
R^2	0.773	0.850	0.825	0.782	0.857	0.821	0.874	0.658	0.836	0.900	0.692
Adjusted R^2	0.619	0.747	0.705	0.634	0.759	0.700	0.788	0.424	0.724	0.832	0.483
Program fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

p-values in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
Each column is a program. Categories are 1) Agriculture, natural resources and conservation 2) Architecture, engineering and related technologies 3) Business, management and public administration 4) Education 5) Health and related studies 6) Humanities 7) Mathematics, computer and information science 8) Other instructional programs 9) Physical and life science and technologies 10) Social and behavioural science and law 11) Visual and performing arts and communication technologies.
Source: PSIS, Statistics Canada and author calculations. Standard Errors are bootstraped with 400 replications.