Optimal Redistributive Capital Taxation with Different Types of Workers

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18 May 2017

Abstract

This paper analyzes the optimality of undertaking income redistribution through capital taxation in capitalist-worker models à la Judd (1985) when there are different types of workers that supply labor inelastically. Two cases are analyzed: one with working capitalists and one in which there are two types of pure workers alongside non-working capitalists. In both cases, taxing capital income to make lump-sum transfers to pure workers is generally optimal. In the case of working capitalists, this is because wage rents enter the savers-taxpayers’ budget constraint. In the other case, such a result is to be ascribed to the fact that consumption of the second type of workers, a function of the capital stock, enters the social welfare function and crowds out consumption of other workers. Finally, by combining the two cases discussed here, the desirability of capital taxation is studied in an economy with illegal workers subject to exploitation.


Keywords: Optimal capital tax; Agent heterogeneity; Redistribution; Pure rents; Different types of workers.

*The author would like to thank Luigi Bonatti for his perceptive comments.
1 Introduction

In heterogeneous-agent economies based on the distinction between capitalists and workers, the redistributive potential of a capital income tax is generally zero. This result is demonstrated by Judd (1985) in infinite-lived neoclassical growth models peopled by capitalists, who pay taxes on capital income, and workers, who do not pay taxes and receive capital tax proceeds from the government. In contexts of this type, setting the capital income tax rate to zero is generally optimal in the long-run even from the viewpoint of the recipients of tax revenue.\(^1\)

Judd (1985) shows that the optimality of a zero redistributive capital income tax is independent of agents' political influence (captured by the weight assigned to each type of agent in the social welfare function), the degree of income inequality, inputs' elasticities, workers' ownership of capital and the endogeneity of the rate of time preference.

By considering the simplest version of the capitalist-worker economy analyzed by Judd (1985) —that is, an economy in which only capitalists save and only workers work—, Lansing (1999) obtains that the steady state optimal tax on capital income differs from zero when the capitalists' utility function is logarithmic and no other suitable policy instruments are at the disposal of the policy maker. In such a case, the government does not have a policy tool that can be used to implement the allocations of capital and consumption consistent with a zero capital tax. Because when the savers' intertemporal elasticity of substitution is equal to one, shifts in the future after-tax interest rate exert no 'anticipation effects' on the capitalists' decision rules as the substitution effect and the income effect that they activate exactly cancel out. For this reason, capital income ought to be taxed.

From a methodological standpoint, the Lansing (1999) result demonstrates that the standard normative approach (which considers the first-order conditions of the private agents’ problems as the constraints for the government’s tax choice) is invalid when preferences of capitalists are logarithmic, since it implicitly assumes that future tax shifts can affect current allocations. Because of the absence of 'anticipation effects', the capitalist consumption function (which states that consumption is a fixed proportion of

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\(^1\)Chamley (1986) obtains the same result as Judd (1985) in a single-type representative-agent economy with an endogenous labor supply, where the emphasis is on factor heterogeneity (labor versus capital) rather than agent heterogeneity, on the one side, and second-best efficiency rather than optimal redistribution, on the other.
the capital stock, the unique asset of the economy) is the correct constraint that the social planner has to take into account when choosing the optimal tax rate.

In the logarithmic case, however, if an additional policy instrument suitable for making redistribution in a less distortionary way —such as government debt or a constant consumption tax imposed on capitalists— were at disposal of the government, the zero redistributive capital tax result would be obtained.

Reinhorn (2014) tries to reconcile the different and apparently contradictory results of Judd (1985) and Lansing (1999) by investigating the dynamic properties of the costate variables of the government’s problem (i.e., the social shadow prices), which in general receive less attention, being unobservable. Reinhorn (2014) shows that when the intertemporal elasticity of substitution of capitalists differs from one, the social shadow prices converge to the steady state. When instead capitalists’ preferences are logarithmic, the costates of the normative problem do not converge to the long-run equilibrium and the dynamic system does not even have a steady state if the standard approach is employed. In this case, the use of the capitalists’ consumption function as a constraint on the social planner’s tax choice (proposed by Lansing, 1999), implying an optimal capital tax rate different from zero, solves the problems of the existence of the steady state equilibrium and of the costates’ convergence.

Departures from the zero redistributive capital tax result are also obtained by Kemp, Van Long and Shimomura (1993), and Straub and Werning (2015). By using a feedback Stackelberg equilibrium with government as a leader, the former contribution finds that a tax on capital can be optimally used for redistributing income across social classes in the long-run. Straub and Werning (2015) obtain that an optimal redistributive capital tax rate does not fall towards zero, but rises over time, converging asymptotically to a positive level, when the intertemporal elasticity of substitution is less than one. For other values of such elasticity, they show numerically that the capital tax rate converges to zero very slowly, after centuries of high tax rates.

Mankiw (2015) notices that if the social planner is concerned about inequality between capitalists and workers —that is, the objective of the government is to increase the ratio of workers’ consumption to capitalists’ consumption—, then it is optimal to tax capital. Capital taxation, whose proceeds are transferred to workers, reduces the consumption level of both
workers and capitalists, but consumption of the latter will be reduced by more, thus raising (for a standard production function) the ratio of worker consumption to capitalist consumption and hence increasing income inequality.

The purpose of this paper is to discuss the optimality of a redistributive capital income tax in capitalist-worker models with different types of workers who supply labor inelastically. Two cases are analyzed in the simplest Judd (1985) economy: one in which capitalists also work and one in which there are two types of pure workers alongside capitalists.

As will be demonstrated below, the consideration of different types of workers supplying labor inelastically has unexplored implications on the role that capital taxation can play in addressing problems of income inequality: the optimal redistributive capital income tax is generally different from zero when no other suitable policy instruments are at disposal of the government.\(^2\)

With working capitalists, capital income taxation is desirable for undertaking income redistribution because pure wage rents (stemming from an exogenous labor supply), which depends on the capital stock, enter the capitalists’ budget constraint.\(^3\)

In such an economy, the case of savers’ logarithmic preferences does not have qualitative consequences on the optimal capital tax rate as the ‘anticipation effect’ of future after-tax interest rate shifts is at work through

\(^2\)In a second-best perspective, the consideration of different types of workers has no consequences on the optimality of a zero capital tax rate when free taxation of all factor incomes is allowed. If such an approach is considered with tax restrictions—as, for example, discussed by Stiglitz (1987) when there is a uniform taxation of wages and income from capital, Jones, Manuelli and Rossi (1997) when different wages are taxed at the same rate, and Reis (2011) in the case in which entrepreneurial labor income and capital income are taxed equally—, the zero capital tax result will be invalidated. This is because tax limitations imply the violation of the Diamond and Mirrlees (1971) and Atkinson and Stiglitz (1972) principles of public finance that justify the second-best optimality of lifting capital income from taxation, as pointed out by Munk (1980) and Correia (1996).

\(^3\)In Judd (1985), the case in which both taxpayers and recipients of capital tax proceeds are capital owners and workers at the same time, and every agent supplies labor endogenously is studied; in such a circumstance, the zero capital tax result is satisfied. The new hypothesis introduced here in the simplest version of the Judd (1985) model, that is conducive to new results, is that capitalists work and supply labor inelastically. In the second-best analysis of capital taxation, Jones, Manuelli and Rossi (1997) show that the presence of rents that cannot be fully taxed results in an optimal non-zero capital tax rate.
the labor income of capitalists. Moreover, if another suitable policy instrument —like a constant consumption tax levied on capitalists or a tax on the capitalist-worker wage or government debt— is used by the government to carry out redistribution, it will be optimal to exempt capital income from taxation.

When there are two types of pure workers, only one type of which receives capital tax revenue from the government, the nonoptimality of the Judd (1985) result is to be ascribed to the fact that consumption of the second type of workers (in practice their wage), which is a function of the capital stock, enters the social welfare function and, by absorbing resources, crowds out consumption of other workers and capital formation.

With two types of pure workers, a problem of existence of the steady state equilibrium, due to the absence of “anticipation effects”, will arise when the capitalists’ utility is logarithmic. After using the Lansing (1999) procedure to overcome such a problem, I obtain that the optimal capital tax rate differs from zero for two reasons: i) the impossibility of implementing the first-best allocation of resources; ii) different types of workers.

The consideration of a second policy instrument suitable for redistribution, optimally chosen by the social planner, can imply an optimal capital tax rate equal to zero. In the case of a non-logarithmic utility of capitalists, a consumption tax imposed on those workers that are not receiving lump-sum transfers from the government is sufficient to obtain a zero optimal capital tax rate. When the intertemporal elasticity of substitution of capitalists is equal to one, two instruments are necessary to set optimal capital taxation to zero; these policy instruments can be a consumption tax levied on capitalists and a consumption tax imposed on those workers that do not receive government transfers.

Finally, by combining the two cases developed here, the desirability of capital taxation is studied in an economy with illegal workers that are exploited because firms hiring them risk paying penalties if discovered. In such an economy, the illegality of some workers implies that there are exploitative rents perceived by capitalists, on the one hand, and the utility of clandestine workers is not taken into account by the social planner, on the other. In this context, the optimality of taxing capital income is confirmed, when there are not additional policy instruments that can bring the capital tax rate to zero.

If capital tax proceeds were distributed in fixed proportions to both types of workers, the qualitative results will be basically unchanged.
The rest of the paper is structured as follows. Section 2 studies the normative properties of redistributive capital taxation in a capitalist-worker model in which capital owners work, supplying labor inelastically, along with pure workers. Section 3 examines the implications of considering two types of pure workers alongside capitalists on optimal capital taxation. By combining the analysis of sections 2 and 3, section 4 investigates an optimal redistributive fiscal policy when there are illegal workers, exploited because of their status. Section 5 concludes.

2 A Judd (1985) economy with working capitalists and pure rents

2.1 The model

Consider a perfectly competitive real economy, peopled by two types of infinite-lived agents (the number of each type is given and equal to one): capitalists-workers—who save and work—and pure workers—who work and do not save. Working capitalists are the taxpayers, while pure workers are the recipients of capital tax proceeds. All the agents supply labor inelastically.

Physical capital, whose stock is denoted by \( k \), is the unique asset of the economy. Capital is assumed to depreciate at a constant rate \( \delta \).

Capital owners, who decide on consumption \( c \) and capital accumulation, solve the following intertemporal problem

\[
\max \int_0^{\infty} U(c)e^{-\rho t} dt \tag{1a}
\]

subject to the flow budget constraint

\[
c + \dot{k} = (1 - \tau_k)(r - \delta)k + vn, \tag{1b}
\]

where \( r \) is the before-tax real interest rate, \( v \) the real wage perceived by working capitalists, \( n \) their labor supply, \( \tau_k \) the proportional tax rate on capital income net of capital depreciation, and \( \rho \) the exogenous rate of time preference. Capital depreciation allowance is assumed. The instantaneous utility function \( U(\cdot) \), strictly increasing and concave in \( c \), is supposed to be of the isoelastic type; that is, \( U(c) = \frac{c^{1-\beta} - 1}{(1-\beta)} \), where \( \beta \) is the elasticity
of the marginal utility (i.e., the inverse of the intertemporal elasticity of substitution in consumption); if $\beta = 1$, $U(c) = \ln c$.

The first-order conditions for the capitalists’ problem (1) are

$$U'(c) = c^{-\beta} = \lambda,$$  \hspace{1cm} (2a)

$$\frac{\dot{\lambda}}{\lambda} = -\beta \frac{c}{c} = \rho - \bar{r},$$ \hspace{1cm} (2b)

where $\lambda$ represents the shadow value of capital and $\bar{r} = (1 - \tau_k)(r - \delta)$. In addition, (1b) and the transversality condition $\lim_{t \to \infty} \lambda ke^{-\rho t} = 0$ must be satisfied at the optimum.

The pure workers’ utility is intertemporally provided by the functional $\int_0^\infty V(x) e^{-\rho t} dt$, where $x$ represents consumption and $V(\cdot)$ the instantaneous well-behaved utility function. Pure workers face a static problem as they simply consume their disposable income, which is given by labor income plus government lump-sum transfers; that is,

$$x = wl + s,$$ \hspace{1cm} (3)

where $w$ is the real wage of pure workers, $l$ their labor supply and $s$ lump-sum transfers received from the government.

Perfectly competitive firms produce output $y$ by using physical capital, savers’ labor and pure workers’ labor. The production function, given by $y = F(k, n, l)$, satisfies the neoclassical properties of regularity and is linearly homogeneous in its three arguments.\footnote{Without implicating the results, $n$ and $l$ are assumed to be imperfectly substitutable in production. This hypothesis can be justified by assuming, for example, that $n$ represents managerial-entrepreneurial labor, while $l$ clerical-manual labor.} It is assumed that capital and labor of any type are Edgeworth complementary: $F_{kn} > 0$ and $F_{kl} > 0$.

Maximum profit implies

$$F_k(k, n, l) = r,$$ \hspace{1cm} (4a)

$$F_n(k, n, l) = v,$$ \hspace{1cm} (4b)

$$F_l(k, n, l) = w.$$ \hspace{1cm} (4c)
Labor of capitalists and labor of pure workers, being both exogenously supplied, are normalized to one: \( n = l = 1 \).

The government follows a balanced-budget policy: proceeds from capital income taxation are lump-sum transferred to pure workers. Therefore, the government budget constraint is given by

\[
s = \tau_k (r - \delta)k. \tag{5}
\]

The equilibrium in the good market requires that output equals aggregate consumption, \( c + x \), plus gross investment in physical capital:

\[
y = c + x + \dot{k} + \delta k. \tag{6}
\]

### 2.2 Optimal redistributive capital taxation

The government chooses the optimal capital tax rate by maximizing the social welfare function — given by the integral of a weighted average of the two agent utility functions discounted at the rate of time preference \( \rho \) — subject to saver and worker optimal decisions. The instantaneous social welfare function is \( \gamma V(x) + U(c) \), where \( \gamma \geq 0 \) represents the weight attributed by the social planner to workers relative to wealth owners.

In formal terms, the government’s optimal tax problem can be formulated as follows

\[
\max_{k,c,x,\tau} \int_0^\infty [\gamma V(x) + U(c)]e^{-\rho t}dt \tag{7a}
\]

subject to

\[
\dot{k} = \tilde{r} k + F_n(k) - c, \tag{7b}
\]

\[
\dot{c} = \frac{c}{\beta} (\tilde{r} - \rho), \tag{7c}
\]

\[
x = F(k) - \delta k - \tilde{r} k - F_n(k), \tag{7d}
\]

\[
\tilde{r} \geq 0, \tag{7e}
\]
together with the transversality condition of capitalists and the condition $k(0) > 0$ given.\footnote{Equation (7b) is obtained by inserting (4b) into (1b) (when $n = l = 1$), while equation (7d) is obtained by plugging (7b) into (6) and then employing the production function with $n = l = 1$. The inequality (7e) ensures that $\tau_k \leq 1$.}

The result of this fiscal policy experiment can be stated as follows:

**Proposition 1** \textit{In a neoclassical growth model à la Judd (1985) with working capitalists, supplying labor inelastically, and pure workers, the optimal capital income tax on savers, $\tau_k^*$, used to finance lump-sum transfers to pure workers generally differs from zero in the long-run: $\text{sgn}(\tau_k^*) = \text{sgn}(\gamma V' - U') \neq 0$.}

**Proof.** The first-order conditions for the social optimum are\footnote{For the sake of simplicity, the arguments of the derivatives are omitted.}

\begin{equation}
-\dot{\Lambda}_1 + \Lambda_1 \rho = \gamma V'(F_k - \delta - \widetilde{r} - F_{nk}) + \Lambda_1 (\widetilde{r} + F_{nk}), \quad (8a)
\end{equation}

\begin{equation}
-\dot{\Lambda}_2 + \Lambda_2 \rho = U' - \Lambda_1 + \frac{\Lambda_2}{\beta} (\widetilde{r} - \rho), \quad (8b)
\end{equation}

\begin{equation}
-(\gamma V' - \Lambda_1)k + \frac{\Lambda_2 c}{\beta} + \Psi = 0, \quad (8c)
\end{equation}

\begin{equation}
\Psi \widetilde{r} = 0, \widetilde{r} \geq 0, \Psi \leq 0, \quad (8d)
\end{equation}

together with the constraints of the problem (7) and the proper transversality conditions. $\Lambda_1$ and $\Lambda_2$ are the costate variables associated with the dynamic constraints (7b) and (7c), respectively, while $\Psi$ is the Kuhn-Tucker multiplier associated with (7e).

Consider the long-run equilibrium. As $\widetilde{r} = \rho > 0$ from (7c), $\Psi = 0$ from (8d). The optimality conditions (8) can be expressed as

\begin{equation}
\gamma V'(F_k - \delta - \widetilde{r}) = (\gamma V' - \Lambda_1) F_{nk}, \quad (8a')
\end{equation}

\begin{equation}
\Lambda_1 = U' - \Lambda_2 \rho, \quad (8b')
\end{equation}

\begin{equation}
\Lambda_2 c = \beta k (\gamma V' - \Lambda_1). \quad (8c')
\end{equation}
The optimal solutions for the capital tax rate and the costate variables are:

\[
\tau^*_k = \frac{c(\gamma V' - U')F_{nk}}{(F_k - \delta)\gamma V'[1 - \beta]\rho k + F_n]}, \quad (9a)
\]

\[
\Lambda^*_1 = \frac{(cU' - \beta \rho k\gamma V')}{[1 - \beta]\rho k + F_n}], \quad (9b)
\]

\[
\Lambda^*_2 = \frac{\beta k(\gamma V' - U')}{[1 - \beta]\rho k + F_n].} \quad (9c)
\]

Thus, \(\tau^*_k\) is in general different from zero; its sign depends on the divergence between the social marginal utility of pure workers and that of working capitalists: \(\tau^*_k \leq 0\) if \(\gamma V' \leq U'\). The departure from the optimal zero tax rate occurs because of labor inelastically supplied by capitalists-taxpayers, whose income, a pure rent, depends on \(k (F_{nk} > 0)\).

When \(U(\cdot)\) is logarithmic (i.e., \(\beta = 1\)), there is no knife-edge result as instead obtained by Lansing (1999) when pure rents are absent. Using the interpretation provided by Reinhorn (2014), this is because with a fixed input owned by capitalists (whose income is untaxed), the steady state equilibrium exists and the costates are clearly defined —see equations (9)—, thus converging to the long-run values.\(^8\)

In order to understand how crucial parameters —for example the agents’ political weight in the social welfare, factors’ income shares, etc.— affect the optimal capital tax rate when \(\beta = 1\), let us calculate the reduced-forms for \(k^*\) and \(\tau^*_k\). Suppose that pure workers have logarithmic preferences —that is, \(V(x) = \ln x\) — and the production function is Cobb-Douglas —that is, \(y = Ak^\theta n^{1-\theta-\eta}\) (where \(A > 0, 0 < \theta < 1\) and \(0 < \eta < 1\) are standard technological parameters).

\(^8\)When \(\beta = 1\), the existence of the steady state equilibrium is guaranteed by the fact that ‘anticipation effects’ are operative. In fact, in such a case, the consumption function of savers is given by

\[
c = \rho (k + h), \quad (i)
\]

where \(h\) is human wealth, which is defined as

\[
h = \int_t^\infty F_n e^{-\int_t^v \tilde{r} du} dv. \quad (ii)
\]

Equations (i) and (ii) imply that the ‘anticipation effects’ of future capital tax changes are at work also when capitalists have logarithmic preferences.
The long-run capital stock —obtained from (8a’), after using (9b), the relationship \( r = \rho \), and the specific functional forms assumed—is given by

\[
k^* = \left\{ \frac{A\theta (1 - \eta) (1 + \gamma)}{\gamma [\rho (1 + \theta) + \delta] + \theta (\rho + \delta)} \right\}^{\frac{1}{1-\theta}}.
\]

(10)

The optimal capital tax rate —derived from the 'modified golden rule' \((1 - \tau_k^*) [A\theta k^*^{\theta - 1} - \delta] = \rho \) after using (10)— is

\[
\tau_k^* = \frac{\gamma [(\theta + \eta) \rho + \eta \delta] - (1 - \theta - \eta) (\delta + \rho)}{\gamma [(1 + \theta) \rho + \eta \delta] + \theta \rho - (1 - \theta - \eta) \delta}.
\]

(11)

When the social planner only cares about pure workers \((\gamma \to +\infty)\), the optimal capital tax rate is: \( \tau_k^* = \frac{(\theta + \eta) \rho + \eta \delta}{(1 + \theta) \rho + \eta \delta} > 0 \). If only savers matter for the government \((\gamma = 0)\), the optimal capital tax rate will be given by \( \tau_k^* = \frac{(1 - \theta - \eta) (\delta + \rho)}{(\theta + \eta) \rho + \eta \delta} \). If \( \gamma = \frac{(1 - \theta - \eta) (\delta + \rho)}{(\theta + \eta) \rho + \eta \delta} \), then \( \tau_k^* = 0 \). \( \square \)

The crucial element driving the result \( \tau_k^* \neq 0 \) is given by the presence of pure rents, perceived by capital-owners-taxpayers. When capitalists work facing elastic labor-leisure choices, instead, capital taxation should be zero since there are no pure rents that accrue to them (see Judd, 1985).

### 2.3 Optimal redistributive capital taxation when another suitable fiscal instrument is available

In this sub-section, the case in which the government can use another policy instrument for redistributive purposes in addition to capital taxation is considered. I show that income redistribution can be accomplished by either exempting capital income from taxation or preferably using a less distortionary fiscal instrument, when this is suitable.

Suppose, for example, that a constant consumption tax, whose tax rate is denoted by \( \bar{\tau}_c \), is levied on capitalists.

In this case, the following result holds:

**Proposition 2** In a neoclassical growth model à la Judd (1985) with working capitalists, supplying labor inelastically, and pure workers, the optimal redistributive capital income tax is zero if an additional tax instrument —like a constant tax on capitalists’ consumption— is employed by the social planner. This result will be confirmed if either a tax on wage rents of capitalists or government debt were the additional fiscal instrument.
Proof. The working capitalists’ budget constraint and consumption of pure workers, who receive all tax proceeds, are given by

\[ k = \tilde{r}k + F_n(k) - (1 + \tilde{\tau}_c)c, \]  
\[ x = F(k) - \delta k - \tilde{r}k - F_n(k) + \tilde{\tau}_c c. \]

Now (12a) and (12b) replace (7b) and (7d), respectively, in the social planner problem (7).

The social planner’s optimal choice of $\tilde{\tau}_c$ entails

\[ \gamma V' = \Lambda_1. \]  

By inserting (13) into (8a'), we get $F_k - \delta - \tilde{r} = \tau^*_k (F_k - \delta) = 0$, which yields $\tau^*_k = 0$.

It is easy to show that also a tax on wage income of capitalists or government debt hold by savers would result in $\gamma V' = \Lambda_1$ and, therefore, in $\tau^*_k = 0$. □

3 A Judd (1985) economy with two types of pure workers

3.1 The model

Consider a perfectly competitive economy peopled by capitalists, who do not work, and two types of workers—workers of type 1 and workers of type 2—, who do not save.⁹ The number of each type of agents is normalized to one, being constant. Workers of the two types differ in terms of labor productivity (hence wages), the amount of lump-sum transfers received by the government and therefore consumption. The government is perfectly informed about who is of type 1 and who of type 2. Capital is the only asset of the economy. Capitalists pay taxes on capital income to finance lump-sum transfers to pure workers. All workers supply labor inelastically.

⁹For the analysis of optimal taxation with the two types of workers, see, for example, Stiglitz (1987).
Since there are no rents that accrue to capitalists (because they do not own fixed inputs or assets), their budget constraint is given by

\[ c + \dot{k} = \tilde{r} k. \]  

(14)

The optimal behavior of capitalists is described by (2a), (2b), (14) and the usual transversality condition.

Worker of type \( i \) (for \( i = 1, 2 \)) faces the following static budget constraint:

\[ x_i = w_i l_i + s_i, \]  

(15)

where \( x_i, w_i, l_i \) and \( s_i \) denote consumption, the wage rate, labor supply and lump-sum transfers of a worker of type \( i \), respectively.

The intertemporal welfare function of a worker of type \( i \) is given by

\[ \int_0^\infty V_i(x_i)e^{-\rho t} dt, \]

where \( V_i(\cdot) \) denotes the momentary utility function. Since the optimal problem of worker of type \( i \) is static, her behavior is entirely described by (15), being her labor supply exogenous.

Output is produced by using capital, labor of type 1 and labor of type 2. A well-behaved and linearly homogeneous technology, given by

\[ y = F(k, l_1, l_2), \]

is used for production. All inputs are assumed to be Edgeworth complementary. Moreover, \( l_1 \) and \( l_2 \) are imperfectly substitutable in production.\(^{10}\)

Profit maximization requires

\[ F_k(k, l_1, l_2) = r, \]  

(16a)

\[ F_{l_1}(k, l_1, l_2) = w_1, \]  

(16b)

\[ F_{l_2}(k, l_1, l_2) = w_2. \]  

(16c)

Labor supplied by each type of workers is normalized to one, being exogenous: \( l_1 = l_2 = 1. \)

The government keeps its budget balanced by distributing tax proceeds to workers in a lump-sum fashion. For simplicity’s sake, lump-sum transfers are entirely distributed to workers of type 1 (nothing is given to workers of type 2):\(^{11}\)

\[ s_1 = \tau_k (r - \delta) k, \]  

(17a)

\(^{10}\)This assumption is inessential for the results obtained below.

\(^{11}\)The case in which tax revenue is distributed in fixed proportions to both types of workers could be easily analyzed. The qualitative results will not change substantially. See footnotes 13, 15, 17 and 19 below.
\[ s_2 = 0. \]  \hspace{2cm} (17b)

The resource constraint is given by
\[ y = c + x_1 + x_2 + \dot{k} + \delta k. \]  \hspace{2cm} (18)

### 3.2 Optimal redistributive capital taxation

The social welfare function corresponds to the integral of the weighted sum of each agent’s momentary utility, given by \( \gamma_1 V_1(x) + \gamma_2 V_2(u) + U(c) \), where \( \gamma_i \geq 0 \) — for \( i = 1, 2 \) — represents the weight attributed by the social planner to workers of type \( i \) relative to capitalists.

The problem faced by the social planner is
\[
\max_{k,c,x_1,x_2,\tilde{r}} \int_0^\infty \left[ \gamma_1 V_1(x_1) + \gamma_2 V_2(x_2) + U(c) \right] e^{-\rho t} dt \tag{19a}
\]
subject to
\[
\dot{k} = \tilde{r}k - c, \tag{19b}
\]
\[
\dot{c} = \frac{c}{\beta}(\tilde{r} - \rho), \tag{19c}
\]
\[
x_1 = F(k) - \delta k - \tilde{r}k - F_{l_2}(k), \tag{19d}
\]
\[
x_2 = F_{l_2}(k), \tag{19e}
\]

The following proposition summarizes the normative results of this case:

**Proposition 3** In an intertemporal optimizing model of capital accumulation with capitalists and two types of pure workers that supply labor inelastically, only one type of whom receives lump-sum transfers by the government, it is optimal to redistribute income through capital taxation. The sign of \( \tau_k^* \) is determined as follows:

i) \( \text{sgn}(\tau_k^*) = \text{sgn}(\gamma_1 V'_1 - \gamma_2 V'_2) \) when \( \beta \neq 1 \);

ii) \( \text{sgn}(\tau_k^*) = \text{sgn}(\rho \gamma_1 V'_1 - \frac{1}{\beta}) + (\gamma_1 V'_1 - \gamma_2 V'_2) F_{l_2 k} \) when \( \beta = 1 \).

\[ \text{The inequality } \tilde{r} \geq 0 \text{ is not imposed because } \tilde{r} = \rho > 0 \text{ in the long-run and therefore the associated Kuhn-Tucker multiplier would be zero.} \]
Proof. The cases $\beta \neq 1$ and $\beta = 1$ are treated separately as different procedures for solving the social planner problem have to be employed. Consider each case in turn.

Case $\beta \neq 1$

In this case, the first-order conditions for the government problem (19) are:

\[ -\dot{\Xi}_1 + \Xi_1 \rho = \gamma_1 V'_1 (F_k - \delta - \tilde{r} - F_{l2k}) + \gamma_1 V'_2 F_{l2k} + \Xi_1 \tilde{r}, \] (20a)

\[ -\dot{\Xi}_2 + \Xi_2 \rho = U' - \Xi_1 + \frac{\Xi_2}{\beta} (\tilde{r} - \rho), \] (20b)

\[ -\gamma_1 V'_1 k + \Xi_1 k + \frac{\Xi_2 c}{\beta} = 0, \] (20c)

together with (19b), (19c), (19d), (19e) and the proper transversality conditions. $\Xi_1$ and $\Xi_2$ are the costate variables associated with the dynamic constraints (19b) and (19c), respectively.

In the steady state, the optimal values of the capital tax rate and the costate variables are:\(^13\)

\[ \tau^*_k = \frac{(\gamma_1 V'_1 - \gamma_2 V'_2) F_{l2k}}{(F_k - \delta) \gamma_1 V'_1}, \] (21a)

\[ \Xi^*_1 = \frac{(U' - \beta \gamma_1 V'_1)}{(1 - \beta)}, \] (21b)

\[ \Xi^*_2 = \frac{\beta (\gamma_1 V'_1 - U')}{(1 - \beta) \rho}. \] (21c)

\(^{13}\)If tax revenue were distributed in fixed proportions to both types of workers (a fraction $\sigma$ to workers of type 1 and a fraction $1 - \sigma$ to workers of type 2), the optimal steady state values of the capital tax rate and the social shadow prices would have been:

\[ \tau^*_k = \frac{(\gamma_1 V'_1 - \gamma_2 V'_2)[F_{l2k} + (1 - \sigma)k F_{kk}]}{(F_k - \delta)[\sigma \gamma_1 V'_1 + (1 - \sigma) \gamma_2 V'_2]}, \] (21a')

\[ \Xi^*_1 = \frac{(U' - \beta [\sigma \gamma_1 V'_1 + (1 - \sigma) \gamma_2 V'_2])}{(1 - \beta)}, \] (21b')

\[ \Xi^*_2 = \frac{\beta [\sigma \gamma_1 V'_1 + (1 - \sigma) \gamma_2 V'_2 - U']}{(1 - \beta) \rho}. \] (21c')
The sign of the optimal capital tax rate is dictated by $\gamma_1 V_1' - \gamma_2 V_2'$, being $F_{t_2k} > 0$. When $\beta \neq 1$, the social marginal utility of capitalists does not play any role for the optimal capital tax rate as there are no rents that enter the budget constraint of capitalists.

**Case $\beta = 1$**

When $\beta = 1$, $\Xi_1^*$ and $\Xi_2^*$ are undefined in the steady state as can be seen from (21b) and (21c). As noticed by Reinhorn (2014), this fact reflects problems of steady state existence and convergence of the shadow prices.\(^{14}\) The normative methodology employed by Lansing (1999), based on the capitalists’ decision rules, permits to overcome such problems.

By taking into account the consumption function of capitalists, given in this case by the relationship $c = \rho k$, the government’s efficient tax choice is obtained by solving the following problem

$$\max_{k, \tau} \int_0^\infty \{\gamma_1 V_1[F(k) - \delta k - F_{t_2}(k)] + \gamma_2 V_2[F_{t_2}(k)] + \ln \rho k\} e^{-\rho t} dt$$

subject to $\dot{k} = (\tilde{r} - \rho)k$ and the usual boundary condition.

The optimal steady state capital tax rate, obtained from the first-order conditions of the social planner problem, is given by\(^{15}\)

$$\tau^*_k = \frac{[(\rho \gamma_1 V_1' - \frac{1}{k}) + (\gamma_1 V_1' - \gamma_2 V_2')F_{t_2k}] / [(F_k - \delta) \gamma_1 V_1']}{(22)}$$

With logarithmic preferences of capitalists, there are two kinds of departures from the zero capital tax rate:

\(^{14}\)By following Reinhorn (2014), this fact can be intuited as follows. By combining (20b) and (20c) under the assumption $\beta = 1$, we obtain that $\gamma_1 V_1' = U'$. This equation determines $k$. But $k$ is also determined by the ‘modified golden rule’. Therefore, there are two independent equations that determine $k$; only accidentally the value of $k$ that satisfies both equations is unique.

\(^{15}\)If lump-sum transfers are distributed in fixed proportions to both types of workers, the optimal capital tax rate will be

$$\tau^*_k = \frac{[\sigma \gamma_1 V_1' + (1 - \sigma) \gamma_2 V_2'][\rho - 1/k + (\gamma_1 V_1' - \gamma_2 V_2')F_{t_2k} + (1 - \sigma) k F_{kk}]}{(F_k - \delta) \sigma \gamma_1 V_1' + (1 - \sigma) \gamma_2 V_2'}. \quad (22')$$
i) one, represented by the term $\rho \gamma_1 V'_1 - \frac{1}{k}$, is due to the absence of 'anticipation effects' of future after-tax interest rate changes, as pointed out by Lansing (1999);

ii) one, represented by the term $(\gamma_1 V'_1 - \gamma_2 V'_2) F_{l2k}$, is to be ascribed to the presence of two types of workers, only one type of whom receives lump-sum transfers, as seen when $\beta \neq 1$.

In order to obtain the reduced-forms for $k^*$ and $\tau_k^*$ when $\beta = 1$, suppose that the two types of workers have logarithmic utilities and the production function takes the Cobb-Douglas form—that is, $y = A k^{\theta} l_1^{1-\theta-\xi} l_2^\xi$, with $A > 0$, $\theta \in (0,1)$ and $\xi \in (0,1)$.

The reduced-form for the steady state capital stock —obtained from (22), after using the specific functional forms assumed— is

$$k^* = \left\{ \frac{A(1-\xi)[1 + \theta (\gamma_1 + \gamma_2)]}{(\delta + \rho)(1 + \gamma_1 + \theta \gamma_2) + \rho \gamma_1} \right\}^{\frac{1}{1-\theta}}. \tag{23}$$

By inserting $k^*$ from (23) into the 'modified golden rule' $(1 - \tau_k^*)[A \theta k^{*\theta-1} - \delta] = \rho$, the following optimal capital tax rate is obtained

$$\tau_k^* = \frac{\theta \gamma_1 [\rho + \xi (\delta + \rho)] - (1 + \theta \gamma_2)(1 - \theta - \xi)(\delta + \rho)}{\theta \gamma_1 (2 \rho + \xi \delta) + (1 + \theta \gamma_2)[\theta \rho - (1 - \theta - \xi)\delta]} \cdot \tag{24}$$

When only workers of type 1 are relevant for the government ($\gamma_1 \rightarrow +\infty$), the optimal capital tax rate is: $\tau_k^* = \frac{[\rho + \xi (\delta + \rho)]}{(2 \rho + \xi \delta)}$. If the social planner only cares about capitalists ($\gamma_1 = \gamma_2 = 0$), the optimal capital tax rate will be given by $\tau_k^* = -\frac{(1-\theta-\xi)(\delta + \rho)}{[\theta \rho - (1-\theta-\xi)\delta]}$. \hfill \Box

### 3.3 Optimal redistributive capital taxation: The role of additional fiscal instruments

This sub-section studies which type of fiscal instrument can be used as an alternative to capital taxation in order to carry out income redistribution when there are two types of pure workers.

Suppose that a proportional consumption tax, $\tau_{x2}$, is imposed on workers of type 2. With such a consumption tax, whose revenue is distributed to

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\(^{16}\)Such an optimal capital tax rate is also obtained when $\gamma_2 \rightarrow +\infty$.  

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workers of type 1, consumption of the two types of workers can be expressed, after using the hypothesis $l_1 = l_2 = 1$, as:

$$x_1 = F(k) - \delta k - \tilde{\gamma}k - \frac{F_{l_2}(k)}{(1+\tau_{x_2})}, \quad (25a)$$

$$x_2 = \frac{F_{l_2}(k)}{(1+\tau_{x_2})}. \quad (25b)$$

These two equations replace (19d) and (19e) in the government problem (19).

The normative results of this case can be summarized as follows:

**Proposition 4** In an intertemporal optimizing model of capital accumulation with capitalists and two types of pure workers supplying labor inelastically, only one type of whom receives lump-sum transfers from the government, the optimal capital tax rate is zero when an optimally chosen consumption tax is imposed on workers who do not receive government transfers and $\beta \neq 1$. When instead $\beta = 1$, such a consumption tax must be accompanied by a constant consumption tax levied on capitalists in order to obtain a zero optimal capital tax rate.

**Proof.** The cases $\beta \neq 1$ and $\beta = 1$ are analyzed in turn.

**Case $\beta \neq 1$**

The optimal choice of $\tau_{x_2}$ entails:

$$\gamma_1 V_1' = \gamma_2 V_2'. \quad (26)$$

Plugging (26) into (21a), we obtain that the optimal capital tax rate is zero.$^{17}$

Notice that a consumption tax imposed on capitalists would not bring the optimal capital tax rate to zero. In fact, if such a tax were introduced, it would imply $\gamma_1 V_1' = U'$, thus having no implications for $\tau^*_k$. This is because a fiscal instrument aimed at replacing capital taxation should be targeted to the nature of the deviation from the Judd (1985) result.

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$^{17}$This result is also satisfied when workers of type 1 and 2 receive tax revenues in fixed proportion, given by $\sigma$ and $1-\sigma$, respectively. Moreover, in such a case, also the optimal choice of $\sigma$ would imply $\gamma_1 V_1' = \gamma_2 V_2'$ and therefore $\tau^*_k = 0$. 

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Case $\beta = 1$

When the capitalists’ utility is logarithmic, the introduction of a consumption tax imposed on workers of type 2 is not sufficient to obtain $\tau_k^* = 0$; this is because another suitable fiscal instrument is necessary to deal with the absence of “anticipation effects“, which is also at the basis of the departure from the Judd (1985) result. Such an instrument could be a constant consumption tax levied on capitalists, $\tau_c$.\(^{18}\)

The optimal choice of $\tau_c$ requires that the following condition must be satisfied in addition to (26):

$$\rho \gamma_1 V_1' = \frac{1}{k}.$$

(27)

Once employed in (22), equations (26) and (27) imply $\tau_k^* = 0$.\(^{19}\) □

4 Combining pure rents with two types of workers: An economy with illegal workers

In this section, the case of pure rents is combined with the hypothesis of two types of pure workers. Consider an economy having the same structure as the one presented in section 3. Suppose that workers of type 1 work legally, while workers of type 2 work illegally. Workers of type 1 are paid their marginal product, while workers of type 2 are paid less than their marginal product because hiring them is risky for firms. In fact, firms employing illegal workers can be detected by the fiscal authority with a given probability and, if discovered, are fined. For this reason, firms pay clandestine workers less than their marginal product to be compensated for the risk of paying fines.

Therefore, $w_1 = F_{l_1}$ and $w_2 = \mu F_{l_2}$, where $\mu \in (0, 1)$ is a parameter that measures the degree of exploitation of illegal workers, implicitly capturing the government policy effort against them.\(^{20}\)

\(^{18}\)The consideration of government debt as the additional fiscal instrument would lead to the same result.

\(^{19}\)The same result holds when both types of workers receive capital tax revenue in fixed proportions.

\(^{20}\)The firm cost of employing illegal workers is given by $(1 + \omega)w_2 l_2$, where $\omega$ represents the probability of being caught employing illegal workers times the fine rate that firms have to pay. The optimal choice of illegal labor is $F_{l_2} = (1 + \omega)w_2$, which implies $w_2 = \mu F_{l_2}$, where $\mu = (1 + \omega)^{-1} < 1$. 

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Capital tax proceeds and revenue from penalties paid by firms that employ clandestine workers are distributed to workers of type 1.

Consumption of illegal workers is given by

\[ x_2 = \mu F_{l_2} l_2. \] (28)

The exploitation of illegal workers implies that there are rents \( \Pi \) going to the owners of firms. The capitalists’ budget constraint is

\[ c + \dot{k} = \tilde{\tau}k + \Pi. \] (29)

Dividends received by capitalists are given by \( \Pi = (1 - \mu)F_{l_2} l_2 \).

By plugging (28) and (29) into the resource constraint (18) and using the condition \( l_1 = l_2 = 1 \), consumption of workers of type 1 can be expressed as

\[ x_1 = F(k) - \delta k - \tilde{\tau}k - F_{l_2}. \] (30)

Since workers of type 2 operate in the illegal sphere of the economy, the government, imperfectly informed about them, does not take into account their utility when the optimal redistributive capital tax rate is chosen; that is, \( \gamma_2 = 0 \) in the social welfare function.

Once (29), (30), the definition of \( \Pi \) and the hypothesis \( \gamma_2 = 0 \) are taken into account, the solutions for the optimal capital tax rate and the social shadow prices are

\[
\tau^*_k = \frac{[(\gamma_1 V'_1' - U')c + \mu(cU' - \beta \rho k \gamma_1 V'_1)] F_{l_2}}{\gamma_1 V'_1(F_k - \delta)[(1 - \beta) \rho k + (1 - \mu) F_{l_2}]},
\] (31a)

\[
\Xi^*_1 = \frac{(cU' - \beta \rho k \gamma_1 V'_1)}{[(1 - \beta) \rho k + (1 - \mu) F_{l_2}]},
\] (31b)

\[
\Xi^*_2 = \frac{\beta k (\gamma_1 V'_1' - U')}{[(1 - \beta) \rho k + (1 - \mu) F_{l_2}]}.
\] (31c)

With illegal workers, we depart from the zero capital tax result for two reasons:

i) the presence of two types of workers, only one type of which is take into account by the government \( \gamma_2 = 0 \);\(^{21}\)

\(^{21}\)If \( \gamma_2 \neq 0 \), the optimal capital tax rate will be given by (31a) when the term \(-\mu \gamma_2 V'_2(c - \beta \rho k)\) is included in the square brackets. In such a case, \( \Xi^*_1 \) and \( \Xi^*_2 \) are given by (31b) and (31c).
ii) the existence of exploitative rents ($\mu < 1$) perceived by capitalists.

When there are illegal workers and exploitative rents, no suitable fiscal instruments can be used to eliminate capital taxation. The introduction of a tax imposed on capitalists’ consumption, whose proceeds are given to workers of type 1, for example, would imply, once optimally chosen, $\gamma_1 V_1' = U'$, thus leading to $\tau^*_k = \frac{\mu F_{ik}}{(F_k - \delta)} > 0$.\(^{22}\)

If $\beta = 1$, problems of existence of steady state equilibrium do not arise as the ‘anticipation effects’ of future capital tax changes are at work through the exploitative rents obtained by capitalists.

5 Conclusion

This paper has shown that capital taxation can be usefully employed to address problems of income inequality in heterogeneous-agent models of capital accumulation when there are different types of workers that supply labor inelastically. By considering capitalist-worker economies à la Judd (1985), the redistributive role of capital taxation has been studied from a normative standpoint in two cases: one in which also capitalists work and one in which there are two types of pure workers alongside capitalists.

The consideration of different types of workers introduces a change in the basic framework developed by Judd (1985) that leads to the nonoptimality of zero capital taxation. The paper results derive from the fact that the wage of the second type of workers, a function of the capital stock, affects the social welfare function and, by absorbing resources, crowds out consumption of other workers.

In the cases examined, the sign of the optimal capital tax rate depends on the differences between the social marginal utilities of the different agents, which are non-zero.

If other suitable policy instruments are employed, the scope of using capital taxation for redistributing income will disappear only when such tools bring the social marginal utilities’ differences to zero, once optimally chosen by the policy maker.

Moreover, an economy in which there are illegal workers, exploited because of their status, has been analyzed. This case represents a combination

\(^{22}\)Also a consumption tax imposed on illegal workers, if implemented, will not lead to $\tau^*_k = 0$.\)
of the two cases studied before as the formal and informal spheres characterize the different types of workers, while the exploitation of illegal workers results in pure rents that accrue to capitalists. Although the optimal capital tax rate is non-zero, the consideration of additional fiscal instruments does not lead to the optimality of exempting capital from taxation because illegal workers are not taken into account by the social planner in the optimal tax problem.

Finally, it is worth underlining that any form of untaxed pure rents that accrue to wealth-owners-taxpayers will undermine the optimality of the zero redistributive capital tax rate, leading to results that are similar to those obtained here in the case of working capitalists. This is, for example, the case in which capitalists own a fixed asset, like land. Consider the simplest Judd (1985) model with one type of workers and suppose that there are two assets: physical capital and unimproved land, whose stock is denoted by \( z \). The budget constraint of capitalists-landowners is \( c + \dot{a} = \tilde{r}a \), where \( a = k + qz \) denotes financial wealth and \( q \) the land value. Output is produced competitively through a neoclassical production function by using capital, land and labor: \( y = F(k, z, l) \). Land, like labor, is inelastically supplied and fully used in production. The land rent \( p \) is equal to the marginal product of land; that is, \( p = F_z \).

Assuming perfect foresight, perfect arbitrage between asset returns implies, when the land rent is untaxed, that \( \tilde{r}q = p + \dot{q} \). By using the definition of financial wealth, the arbitrage condition between asset returns and the hypothesis of a given amount of land, the budget constraint of capitalists-landowners can be written as \( c + \dot{k} = \tilde{r}k + pz \), which is equivalent to (1a) if \( z = n = 1 \) and \( p = v \). Therefore, the analysis of sub-section 2.2 applies.

The normative problem of the social planner is given by (7) and the optimal capital tax rate by (9a).

\[ \text{23} \]

\[ \text{24} \]

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\[ \text{23} \] Even the relevant role played by the value of a fixed asset to explain the rising inequality experienced by advanced countries in the last decades, see, for example, Bonnet et alii (2014) and Stiglitz (2015).

\[ \text{24} \] See Feldstein (1977), for a model with similar supply and asset sides.
References


