Oligopolistic competition for the provision of hospital care

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Abstract

Competition in the market for health care has followed different patterns, and some health care systems have opted for mixed markets where public organisations compete alongside private ones. Empirical evidences on these market structures are however mixed. In this article we argue that public hospitals, which have different objectives from private ones and face different constraints, may also be perceived differently by patients. For this reason we model the market for hospital care as Salop circle with a centre where the public hospital is located; private providers are located on the circle. We show that, depending on the difference in the productivity advantage, mixed markets may outperform both the benchmark (one public hospital at the centre) and private competition ($N$ private providers competing along the circle), but the welfare distribution of these improvements should be carefully analysed since they may derive from the profit of private hospitals. In these cases a monopoly franchise on the mixed market should be introduced to redistribute these benefits.

Keywords: Private hospitals, Public hospitals, Salop competition.

JEL: D47, H44, L51
1 Introduction

The provision of hospital care has been reshaped in several countries where escalating health care costs and shrinking public resources undermine the long run sustainability of public health care.

In the quest to reduce cost and enhance quality, competition and privatisation have been introduced in hospital provision. This process has followed different patterns and some health care systems have created mixed markets where public organisations compete alongside private ones. In most European countries, the share of public hospitals is decreasing, (EUROSTAT, 2016), in spite of mixed evidences on the performance of competition (Duggan, 2004; Amirkhanyan, 2008; Gaynor et al., 2015). The empirical literature does in fact agree neither on the estimation of the productivity differential between organizations pursuing different objectives (Chang et al., 2004; Tiemann et al., 2012), nor on the evaluation of quality by users (Gravelle et al., 2014; Gaynor et al., 2015; Goddard, 2015). The theoretical literature shows that there is a limited scope for these markets: Brekke et al. (2012) suggest that the reduction in efficiency produced by the presence of profit constraints outweighs the altruistic motivation; Herr (2011) shows that public providers may be better than private one while Levaggi (2007) and Levaggi and Levaggi (2017) show that altruistic providers may outperform private providers only in symmetric markets.

Most models in this literature assume that hospitals compete for patients in settings where providers may be asymmetric in their objectives, but are on the same level as concerns patients’ evaluation. However, this may not be the case because patients’ choices are influenced by past experiences (which may create a bias towards public hospitals, given the recent introduction of private providers), and they are concerned by the cost reducing drive of private hospitals (Gu and Johar, 2017). Furthermore, information on quality regulators disclose is quite different across countries (Siciliani et al., 2017).

We argue that a more reasonable model for competition in this market is to consider private hospitals competing on the same level with a reference supplier represented by the public hospital.

Following Bouckaert (2000) and Madden and Pezzino (2011), we model a market for hospital care where patients may choose between being treated by a public hospital located at the centre of a circle or by \( N \) horizontally differentiated private suppliers that choose their location on the circle. Due to the nature of the service, the user charge paid by patients covers only a fraction (if any) of the cost, hence providers must be subsidised.
Our model shows that a mixed market for hospital care may improve welfare, if some conditions are met. These conditions relate to the productivity differential, the level of intrinsic motivation of public providers, the cost structure of the service involved and patients quality evaluation. From a policy point of view interesting results are emerging: the welfare improvement brought about by the mixed market is often driven by the profit of the private hospitals rather than by quality improvements. In this case, the use of a monopoly franchise to regulate market entry may be advisable to compensate for the deficit of the public hospital.

The paper will be organised as follows: in Section 2 we present the main features of the model proposed and the most important features of the market structures analyses; in Section 3 we analyse the main determinants of the difference in welfare of the models proposed and in Section 4 we present the lines that a regulator should follow in choosing the preferred market alternative and some concluding remarks.

2 The model

We model the decision process of a regulator wishing to design the architecture of the market for hospital care $Q$. The provision can be granted using two types of potential providers: profit-maximizing firms ($P$) and public providers with altruistic preferences ($A$). Productivity level and service quality can be observed by the regulator, but they are not verifiable. $P$-s hospitals are more efficient in cost-reducing activities; $A$-s pursue different objectives and share with users preferences for quality enhancement. (Brekke et al., 2012; Levaggi and Levaggi, 2017).

2.1 The environment

A community consisting of a mass of patients (normalised to 1 for simplicity) is uniformly distributed on a circle of unit length.

They are all entitled to receive one unit of hospital care $^1$ from one of the $N$ private hospitals on the circle or from a public hospital located at the centre. $Q$ is supplied for free, but users incur distance costs to acquire it. Individuals choose the preferred provider by comparing service quality net of travel costs. For each unit of care the provider receives a fixed payment $T$. The objective function of a

$^1$See Levaggi and Levaggi (2011) for an alternative formulation.
generic provider $h$ is:

$$V_h = (1 - \rho_h)\Pi_h + \rho_h \varphi q_h$$

where $\Pi_h$ is the gross surplus (i.e. the difference between revenue and running costs), $q_h$ is quality of care, $\varphi$ is patients evaluation of quality and $\rho_h$ is a measure of privatisation of the hospital. Public hospitals cannot retain surplus ($\rho_h = 1$) and pursue quality enhancement; private hospitals retain their surplus ($\rho_h = 0$), but for them quality is a means to attract patients.

The unit cost to produce care with a minimum verifiable level of quality ($\bar{q} = 0$ for simplicity) is $\beta$. It can be reduced through activities that, in line with the literature (Brekke et al., 2012; Levaggi and Levaggi, 2017; Levaggi, 2007), are provider-dependent; cost reduction will be denoted by $(1 - \rho_h)\theta_h$.

Costs for quality enhancement depend on intrinsic motivation. If the staff is intrinsically motivated, the cost to produce quality is non monetary, coincides with the disutility of the effort and is quadratic in the quality. In line with the literature on motivation crowding out (Frey and Jegen, 2001; Bertelli, 2006), we assume that intrinsic motivation is a latent characteristic of the staff which is inversely related to profit retainment. If the hospital is private or gets privatised, intrinsic motivation is crowded out and to increase quality the management has to pay higher salaries. We assume that this cost depends linearly on the quality at rate $k$. With these assumptions, for a generic hospital $h$ increasing quality causes a production cost $C_h$ and a disutility $E_h$ equal to:

$$C_h = (\beta - (1 - \rho_h)\theta_h)D_h + (1 - \rho_h)kq_hD_h, \quad E_h = \rho_h \frac{\theta_h}{2} q_h^2, \quad (1)$$

where $D_h$ is the demand for hospital $h$ and $1 - \rho_h$ is the fraction of profit that providers are allowed to retain.

Finally, we assume the existence of a fixed entry cost $H$, which represents the investment in technology necessary to run the hospital and is independent on the type of provider. The demand $D_h$ is derived from the choices of patients. The utility of a generic individual choosing hospital $h$ located at a distance $x$ is:

$$U_h = v + \varphi q_h - mx, \quad (2)$$

where $v$ is the intrinsic utility of health care, sufficiently high to make any user access the service from some provider. The parameter $\varphi > 0$ is the evaluation quality, thus $\varphi q_h$ is the monetary equivalent gain derived from using the service offered by hospital $h$. Finally, patients that address their demand to a private
provider incur a cost proportional to the distance they have to travel, i.e. \( mx \). Patients that choose to be treated by the public provider located at the centre of the circle incur a fixed travel cost, independent of their position. To simplify notation we denote it by \( \delta \), so that the utility these patients receive is:

\[
U_c = v + \varphi q_c - \delta
\]

where \( q_c \) is the quality offered by the public provider.

Since \( T \) is the unit reimbursement, from (1) the gross surplus of each provider is \( \Pi_h = T D_h - C_h \). Defining \( S = T - \beta \) we then write the objective function of a generic provider \( h \) as:

\[
V_h = (1 - \rho_h) \Pi_h + \rho_h \varphi q_h - E_h - H
\]

\[
= (1 - \rho_h) (S + \theta_h - k(1 - \rho_h)q_h) D_h + \rho_h \left( \phi q_h - \frac{\theta_h q_h}{2} \right) - H.
\]

The regulator that has to decide which market structure maximises welfare (defined as the sum of patients’ utility and private providers’ profit, net of potential losses of the public hospital) among three different scenarios:

- **benchmark**: a public hospital located at the centre of a circle supplies care as a monopolist;
- **mixed market**: \( N_M \) private suppliers are allowed on the circle. By assumption, they are symmetric players and their location will therefore be symmetric. The public hospital retains its position at the centre;
- **private market**: \( N_P \) private hospitals enter the market and decide their position on the circle. The public hospital is either privatised (hence becoming a private competitor on the circle) or it is closed down.

The regulator will evaluate the best reply functions of the available providers (public and private) in different competition settings and compare their welfare properties. We assume that information about the parameters of the environment (e.g. \( v, \varphi, m \)), the market (e.g. the costs \( \beta, k \) and \( H \)) and the providers (e.g. \( \theta_h \)) is available. In what follows we present the optimal solution for each market alternative.

### 2.2 Benchmark

The benchmark represents the status quo, where a public hospital supplies care as a monopolist to the patients around the circle. The optimal level of quality \( q_b \) in
this setting is derived from the maximisation of the objective function in (4) for a
unique provider with \( \rho_h = 1 \) and \( D_h = 1 \):

\[
\max_q \left( \varphi q - \frac{\theta_c}{2} q^2 - H \right).
\]

The optimal solution is readily derived using the F.O.C.s:

\[ q_b = \frac{\varphi}{\theta_c}. \tag{5} \]

Given that quality does not depend on the reimbursement, the regulator sets \( T \)
to the minimum level that allows the public hospital to be budget balanced, i.e.
\( T = \beta + H \). In the notations of (4) this is equivalent to \( S = H \) and total welfare is:

\[ W_B = v - \beta - H + \frac{\varphi^2}{\theta_c} - \delta. \tag{6} \]

### 2.3 Salop competition with a public firm at the centre

In this case the regulator opens to competition and \( N_M \) private providers are al-
lowed to enter the market and decide their location on the circle. From now on,
the index \( c \) denotes quantities and parameters related to the public hospital and
the index \( i \) those of private ones. We develop a two-stage game. In stage one,
private hospitals decide whether or not to enter the market. In stage two, hospit-
als compete in quality. As in Bouckaert (2000), we solve the game by backward
induction.

Depending on the productivity differential of the providers and travelling costs,
three different outcomes are possible.

**a.** The hospital at the centre gets all the market. This occurs if \( q_c \) (the qual-
ity of the public provider) and \( q_i \) (the quality of private hospitals) are suf-
ficiently different. A consumer on the circle receives the maximum utility
when transport costs are equal to zero, i.e. \( v + \varphi q_i \). Travelling to the centre
has a cost equal to \( \delta \), thus the utility for being treated by the public hospital
is \( v + \varphi q_c - \delta \). If \( q_c \geq q_i + \frac{\delta}{\varphi} \) all the consumers go to the centre.

**b.** The hospital at the centre gets no market. The public hospital can best attract
consumers located farther away from providers on the circle. In a symmet-
ric model with \( N_M \) firms on the circle the maximum distance to the nearest

\[ ^2 \]Entry may be furtherly restricted by the provider.
private provider is \( \frac{1}{2NM} \) and the net utility is \( v + \phi q_i - \frac{m}{2NM} \). Utility to be treated at the centre is \( v + \phi q_c - \delta \). If \( q_c \leq q_i + \frac{\delta}{\phi} - \frac{m}{2NM} \) all patients choose private hospitals.

c. The hospital at the centre attracts part of the consumers on the circle. Let us consider a representative private hospital \( i \). The position \( x \) of the indifferent patient is found by equating the net utilities received from the nearest private hospital and the public one, i.e. \( q_c - \delta = q_i - mx \), i.e.

\[
x_M = \frac{\phi(q_i - q_c) + \delta}{m}.
\]  

Each private hospital has a market share equal to

\[
D_i^M = 2x_M = 2 \frac{\phi(q_i - q_c) + \delta}{m},
\]

while the market share of the public hospital is

\[
D_c^M = 1 - 2NMx_M.
\]

We examine the equilibrium in the last case and will then state the conditions for the existence of an internal solution. From (4) hospitals on the circle maximise the following objective function:

\[
V_i = \Pi_i = (S + \theta_i - kq_i)2x_M - H,
\]

while for the public hospital the objective function is:

\[
V_c = \phi q_c - \frac{\theta_c}{2} q_c^2 - H.
\]

The game is solved in Appendix A.1 and the solution can be written as:

\[
q^M_c = \frac{\phi}{\theta_c},
\]

\[
q^M_i = \frac{\theta_i + S}{2k} + \frac{1}{2} \left( \frac{\phi}{\theta_c} - \frac{\delta}{\phi} \right),
\]

\[
x_M = \frac{\phi}{2m} \left( \frac{\theta_i + S}{k} + \frac{\delta}{\phi} - \frac{\phi}{\theta_c} \right),
\]

\[
\Pi^M_i = \frac{\phi k}{2m} \left( \frac{\theta_i + S}{k} + \frac{\delta}{\phi} - \frac{\phi}{\theta_c} \right)^2 - H.
\]
The market is feasible if each competitor has a market share and if the profits of private providers are non-negative. As shown in Appendix A.1 the existence conditions can be translated into the following set of inequalities:

\[ H < \frac{mk}{2\varphi^2}, \quad N_M < \sqrt{\frac{mk}{2H\varphi^2}}, \quad \theta_i^L \leq \theta_i < \theta_i^H, \quad (11a) \]

\[ \theta_i^L := k\left(\frac{\varphi}{\theta_c} - \frac{\delta}{\varphi}\right) + \sqrt{\frac{2Hmk}{\varphi}} - S, \quad (11b) \]

\[ \theta_i^U := k\left(\frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} + \frac{m}{\varphi N_M}\right) - S. \quad (11c) \]

The conditions on \( \theta_i \) imply that quality should be sufficiently homogeneous for both providers to have a market share. The number of competitors on the market affects the range of values for which the mixed market exists, but there is not a market clearing condition. The regulator has to decide how many competitors are allowed to enter. \( N_M \) is a strategic variable in this context, but its choice is constrained by the conditions in (11). If \( N \) is the greatest integer for which \( N < \sqrt{\frac{mk}{2H\varphi}} \) and \( n \in \{1, \ldots, N\} \) we define:

\[ \theta_i^U(n) = k\left(\frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} + \frac{m}{\varphi n}\right) - S. \]

A mixed market is feasible only if \( \theta_i \in [\theta_i^L, \theta_i^U(1)] \) and the number \( N_M \) has to satisfy the following constraints:

\[ \theta_i \in [\theta_i^L, \theta_i^U(n)] \Rightarrow 1 \leq N_M \leq n. \]

Therefore on each subinterval of the feasibility set for \( \theta_i \) different choices for \( N_M \) are allowed.

The welfare function is the sum of the utility of the users, the profits of the private providers and the possible losses of the public provider. If \( S = H \) (as in benchmark), with the market share equal to \( 1 - 2N_Mx_M \), the public hospital is not able to fully cover the fixed cost \( H \). The ensuing deficit, equal to \( 2HN_Mx_M \), has to
be covered using public funds. Thus the welfare is:

\[ W_M = v - \beta - H + 2N_M \int_0^{x_M} (\varphi q_i^M - mz) dz + (1 - 2N_M x_M) (\varphi q_c - \delta) - 2HN_M x_M + \Pi^M_i N_M \]

\[ = v - \beta - H + \frac{\varphi^2}{\theta_c} - \delta + 2N_M x_M \left( m x_M - \frac{1}{2} m x_M - H \right) + \Pi^M_i N_M \]

\[ = W_B + N_M (x_M(m x_M - 2kq_i^M + 2\theta_i) - H). \]

\[ (12) \]

### 2.4 Salop competition among private providers

If the public hospital at the centre is too inefficient or if the regulator decides to privatise it completely, the market is modelled by a standard Salop competition. In this case the solution can be obtained using backward induction starting as in the mixed model from stage two. Let us assume that \( N_P \) competitors are located around the circle. Hospitals are identical, are placed at equal distance along the circle and maximise their profit, as in (8). The market share for each hospital is determined by the position \( x_P \) of the consumer who is indifferent between hospital \( i \) and hospital \( j \):

\[ x_P = \frac{\varphi (q_i - q_j)}{2m} + \frac{1}{2N_P} \cdot \]

The game is solved in Appendix A.2 and gives the following solution:

\[ q_i^P = \frac{\theta_i + S}{k} - \frac{m}{\varphi N_P}, \quad x_P = \frac{1}{2N_P}. \]

\[ (13) \]

The profit of each provider amounts to

\[ \Pi_i^P = \frac{mk}{\varphi N_P^2} - H. \]

If market entry is free, the number of hospitals is determined by the condition \( \Pi_i^P \geq 0 \), i.e.\(^3\)

\[ N_P = \left\lfloor \sqrt{\frac{mk}{H \varphi}} \right\rfloor. \]

\[ (14) \]

\(^3\)The standard notation \( \lfloor x \rfloor \) is used to denote the greatest integer that is less than or equal to \( x \).
<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Private market</th>
<th>Mixed Market</th>
</tr>
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<tbody>
<tr>
<td>quality centre</td>
<td>$\frac{\phi}{\theta_c}$</td>
<td>$\frac{\phi}{\theta_c}$</td>
</tr>
<tr>
<td>quality border</td>
<td>$\frac{\theta_i + H}{k} - \frac{m}{\phi N_p}$</td>
<td>$\frac{\theta_i + H}{2k} + \frac{1}{2} \left( \frac{\phi}{\theta_c} - \frac{\delta}{\phi} \right)$</td>
</tr>
<tr>
<td>$N$</td>
<td>$N_P = \left\lfloor \sqrt{\frac{mk}{H \phi}} \right\rfloor$</td>
<td>$N_M &lt; \sqrt{\frac{mk}{2H \phi}}$ - set by the regulator</td>
</tr>
<tr>
<td>demand</td>
<td>$D_P = \frac{1}{N_P}$</td>
<td>$D_i^M = \frac{\phi}{m} \left( \frac{\theta_i + H}{k} + \frac{\delta}{\phi} - \frac{\phi}{\theta_c} \right)$</td>
</tr>
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<td></td>
<td></td>
<td>$D_c^M = 1 - N_M D_i^M$</td>
</tr>
<tr>
<td>existence</td>
<td>$H \leq \frac{km}{4 \phi}$</td>
<td>$H \leq \frac{km}{2 \phi}$</td>
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<tr>
<td></td>
<td>$\theta_i &gt; \frac{mk}{\phi N_p} - H$</td>
<td>$k \left( \frac{\phi}{\theta_c} - \frac{\delta}{\phi} \right) + \sqrt{\frac{2Hmk}{\phi}} - H \leq \theta_i$</td>
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<td></td>
<td>$\theta_i &lt; k \left( \frac{\phi}{\theta_c} - \frac{\delta}{\phi} + \frac{m}{\phi N_M} \right) - H$</td>
</tr>
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</table>

Table 1: Relevant quantities and existence conditions for the different market structures for $S = H$.

Thus, from (13) and (14) the market is feasible if the following conditions are satisfied:

$$\theta_i > \frac{mk}{\phi N_p} - S, \quad H \leq \frac{mk}{4 \phi}. \quad (15)$$

For $S = H$ the welfare function in this case is:

$$W_P = v - \beta - H + 2 N_P \int_{0}^{\frac{1}{2} \sqrt{\frac{mk}{H \phi}}} \left( \phi q_i^P - mz \right) dz + N_P \Pi_i^P$$

$$= v - \beta - H + \phi q_i^P - \frac{m}{4 N_P} + \left( \frac{mk}{\phi N_p^2} - H \right) N_P$$

$$= v - \beta - H + \phi q_i^P - \frac{m}{4 N_P} + \left( \frac{mk}{\phi N_p^2} - H \right) N_P. \quad (16)$$

The results are summarised in Table 1.

The quality offered by the public hospital is independent of what private providers do and is equal to the benchmark, thus average quality can be improved
only if the efficiency of the private provider is sufficiently high. For the mixed market, this condition is verified if \( \theta_i > k \left( \frac{\phi}{\theta_c} + \frac{\delta}{\phi} \right) - H \), a value that is compatible with the existence interval. For a private market the condition can be written as \( \theta_i > \frac{mk}{\theta_c} - H + \frac{\phi_k}{\theta_c} \), i.e. efficiency should be higher than what required for the existence of private market. However, quality is just a measure that can be used to compare the different markets. In the following section we present the full welfare comparison.

3 Welfare analysis

Total welfare is defined as the aggregation of consumers’ welfare and private providers’ profits, net of the costs deriving from the reimbursement and the possible covering of losses. In what follows we analyse the case where \( S = H \), i.e. the reimbursement is equal to the one in benchmark. We present the analysis for the three elements that are mostly relevant for the regulator, namely:

- public expenditure;
- individual net utility;
- profit of the private hospitals.

In what follows we summarise the main technical results, which will then be discussed in Section 4; their derivation can be found in Appendix B. The indices \( B \), \( M \) and \( P \) will be used to distinguish between the three different scenarios (benchmark, mixed market and private market).

3.1 Public expenditure

Public expenditure (denoted by \( G \)) is the cost to provide the service. For \( S = H \), we can write:

\[
G_B = \beta + H, \\
G_M = \beta + H + 2HN_M, \\
G_P = \beta + H.
\]

Public expenditure is higher in the mixed market because of the deficit of the public hospital. \( G_M \) is linear and increasing in \( N_M \), it depends on \( \theta_i \) through the term \( x_M \) and satisfies the following bounds: \( \beta + H \left( N_M \sqrt{\frac{2H \phi}{mk}} + 1 \right) \leq G < \beta + 2H \).
3.2 Patients net utility

Let us examine the money equivalent utility derived from the service, net of the transport costs ($\Phi$) which may also be interpreted as an overall quality measure. From (10) and (13) we have:

$$\Phi_B = v + \frac{\phi^2}{\theta_c} - \delta,$$

$$\Phi_M = v + \frac{\phi^2}{\theta_c} - \delta + N_M \frac{\phi^2}{4m} \left( \frac{\theta_i + H}{k} + \frac{\delta}{\phi} - \frac{\phi}{\theta_c} \right)^2,$$

$$\Phi_P = v + \frac{\theta_i + H}{k} - \frac{5m}{4N_P}.$$

In a mixed market patients net utility is higher than in benchmark. Note also that $\Phi_M$ is increasing in $N_M$, hence the highest level of $\Phi_M$ is achieved by choosing the highest number of private competitors compatible with the existence conditions. Proposition 1 summarises the results for the comparison between $\Phi_M$ and $\Phi_P$, conditional upon the optimal level of $N_M$.

**Proposition 1.** Let the existence conditions in equation (11) and (15) be satisfied. If $5N_M \geq 3N_P$ $\Phi_M > \Phi_P$ for all $\theta_i \in [\theta_i^L, \theta_i^U]$. If

$$2 \sqrt{\frac{2mk}{H \phi}} - \frac{5mk}{2\phi N_P H} < N_M < \frac{3}{5} N_P. \quad (17)$$

there exists $\theta_i^* \in [\theta_i^L, \theta_i^U]$ such that $\Phi_M \geq \Phi_P$ for $\theta_i \in [\theta_i^L, \theta_i^*]$ and $\Phi_M < \Phi_P$ for higher values of $\theta_i$. Otherwise $\Phi_M < \Phi_P$ for all $\theta_i \in [\theta_i^L, \theta_i^U]$.

**Proof.** See Appendix B.1. □

The above result implies that for $N_M = 1$ the inequality $\Phi_P < \Phi_M$ is satisfied for some range of (low) values of $\theta_i$ only if $N_P \leq 3$. For higher numbers of competitors it depends on the ratio between $N_P$ and $\sqrt{\frac{mk}{H \phi}}$: if the latter is sufficiently high (i.e. profits for private providers are low) $\Phi_P$ is always higher than $\Phi_M$ with $N_M = 1$.  

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3.3 Profit of private hospitals

In a private market with free entry the number of competitors $N_p$ is the highest integer compatible with the non negative profit condition, hence providers may have a surplus due to the rounding effect. The total profit $\Pi_p = N_p \Pi^p_i$ does not depend on $\theta_i$, is a discontinuous function of $H$ that is zero whenever $H = \frac{mk}{K^2 \varphi}$ (with $K$ integer), and increases on each interval between two roots. However, as $H$ tends to zero it becomes negligible (see Appendix B.2 for a formal proof).

The total profit in the mixed market is equal to

$$\Pi_m = N_m \Pi^M_i = N_m ((H + \theta_i - k q^M_i) 2 x_M - H);$$

it is increasing and linear in $N_m$, and is a quadratic, convex and increasing function of $\theta_i$ for each fixed $N_m$. For each fixed $\theta_i$ profit is maximised by choosing the highest number of competitors $N_m$. It should however be noted that a higher number of competitors on the circle (an increase in $N_m$) is only compatible with decreasing values of $\theta_i$, hence profits decrease as $N_m$ increases (see Appendix B.2).

The difference $\Pi_p - \Pi_m$ is negative for any $H = \frac{mk}{K^2 \varphi}$ with $K$ integer and by continuity it is negative for almost all values of $\theta_i$ if $H$ is just above this threshold. As $H$ decreases to the next jump of $\Pi_p$ the difference $\Pi_p - \Pi_m$ increases and it is negative only for $\theta_i$ high enough (see Appendix B.2).

“Net profits” in the mixed market, i.e $\Pi_m$ net of the losses of the public hospital, we have:

$$\hat{\Pi}_m := -2 H N_M x_M + \Pi_m = N_M ((\theta_i - k q^M_i) 2 x_M - H).$$

This function is linear in $N_M$, quadratic, increasing and convex as a function of $\theta_i$ for a fixed $N_M$. Its minimum is always negative; its maximum is positive if $\frac{mk}{2(n+1)^2 \varphi} \leq H < \frac{mk}{2n(n+1) \varphi}$ for some integer $n$ (see Appendix B.2).

3.4 Welfare difference: benchmark vs mixed market

The total welfare in a mixed market is given by:

$$W_m = W_B + N_M (x_M (m x_M - 2 k q^M_i + 2 \theta_i) - H),$$

where $W_B$ is the welfare of the benchmark scenario. Thus, if $x_M (m x_M - 2 k q^M_i + 2 \theta_i) - H$ is positive, welfare is higher in the mixed market and the highest welfare level is reached by allowing the maximum number of private providers into the market. The formal result is summarised in Proposition 2.
Proposition 2. Let the existence conditions in equation (11) be satisfied. If
\[ H \leq \frac{m\varphi}{8k} \]
the inequality \( W_M > W_B \) holds for all \( \theta_i \in [\theta_i^L, \theta_i^U] \).

If \( \frac{m\varphi}{8k} < H < \frac{m(\varphi+2k)}{4\varphi N M (N M + 1)} \), there exists \( \hat{\theta}_i \in (\theta_i^L, \theta_i^U) \) such that \( W_M > W_B \) if \( \theta_i > \hat{\theta}_i \) and \( W_M < W_B \) for \( \theta_i < \hat{\theta}_i \).

Otherwise \( W_M < W_B \) for all values of \( \theta_i \) for which a mixed market exists.

Proof. See Appendix B.3.

Corollary 1. Let the hypotheses in Proposition 2 be satisfied. If \( \varphi \geq 2k \) it is \( W_M > W_B \) for all \( \theta_i \in [\theta_i^L, \theta_i^U] \).

Proof. If \( \varphi \geq 2k \) it is \( \frac{m k}{2\varphi} \leq \frac{m\varphi}{8k} \), therefore the first case of Proposition 2 applies to all values of \( H \) for which a mixed market exists.

3.5 Welfare difference: benchmark vs private market

Let us now turn to the difference \( W_P - W_B \). In this case the welfare level in the private market is always higher than in benchmark for the the range of values of \( \theta_i \) for which a mixed market exists, as shown below.

Proposition 3. Let the conditions in equation (15) be satisfied and let \( N_P = \varepsilon \sqrt{\frac{mk}{H c}} \) for some \( 0 < \varepsilon \leq 1 \). Then if

\[ \frac{1}{\varepsilon} \left( 1 - \varepsilon^2 \right) \sqrt{\frac{H m k}{\varphi} - \frac{1}{4\varepsilon} \sqrt{\frac{H m \varphi}{k}} - \left( \frac{\varphi^2}{\theta_c} - \delta \right) } \geq 0 \]

the inequality \( W_P - W_B > 0 \) holds for all admissible values of \( \theta_i \). Otherwise there exists \( \tilde{\theta}_i \) such that \( W_P - W_B > 0 \) for \( \theta_i > \tilde{\theta}_i \) and \( W_P - W_B > 0 \) if \( \theta_i < \tilde{\theta}_i \).

Moreover \( W_P - W_B > 0 \) for any \( \theta_i \geq \theta_i^L \).

Proof. See Appendix B.4.

3.6 Welfare difference: mixed vs private market

From Proposition 3 if both conditions in (11) and (15) are satisfied the welfare level of a private market is higher than that of the benchmark solution. From Proposition 2 it is then necessary to compare the welfare levels of a private and a mixed market only when \( W_M > W_B \). Also, the highest level of welfare in the
mixed market will in these cases be achieved by choosing the greatest possible number of private competitors on the border.

In the comparison between $W_P$ and $W_M$ and the ratio $\frac{\phi}{k}$ plays a fundamental role. Let us study the behaviour of these two functions wrt $\theta_i$. We have:

$$W_P = \Phi_P + \Pi_P, \quad W_M = \Phi_M + \Pi_M - 2HN_M.$$  

From the previous analysis $\Phi_P$ grows linearly in $\theta_i$ at a rate equal to $\frac{\phi}{k}$, while $\Phi_M$ is quadratic and increasing in $\theta_i$ with a slower growth. The term $\Pi_P$ does not depend on $\theta_i$, while the sum $\Pi_M - 2HN_M$ is convex and increasing in this variable for each fixed $N_M$. The maximum value of the derivative is $\frac{1}{2} \phi \frac{H}{k} N_M$, which is greater than $\frac{\phi}{k}$ if $\frac{\phi}{k} < \frac{2m}{m + 2HN_M}$. Thus if $\frac{\phi}{k}$ is low there exist values of $\theta_i$ for which the term $\Pi_M - 2HN_M$ grows faster than $\Phi_P$. As a result the same behaviour can be shown in comparing $W_P$ and $W_M$. In fact:

$$\frac{\partial W_M}{\partial \theta_i} = N_M \left( \frac{\phi}{k} \left( \frac{\phi}{k} + 2 \right) \theta_i + H \frac{\phi^2}{k^2} - 2 + \frac{\phi}{k} \left( \frac{\phi^2}{\theta_i} - \delta \right) \right) \leq \frac{\partial W_M}{\partial \theta_i} |_{\theta_i = \theta_i^U} = \frac{1}{2} \left( 2 + \frac{\phi}{k} \right) - N_M \frac{\phi H}{km}.$$  

The above quantity is greater than $\frac{\phi}{k}$ whenever $\frac{\phi}{k} < \frac{2m}{m + 2HN_M}$, so that if this condition is satisfied the total welfare in the mixed market grows faster than the one in the private market for high values of $\theta_i$.

4 Discussion and Conclusions

After observing the parameters that characterise the environment, the regulator shapes the market by choosing the setting that maximises its objectives, given the constraints of the problem. In what follows we examine its choice. If $H \geq \frac{mk}{2\phi}$, only a public firm at the centre can be a viable solution.

For $H < \frac{mk}{2\phi}$ the benchmark is not the only viable alternative. In this case, also $\theta_i$ and $N_M$ determine which market organisations are feasible. Figure 1 presents the available choices. On the horizontal axis we measure the entry cost $H$, while on the vertical axis we measure $\theta_i$. The red and blue curves delimit the area where a mixed market with $N_M = 1$ is a viable solution. Higher values of $N_M$ are feasible only for lower values of $H$ and for smaller ranges of $\theta_i$ (between the blue curve and the coloured lower curves). As competition increases, market shares shrink
and competitive private providers find it easier to push the public provider out of the market.

For \( \frac{mk}{4\varphi} < H < \frac{mk}{2\varphi} \), in the area delimited by the red and blue curves only benchmark and a mixed market with \( N_M = 1 \) are feasible alternatives. In this case the best option is given by the result in Proposition 2. If \( \frac{\varphi}{k} \) is sufficiently high, from Corollary 1, a mixed market is always preferable. Above and below the two curves, only benchmark is a feasible alternative.

If \( H \leq \frac{mk}{4\varphi} \) also a private market is a feasible alternative. Let us first analyse the combinations of parameters that fall between the curves. From Proposition 3 if both conditions in (11) and (15) are satisfied the welfare level of a private market is higher than in benchmark, hence we can compare this form with a mixed market with the greatest possible number of private competitors (see Section 3.4). Several countervailing effects determine the sign of the welfare difference as shown in the previous section. One of the key parameters is the ratio \( \frac{\varphi}{k} \), which is the ratio of marginal utility of quality to its cost in the private market. Patients’ net utility is
increasing in this parameter, while the profit of private providers in both markets decreases as this ratio increases. This means that, other things being equal, the

\[ \varphi = 2k \]

welfare composition changes according to the value of this ratio. At the same time, the feasible area and the number of possible competitors in the mixed market shrinks as this ratio increases. For this reason, it is not possible to determine easily verifiable necessary or sufficient conditions on \( H \) and \( \theta_i \) for which one solution is superior to the other one.

However, some conclusions are possible: if \( \varphi \) is high the market will be shared among a restricted number of providers and \( N_M \) and \( N_P \) are quite close to one another. In this case, from Proposition 1 patients net utility can be higher in the mixed market, but in these cases, the cost in terms of deficit is higher than this increase and a private market should be chosen. This case is presented in Figure 2. On the horizontal axis we show the feasible intervals of \( \theta_i \) compatible with alternative levels of private firms on the circle (\( N_M \)). As shown in Figure 1, the intervals shrink as \( N_M \) increases and only relatively low values of \( \theta_i \) are compatible with an increasing number of competitors on the circle. For \( \theta_i \) close to its minimum level (\( \theta_i^L \)), two competitors may be allowed on the circle and total welfare
in the mixed market is superior, as well as patients net utility. As $\theta_i$ increases, the performance of the private market improves. Initially, the total welfare is higher because of profit, but as $\theta_i$ approaches $\theta_i^{U}$ also patients utility is higher in the private market. As the ratio $\frac{\Phi}{k}$ decreases, more private providers can access the market and total welfare decreases because quality is less and less important. The gain in terms of patients welfare decreases in both markets, but more rapidly in the mixed one, hence the difference $\Phi_M - \Phi_P$ may become negative. However, the profit of private providers in the mixed market (where entry is regulated) may increase to the point that total welfare in the mixed market becomes higher than the one in a private setting as shown in Figure 3. For values of $\theta_i$ close to $\theta_i^{L}$, patients net utility is higher in the mixed market, but total welfare is maximised by a private market. As $\theta_i$ increases, the profit of the private provider in the mixed market increases and as $\theta_i$ approaches $\theta_i^{U}$ total welfare is higher in the mixed market; however, patients net utility is maximised by a private market, i.e. the welfare

Figure 3: Welfare comparison $\frac{\Phi}{k}$ low.
improvement is simply due to a shift of resources from the public to the private sector.

Finally, if \( H \leq \frac{mk}{4\phi} \), but a mixed market is not feasible (points outside the area delimited by the red and blue curves) the comparison should be done between benchmark and a private market. In this case, if conditions in (11) and (15) are satisfied the welfare level of a private market is higher than in benchmark otherwise the latter should be chosen.

Our analysis shows that the regulator is facing several trade-offs when choosing the best market structure. Competition in the market for health care was firstly introduced with the aim to improve quality of care and/or reduce costs. In this respect our model shows that quality may be improved only if the efficiency of private providers is sufficiently high. In the other cases, the average quality level will be lower, but this does not necessarily mean that welfare decreases. For this reason, we have decomposed several aspect of welfare that the regulator may want to keep separate, namely patients net utility, public expenditure and profit of the private hospitals. The analysis of the welfare decomposition shows that only for some combinations of parameters a market alternative is dominating both in terms of patients net utility and “net profit”. In the other cases a trade off emerges and the choice may also depend on the relative importance that the regulator assigns to the different components of welfare. When they all have the same weight, the regulator will choose the combination that maximises welfare. In this case when a mixed market is chosen, it may be advisable to regulate entry through a monopoly franchise. This device allows to avoid the transformation of too much income generated by taxation (which has a cost in terms of welfare) into profit of private providers, especially when preferences for quality improvement beyond the verifiable level are rather low while costs to obtain the latter are relatively high (\( \frac{\phi}{k} \) low, see Figure 3). Monopoly franchises are quite common in other regulated industries and perhaps their use should be also introduced in the market for health care. A further argument in favour of franchises is that in the mixed market the presence of a profit by private providers may be accompanied by a deficit of the public hospital (or an increase in public expenditure); the revenue from the franchise could partially mitigate this problem.

The analysis just presented allows also to draw an interesting conclusion from a policy point of view. Mixed markets with a substantial number of private providers are feasible if, other things being equal, providers are relatively inefficient which in turns means that, even when patients’ net utility is higher than in benchmark, the improvement is mainly related to a reduction in transport costs.

The final consideration relates to soft budget constraint policies. The ana-
ysis presented in this paper allows to draw only some preliminary conclusions, given that the welfare comparison has been performed for the price that covers the cost of the public hospital in benchmark. If the regulator has financial constraints the use of soft budget is the best alternative, in other cases an overall conclusion cannot be drawn. An increase in the fee paid for each patient treated increases quality. This effect is certainly more important in the private market than in the mixed one. $T$ has also important effects on the profit of private providers, especially in the mixed market and for this reason the result in terms of welfare is not clear-cut; this aspect could be the object of further analysis. Another interesting extension of this model would be to consider the existence of a reference hospital (public or private) and the mechanisms leading a provider to become the reference supplier for the market.

**References**


A Appendix

A.1 Salop competition with a public hospital at the centre

Let us start by assuming that an interior solution exists; in this case the indifferent patient is located at $x_M = \frac{\varphi(q_i - q_c) + \delta}{m}$. The objective of the firms on the circle is:

$$\max_{q_i} (S + \theta_i - kq_i) 2x_M - H.$$ 

From the F.O.C. the reaction function is:

$$q_i = \frac{q_c}{2} + \frac{\varphi \theta_i - \delta k + \varphi S}{2k \varphi}.$$ 

The firm at the centre maximises the following function:

$$V_c = \left( \varphi q_c - \frac{\theta_c}{2} q_c^2 \right) - H.$$ 

Again, from the F.O.C. the following reaction function is derived:

$$q_c = \frac{\varphi}{\theta_c}.$$ 

Using the above results it is possible to obtain the Nash equilibrium presented in equation (10). However, depending on the level of $\theta_i$ the above solution may be feasible or not. As discussed in the text, the following conditions have to be satisfied:

$$q_c > q_i + \frac{\delta}{\varphi} - \frac{m}{2NM\varphi}, \quad q_c < q_i + \frac{\delta}{\varphi}.$$ 

The first condition is satisfied for all values of $\theta_i$ whenever $S > k \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} \right)$, otherwise it is sufficient to have:

$$\theta_i > k \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} \right) - S.$$ 

The second is true if

$$\theta_i < k \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} + \frac{m}{\varphi NM} \right) - S.$$ 

(18)
Apart from the market shares, the market is feasible only if the profit of private hospitals entering the market is not negative, that is if:

$$\Pi^M_i = (S + \theta_i - kq_i) 2x_M - H \geq 0.$$  

Substituting the relevant quantities in the above equation we get:

$$\Pi^M_i = \phi k \left( \frac{\theta_i + S}{k} + \frac{\delta}{\phi} - \frac{\phi}{\theta_c} \right)^2 - H,$$

which can be translated into a further condition on $\theta_i$:

$$\theta_i \geq k \left( \frac{\phi}{\theta_c} - \frac{\delta}{\phi} \right) + \sqrt{\frac{2Hmk}{\phi}} - S. \quad (20)$$

Obviously condition (20) is more restrictive than condition (18), thus the lowest possible value is the one above. In order for the mixed market to be feasible it is then sufficient that the following inequality is satisfied:

$$k \left( \frac{\phi}{\theta_c} - \frac{\delta}{\phi} \right) + \sqrt{\frac{2Hmk}{\phi}} - S < k \left( \frac{\phi}{\theta_c} - \frac{\delta}{\phi} + \frac{m}{\phi N_M} \right) - S.$$  

The above is equivalent to the following condition on $N_M$:

$$N_M < \sqrt{\frac{mk}{2H\phi}}.$$  

### A.2 Salop competition with private hospitals on the circle

The indifferent patient is located at $x_P = \frac{\phi(q_i-q_j)}{2m} + \frac{1}{2N_P}$. From (8) the objective of hospital $i$ sitting between hospital $i-1$ and hospital $i+1$ is:

$$\max_{q_i} (S + \theta_i - kq_i) \left( \frac{1}{N_P} + \frac{\phi}{2m}(2q_i - q_{i+1} - q_{i-1}) \right)$$

Under the feasibility condition $\frac{1}{N_P} + \frac{\phi}{2m}(2q_i - q_{i+1} - q_{i-1}) > 0$ from the F.O.C. the following reaction function is easily derived:

$$q_i = \frac{1}{4} (q_{i+1} + q_{i-1}) + \frac{\theta_i + S}{2k} - \frac{m}{2\phi N_P}.$$  

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If providers are symmetric it must hold \( q_i = q_j \) for all \( i \) and \( j \), thus

\[
q^P_i = \frac{\theta_i + S}{k} - \frac{m}{\phi N_P},
\]

which is meaningful only if

\[
\theta_i > \frac{mk}{\phi N_P} - S.
\]

Substituting the value of \( q^P_i \) back into (8) the profit of each provider is found:

\[
\Pi^P_i = \frac{mk}{\phi N^2_P} - H.
\]

Obviously, the condition \( \Pi^P_i \geq 0 \) has to be satisfied, meaning that the number of private providers is \( N_P = \left\lfloor \sqrt{\frac{mk}{H\phi}} \right\rfloor \). In order to have \( N_P \geq 2 \) the condition \( H \leq \frac{mk}{4\phi} \) must hold.

**B  Welfare comparison**

Total welfare is defined as the aggregation of consumers’ welfare and private providers’ profits, net of the costs deriving from the reimbursement and the possible covering of losses. Apart from a common constant part, the terms to be analysed are the net welfare derived from the service and the difference between profits and losses of the providers. In what follows we analyse the case where \( S = H \), i.e. the reimbursement is equal to the one in benchmark also when competition is allowed.

**B.1  Comparison of patients net utility**

The first comparison will be made examining only the money equivalent welfare derived from the service, net of the transport costs (in what follows it will implicitly be assumed that the related existence conditions hold). From (10) and (13) in the three examined scenarios (benchmark, mixed and private market) this is equal
to:

\[ \Phi_B = v + \frac{\varphi^2}{\theta_c} - \delta \]
\[ \Phi_M = v + (1 - 2N_Mx_M)(q_c\varphi - \delta) + 2N_Mx_M\varphi q_i^M - 2mN_M \frac{1}{2} x_M^2 \]
\[ = v + \frac{\varphi^2}{\theta_c} - \delta + N_M \frac{\varphi^2}{4m} \left( \frac{\theta_i + H}{k} + \frac{\delta - \varphi}{\theta_c} \right)^2 \]
\[ \Phi_P = v + \varphi q_i^P - mN_P \bar{x}_P \]
\[ = v + \varphi \frac{\theta_i + H}{k} - \frac{5m}{4N_P}. \]

\( \Phi_M \) is increasing in \( N_M \) and is higher than \( \Phi_B \).

\( \Phi_P \) depends linearly on \( \theta_i \), while \( \Phi_M \) is a quadratic convex function of \( \theta_i \). Its derivative wrt \( \theta_i \) is:

\[ N_M \frac{\varphi}{2mk} \frac{\varphi}{k} \left( \frac{\varphi}{k} \theta_i + \frac{\varphi}{k} H - \left( \frac{\varphi^2}{\theta_c} - \delta \right) \right). \]

The upper bound of the derivative is found by substituting \( \theta_U \) in the above equation. From (11c) this value is \( \frac{\varphi}{2k} \), while the derivative of \( \Phi_P \) wrt \( \theta_i \) is \( \frac{\varphi}{k} \). From Lagrange’s theorem this means that the graphs of \( \Phi_P \) and \( \Phi_M \) have at most a common point. Since both functions are increasing, when the value of \( \Phi_M - \Phi_P \) for \( \theta_i = \theta_U \) is positive this part of the welfare will be greater in a mixed market for all the values of \( \theta_i \) that allow its existence. Since

\[ (\Phi_M - \Phi_P)|_{\theta_i = \theta_U} = \frac{m}{4N_MN_P}(5N_M - 3N_P) \]

if \( 5N_M \geq 3N_P \) the condition \( \Phi_M > \Phi_P \) will be satisfied for \( \theta_i \in [\theta_L, \theta_U] \).

The value of the difference for \( \theta_L \) is:

\[ (\Phi_M - \Phi_P)|_{\theta_i = \theta_L} = N_M \frac{H \varphi}{2k} - \sqrt{\frac{2mH \varphi}{k}} + \frac{5m}{4N_P}. \]

(21)

When \( 5N_M < 3N_P \) and the above value is negative, \( \Phi_P \) is higher than \( \Phi_M \) for all the admissible values of \( \theta_i \), if it is positive the graphs of \( \Phi_P \) and \( \Phi_M \) will intersect at some point. Since the rhs of (21) is zero when \( N_M \) is equal to \( \frac{2\sqrt{\frac{2mk}{H\varphi}} - \frac{5mk}{2\varphi N_PH}}{2} \),
the intersection point exists if
\[2\sqrt{\frac{2mk}{H\varphi}} - \frac{5mk}{2\varphi N_P H} < N_M < \frac{3}{5} N_P.\] (22)

Thus the following result proves Proposition 1.

Note that if \(N_M = 1\) the inequality \(\Phi_P > \Phi_M\) is always satisfied if \(\theta_i\) is high enough. Also, by standard algebra the interval in (22) is non empty if \(N_P < \frac{5\sqrt{2}}{6} \sqrt{\frac{mK}{H\varphi}}\), which by (14) is always true. However, \(N_M\) is an integer, therefore whether the condition is true or not depends on the parameters. Since \(\theta_i^U\) decreases with \(N_M\), for high values of \(\theta_i\) only a mixed market with one private provider is feasible and (22) is valid for \(N_M = 1\) only if \(2\sqrt{\frac{2mk}{H\varphi}} - \frac{5mk}{2\varphi N_P H} < 1\). Since \(N_P \leq \sqrt{\frac{mK}{H\varphi}}\) the latter equation is verified for any \(N_P \leq 3\), but for higher numbers of competitors it depends on the ratio between \(N_P\) and \(\sqrt{\frac{mK}{H\varphi}}\): if the latter is sufficiently high (i.e. profits for private providers are low) \(\Phi_P\) is always higher than \(\Phi_M\) with \(N_M = 1\).

### B.2 Profits comparison

**Profits in the private market**

The total profits of providers on a private market are:
\[N_P \Pi_i^P = \left(\frac{mk}{\varphi N_P^2} - H\right) N_P\]
and do not depend on \(\theta_i\). Since \(N_P = K\) when \(\frac{mk}{(K+1)^2\varphi} < H \leq \frac{mk}{K^2\varphi}\) the following inequality holds:
\[0 \leq N_P \left(\frac{mk}{N_P^2\varphi} - H\right) < N_P \left(\frac{mk}{N_P^2\varphi} - \frac{mk}{(N_P + 1)^2\varphi}\right) = \frac{mk}{\varphi} \frac{2N_P + 1}{N_P(N_P + 1)^2} - N_P(N_P + 1)^2.\]
Profits are therefore bounded by \(\frac{5mk}{18\varphi}\) and tend to zero as \(H \to 0\).

**Profits in the mixed market**

Total profits of private providers in the mixed market are:
\[\Pi_M = \Pi_i^m N_M = N_M((H + \theta_i - kq_i^M)2x_M - H).\]
\( \Pi_M \) increases linearly with \( N_M \) and is a quadratic, convex and increasing function of \( \theta_i \). If \( \frac{mk}{2(N+1)^2} \leq H < \frac{mk}{2N^2} \) for some integer \( N \), on each interval \( \theta_i^L \leq \theta_i < \theta_i^U(n) \) with \( n \leq N \) the number of private competitors \( N_M \) can be chosen between 1 and \( n \) and for each fixed value of \( \theta_i \) the maximum profit is obtained for \( N_M = n \). Also, on each interval the value of \( \Pi_M \) is comprised between 0 and \( n \left( \frac{mk}{2\phi n^2} - H \right) \), which is a decreasing function of \( n \).

**Profit difference**

From the above analysis the sign of \( \Pi_P - \Pi_M \) depends on several factors: if \( H \) is near to the threshold \( \frac{mk}{K^2\phi} \) for some integer \( K \) then \( \Pi_P \) is near to zero and the difference will therefore be negative for almost all values of \( \theta_i \) for any choice of \( N_M \). As \( H \) decreases towards \( \frac{mk}{(K+1)^2\phi} \) the term \( \Pi_P - \Pi_M \) gradually increases and only for high values of \( \theta_i \) the difference may be negative. In fact for \( \theta_i \) tending to some \( \theta_i^U(n) \) from the left the difference is less than:

\[
\frac{mk}{\phi} \frac{2N_P + 1}{N_P(N_P + 1)^2} - \frac{mk}{2n\phi} + nH \leq \frac{mk}{\phi} \left( \frac{2N_P + 1}{N_P(N_P + 1)^2} - \frac{1}{2n} + \frac{n}{N_P^2} \right). \tag{23}
\]

By standard algebraic calculations it turns out that if \( n = 1 \) the quantity above is negative whenever \( N_P > 2 \), therefore if \( H \leq \frac{mk}{2\phi} \Pi_M \) is higher than \( \Pi_P \) for values of \( \theta_i \) high enough. More generally, the sign of the rhs of (23) equals that of:

\[-N_P^4 - 2N_P^3 + (2n^2 + 4n - 1)N_P^2 + (4n^2 + 2n)N_P + 2n^2\]

and by Descartes rule the above polynomial has a unique positive root in the variable \( N_P \) whose value increases with \( n \). This means that as \( H \) decreases the difference \( \Pi_P - \Pi_M \) can be negative for \( \theta_i \) sufficiently high also for values of \( n > 1 \). Note also that from (23) this difference depends linearly on \( \frac{k}{\phi} \).

**“Net profit” in the mixed market**

In the mixed market the loss of the public hospital equals \( 2HN_M \), which from (10) for each fixed value of \( N_M \) is increasing in \( \theta_i \) from the value \( HN_M \sqrt{\frac{2H\phi}{mk}} \) to \( H \). The “net profit” in the mixed market is:

\[-2HN_M \chi_M + N_M \Pi^M_i = N_M ((\theta_i - kq^M_i)2\chi_M - H). \tag{24}\]

From (10) the above quantity can be written as a quadratic, convex function of \( \theta_i \). Its derivative for \( \theta_i = \theta_i^L \) is equal to \( N_M \frac{q}{mk} \left( \sqrt{\frac{2Hmk}{\phi}} - H \right) > 0 \) thus the function is
increasing in $\theta_i$. Its value for $\theta_i = \theta_i^L$ is $-N_M\sqrt{\frac{2Hmk}{\phi}}$, thus negative; for $\theta_i = \theta_i^U$ it is equal to $-H(N_M + 1) + \frac{mk}{2N_M\phi}$, which is positive when $N_M < \sqrt{\frac{1}{4} + \frac{mk}{2H\phi}} - \frac{1}{2}$. This condition is verified if $\frac{mk}{2(n+1)^2\phi} \leq H < \frac{mk}{2n(n+1)\phi}$ for some integer $n$.

Note also that the maximum value of the derivative is $1 - N_M\sqrt{\frac{H}{mk\phi}}$ and decreases with the ratio $\sqrt{\phi \frac{k}{\phi}}$, while the minimum value of the derivative increases with this parameter.

### B.3 Welfare comparison: benchmark vs mixed

The total welfare in a mixed market is given by:

$$W_M = W_B + N_M(x_M(mx_M - 2kq_i^M + 2\theta_i) - H),$$

where $W_B$ is the welfare of the benchmark scenario. Thus, if the term $x_M(mx_M - 2kq_i^M + 2\theta_i) - H$ is positive, a mixed market will give a higher welfare level than the benchmark and at the same time the highest welfare level is reached by allowing the maximum number of private providers into the market. The term $x_M$ is linear and increasing in $\theta_i$, while from (10) it is:

$$mx_M - 2kq_i^M + 2\theta_i = \left(\frac{\phi}{2k} + 1\right)\theta_i + \left(\frac{\phi}{2} + k\right)\left(\frac{\delta}{\phi} - \frac{\phi}{\theta_c}\right) + \frac{H\phi}{2k} - H,$$

(25)

so the same is also true also for this quantity. Its minimum value is achieved for $\theta_i = \theta_i^L$ and is equal to:

$$\sqrt{\frac{2Hmk}{\phi}}\left(\frac{\phi}{2k} + 1\right) - 2H,$$

which is non negative for all values of $H$ for which the mixed market exists, thus $W_M$ is increasing in $\theta_i$. Substituting the minimum value for $\theta_i$ we get:

$$x_M(mx_M - 2kq_i^M + 2\theta_i) \geq \frac{\phi H}{k}\left(\frac{1}{2} - \frac{1}{m}\sqrt{\frac{2Hmk}{\phi}}\right)$$

thus if $H \leq \frac{m\phi}{8k}$ for all $\theta_i$ it is $W_M > W_B$. This is true for all values of $H$ that allow the existence of a mixed market if $\phi \geq 2k$.

Since we also have

$$W_M - W_B \leq \frac{m}{4N_M}\left(1 + \frac{2k}{\phi}\right) - H(N_M + 1)$$
there exist values of $\theta_i$ for which $W_M \geq W_B$ only if $H < \frac{m(\varphi + 2k)}{4\varphi M (N_M + 1)}$. The result in Proposition 2 is thus proved.

**B.4 Welfare comparison: benchmark vs private**

In the private market setup total welfare is equal to:

$$W_P = v - \beta - H + \phi \frac{\theta_i + H}{k} - \frac{5m}{4N_P} + \left( \frac{mk}{\varphi N_P} - H \right) N_P.$$  

From (14) if $\epsilon = \frac{N_P}{\sqrt{mk \varphi}}$ it is

$$\frac{1}{2} \leq 1 - \frac{1}{\sqrt{\frac{mk}{\varphi N}}} < \epsilon = \frac{\sqrt{\frac{mk}{\varphi N}}}{\sqrt{\frac{mk}{\varphi}}} \leq 1,$$  

and

$$W_P = v - \beta - H + \phi \frac{\theta_i + H}{k} + \sqrt{\frac{H mk}{\varphi}} \left( 1 - \epsilon^2 - \frac{5 \varphi}{4k} \right)$$

thus

$$W_P - W_B = \phi \frac{\theta_i + H}{k} - \left( \frac{\varphi^2}{\theta_c} - \delta \right) + \sqrt{\frac{H mk}{\varphi}} \left( 1 - \epsilon^2 - \frac{5 \varphi}{4k} \right).$$

The difference is increasing in $\theta_i$ and from (15) has the following lower limit:

$$\frac{1}{\epsilon} \left( 1 - \epsilon^2 \right) \sqrt{\frac{H mk}{\varphi}} - \frac{1}{4\epsilon} \sqrt{\frac{H m \varphi}{k}} - \left( \frac{\varphi^2}{\theta_c} - \delta \right)$$

whose sign depends on the parameters. Since it is reasonable to assume that $\frac{\varphi^2}{\theta_c} - \delta > 0$, if the ratio $\frac{\varphi}{k}$ is sufficiently low there can exist values for $\epsilon$ and $\theta_c$ for which the above quantity is positive. In these cases $W_P - W_B > 0$ for all admissible values of $\theta_i$. Note however that if $\epsilon = 1$, that is if $N_P = \sqrt{\frac{mk}{\varphi}}$ and the profit of private providers is zero, the above quantity is negative. In these cases there exists a threshold for $\theta_i$ below which the welfare in benchmark is higher than in the private market and lower for higher values of this parameter.
Evaluating the difference $W_p - W_B$ for the lowest threshold that allows the existence of a mixed market we have:

$$(W_p - W_B)_{\theta_i = \theta^L_i} = \sqrt{\frac{H m k}{\varphi \epsilon}} \frac{1}{4 k \epsilon} (4\sqrt{2} \epsilon \varphi - 4\epsilon^2 k + 4k - 5 \varphi).$$

By standard algebra, the last term is positive if $\epsilon < \frac{1}{2k}(\sqrt{2} \varphi + \sqrt{4k^2 - 5k \varphi + 2\varphi^2})$, but this condition is always verified since the quantity on the rhs is greater than 1, therefore the quantity in the above equation is positive. This proves Proposition 3.