Leadership in Tax and Public Investment Competition

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ABSTRACT

In this paper, we focus on tax and public investment competition in a sequential game set up. We demonstrate that, in a timing game, regions choose simultaneous move equilibrium outcomes, if the effect of public investment (strategic substitutes) is higher than the tax rate (strategic complements) effect. On the other side, regions opt for sequential move game outcomes if tax rate effect is stronger than public investment effect. Moreover, there are multiple equilibria in case of sequential move and we cannot select any particular equilibrium based on Pareto dominance or risk dominance criterion. We also show that in sequential move equilibrium, race-to-the-bottom in tax rates is restricted, compared to the simultaneous move game. Further, regions get higher welfare level in sequential move game as compared to simultaneous move game.

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1. INTRODUCTION

In the fiscal competition for mobile capital, governments compete to attract capital flows using tax (Wilson, 1986; Zodrow and Mieszkowski, 1986; Wildasin, 1988) as well as non-tax instruments\(^2\) (Hindriks, Peralta and Weber, 2008; Dembour and Wauthy, 2009; Pieretti and Zanaj, 2011). There is plenty of empirical evidence for use of tax and non-tax instruments to attract mobile capital across countries and regions within a country. Hauptmeier et al. (2012), analyze the strategic nature of tax and public input competition, using the data from Germany municipal regions. They find that tax rate in a region is inversely affected by public input provision in the other region. To counter the tax rate decrease in a region, other region can decrease the tax rate or can increase its provision for public input good. Benassy-Quere et al. (2007), in the context of European countries, provide empirical support for the joint role of taxes and public input goods in attracting foreign direct investment. They document that the role of productivity enhancing public goods is as large as the role of taxes in determining capital flows. Further, countries, which set high tax rates, can attract capital by providing higher level of public good. Bellak et al. (2009) corroborates the findings of Benassy-Quere et al. (2007), that taxes and infrastructure (telecommunication, electricity, transport and production facilities) are determinants of FDI in central and Eastern European Countries (CEEC). They also show that tax elasticities are decreasing function of infrastructure facilities. This means that for higher infrastructure levels, regions can charge higher taxes, vice-versa. Venkatesan and Verma (1998), in the context of Indian federal system, demonstrates that there is evidence of industrial policy based competition among Indian states, using industrial policy and FDI data from 1991-97. This

\(^2\) One of the widely used instruments apart from taxation is public investment in terms of infrastructure, electricity, law and order conditions, ease of doing business and so forth. These goods are considered to be productivity enhancing and therefore making the region attractive for capital inflows.
study documents that there is wide variation in terms of tax rate as well as in terms of non tax incentives\(^3\) provided by state governments, which determine the capital flows to these states and there is positive correlation between the incentives provided by the governments and inflow of capital. It is evident from these empirical studies that both tax rates and public investments provided by the governments are important factors in determining allocation of mobile capital across regions.

This paper analyzes the nature and consequences of interregional competition for foreign owned mobile capital in terms of both tax rates and levels of public investments, focusing on joint strategic effects of tax rates and levels of public investments. We also endogenize the timings moves by regions in tax and public investment competition. This study contributes to the literature in the following manner. First, we consider the joint and simultaneous decision of tax and public investment as opposed to sequential decision for public investment and taxation as in Hindriks, Peralta and Weber (2008), Dembourn and Wauthy (2009) and Pieretti and Zanaj (2011). This helps us in understanding the interaction of tax and public investment strategies and their joint impact on attracting mobile capital. Second, we endogenize the leadership in tax and public investment competition by extending the formulation of Kempf and Rota-Graziosi (2010)’s timing game for multiple strategy space. Lastly, but not least, we also contribute to the literature on the nature and classification of strategic effects in competitive environment, based on the seminal work of Bulow, Geanakoplos, and Klemperer (1985). We analyze the joint effect of tax and public investment which individually are of different strategic nature i.e. strategic

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\(^3\) They characterizing these incentives in three groups: a) financial incentives (investment subsidy; financing of technology, land etc; special packages for large projects), b) fiscal incentives (export based incentives, sales tax incentives, stamp and registration duty subsidy, free electricity etc.) and c) other incentives (single window clearance, road and transport facilities etc.).
complements and strategic substitutes respectively. To the best of our knowledge, this is the first paper that analyzes the joint effect of opposite type of strategic variables in a model.

The main findings from our model are as follows. We show that, under plausible parametric conditions, sequential move subgame perfect Nash equilibrium is welfare improving for both the regions and, therefore, the two regions prefer to move sequentially in equilibrium. We demonstrate that in tax and public investment competition there is a trade off in tax and public investment effect and there are possibility of both first mover and second mover advantage. We formalize the concepts of joint strategic substitutes (i.e. strategic substitute effect dominates strategic complement effect) and joint strategic complements (i.e. strategic complement effect dominates strategic substitute effect), which drive the results in timing game. We show that if joint strategic substitute effect holds, then simultaneous move equilibrium prevails and if joint strategic complements effect holds, then two sequential move equilibria hold and are Pareto superior for both the regions compared to the simultaneous move equilibrium. We discuss the issue of equilibrium selection using Pareto dominance and risk dominance criteria (proposed by Harsanyi and Selten, 1986). We are not able to select any of these sequential equilibria based on risk or Pareto dominance criteria due to symmetric nature of outcomes. Further, we show that in sequential game equilibrium, welfare as well as tax rates in both the regions are higher compared to simultaneous move game.

The rest of the paper is organized as follows. Section 2 explains the basic framework of the model and properties of tax and public investment strategies. Section 3 examines the implications of tax and public investment competition in both simultaneous and sequential moves by the regions. Section 4 deals with the issue of endogenous leadership in case of tax and
public investment competition and analyzes the joint strategic effects of multiple strategies. Section 5 concludes with policy implications and possible extensions.

2. THE BASIC MODEL

We consider two regions: region-1 and region-2, competing for foreign owned mobile capital. Total available mobile capital is assumed to be 1, which is exogenously determined. Both the regions strategically decide the level of public investment ($g$) and the tax rate ($t$) on investment capital ($x$) in order to maximize their respective welfare. Public investment ($g$) facilitates production and thus enhances the productivity of investment capital ($x$); whereas higher tax ($t$) discourages capital inflows. Let $(g_i, t_i)$ denote the pair of public investment and tax rate to be chosen by region $i$, and $x_i$ denote the investment capital attracted in region $i$, where $i = 1, 2$. Following Hindriks et al. (2008), we consider that the production functions of region-1 and region-2 are given by (1) and (2), respectively.

$$F_1(x_1, g_1) = (\gamma + g_1 + \theta g_2)x_1 - \frac{\delta}{2} x_1^2$$  \hspace{1cm} (1)

$$F_2(x_2, g_2) = (\gamma + g_2 + \theta g_1)x_2 - \frac{\delta}{2} x_2^2$$  \hspace{1cm} (2)

In equations (1) and (2), $\gamma$ ($> 0$) is the technology parameter, $\delta$ ($> 0$) is the rate of decline in the marginal productivity of capital and $\theta$ ($0 \leq \theta \leq 1$) is the spillover effect of public investment in one region on the other region. Here, $\theta = 1$ ($\theta = 0$) corresponds to the case of perfect spillover (no spillover). Note that the capital and public investment are complements. Clearly, the production functions are increasing, twice continuously differentiable and concave in the level of capital.
The provision of productivity enhancing public investment ($g$) by both regions, involves cost which is assumed to be a convex quadratic function, $c_i(g_i) = \frac{g_i^2}{2}$, indicating an increasing marginal cost of provision for public investment.

Following Laussel and Le Breton (1998), Hindriks, Peralta and Weber (2008) and Kempf and Rota-Greziosi (2010), the objective functions of region-1 and region-2, in terms of social welfare, are as follows:

\[
W_1 = \left( F_1 - x_1 F_{1,x_1}(x_1, g_1) \right) + \left( t_1 x_1 \right) - \left( \frac{g_1^2}{2} \right) = \frac{\delta}{2} x_1^2 + \left( t_1 x_1 \right) - \left( \frac{g_1^2}{2} \right) \tag{3}
\]

\[
W_2 = \left( F_2 - x_2 F_{2,x_2}(x_2, g_2) \right) + \left( t_2 x_2 \right) - \left( \frac{g_2^2}{2} \right) = \frac{\delta}{2} x_2^2 + \left( t_2 x_2 \right) - \left( \frac{g_2^2}{2} \right) \tag{4}
\]

Each region is maximizing the sum of return to immobile factors and tax revenue, net of the cost of public investment provision. The regions are maximizing only returns on immobile factors and tax revenue, since capital is assumed to be foreign owned.\(^4\)

We assume that the capital market is perfectly competitive and there is no arbitrage possibility. Therefore, the capital market clearance condition can be written as follows.

\[
F_{1,x_1}(x_1, g_1) - t_1 = F_{2,x_2}(x_2, g_2) - t_2 > 0 \tag{5}
\]

\[
x_1 + x_2 = 1 \tag{6}
\]

Condition (5) states that the net marginal returns to capital for both the regions should be equal for capital market clearing. Further, marginal return to capital is considered to be positive,\(^4\)

\(^4\) Laussel and Le Breton (1998) argues that such objective functions can also be justified by considering that majority of the citizens are consumers (labour) and not investment capital owners. Therefore the regions are concerned about the welfare of the representative median voter.
leading to condition (6) i.e. all the capital is allocated between the regions and no idle capital is available.

In the next sub-section, we analyze the properties of tax rate and public investment individually in the context of fiscal competition between the regions.

3.1 Fiscal competition between regions: tax rates vs. public investment

We characterize the equilibrium corresponding to simultaneous move competition game in terms of tax rates (public investment), assuming public investments (tax rates) exogenously given. We focus on these aspects to understand the joint implications of tax and public investment competition clearly. The stages of the game involved are as follows: a) Stage 1: Each region decide its tax rate (public investment), considering exogenously given public investments (tax rates), simultaneously and independently; b) Stage 2: Mobile capital is allocated between the two regions, in the presence of perfectly competitive capital market. We solve the game by standard backward induction method. Note that, irrespective of whether the two regions are engaged in tax competition or public investment competition, stage 2 equilibrium would be the same. To solve this stage we optimize (5) with constraints (1), (2) and (6). The stage 2 equilibrium capital allocation is as follows:

\[
x_1 = \frac{\delta + (1 - \theta)(g_1 - g_2) - t_1 + t_2}{2\delta} \quad (7a)
\]

\[
x_2 = \frac{\delta + (1 - \theta)(g_2 - g_1) - t_2 + t_1}{2\delta} \quad (7b)
\]

For any given tax rate and level of public investment, amount of mobile capital allocated in region-1 and region-2 are given by (7a) and (7b), respectively. Clearly, we can see that, \( \frac{\partial x_1}{\partial t_i} < \)
\[ \frac{\partial x_i}{\partial t_j} > 0 \forall i \neq j, \text{ i.e. higher tax rate in a region leads to lower capital allocation in that region and higher capital allocation in the other region. Also note that if } \theta \neq 1, \text{ i.e., if spillover of public investment is not perfect, } \frac{\partial x_i}{\partial g_i} > 0, \frac{\partial x_i}{\partial g_j} < 0 \forall i \neq j. \text{ It implies that public investment in a region positively (negatively) affects level of capital allocated to that region (other region), unless the spillover effect of public investment is perfect. Therefore, it is evident that, tax rate and level of public investment of a region have opposing effects on the inflow of investment capital in that region, as argued before. We can also see that, if tax rate in region-2 (say) decreases, then region-1 has an option not to decrease its tax rate and instead spend more on public investment. This indicates that the regions have alternate choices in the presence of public investment, other than entering into a tax undercutting war leading to a race to the bottom. We discuss this issue in detail in subsequent section.}

Now, we consider (a) the properties of tax competition corresponding to exogenously given public investment and (b) properties of public investment competition corresponding to exogenously given tax rates, in stage 1, separately.

Substituting the expressions for \( x_1 \) and \( x_2 \) from (7a) and (7b) in (3) and (4), we obtain \( W_1(t_1, t_2) \) and \( W_1(t_1, t_2) \). From the first order condition with respect to tax rates, we get the tax reaction functions of region-1 and region-2, as follows:

\[
\begin{align*}
TR_1: \quad t_1(t_2) &= \frac{1}{3}(\delta + (1 - \theta)(g_1 - g_2) + t_2) \\
TR_2: \quad t_2(t_1) &= \frac{1}{3}(\delta - (1 - \theta)(g_1 - g_2) + t_1)
\end{align*}
\]

(8a)

(8b)

Based on tax reaction functions, we characterize some properties of tax rates.
Lemma 1 (Complements): Tax rates are complements in nature. Welfare of region-1 is increasing in $t_2$ and welfare of region-2 is increasing in $t_1$.

Proof: See appendix A1.

This means that if region-1 (say) increases tax rate, region-2’s welfare increases due to higher capital flow to region 2.

Lemma 2 (Strategic complements): Tax rates are strategic complements and the two regions reaction functions are positively sloped.

Proof: See appendix A2.

Therefore, interregional fiscal competition in terms of tax rates is similar to that of price (Bertrand) competition between firms. A decrease in tax rate in region-1 (region-2) induces the region-2 (region-1) to set lower tax rate. Also, note that lower tax rate in one region, compared to that in its rival region, leads to higher capital flow in that region and that, in turn, leads to higher welfare of that region, ceteris paribus. It implies that, in case of tax competition, there is race-to-the bottom in tax rates.

Now, let’s consider that the two regions set their tax rates sequentially, in stage 1. That is, one region decides its tax rate before the other region. The region which sets the tax rate first is the leader and the other region is the follower. In case of such sequential tax setting by the regions, the follower region’s problems remains the same as that in case of simultaneous move tax competition. The leader region takes the tax reaction function of follower in to account while deciding its tax rate. Given that tax rates of the regions are strategic complements, we can say that:

Proposition 1(Second mover advantage): Under lemma 1 and Lemma 2, region-1 as well as region-2 always prefers to be the follower, rather than a leader, in case of sequential move tax competition, when public investment are exogenously given.

Proof: See A3 in appendix A.

Proposition 1 implies that both the regions have unilateral incentive to become follower in a tax competition game, when public investments are exogenously determined.
Next, we consider the properties of public investment in a fiscal competition game. Let us first consider that, given the tax rates, the two regions decide their levels of public investments simultaneously and independently. As before, substituting the expressions for $x_1$ and $x_2$ from (7a) and (7b) in (3) and (4), we obtain $W_1(g_1, g_2)$ and $W_2(g_1, g_2)$. Differentiating the welfare functions $W_1(.)$ and $W_2(.)$ and with respect to public investment level $g_1$ and $g_2$ respectively, we get the resultant public investment reaction functions of region 1 and region 2, respectively, as follows.

$$G1: g_1(g_2) = \frac{(1-\theta)(\delta - (1-\theta)g_2 + t_1 + t_2)}{4\delta - (1-\theta)^2}$$  \hspace{1cm} (9a)$$

$$G2: g_2(g_1) = \frac{(1-\theta)(\delta - (1-\theta)g_1 + t_1 + t_2)}{4\delta - (1-\theta)^2}$$  \hspace{1cm} (9b)$$

It is easy to check that the reactions functions are negatively sloped. We depict the public investment reaction functions of the two regions in Figure 1. Note that the $\delta$ parameter only affects the intercept term in the reaction function and causes parallel shifts of the reaction function.

![Figure 1: Public investment reaction functions](image-url)
function. Higher $\delta$ leads to outward shift of the reaction functions and higher spending on public investment and lower welfare. If $\theta$ increases, the reaction functions rotates inwards, as in Figure 3.

Now, we show some important characteristics of public investment in the given framework, which are useful for further analysis.

**Lemma 3 (Substitutes):** Levels of public investment are substitutes. Welfare of region-1, $W_1$, is decreasing in $g_2$ and welfare of region-2, $W_2$, is decreasing in $g_1$.

**Proof:** See A4 in appendix A.

**Lemma 4 (Strategic Substitutes):** Levels of public investments are strategic substitutes and the reaction functions of the two regions are downward sloping.

**Proof:** See A5 in appendix A.

Now, we show that if regions engage in public investment competition in a sequential move game, then regions prefer to be the leader compared to being the follower.

**Proposition 2 (First mover advantage):** Under Lemma 3 and Lemma 4, both the regions always prefer to be the leader rather than a follower in interregional fiscal competition in terms of public investments, given the exogenously determined tax rates.

**Proof:** See A6 in appendix A.

Clearly, we can say that both the regions prefer to be the leader in a public investment competition game, because of the strategic substitute nature of the public investment variable. This is similar to that of the standard Cournot type quantity competition among firms.

In the next section, we analyze the combined effect of joint choice of tax and public investment in a simultaneous move fiscal competition game.
3. TAX AND PUBLIC INVESTMENT COMPETITION

In this section, we analyze multidimensional nature of fiscal competition for mobile capital between the two regions. That is, we consider that the two regions compete in terms of both tax rate and public investment. As we have seen in the previous section, in case of pure tax competition, each region wants to be the follower, since tax rates are strategic complements. In contrast, in case of pure public investment competition, each region wants to be the leader, since levels of public investments are strategic substitutes. The question is, when each region has two strategies (tax rate and level of public investment) of opposite nature, will any region prefer to be the leader/follower? Will sequential move game be preferred to simultaneous move game? In this section, we analyze the interplay of these two strategies, which are of opposite nature.

The stages of the multidimensional competition game involved are as follows.

Stage 1: Region-1 and region-2 decide the respective tax rates, \((t_1, t_2)\), and amount of public investment, \((g_1, g_2)\). Each region decides its tax rate as well as the level of public investment at the same time.

a) If the regions move simultaneously, then tax rates and levels of public investments are decided by both the regions simultaneously and independently.

b) Alternatively, if the regions move sequentially, the leader region (say region 1) decides its tax rate and level of public investment first, and the follower region (say region 2) decides its tax rate and level of public investment next.

Stage 2: Mobile capital is allocated between the two regions through a perfectly competitive capital market, based on their respective tax rates and public investment levels.

We solve the game using the standard backward induction method, considering (a) simultaneous move game and (b) sequential move game, in stage 1, separately.

First, we consider the simultaneous move game in stage 1. The outcomes of this game are used for the comparison of the results of sequential move game.
3.1 Simultaneous move game

Note that, in this case also allocation of mobile capital in stage 2 is given by (7a) and (7b) from previous section.

\[ x_1(t_1, t_2, g_1, g_2) = \frac{\delta + (1 - \theta)(g_1 - g_2) - t_1 + t_2}{2\delta} \] (7a)

\[ x_2(t_1, t_2, g_1, g_2) = \frac{\delta + (1 - \theta)(g_2 - g_1) - t_2 + t_1}{2\delta} \] (7b)

Now, from (7a), (7b), (3) and (4) we get \( W_1(t_1, t_2, g_1, g_2) \) and \( W_2(t_1, t_2, g_1, g_2) \), respectively.

Therefore, the problem of the region-1 and the region-2 can be written as follows:

\[
\begin{align*}
\text{Max}_{t_1, g_1} W_1(t_1, t_2, g_1, g_2) &= \frac{1}{8\delta}((-4\delta + (1 - \theta)^2)g_1^2 - (1 - \theta)g_1(2\delta - 2(1 - \theta)g_2 + 2t_1 + 2t_2) \\
&\quad + (\delta + (-1 + \theta)g_2 - t_1 + t_2)(\delta + (-1 + \theta)g_2 + 3t_1 + t_2) \\
\text{Max}_{t_2, g_2} W_2(t_1, t_2, g_1, g_2) &= \frac{1}{8\delta}((1 - \theta)^2g_2^2 + (-4\delta + (1 - \theta)^2)g_2^2 + (-1 + \theta)g_2(-2\delta - 2t_1 - 2t_2) + (-\delta - t_1 \\
&\quad - 3t_2)(-\delta - t_1 + t_2) + (-1 + \theta)g_1(2\delta - 2(-1 + \theta)g_2 + 2t_1 + 2t_2))
\end{align*}
\]

The first order conditions for maximization of the above two problems are as follows.

\[
\begin{align*}
\frac{\partial W_1}{\partial t_1} &= 0 \Rightarrow t_1 = \frac{(\delta + (1 - \theta)(g_1 - g_2) + t_2)}{3} \\
\frac{\partial W_2}{\partial t_2} &= 0 \Rightarrow t_2 = \frac{(\delta - (1 - \theta)(g_1 - g_2) + t_1)}{3} \\
\frac{\partial W_1}{\partial g_1} &= 0 \Rightarrow g_1 = \frac{(1 - \theta)(\delta - (1 - \theta)g_2 + t_1 + t_2)}{4\delta - (1 - \theta)^2} \\
\frac{\partial W_2}{\partial g_2} &= 0 \Rightarrow g_2 = \frac{(1 - \theta)(\delta - (1 - \theta)g_1 + t_1 + t_2)}{4\delta - (1 - \theta)^2}
\end{align*}
\]
For satisfying the second order and stability conditions, we assume that, $0 \leq \theta \leq 1$ and $\delta > \frac{(1-\theta)^2}{2}$. These conditions also satisfy the non negativity constraints on outcomes of public investment tax rates, capital and welfare level.

The values of the capital, tax rates, public investment and welfare for both the regions in subgame perfect Nash equilibrium, are as follows:

\[
t_1^N = t_2^N = \frac{\delta}{2}; \ g_1^N = g_2^N = \frac{\theta}{2}; \ x_1^N = x_2^N = \frac{1}{2} \text{ and } W_1^N = W_2^N = \frac{1}{8}(3\delta - (1 - \theta)^2)
\] (12)

Since the two regions are symmetric, each region get equal share of mobile capital in equilibrium. Tax rates and levels of public investments are also same in the two regions.

3.2 Sequential move game

Next, we consider a sequential move game in stage 1. Kempf and Rota-Greziosi (2010) demonstrates that in a sequential move pure tax competition for mobile capital, both the regions levy higher tax rates compared to that in case of simultaneous move pure tax competition. It is argued that, due to change in the timing of the moves, regions improve their welfare level and there is restriction on the race to the bottom in tax rates. Does this result go through in case of multidimensional competition? We attempt to answer this question. The present analysis is particularly interesting because tax rates and levels of public investments are of opposite strategic nature. If both the strategies were strategic complements as the tax rates, it would have been easy to see that the results of Kempf and Rota-Greziosi (2010) likely to hold even in case of tax and public investment competition. However, it is not straightforward to understand the implications of multidimensional competition, when strategies are of opposite nature. Without any loss of generality, we assume that region-1 is the leader and region-2 is the follower, since the two regions are assumed to be symmetric. Needless to mention here, that the equilibrium
outcomes of stage 2 remain the same as before. Now, in stage 1, we solve the problem of the follower region first, using backward induction method.

Note that the problem of the follower (region-2) is the same as in the case of the simultaneous move game. The follower region assumes that the tax rate and public investment of the leader region as given and then chooses its own optimal level of public investment and tax rate.

We get the reaction function of the follower (region-2) from first order conditions- (10b) and (11b). Now, we solve the problem of the leader (region-1). The leader anticipates the strategies of the follower region and includes the reaction function of the follower in its problem, then chooses for the optimal tax rate and public investment to maximize its regional welfare.

\[
\text{Max}_{t_1, g_1} W_1(t_1, t_2, g_1, g_2)
\]
\[
= \frac{1}{32\delta} \left(4(-4\delta + (1 - \theta)^2)g_1^2 - 4(-1 + \theta)g_1 \left(2\delta + 2(-1 + \theta)g_2 + 2t_1 + 2t_2\right) + \left(2\delta + 2(-1 + \theta)g_2 - 2t_1 + 2t_2\right)(2\delta + 2(-1 + \theta)g_2 + 6t_1 + 2t_2)\right)
\]

Subject to the constraints (10b) and (11b).

Solving the above problem, we get the equilibrium level of public investment and tax rate of the leader region as follows.

\[
g_{1L}^* = \frac{(2\delta - (1-\theta)^2)(1-\theta)}{(5\delta - 3(1-\theta)^2)} \quad \text{and} \quad t_{1L}^* = \frac{(2\delta - (1-\theta)^2)^2}{(5\delta - 3(1-\theta)^2)} \quad (13a)
\]

Substituting the equilibrium values of leader region’s tax rate and public investment in (10b) and (11b), we get the optimal level of public investment and the tax rate of the follower region as follows.

\[
g_{2F}^* = \frac{(3\delta - 2(1-\theta)^2)(1-\theta)}{(5\delta - 3(1-\theta)^2)} \quad \text{and} \quad t_{2F}^* = \frac{\delta(3\delta - 2(1-\theta)^2)}{(5\delta - 3(1-\theta)^2)} \quad (13b)
\]

The corresponding welfare and capital allocation, in equilibrium, are

\[
W_{1L}^* = \frac{(2\delta - (1-\theta)^2)^2}{2(5\delta - 3(1-\theta)^2)} \quad \text{and} \quad W_{2F}^* = \frac{(3\delta - (1-\theta)^2)(-6\delta + 4(1-\theta)^2)^2}{8(5\delta - 3(1-\theta)^2)^2} \quad (13c)
\]

\[
x_{1L}^* = \frac{(2\delta - (1-\theta)^2)^2}{(5\delta - 3(1-\theta)^2)} \quad \text{and} \quad x_{2F}^* = \frac{(3\delta - 2(1-\theta)^2)}{(5\delta - 3(1-\theta)^2)} \quad (13d)
\]
To satisfy the stability condition, we need \( \delta > \frac{(1-\theta)^2}{2} \). and for the non negativity constraints we must have \( \delta > \frac{2(1-\theta)^2}{3} \) and \( 0 \leq \theta < 1 \). Therefore, we assume that \( \delta > \frac{2(1-\theta)^2}{3} \) and \( 0 \leq \theta < 1 \). In the next section, we use the results of this section from the simultaneous and sequential move games, to solve a timing game between the regions to endogenize the choice of leadership.

4. ENDOGENIZING LEADERSHIP IN FISCAL COMPETITION: A TIMING GAME

Now, we turn to endogenize the regions’ decision to be a leader or a follower in case of sequential move multidimensional competition. First, we analyze whether regions are better off in case of sequential move competition than in case of simultaneous move competition and address the issue of endogenous leadership, by considering a timing game. Next, we further analyze the interplay of the two strategic variables – tax rate and public investment and offer explanation for the equilibrium outcomes of the timing game. In the timing game, in the initial stage (say, stage 0), both the regions simultaneously and independently decide whether to be the leader (early) or to be the follower (late). That is, in stage 0, each region decides whether to move early or late.

If both the regions decide to move early or they both decide to move late then, the game becomes a simultaneous move game in stage 1. Alternatively, if one region opts to move early and the other region opts to move late, we have a sequential move game (i.e. Stackelberg game) in stage 1. The player who moves early becomes the leader and the player moving late is the follower. Finally, in stage 2, allocation of mobile capital between the two regions is determined through competitive capital market as in section 3. Since the decision in stage 0 is about the time of move, we refer this extended game as timing game.

We solve the above mentioned timing game by standard backward induction method. Note that the stage 2 equilibrium outcomes would be same as that in Section 3. In stage 1, (a) the equilibrium outcomes are same as given by (12), if both the regions decide to move early or both decide to move late in stage 0; (b) the equilibrium outcomes are as given by (13a)-(13d), if one
region decides to move *early* and the other region decides to move *late* in stage 0. Note that, since the regions are symmetric, the subscripts 1 and 2 (in 1L and 2F) are interchangeable in (13a)-(13d).

Now, we can represent the stage 0 of the timing game in the normal form as follows.

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<tbody>
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<td></td>
<td>Early</td>
</tr>
<tr>
<td>Region-1 Early</td>
<td>(W_{1N}, W_{2N})</td>
</tr>
<tr>
<td>Late</td>
<td>(W_{1F}, W_{2L})</td>
</tr>
</tbody>
</table>

Comparing the payoffs of the above game corresponding to alternative pairs of strategies, we observe that

(a) if \(\delta_1 < \delta \leq \delta_2\) and \(0 \leq \theta < 1\), \(W_{il} > W_{iN} = W_{jF} > W_{jF}\); and

(b) if \(\delta > \delta_2\) and \(0 \leq \theta < 1\), \(W_{jF} > W_{il} > W_{iN} = W_{jN}\),

where \(\delta_1 = \frac{2}{3}(1 - \theta)^2\) and \(\delta_2 = (1 - \theta)^2\) and \(i \neq j = 1, 2\). Note that, by assumption, \(\delta > \delta_1\) and \(0 \leq \theta < 1\). Therefore, \(\delta_2\) is the critical value of interest. Note that, welfare of any region \(i\) \((i = 1, 2)\) can be expressed as \(W_i = \frac{\delta x_i^2}{2} + t_i x_i - \frac{\theta x_i^2}{2}\). Therefore, \(\delta = \frac{\partial^2 W_i}{\partial x_i^2}\). That is, we can interpret \(\delta\) as the rate of increase in marginal welfare of a region due to change in allocated capital in that region. So, higher is the value of \(\delta\), higher is the increase in welfare (both at level and in marginal terms) due to increase in capital flow to a region. In other words, regions value the mobile capital more, if \(\delta\) is higher.

So, if the regions value mobile capital less than a critical level and the spillover effect of public investment is less than perfect, i.e., when \(0 \leq \theta < 1\) and \(\delta_1 < \delta \leq \delta_2\), the leader gets higher payoff (welfare) compared to the simultaneous move game outcome as well as the follower region’s payoffs, but the follower gets lesser payoff compared to the simultaneous case. Clearly, in this case there is first mover advantage. So both the regions would want to become the leader.
in sequential game. In such a situation, the game in stage 1 turns out to be simultaneous move game, since in stage 0 the equilibrium strategy pair of the two region is \((\text{early}, \text{early})\).

Alternatively, if the regions value mobile capital more than a critical level and the spillover effect of public investment is less than perfect, i.e., when \(0 \leq \theta < 1 \text{ and } \delta > \delta_2\), both the leader and follower get higher payoffs compared to the simultaneous move situation. So both the regions prefer to have a sequential move game. In this case, the follower’s welfare is higher compared to the leader’s welfare. Clearly, there is a second move advantage in this case. So, each region wants to become a follower. But, even if one region becomes leader, it attains a higher welfare level compared to the simultaneous move game. If one region opts to move \textit{early}, then other region prefers to move \textit{late} and vice versa. In this situation there are two subgame perfect Nash equilibria- \((\text{late, early})\) and \((\text{early, late})\). We summarize these results in the following proposition.

**Proposition 3:** In the timing game with two choice variables of each region, there are three equilibria as follows.

a) If \(0 \leq \theta < 1 \text{ and } \delta_1 < \delta \leq \delta_2\), there is only one subgame perfect Nash equilibrium, \((\text{early, early})\).

b) If \(0 \leq \theta < 1 \text{ and } \delta > \delta_2\), there are two subgame perfect Nash equilibria, \((\text{late, early})\) and \((\text{early, late})\). These are also the Stackelberg equilibria of the game and are Pareto improving equilibria for both the regions.

We have shown that, since levels of public investments are strategic substitutes, there is first mover advantage in case of pure public investment competition (Proposition 2). On the other hand, since tax rates are strategic complements, there is second mover advantage in pure tax competition (Proposition 1). Now, Proposition 3 implies that, in case of competition in tax rates and levels of public investments, if the regions value mobile capital less than a critical level \((\delta \leq \delta_2)\), regions have a first mover advantage and the resultant game is simultaneous move game. In contrast, if the regions value mobile capital more than a critical level \((\delta > \delta_2)\), regions have the second mover advantage and the resultant game is a sequential move game. Therefore, it is not necessary that there will be second mover advantage for at least one region and sequential move
game need not necessarily be Pareto superior to simultaneous move game, if we allow for larger strategy space that includes both tax rate and level of public investment. Clearly, the results of Kempf and Rota-Graziosi (2010), where pure tax competition has been considered, emerge as a special case in the present analysis. In case of multidimensional competition for mobile capital, whether Stackelberg equilibrium will emerge as the subgame perfect Nash equilibrium or not, that depends on the rate of change in marginal welfare of the regions due to change in capital allocation.

Note that, we can say that whether a region is a leader (loses in tax rates decision and gains in public investment decision) or a follower (loses in public investment decision and gains in tax rate decision), it loses in one choice variable's decision and gains in the other choice variables decision. It indicates that the relative strength of the two strategic effects, strategic substitute effect via public investment and strategic complement effect via tax rate, determines the equilibrium of the timing game.

Therefore, in order to understand the mechanism behind the changes in the equilibrium as the magnitude of the parameter δ changes, we need to examine the interplay between the strategic complement effect of tax rates and strategic substitute effect of levels of public investments. We suspect that when strategic substitutes effect dominates strategic complement effect, we get the first equilibrium (early, early) because of first mover advantage; on the other hand, when strategic complements effect is stronger than the strategic substitute effect, there is second mover advantage and we get two Stackelberg equilibria, (late, early) and (early, late). These results can be compared with industrial organization literature (Hamilton and Slutsky, 1990; Robson, 1990). In case of endogenous timing price (quantity) competition game between two firms, Stackelberg (Cournot) outcomes are Pareto superior for both the both firms. But our results differentiate from them in the context of strategy space (we consider two strategic variables, instead of only one strategic variable).

In the next sub-section, we examine the joint effect of the two strategies. We note here that, to the best of our knowledge in the relevant literature, the nature and impact of joint effect of two or more number of opposite strategies has not been discussed. Therefore, from theoretical point view as well, it is important to formalize the concept of joint effect of multiple strategies, which
are of opposite nature, and to examine the implication of joint effect of multiple strategies to equilibrium outcomes.

4.1 Joint Strategic Substitute and Joint Strategic Complement

Let us first define the ‘Joint strategic substitute’ and ‘Joint strategic complement’ as follows.

**Definition:** In case of multidimensional competition between players, i.e., when players can choose more than one variable (of opposite nature) strategically, if strategic substitute effect dominates the strategic complements effect, the joint nature of strategic variables is ‘Joint Strategic Substitute (JSS)’ and their joint effect is ‘Joint Strategic Substitute Effect (JSSE)’. On the other hand, if strategic complement effect dominates strategic substitute effect, the joint nature of strategic variables is ‘Joint Strategic Complement (JSC)’ and their joint effect is ‘Joint Strategic Complement Effect (JSCE)’.

Now, in the multidimensional competition game, there are two choices where regions either prefer to become leader or follower. Clearly, the joint effect of the two strategic variables, tax rate and level of public investment, is the difference between the leader and the follower region’s payoffs in the sequential move game. We can express the joint effect (JE) of the two strategic variables on region i’s ($i = 1, 2$) welfare as follows.

$$JE_i = W_i( g_{iL}, t_{iL}, g_{jF}, t_{jF} ) - W_i( g_{iF}, t_{iF}, g_{jL}, t_{jL} ); i, j = 1, 2, \ i \neq j$$

(14)

Without any loss of generality, let us consider that $i = 1$ and $j = 2$. Now using the welfare function of region-1, we can decompose the $JE_1$ as follows.

$$JE_1 = W_1( g_{1L}, t_{1L}, g_{2F}, t_{2F} ) - W_1( g_{1F}, t_{1F}, g_{2L}, t_{2L} )$$

$$= [W_1( g_{1L}, t_{1L}, g_{2F}, t_{2F} ) - W_1( g_{1F}, t_{1F}, g_{2L}, t_{2L} )] - [W_1( g_{1F}, t_{1L}, g_{2L}, t_{2F} ) - W_1( g_{1F}, t_{1F}, g_{2L}, t_{2F} )]$$

$$= [SSE_1] - [SCE_1]$$

Clearly, $JE_1 > 0$, if $SSE_1 > SCE_1$. Now, plugging the values of welfare we get the following.
\[ JE_1 = \left\{ \left( -\delta + (1-\theta)^2 \right) \left( 7\delta^2 - 6\delta(1-\theta)^2 + (1-\theta)^4 \right) \right\} \left( \frac{1}{2\delta(5\delta - 3(1-\theta)^2)^2} \right) \]

\[ = JE_1 \left( -7\delta^3 + 13\delta^2(1-\theta)^2 - 7\delta(1-\theta)^4 + (1-\theta)^6 \right) \]

It is easy to check that,

(a) \[ JE_1 > 0, \text{ if } \delta < \delta_2 = (1-\theta)^2. \]

(b) \[ JE_1 = 0, \text{ if } \delta = \delta_2 = (1-\theta)^2. \]

(c) \[ JE_1 < 0, \text{ if } \delta > \delta_2 = (1-\theta)^2. \]

Therefore, if \( \delta < \delta_2 \), SSE dominates SCE, i.e., there is JSSE, and the regions prefer to be the leader. In contrast, if \( \delta > \delta_2 \), SCE dominates SSE, i.e., there is JSCE, and regions prefer to be follower. Note that, this is true for region 2 as well.

To illustrate it further, note that tax reaction functions are strictly upward sloping and public investment reaction functions are strictly downward sloping (see Lemma 2 and Lemma 4). The slopes of tax reaction function in \( t_1-t_2 \) plane and the slope of the public investment reaction function in \( g_1-g_2 \) plane, of region 1 are, respectively, \( \frac{\partial t_2}{\partial t_1} = 3 \) and \( \frac{\partial g_2}{\partial g_1} = -\frac{4\delta-(1-\theta)^2}{(1-\theta)^2} \). And, the slopes of tax reaction function in \( t_1-t_2 \) plane and the slope of the public investment reaction function in \( g_1-g_2 \) plane, of region 1 are, respectively, \( \frac{\partial t_2}{\partial t_1} = \frac{1}{3} \) and \( \frac{\partial g_2}{\partial g_1} = -\frac{(1-\theta)^2}{4\delta-(1-\theta)^2} \). Note that, in \( t_1-t_2 \) plane, flatter (steeper) the tax reaction function of region 1 (region 2), greater is the response of region 1 (region 2) for a unit change in its rival’s tax rate. Similarly, in \( g_1-g_2 \) plane, flatter (steeped) the public investment reaction function of region 1 (region 2), greater is the response of region 1 (region 2) for a unit change in its rival’s public investment.

In general, we can write the slope of the tax reaction functions of region 2, in \( t_1-t_2 \) plane, as

\[ \frac{\partial t_2}{\partial t_1} = -\left( \frac{\partial^2 W_2}{\partial t_1 \partial t_2} \right) \]
On the RHS, the denominator must be negative (due to the second order condition) and the numerator is the strategic complement effect of tax rates, which is positive.

Similarly, we can write the slope of the public investment reaction function of region 2, in $g_1$-$g_2$ plane, as $\frac{\partial g_2}{\partial g_1} = -\left(\frac{\partial^2 W_2}{\partial g_1 \partial g_2}\right)$. Here, In the RHS, the denominator must be negative (by the second order condition for maximization) and the numerator is the strategic substitute effect of tax rates, which is negative.

Now, we argue that the relative magnitude of strategic effect of a choice variable, relative to that of the other choice variable, can be shown to be related to the relative magnitudes of the reaction functions.

**Proposition 4:** In the sequential move multidimensional competition, the relation between the joint strategic effects and slopes of the reactions functions are as follows.

a) **Joint Strategic Substitutes Effect (JSSE)** holds, if the absolute slope of the public investment reaction function of region 2 (region 1) in $g_1$-$g_2$ plane is greater (smaller) than the absolute slope of the tax reaction function of region 2 (region1) in $t_1$-$t_2$ plane.

b) **Joint Strategic Complements Effect (JSCE)** holds, if the absolute slope of the public investment reaction function of region 2 (region 1) in $g_1$-$g_2$ plane is smaller (greater) than the absolute slope of the tax reaction function of region 2 (region 1) in $t_1$-$t_2$ plane.

**Proof:** It is sufficient to show that the Proposition 4 is true for any one of the two regions due to symmetric nature. Let us consider region 2’s reaction functions. Now, the absolute value of slope of the public investment reaction function of region 2, in $g_1$-$g_2$ plane, is greater than the absolute value of the slope of the tax reaction function of region 2, in $t_1$-$t_2$ plane, if

$$\left| \frac{\partial g_2}{\partial g_1} \right| > \left| \frac{\partial t_2}{\partial t_1} \right|$$

$$\Rightarrow -\left| \frac{\partial^2 W_2}{\partial g_1 \partial g_2} \right| > -\left(\frac{\partial^2 W_2}{\partial t_1 \partial t_2}\right)$$
\[ \Rightarrow \left( \frac{\partial^2 W_2}{\partial g_1 \partial g_2} \right) > - \left( \frac{\partial^2 W_2}{\partial g_1 \partial t_2} \right) \]
\[ \Rightarrow \left( \frac{\partial^2 W_2}{\partial g_1 \partial g_2} \right) + \left( \frac{\partial^2 W_2}{\partial g_2 \partial t_2} \right) > 0 \]
\[ \Rightarrow \left( \frac{(1 - \theta)^2}{4\delta - (1 - \theta)^2} \right) - \frac{1}{3} > 0 \]
\[ \Rightarrow \delta < (1 - \theta)^2 \]
\[ \Rightarrow \text{SSE} > \text{SCE} \]

That is, if for region 2 \( \frac{\partial g_2}{\partial g_1} > \frac{\partial t_2}{\partial t_1} \), there is JSSE. Otherwise, if \( \frac{\partial g_2}{\partial g_1} < \frac{\partial t_2}{\partial t_1} \), there is JSCE.

QED

Note that, if all the choice variables are strategic complements in nature, JSCE will always hold true. In Kempf and Rota-Graziosi (2010), each region has one choice variable (tax rate) and tax rates are strategic complements, so there is JSCE and, thus, each region has second mover advantage. However, in general, JSCE effect need not necessarily hold true. In the present model, JSCE holds true, only if the absolute slope of the public investment reaction function of region 2 (region 1) in \( g_1 - g_2 \) plane is smaller (greater) than the absolute slope of the tax reaction function of region 2 (region 1) in \( t_1 - t_2 \) plane. Otherwise, JSSE holds true.

### 4.2 Equilibrium Selection: Pareto vs. Risk Dominance

As noted before, there are multiple equilibria of the timing game, when JSCE holds, (Proposition 3, part (b)). Now the question arises, how do we select one of the two equilibria? In the literature on selection of equilibrium, there are two criteria which are widely used to rank the equilibria. One is Pareto dominance criterion and another is the risk dominance criterion. Pareto dominance
criterion is applicable when at least one player is better off without worsening the others. On the other hand, an equilibrium risk-dominates the other equilibrium when the former is less risky than the latter, that is the risk-dominant equilibrium is the one for which the product of the deviation losses is the largest (Harsanyi and Selten (1988)).

In our case, from Proposition 3 part (b), equilibrium (early, late) (region-1 leads and region-2 follows) risk dominates equilibrium (late, early) (region-2 leads and region-1 follows), if the former is associated with larger product of deviation losses: \( \varphi = (W_{1L} - W_{1N})(W_{2F} - W_{2N}) - (W_{1F} - W_{1N})(W_{2L} - W_{2N}) > 0 \). That is, the equilibrium (early, late) risk dominates (late, early), if the product of welfare losses of the two regions by deviating from (early, late) is greater than the product of welfare losses of the two regions by deviating from (late, early), to the (early, early) or (late, late).

Now, since the regions are assumed to be symmetric, \( W_{1L} = W_{2L}, W_{1F} = W_{2F} \) and \( W_{1N} = W_{2N} \). Therefore, in the present context, we have \( \varphi = 0 \). It implies that none of the two equilibrium risk dominates the other. Therefore, we cannot select any one of the two equilibria by the risk dominance criterion. Also, it is straightforward to observe that we cannot select any equilibrium on the basis of the Pareto dominance criterion either.

**4.3 Comparison of equilibrium outcomes: tax rate, public investment, capital allocation and social welfare**

Finally, we turn to compare the equilibrium outcomes under alternative scenarios. In Table 1, we report the rankings welfare, capital allocation, level of public investment and tax rate of region 1 and region 2, in (early, late), (late, early) and simultaneous move equilibrium, assuming \( 0 \leq \theta < 1 \), \( \frac{2}{3}(1 - \theta)^2 = \delta_1 \leq \delta \).

<table>
<thead>
<tr>
<th>Table 1: Ranking of equilibrium outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
</tr>
<tr>
<td>Welfare</td>
</tr>
<tr>
<td>Capital</td>
</tr>
<tr>
<td>Pub. Investment</td>
</tr>
</tbody>
</table>
Case 1: $\delta \leq \delta_2$

In this case, tax rates (strategic complement) effect is less than public investments (strategic substitute) effect and, thus, joint strategic substitute effect holds true. So, there is first mover advantage. If region 1 moves early and region 2 moves late, public investment in region 1 is higher than that in region 2. The reason being that region 1 is more aggressive in setting public investment due to JSSE. But, both leader and follower region’s tax rates are lower as compared to the simultaneous move game. The reason is that, in region 1, the capital elasticity of tax rates in the sequential game is higher than simultaneous move game ($e_{seq}(x_{t1}) = \frac{(1-\delta)^2}{2\delta} - 1 > e_{sim}(x_{t1}) = -\frac{1}{2}$). For region 2 which has the same level of capital elasticity of tax in both the games ($e_{seq}(x_{t2}) = e_{sim}(x_{t2}) = -\frac{1}{2}$), the level of public investment is lower than the earlier game, so region 2 being a follower charges tax rate even lower than region 1. Moreover region 2 is the follower in tax rates which causes lower tax rate as compared to region 1. In total, the strategic effect of tax rates (SCE) is dominated by strategic effect of public investment (SSE). So, the higher public investment in region 1 over compensates the negative effect of tax rate. This causes higher allocation of capital in region 1 and lower in region 2, compared to the simultaneous move game. These are new results.

Case 2: $\delta > \delta_2$

In this case, we have a Pareto superior situation in the sequential move game, because both the regions are getting higher payoffs compared to their simultaneous game payoffs, irrespective of the role of the region as a leader or a follower. For this case, we have joint strategic complements effect and regions prefer to become the follower. As we can see that region 2 is providing higher level of public investment than region 1. There are higher tax rates because, regions’ tax reaction functions shifts outwards due higher $\delta$. So region 1 as well as region 2 charges higher tax rates than the simultaneous move game. Moreover, region 2 is levying lower tax rate than region 1, due to higher sensitivity of capital to tax rates in region 2. This leads to a higher capital

| Taxes          | $t_{1N} = t_{2N} \geq t_{1L} \geq t_{2F}$ | $t_{1L} > t_{2F} > t_{1N} = t_{2N}$ |
allocation to region 2 \( \frac{\partial x_{2f}}{\partial g_{2f}} > 0, \frac{\partial x_{2f}}{\partial t_{2f}} < 0 \) as well as higher welfare levels in region 2. These results are similar to Kempf and Rota-Graziosi (2010), in the sense that sequential game equilibrium is providing higher welfare level for both the regions, irrespective of the role as a leader or a follower.

5. CONCLUSION

We demonstrate the nature of equilibrium in a multi dimensional fiscal competition for mobile capital if the strategic choices (tax and public investment) are of opposite nature. We first show that in a pure tax competition game, tax rates are strategic complements and each of the two regions prefers to be follower, because of second mover advantage. In contrast, public investments are strategic substitutes and the regions prefer to be leader in sequential move game.

In a fiscal competition with both tax rate and public investment, we examine the consequences of interplay between these two choice variables. We demonstrate that, since tax rate and level of public investment are of opposite strategic nature, there is a trade off involved. As a result, the possibility of both first mover advantage and second mover advantage exists, unlike in case of uni-dimensional competition. This is a new result.

Next, we formalize the concepts of joint strategic substitute effect and joint strategic complements effect, and establish their relation with the slopes of the reaction function of the regions. Further, we show that if there is a joint strategic substitute effect, both the regions prefers to be the leader and, thus, the resultant outcome is simultaneous move game. On the other hand, if there is a joint strategic complement effect, sequential move equilibrium is Pareto superior than the simultaneous move equilibrium. However, in the later case there are two sequential move equilibria, which are identical from both Pareto dominance criterion’s point of view as well as risk dominance criterion’s point of view.

We also provide ranking of the equilibrium outcomes in alternative scenarios. We show that, if there is joint strategic complements effect, capital allocated and level of public investment in the follower (leader) region is higher than that in case of simultaneous move equilibrium. But, the equilibrium tax rate (welfare) of leader region is higher (lower) than that of the follower region,
and equilibrium tax rate and welfare of both leader region and follower region are higher than that in case of simultaneous move game. These results are similar to that in Kempf and Rota-Graziosi (2010). However, if there is joint strategic substitutes effect, the equilibrium welfare, allocated capital and public investment of the leader (follower) region are higher (lower) than that in case of simultaneous move equilibrium. But, the tax rate of the follower region is lower than that in the leader region, and the leader region’s tax rate is lower than the simultaneous move equilibrium tax rate. It indicates that, sequential move interregional competition for mobile capital need not necessarily control race-to-the-bottom in tax rates, unlike as argued in the existing literature. In fact, race-to-bottom in tax rates may be intensified in case of sequential move than that in case of simultaneous move, unless timing of move is endogenously determined by the competing regions.
REFERENCES


APPENDIX A

A1: Tax rates are complements in nature.

**Proof:** We have, \( W_1 = \left( \frac{\delta x_1^2}{2} \right) - \left( \frac{g_1^2}{2} \right) + (t_1 x_1) \) and \( W_2 = \left( \frac{\delta x_2^2}{2} \right) - \left( \frac{g_2^2}{2} \right) + (t_2 x_2) \). Differentiating \( W_1 \) and \( W_2 \) with respect to \( t_2 \) and \( t_1 \) respectively, we get

\[
\frac{\partial W_1}{\partial t_2} = (t_1 + \delta x_1) \frac{\partial x_1}{\partial t_2} \quad (9a)
\]

\[
\frac{\partial W_2}{\partial t_1} = (t_2 + \delta x_2) \frac{\partial x_2}{\partial t_1} \quad (9b)
\]

Now, from equation (7a) and (7b), we get \( \frac{\partial x_1}{\partial t_2} \frac{\partial x_1}{\partial t_1} = \frac{1}{2\delta} > 0 \). Substituting these values in (9a) and (9b), we have, \( \frac{\partial W_1}{\partial t_2} = \frac{1}{2\delta} (t_1 + \delta x_1) > 0 \) and \( \frac{\partial W_2}{\partial t_1} = \frac{1}{2\delta} (t_2 + \delta x_2) > 0 \) . QED.

A2: (Strategic complements)

**Proof:** It is easy to check that \( \frac{\partial^2 W_1}{\partial t_2 \partial t_1} = \frac{1}{2 \delta} \frac{\partial x_1}{\partial t_2} \frac{\partial x_1}{\partial t_1} > 0 \) and \( \frac{\partial^2 W_2}{\partial t_1 \partial t_2} = \frac{1}{2 \delta} \frac{\partial x_2}{\partial t_1} \frac{\partial x_2}{\partial t_2} > 0 \). That is, if there is an increase in region-2’s (region-1’s) tax rate, the negative marginal effect of region-1’s (region-2’s) tax rate on its own welfare decreases. Therefore, tax rates are strategic complements. From (10a) and (10b), we get the slopes of the tax reaction functions of region 1 and region 2, respectively, as follows. find that, \( \frac{\partial t_2}{\partial t_1} \big|_{T_1} = 3 > 0 \) and \( \frac{\partial t_2}{\partial t_1} \big|_{T_2} = \frac{1}{3} > 0 \). QED.

A3: (First mover advantage)

**Proof:** Since the two regions are symmetric, it is sufficient to show that region-1 always prefers to be the follower in sequential move tax competition. That is, we have to prove that \( W_1( t_{1F}, t_{2L} ) > W_1( t_{1L}, t_{2F} ) \), where (a) \( t_{1F} \) and \( t_{2L} \) are the equilibrium tax rates of region-1 and region-2, respectively, when region-1 is the follower and region-2 is the leader and (b) \( t_{1L} \) and \( t_{2F} \) are the equilibrium tax rates of region-1 and region-2, respectively, when region-1 is the leader and region-2 is the follower.
Now, to prove that \( W_1(t_{1F}, t_{2L}) > W_1(t_{1L}, t_{2F}) \), we need to show that

\[
t_{2L} \geq T2(t_{1F}). \tag{P1.1}
\]

Let us assume that,

\[
t_{2L} < T2(t_{1F}). \tag{P1.2}
\]

Applying the function \( T1(.) \) to both sides of the inequality, we get the following.

\[
t_{1F} = T1(t_{2L}) < T1(T2(t_{1F})) \tag{P1.3}
\]

In (P1.3), the first equality follows from the Stackelberg equilibrium definition, and the second inequality follows from Lemma 2.

Using the above inequalities we can show that

\[
W_1(t_{1F}, t_{2L}) \leq W_1(t_{1F}, T2(t_{1F})) < W_1(T1(T2(t_{1F})), T2(t_{1F})), \quad \text{where the first inequality follows from (P1.2) and Lemma 1, and the second inequality follows from (P4.3).}
\]

This contradicts our statement that \( W_1(t_{1F}, t_{2L}) \) is the Stackelberg equilibrium. So, our assumption (P1.2) is false. That is, we are able to show by contradiction that \( t_{2L} \geq T2(t_{1F}) \) is true.

Since the regions are symmetric, we can write \( W_1(t_{1F}, t_{2L}) = W_2(t_{2F}, t_{1L}) \) and \( W_1(t_{1L}, t_{2F}) = W_2(t_{1F}, t_{2L}) \).

Now, we can show that every region wants to be the follower when Lemma 1 and Lemma 2 hold true,

\[
W_1(t_{1F}, t_{2L}) \geq W_1(t_{1F}, T2(t_{1F})) \geq W_2(t_{1F}, t_{2L}) = W_1(t_{2F}, t_{1L}), \quad \text{where the first inequality follows from (P1.1), and the second inequality follows from symmetry and (P1.1), and the third equality follows from the symmetric equilibrium outcomes.}
\]

Clearly, both the regions prefer to be the follower than to be the leader in case of pure tax competition. QED
A4: (Public investments are Substitutes)

Proof: We have \( W_1 = \left(F_1 - x_1 f_{1,x_1}(x_1, g_1)\right) - \frac{(g_1^2)}{2} + (t_1 x_1) \) and \( W_2 = \left(F_2 - x_2 f_{2,x_2}(x_2, g_2)\right) - \frac{(g_2^2)}{2} + (t_2 x_2) \). Using the production function (1) and (2), we can write the following.

\[
W_1 = \left(\frac{\delta x_1^2}{2}\right) - \frac{(g_1^2)}{2} + (t_1 x_1)
\]

\[
W_2 = \left(\frac{\delta x_2^2}{2}\right) - \frac{(g_2^2)}{2} + (t_2 x_2)
\]

Now differentiating \( W_1, W_2 \) with respect to \( g_2 \) and \( g_1 \) respectively, we get,

\[
\frac{\partial W_1}{\partial g_2} = (t_1 + \delta x_1) \frac{\partial x_1}{\partial g_2}
\]

\[
\frac{\partial W_2}{\partial g_1} = (t_2 + \delta x_2) \frac{\partial x_2}{\partial g_1}
\]

Differentiating (7a) and (7b) w.r.t. \( g_2 \) and \( g_1 \), respectively, we get

\[
\frac{\partial x_1}{\partial g_2} = \frac{\partial x_2}{\partial g_1} = \frac{-(1-\theta)}{2\delta} < 0
\]

Substituting these values in above equations, we get

\[
\frac{\partial W_1}{\partial g_2} = \frac{-(1-\theta)}{2\delta} (t_1 + \delta x_1) < 0 \quad \text{and} \quad \frac{\partial W_2}{\partial g_1} = \frac{-(1-\theta)}{2\delta} (t_2 + \delta x_2) < 0 . \text{QED}
\]

A5: (Strategic Substitutes)

Proof: It is easy to check that \( \frac{\partial^2 W_1}{\partial g_2 \partial g_1} = \frac{\partial x_1}{\partial g_1} \frac{\partial x_1}{\partial g_2} < 0 \) and \( \frac{\partial^2 W_2}{\partial g_1 \partial g_2} = \frac{\partial x_2}{\partial g_2} \frac{\partial x_2}{\partial g_1} < 0 \). Therefore, levels of public investments are strategic substitutes. Now, using the public investment reaction functions of region-1 and region-2 from (12a) and (12b), we obtain,

\[
\frac{\partial g_2}{\partial g_1} = -\frac{(1-\theta)^2}{4\delta - (1-\theta)^2} < 0
\]
\[ \frac{\partial g_1}{\partial g_2} = -\frac{(1 - \theta)^2}{4\delta - (1 - \theta)^2} < 0 \]

We must have, \(2\delta > (1 - \theta)^2\) to satisfy the second order, stability and non negativity conditions. So, we can say that reaction curves in public investment choice are downward sloping. QED.

**A6: Proposition 2 (First mover advantage)**

**Proof:** Under Lemma 3 and Lemma 4, we can show that both the regions prefer to be the leader.

Let \(W_1( g_{1L}, g_{2F}) = W_1( g_{1L}, G2( g_{1L}))\) be the Stackelberg equilibrium when firm 1 leads. Then, we have to show that \(W_1( g_{1L}, g_{2F}) > W_1( g_{1F}, g_{2L})\), where (a) \(g_{1F}\) and \(g_{2L}\) are the equilibrium levels of public investments of region 1 and region 2, respectively, when region 1 is the follower and region 2 is the leader and (b) \(g_{1L}\) and \(g_{2F}\) are the equilibrium levels of public investments of region 1 and region 2, respectively, when region 1 is the leader and region 2 is the follower.

As we have symmetric regions, we can write the following

\[ W_1( g_{1L}, G2( g_{1L})) = W_2(G1( g_{2L}), g_{2L}) \]
\[ W_1(G1( g_{2L}), g_{2L}) = W_2( g_{1L}, G2( g_{1L})) \]

First, we need to show that;

\[ g_{1L} \geq G1( g_{2F}) \quad \text{(P2.1)} \]

Now, let’s assume that

\[ g_{1L} < G1( g_{2F}) \quad \text{(P2.2)} \]

Applying the function \(G2(\cdot)\) to both sides of the equation (P2.2), we get,

\[ g_{2F} = G2(g_{1L}) > G2(G1( g_{2F})) \quad \text{(P2.3)} \]

We get the first equality, from the definition of Stackelberg equilibrium and the second inequality, from Lemma 4 because the \(G2(\cdot)\) function is strictly decreasing.
Using these relations we can show that

$$W_1(g_{1L}, g_{2F}) \leq W_1(G_1(g_{2F}), g_{2F}) < W_1(G_1(g_{2F}), G_2(G_1(g_{2F})))$$

where inequality (1) follows from the assumption (P2.2) and inequality (2) follows from (P2.3) and Lemma 4. So we have the equilibrium $G_1(g_{2F}), G_2(G_1(g_{2F}))$, which yields higher welfare compared to the $(g_{1L}, G_2(g_{1L}))$ equilibrium. This contradicts the claim that $(g_{1L}, G_2(g_{1L}))$ is the Stackelberg equilibrium. This contradiction refutes our assumption that $g_{1L} < G_1(g_{2F})$. So, we have proved by contradiction that $g_{1L} \geq G_1(g_{2F})$.

Now we can show that,

$$W_1(g_{1L}, g_{2F}) = W_1(g_{1L}, G_2(g_{1L})) \geq W_1(G_1(g_{2F}), g_{2F}) \geq W_2(g_{1L}, g_{2F}) = W_1(g_{1F}, g_{2L})$$

First equality follows from the definition of Stackelberg equilibrium, the second inequality follows from $g_{1L} \geq G_1(g_{2F})$, and the third inequality follows from symmetric equilibriums and $g_{1L} \geq G_1(g_{2F})$. The last equality follows from symmetry.

So, we have proved that region-1 always prefers to be the leader under Lemma 3 and lemma 4, when tax rates are constant. Similarly, region-2 also prefers to be the leader compared to being the follower. QED