Consumption Norms with Endogenous Norm Beliefs – Implications for Welfare, Commodity Taxation and Income Redistribution*

Tomas Sjögren**
Department of Economics
Umeå School of Business and Economics
Umeå University, SE - 901 87 Umeå, Sweden

Abstract
This article concerns commodity taxation and income redistribution when agents derive status from living up to a consumption norm that gives rise to positional preferences. The consumption norm relates an individual agent’s visible expenditure on a given prestige good to the visible expenditure on the same prestige good made by a reference group (high-income agents). The novelty in this paper is to treat the agents’ personal belief in the consumption norm as endogenous. In line with Cognitive Dissonance Theory it is assumed that if an agent lives up to (does not live up to) a given norm, then the agent’s belief in the norm increases (decreases) over time. This behavior can be used by a policy-maker to change the belief in a norm that the policy-maker perceives as distortionary. As such, the approach in this paper points to an alternative method of correcting for consumption externalities (or for imperfect consumption behavior) and we characterize how this policy incentive affects commodity taxation and income redistribution.

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** E-mail: tomas.sjogren@econ.umu.se, Phone: +46 (0)90 – 786 99 94.
1. Introduction

Today, there is a large literature which emphasizes that individuals are concerned with relative consumption comparisons and experience happiness if they do well in comparison with some reference group(s). Empirical support for the idea that agents have these types of positional preferences comes from different fields such as happiness research,\(^1\) questionnaire-based experiments\(^2\) and brain science.\(^3\) As a consequence, many theoretical models today incorporate relative consumption comparisons in various contexts such as renewable resource extraction,\(^4\) economic growth,\(^5\) asset pricing\(^6\) and optimal taxation.\(^7\) An economic theoretical underpinning for why agents are concerned with the consumption of others comes from the Theory of Social Custom\(^8\) (TSC) which has been used to explain the development of consumption norms.\(^9\) The TSC has also been applied in other contexts\(^10\) and a modification used in a few applications (but not in the context of consumption norms) is to treat an individual agent’s *personal belief* in a given norm as an endogenous variable which may change over time.\(^11\) The latter approach can be motivated by research in psychology\(^12\) which emphasizes that if an agent takes an action which violates a deep-rooted belief (e.g. in a norm) then she tends to experience psychological discomfort, also referred to as cognitive dissonance. To reduce her cognitive dissonance an agent may, over time, degrade her deep-rooted belief. This suggests an alternative method of correcting for the externalities or imperfections that arise as a consequence of consumption norms; implement a tax policy which over time erodes away the belief in “bad” norms and which reinforces the belief in “good” norms. The purpose of this paper is to analyze this issue by incorporating the ideas from Cognitive Dissonance Theory (CDT) into an optimal tax framework. This makes it possible to address the following broad questions: *What are the welfare effects of*

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6 Abel (1999), Campbell and Cochrane (1999), and Dupor and Liu (2003).
8 Akerlof (1980).
9 See Corneo and Jeanne (1997).
11 See Vendrik (2003), and Mannberg and Sjögren (2015).
12 For some key references, see Festinger (1957, 1962) and Aronson (1968, 1992).
changing the private agents’ belief in a consumption norm? Can public policy be used to influence the belief in a consumption norm which a policy-maker perceives as distortionary? If this is the case, how does this affect commodity taxation and income redistribution, both in the short-run and in the long-run?

The idea that agents are concerned with positional preferences is not new. The classic reference to the idea of “conspicuous consumption” (according to which individuals signal wealth, or more generally, status via their consumption behavior) is Veblen (1899). Also Duesenberry (1949) argued that the pursuit of self-esteem and social prestige could generate a demand for socially visible goods that had no tangible utility in conventional terms. The argument was that in communities where systems of ascribed status and social stratification were weakening because of increased opportunities for social mobility, status was increasingly being conferred on the basis of socially visible consumption where agents were making comparisons between their own standards of living and those in higher or lower status groups.

If agents make consumption comparisons they impose consumption externalities on each other. This observation has received much attention in the optimal tax literature and as mentioned above, there are today several studies which are concerned with how positional preferences affect income and commodity taxation. The bulk of this literature is based on the assumption that all agents contribute equally much to the reference consumption level. However, a more realistic approach is to allow some agents to contribute more to the reference consumption than others. This happens if, for example, some group of agents aspires to mimic the consumption of some elite group (aspiration effect) or if some type of consumption becomes more attractive because it can be used by an agent to out-shine her peers (distinction effect). Eckerstorfer and Wendner (2013) have used this idea to analyze commodity taxation in the presence of negative consumption externalities. In their model, negative consumption externalities arise because agents compare their personal consumption of a status good with a reference level and Eckerstorfer and Wendner refer to the externality that arises when some agents contribute more

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13 Typically, it is assumed that the reference consumption is the average consumption in the economy, and that all agents use this average level as the point of reference in their consumption comparisons.

14 As Eckerstorfer and Wendner (2013) put it: “… sometimes I see my neighbor drinking Nespresso. Other times, on TV, I see George Clooney drinking Nespresso. For some reason, one impresses me more than the other.”

to the reference consumption than others as a *non-atmospheric externality.*\(^{16}\) Eckerstorfer and Wendner show that if personalized commodity taxes are available, then it is possible to achieve a first-best outcome where the policy-maker imposes positive consumption taxes on the agents whose behavior needs to be corrected whereas the policy-maker imposes zero consumption taxes on the other agents. If the policy-maker instead is restricted to use uniform commodity taxes, a first-best outcome is no longer attainable and in a second-best optimum the taxes on both the status good(s) and on the non-status good(s) will, in general, be non-zero.

In this paper we interpret preferences for relative consumption as arising from a consumption norm (or equivalently, a status norm) and ask whether a policy-maker who has too few policy instruments at her disposal (to implement the first-best outcome) instead can use the available policy instruments to change the agents’ belief in the status norm? To illustrate the idea, consider a situation where only uniform commodity taxes are available and where the purchase of a certain good (e.g. a car) is associated with status in the sense that people compare the quality of their particular car with the quality of the cars used by some reference group.\(^{17}\) If all agents are identical, make similar consumption comparisons and contribute equally much to the reference consumption, then a uniform commodity tax on cars is sufficient to correct for the distortion/externality that the consumption comparison generates. In this case a first-best outcome is attainable. However, if the agents are heterogeneous, either in the sense that some (but not all) of them associate cars with status, or in the sense that not all agents contribute equally much to the reference consumption, then a first-best outcome is unattainable as long as the policy-maker is restricted to use uniform commodity taxes. The reason is that a uniform tax on cars distorts the consumption choices for those agents who do not treat the car as a status good and/or for those agents who do not contribute to the consumption externality. On the other hand, if the policy instruments instead could be used to influence the degree by which agents believe in the status norm then a policy-maker could, in principle, use the tax instruments at her disposal to (over some time period) erode away the belief in the status norm within the sub-group of agents who

\(^{16}\) In contrast, the case when all agents contribute equally much to the reference consumption level is referred to as an *atmospheric externality.*

\(^{17}\) A relevant question in this context is why the policy-maker should be restricted to use only uniform commodity taxes? Why not tax expensive cars at a different rate than cheap cars? If it is high-income agents who buy expensive cars then differentiated commodity taxes would be equivalent with type-specific/personalized commodity taxes. One reason for why this may not be so easy to do in practice is that if there are many different agent types then this could, potentially, call for a highly differentiated commodity tax structure. A differentiated commodity tax structure may, in turn, provide incentives for tax evasion which means that the cost of monitoring could potentially be very high.
associate cars with status. After the norm belief has been eroded away, it would be possible to implement a first-best policy which includes a zero commodity tax on cars.

Is it then reasonable to assume that norm beliefs are endogenous? According to Cognitive Dissonance Theory (CDT) in psychology (see e.g. Festinger 1957, 1962 and Aronson 1968, 1992) the answer is yes. The argument is that if an agent takes an action which violates a personal norm\textsuperscript{18} then she will experience a sense of unease (cognitive dissonance). This creates an incentive to restore consistency which can be accomplished either by changing the behavior\textsuperscript{19} or by degrading the value/belief of that particular norm. Vendrik (2003) uses this idea in a continuous version of the TSC to model the impact of a traditional household norm on married women’s labour supply. He shows that an increase in the market wage for women may initiate a process where women make the full transition from housewives to career women due to the existence of cognitive dissonance.

In the present paper the TSC and the CDT are combined to model consumption norms which give rise to relative consumption comparisons. The economy consists of low- and high-income agents who purchase a prestige good and there is a positive relationship between the price of the prestige good and its visible quality content. The low-income agents look at the high-income agents to form their reference consumption (the aspiration effect mentioned above is at work here) whereas the high-income agents look at the other high-income agents to form their reference consumption (the distinction effect is assumed to be at work here). The consumption of the status good made by each high-income agent is a social cost to the other agents in the economy because they have to buy a prestige good which is more expensive (than in the absence of the status norm) in order to live up to their personal norm and in order to preserve their social status. This gives rise to wasteful consumption and implies that public intervention via taxes can improve overall welfare. In the first part of the paper we characterize the optimal tax problem facing the policy-maker in a static context where norm beliefs are fixed. Here we also look at how exogenous changes in the norm beliefs would affect welfare. We consider both a paternalistic government\textsuperscript{20} (which ignores the agents’ preferences for status) and a non-
paternalistic government (which respects the agents´ preferences for status). The analysis is conducted both in a first-best setting (where personalized commodity taxes are available) and in a second-best setting (where only uniform commodity taxes can be used by the policy-maker). The analysis in the first part of the paper provides a motivation for why a policy-maker will have an incentive to influence the private agents´ norm beliefs. This analysis also constitutes a point of reference against which we may compare the results derived in the second part of the paper where the norm beliefs are endogenous. In the second part of the paper we allow an agent´s norm belief to be endogenous and to be determined by how well the agent lives up to the consumption norm. If the agent lives up to the norm and thereby experiences positive utility, then the agent will further increase her belief in the norm but if the agent does not live up to the norm, then she instead has an incentive to reduce her belief in the norm. The change in the norm belief is modelled as a dynamic process in continuous time and we characterize the policy-maker’s decision problem conditional on that the policy-maker recognizes how its tax instruments may affect the formation/erosion of norm beliefs. By using this approach, we show that a paternalistic policy-maker may have fundamentally different incentives regarding in what direction norm beliefs should be changed compared with the incentives facing a non-paternalistic policy-maker.

We also characterize how the policy-maker can use the available tax instruments to create/reinforce a process whereby the belief in the consumption norm is either increased/decreased to some desired level or eroded away.

This study makes three contributions to the literature. It is the first study which incorporates cognitive dissonance theory and endogenous norm beliefs into a dynamic model of optimal commodity taxation and income redistribution. As far as we know, the only previous studies which consider endogenous personal norm beliefs in a continuous time framework are Vendrik (2004) and Mannberg and Sjögren (2015). However, both these studies analyze endogenous norm beliefs related to the labor market and neither of them are concerned with optimal taxation.²¹ The

２１There are a few models which have considered endogenous social norms in other contexts than the one addressed in this paper. However, neither of these studies is based on CDT, nor do these studies treat the personal norm beliefs as endogenous. Corneo and Jeanne (1997) develop a model where the consumption norm is endogenous. In their model, there is a loss of reputation if an agent does not consume a certain status good and the size of this utility loss depends on how many agents there are which attribute a symbolic value to the good. Over time, the reputational loss will change if the number of agents who attribute value to the status good changes. This mechanism can be used by producers and it is shown that norms may be created only if producers have enough market power. There are also studies which have looked at how public policy can be used to influence consumption norms. Nyborg and Rege (2003) study the formation of social norms for considerate smoking behavior and they show that the introduction of a smoking regulation may influence the social norm that is associated with considerate smoking behavior, so that there
second contribution is to suggest an alternative method for dealing with the externalities/imperfections that may arise from social norms. Instead of implementing Pigouvian taxes to internalize the norm related externalities, the approach in this paper suggests that a policy-maker may instead use the policy instruments at her disposal to change the belief in a distortionary norm. The latter approach may be especially effective if the policy-maker has too few policy instruments at her disposal. A third contribution is to show that both the short-run policy and the long-run outcome may differ dramatically between a paternalistic and a non-paternalistic policy-maker when norm beliefs are endogenous.

The outline of the study is as follows: Section 2 presents the basic model and characterizes the private sector in a static framework where the norm beliefs are fixed. Section 3 characterizes the optimal tax policy implemented both by a paternalistic and by a non-paternalistic government in a static framework, and here we also characterize the welfare effects of exogenous changes in the norm beliefs. In Section 4, we propose a model for how norm beliefs may change over time and in Section 5 we characterize the optimal tax policy in a continuous framework where the norm beliefs are endogenous. Section 6 summarizes and concludes the article. Proofs are presented in the Appendix.

2. The Basic Model

Following the discussion in the introduction, we want the model to have two key ingredients; (i) the agent types may differ in terms of how much they care about a consumption norm and (ii) one agent type contributes more than the other agent type to the reference consumption associated with the consumption norm. To achieve this, consider a model economy made up of two agent types $i = 1, 2$ where the number of type $i$ agents is given by $N_i$. An agent of type $i$ supplies one unit of labor in return for an exogenously given before-tax income $w_i$, and it is assumed that $w_1 < w_2$. The after-tax income is given by $w_i^n = w_i - T_i$, where $T_i$ is a lump-sum...
tax. There are two consumer goods; a numeraire good and a prestige good. An agent of type \(i\) uses her after-tax income to purchase \(c_i\) units of the numeraire good and one unit of the prestige good. Even though there is only one prestige good, there are many different variants of the prestige good and these variants differ in quality and in price. The quality of the prestige good is measured by a continuous quality indicator \(y\) and it is assumed that the price, \(p(y)\), that a producer charges for the prestige good is monotonously increasing and convex in \(y\). The agents also derive utility from using a public good, and agent type \(i\)'s material utility function is given by \(u(c_i, y_i) + \phi(G)\). The material utility function is twice continuously differentiable, increasing and concave in its arguments, and it is assumed that the cross derivative in \(u(c_i, y_i)\) is non-negative. It is also assumed that the two agent types have identical material preferences w.r.t. \(c\), \(y\) and \(G\).

The agents also have preferences for conspicuous consumption which gives rise to non-material utility. The non-material utility depends on how well agent type \(i\) lives up to a consumption norm which relates the quality content of the individual agent’s prestige good to a reference level. The non-material utility is made up of two parts. In line with Cognitive Dissonance Theory one part depends on how strongly the agent herself believes in, and lives up to, the prevailing consumption norm. In line with the Theory of Social Custom, the second part of the non-material utility comes from the agent’s status within the community and also this part depends on how well the agent lives up to the consumption norm. The consumption norm arises from a comparison of agent type \(i\)’s personal consumption of the prestige good with the consumption of the prestige good made by a reference group. For the low-income agents, we assume that the aspiration effect is at work which implies that the high-income agents constitute the point of reference for the low-income agents. More specifically, a low-income agent compares the quality content, \(y_1\), of her prestige good with the average quality content of the prestige good within the group of high-income agents, denoted \(\bar{y}_2\). As for the high-income agents, we assume that it is the distinction effect which is the key driver behind the preferences for conspicuous consumption. This means that each high-income agent compares the quality

\[22\] For example, in the car market there are many car brands which differ in quality and price.
\[23\] The motive for including a public good into the model is that it simplifies the analysis below.
\[24\] The low-income agents want to mimic the consumption of some elite group.
\[25\] Continuing with the car example, this comparison could involve comparing car brands, horse-powers etc, all of which are summarized by the quality indicator \(y\).
\[26\] Each high-income agent wants to outshine her peers.
content of her prestige good, $y_2$, with the average quality content among the other high-income agents, $\hat{y}_2$. As such, $\hat{y}_2$ constitutes the point of reference for both agent types in this model economy. The quality level $\hat{y}_2$ will be referred to as the consumption norm associated with the prestige good and since all agents of a given type are assumed to be identical, it follows that $\hat{y}_2 = y_2$.

The non-material utility, $NU_i$, experienced by an agent of type $i$ is captured by the function

\[ NU_i = \frac{0.5h_i(n_i)\Delta_i}{NU_i^p} + \frac{0.5h_i(\bar{n}_i)\Delta_i}{NU_i^s} \]

where $\Delta_i = y_i - \hat{y}_2$ is agent type $i$’s relative quality content of the prestige good compared with the consumption norm. The consumption norm is treated as exogenously given by each individual agent and it is worth emphasizing that even though $\Delta_2 = y_2 - \hat{y}_2$ will be zero in equilibrium for the high-income agents (because $\hat{y}_2 = y_2$), this is not perceived by the individual high-income agent who treats $y_2$ as a decision variable while $\hat{y}_2$ is treated as exogenously given.

The first term in equation (1), $NU_i^p = 0.5h_i(n_i)\Delta_i$, is the non-material utility associated with the personal satisfaction/dissatisfaction that the agent experiences by living up to/not living up to the consumption norm. It arises as a consequence of the agent’s personal belief in the consumption norm. The second term, $NU_i^s = 0.5h_i(\bar{n}_i)\Delta_i$, is the non-material utility associated with the status that an individual agent of type $i$ achieves within her social group (i.e. among the other type $i$ agents) by living up to the consumption norm. It arises if other persons in the agent`s social group believe in the consumption norm because then they praise the individual agent if she lives up to the norm but rebukes the agent if she violates the norm.

Let us take a closer look at the non-material utility that agent type $i$ associates with the personal norm; $NU_i^p = 0.5h_i(n_i)\Delta_i$. Here the term $n_i$ indicates the degree by which the agent herself believes in the consumption norm while the term $h_i(n_i)$ will be referred to as the norm function. If the agent’s personal norm belief ($n_i$) is zero then the agent does not care about the norm which means that she does not experience any utility or disutility from living up to ($\Delta_i > 0$), or not living up to ($\Delta_i < 0$), the norm. In this case, the norm function takes the value zero ($h_i(0) = 0$).

On the other hand, if the agent has some belief in the norm ($n_i > 0$), then the agent experiences positive utility if she lives up to the norm but she experiences negative utility if she violates the norm. These characteristics imply that the norm function is increasing in the norm belief. When
Vendrik (2003) analyzed the belief in a traditional household norm, he assumed that the norm function was linear and he used the normalization that a value of \( n = 1 \) was equivalent with full norm belief whereas a value of \( n = 0 \) was equivalent with no belief in the norm. In this paper, it will be assumed that the norm function is continuous and that \( h_i(n_i) \) approaches an upper limit for large \( n_i \), that is \( \lim_{n_i \to \infty} h_i(n_i) = h_i^{\text{max}} \), where \( h_i^{\text{max}} \) is a positive finite number.\(^{27}\) It is also assumed that \( dh_i(0)/dn_i \) is close, but not equal, to zero. This assumption reflects that if an agent initially has little belief in the norm (i.e. \( n_i \) is close to zero), then a marginal increase in \( n_i \) will have a negligible effect on the agent’s marginal non-material utility. Taken together, these assumptions imply that \( h_i(n_i) \) has an S-shape in the first quadrant in \((n_i,h_i)\) space.

In this paper we also allow \( n_i \) to take on negative values and we interpret a negative norm belief as if the agent does not believe in the norm. To exemplify what we mean with disbelieving a norm, assume that the prevailing norm prescribes that a successful person should drive a brand new status car. Assume now that some persons in the community find this norm utterly ridiculous. To signal their discontent (maybe even contempt) they may deliberately buy a car which is less sophisticated than they otherwise would have acquired just to disassociate themselves from the prevailing norm. Since a disbeliever experiences increased personal satisfaction by deviating from the status norm, this is equivalent with stating that the agent’s personal non-material utility, \( NU_i^P = 0.5h_i(n_i)\Delta_i \), will increase as \( \Delta_i \) decreases. This feature implies that the norm function \( h_i(n_i) \) is negative whenever \( n_i < 0 \) and we impose a lower bound on the norm function; \( \lim_{n_i \to -\infty} h_i(n_i) = h_i^{\text{min}} \) where \( h_i^{\text{min}} \) is a negative finite number. Since it is assumed that \( dh_i(0)/dn_i \) is close to zero, it follows that \( h_i(n_i) \) is S-shaped also in the third quadrant in \((n_i,h_i)\) space.

The definition of \( n_i \) as the personal belief (disbelief) in the consumption norm implies that the personal non-material utility \( (NU_i^P) \) reflects the individual agent’s personal satisfaction (dissatisfaction) of living up to (not living up to) the norm. If \( NU_i^P = 0.5h_i(n_i)\Delta_i > 0 \), the agent is said to experience cognitive consonance because then she acts in accordance with her personal norm. Hence, cognitive consonance occurs if \( \Delta_i > 0 \ and \ n_i > 0 \), or if \( \Delta_i < 0 \ and \ n_i < 0 \).

\(^{27}\) This specification has the same qualitative properties as the one used by Vendrik (2003) but our assumption of continuity will simplify the analysis to be conducted below. Note also that if there would be no upper limit on either \( n_i \) or \( h_i(n_i) \), then \( h_i(n_i) \) may grow to infinity in which case the preferences for status would eventually dominate in the agent’s overall utility function defined below in equation (2). To rule out this extreme outcome, we impose an upper limit on the norm function. The argument for imposing a lower limit on the norm function is analogous.
Conversely, the agent is said to experience cognitive dissonance if $NU_i^p < 0$, because here the agent acts in discord with her personal norm and experiences negative non-material personal utility. This happens if $\Delta_i < 0$ and $n_i > 0$, or if $\Delta_i > 0$ and $n_i < 0$.

Let us now turn to the second term on the right hand side (RHS) of equation (1); $NU_i^\theta = 0.5h_i(\tilde{n}_i)\Delta_i$. It reflects the non-material utility that comes from the agent’s status within her social group and depends on how well the agent lives up to the consumption norm. Here the term $\tilde{n}_i$ reflects the average belief in the consumption norm within the agent’s social group and since an agent of type $i$’s social group is made up of the other type $i$ agents, it follows that $\tilde{n}_i = n_i$. This implies that equation (1) reduces to $NU_i = h_i(n_i)\Delta_i$ and the latter result means that we can write the overall utility function for an agent, $U_i$, as

$$U_i = u(c_i, y_i) + \phi(G) + h_i(n_i)\Delta_i \quad (2)$$

The government levies a uniform proportional commodity tax, $\tau$, on the prestige good (this assumption is briefly relaxed in Section 3.1 where type-specific commodity taxes are considered). This means that the consumer price of one unit of the prestige good with a quality content of $y$ will be given by $q(y, \tau) = p(y)(1 + \tau)$. The budget constraint facing an agent of type $i$ will then be given by $w_i^n = c_i + q(y_i, \tau)$. Conditional on her private budget constraint, an agent chooses $c_i$ and $y_i$ to maximize $U_i$ and the solution to this maximization problem implicitly defines $y_i$ as a function of the vector $(w_i^n, \tau, n_i)$. Let us briefly consider agent type $i$’s first-order condition w.r.t. the quality of the prestige good, which can be written as

$$\frac{MRS_i}{dp/\partial y_i} + \frac{NMRS_i}{dp/\partial y_i} = (1 + \tau) \quad (3)$$

where $MRS_i = (\partial u_i/\partial y_i)/(\partial u_i/\partial c_i)$ is the marginal rate of substitution between $y_i$ and $c_i$ w.r.t. the material utility while $NMRS_i = h_i(n_i)/(\partial u_i/\partial c_i)$ is the marginal rate of substitution between $y_i$ and $c_i$ w.r.t. non-material utility. Equation (3) shows that the consumption norm provides an agent with an incentive to buy a prestige good which is more expensive than what is motivated from a strictly material utility perspective, and as long as the second-order condition is fulfilled it is straightforward to show that $\partial y_i/\partial n_i > 0$ and $\partial y_i/\partial w_i^n > 0.28$

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28 The latter result holds as long as the cross derivative in $u(c_i, y_i)$ is non-negative.
The production sector is modelled as simply as possible. Each agent supplies one unit of labor in return for the gross wage \( w_i \) and this labor input is employed in the production of the numeraire good. The aggregate production of the numeraire good, \( Y_c \), takes place using a linear production technology, \( Y_c = w_1N_1 + w_2N_2 \), where \( w_1 \) and \( w_2 \) are the fixed marginal products of labor for agent type 1 and 2, respectively. The net export of the numeraire good is given by \( Y_c - c_1N_1 - c_2N_2 \) and the export revenues are used to finance the import of the two prestige goods which are produced abroad.

3. Optimal Policy in a Static Framework

To obtain a point of reference against which we may compare the policies implemented when the norm beliefs are endogenous, it is instructive to begin by characterizing the outcome in a static framework where the norm beliefs are fixed. Here we will also look at how exogenous changes in the norm beliefs will affect welfare. Two government objective functions will be considered

\[
W^P = N_1[u(c_1, y_1) + \phi(G)] + N_2[u(c_2, y_2) + \phi(G)]
\]

\[
W^{NP} = N_1[u(c_1, y_1) + \phi(G) + h_1(n_1)\Delta_1] + N_2[u(c_2, y_2) + \phi(G) + h_2(n_2)\Delta_2]
\]

The objective function \( W^P \) defined in (4a) is Paternalistic in the sense that it ignores the non-material part of agent type \( i \)'s utility\(^{29}\) whereas the objective function \( W^{NP} \) defined in (4b) is Non-Paternalistic and fully respects the preferences of the agents. The government uses the lump-sum taxes and the commodity tax on the prestige good to finance redistribution of income and the provision of the public good. There are no information asymmetries and let us begin by characterizing the policies which maximize the welfare in each policy regime.

3.1 First-Best Policies

The first-best outcome in policy regime \( j = P, NP \) can be retrieved by asking what policy the government will implement if it can use type-specific commodity taxes. If type-specific commodity taxes are available, then the public budget constraint will be given by

\(^{29}\) As mentioned in the introduction, the motivation for paternalism is that preferences for status can be viewed as a form of envy. This has motivated some authors to argue that it is doubtful to incorporate such antisocial preferences into a social welfare function (see e.g. Harsanyi 1982).
\[ G = \sum_{i=1}^{\delta} N_i \left[ T_i + \tau_i^j p(y_i) \right] \]  

where \( \tau_i^j \) is the type specific commodity tax implemented for agent type \( i \) in policy regime \( j \). If we introduce the short notation \( \gamma = (N_1 + N_2) \partial \phi / \partial G \) then the solution to the government’s maximization problem in policy regime \( j \) can be summarized as follows;

**Proposition 1:** The first-best policy implemented by a paternalistic government is determined by the following equations

\[ \tau_i^P = \frac{h_i(n_i)}{\gamma(dp/dy)} \quad \text{for } i = 1,2, \quad \frac{\partial u_1}{\partial c_1} = 1, \quad \frac{MRS_1}{dp/dy_1} = \frac{MRS_2}{dp/dy_2} = 1 \]  

This policy implies complete income redistribution in the sense that \( c_1 = c_2, \ y_1 = y_2, \ \Delta_1 = 0, \ u_1 = u_2, \ w_1^n = w_2^n \) and \( U_1 = U_2 \).

The first-best policy implemented by a non-paternalistic government is determined by the following equations

\[ \tau_1^{NP} = 0, \quad \tau_2^{NP} = \frac{N_1 h_1(n_1) + N_2 h_2(n_2)}{\gamma_n(dp/dy_2)}, \quad \frac{\partial u_1}{\partial c_1} = 1 \]  

\[ \frac{MRS_1}{dp/dy_1} = 1 - \frac{h_1(n_1)}{\gamma(dp/dy_1)}, \quad \frac{MRS_2}{dp/dy_2} = 1 + \frac{N_1 h_1(n_1)}{N_2 \gamma(dp/dy_2)} \]  

This policy implies

(i) if \( n_1 > 0 \) and \( \partial^2 u_1 / \partial c_1 \partial y_1 \geq 0 \), then there will be over-redistribution of income in the sense that \( c_1 \geq c_2, \ y_1 > y_2, \ \Delta_1 > 0, \ \Delta_2 \) and \( u_1 > u_2 \).

(ii) if \( n_1 < 0 \) and \( \partial^2 u_1 / \partial c_1 \partial y_1 \geq 0 \), then there will be under-redistribution of income in the sense that \( c_1 \leq c_2, \ y_1 < y_2, \ \Delta_1 < 0 \) and \( u_1 < u_2 \).

(iii) if \( n_1 = 0 \), then there will be complete income redistribution in the sense that \( c_1 = c_2, \ y_1 = y_2, \ \Delta_1 = 0 \) and \( u_1 = u_2 \).
3.1.1 First-Best Policy in the Paternalistic Regime

The paternalistic government perceives that if \( n_i > 0 \), then the non-material part of the private utility function entices each agent type to buy a unit of the prestige good which is too expensive compared with what is motivated from a strictly material utility perspective. This provides the paternalistic government with an incentive to implement a positive commodity tax on each agent type to eliminate the “overspending” on the prestige good. By substituting the tax formula for \( \tau_i^P \) defined in (6) into agent type i’s first-order condition for \( y_i \) while using that \( \gamma = \partial u_i / \partial c_i \), the private first-order condition in equation (3) reduces to \( MRS_i / (dp/dy_i) = 1 \). The latter expression is the condition for an optimal choice of \( y_i \) from the perspective of the paternalistic government.

Conditional on the private and the public budget constraints, which effectively summarize the overall resource constraint in this model economy, there is a unique resource allocation \((c_1^*, y_1^*, c_2^*, y_2^*, G^*)\), where \( c_1^* = c_2^* \) and \( y_1^* = y_2^* \), which maximizes the paternalistic welfare function \( W^P \). This maximum welfare is attained by implementing the first-best tax policy defined in Proposition 1 and let us denote this maximum welfare level by \( W^* \). Since the paternalistic government does not incorporate the private agents’ non-material preferences for conspicuous consumption into the welfare function, the maximum welfare level \( W^* \) is independent of \( n_1 \) and \( n_2 \). This means that if the norm belief would change, then the paternalistic government would respond by adjusting the taxes so as to keep the resource allocation \((c_1^*, y_1^*, c_2^*, y_2^*, G^*)\), and hence the level of welfare \( W^* \), unchanged.

3.1.2 First-Best Policy in the Non-Paternalistic Regime

The non-paternalistic government recognizes that the individual high-income agent does not take into account that her choice of how much to spend on the prestige good imposes a negative consumption externality on the other agents in the economy. Therefore, the non-paternalistic government implements a corrective (Pigouvian) tax on good y for the high-income agents (in 7a, \( \tau_{2}^{NP} \) is positive if \( n_i > 0 \) for \( i = 1, 2 \)).

Note also (in contrast to the optimality conditions summarized in (6) for the paternalistic regime) that the optimality conditions summarized in (7b) contain the personal norm belief of the

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\(^{30}\) It can be shown that \( \gamma = (N_1 + N_2) \partial \phi / \partial G = \partial u_i / \partial c_i \) for \( i = 1, 2 \) in the first-best setting.
low-income agents. This implies that the first-best consumption bundle in the non-paternalistic regime will be a function of \( n_1 \). To explain this feature of the non-paternalistic policy, observe that if \( n_1 \) would go up, then a low-income agent would increase her spending on good \( y \) at the expense of her consumption of the numeraire good. Since the non-paternalistic government respects the preferences of the individual agents, and since the optimal redistribution policy features \( \partial u_1 / \partial c_1 = \partial u_2 / \partial c_2 \), the non-paternalistic government would respond to such a change in the preferences by allocating more resources to the low-income agents at the expense of the high-income agents. This mechanism explains the over-redistribution result in part (i) in Proposition 1 and an analogous argument can be used to explain the under-provision result in part (ii). These results imply that the level of welfare in the non-paternalistic regime will be a function of the low-income agent’s norm belief; \( W^{NP}(n_1) \). To evaluate how a change in \( n_1 \) affects the first-best level of welfare in the non-paternalistic regime, let us use that we can write the first-best taxes as functions of the norm beliefs, i.e. \( \tau_i^{NP}(n_1, n_2), T_i^{NP}(n_1, n_2) \) for \( i = 1, 2 \). If we substitute the private and the public budget constraints into equation (4b), and then substitute the private demand functions and the functions \( \tau_i^{NP}(n_1, n_2) \) and \( T_i^{NP}(n_1, n_2) \) into the resulting expression, and then differentiate the resulting expression w.r.t. \( n_1 \) while using the envelope properties of the welfare function, we obtain

\[
\frac{\partial W^{NP}}{\partial n_1} = (y_1 - y_2)N_1 \frac{d h_1}{d n_1}
\]

(8)

Combining equation (8) with parts (i) – (iii) in Proposition 1 produces the following result;

**Corollary 1:** If \( \partial^2 u_i / \partial c_i \partial y_i \geq 0 \), then the first-best welfare in the non-paternalistic regime, \( W^{NP}(n_1) \), is an increasing (decreasing) function of \( n_1 \) if \( a n_1 > 0 \) (\( n_1 < 0 \)). This implies that the first-best welfare in the non-paternalistic regime is at a minimum when \( n_1 = 0 \).

The result in Corollary 1 is a direct consequence of the over-redistribution that takes place when \( n_1 > 0 \) (which causes \( y_1 \) to exceed \( y_2 \)) and of the under-redistribution that takes place when \( n_1 < 0 \) (which causes \( y_1 \) to fall short of \( y_2 \)). Note in particular that Corollary 1 implies that the non-paternalistic government would have an incentive to increase (decrease) the low-income
agents’ norm belief (if that would be possible) if \( n_1 > 0 \) (\( n_1 < 0 \)) holds initially. If \( n_1 = 0 \) holds initially, then any change in \( n_1 \) would have a positive effect on the welfare.

Observe also that the optimality conditions in (7b) do not contain the norm belief of the high-income agents, \( n_2 \). This means that the first-best level of welfare in the non-paternalistic regime is invariant to changes in \( n_2 \) and this is verified if we use the same approach as when we derived equation (8). Differentiating the non-paternalistic welfare function w.r.t. \( n_2 \) while using the envelope properties of this function produces

\[
\frac{\partial W^{NP}}{\partial n_2} = (y_2 - \hat{y}_2) N_2 \frac{dh_2}{dn_2} = 0
\]  

(9)

where we have used that \( \hat{y}_2 \) is equal to \( y_2 \) to obtain the last equality on the RHS of equation (9).

To explain this invariance result, observe that if \( n_2 \) would increase then, all else equal, each high-income agent would increase her spending on the quality content of the prestige good. However, in response to this, the non-paternalistic government would increase the type-specific commodity tax \( \tau_2^{NP} \) so as to choke off the negative consumption externality that an increase in \( \hat{y}_2 \) would create (while at the same time adjusting the lump-sum taxes so as to maintain the equality \( \partial u_1 / \partial c_1 = \partial u_2 / \partial c_2 \)). This means that the direct positive effect that an increase in \( n_2 \) has on \( y_2 \) is counter-balanced by the direct negative effect that the subsequent increase in \( \tau_2^{NP} \) has on \( y_2 \). In the non-paternalistic first-best equilibrium, these two effects cancel out which means that \( y_2 \) is not affected by a change in \( n_2 \) once the non-paternalistic government has adjusted its tax policy. Since \( y_2 \) is unaffected by \( n_2 \) once \( \tau_2^{NP} \) and the lump-sum taxes have been adjusted, it follows that neither \( c_1, c_2, y_1, G \) and \( W^{NP} \) will be affected by a change in \( n_2 \). This is summarized as follows;

**Corollary 2:** In the non-paternalistic regime, the first-best welfare is invariant to changes in \( n_2 \).

The analysis above implies that the non-paternalistic government responds differently to a change in the norm belief depending on whether the change comes from agent type 1 or agent type 2. An exogenous increase in \( n_1 \) (if \( n_1 > 0 \) holds initially) would induce the non-paternalistic government to redistribute more resources to agent type 1 while a corresponding increase in \( n_2 \) would have no effect on redistribution. More fundamentally, these asymmetric policy responses reflect that the low-income agents’ status norm is a consequence of the aspiration effect (where
each type 1 agent wants to mimic the consumption of some elite group, here agent type 2) while the high-income agents’ status norm arises from the distinction effect (where each type 2 agent wants to outshine her peers, here the other type 2 agents). We can therefore draw the following general conclusions;

**Proposition 2:** Consider an economy made up of two agent types where the agents of a given type are identical and where a non-paternalistic government implements a first-best policy. Then

- if the consumption norm of one of the agent types arises from the aspiration effect, then an increase in the norm belief of this agent type induces the non-paternalistic government to redistribute more resources to this agent group but
- if the consumption norm of one of the agent types instead arises from the distinction effect, then a change in the norm belief of this agent type has no effect on the redistribution.

Recall that this asymmetric treatment of the two agent types is not present in the first-best policy implemented by a paternalistic government.

### 3.2 Second-Best Policies

Let us now compare the policies implemented in the first-best regimes with the second-best policies that will be implemented when the government is restricted to use a uniform commodity tax on the prestige good. In this case the public budget constraint defined in equation (5) is modified to read \( G = \sum_{i=1}^{2} N_i [T_i + \tau^j p(y_i)] \) where \( \tau^j \) is the uniform commodity tax on the prestige good in policy regime \( j \). Let us also introduce the following short notations

\[
\frac{\partial y_i}{\partial \tau} = \frac{\partial y_i}{\partial \tau} - p_i \frac{\partial y_i}{\partial T_i} < 0, \quad \alpha_i = \frac{N_i \partial y_i}{\partial \tau} \frac{dp}{dy_i}, \quad \rho = N_1 \frac{\partial y_1}{\partial \tau} \frac{dp}{dy_1} + N_2 \frac{\partial y_2}{\partial \tau} \frac{dp}{dy_2} < 0
\]

where \( p_i = p(y_i), 0 < \alpha_i < 1 \) and where \( \alpha_1 + \alpha_2 = 1 \). The term \( \partial y_i / \partial \tau \) will be referred to as a compensated price effect and the negative sign of this term follows from a comparative static analysis of the solution to agent type \( i \)´s maximization problem. After these preliminaries, consider the following result;
Proposition 3: The second-best commodity tax in policy regime $j = P, NP$ is given by

$$\tau_j = \alpha_1 \tau_{1j} + \alpha_2 \tau_{2j}$$

(10)

where $\tau_{1j}$ and $\tau_{2j}$ are the first-best tax rules defined in Proposition 1.

(a) The redistribution policy implemented in the paternalistic regime can be summarized as follows:

(i) If $n_1 = n_2$, then there will be complete income redistribution in the sense that $c_1 = c_2$, $y_1 = y_2$, $\Delta_1 = 0$ and $u_1 = u_2$. In this case the second-best welfare coincides with the first-best welfare level ($W^*$).

(ii) If $n_1 > n_2$ in the special case where $\partial^2 u_i / \partial c_i \partial y_i = 0$ and where the price function $p(y)$ is linear, then there will be under-redistribution of income in the sense that $c_1 < c_2$, $y_1 < y_2$, $\Delta_1 < 0$, $\Delta_2$ and $u_1 < u_2$.

(iii) If $n_1 < n_2$ in the special case where $\partial^2 u_i / \partial c_i \partial y_i = 0$ and where the price function $p(y)$ is linear, then there will be over-redistribution of income in the sense that $c_1 > c_2$, $y_1 > y_2$, $\Delta_1 > 0$ and $u_1 > u_2$.

(b) In the special case where $\partial^2 u_i / \partial c_i \partial y_i = 0$ and where the price function $p(y)$ is linear, the redistribution policy implemented in the non-paternalistic regime can be summarized as follows:

(i) If $N_1 h_1 + N_2 h_2 > 0$ and if $n_1 \geq n_2$, then there will be over-redistribution of income in the sense that $c_1 > c_2$, $y_1 > y_2$, $\Delta_1 > 0$, $\Delta_2$ and $u_1 > u_2$.

(ii) If $N_1 h_1 + N_2 h_2 < 0$ and if $n_1 \leq n_2$, then there will be under-redistribution of income in the sense that $c_1 < c_2$, $y_1 < y_2$, $\Delta_1 < 0$, $\Delta_2$ and $u_1 < u_2$.

From equation (10), we see that the second-best commodity tax on the prestige good is a compromise (a weighted average) between providing the correct incentive (from government $j$’s point of view) for agent type 1 and for agent type 2, respectively. Furthermore, since we know from Proposition 1 that $\tau_{1NP} = 0$, it follows that the second-best commodity tax in the non-paternalistic regime actually reduces to $\tau_{NP} = \alpha_2 \tau_{2NP}$.

The compromise of objectives implied by the tax formula in equation (10) will have implications for redistribution, and parts (a) and (b) in Proposition 3 show that the redistribution
policies will differ substantially between the two policy regimes. Beginning with the paternalistic regime, it follows that if \( n_1 = n_2 \) then the government can implement a policy which replicates the first-best outcome where the resource allocation is given by \((c_1^*, y_1^* = y_2^*, G^*)\), and where the level of welfare is \( W^* \). The reason is that when the two agent types have identical preferences, then they will make identical consumption choices if income is redistributed evenly. This, in turn, implies that the first-best tax rules will be the same for both agent types (i.e. \( \tau_1^P = \tau_2^P \)) and by setting the uniform second-best commodity tax, \( \tau^P \), equal to \( \tau_i^P \) the first-best outcome is attainable where the level of welfare is equal to \( W^* \). If \( n_1 \neq n_2 \), the argument above does not hold. If, for example, \( n_1 > n_2 \) so that \( \tau_1^P > \tau_2^P \), then it follows from equation (10) and from the first-best rule defined in (6) that \( \tau_2^P < \tau^P < \tau_1^P \). Here the type 2 agent faces a commodity tax on the prestige good which exceeds the first-best rate whereas the type 1 agent faces a commodity tax which is below the first-best rate. In this situation, the paternalistic government compensates the type 2 agent by redistributing some income from the type 1 agent. This explains point (ii) in part (a) in Proposition 3. An analogous argument can be applied to explain point (iii) in part (a).

As for the non-paternalistic government, part (b) in Proposition 3 shows that the over-redistribution result that occurs in the non-paternalistic first-best regime when \( n_1 > 0 \) partly carries over into the second-best framework because there will be over-redistribution of income when \( n_1 \geq n_2 \) and \( N_1 h_1 + N_2 h_2 > 0 \). Note in particular that this over-redistribution result is quite the opposite compared with the redistribution policy implemented in the paternalistic second-best regime where, instead, there will be under-redistribution of income when \( n_1 > n_2 \).

Let us also evaluate how changes in \( n_1 \) and \( n_2 \) affect the level of welfare in policy regime \( j \). By using the envelope properties of the second-best welfare function \( W^j \), it can be shown that

\[
\frac{\partial W^p}{\partial n_1} = (\tau_2^P - \tau_1^P) \alpha_2 \gamma N_1 \frac{dp}{dy_1} \frac{\partial y_1}{\partial n_1} \quad (11a)
\]

\[
\frac{\partial W^p}{\partial n_2} = -(\tau_2^P - \tau_1^P) \alpha_1 \gamma N_2 \frac{dp}{dy_2} \frac{\partial y_2}{\partial n_2} \quad (11b)
\]

\[
\frac{\partial W^{NP}}{\partial n_1} = \alpha_2 \tau_2^{NP} \gamma N_1 \frac{dp}{dy_1} \frac{\partial y_1}{\partial n_1} + N_1 (y_1 - y_2) \frac{d h_1}{dn_1} \quad (11c)
\]

\[
\frac{\partial W^{NP}}{\partial n_2} = -\tau_2^{NP} \alpha_1 \gamma N_2 \frac{dp}{dy_2} \frac{\partial y_2}{\partial n_2} \quad (11d)
\]
Equations (11a) and (11b) confirm point (i) in part (a) in Proposition 3; in the paternalistic regime the second-best welfare has a maximum when \( n_1 = n_2 \). Turning to the non-paternalistic government, equation (11c) shows that \( \partial W^{NP} / \partial n_1 \) is positive (negative) if \( \tau_2^{NP} > 0 \) (\( \tau_2^{NP} < 0 \)) and if \( y_1 > y_2 \) \( (y_1 < y_2) \) whereas equation (11d) shows that \( \partial W^{NP} / \partial n_2 \) is positive (negative) if \( \tau_2^{NP} < 0 \) (\( \tau_2^{NP} > 0 \)). Combining these observations with equation (10) and points (i) and (ii) in part (b) in Proposition 3, we can summarize these observations as follows;

**Corollary 3:** The welfare in the paternalistic second-best regime is maximized if \( n_1 = n_2 \). The welfare in the non-paternalistic second-best regime has the following properties:

\[
\frac{\partial W^{NP}}{\partial n_1} > 0 \text{ if } N_1 h_1(n_1) + N_2 h_2(n_2) > 0 \text{ and } n_1 \geq n_2 \quad (12a)
\]

\[
\frac{\partial W^{NP}}{\partial n_1} < 0 \text{ if } N_1 h_1(n_1) + N_2 h_2(n_2) < 0 \text{ and } n_1 \leq n_2 \quad (12b)
\]

\[
\frac{\partial W^{NP}}{\partial n_1} = 0 \text{ if } n_1 = n_2 = 0 \quad (12c)
\]

\[
\frac{\partial W^{NP}}{\partial n_2} > 0 \text{ if } n_2 < 0, \quad \frac{\partial W^{NP}}{\partial n_2} < 0 \text{ if } n_2 > 0, \quad \frac{\partial W^{NP}}{\partial n_2} = 0 \text{ if } n_2 = 0 \quad (12d)
\]

Corollary 3 implies that if the personal norm beliefs would be endogenous, then a paternalistic government would have an incentive to influence \( n_1 \) and \( n_2 \) in such a way that they become equalized. A non-paternalistic government would instead have an incentive to push up (push down) \( n_1 \) if \( n_1 \geq n_2 > 0 \) \( (n_1 \leq n_2 < 0) \) holds initially. We also see that \( W^{NP} \) has a maximum w.r.t. \( n_2 \) when \( n_2 = 0 \). Hence, if the personal norm beliefs were endogenous, then the government in each policy regime would have an incentive to influence these beliefs. This begs the question if it is reasonable to assume that the personal norm beliefs are endogenous? To address this question recall that agents, according to Cognitive Dissonance Theory, have an incentive to maintain a consistent self-image. If an agent’s actual behavior violates a personal norm that she believes in, then the agent has an incentive to downplay the importance of that norm but if the agent instead lives up to a personal norm that she believes in, then she has an incentive to reinforce the belief in that norm. In the next section, we will incorporate this idea into the model framework presented above so as to develop a model where the norm beliefs are endogenous. In line with Vendrik (2003), it will be assumed that norm beliefs do not adjust
instantaneously but need some time for that. Therefore we need to extend the analysis into a dynamic framework.

4. Endogenous Norm Beliefs

To make the norm beliefs endogenous, we will use the following approach. Substitute the demand function \( y_i(w_i^n, \tau, n_i) \) into the overall utility function \( U_i \) which was defined in equation (2). Then differentiate the resulting expression w.r.t. \( n_i \) (conditional on the reference consumption \( \hat{y}_2 \) which the private agents treat as exogenously given) and use the private first-order conditions to simplify the resulting expression. This produces \( \frac{\partial U_i}{\partial n_i} = \frac{dh_i}{dn_i} \Delta_i \). If \( \frac{\partial U_i}{\partial n_i} > 0 \), then the agent has an incentive to increase her belief in norm \( i \) over time but if \( \frac{\partial U_i}{\partial n_i} < 0 \), then agent type \( i \) instead has an incentive to reduce her belief in norm \( i \) over time. This argument implies that the change in \( n_i \) should be proportional to the term \( \frac{dh_i}{dn_i} \Delta_i \).

Therefore, we write the differential equation for \( n_i \) as follows

\[
\dot{n}_i = k_i \Delta_i \frac{dh_i}{dn_i}
\]

where \( \dot{n}_i \) is the time derivative of \( n_i \) and where \( k_i \) is a positive constant. It is also assumed that the before-tax income, \( w_i \), and the price function, \( p(y) \), are time-invariant.\(^{31}\)

The specification in equation (13) is consistent with Cognitive Dissonance Theory. To see this, recall that agent type \( i \)'s non-material utility is given by \( NU_i^p = 0.5h_i(n_i) \Delta_i \). If \( n_i \) is allowed to change over time, then the agent has an incentive to change \( n_i \) in the direction that increases her non-material utility. If \( NU_i^p > 0 \) holds initially, then the agent experiences cognitive consonance and has an incentive to increase \( n_i \) further but if \( NU_i^p < 0 \) holds initially, then the agent experiences cognitive dissonance and has an incentive to reduce \( n_i \). These arguments imply that the specification in equation (13) is consistent with CDT if the differential equation for \( n_i \) satisfies the following criteria

(i) If an agent believes in norm \( i \) and lives up to it (\( n_i > 0, \Delta_i > 0 \Rightarrow NU_i^p > 0 \)) then the agent experiences cognitive consonance and has an incentive to reinforce (increase) her belief in norm \( i \), i.e. \( \dot{n}_i > 0 \).

\(^{31}\) The latter two assumptions make it possible to define a steady-state in the analysis below.
(ii) If an agent believes in norm $i$ but does not live up to it ($n_i > 0$, $\Delta_i < 0 \Rightarrow NU^p_i < 0$) then the agent experiences cognitive dissonance and has an incentive to downplay (reduce) her belief in norm $i$, i.e. $\dot{n}_i < 0$.

(iii) If an agent does not believe in norm $i$ and violates it ($n_i < 0$, $\Delta_i < 0 \Rightarrow NU^p_i > 0$) then the agent experiences cognitive consonance and has an incentive to increase her disbelief in norm $i$ (the agent has an incentive to reduce her belief in norm $i$), i.e. $\dot{n}_i < 0$.

(iv) If an agent does not believe in norm $i$ but lives up to it ($n_i < 0$, $\Delta_i < 0 \Rightarrow NU^p_i < 0$) then the agent experiences cognitive dissonance and has an incentive to reduce her disbelief in norm $i$ (the agent has an incentive to increase her belief in norm $i$), i.e. $\dot{n}_i > 0$.

The specification in equation (13) does satisfy these criteria.

Let us take a brief look at how the government’s policy instruments will influence the speed at which the norm beliefs change over time. Beginning with the low-income agents, the comparative static properties of the $\dot{n}_1$ locus are given by

$$\frac{\partial n_1}{\partial w_1^n} = k_1 \frac{\partial y_1}{\partial w_1^n} \frac{dh_1}{dn_1} > 0, \quad \frac{\partial n_1}{\partial w_2^n} = -k_1 \frac{\partial y_2}{\partial w_2^n} \frac{dh_1}{dn_1} < 0, \quad \frac{\partial n_1}{\partial \tau} = k_1 \left( \frac{\partial y_1}{\partial \tau} - \frac{\partial y_2}{\partial \tau} \right) \frac{dh_1}{dn_1}$$  \hspace{1cm} (14)

These properties imply that an increase in $w_1^n$ and/or a reduction in $w_2^n$ will shift the $\dot{n}_1$ locus upwards in $(n_1, \dot{n}_1)$ space. Taken together, these results imply that traditional income redistribution (from high-income to low-income agents) has a positive effect on $\dot{n}_1$. From the third comparative static result in (14) it follows that an increase in the uniform commodity tax $\tau$ has an ambiguous effect on the speed at which $\dot{n}_1$ changes. Turning to the high-income agents, recall that since all high-income agents are identical, it follows that $\Delta_2 = y_2 - \hat{y}_2 = 0$. As a consequence, $\dot{n}_2 = k_2 \Delta_2 (\partial h_2 / \partial n_2)$ will be zero at each point in time. This means that the high-income agents’ norm belief will be constant over time and the differential equation for $n_2$ is therefore redundant.

5. Second-Best Policies in a Dynamic Framework

The comparative static properties in (14) imply that government policy can be used to influence the dynamics of the low-income agents’ norm belief. To understand how the incentive to
influence \( n_1 \) will affect tax policy, let us turn to the dynamic optimization problem facing the government in policy regime \( j = P, NP \). To keep the mathematical complexity at a minimum, we abstract from saving and assume that both the private agents and the government balance their respective budgets at each point in time. Furthermore, since the change in an agent’s norm belief is likely to be a sub-conscious process (see Vendrik 2003), this change is not incorporated into the private agents’ optimization problems.\(^{32}\) These assumptions imply that the private agents behave as in the static framework outlined in Section 2.

The government in regime \( j \) aims to maximize the present value of the discounted welfare

\[
\Phi^j = \int_0^\infty W^j(t)e^{-\theta t} dt
\]

where \( \theta \) is the government’s discount rate and where \( W^j \) is the instantaneous welfare function defined in equation (4). The maximization of \( \Phi^j \) takes place subject to the following restrictions

\[
\begin{align*}
\dot{n}_1(t) &= k_1(y_1(t) - y_2(t)) \frac{dh_1(n_1(t))}{dn_1} \quad (16a) \\
G(t) &= \sum_{i=1}^2 N_i[T_i(t) + \tau(t)p(y_i(t))] \quad (16b) \\
c_i(t) &= w_i - T_i(t) - p(y_i(t))(1 + \tau(t)) \quad \text{for } i = 1, 2 \quad (16c) \\
y_1(t) &= y_1(w_1 - T_1(t), \tau(t), n_1(t)) \quad (16d) \\
y_2(t) &= y_2(w_2 - T_2(t), \tau(t), \bar{n}_2) \quad (16e)
\end{align*}
\]

where \( \bar{n}_2 \) is the initial level of agent type 2’s norm belief which is fixed. As can be seen in (16a), the problem is formulated such that the government treats the low-income agent’s norm belief, \( n_1 \), as a state variable. The current-value Hamiltonian associated with this unconstrained optimization problem can be written as

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\(^{32}\) There are a few models which do operationalize this sub-conscious process as a dynamic optimization problem. In both Vendrik (2003) and in Mannberg and Sjögren (2015) the agent may “invest/disinvest” in the belief in a labor supply norm. In their respective models, the differential equation for the belief in the labor supply norm, \( n \), is given by \( \dot{n} = I \), where \( I \) is the net “investment” in \( n \). This “investment” takes place subject to a convex cost function which reflects psychological adjustment costs associated with re-evaluating a deep-rooted belief. In both Vendrik (2003) and in Mannberg and Sjögren (2015), the solution to the dynamic optimization problem turns out to have the same qualitative property as that implied by the differential equation used in this paper; at each point in time \( t \) the differential equation which governs \( n \) is a function of whether the instantaneous utility at time \( t \) is increasing or decreasing in \( n \), or put differently, whether the agent at time \( t \) experiences cognitive consonance or cognitive dissonance.
\[ H^j(t) = W^j(t) + \lambda^j(t)k_1 [y_1(t) - y_2(t)] \frac{dh_1(n_1(t))}{dn_1} \]  

(17)

where \( \lambda^j \) is the shadow price associated with \( n_1 \). Using the Maximum Principle, the necessary conditions can be written as

\[
\frac{\partial H^j}{\partial \tau_1} = \frac{\partial W^j}{\partial \tau_1} + \lambda^j(k_1 \frac{\partial y_1}{\partial \tau_1} \frac{dh_1}{dn_1} = 0 \quad (18a)
\]

\[
\frac{\partial H^j}{\partial \tau_2} = \frac{\partial W^j}{\partial \tau_2} - \lambda^j(k_1 \frac{\partial y_2}{\partial \tau_2} \frac{dh_1}{dn_1} = 0 \quad (18b)
\]

\[
\frac{\partial H^j}{\partial \tau} = \frac{\partial W^j}{\partial \tau} + \lambda^j(k_1 \frac{\partial y_1}{\partial \tau} - \frac{\partial y_2}{\partial \tau}) \frac{dh_1}{dn_1} = 0 \quad (18c)
\]

\[
\dot{\lambda^j} - \theta \lambda^j = - \frac{\partial W^j}{\partial n_1} - \lambda^j(k_1 \frac{\partial y_1}{\partial n_1} \frac{dh_1}{dn_1} - \lambda^j(k_1 \Delta_1 \frac{d^2 h_1}{dn_1^2} \quad (18d)
\]

\[
\lim_{t \to \infty} \lambda^j(n_1 e^{-\theta t} = 0 \quad (18e)
\]

where equation (18e) is the transversality condition and where we omit the time indicator. Below we will take a closer look at what these necessary conditions imply for the optimal policy implemented in each of the two policy regimes. We begin with the paternalistic regime.

### 5.1 The Steady-State in the Paternalistic Regime

Recall from part (a) in Proposition 3 that in the static second-best setting, it is possible for the paternalistic government to replicate the first-best policy if \( n_1 = \bar{n}_2 \). This indicates that the solution to the paternalistic government’s dynamic optimization problem features a time path for the economy which leads to \( n_1 = \bar{n}_2 \). It is therefore natural to begin the analysis by looking at whether \( n_1 = \bar{n}_2 \) is a steady-state solution which satisfies the necessary conditions in (18a) – (18e). The answer is yes and the optimal policy implemented in this steady-state has the following characteristics;
Proposition 4: The steady-state where \( n_1 = \bar{n}_2 \) does satisfy the necessary conditions in (18a) – (18e) for \( j = P \). In this steady-state, \( \lambda^P = 0 \) and the paternalistic government is able to implement a policy which replicates the first-best outcome in the static framework where

\[
\tau^P = \tau^P_i, \quad \tau^P_i = \frac{h_1(n_1)}{y(dp/dy_1)}, \quad \frac{\partial u_1/\partial c_1}{\partial u_2/\partial c_2} = 1, \quad \frac{MRS_1}{dp/dy_1} = \frac{MRS_2}{dp/dy_2} = 1
\]  

This policy implies complete income redistribution in the sense that \( c_1 = c_2, \ y_1 = y_2, \ \Delta_1 = 0, \ u_1 = u_2, \ w_1^n = w_2^n \) as well as \( U_1 = U_2 \) and \( W^P = W^* \).

The intuition for why the paternalistic government implements this policy in the steady-state is that once the economy reaches the steady state, then the intertemporal objective function defined in equation (15), \( \Phi^P \), is maximized if the instantaneous welfare function \( W^P \) is maximized at each point in time \( t \). The latter maximization problem is analogous to the static second-best maximization problem described in Section 3.2 and from that analysis we know that when \( n_1 = \bar{n}_2 \), then the second-best maximum coincides with the first-best maximum. Note also that the shadow price is zero in the steady-state. This reflects that since there is no direct utility cost associated with changing the norm belief, the marginal benefit of changing \( n_1 \) will be zero in the steady-state.

5.1.1 Local Stability Analysis in the Paternalistic Regime

To analyze under what conditions the steady-state defined in Proposition 4 is locally saddle-point stable, we proceed as follows. Use that equations (18a) – (18c) implicitly define \( T_1, \ T_2 \) and \( \tau \) as functions of \( \lambda \) and \( n_1 \); \( T_1(\lambda, n_1), \ T_2(\lambda, n_1) \) and \( \tau(\lambda, n_1) \). Then, substitute these functions into equation (16a) and into equation (18d), respectively, for \( j = P \). This produces the following system of equations

\[
\dot{\lambda}(\lambda, n_1) = \theta \dot{\lambda} + \left( h_1 N_1 - \tau y N_1 \frac{dp}{dy_1} - \lambda k_1 \frac{dh_1}{dn_1} \right) \frac{dy_1}{dn_1} - \lambda k_1 (y_1 - y_2) \frac{d^2 h_1}{dn_1^2} \tag{20a}
\]

\[
\dot{n}_1(\lambda, n_1) = k_1 (y_1 - y_2) \frac{dh_1}{dn_1} \tag{20b}
\]

\[33\] We omit the superindex \( j = P \) in the remainder of this subsection.
To examine the dynamic properties of this equation system, we make a linear approximation of equations (20a) and (20b) in a close neighborhood around the steady-state in the special case where \( \bar{n}_2 = 0 \) (which in turn implies that \( \lambda = n_1 = \tau = \tau_1^p = \tau_2^p = 0 \) in the steady-state). Differentiating equations (20a) and (20b) w.r.t. \( \lambda \) and \( n_1 \) allows us to write the Jacobian matrix of first-order partial derivatives as

\[
J = \begin{bmatrix}
\frac{\partial \lambda}{\partial \lambda} & \frac{\partial \lambda}{\partial n_1} \\
\frac{\partial n_1}{\partial \lambda} & \frac{\partial n_1}{\partial n_1}
\end{bmatrix}
\]  

(21)

where these partial derivatives are evaluated in the steady-state. Local stability around the steady-state is directly related to the determinant of the Jacobian matrix which is given by

\[
|J| = \left. \frac{\partial \lambda}{\partial \lambda} \frac{\partial n_1}{\partial n_1} - \frac{\partial \lambda}{\partial n_1} \frac{\partial n_1}{\partial \lambda} \right|_{|\lambda=0,-n_1=0|} = \left. \frac{\partial \lambda}{\partial n_1} \right|_{|\lambda=0,n_1=0} \left. \frac{\partial n_1}{\partial \lambda} \right|_{|\lambda=0,n_1=0}
\]  

(22)

where we in the second step have used that the slopes of the \( \dot{\lambda} = 0 \) and the \( \dot{n}_1 = 0 \) loci in \((n_1, \lambda)\) space are given by

\[
\left. \frac{\partial \lambda}{\partial n_1} \right|_{\lambda=0} = \frac{\partial \lambda}{\partial \lambda} / \frac{\partial n_1}{\partial \lambda}, \quad \left. \frac{\partial \lambda}{\partial n_1} \right|_{n_1=0} = \frac{\partial n_1 / \partial n_1}{\partial n_1 / \partial \lambda}
\]  

(23)

As long as the characteristic roots associated with the system of linearized differential equations are real numbers, a sufficient condition for local saddle-point stability is that \( |J| < 0 \). From equation (22), it follows that if we can sign \( \partial \dot{\lambda} / \partial \lambda \) and \( \partial \dot{n}_1 / \partial \lambda \), then local saddle-point stability can be linked to whether the \( \dot{\lambda} = 0 \) locus has a steeper or flatter slope than the \( \dot{n}_1 = 0 \) locus in a close neighborhood around the steady-state. Evaluating the partial derivatives of \( \dot{\lambda} / \partial \lambda \), \( \partial \dot{n}_1 / \partial \lambda \) and \( \dot{\lambda} / \partial n_1 \) in the steady-state (where \( n_1 = \bar{n}_2 = \tau = \tau_1^p = \tau_2^p = 0 \), which implies that \( y_1 = y_2, w_1^n = w_2^n \) and \( \partial y_1 / \partial w_1^n = \partial y_2 / \partial w_2^n \)) produces

\[
\frac{\partial \dot{\lambda}}{\partial \lambda} = \theta - k_1 \frac{\partial y_1}{\partial n_1} \frac{\partial h_1}{\partial n_1} > 0
\]  

(24)

\[
\frac{\partial \dot{n}_1}{\partial \lambda} = k_1 \left( \frac{\partial \tau_2}{\partial \lambda} - \frac{\partial \tau_1}{\partial \lambda} \right) \frac{\partial y_1}{\partial n_1} \frac{\partial h_1}{\partial n_1} > 0
\]  

(25)

\[
\frac{\partial \dot{\lambda}}{\partial n_1} = (1 - \alpha_1) N_1 \frac{\partial y_1}{\partial n_1} \frac{\partial h_1}{\partial n_1} > 0
\]  

(26)
where the assumption that $dh_1(0)/dn_1$ is close to zero has been used to sign equation (24), and where the signs of equations (24) and (26) imply that the $\dot{\lambda} = 0$ locus has a negative slope. We can now combine equations (22) – (25) to make the following claim: \textsuperscript{34}

**Proposition 5:** Assume that $\partial T_1(\lambda, n_1)/\partial \lambda < 0$, $\partial T_2(\lambda, n_1)/\partial \lambda > 0$ and $\bar{n}_2 = 0$. In this situation the steady-state in the paternalistic regime is locally saddle-point stable if the slope of the $\dot{n}_1 = 0$ locus exceeds the slope of the $\dot{\lambda} = 0$ locus.

There are two possible outcomes and they are illustrated in Figures 1a and 1b below. As illustrated, both outcomes are associated with a negatively sloped saddle-path which means that $\lambda$ is negative (positive) along the optimal time path towards the steady-state if the initial value of $n_1$ is larger (smaller) than $\bar{n}_2 = 0$.

**FIGURES 1a and 1b HERE**

The basic intuition for why the saddle-path has a negative slope is straightforward. If $n_1$ initially exceeds $\bar{n}_2$, and therefore exceeds the first best-level ($n_1 = \bar{n}_2$), then the government has an incentive to implement a tax and expenditure policy which reduces the norm belief $n_1$ towards $\bar{n}_2$. Along an optimal time path where $n_1$ is reduced, the valuation of $n_1$ (i.e. the shadow price $\lambda$) must be below the value of the shadow price in the steady state (which is zero), i.e. $\lambda < 0$. This saddle-path is observed in the fourth quadrant in Figures 1a and 1b. An analogous argument can be used to explain the saddle-path observed in the second quadrant in Figures 1a and 1b where $\lambda > 0$.

**5.2 Long-Run Outcomes in the Non-Paternalistic Regime**

Let us now turn to the long-run outcome in the second-best regime with a non-paternalistic government and we begin by posing the question if the long-run outcome could feature an

\textsuperscript{34}The assumption in Proposition 5 that $\partial T_1(\lambda, n_1)/\partial \lambda < 0$ reflects that the larger $\lambda$ is (i.e. the higher the valuation of $n_1$ is), the stronger is the government’s incentive to set $T_1$ marginally lower than otherwise because this has a positive effect on the speed at which $n_1$ increases over time (follows from the second comparative static result defined in 14). Analogously, the assumption that $\partial T_2(\lambda, n_1)/\partial \lambda > 0$ reflects that the larger $\lambda$ is the stronger is the government’s incentive to set $T_2$ marginally higher than otherwise because this has a positive effect on the speed at which $n_1$ increases over time (follows from the third comparative static result defined in 14).
interior steady-state where \( n_1 = \bar{n}_2 \)? The answer is no because any nonzero value of \( n_1 = \bar{n}_2 \) does not satisfy the necessary conditions in steady-state.\(^{35}\) If, on the other hand, \( n_1 = \bar{n}_2 = 0 \), then the second-order conditions which will not be satisfied.\(^{36}\) This means that the optimal policy in the non-paternalistic regime will either feature a time path where \( n_1 \) increases over time towards the upper corner solution (where \( h_1(n_1) \) approaches the upper level \( h_1^{max} > 0 \)), or the optimal policy will feature a time path where \( n_1 \) decreases over time towards the lower corner solution (where \( h_1(n_1) \) approaches the lower level \( h_1^{min} < 0 \)). It can be shown that in each of these two corner solutions, the shadow price approaches zero\(^{37}\) and the optimal policies implemented in these two corner solutions are summarized in Proposition 6:

**Proposition 6:** Assume that the optimal policy in the non-paternalistic regime features a time-path towards the upper steady-state where \( h_1 = h_1^{max} \geq h_2(\bar{n}_2) \). In the upper steady-state the uniform commodity tax is given by

\[
\tau^{NP} = \alpha_2 \frac{N_1 h_1^{max} + N_2 \bar{n}_2}{\gamma N_2 (dp/dy_2)}
\]  

(27)

If \( N_1 h_1^{min} + N_2 \bar{n}_2 > 0 \), \( \partial^2 u_i / \partial c_i \partial y_i \geq 0 \) and if the price function \( p(y) \) is linear, then there will be over-redistribution of income in the upper steady-state in the sense that \( c_1 \geq c_2 \), \( y_1 > y_2 \), \( \Delta_1 > 0 \), \( \Delta_2 \) and \( u_1 > u_2 \).

Assume instead that the optimal policy in the non-paternalistic regime features a time-path towards the lower steady-state where \( h_1 = h_1^{min} \leq h_2(\bar{n}_2) \). In this steady-state the uniform commodity tax is given by

\[
\tau^{NP} = \alpha_2 \frac{N_1 h_1^{min} + N_2 \bar{n}_2}{\gamma N_2 (dp/dy_2)}
\]  

(28)

\(^{35}\) If there would exist an interior steady-state where \( n_1 = \bar{n}_2 \neq 0 \), then equation (18d) implies that \( \partial W^{NP} / \partial n_1 \) would have to be zero because \( \lambda^{NP} = \lambda^{NP} = 0 \) in that steady-state. Since \( y_1 \) has to be equal to \( y_2 \) in an interior steady-state, the second term on the RHS of equation (11c) (which defines \( \partial W^{NP} / \partial n_1 \)) will be zero but since \( \tau^p_2 \neq 0 \) whenever \( n_1 = \bar{n}_2 \neq 0 \), it follows that \( \partial W^{NP} / \partial n_1 \) cannot be zero as long as \( n_1 = \bar{n}_2 \neq 0 \).

\(^{36}\) If there would exist a steady-state where \( n_1 = \bar{n}_2 = 0 \), then it follows from Corollary 3 that \( W^{NP} \) has a minimum w.r.t. \( n_1 \) at that point (i.e. \( W^{NP} \) is convex w.r.t. \( n_1 \) at that point), which violates the second-order condition that the Hamiltonian should be concave at the optimum.

\(^{37}\) As \( h_1 \to h_1^{max} \), it follows that \( dh_1 / dn_1 \to 0 \) and that \( dy_1 / dn_1 \to 0 \). The latter results imply that \( \partial W^{NP} / \partial n_1 \to 0 \) (follows from equation 11c). We also have that \( \text{lim}_{n_1 \to \infty} \partial^2 h_1 / \partial n_1^2 \to 0 \). Taken together, these results imply that the RHS of (18d) approaches zero as \( h_1 \to h_1^{max} \). This means that the only value of \( \lambda^{NP} \) which satisfies equation (18d) in the steady-state (where \( \lambda^{NP} = 0 \)) is \( \lambda^{NP} = 0 \). An analogous argument can be applied when \( h_1 \to h_1^{min} < 0 \).
If $N_1 h_1^\text{min} + N_2 \tilde{h}_2 < 0$, $\partial^2 u_i / \partial c_i \partial y_i \geq 0$ and if the price function $p(y)$ is linear, then there will be under-redistribution of income in the lower steady-state in the sense that $c_1 < c_2$, $y_1 < y_2$, $\Delta_1 < 0$, $\Delta_2$ and $u_1 < u_2$.

A general conclusion from Propositions 5 and 6 is that the long-run outcomes will differ between the two policy regimes; in the paternalistic regime the long-run outcome features an interior steady-state whereas the long-run outcome in the non-paternalistic regime features one of the two corner steady-states.

### 5.3 Optimal Policy along the Saddle-Path

In this section, we characterize the policy implemented by the government in policy regime $j$ along an optimal time path towards steady-state. Let us introduce the following short notations

$$\kappa_1 = \frac{k_1}{\gamma N_1} \frac{dh_1/\partial n_1}{dp/\partial y_1} > 0,$$

$$\kappa_2 = \frac{k_1}{\gamma N_2} \frac{dh_1/\partial n_1}{dp/\partial y_2} > 0$$

and then consider the following result;

**Proposition 7:** Along the saddle-path towards a steady-state, the uniform commodity tax on the prestige good implemented in policy regime $j = N, NP$ is given by

$$\tau^j = \alpha_1 (\tau^j - \lambda^j \kappa_1) + \alpha_2 (\tau^j + \lambda^j \kappa_2)$$

(29)

Note that if $\lambda^j = 0$ then equation (29) reduces to $\tau^j = \alpha_1 \tau^j_1 + \alpha_2 \tau^j_2$ which corresponds to the second-best tax rule in the static framework that was defined in part (a) in Proposition 3. The additional terms that appear when $\lambda^j \neq 0$ arise as a consequence of the government’s incentive to influence the time path of $n_1$. To see this incentive more clearly, let us use that equation (29) can be rewritten to read

$$\tau^j = \alpha_1 \tau^j_1 + \alpha_2 \tau^j_2 + \lambda^j \kappa \left( \frac{\partial q_1}{\partial \tau} - \frac{\partial q_2}{\partial \tau} \right) \frac{dh_1}{\partial n_1}$$

(29’)

29
where\(^3\) \(\kappa = -k_1/\gamma \rho > 0\). The third term on the RHS of equation (29) reflects that the commodity tax can be used by the government to influence the speed at which \(n_1\) changes over time towards a steady-state. Recall first that the compensated price effects that appear inside the brackets in equation (29) are negative. Then, if the low-income agent is more price sensitive than the high-income agent in the sense that \(\partial \tilde{y}_1/\partial \tau < \partial \tilde{y}_2/\partial \tau\), it follows that an increase in \(\tau\) has a negative effect on \(\tilde{n}_1 = k_1(y_1 - y_2)dh_1/dn_1\). This implies that if the government has a negative valuation of \(n_1 (\lambda^l < 0)\) and therefore wants to reduce the low-income agents´ norm belief, then the second term on the RHS of equation (29) is positive. In this situation the paternalistic government implements a tax on the prestige good which is higher than the second-best tax in the static framework. When \(\lambda^l < 0\), this tax policy is welfare improving because \(n_1\) (after the increase in \(\tau\)) now erodes away at a faster rate than before. On the other hand, if \(\partial \tilde{y}_1/\partial \tau > \partial \tilde{y}_2/\partial \tau\) then the argument for a tax which is set below the tax second-best rule in the static framework is analogous as long as \(\lambda^l < 0\). Analogous arguments can be used to interpret the outcome when \(\lambda^l > 0\). We can summarize these results as follows;

**Corollary 4:** In policy regime \(j = P, NP\) the commodity tax policy has the following properties;

- If \(\partial \tilde{y}_1/\partial \tau < \partial \tilde{y}_2/\partial \tau\) and \(\lambda^l < 0 (\lambda^l > 0)\), then the government will, at each point in time before the steady-state is reached, implement a higher (lower) commodity tax on the prestige good compared with the second-best tax in the static framework.

- If \(\partial \tilde{y}_1/\partial \tau > \partial \tilde{y}_2/\partial \tau\) and \(\lambda^l < 0 (\lambda^l > 0)\), then the government will, at each point in time before the steady-state is reached, implement a lower (higher) commodity tax on the prestige good compared with the second-best tax in the static framework.

These results imply that the tax policy along a time path towards a steady-state may differ considerably between the two policy regimes. Consider, for example, the case when \(n_1 > \bar{n}_2 > 0\) holds initially. In this situation, a paternalistic government has an incentive to implement a tax policy which reduces \(n_1\) over time (i.e. \(\lambda^P < 0\)) towards the steady-state where \(n_1 = \bar{n}_2\). In the same situation a non-paternalistic government is instead likely to have an incentive to implement a policy which supports the growth of \(n_1\) over time (i.e. \(\lambda^{NP} > 0\)). From Corollary 4, we see that

---

38 The term \(\rho < 0\) was defined in Section 3.2.
these different policy incentives induce the paternalistic and the non-paternalistic government to implement diametrically opposite commodity tax policies (in comparison with the commodity taxes implemented in a static framework).

Let us also look at income redistribution along an optimal time path;

**Proposition 8:** If the economy is on a time path where $\lambda^l > 0$, then the intention to increase the low-income agents’ belief in the consumption norm will, at each point in time before the steady-state is reached, provide the government with an incentive to redistribute more income from the high-income agents to the low-income agents than otherwise.

On the other hand, if the economy is on a time path where $\lambda^l < 0$, then the intention to erode away the low-income agents’ belief in the consumption norm will, at each point in time before the steady-state is reached, provide the government with an incentive to redistribute less income from the high-income agents to the low-income agents than otherwise.

The intuition for the first part of Proposition 8 is straightforward if we again look at the differential equation which governs the time path of the low-income agents’ norm belief;

$$\dot{n}_1 = k_1(y_1 - y_2)dh_1/dn_1.$$ By redistributing income from the high-income agents towards the low-income agents, $y_1$ increases and $y_2$ is reduced as long as the prestige good is a normal good. All else equal, this redistributive policy has a positive effect on $\dot{n}_1$ and if $\lambda^l > 0$, the government has an incentive to support this process by setting $T_2$ higher, and $T_1$ lower, than otherwise.\(^{39}\) The intuition for the second part of Proposition 8 is analogous.

**5.4 Optimal Taxation with Two Taxable Commodities**

So far, the model only includes one taxable commodity; the status good. From earlier research\(^{40}\) it is, however, well known that also commodity taxes on other non-status goods will be non-zero in a second-best optimum with consumption externalities. This suggests that the incentive to

\(^{39}\) Formally, the result in Proposition 8 follows from the necessary conditions associated with $T_1$ and $T_2$. In equation (18a), the final term on the RHS is negative as long as $\lambda^l > 0$ and $\partial(y_1)/\partial T_1 < 0$. This means that the government, all else equal, has an incentive to set $T_1$ lower than otherwise. Analogously, in equation (18b), the final term on the RHS is positive as long as $\lambda^l > 0$ and $\partial y_2/\partial T_2 < 0$. All else equal, this provides the government with an incentive to set $T_2$ higher than otherwise. Taken together, these two arguments lead to the conclusion in the first part in Proposition 8. The second part in Proposition 8 is analogous.

\(^{40}\) See e.g. Eckerstorfer and Wendner (2013).
influence the time path of \( n_1 \) will also have implications for how other non-status goods will be taxed in a second-best setting. Let us, therefore, extend the model to include an additional non-status good which can be taxed by the government (recall that the numeraire good \( c \) is not taxed) and analyze how the incentive to erode away the low-income agents’ norm belief will affect the taxation of this good in a second-best setting. To conduct this analysis, we extend the model and assume that the agents also have preferences for an additional good \( x \) so that the utility function defined in equation (2) is modified to read

\[
U_i = u(c_i, y_i, x_i) + \phi(G) + h_i(n_i)\Delta_i
\]

(30)

where \( x_i \) is agent type \( i \)’s consumption of the additional good. The utility function is increasing and concave in good \( x \) and it is assumed that this good is not associated with status. The private budget constraint is now given by \( w_i^n = c_i + q_x x_i + p(y)(1 + \tau_y) \), where \( q_x = p_x(1 + \tau_x) \) is the consumer price of good \( x \), \( p_x \) is the fixed producer price of good \( x \), and where \( \tau_x \) and \( \tau_y \) are the commodity taxes on good \( x \) and on good \( y \), respectively. The solution to the private agents’ optimization problems implicitly define \( x_i \) and \( y_i \) as functions of \( w_i^n, \tau_x, \tau_y \) and \( n_i \). Good \( x \) is assumed to be imported from abroad and the government’s budget constraint is now given by

\[
G = \sum_{i=1}^{2} N_i \left[ T_i + \tau_x x_i + \tau_y p(y_i) \right].
\]

The model is unchanged in all other aspects. Let us introduce the following short notations;

\[
\bar{\alpha}_1 = \frac{A_1}{\bar{\rho}}, \quad \bar{\alpha}_2 = \frac{A_2}{\bar{\rho}}, \quad \bar{\rho} = A_1 + A_2 > 0
\]

(31a)

\[
A_1 = N_1 N_1 p_x \frac{dp}{dy_1} \left( \frac{\partial \xi_1}{\partial \tau_x} \frac{\partial \eta_1}{\partial \tau_y} - \frac{\partial \xi_1}{\partial \tau_x} \frac{\partial \eta_1}{\partial \tau_y} \right) + N_1 N_2 p_x \frac{dp}{dy_1} \left( \frac{\partial \xi_2}{\partial \tau_x} \frac{\partial \eta_1}{\partial \tau_y} - \frac{\partial \xi_2}{\partial \tau_x} \frac{\partial \eta_1}{\partial \tau_y} \right) > 0
\]

(31b)

\[
A_2 = N_1 N_2 p_x \frac{dp}{dy_2} \left( \frac{\partial \xi_3}{\partial \tau_x} \frac{\partial \eta_2}{\partial \tau_y} - \frac{\partial \xi_3}{\partial \tau_x} \frac{\partial \eta_2}{\partial \tau_y} \right) + N_2 N_2 p_x \frac{dp}{dy_2} \left( \frac{\partial \xi_4}{\partial \tau_x} \frac{\partial \eta_2}{\partial \tau_y} - \frac{\partial \xi_4}{\partial \tau_x} \frac{\partial \eta_2}{\partial \tau_y} \right) > 0
\]

(31c)

\[
\varepsilon_1 = -\frac{N_1 N_2}{\bar{\rho}} \frac{dp}{dy_1} \frac{dp}{dy_2} \frac{\partial \eta_1}{\partial \tau_y} > 0, \quad \varepsilon_2 = -\frac{N_1 N_2}{\bar{\rho}} \frac{dp}{dy_1} \frac{dp}{dy_2} \frac{\partial \eta_1}{\partial \tau_y} > 0
\]

(31d)

\[
\eta_1 = -\frac{k_1}{\gamma \bar{\rho} N_1} \left( N_2 \frac{dp}{dy_2} + N_1 \frac{dp}{dy_1} \right) \frac{\partial \eta_2}{\partial \tau_y} > 0, \quad \eta_2 = -\frac{k_1}{\gamma \bar{\rho} N_2} \left( N_1 \frac{dp}{dy_1} + N_2 \frac{dp}{dy_2} \right) \frac{\partial \eta_2}{\partial \tau_y} > 0
\]

(31e)

\[
\frac{\partial \xi_1}{\partial \tau_x} = \frac{\partial x_i}{\partial \tau_x} - p_x x_i \frac{\partial x_i}{\partial T_i} < 0, \quad \frac{\partial \xi_1}{\partial \tau_y} = \frac{\partial x_i}{\partial \tau_y} - p_i \frac{\partial x_i}{\partial T_i} < 0
\]

(31f)
\[
\frac{\partial \tilde{y}_i}{\partial \tau_x} = \frac{\partial y_i}{\partial \tau_x} - p_x x_i \frac{\partial y_i}{\partial \tau_i} < 0, \quad \frac{\partial \tilde{y}_i}{\partial \tau_y} = \frac{\partial y_i}{\partial \tau_y} - p_i \frac{\partial y_i}{\partial \tau_i} < 0
\]

where \(0 < \tilde{a}_i < 1\) for \(i = 1, 2\). The signs in (31a) - (31g) follow from comparative static results derived in the special case when \(u(c_i, y_i, x_i)\) is additively separable in its components. Recall also that the terms \(\kappa_1 > 0\) and \(\kappa_2 > 0\) were defined in Section 5.3. Finally, let us denote the first-best commodity tax rules for good \(y\) in policy regime \(j\) in this three-good model context by \(\tau_{1,y}^j\) and \(\tau_{2,y}^j\).\(^{41}\) Now, consider the following results;

**Proposition 9:** When the government is able to tax two commodities, a status good \(y\) and a non-status good \(x\), then the second-best commodity taxes \(\tau_{1,y}^j\) and \(\tau_{x}^j\) along the saddle-path towards a steady-state in policy regime \(j = P, NP\) are given by

\[
\tau_{1,y}^j = \tilde{a}_1 (\tau_{1,y}^j - \lambda^j \kappa_1) + \tilde{a}_2 (\tau_{2,y}^j + \lambda^j \kappa_2)
\]

\[
\tau_{x}^j = \varepsilon_1 (\tau_{2,y}^j - \tau_{1,y}^j) \frac{\partial \tilde{y}_1}{\partial \tau_x} + \varepsilon_2 (\tau_{1,y}^j - \tau_{2,y}^j) \frac{\partial \tilde{y}_2}{\partial \tau_x} + \lambda^j \eta_1 \frac{\partial \tilde{y}_1}{\partial \tau_x} - \lambda^j \eta_2 \frac{\partial \tilde{y}_2}{\partial \tau_x}
\]

Observe first that the formula for the commodity tax on the status good, \(\tau_{1,y}^j\), takes the same general form, and can be interpreted in an analogous way, as the corresponding tax formula in the two-good model which was presented in equation (29). As for the commodity tax formula for the non-status good, it is made up of four components. The first two terms on the RHS of equation (33) correspond to the optimal second-best tax rule that would apply in a static framework. To interpret these two terms, note that in a static framework (or in a steady-state where \(\lambda^j = 0\)), the commodity tax for good \(y\) reduces to \(\tau_{1,y}^j = \tilde{a}_1 \tau_{1,y}^j + \tilde{a}_2 \tau_{2,y}^j\). Then, since the second-best commodity tax \(\tau_{1,y}^j\) is a weighted average of the type-specific first-best tax rates, it follows that (i) if \(\tau_{1,y}^j < \tau_{2,y}^j\) then \(\tau_{1,y}^j < \tau_{1,y}^j\) and (ii) if \(\tau_{1,y}^j > \tau_{2,y}^j\) then \(\tau_{1,y}^j > \tau_{2,y}^j\). In case (i), the low-income agents face a second-best tax on the prestige good which exceeds the first-best tax rate (\(\tau_{1,y}^j\)). As a consequence, the low-income agents include less quality in good \(y\) than what is desired by the government. This provides the government with an incentive to use other tax instruments (here

\(^{41}\)The first-best tax rules in the three-good model take the same form as the first-best tax rules defined in Proposition 1 in the two-good model (i.e. \(\tau_{1,y}^j\) in the three-good model corresponds to, and is equal to, \(\tau_{1,y}^j\) in the two-good model).
the commodity tax on good \( x \) as tools to indirectly stimulate the low-income agents´ expenditure on good \( y \). Therefore, that if the cross-price effect \( \partial \tilde{y}_1/\partial \tau_x \) is positive, then the government (on the margin) has an incentive to set \( \tau_x \) higher than otherwise. This is captured by the first term on the RHS in equation (33) which is positive if \( \partial \tilde{y}_1/\partial \tau_x > 0 \). On the other hand, if \( \partial \tilde{y}_1/\partial \tau_x \) is negative then the opposite argument applies and if \( \partial \tilde{y}_1/\partial \tau_x = 0 \), then the ability to use \( \tau_x \) to indirectly influence the low-income agent´s expenditure on good \( y \) is no longer present. An analogous argument can be used to interpret case (ii). The second term on the RHS of equation (34) can be interpreted along similar lines.

Let us finally turn to the third and fourth terms on the RHS of equation (33) which arise as a direct consequence of the government´s desire to influence the time path of the low-income agents´ norm belief. As in Section 5.3, the key to interpreting these types of terms comes from looking at how \( \tau_x \) affects \( \dot{n}_1 = k_1(y_1 - y_2)d h_1/d n_1 \). If \( \lambda^j > 0 \), then the government wants to increase the speed at which \( n_1 \) grows over time and this is (on the margin) achieved by increasing \( y_1 \). If the low-income agent’s cross-price effect \( \partial \tilde{y}_1/\partial \tau_x \) is positive (negative), then this policy incentive induces the government to set \( \tau_x^j \) higher (lower) than otherwise. This is captured by the third term on the RHS of equation (33) which is positive (negative) when \( \lambda^j > 0 \) and when \( \tilde{y}_1/\partial \tau_x > 0 \) (\( \partial \tilde{y}_1/\partial \tau_x < 0 \)). This tax policy is welfare improving because \( n_1 \) now grows at a faster rate than otherwise. Observe that the government can also increase the speed at which \( n_1 \) grows over time by reducing \( y_2 \). If the high-income agent’s cross-price effect \( \partial \tilde{y}_2/\partial \tau_x \) is positive (negative), then this policy incentive induces the government to set \( \tau_x^j \) lower (higher) than otherwise. This is reflected by the fourth term on the RHS of equation (33) which is negative (positive) in this situation. If, instead, \( \lambda^j < 0 \) the arguments presented above go in the opposite direction because then it is welfare improving to increase the speed at which \( n_1 \) erodes away over time.

6. Concluding Discussion

This article concerns commodity taxation and income redistribution in the presence of consumption norms when an agent´s personal belief in a given consumption norm is endogenous. The analysis is motivated by Cognitive Dissonance Theory which argues that agents, over time, may increase (decrease) their personal belief in a given norm if they live up to (violate) the norm.
A government which recognizes this mechanism may use its policy instruments to erode away “bad” norms and to reinforce “good” norms, and the main question addressed in this paper is how these incentives will influence government policy. Two government objectives were considered; a paternalistic and a non-paternalistic objective function. A paternalistic government does not incorporate the individual agents’ preferences for status into the welfare function whereas a non-paternalistic government fully respects the agents’ preferences for status. In this model there are of two types of agents, low- and high income agents and they differ in terms of their productivity on the labor market.

We began by analyzing what type of policy the government in each policy regime would implement in a static first-best setting where, in addition to lump-sum taxes, agent type-specific commodity taxes are also available. We found that if the agents believe in the status norm, then both the paternalistic and the non-paternalistic government implement positive type-specific commodity taxes on the prestige good but these taxes differ between the two policy regimes. In the paternalistic regime, the government perceives that status-seeking leads to overspending on the prestige good, and this overspending is inhibited by implementing a positive type-specific commodity tax on both agent types. For the non-paternalistic government the motive is instead to internalize the consumption externality that arises from status-seeking. This means that the agents who constitute the point of reference in the consumption norm (the high-income agents) will face a positive commodity tax whereas the low-income agents will face a zero commodity tax. The different treatments of the two agent types in the two policy regimes have implications for redistribution of income. The paternalistic government implements a policy where the after-tax income is equalized between the two agent types but this is not the case for the non-paternalistic government. Instead, the non-paternalistic government will redistribute more resources from the high-income to the low-income agents the stronger the preference for status is among the low-income agents. However, a stronger preference for status among the high-income agents will not (in the non-paternalistic regime) lead to any redistribution of income. Hence, there is an asymmetric treatment of the two agent types in the non-paternalistic first-best regime.

Since type-specific commodity taxes are not commonly observed empirically, the main analysis was conducted in a setting where the government is restricted to use a uniform commodity tax on the status good. In this second-best setting, it was shown that the commodity tax on the prestige good can be viewed as a weighted average of the first-best tax rules associated with each agent.
type. It was also shown that if the agents believe in the consumption norm, then there is a tendency for over-redistribution in both policy regimes. Finally, we also looked at how the second-best welfare would be affected by exogenous changes in the personal norm beliefs. Here, one conclusion is that the welfare in the paternalistic regime is maximized if the norm beliefs of the two agent types are equalized. In the non-paternalistic regime the welfare may, instead, be an increasing function of the low-income agents´ norm belief if the initial norm beliefs are positive. Taken together, these results suggest that if the private norm beliefs would be endogenous, then a government may indeed (in both policy regimes) have an incentive to influence these beliefs.

In the second part of the paper, we allowed the personal norm beliefs to change according to the principles of Cognitive Dissonance Theory. In that context, the norm beliefs of the high-income agents are constant over time (a result of assuming that all high-income agents are identical and compare themselves with the other high-income agents) whereas the evolution of the low-income agents´ norm belief is a function of the fiscal policy instruments. In particular it was shown that income redistribution towards the low-income agents tends to reinforce their belief in the status norm. This mechanism can be used by a government to influence the low-income agents´ preferences for status. Another key result was that the long-run outcome (steady-state) in the paternalistic regime is likely to differ from the long-run outcome in a non-paternalistic regime. In the former regime, the steady-state will be a first-best outcome where the two agent types have equal norm beliefs whereas the government in the non-paternalistic regime (depending on the initial conditions) either implements a policy which leads to maximum norm belief, or implements a policy which leads to minimum norm belief for the low-income agents.

If the time path towards steady-state is associated with eroding away the low-income agents´ belief in the status norm, then it was shown that this involves a higher (lower) commodity tax on the status good if the low-income agents´ compensated demand function for the status good is less (more) sensitive to a price change than that of the high-income agents. Along such a time path it was also shown that the intention to erode away the low-income agents´ belief in the status norm will, all else equal, provide the government with an incentive to redistribute less income from the high-income to the low-income agents. If, instead, the time path towards steady-state is associated with an increase in the low-income agents´ norm belief then the opposite policy applies. The latter policy features a lower (higher) commodity tax on the status good if the low-income agents´ compensated demand function for the status good is more (less) sensitive to a...
price change than that of the high-income agents. Along this time path the intention to increase the low-income agents’ belief in the status norm will, all else equal, provide the government with an incentive to redistribute more income from the high-income to the low-income agents. Finally, we also looked at how the taxation of another non-status good would be affected by the incentive to influence norm beliefs along a time path towards steady-state. Here we provided tax rules which relate the commodity tax on the non-status good both to the difference of the first-best tax rules for the status good and to whether the agents’ consumption of the status good is complimentary or substitutable with the non-status good.

Finally, although this article is a first attempt to address the implications of endogenous norm beliefs in a dynamic context, there are many important aspects left to explore. Examples include incorporating saving and capital formation into the model, allowing for asymmetric information regarding ability and relaxing the assumption that all agents of a given type are identical.

Appendix

First-Best Policy in the Non-Paternalistic Regime
Substitute the private and the public budget constraints into $W^{NP}$. If we use that $\hat{y}_2 = y_2$, the first-order conditions become

\[
\begin{align*}
\frac{\partial W^{NP}}{\partial \tau_1} &= (N_1 + N_2) \frac{\partial \phi}{\partial \tau_1} N_1 \left( 1 + \tau_1 \frac{dp}{dy_1 \partial \tau_1} \right) - N_1 \frac{\partial u_1}{\partial c_1} = 0 \\
\frac{\partial W^{NP}}{\partial \tau_2} &= (N_1 + N_2) \frac{\partial \phi}{\partial \tau_2} N_2 \left( p_1 + \tau_1 \frac{dp}{dy_1 \partial \tau_1} \right) - N_1 p_1 \frac{\partial u_1}{\partial c_1} = 0 \\
\frac{\partial W^{NP}}{\partial \tau_2} &= (N_1 + N_2) \frac{\partial \phi}{\partial \tau_2} N_2 \left( p_2 + \tau_2 \frac{dp}{dy_2 \partial \tau_2} \right) - N_2 p_2 \frac{\partial u_2}{\partial c_2} - (N_1 h_1 + N_2 h_2) \frac{\partial y_2}{\partial \tau_2} = 0
\end{align*}
\]

(A.1) (A.2) (A.3)

Multiply $\partial W^P / \partial \tau_i = 0$ by $p_i$ and subtract the resulting expression from $\partial W^{NP} / \partial \tau_i = 0$. This produces

\[
\begin{align*}
\tau_1 (N_1 + N_2) \frac{\partial \phi}{\partial \tau_1} N_1 \left( \frac{\partial y_1}{\partial \tau_1} - p_1 \frac{\partial y_1}{\partial \tau_1} \right) \frac{dp}{dy_1} &= 0 \\
\tau_2 (N_1 + N_2) \frac{\partial \phi}{\partial \tau_2} N_2 \left( \frac{\partial y_2}{\partial \tau_2} - p_2 \frac{\partial y_2}{\partial \tau_2} \right) \frac{dp}{dy_2} - (N_1 h_1 + N_2 h_2) \left( \frac{\partial y_2}{\partial \tau_2} - p_2 \frac{\partial y_2}{\partial \tau_2} \right) &= 0
\end{align*}
\]

(A.5) (A.6)

Solving for $\tau_1$ and $\tau_2$ produces the tax formulas for $\tau_1^P$ and $\tau_2^P$ in (7a). By substituting $\tau_1^P = 0$ into (A.1), it follows that $\gamma = \partial u_1 / \partial c_1$. Similarly, substituting the tax formula for $\tau_2^P$ into (A.3) produces $\gamma = \partial u_2 / \partial c_2$. These results imply $\partial u_1 / \partial c_1 = \partial u_2 / \partial c_2$. To derive the equations in (7b) we substitute the type specific commodity taxes defined in (7b) into the private first-order condition for $y_i$. This gives
\[
\frac{\partial u_1/\partial y_1}{dp/dy_1} + \frac{h_1}{dp/dy_1} = \frac{\partial u_1}{\partial c_1}
\]
(A.7)
\[
\frac{\partial u_2/\partial y_2}{dp/dy_2} + \frac{h_2}{dp/dy_2} = \frac{\partial u_2}{\partial c_2} + \frac{N_1 h_1 + N_2 h_2}{N_2(dp/dy_2)}
\]
(A.8)

These equations imply the equalities in (7b). To verify the claims made at the end of Proposition 1, we combine (A.7) and (A.8), while we use that \( \gamma = \partial u_1/\partial c_1 = \partial u_2/\partial c_2 \). This produces
\[
\frac{\partial u_1/\partial y_1}{dp/dy_1} - \frac{\partial u_2/\partial y_2}{dp/dy_2} = -\frac{h_1}{dp/dy_1} - \frac{N_1 h_1}{N_2(dp/dy_2)} < 0
\]
(A.9)

If \( h_1 > 0 \), the expression in (A.9) is negative. As long as \( \partial^2 u_i/\partial c_i \partial y_i \geq 0 \), and by using that \( \partial u_1/\partial c_1 = \partial u_2/\partial c_2 \), it follows that \( c_1 \geq c_2, y_1 > y_2 \) and \( \Delta_1 = y_1 - y_2 > 0 \). This produces part (i) in Proposition 1. Using analogous arguments for \( h_1 < 0 \) and \( h_1 = 0 \) produces parts (ii) and (iii) in Proposition 1.

The first-best policy in the paternalistic regime is derived analogously.

**Second-Best Policy with a Paternalistic Government**

The first-order conditions become (after using the private first-order conditions)
\[
\frac{\partial W^P}{\partial T_1} = (N_1 + N_2) \frac{\partial \phi}{\partial c} N_1 \left( 1 + \tau \frac{dp}{dy_1} \frac{\partial y_1}{\partial T_1} \right) - N_1 \frac{\partial u_1}{\partial c_1} - N_1 \frac{h_1}{\partial T_1} = 0
\]
(A.10)
\[
\frac{\partial W^P}{\partial T_2} = (N_1 + N_2) \frac{\partial \phi}{\partial c} N_2 \left( 1 + \tau \frac{dp}{dy_2} \frac{\partial y_2}{\partial T_2} \right) - N_2 \frac{\partial u_2}{\partial c_2} - N_2 \frac{h_2}{\partial T_2} = 0
\]
(A.11)
\[
\frac{\partial W^P}{\partial \tau} = (N_1 + N_2) \frac{\partial \phi}{\partial c} N_1 \left( p_1 + \tau \frac{dp}{dy_1} \frac{\partial y_1}{\partial \tau} \right) - N_1 p_1 \frac{\partial u_1}{\partial c_1} - N_1 \frac{h_1}{\partial \tau} + (N_1 + N_2) \frac{\partial \phi}{\partial c} N_2 \left( p_2 + \tau \frac{dp}{dy_2} \frac{\partial y_2}{\partial \tau} \right) - N_2 p_2 \frac{\partial u_2}{\partial c_2} - N_2 \frac{h_2}{\partial \tau} = 0
\]
(A.12)

Multiply \( \partial W^P/\partial T_1 = 0 \) by \( p_1 \) and multiply \( \partial W^P/\partial T_2 = 0 \) by \( p_2 \), then subtract the resulting expressions from \( \partial W^P/\partial \tau = 0 \). Solving for \( \tau \) in the resulting equation, and using the definitions of the first-best taxes in policy regime \( j = P \) produces the tax formula for \( j = P \) in Proposition 3. As for redistribution, substitute the definition of the first-best tax rule into (A.10) and (A.11), and then rewrite \( \partial W^P/\partial T_1 \) and \( \partial W^P/\partial T_2 \) read
\[
\frac{\partial W^P}{\partial T_1} \frac{1}{\gamma N_1} = 1 + (\tau^P - \tau_1^P) \frac{dp}{dy_1} \frac{\partial y_1}{\partial T_1} - \frac{\partial u_1/\partial c_1}{\gamma} = 0
\]
(A.13)
\[
\frac{\partial W^P}{\partial T_2} \frac{1}{\gamma N_2} = 1 + (\tau^P - \tau_2^P) \frac{dp}{dy_2} \frac{\partial y_2}{\partial T_2} - \frac{\partial u_2/\partial c_2}{\gamma} = 0
\]
(A.14)

Let us now pose the following question: If \( T_1, T_2 \) and \( \tau \) have been chosen optimally, could the optimal solution feature a redistribution policy where \( \partial u_1/\partial c_1 = \partial u_2/\partial c_2 \)? Subtracting (A.14) from (A.13) produces
\[
\frac{\partial W^P}{\partial T_1} \frac{1}{\gamma N_1} - \frac{\partial W^P}{\partial T_2} \frac{1}{\gamma N_2} = (\tau^P - \tau_1^P) \frac{dp}{dy_1} \frac{\partial y_1}{\partial T_1} - (\tau^P - \tau_2^P) \frac{dp}{dy_2} \frac{\partial y_2}{\partial T_2} + \frac{1}{\gamma} \left( \frac{\partial u_2}{\partial c_2} - \frac{\partial u_1}{\partial c_1} \right) = 0
\]
(A.15)

Observe first that if \( n_1 = n_2 \) and if there is complete income redistribution, then \( \tau_1^P = \tau_2^P \) so that \( \tau^P = \tau_1^P = \tau_2^P \). This implies that the first two terms on the RHS of (A.15) are zero which, in turn, means that \( \partial u_1/\partial c_1 = \partial u_2/\partial c_2 \) and there is complete income redistribution. Hence, when
\( n_1 = n_2 \), then the second-best policy in the paternalistic regime replicates the first-best policy in the paternalistic regime in which case the level of welfare is \( W^* \).

Assume instead that \( n_1 \neq n_2 \) and consider the special case where the cross-derivative in \( u(c, y) \) is zero and where the price function \( p(y) \) is linear. Let us now ask the following question. If \( \tau^p \) is chosen according to the second-best rule, could then an optimal policy feature \( c_1 = c_2 \) and \( y_1 = y_2 \)? In this special case \( \partial y_1 / \partial T_1 = \partial y_2 / \partial T_2 \) and \( \partial u_1 / \partial c_1 = \partial u_2 / \partial c_2 \), in which case (A.15) reduces to

\[
\frac{\partial w^p}{\partial T_1} \frac{1}{yN_1} - \frac{\partial w^p}{\partial T_2} \frac{1}{yN_2} = (\tau^p_2 - \tau^p_1) \frac{dp}{dy} \frac{\partial y_1}{\partial T_1} \tag{A.16}
\]

Beginning with the case where \( n_1 < n_2 \), it follows that \( h_1(n_1) < h_2(n_2) \) and that \( \tau^p_1 < \tau^p_2 \) when \( c_1 = c_2 \) and \( y_1 = y_2 \). In this case, the RHS of (A.16) is negative which indicates that either \( \partial W^p / \partial T_1 < 0 \) and/or \( \partial W^p / \partial T_2 > 0 \). The conclusion is that if \( c_1 = c_2 \) and \( y_1 = y_2 \), then either \( T_1 \) is set too high and/or \( T_2 \) is set too low when \( n_1 < n_2 \). Hence, the optimal choices of \( T_1 \) and \( T_2 \) must imply that \( c_1 > c_2 \) and \( y_1 > y_2 \) when \( n_1 < n_2 \). When \( n_1 > n_2 \), the opposite arguments apply in which case \( c_1 < c_2 \) and \( y_1 > y_2 \).

**Second-Best Policy with a Non-Paternalistic Government**

The first-order conditions become (after using the private first-order conditions)

\[
\frac{\partial w^{NP}}{\partial T_1} = (N_1 + N_2) \frac{\partial \phi}{\partial y} N_1 \left( 1 + \tau \frac{dp}{dy} \frac{\partial y_1}{\partial T_1} \right) - N_1 \frac{\partial u_1}{\partial c_1} = 0 \tag{A.17}
\]

\[
\frac{\partial w^{NP}}{\partial T_2} = (N_1 + N_2) \frac{\partial \phi}{\partial y} N_2 \left( 1 + \tau \frac{dp}{dy} \frac{\partial y_2}{\partial T_2} \right) - N_2 \frac{\partial u_2}{\partial c_2} - (N_1 h_1 + N_2 h_2) \frac{\partial y_2}{\partial T_2} = 0 \tag{A.18}
\]

\[
\frac{\partial w^{NP}}{\partial \tau} = (N_1 + N_2) \frac{\partial \phi}{\partial y} N_1 \left( p_1 + \tau \frac{dp}{dy} \frac{\partial y_1}{\partial \tau} \right) - N_1 p_1 \frac{\partial u_1}{\partial c_1} + (N_1 + N_2) \frac{\partial \phi}{\partial y} N_2 \left( p_2 + \tau \frac{dp}{dy} \frac{\partial y_2}{\partial \tau} \right) - N_2 p_2 \frac{\partial u_2}{\partial c_2} - (N_1 h_1 + N_2 h_2) \frac{\partial y_2}{\partial \tau} = 0 \tag{A.19}
\]

Multiply \( \partial W^{NP} / \partial T_1 = 0 \) by \( p_1 \) and multiply \( \partial W^{NP} / \partial T_2 = 0 \) by \( p_2 \), then subtract the resulting expressions from \( \partial W^{NP} / \partial \tau = 0 \). Solving for \( \tau \) in the resulting equation, and using the definitions of the first-best taxes in policy regime \( j = NP \) produces the tax formula for \( j = NP \) in Proposition 3. As for redistribution, rewrite the (A.17) and (A.18) to read

\[
\frac{\partial u_1 / \partial c_1}{\gamma} = 1 + \tau^{NP} \frac{dp}{dy_1} \frac{\partial y_1}{\partial T_1} \tag{A.20}
\]

\[
\frac{\partial u_2 / \partial c_2}{\gamma} = 1 + (\tau^{NP} - \tau^{NP}_2) \frac{dp}{dy_2} \frac{\partial y_2}{\partial T_2} \tag{A.21}
\]

Take the difference between these two expressions and use that \( \tau^{NP} = \alpha_2 \tau^{NP}_2 \). This produces

\[
\frac{\partial u_1 / \partial c_1}{\gamma} - \frac{\partial u_2 / \partial c_2}{\gamma} = \left[ \alpha_2 \frac{dp}{dy_1} \frac{\partial y_1}{\partial T_1} + (1 - \alpha_2) \frac{dp}{dy_2} \frac{\partial y_2}{\partial T_2} \right] \tau^{NP}_2 \tag{A.22}
\]

Since the sum of the terms inside square brackets on the RHS of (A.22) is negative, it follows that if \( N_1 h_1 + N_2 h_2 > (\ast) \) (which implies \( \tau^{NP}_2 > (\ast) \)) then \( \partial u_1 / \partial c_1 < (\ast) \partial u_2 / \partial c_2 \). Therefore, if \( \partial^2 u_1 / \partial c_1 \partial y_1 = 0 \), it follows that \( c_1 < (\ast) c_2 \). On the other hand, if \( N_1 h_1 + N_2 h_2 = 0 \) then \( \partial u_1 / \partial c_1 = \partial u_2 / \partial c_2 \) and \( c_1 = c_2 \). Next, we substitute the uniform commodity tax into the private first-order condition for \( y_1 \). This gives
\[
\frac{\partial u_1}{\partial y_1} + \frac{h_1}{dp/dy_1} \frac{\partial y_1}{\partial c_1} + \alpha_2 \frac{N_1 h_1 + N_2 h_2}{N_2 (dp/dy_2)} \frac{\partial u_1}{\partial c_1} = \frac{\partial u_1}{\partial c_1} + \alpha_2 \frac{N_1 h_1 + N_2 h_2}{N_2 (dp/dy_2)} \frac{\partial u_1}{\partial c_1} \gamma \] (A.23)

\[
\frac{\partial u_2}{\partial y_2} + \frac{h_2}{dp/dy_2} \frac{\partial y_2}{\partial c_2} + \alpha_2 \frac{N_1 h_1 + N_2 h_2}{N_2 (dp/dy_2)} \frac{\partial u_2}{\partial c_2} = \frac{\partial u_2}{\partial c_2} + \alpha_2 \frac{N_1 h_1 + N_2 h_2}{N_2 (dp/dy_2)} \frac{\partial u_2}{\partial c_2} \gamma \] (A.24)

Taking the difference between these equations produces

\[
\frac{\partial u_1}{\partial y_1} - \frac{\partial u_2}{\partial y_2} = (1 + \alpha_2 \frac{N_1 h_1 + N_2 h_2}{N_2 (dp/dy_2)}) \left( \frac{\partial u_1}{\partial c_1} - \frac{\partial u_2}{\partial c_2} \right) + \frac{h_2}{dp/dy_2} - \frac{h_1}{dp/dy_1} \] (A.25)

This expression is negative (positive) if \( N_1 h_1 + N_2 h_2 > (<) 0 \), \( n_1 > (<) n_2 \) and if the pricing function \( p(y) \) is linear. If \( \partial^2 u_i / \partial c_i \partial y_i = 0 \), then \( y_1 > (<) y_2 \) and \( \Delta_1 = y_1 - y_2 > (<) 0 \).

**Derivation of Equations (11c) and (11d)**

Use that (A.17) – (A.19) implicitly define the tax instruments as functions of \( n_1 \) and \( n_2 \), i.e. \( T_1(n_1, n_2) \), \( T_2(n_1, n_2) \) and \( \tau(n_1, n_2) \) and then substitute these functions into the welfare function. Differentiating the resulting expression w.r.t. \( n_1 \) and \( n_2 \) produces

\[
\frac{\partial W^{NP}}{\partial n_1} = \left[ \frac{\partial u_1}{\partial y_1} - (1 + \tau) \frac{dp}{dy_1} \frac{\partial u_1}{\partial c_1} + h_1 + \tau \gamma \frac{dp}{dy_1} \right] N_1 \frac{\partial y_1}{\partial n_1} + N_1 \frac{dh_1}{dn_1} (y_1 - y_2) + \sum_{i=1,2} \frac{\partial W^{NP}}{\partial T_i} \frac{\partial \tau_i}{\partial n_1} \] (A.26)

\[
\frac{\partial W^{NP}}{\partial n_2} = \left[ \frac{\partial u_2}{\partial y_2} - (1 + \tau) \frac{dp}{dy_2} \frac{\partial u_2}{\partial c_2} + h_2 + \tau \gamma \frac{dp}{dy_2} \right] N_2 \frac{\partial y_2}{\partial n_2} + N_2 \frac{dh_2}{dn_2} (y_2 - y_1) - (N_1 h_1 - N_2 h_2) \frac{\partial y_2}{\partial n_2} + \sum_{i=1,2} \frac{\partial W^{NP}}{\partial T_i} \frac{\partial \tau_i}{\partial n_2} \] (A.27)

Using that (A.17) – (A.19) imply \( \partial W^{NP} / \partial T_i = 0 \) and \( \partial W^{NP} / \partial \tau = 0 \), and using the private first-order conditions for \( y_1 \) and \( y_2 \), to simplify the resulting expressions, gives

\[
\frac{\partial W^{NP}}{\partial n_1} = \tau_1^{NP} \gamma N_1 \frac{dp}{dy_1} \frac{\partial y_1}{\partial n_1} + N_1 \frac{dh_1}{dn_1} (y_1 - y_2) \] (A.28)

\[
\frac{\partial W^{NP}}{\partial n_2} = -(1 - \alpha_2) \tau_2^{NP} \gamma N_2 \frac{dp}{dy_2} \frac{\partial y_2}{\partial n_2} \] (A.29)

where we also have used the definition of \( \tau_2^{NP} \) and that \( \tau^{NP} = \alpha_2 \tau_2^{NP} \). These are equations (11c) and (11d) in the paper. Equation (11a) and (11b) can be derived analogously.

**Proposition 4**

To prove that the steady-state solution features \( \tau^P = \tau_1^P = \tau_2^P \) and complete income redistribution when \( n_1 = n_2 \), let us check whether this solution satisfies equation system (19a) – (19e). To do this, we first observe that since \( \dot{\lambda}^P = 0 \) holds in steady-state, and since the proposed solution implies that \( \partial W^P / \partial n_1 = 0 \), it follows that (19d) reduces to

\[
0 = \lambda^P \left( \theta - k_1 \frac{\partial y_1}{\partial n_1} \frac{dh_1}{dn_1} - k_1 \Delta_1 \frac{d^2 h_1}{dn_1^2} \right) \] (A.30)

Since the expression inside brackets on the RHS of (A.30) can take on any value, equation (A.30) can only be satisfied if \( \lambda^P = 0 \). Using that \( \lambda^P = 0 \) in (19a) – (19c) implies that the necessary conditions become identical to those in the paternalistic static second-best setting where we know
from part (a) in Proposition 3 that the paternalistic first-best outcome is attainable in the special case where \( n_1 = \bar{n}_2 \). Finally, observe that (19e) is automatically satisfied when \( \lambda^p = 0 \).

**Second-Best Policy in a Dynamic Framework with Two Taxable Commodities**

The necessary conditions w.r.t. \((T_1, T_2, \tau_x, \tau_y)\) become

\[
0 = (N_1 + N_2) \frac{\partial \phi}{\partial G} N_1 \left( 1 + \tau_x p_x \frac{\partial x_1}{\partial \tau_x} + \tau_y \frac{\partial y_1}{\partial \tau_y} \right) - N_1 \frac{\partial u_1}{\partial c_1} + \lambda k_1 \frac{\partial y_1}{\partial \tau_1} \frac{dh_1}{dn_1}
\]

\[
0 = (N_1 + N_2) \frac{\partial \phi}{\partial G} N_2 \left( 1 + \tau_x p_x \frac{\partial x_2}{\partial \tau_x} + \tau_y \frac{\partial y_2}{\partial \tau_y} \right) - N_2 \frac{\partial u_2}{\partial c_2} - (N_1 h_1 + N_2 h_2) \frac{\partial y_2}{\partial \tau_2} - \lambda k_1 \frac{\partial y_2}{\partial \tau_2} \frac{dh_1}{dn_1}
\]

Next, multiply \( \partial W^{NP}/\partial T_1 = 0 \) by \( p_x x_1 \) and multiply \( \partial W^{NP}/\partial T_2 = 0 \) by \( p_x x_2 \), then subtract the resulting expressions from \( \partial W^{NP}/\partial \tau_x = 0 \). Similarly, multiply \( \partial W^{NP}/\partial T_1 = 0 \) by \( p_1 \) and multiply \( \partial W^{NP}/\partial T_2 = 0 \) by \( p_2 \), then subtract the resulting expressions from \( \partial W^{NP}/\partial \tau_y = 0 \). This produces

\[
\tau_x \left( N_1 p_x \frac{\partial x_1}{\partial \tau_x} + N_2 p_x \frac{\partial x_2}{\partial \tau_x} \right) + \tau_y \left( N_1 \frac{dp}{dy_1} \frac{\partial y_1}{\partial \tau_x} + N_2 \frac{dp}{dy_2} \frac{\partial y_2}{\partial \tau_x} \right) = \frac{(N_1 h_1 + N_2 h_2)}{\gamma} \frac{\partial y_2}{\partial \tau_x} \frac{dh_1}{dn_1}
\]

\[
\tau_x \left( N_1 p_x \frac{\partial x_1}{\partial \tau_y} + N_2 p_x \frac{\partial x_2}{\partial \tau_y} \right) + \tau_y \left( N_1 \frac{dp}{dy_1} \frac{\partial y_1}{\partial \tau_y} + N_2 \frac{dp}{dy_2} \frac{\partial y_2}{\partial \tau_y} \right) = \frac{(N_1 h_1 + N_2 h_2)}{\gamma} \frac{\partial y_2}{\partial \tau_y} \frac{dh_1}{dn_1}
\]

Write these equations in matrix form

\[
\begin{bmatrix}
\tau_x \\
\tau_y
\end{bmatrix}
= \begin{bmatrix}
\frac{(N_1 h_1 + N_2 h_2)}{\gamma} \frac{\partial y_2}{\partial \tau_x} \\
\frac{(N_1 h_1 + N_2 h_2)}{\gamma} \frac{\partial y_2}{\partial \tau_y}
\end{bmatrix}
\begin{bmatrix}
N_1 \frac{dp}{dy_1} \frac{\partial y_1}{\partial \tau_x} + N_2 \frac{dp}{dy_2} \frac{\partial y_2}{\partial \tau_x} \\
N_1 \frac{dp}{dy_1} \frac{\partial y_1}{\partial \tau_y} + N_2 \frac{dp}{dy_2} \frac{\partial y_2}{\partial \tau_y}
\end{bmatrix}
\]

where

\[
b_1 = \frac{(N_1 h_1 + N_2 h_2)}{\gamma} \frac{\partial y_2}{\partial \tau_x} - \frac{\lambda}{\gamma} k_1 \left( \frac{\partial y_1}{\partial \tau_x} - \frac{\partial y_2}{\partial \tau_x} \right) \frac{dh_1}{dn_1}, \quad b_2 = \frac{(N_1 h_1 + N_2 h_2)}{\gamma} \frac{\partial y_2}{\partial \tau_y} - \frac{\lambda}{\gamma} k_1 \left( \frac{\partial y_1}{\partial \tau_y} - \frac{\partial y_2}{\partial \tau_y} \right) \frac{dh_1}{dn_1}
\]

and where \(|A| = \bar{\rho} \). Solving for the tax rates while using the definitions of the first-best tax rates and the definitions in (32a) – (32g) produces the tax formulas in (33) and (34) associated with the non-paternalistic regime.

An analogous approach can be used to derive the corresponding tax rates in the paternalistic regime.
References


