Optimal Unemployment Insurance
and Redistribution

Robin Boadway
Queen’s University
boadwayr@econ.queensu.ca

Katherine Cuff
McMaster University
cuffk@mcmaster.ca

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Abstract

We characterize optimal income taxation and unemployment insurance in a search-matching framework where both voluntary and involuntary unemployment are endogenous and Nash bargaining determines wages. Individuals differ in their utility from being voluntarily unemployed (non-participants in the labour market) and decide whether to participate as a job seeker and if so, how much job search effort to exert. Unemployment insurance trades off insurance versus moral hazard due to search. We first consider the efficiency-only case where all individuals have the same productivity and show that is optimal to have a positive linear income tax even if search is efficient so the Hosios condition is satisfied. We then allow for different productivity types so there is a redistributive role for the income tax between individuals of differing productive abilities. Our analysis embeds optimal unemployment insurance into an extensive-margin optimal redistribution framework where transfers to the involuntary and voluntary unemployed can differ. Finally, we incorporate a role for self-insurance through a savings decision by extending the static framework to two periods.

Key Words: Optimal Income Taxation, Unemployment Insurance

JEL: H21, H3, J6
1 Introduction

Governments engage in both redistribution and social insurance. They redistribute among employed workers earning different incomes and those who choose not to work, and they provide unemployment insurance to those unable to work. Optimal redistribution and optimal unemployment insurance have typically been analyzed separately, the former in models with workers with heterogeneous productivities and the latter in single-productivity settings. The purpose of this paper is to embed optimal unemployment insurance analysis into an optimal income tax model of redistribution. This entails policy-makers making a distinction between transfers to the voluntary unemployed (non-participants) and the involuntary unemployed.

There is a large literature on optimal redistribution following the original approach of Mirrlees (1971), and it has been extended in several directions. See, for example, the summaries in Banks and Diamond (2010), Boadway (2012), Golosov and Tsyvinski (2015) and Tuomala (2016). Of relevance for us is the extension to allow for involuntary unemployment due to search frictions (Hungerbühler et al, 2006; Lehmann et all, 2011; Jacquet, et al, 2014; Kroft, et al 2015). In these papers, following Diamond (1980) and Saez (2002), individuals can vary their labour supply only along the extensive margin: they decide between searching for work in a job market specific to their productivity or being voluntarily unemployed. Employment in each job market is determined by a static matching function (see Mortensen, 1977; Pissarides, 1990; and the survey in Mortensen and Pissarides, 1999), and wages are the outcome of bargaining between each hired worker and a firm. The government observes wages and chooses wage-specific taxes as well as a uniform transfer to all voluntary and involuntary unemployed. Workers are risk-neutral, which obviates the need for unemployment insurance. The analysis characterizes the pattern of optimal taxes and transfers, and compares them with those in the absence of involuntary unemployment. Redistributive taxes take into account their effect both on participation decisions and on wage-setting.
The literature on optimal unemployment insurance is also long-standing (Topel and Welch, 1980; Karni, 1999; Coles and Masters, 2006) and has recently been revisited by Chetty (2008). It focuses on the trade-off between insurance against involuntary unemployment and moral hazard due to search effort, when there is no market for unemployment insurance and workers may not be able to self-insure because of liquidity constraints. This approach has been extended by Landais et al 2015 to allow for endogenous search unemployment. Unemployment insurance continues to trade off insurance against search incentives, and they show how this trade-off varies over the macroeconomic business cycle. As well, the fact that unemployment insurance can affect the wage rate and that search can be inefficient leads to indirect — or macro — effects that must be taken into consideration. Since all workers are ex ante identical, there is no redistributive policy motive affecting the choice of unemployment insurance.

We analyze optimal redistribution and unemployment insurance jointly in a model that includes the main features of both the above approaches. We adopt an extensive-margin approach to the labour market augmented by search frictions. Individuals endowed with given productivity decide whether to search for a job at their skill level, and if so how intensively to search. There is a perfectly elastic supply of firms offering jobs at each skill level, and free entry subject to a zero-expected profit constraint determined the number of jobs offered. Successful matches are determined by a matching function, and wages are determined by Nash bargaining. Workers are risk-averse so value unemployment insurance. The government observes wages and therefore worker types, and imposes skill-specific taxes and unemployment insurance. The skill of those who choose not to participate is not observable, so all are offered the same transfer. We characterize optimal policies first in a static context and then in a two-period setting. To clarify the relation between unemployment insurance and the tax-transfer system and how they interact when wages are endogenous, we begin with the case where all workers have the same productivity.

The remainder of the paper is organized as follows. In the next section, we outline
the general static model, and consider the special case of all workers having the
same productivity in Section 3. In Section 4, we allow for a discrete distribution
of productivity types. In Section 5, we return to the single productivity case and
consider a simple dynamic setting. Finally, we conclude in Section 6.

2 The Static Model

There is a discrete distribution of skill-types in the economy indexed by $i = 1, ..., N$
where type $i + 1$ has higher skill than type $i$. Following the pure extensive-margin
labour supply model of Diamond (1980) and Saez (2002), there is job suitable for
each skill-type. Denote $y_i$ as the output of a type $i$ skilled worker if employed, where
$y_{i+1} > y_i$. For simplicity, the number of individuals of each type is normalized to
unity.\footnote{Allowing for different population sizes across skill-types would not affect the qualitative results.}
Individuals of a given skill-type only differ in their utility if they choose not
to participate, so all labour market participants (of a given skill-type) are identical.
Among individuals of type $i$, $n_i$ choose to participate and search for a job while
$1 - n_i$ choose to be voluntarily unemployed. The $n_i$ labour market participants or
job-seekers decide how much search effort, denoted by $s_i$, to undertake.

Following Hungerbühler et al. (2006), there is a separate job matching market for
each skill-type. In each market, there is free (costless) entry of firms. Each firm posts
one vacancy and the total number of vacancies posted in the labour market for type $i$ workers is denoted by $o_i$ (for offers). Assuming each firm who enters posts one vacancy is for simplicity and does not restrict the analysis.\footnote{With constant returns to scale in production, focusing on a single hire by each firm is not restrictive. The underlying assumption is that firms can fully insure against not filling a vacancy and getting negative ex post profits. In the background is a competitive lender who is risk averse and fully insures firms when the lender gives credit to finance the costs of posting a vacancy.} The $n_i$ job seekers are matched to job vacancies by a constant returns to scale matching function, $m(s_i n_i, o_i)$, which depends on aggregate search effort in the labour market, $s_i n_i$, and total vacancies $o_i$. 

\begin{table}
\begin{tabular}{|c|c|c|}
\hline
Skill-Type & Participation & Search Effort \\
\hline
$i$ & $n_i$ & $s_i$ \\
\hline
\end{tabular}
\end{table}
The matching function is assumed to be differentiable and increasing in both of its arguments and for simplicity is the same in all markets.

Define labour market tightness (from the firm’s perspective) as the ratio of total vacancies to aggregate search effort, \( \theta_i = o_i / (s_i n_i) \). Given constant returns to scale in the matching function, we can write total matches as \( \ell_i = m(s_i n_i, o_i) \equiv s_i n_i f(\theta_i) \) where \( f'(\theta_i) > 0 \). The probability that a job seeker finds a job is given by \( p(s_i, \theta_i) \equiv \ell_i / n_i = s_i f(\theta_i) \).

The probability that a vacancy is filled is given by \( q(\theta_i) \equiv m(s_i n_i, o_i) / o_i = m(1/\theta_i, 1) = f(\theta_i) / \theta_i \) where \( q'(\theta_i) < 0 \). Following Landais et al. (2015), we define \( 1 - \eta_i = \theta_i f'(\theta_i) / f(\theta_i) > 0 \) and \( \eta_i = -\theta_i q'(\theta_i) / q(\theta_i) > 0 \), as the elasticities of \( f(\theta_i) \) and \( q(\theta_i) \), respectively. A skill-type \( i \) individual who is matched to a job produces \( y_i \) units of output and is paid wage \( w_i \).

As in Saez (2002), the government observes individual wage rates or incomes \( w_i \), and we also assume that the government knows in which market workers search. It cannot observe the skill-types of the voluntary unemployed. Government policies include a type-specific linear income tax rate \( \tau_i \), a transfer \( b_i \) given to working individuals, an unemployment benefit \( b_I^i \) given to the involuntarily unemployed individuals, and a transfer \( b_V^i \) given to non-participants, where transfers can be positive or negative. The policy instruments \( \tau_i, b_i \) and \( b_I^i \) are equivalent to a type-specific linear progressive income tax in which the lump-sum component can differ by employment status.

### 2.1 Individual Behaviour

In this static setting individuals make two decisions: whether to participate in the labour market and if they choose to participate how much search effort to undertake. We can think of these as the extensive and intensive search decisions. We charac-

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3Landais et al. (2015) assume a similar matching function in aggregate search effort, but the number of job seekers is fixed at unity and there is a single ability type. They define \( f(\theta_i) \) as the rate a job seeker finds a job per unit of search effort, i.e., \( f(\theta_i) = m(s_i n_i, o_i) / s_i n_i = m(1, \theta_i) \).
terize in sequence an individual’s optimal search and participation decisions. When we consider an intertemporal version of our model later, savings decisions become relevant.

2.1.1 Search Effort Decision

A type−$i$ individual who chooses to participate in the labour market takes as given the tightness of the type−$i$ labour market $\theta_i$, the market wage rate $w_i$ and government policies, and chooses search effort $s_i$ to maximize:

$$s_if(\theta_i)v((1-\tau_i)w_i+b_i+a_i) + (1-s_if(\theta_i))u(b'_i + a_i) - \phi(s_i)$$

where $a_i$ is the individual’s initial assets and $\phi(s_i)$ is the increasing and convex search cost function. The utility of consumption while in the employed state, $v(\cdot)$, can differ from the utility of consumption while in the involuntarily unemployed state, $u(\cdot)$, possibly reflecting some positive disutility of working. Any disutility of working is implicitly assumed to be the same for all workers in this formulation. All utility of consumption functions are assumed to be increasing and strictly concave.\(^4\)

The first-order condition is

$$f(\theta_i)[v((1-\tau_i)w_i+b_i+a_i) - u(b'_i + a_i)] - \phi'(s_i) = 0$$

and the second-order condition, $-\phi''(s_i) < 0$, is satisfied. Provided the left-hand side of (2) evaluated at $s_i = 0$ is positive, then the unique interior optimum will have positive search effort. We assume that at the optimum employed individuals are strictly better off than involuntarily unemployed individuals. This implies that any individual will accept a job if they are matched to one.\(^5\) The solution to (2) gives

\(^4\)It is also assumed that the marginal utility of consumption tends to infinity as consumption tends to zero in all employment/unemployment states.

\(^5\)Without government intervention individuals would only exert a positive search effort if $v(w_i + a_i) > u(a_i)$. In reality with unemployment benefits, this condition may be violated, so governments require unemployed persons to search for a job and accept jobs that are offered. If job
optimal search effort \( s(\tau_i, b_i, b_i^l; \theta_i, w_i, a_i) \). Comparative statics yields

\[
\frac{\partial s(\cdot)}{\partial \tau_i} = -\frac{w_i v'((1 - \tau_i)w_i + b_i + a_i)f(\theta_i)}{\phi''(s_i)} < 0; \quad \frac{\partial s(\cdot)}{\partial b_i} = \frac{v'((1 - \tau_i)w_i + b_i + a_i)f(\theta_i)}{\phi''(s_i)} > 0; \\
\frac{\partial s(\cdot)}{\partial w_i} = \frac{(1 - \tau_i)v'((1 - \tau_i)w_i + b_i + a_i)f(\theta_i)}{\phi''(s_i)} > 0; \quad \frac{\partial s(\cdot)}{\partial a_i} = \frac{\partial s(\cdot)}{\partial b_i} + \frac{\partial s(\cdot)}{\partial b_i^l} \approx 0
\]

where

\[
\Delta v_i = v'((1 - \tau_i)w_i + b_i + a_i) - u(b_i^l + a_i)
\]

is the difference in the utility of consumption when employed and when involuntarily unemployed.

### 2.1.2 Labour Market Participation Decision

Individuals differ in the utility they receive if they do not participate in the labour market, which could reflect differences in home productivities.\(^6\) Let \( \delta \) be the utility benefit of not participating in the labour market and \( \mu(b^V) \) be the utility of consumption when voluntarily unemployed (or a non-participant), where as mentioned \( b^V \) is the income transfer to the voluntary unemployed. The cumulative distribution of \( \delta \) is given by \( G(\delta) \) with positive density \( g(\delta) \), where \( \delta \) is distributed on \([\delta_{\text{min}}, \delta_{\text{max}}]\).\(^7\)

An individual choosing whether to participate in the labour market anticipates search/acceptance behaviour is unobservable, then the government must monitor for these activities. See, for example, Boadway and Cuff (1999, 2014).

\(^6\)Alternate interpretations such as differences in fixed utility costs of participating have different social welfare implications that would need to be considered. Under the current formulation, non-participants are strictly better off than all of the labour market participants. If instead \( \delta \) is a fixed utility cost of participating in the labour market, then all of the labour market participants are strictly better of than the non-participants. In both cases, the marginal participant is the worst-off individual.

\(^7\)We assume the same distribution for \( \delta \) for both skill-types for simplicity, but this could be relaxed without changing the qualitative results. In the latter case, the \( \tilde{\delta} \) function would depend on the assumed type-specific distribution.
optimal search effort $s(\tau_i, b_i, b^V_i; \theta_i, w_i, a_i)$, and will participate if and only if

$$s(\cdot)f(\theta_i)v((1-\tau_i)w_i+b_i+a_i)+(1-s(\cdot)f(\theta_i))u(b^V_i+a_i)-\phi(s(\cdot)) \geq \mu(b^V+a_i)+\delta. \quad (5)$$

The participation constraint (5) is assumed to bind at some $\delta_i = \tilde{\delta}(\tau_i, b_i, b^V_i; \theta_i, w_i, a_i) \in (\delta_{\min}, \delta_{\max})$ so individuals with $\delta \leq \delta_i$ participate and those with $\delta > \delta_i$ choose not to participate where$^8$

$$\frac{\partial \tilde{\delta}(\cdot)}{\partial \tau_i} = -w_is_i f(\theta_i)v'(\cdot) < 0; \quad \frac{\partial \tilde{\delta}(\cdot)}{\partial b_i^V} = s_if(\theta_i)v'(\cdot) > 0;$$

$$\frac{\partial \tilde{\delta}(\cdot)}{\partial b_i} = (1-s_if(\theta_i))u'(\cdot) > 0; \quad \frac{\partial \tilde{\delta}(\cdot)}{\partial \theta_i} = s_if'(\theta_i)\Delta v_i > 0; \quad \frac{\partial \tilde{\delta}(\cdot)}{\partial w_i} = (1-\tau_i)s_if(\theta_i)v'(\cdot) > 0;$$

$$\frac{\partial \tilde{\delta}(\cdot)}{\partial a_i} = \frac{\partial \tilde{\delta}(\cdot)}{\partial b_i} + \frac{\partial \tilde{\delta}(\cdot)}{\partial b^V_i} + \frac{\partial \tilde{\delta}(\cdot)}{\partial b^V} \geq 0.$$  \quad (6)

The number of job seekers will be given by $n(\tau_i, b_i, b^V_i; \theta_i, w_i, a_i) = G(\tilde{\delta}(\cdot))$, so $1-n(\tau_i, b_i, b^V_i; \theta_i, w_i, a_i)$ is the number of non-participants or voluntarily unemployed.

For any given variable $x \in \{\tau_i, b_i, b^V_i, \theta_i, w_i, a_i\}$, we have

$$\frac{\partial n(\cdot)}{\partial x} = \frac{\partial \tilde{\delta}(\cdot)}{\partial x}g(\tilde{\delta}). \quad (7)$$

As expected, an increase in $b^V_i$ or $b_i$, and a decrease in $b^V$ or $\tau_i$ will increase labour market participation. Labour market participation will also be higher, the greater the wage rate and the more likely a job seeker will find a job (as determined by the tightness of the labour market, $\theta_i$).

### 2.2 Firms

Firms incur a cost of $k$ per vacancy posted, and we assume this is the same in all labour markets. Given the probability $q(\theta_i)$ that a vacancy is filled, profits per vacancy are

$^8$We could assume, as in Saez (2002), that there is a separate group of persons none of whom choose to participate. These would include those unable to work. For simplicity we exclude them from our analysis. If one could observe these persons, it might be desirable to offer them a different transfer. This would involve invoking some mechanism such as screening to identify them.
$q(\theta_i)(y_i - w_i) - k$. Increases in vacancies/offers cause $\theta_i$ to rise, and that reduces $q_i$ for a given $w_i$ since $q'(\theta_i) < 0$. “Entry” of vacancies occurs until firms earn zero expected profits, $q(\theta_i)(y_i - w_i) = k$, so in equilibrium,

$$q(\theta_i) = \frac{k}{y_i - w_i} = \frac{f(\theta_i)}{\theta_i}$$

(8)

using $q(\theta_i) = f(\theta_i)/\theta_i$ from above. This implicitly yields

$$\theta \left( \frac{k}{y_i - w_i} \right) \equiv q^{-1}(\theta_i), \quad \text{with} \quad \theta' \left( \frac{k}{y_i - w_i} \right) < 0$$

(9)

since $q'(\theta_i) < 0$. Therefore, we can write $\theta_i = \theta(w_i; y_i)$ which will be decreasing in $w_i$ and increasing in $y_i$. More explicitly, differentiating (8) and using $1 - \eta_i = \theta_i f'(\theta_i)/f(\theta_i)$ from above, we can derive

$$\frac{\partial \theta}{\partial w_i} = -\frac{\partial \theta}{\partial y_i} = -\frac{\theta_i}{\eta_i(y_i - w_i)} < 0$$

(10)

The probability $q(\theta_i)$ that a vacancy is filled can be written as $q\left(\theta(k/(y_i - w_i))\right)$, or simply as $q(w_i)$ with some abuse of notation, where $q'(w_i) > 0$. Intuitively, an increase in $w_i$ reduces job offers and therefore market tightness $\theta_i$, so the probability of filling a job $q_i$ increases.

Recall that the probability of a job-seeker getting a job is $p(s_i, \theta_i) = s_i f(\theta_i)$. Therefore, using (9) and writing $p(s_i, \theta(w_i; y_i))$ as $p(s_i, w_i; y_i)$ for simplicity, we have:

$$p(s_i, w_i; y_i) = s_i f \left( \theta_i \left( \frac{k}{y_i - w_i} \right) \right), \quad \text{with} \quad \frac{\partial p}{\partial s_i}, \frac{\partial p}{\partial y_i} > 0, \frac{\partial p}{\partial w_i} < 0$$

(11)

since $f'(\theta_i) > 0$.$^9$

$^9$Kroft et al (2015) refer to $p(s_i, w_i)$ as ‘the labour demand relation’ (p.16).
2.3 Wage Determination Process

Following Lehmann and Van der Linden (2007), we assume the wage is determined by asymmetric Nash bargaining after a match is made. Therefore, the wage $w_i$ satisfies
\[
\max_{w_i} \ (\Delta v_i)^\beta (y_i - w_i)^{1-\beta} \tag{12}
\]
where $\beta$ is the worker’s bargaining power, assumed to be the same for all worker-types. Note that the solution to this problem is not affected by affine transforms of the utility functions, e.g. $a + kv(\cdot), a + ku(\cdot)$, and we exploit this by assuming that utility functions are cardinal orderings. Note also that $v((1 - \tau_i)w_i + b_i + a_i)$ is the same for all workers in a given search market so the same bargaining applies to all.\footnote{Landais et al. (2015) and Kroft et al. (2015) assume the wage is determined by proportional bargaining where workers get a share $\beta$ of the surplus and the firm gets $1 - \beta$. Total surplus per match is $\Delta v_i + y_i - w_i$. However, the solution to this is not independent of the cardinalization of worker utility, as shown by l’Haridon, Malherbet and Pérez-Duarte (2013). That is, a common affine transformation of utility functions $v(\cdot)$ and $u(\cdot)$ change $\Delta v_i$ and therefore the wage rate. As noted in the text, the Nash bargaining solution is independent of affine transformations of utility.}

From the first-order condition for problem (12) we solve for the following wage:
\[
w(\tau_i, b_i, b_i^f, a_i; y_i) = y_i - \frac{1 - \beta}{\beta} \frac{\Delta v_i}{(1 - \tau_i)v'((1 - \tau_i)w_i + b_i + a_i)}. \tag{13}
\]
Differentiating (13) we obtain:
\[
\begin{align*}
\frac{\partial w(\cdot)}{\partial \tau_i} &= \frac{\beta(y_i - w_i)v_i' - (1 - \beta)v_i'w_i + \beta(1 - \tau_i)(y_i - w_i)v_i''w_i}{D_i} \geq 0; \\
\frac{\partial w(\cdot)}{\partial b_i} &= \frac{(1 - \beta)v_i' - \beta(1 - \tau_i)(y_i - w_i)v_i''}{D_i} < 0; \quad \frac{\partial w(\cdot)}{\partial b_i^f} = \frac{-(1 - \beta)u_i'}{D_i} > 0; \\
\frac{\partial w(\cdot)}{\partial a_i} &= \frac{(1 - \beta)[u_i' - u_i] - \beta(1 - \tau_i)(y_i - w_i)v_i''}{D_i} \geq 0 \tag{14}
\end{align*}
\]

where
\[
D = -(1 - \tau_i)v_i' + \beta(1 - \tau_i)^2(y_i - w_i)v_i'' < 0
\]
and
\[
\frac{\partial w(\cdot)}{\partial y_i} = \frac{\beta}{1 - (1 - \beta)\Delta v_i(v_i''/(v_i')^2)} \in (0, 1). \tag{15}
\]

From (14), we have
\[
\frac{\partial w(\cdot)}{\partial \tau_i} = \frac{\beta(y_i - w_i)v_i'}{D_i} - w_i \frac{\partial w(\cdot)}{\partial b_i} < -w_i \frac{\partial w(\cdot)}{\partial b_i}. \tag{16}
\]
Thus, a change in \(\tau_i\) has a different effect on the wage rate than a change in \(b_i\), and this relationship will be useful in what follows. For \(\tau_i > 0\) the surplus falls when \(w_i\) rises, and a higher \(\tau_i\) should tend to a lower wage. That is, a rise in \(w_i\) does not just imply a transfer from firm to worker: part of the increase in \(w_i\) goes to the government due to higher taxes. That is not so with \(b_i\). For any \(b_i\), an increase in \(w_i\) is a pure transfer from firm to worker. In fact the comparative statics show that an increase in \(b_i\) (which is like a reduction in tax payments) reduces \(w_i\). However, the effect of \(\tau_i\) on \(w_i\) is ambiguous.\(^{12}\)

By substituting (13) into (9), we can write \(\theta(w(\tau_i, b_i, b_i^I, a; y_i); y_i)\) where it follows from (10) and (13) that
\[
\frac{\partial \theta(\cdot)}{\partial w_i} = -\frac{\partial \theta(\cdot)}{\partial y_i} = -\frac{\theta_i \beta (1 - \tau_i) v_i'}{\eta (1 - \beta) \Delta v_i} \tag{17}
\]
which will be useful in characterizing the optimal policies.

\subsection*{2.4 Labour Market Equilibrium}

The individual’s optimal search effort and the number of job seekers can be written solely as functions of the policy parameters, that is,
\[
s_i(\tau_i, b_i, b_i^I) = s(\tau_i, b_i, b_i^I, \theta(w(\tau_i, b_i, b_i^I; y_i); y_i), w(\tau_i, b_i, b_i^I; y_i)) \tag{18}
\]
\(^{12}\)In the case of proportional bargaining, \(\partial w_i/\partial \tau_i = -w_i \partial w_i/\partial b_i > 0\). With Nash bargaining, a sufficient condition for \(w_i\) to be increasing in \(\tau_i\) is if \(-(1 - \tau_i)w_i v_i''/v_i' \geq 1\) where the left-hand side is the measure of relative risk aversion when \(b_i = a_i = 0\).
and

\[ n_i(\tau_i, b_i, b_i^I, b^V) = G(\delta_i(\tau_i, b_i, b_i^I, b^V)) \]  \hspace{1cm} (19)

where

\[ \delta_i(\tau_i, b_i, b_i^I, b^V) = \delta(\tau_i, b_i, b_i^I, b^V, \theta(w(\tau_i, b_i, b_i^I; y_i); y_i), w(\tau_i, b_i, b_i^I; y_i)). \]

The linear progressive tax system \( \{\tau_i, b_i, b_i^I\} \) affects both the individual’s participation and search effort decisions. Tax/transfer policies have a direct effect on these decisions as well as indirect or macro effects through their impact on the labour market equilibrium as given by the equilibrium wage rate and tightness of the market. The overall impact of these policies on an individual’s decisions depends on the relative magnitude of these various effects. The transfer to the voluntary unemployed \( b^V \) does not affect the surplus of a match and therefore, does not affect the equilibrium wage or tightness of the labour market. The voluntary unemployment benefit only affects the participation decision (or, the extensive margin of the search decision) and not the decision about how much to search (the intensive margin). If the government could not distinguish between non-participants and job-seekers (as in Kroft et al. (2015)), then it would be constrained to give the same transfer to both in which case this transfer to both involuntarily and voluntarily unemployed individuals would affect the equilibrium wage and tightness of the labour market. This special case is considered below.

From (3), (6) and (7) we have that

\[ \frac{\partial n(\cdot)}{\partial w_i} = \frac{\partial s(\cdot)}{\partial w_i} = \frac{(1 - \tau_i)f(\theta_i)v_i'}{f'(\theta_i)\Delta v_i}. \]

Consequently, using (10) we can write for any \( x = \{\tau_i, b_i, b_i^I\} \)

\[ \frac{ds_i}{dx} = \frac{\partial s(\cdot)}{\partial x} + \frac{\partial s(\cdot)}{\partial w_i} \left[ 1 - \frac{\beta(1 - \eta_i)}{\eta_i(1 - \beta)} \right] \frac{\partial w(\cdot)}{\partial x}, \]  \hspace{1cm} (20)

\[ \frac{dn_i}{dx} = \frac{\partial n(\cdot)}{\partial x} + \frac{\partial n(\cdot)}{\partial w} \left[ 1 - \frac{\beta(1 - \eta_i)}{\eta_i(1 - \beta)} \right] \frac{\partial w(\cdot)}{\partial x}. \]  \hspace{1cm} (21)
The bracketed term on the right-hand sides captures the macro effects of a change in the policy variables $\tau_i$, $b_i$ and $b'_i$ on search behaviour and participation decisions. Its sign will be $\geq 0$ as $\eta_i \geq \beta$.

Note that $\eta_i = \beta$ is the Hosios (1990) condition for job market $i$. With risk-neutral workers, if the Hosios condition is satisfied, search and participation are efficient in the sense that they maximize expected surplus. With risk aversion, things are a bit more complicated, but it is still possible to show that search and participation maximize expected social surplus (normalized expected utility less total costs of job postings and search costs) when the Hosios condition is satisfied. Specifically, with free entry of firms who post costly vacancies and individuals who make participation and search effort decision, the resulting tightness of the labour market $\theta_i = o_i/(n_is_i)$ in the private market equilibrium maximizes expected social surplus when $\eta_i = \beta$. If this condition does not hold, then the market equilibrium $\theta_i$ will be inefficient. With $\eta_i \neq \beta$, externalities arise from the firm’s job posting decision and from the individuals' participation and search effort decisions. If $\eta_i < \beta$, market tightness $\theta_i$ is inefficiently low. A subsidy on the cost of job posting, and a tax on both search effort and participation would result in the efficient $\theta_i$, and the converse is true when $\eta_i > \beta$. Details of these results are provided in Appendix A.

### 2.5 The Government Problem

Recall that the government observe wages and chooses type-specific proportional income taxes and lump-sum transfers to participants $\tau_i$, $b_i$ and $b'_i$, as well as uniform lump-sum transfers to the voluntary unemployed, $b^V$. We relax the assumption that the transfer to the involuntary unemployed can be conditioned on skill-type below. For simplicity, we also suppress $a_i$ from the model. Financial assets become relevant

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13 As shown in Appendix A, this assumes that the social optimum is evaluated at the wage determined in the private market equilibrium by Nash bargaining. With risk-averse workers, changes in $w$ redistribute surplus between the worker and the firm. This affects social welfare unlike in the risk-neutral case where total surplus is all that counts.
when we consider a dynamic setting with individual savings. Policies are chosen to maximize the following utilitarian social welfare function:

\[
\sum_{i=1}^{N} \left( n_i(\tau_i, b_i, b'_i, b^V) \left[ s_i(\tau_i, b_i, b'_i) f(\theta_i(\tau_i, b_i, b'_i)) v((1 - \tau_i)w_i(\tau_i, b_i, b'_i) + b_i) + (1 - s_i(\tau_i, b_i, b'_i) f(\theta_i(\tau_i, b_i, b'_i))) u(b'_i) - \phi(s_i(\tau_i, b_i, b'_i)) \right] \right)
\]

\[
+ \int^{\delta_{\text{max}}}_{\delta_i(\tau_i, b_i, b'_i, b^V)} (\mu(b^V) + \delta_i(g(\delta))d\delta)
\]

where the three terms represent the employed, the involuntary unemployed and the voluntary unemployed. The government’s budget constraint is:

\[
\sum_{i=1}^{N} n_i(\tau_i, b_i, b'_i, b^V) \left( s_i(\tau_i, b_i, b'_i) f(\theta_i(\tau_i, b_i, b'_i)) b_i + (1 - s_i(\tau_i, b_i, b'_i) f(\theta_i(\tau_i, b_i, b'_i))) b'_i \right)
\]

\[
+ \sum_{i=1}^{N} (1 - n_i(\tau_i, b_i, b'_i, b^V)) b^V = \sum_{i=1}^{N} n_i(\tau_i, b_i, b'_i, b^V) s_i(\tau_i, b_i, b'_i) f(\theta_i(\tau_i, b_i, b'_i)) \tau w_i(\tau_i, b_i, b'_i)
\]

Using the Envelope Theorem from the individual’s optimal search effort and participation decisions, the first-order conditions on \(\tau_i, b_i, b'_i\) for \(i = 1, \cdots, N\) and \(b^V\) can be written as follows, where \(\lambda\) is the multiplier on the government’s budget constraint:

\[
-n_i s_i f(\theta_i) v_i' \cdot w_i + A_i \frac{\partial w_i}{\partial \tau_i} - \lambda \left( -n_i s_i f(\theta_i) w_i - t_{pi} \frac{dn_i}{\partial \tau_i} - t_{ei} n_i f(\theta_i) \frac{ds_i}{\partial \tau_i} \right)
\]

\[
+ \left[ -t_{ei} n_i s_i f'(\theta_i) \frac{\partial \theta_i}{\partial w_i} - n_i s_i f(\theta_i) \tau_i \frac{\partial w_i}{\partial \tau_i} \right] = 0
\]

\[
n_i s_i f(\theta_i) v_i' + A_i \frac{\partial w_i}{\partial b_i} - \lambda \left( n_i s_i f(\theta_i) - t_{pi} \frac{dn_i}{\partial b_i} - t_{ei} n_i f(\theta_i) \frac{ds_i}{\partial b_i} \right)
\]

\[
+ \left[ -t_{ei} n_i s_i f'(\theta_i) \frac{\partial \theta_i}{\partial w_i} - n_i s_i f(\theta_i) \tau_i \frac{\partial w_i}{\partial b_i} \right] = 0
\]

\[
n_i \left( 1 - s_i f(\theta_i) \right) u'(b'_i) + A_i \frac{\partial w_i}{\partial b'_i} - \lambda \left( n_i \left( 1 - s_i f(\theta_i) \right) - t_{pi} \frac{dn_i}{\partial b'_i} - t_{ei} n_i f(\theta_i) \frac{ds_i}{\partial b'_i} \right)
\]

\[
+ \left[ -t_{ei} n_i s_i f'(\theta_i) \frac{\partial \theta_i}{\partial w_i} - n_i s_i f(\theta_i) \tau_i \frac{\partial w_i}{\partial b'_i} \right] = 0
\]
\[
\sum_{i=1}^{N} \left( (1 - n_i)\mu^i(\cdot) - \lambda \left( (1 - n_i) - t_{pi} \frac{\partial n_i}{\partial b^i} \right) \right) = 0 \quad (27)
\]

where
\[
A_i = n_i s_i f'(\theta_i) \frac{\partial \theta_i}{\partial w_i} \Delta v_i + n_i s_i f(\theta_i) v_i'(\cdot)(1 - \tau_i)
\]
\[
= n_i s_i f(\theta_i) v_i'(\cdot)(1 - \tau_i) \left[ 1 - \beta \frac{(1 - \eta_i)}{\eta_i(1 - \beta)} \right] \geq 0 \quad \text{for} \quad \eta_i \geq \beta. \quad (28)
\]

with the second equality following from using (17). The interpretation of \( A_i \) is as follows. Holding participation and search effort constant, \( A_i \) shows what happens to the government’s objective with a marginal increase in the equilibrium type—\( i \) wage \( w_i \). Social welfare (given \( n_i \) and \( s_i \)) may be increasing or decreasing in \( w_i \) depending on whether the elasticity of the probability of filling a vacancy is greater or less than the worker’s bargaining power.

The variable \( t_{pi} \) is the participation tax on ability-type \( i \) and is defined by
\[
t_{pi} = - \left[ s_i f(\theta_i) (b_i - \tau_i w_i) + (1 - s_i f(\theta_i)) b_i' - b_i^V \right] \quad (29)
\]

and \( t_{ei} \) is the employment tax defined as
\[
t_{ei} = - \left[ b_i - b_i' - \tau_i w_i \right]. \quad (30)
\]

The participation tax describes the net effect on government revenue as an individual moves from non-participation (voluntary unemployment) to participation in the labour market. The government no longer transfers \( b_i^V \), it raises revenue \( \tau_i w_i - b_i \) from an employed worker with probability \( s_i f_i \), and it pays out an involuntary unemployment transfer \( b_i' \) with probability \( (1 - s_i f_i) \). The employment tax describes what happens to government revenue when a participating worker moves from being involuntary unemployed to employed. The government raises revenue \( \tau_i w_i - b_i \) and no longer pays out \( b_i' \).

It is useful to begin by setting aside redistribution across skill-types to focus on the efficient design of a tax-transfer system that differentiates among the employed, the
involuntary unemployed and the voluntary unemployed. This yields insight into how endogenous wage formation influences the optimal unemployment insurance system and is relevant when we incorporate redistributive considerations in Section 4.

3 One Productivity Skill-Type

Assume that all individuals have the same productivity by suppressing the skill-type index \( i \), so there is effectively only one skill type and the population has been normalized to unity. We investigate optimal policies chosen by a social welfare maximizing government. We assume that the population is large enough such that the government faces no uncertainty. It knows the shares of the population made up of the employed and the voluntary and involuntary unemployed, and therefore it knows its total tax revenues and transfer payments.

3.1 Optimal Policies

The government problem is as above with objective function (22) and budget constraint (23), both with \( i \) suppressed. The first-order conditions of the government’s problem (24)–(27) can be written as follows using (20), (21), the expressions for \( \partial s/\partial w \) and \( \partial n/\partial w \) from (3) and (7), respectively, and \( A, t_p \) and \( t_e \) defined as in (28), (29) and (30), all with \( i \) suppressed:

\[
-nsf(\theta)v'(\cdot)w + A \frac{\partial w}{\partial \tau} + \lambda \left[ nsf(\theta)w + t_p \frac{\partial n(\cdot)}{\partial \tau} + t_e nsf(\theta) \frac{\partial s(\cdot)}{\partial \tau} \right]
\]

\[
+ \left( t_p A \frac{g(\delta)}{G(\delta)} + t_e A \frac{f(\theta)}{s \phi'(s)} + nsf(\theta) \left[ -t_e \frac{\beta(1 - \eta)}{\eta(1 - \beta)} \frac{(1 - \tau)v'}{\Delta v} + \tau \right] \right) \frac{\partial w}{\partial \tau} = 0 \tag{31}
\]
\[
nsf(\theta)v(\cdot) + A \frac{\partial w}{\partial b} + \lambda \left[ -nsf(\theta) + tp \frac{\partial n(\cdot)}{\partial b} + te \frac{\partial f(\theta) \partial s(\cdot)}{\partial b} \right] + \left( tp A \frac{g(\delta)}{G(\delta)} + te A \frac{f(\theta)}{s \phi'(s)} + nsf(\theta) \left[ -te \frac{\beta (1 - \eta) (1 - \tau) v'}{\eta (1 - \beta) \Delta v} + \tau \right] \right) \frac{\partial w}{\partial b} = 0 \quad (32)
\]

\[
n(1 - sf(\theta))u(\cdot) + A \frac{\partial w}{\partial b^I} + \lambda \left[ -n(1 - sf(\theta)) + tp \frac{\partial n(\cdot)}{\partial b^I} + te \frac{\partial f(\theta) \partial s(\cdot)}{\partial b^I} \right] + \left( tp A \frac{g(\delta)}{G(\delta)} + te A \frac{f(\theta)}{s \phi'(s)} + nsf(\theta) \left[ -te \frac{\beta (1 - \eta) (1 - \tau) v'}{\eta (1 - \beta) \Delta v} + \tau \right] \right) \frac{\partial w}{\partial b^I} = 0 \quad (33)
\]

\[
(1 - n)\mu(\cdot) + \lambda \left[ -(1 - n) + tp \frac{\partial n(\cdot)}{\partial b^\tau} \right] = 0 \quad (34)
\]

Using these first-order conditions, we can derive a series of results characterizing the optimal tax-transfer system in this case with a single skill-type.

**Result 1** With an exogenous wage rate, a proportional wage tax \( \tau \) and a transfer to the employed \( b \) are perfect policy substitutes.

Result 1 follows directly from the two first-order conditions (31) and (32) assuming that \( w \) is fixed and given the following two relationships by (3) and (6): \( \partial s(\cdot)/\partial \tau = -w \partial s(\cdot)/\partial b \) and \( \partial n(\cdot)/\partial \tau = -w \partial n(\cdot)/\partial b \). The first-order condition on \( b \) is equivalent to \((-w)\) times the first-order condition on \( \tau \) implying that the two policy instruments are perfect policy substitutes.

A corollary of Result 1 is that if, in addition to fixed wages, there were no search effort and participation decisions, the government could provide full unemployment insurance (that is, equalize the marginal utility of consumption for employed, involuntarily unemployed and voluntarily unemployed individuals, \( v' = u' = \mu' \)).

Given Result 1, this first-best outcome can be achieved by using only \( b \) and \( b^I \) as policy instruments: if there is under-insurance, so \( w + b \) is too high relative to \( b^I \), \( b^I \) can

\[14\]This first-best outcome follows directly from (32)–(34) when \( w, s \) and \( n \) are all fixed.
be increased and \( b \) reduced until full insurance is achieved and \( w \) is not affected. This logic continues to hold when individuals decide both whether to participate in the labour market and how hard to look for work, so moral hazard prevents the government from providing full unemployment insurance.

When the wage rate is endogenous, increasing \( b' \) and reducing \( b \) increases \( w \) by (14), and changes in \( w \) will have effects on social welfare both directly and indirectly through changes in government revenue. Thus, moving toward full insurance using \( b \) and \( b' \) cannot be guaranteed to be optimal, and this will be the case even without any search or participation decisions. Allowing for a third policy instrument, \( \tau \), gives the government an additional degree of freedom. With an endogenous wage, there is now a separate role for the proportional income tax rate alongside \( b \), \( b' \) and \( b'' \) as stated in Result 2.\(^{15}\)

**Result 2** With an endogenous wage rate, the optimal proportional income tax \( \tau \) ensures that income transfers to the employed, \( b \), and to the involuntary unemployed, \( b' \), are both independent of changes in the equilibrium wage rate and the tightness of the labour market.

To see this result, rewrite (31), the first-order condition on \( \tau \), using the two relationships from (3) and (6) noted above as well as the relationship in (16) as follows (where all arguments of functions have been suppressed):

\[
- n s f v' w + A \left( \frac{\beta(y - w)v'}{D} - w \frac{\partial w}{\partial b} \right) + \lambda \left[ n s f w - t_p w \frac{\partial n}{\partial b} - w t_e n f \frac{\partial s}{\partial b} \right] + \left( t_p A \frac{g}{G} + t_e A \frac{f}{s \phi'(s)} \right) + n s f \left[ - \frac{\beta(1 - \eta)(1 - \tau)v'}{\eta(1 - \beta)} \phi''(s) + \tau \right] \left( \frac{\beta(y - w)v'}{D} - w \frac{\partial w}{\partial b} \right) = 0
\]

\(^{15}\)This arises from our assumed wage determination process. Under Nash bargaining, the proportional income tax rate and the income transfer to the employed differentially affect the bargained wage. This would not be true, say, for example under a proportional sharing rule. Under this wage determination process, changes in \( \tau \) and \( b \) would have the same affect on the bargained wage.
Substituting the first-order condition for $b$, (32), into the above yields:

$$
A + \lambda \left( t_p \frac{g(\delta)}{G(\delta)} + t_e A \frac{f(\theta)}{s \phi''(s)} + n s f(\theta) \left[ - t_e \frac{\beta(1-\eta)(1-\tau)}{\eta(1-\beta)} \frac{\Delta v}{\Delta v} + \tau \right] \right) = 0 \quad (35)
$$

Eq. (35) represents the difference in the effect of $\tau$ on the Lagrangian expression relative to $b$. Note in particular that (35) is the coefficient of the terms involving the direct effects of $\tau$, $b$ and $b^I$ on the wage rate $w$ in the first-order conditions for these variables, (31)–(33), the so-called macro effects of these policies (Landais et al. (2015)). Equivalently, (35) is derivative of the Lagrangian of the government problem with respect to the wage rate $w$. It captures all of the effects of a change in $w$ on the Lagrangian including the direct effect on social welfare, as given by $A$, and the various revenue effects through a) changes in participation, $n$, as given by first term in the brackets, b) changes search effort, $s$, which affects employment as given by the second term in the brackets, c) changes in the tightness of the labour market which also affects employment, as given by the first term in the square brackets as well as d) the direct revenue effect of an increase in $w$ as given by the last term in square brackets.

Substituting (35) into (32) and (33) yields

$$
n s f' + \lambda \left[ - n s f + t_p \frac{\partial n}{\partial b} + t_e n f \frac{\partial s}{\partial b} \right] = 0, \quad (36)
$$

$$
n(1 - s f) u' + \lambda \left[ - n(1 - s f) + t_p \frac{\partial n}{\partial b^I} + t_e n f \frac{\partial s}{\partial b^I} \right] = 0. \quad (37)
$$

These simplified first-order conditions on $b$ and $b^I$ are independent of changes induced in $w$, implying that macro effects do not affect the conditions governing the choice of $b$ or $b^I$ as stated in Result 2. The choice of $b^V$, as given by (34), remains unaffected by the macro effects.

A corollary of Result 2 is that the government can ensure that with endogenous
wages full unemployment insurance is obtained in the case of no moral hazard.\textsuperscript{16} As long as the first-order condition on the optimal $\tau$ holds, any induced changes in $w$ will have no net effect on social welfare. The values for $b$, $b^I$ and $b^V$ can be changed to obtain optimal unemployment insurance with moral hazard as well as optimal transfers to the voluntary unemployed. The induced effects of changes in these transfers on the wage rate — the macro effects — are no longer operational since these are neutralized by the choice of $\tau$.

Result 2 relies on our assumption that the equilibrium wage is determined by Nash bargaining, which ensures that the equilibrium wage will be independent of affine transformations of utility. This is important for the following reason. With a utilitarian social welfare function, the social ordering is unaffected by a common affine transformation of individual utilities. To ensure that the optimal allocation is also unchanged with affine utility transformations, wage bargaining outcomes must also be unaffected. That will be satisfied by Nash bargaining. Under proportional bargaining (Landais et al. 2015, Kroft 2015), as noted above, this is no longer the case. Affine transformations of utility will affect the equilibrium wage. Further, under this wage determination process the proportional income tax rate and the transfer to the employed continue to be perfect policy substitutes. Consequently, changes in the unemployed transfer must take macro effects into account.

\textsuperscript{16}Appendix B shows explicitly in the case without a search or participation decision and an endogenous wage that the first-order condition on $\tau$ when combined with the first-order condition on $b$ reduces to the requirement that a change in $w$ has a zero net effect on social welfare (i.e., on the value of the government’s Lagrangian expression): that is, it ensures that the macro effects of changes in $b$ and $b^I$ are zero. As a consequence, $b^I$ can be increased and $b$ reduced until full employment is achieved without inducing any welfare effects through changes in $w$, and full insurance can be achieved. Note that this neutralizing effect of $\tau$ on changes in the wage rate apply regardless of whether the Hosios conditions are satisfied as discussed further below.
nate $\lambda$ from (36) and (37) and simplify to give:\(^{17}\)

$$
(1 - sf)sf \frac{u' - v'}{v'} = -t_e \frac{\partial s}{\partial b^I}
$$

Equation (38) is independent of the participation decision and would thus also hold if all individuals are assumed to be participants and decide only how much search effort to undertake.\(^{18}\) This corresponds to the case studied by Chetty (2008) and

\(^{17}\)Solving for $\lambda$ and setting the two expressions equal, the resulting terms involving $\partial n/\partial b$ and $\partial n/\partial b^I$ cancel since an increase in $b$ and $b^I$ both positively affect participation. An increase in $b$ and $b^I$ have opposing effects on search effort. The government therefore faces a trade-off in its choice of $b$ and $b^I$ between insuring individuals against unemployment and inducing them to search for work.

\(^{18}\)The terms involving changes in $n$ disappear from the first order conditions (31)–(33) (including the term in $g(\bar{\delta})$) and again using the first-order condition on $\tau$ (rewritten as the macro condition (35)) and the first-order conditions on $b$ and $b^I$, (36) and (37), we obtain (38).
Landais et al. (2015). In this case, since there are no voluntary unemployed, $b^V$ is no longer a policy variable and by the government budget constraint, we have $(b - b^I - \tau w) = -t_c = -b^I / (sf)$, so (38) becomes

$$(1 - sf) sf \frac{u' - v'}{v'} = - \frac{b^I}{s} \frac{\partial s}{\partial b^I}$$

Since $\partial s / \partial b^I < 0$, we again have that $u'(b^I) > v'(w(1 - \tau) + b)$.

This special case with no participation decision illustrates that if the individual could smooth consumption over the two states so as to equalize the marginal utility of consumption when employed and when unemployed (i.e., $v' = u'$), then there would only be a moral hazard effect of a transfer to the involuntarily unemployed (given by the right-hand side (38)). In this case, no unemployment income transfer should be made. Individuals are already full insured against unemployment.\(^\text{19}\) If, on the other hand, individuals are not fully insured so the marginal utility of consumption when unemployed is higher than when employed, potentially as a result of liquidity constraints (i.e., $v' < u'$ or equivalently, $\partial s / \partial a < 0$), then there is some benefit to providing a transfer in the unemployed state.

To make this even more explicit, following Chetty (2008), we can define

$$\epsilon_I = \frac{b^I}{1 - sf} \frac{d(1 - sf)}{db^I} = -f \frac{b^I}{1 - sf} \frac{\partial s}{\partial b^I} > 0$$

as the elasticity of the probability an individual does not find a job with respect to the involuntary unemployment transfer. Using the expressions for the partials in (3), (39) can be rewritten as

$$\frac{1 - sf}{sf} \left[ sf \frac{-\partial s / \partial a}{\partial s / \partial w} - \frac{\epsilon_I}{sf} \right] = 0$$

\(^\text{19}\)Note that with full insurance, it is likely that $u(\cdot) > v(\cdot)$ so that individuals are better off involuntarily unemployed than working. If the government cannot prevent workers from claiming to be involuntarily unemployed by, for example, not accepting a job offer, full insurance would not be sustainable. The government would have to address this problem by monitoring workers to ensure that they engage in job search, they accept jobs that are offered, and they do not quit jobs. These issues are addressed in Boadway and Cuff (1999, 2014).
which is similar to expression (10) in Chetty (2008).\textsuperscript{20}

Finally, recall the important point that unlike Landais et al. (2015) macro effects of policy on the wage rate do not affect the conditions on transfers \( b \) and \( b' \), and therefore on unemployment insurance. All effects arising from \( A \neq 0 \) are addressed by the choice of marginal tax rate \( \tau \). If, as in Chetty (2008), pre-tax wages are fixed and not affected by policies, then having a wage tax \( \tau \) alongside an income transfer to the employed would be redundant.

Although the participation decision does not affect the incentive-insurance trade-off the government faces, the optimal participation tax rule is affected by the inability of the government to provide complete unemployment insurance with endogenous search effort, which is Result 4.

**Result 4** Endogenous search effort tends to reduce the optimal participation tax.

To see this, consider first the case where search is fixed. If \( \tau \) is chosen optimally it follows immediately from the first-order conditions on \( b \) and \( b' \), (36) and (37) respectively, that the government would provide full unemployment insurance. If the government is only trying to induce individuals into the labour market and individuals cannot affect the likelihood they find work, then by providing full unemployment insurance \((v' = u')\) the government makes it more attractive to participate in the labour market.

In this special case of fixed search effort, the optimal participation tax can then be derived directly from either of these two first-order conditions and will be given

\textsuperscript{20}The two expressions differ by \( sf \) in first term in square brackets. In the static case of Chetty (2008), the search effort decision is assumed to depend only on the unemployment transfer and the wage tax is determined from the binding government budget constraint given individuals’ optimal search behaviour. The government then chooses the unemployment transfer subject to its budget constraint. In the above, the search effort decision depends on both the unemployment transfer and the wage tax (or, equivalently \( b \)) and the government chooses both of these policy variables subject to its budget constraint.
by:

$$t_p = nsf(\theta) \frac{1 - v'/\lambda}{\partial n/\partial b}$$

(41)

This optimal participation tax rule is equivalent in interpretation to the one obtained by Saez (2002) who does not allow for involuntary unemployment. The expression $v'/\lambda$ in the numerator is the value in terms of government revenue of an increment in transfer to an employed individual and is the same as social welfare weight, $g$, defined in Saez (2002). The expression in the denominator reflects the responsiveness of the tax base (participation) to the tax.\textsuperscript{21} The sign of $t_p$ follows from the government’s budget constraint, which implies that $t_p = sf(b - \tau w) + (1 - sf)b^t + b^v = b^v/n$. This will be positive if $b^v > 0$, which will be the case as long as $\mu'(0) = \infty$, which we assume. With fixed search effort, $v' = u'$, and it follows from the first-order conditions on $b$ and $b^v$ that the sign of $1 - v'/\lambda$ is the same sign as $1 - \mu'/\lambda$. Together these imply that $t_p > 0$ and $v' < \lambda < \mu'$.

We can now highlight how allowing for endogenous search affects the optimal participation tax. First note that first-order condition (36) on $b$ can be rewritten as

$$sf(\theta) \frac{v'}{\lambda} + t_p \frac{\partial n}{\partial b} \frac{1}{n} + t_e f(\theta) \frac{\partial s}{\partial b} = sf(\theta)$$

(42)

which equates the benefits and costs of an increase in $b$ in terms of government revenue. The benefits are on the left-hand side. The first term is the so-called mechanical effect of the change, and includes the social value of the change in $b$ to existing employed, $sf(\theta)v'/\lambda$. The second term is the additional revenue raised by the

\textsuperscript{21}We can re-express (41) as

$$\frac{T_P}{1 - T_P} = \left(1 - \frac{v'}{\lambda}\right) \frac{1}{\epsilon_P}$$

which is equivalent to the optimal tax rule obtained in the pure extensive model of Saez (2002) where $T_P = t_p/sf(\theta)w$ is the share of expected wage individuals retain if they choose to participate. That is, $c-c_V = (1-T_P)sf(\theta)w$, where $c = sf(\theta)((1-\tau)w+b)+(1-sf(\theta))b^t$ is expected consumption from participating and $c_V = b^v$ is consumption from not participating and $\epsilon_P = (dn/d(c-c_V))(c-c_V)/n$ is the participation elasticity. The tax rule is obtained by considering the effect of a marginal change in $b$ on the difference in consumption between participating and not, $d(c - c_V) = sf(\theta)db$, starting from the optimal tax/transfer system.
increase in participation induced by the change in $b$, and the last term is the increase in revenue induced by an increase in employment resulting from more intensive search. The right-hand side is the revenue cost of the increased transfer to the $sf(\theta)$ employed workers.

Solving for the optimal participation tax yields:

$$tp = nsf(\theta)\frac{1 - v'/\lambda}{\partial n/\partial b} - t_e nf(\theta)\frac{\partial s(\cdot)}{\partial b}\frac{\partial n(\cdot)}{\partial b}.$$  \hspace{1cm} (43)

Recall that the employment tax will be positive (with $b^I, b^V > 0$) and that a higher transfer to the employed induces greater search effort and more participation. Then, the last term on the right-hand side will be positive. Moral hazard arising from the search effort decision puts downward pressure on the optimal participation tax relative to the case with fixed search effort, confirming Result 4.

Finally, dividing each of the first-order conditions for $b$, $b^I$ and $b^V$ — (36), (37), and (34) — by the relevant marginal utility, using the expressions for the changes in $s$ and $n$ in (3) and (7), and summing up yields:

$$\frac{1}{\lambda} = \frac{nsf}{v'} + \frac{n(1 - sf)}{u'} + \frac{1 - n}{\mu'}.$$  \hspace{1cm} (44)

where the right-hand side of the weighted average of the inverse of the marginal utilities of consumption. The inverse of the marginal utility of consumption is the marginal cost in terms of consumption of increasing utility by one util. Multiplying this inverse by the number of individuals of a given type (employed, involuntarily unemployed, voluntarily employed) yields the total amount of consumption needed to increase the utility of all of these individuals by one util. The social benefit in terms of consumption of increasing everyone’s utility by one util is given the total population divided by the marginal cost of public funds, $\lambda$.

It is useful to consider some special cases of the above analysis.
3.2 Hosios Condition Satisfied

In the analysis of Hungerbühler et al (2006) where workers are risk-neutral, wage bargaining yields the optimal wage when the Hosios condition is satisfied. There is no need for policies to correct the wage rate. That is no longer the case with risk-averse workers. To show this, note that the Hosios condition implies that η = β, so A = 0 from (28), and the first-order condition on τ, (35), reduces to:

\[
\lambda \left( nsf(\theta) \left[ - t_e \frac{(1 - \tau)v'}{\Delta v} + \tau \right] \right) = 0
\] (45)

As discussed above, the proportional tax rate τ is chosen such that the various effects of changes on w on the Lagrangian sum to zero. When the Hosios condition is satisfied, the equilibrium wage affects only revenues and it does so through two effects — a direct positive effect (given by τ times the number of employed) and a negative indirect effect through its effect on the tightness of the labour market (given by the employment tax times the change in the number of employed with a change in θ as a result of a change in w). The optimal income tax rate ensures these two effects exactly offset one another.

To see this more clearly, use (17) in (45) to obtain:

\[
t_e nsf'(\theta)\theta'(w) + nsf(\theta)\tau = 0
\] (46)

The first term is the employment tax \( t_e \) times the change in employment when w increases (which is negative), that is, the indirect effect of a change in w on tax revenues via the induced change in employment. The second term is the direct effect of a change in w on income tax revenues from all nsf(θ) employed workers. Note that even if the Hosios condition is satisfied so search is optimal, an induced change in the wage rate will still affect maximized social welfare through its effect on government revenues. That is, the macro effect does not disappear when the Hosios condition is satisfied.
Using the expression for the employment tax from the government’s budget constraint, \( t_e = -(b - b^l - \tau w) = b^l/(sf) + (1 - n)b^V/(nsf) \). Using this, (45) can be solved for the optimal income tax rate to yield:

\[
\frac{\tau}{1 - \tau} = \left( \frac{b^l}{sf} + \frac{(1 - n)b^V}{nsf} \right) \frac{v'}{\Delta v}.
\]

Therefore, if \( b^l, b^V > 0 \), then the above implies that \( \tau > 0 \).

This can be contrasted with the case where the Hosios condition does not apply where \( \tau \) is not necessarily positive. To see this, rewrite (35) as follows using the definition of \( A \) given by (28):

\[
\frac{\tau}{1 - \tau} = \frac{v'}{\Delta v} \frac{\beta(1 - \eta)}{\eta(1 - \beta)} - \left( \frac{1 - \beta(1 - \eta)}{\eta(1 - \beta)} \right) \left( \frac{1}{\lambda} + t_p \frac{g(\delta)}{G(\delta)} + t_e f(\theta) \frac{t_p}{\delta''(s)} \right).
\]

The first-term on the right-hand side is positive for any values of \( \eta \) and \( \beta \). The last term is negative if \( \eta > \beta \), and could be large enough to cause \( \tau < 0 \) overall. The point is that \( \tau \) is now used to correct for search externalities and they can be positive or negative overall.

### 3.3 Same Transfer to all Unemployed

Some papers assume that the same transfer is made to the voluntary and involuntary unemployed (e.g., Kroft et al. (2015)) as the government may not be able to distinguish between individuals who choose not to look for work and those who did not find a job, so \( b^l = b^V \). Denote the transfer to all unemployed individuals as \( b^U \). The individual’s optimal search decision is as before with \( b^l \) replaced by \( b^U \). Optimal search effort will be \( s(\tau, b, b^U; \theta, w, a) \) and the partials will be given by (3) with \( b^l \) replaced by \( b^U \).

An individual will participate in the labour market provided:

\[
s(\cdot, f(\theta))v(w(1 - \tau) + b + a) + (1 - s(\cdot, f(\theta)))u(b^U + a) - \phi(s(\cdot)) \geq \mu(b^U + a) + \delta \tag{47}
\]
which is assumed to bind at some \( \bar{\delta}(\tau, b, b^U; \theta, w, a) \in (\delta_{\min}, \delta_{\max}) \) so individuals with \( \delta \leq \bar{\delta} \) participate and those with \( \delta > \bar{\delta} \) choose not to participate. The partials are given by (6) with \( b^I = b^V = b^U \) and
\[
\frac{\partial \bar{\delta}(.)}{\partial b^U} = \frac{\partial \bar{\delta}(.)}{\partial b^I} + \frac{\partial \bar{\delta}(.)}{\partial b^V} = (1 - sf(\theta))u'(\cdot) + \mu'(\cdot)
\]
(48)

The number of job seekers will be given by \( n(\tau, b, b^U; \theta, w, a) = G(\bar{\delta}(\cdot)) \) and \( 1 - n(\tau, b, b^U; \theta, w, a) \) is the number of non-participants or voluntarily unemployed. For any given variable \( x \in \{\tau, b, b^U, \theta, w, a\} \), we have
\[
\frac{\partial n(.)}{\partial x} = \frac{\partial \bar{\delta}(\cdot)}{\partial x} g(\bar{\delta}).
\]
(49)

The equilibrium wage will now be \( w(\tau, b, b^U, a) \) with partials given by (14) with \( b^I \) replaced by \( b^U \). The relationship between \( \theta \) and \( w \) is unaffected by the unemployment transfers and continues to be given by (17). Taken together, the individual’s optimal search effort and the number of job seekers can be written solely as functions of the policy parameters, that is,
\[
s(\tau, b, b^U; a) = s(\tau, b, b^U, \theta(w(\tau, b, b^U); a), w(\tau, b, b^U; a); a) \quad \text{and}
\]
\[
n(\tau, b, b^U; a) = G(\bar{\delta}(\tau, b, b^U; a)) \quad \text{where}
\]
\[
\bar{\delta}(\tau, b, b^U; a) = \delta(\tau, b, b^U, \theta(w(\tau, b, b^U); a), w(\tau, b, b^U; a); a).
\]
The relationships expressed in (20) and (21) hold for \( b^U \) as well as continue to hold for the other policy variables.

The government maximizes the following objective (where \( a \) has been suppressed):
\[
\max_{\{\tau, b, b^U\}} n(\tau, b, b^U) \left[ s(\tau, b, b^U)f(\theta(w(\tau, b, b^U)))v((1 - \tau)w(\tau, b, b^U) + b)
\right.
\]
\[
+ (1 - s(\tau, b, b^U)f(\theta(w(\tau, b, b^U))))u(b^U) - \phi(s(\tau, b, b^U))
\]
\[
\left. + \int_{\delta(\tau, b, b^U)}^{\delta_{\max}} (\mu(b^U) + \delta)g(\delta)d\delta \right]
\]
subject to the government’s budget constraint
\[
n(\tau, b, b^U)s(\tau, b, b^U)f(\theta(w(\tau, b, b^U)))b + (1 - n(\tau, b, b^U))s(\tau, b, b^U)f(\theta(w(\tau, b, b^U)))b^U
\]
\[
= n(\tau, b, b^U)s(\tau, b, b^U)f(\theta(w(\tau, b, b^U)))\tau w(\tau, b, b^U)
\]
(51)
where λ is the Lagrange multiplier on the government’s budget constraint (51).

Unlike the case when the government can perfectly distinguish the voluntary from the involuntary unemployed, the government does not have enough policy instruments to affect separately the search and participation decision. In this case, there is a single employment/participation tax defined as \( t_P = -[b - b^U - \tau w] \). Consequently, the first-order conditions on \( \tau \) and \( b \) are unchanged from the case when \( b^I \) can be chosen separately from \( b^U \), except that \( t_e \) and \( t_p \) in (31) and (32) are both replaced by \( t_P \). Consequently, Results 1 and 2 continue to hold with \( b^I = b^V = b^U \).

The first-order condition on \( b^U \) is:

\[
\begin{align*}
  n(1 - sf(\theta))u'(\cdot) + (1 - n)\mu'(\cdot) + A \frac{\partial w}{\partial b^U} + \lambda \left[ - (1 - nsf(\theta)) + t_Psf(\theta) \frac{\partial n(\cdot)}{\partial b^U} + t_Pnf(\theta) \frac{\partial s(\cdot)}{\partial b^U} \right] \\
  + \left( t_Psf(\theta)A \frac{g(\delta)}{G(\delta)} + t_PA \frac{f(\theta)}{s\phi''(s)} + nsf(\theta)\left[ - t_P \frac{\beta(1 - \eta)}{\eta(1 - \beta)} \frac{(1 - \tau)v'}{\Delta v} + \tau \right] \right) \frac{\partial w}{\partial b^U} = 0
\end{align*}
\]

(52)

Given Result 2, the first-order conditions on \( b \) and \( b^U \), (32) (with \( t_e = t_p = t_P \)) and (52) respectively, can be written as:

\[
\begin{align*}
  nsf(\theta)v'(\cdot) + \lambda \left[ - nsf(\theta) + t_Psf(\theta) \frac{\partial n(\cdot)}{\partial b} + t_Pnf(\theta) \frac{\partial s(\cdot)}{\partial b} \right] = 0, \quad (53) \\
  n(1 - sf(\theta))u'(\cdot) + (1 - n)\mu'(\cdot) + \lambda \left[ - (1 - nsf(\theta)) + t_Psf(\theta) \frac{\partial n(\cdot)}{\partial b^U} + t_Pnf(\theta) \frac{\partial s(\cdot)}{\partial b^U} \right] = 0.
\end{align*}
\]

(54)

With a common transfer to all the unemployed, the incentive-insurance trade-off faced by the government is now affected by the participation decision and Result 3 no longer holds. To see this most clearly, suppose \( \mu(b^U) = u(b^U) \) so the utility of consumption is the same regardless of the reason for being unemployed — voluntary or involuntary — and \( u' = \mu' \).\(^{22}\) Eliminating \( \lambda \) from (53) and (54) and using (48) and

\(^{22}\)The voluntary unemployed will, however, have a different level of utility than the involuntary unemployed.
the expressions for $\partial n/\partial b$ and $\partial s/\partial b$, yields
\[
(1 - nsf) \frac{u' - v'}{v'} = -t_P \left( \frac{\partial n}{\partial b} \frac{1}{n} + \frac{\partial s}{\partial b} \frac{1}{s} \right).
\] (55)

With a single unemployment transfer to both types of unemployed, moral hazard arises from both the search effort decision (as before) and the participation decision. Further with $u' = \mu'$ an increase in $b^U$ reduces both search effort and participation. From the government’s budget constraint, $t_P = -[b - b^U - \tau w] = b^U/(nsf)$, which will be positive given $b^U > 0$ and implies that the government cannot fully insure individuals against unemployment, $u' > v'$. Finally, it is straightforward to see from the first-order condition on $b$ (53) that Result 4 continues to hold.

4 Allowing for Many Productivity Skill-Types

We now return to the multiple-type case as outlined in Section 2 to consider the redistributive role played by the tax/transfer system when there are different skill-types. Tax and transfers can now be used to redistribute between different employment states for a given ability type as well as between employed or unemployed persons of differing skill-types.

The first thing to note is that Result 1 continues to hold with multiple skill-types.\(^{23}\) To highlight the different roles played by the tax/transfer system when there are multiple skill-types, we first consider the special case of no participation or search effort decisions with endogenous wages. We then allow for endogenous search and participation decisions with fixed wages which captures as special cases those considered by Saez (2002) and Chetty (2008) before turning to the general case with endogenous wages, participation and search effort. Finally, we restrict the government to provide the same transfer to all of the involuntarily unemployed, regardless of type.

\(^{23}\)This follows directly from noting that the first-order conditions on $\tau_i$ and $b_i$, (24) and (25), respectively are equivalent when wages are fixed.
4.1 Endogenous Wage with Fixed Participation and Search

In this case, it is assumed that $n_i$ and $s_i$ are fixed. The government may still transfer income to some fixed number of non-market participants, $1 - n_i$. By Result 1, if wages were also fixed, then the first-order conditions on $\tau_i$ and $b_i$ would be equivalent and either policy instrument could be used. Furthermore, the government could achieve the first-best: the marginal utility of consumption would be equalized across both ability types and employment/unemployment states, that is, $v'(1 - \tau_i)w_i + b_i = u'(b_i^*) = \mu'(b^*) = \lambda$ for $i = 1, \ldots, N$. The government would choose to set the same $b_i^*$ for all $i = 1, \ldots, N$ as the utility of consumption when unemployed is assumed independent of ability-type.

Now suppose wages are endogenous. The first-order conditions on $\tau_i$, $b_i$, and $b_i^*$ for $i = 1, \ldots, N$ reduce to the following:

$$- n_i s_i f(\theta_i) v'_i w_i + \lambda n_i s_i f(\theta_i) w_i + \Omega_i \frac{\partial w_i}{\partial \tau_i} = 0 \quad (56)$$

$$n_i s_i f(\theta_i) v'_i - \lambda n_i s_i f(\theta_i) + \Omega_i \frac{\partial w_i}{\partial b_i} = 0 \quad (57)$$

$$n_i (1 - s_i f(\theta_i)) u'(b_i^*) - \lambda n_i (1 - s_i f(\theta_i)) + \Omega_i \frac{\partial w_i}{\partial b_i^*} = 0 \quad (58)$$

where

$$\Omega_i = A_i + \lambda \left[ t_i n_i s_i f'(\theta_i) \frac{\partial \theta}{\partial w_i} + n_i s_i f(\theta_i) \tau_i \right]. \quad (59)$$

is the derivative of the government’s Lagrangian with respect to the wage rate $w_i$ when there is no search and participation decisions.

Using (16) we can rewrite the first-order condition on $\tau_i$ (56) as

$$- w_i \left( n_i s_i f(\theta_i) v'_i - \lambda n_i s_i f(\theta_i) + \Omega_i \frac{\partial w_i}{\partial b_i} \right) + \Omega_i \frac{\beta(y_i - w_i) v'_i}{D_i} = 0 \quad (60)$$

The binding first-order condition on $b_i$ (57), implies that the first term is zero and therefore, the optimal type-specific proportional wage tax is chosen to ensure that the
Lagrangian of the government’s problem is maximized with respect to the equilibrium wage \( w_i \), that is, \( \Omega_i = 0, \forall i \).

Using this result, it follows immediately from the first-order conditions (57) and (58) on \( b_i \) and \( b_i^f \) that \( v'((1 - \tau_i)w_i + b_i) = u'(b_i^f) = \mu'(b^V) = \lambda, \forall i \). With endogenous wages, type-specific linear income taxes can be used to ensure that the macro effects arising from endogenous wages does not affect the choice of income transfers. Furthermore, given that there is no moral hazard with fixed participation and fixed search effort, the government can achieve full equality of the marginal utility of consumption across both skill-types and employment/unemployment states.

**Result 5** With multiple productivity types when participation and search effort is exogenous, the government can achieve the first-best with equal marginal utility of consumption across all skill-types and employment using type-specific linear income taxes.

### 4.2 Endogenous Participation and Search with Fixed Wages

By Result 1, \( \tau_i \) and \( b_i \) are equivalent policy instruments and the government could use either policy instruments. Focus on the use of \( b_i \). With fixed wages, the first-order conditions on \( b_i \) and \( b_i^f \) for \( i = 1, \ldots, N \) and \( b^V \) become:

\[
    n_i s_i f(\theta_i) v'_i - \lambda \left( n_i s_i f(\theta_i) - t_{pi} \frac{\partial n_i}{\partial b_i} - t_{ei} n_i f(\theta_i) \frac{\partial s_i}{\partial b_i} \right) = 0 \tag{61}
\]

\[
    n_i (1 - s_i f(\theta_i)) u'(b_i^f) - \lambda \left( n_i (1 - s_i f(\theta_i)) - t_{pi} \frac{\partial n_i}{\partial b_i^f} - t_{ei} n_i f(\theta_i) \frac{\partial s_i}{\partial b_i^f} \right) = 0 \tag{62}
\]

\[
    \sum_{i=1}^{N} \left( (1 - n_i) \mu'(b^V) - \lambda \left( (1 - n_i) - t_{pi} \frac{\partial n_i}{\partial b^V} \right) \right) = 0 \tag{63}
\]

It is straightforward to see that Result 3 continues to hold with multiple skill-types. Dividing (61) and (62) by the relevant marginal utility of consumption and then
eliminating $\lambda$ from the two conditions and using (3) and (6) yields

$$s_i(1 - s_i f(\theta_i)) \frac{v'_i - u'(b'_i)}{v'_i} = t_{ei} \frac{\partial s_i}{\partial b'_i}.$$  \hspace{1cm} (64)

This is analogous to (38) and has the same interpretation as in single skill-type case. The optimal employment tax rule is not affected by the participation decision and the same expression would be obtained if $n_i$ were assumed fixed as in Chetty (2008). The employment tax for any given skill-type will be positive since individuals are choosing how much search effort to exert and consequently, moral hazard prevents the government from providing full unemployment insurance.\footnote{Recall, an individual will only exert positive search effort if he is better off working than being involuntarily unemployed, that is, if $\Delta v_i = v((1 - \tau_i)w_i + b_i) - u(b'_i) > 0$. We have left unspecified the relationship between $v$ and $u$, but under reasonable assumptions, e.g., $v(c) = u(c - \rho)$ or $v(c) = u(c) - \rho$ where $\rho \geq 0$ is some disutility from working, it follows directly from $v_i > u(b'_i)$ that $v'_i < u'_i$ and individuals are not fully insured against involuntary unemployment. Consequently, by (38), the employment taxes are all positive.}

As in the single skill-type case, the optimal participation tax rule is again modified to account for the inability of the government to provide complete unemployment insurance with endogenous search effort. Unlike the single skill-type, some optimal participation taxes may be negative. This is most easily seen in the case of fixed search effort. In this case, the government can provide full unemployment insurance ($v'_i = u'$) and the optimal participation tax can be derived from the first-order condition on $b'\hspace{1cm} (61)$:

$$t_{pi} = n_i s_i f(\theta_i) \frac{1 - v'_i/\lambda}{\partial n_i/\partial b_i}.$$  \hspace{1cm} (65)

This optimal participation tax rule is equivalent in interpretation to the one obtained by Saez (2002) who does not allow for involuntary unemployment, and is analogous to the one obtained in the single skill-type case (41). The expression in the denominator reflects the responsivity of the tax base (participation) to the tax. The expression $v'_i/\lambda$ in the numerator is the value in terms of government revenue of an increment in transfer to type $-i$ individuals and is the same as social welfare weight, $g_i$, defined in

24
Saez (2002). With multiple skill-types, \( v'_i > \lambda \) for some \( i \) and the optimal participation tax is negative as highlighted in Saez (2002).

As in the single skill-type case, endogenous search tends to reduce the optimal participation tax. To see this, solving for the optimal participation tax from (61) to yield the analogue of (43):

\[
t_{pi} = n_i s_i f(\theta_i) \frac{1 - v'_i/\lambda}{\partial n_i/\partial b_i} - t_{ei} n_i f(\theta_i) \frac{\partial s_i/\partial b_i}{\partial n_i/\partial b_i}
\]

where again the positive employment taxes puts downward pressure on the participation tax as a higher transfer to the employed, \( b_i \), increases both search effort and participation and we have Result 6.

**Result 6** With multiple productivity types and endogenous participation, the optimal participation tax at the bottom of the wage distribution will be negative. Endogenous search effort accentuates the negativity of the optimal participation tax at the bottom.

### 4.3 General Case

With type-specific linear income taxes, the government can again use the proportional tax to ensure that the equilibrium wage is optimal, that is, the government’s Lagrangian is maximized with respect to \( w_i \) for all \( i = 1 \ldots N \) as in the single skill-type case and Result 2 holds. This result has already been demonstrated above in the case without any moral hazard, and it is straightforward to show it continues to hold with participation and search decisions. Substituting the first-order condition on \( b_i \) into the first-order condition on \( \tau_i \) in the general case and using the following relationship in (16) yields the following first-order condition on \( \tau_i \):

\[
\Omega_i + \lambda \left( t_{pi} A_i \frac{g(\delta_i)}{G(\delta_i)} + t_{ei} A_i \frac{f(\theta_i)}{s_i \phi'(s_i)} \right) = 0
\]

(66)

where as in the single-type case, the left-hand side represents the change in the government’s Lagrangian with respect to \( w_i \). Relative to the case with no moral
hazard, there are two additional terms in this expression reflecting the effect of a change in \( w_i \) on individuals’ search and participation decision and the ultimate effect of these decisions on revenue. Both of these terms depend on \( A_i \) and will be zero if the Hosios condition holds. Given that \( \tau_i \) is chosen to ensure the optimality of \( w_i \), the first-order conditions on \( b_i \) and \( b_I \) will again be the same as the case with fixed wages as given above.

### 4.4 Same Transfer to All Involuntarily Unemployed

The assumption that the government can condition the transfers to the involuntary unemployed on skill-type requires that it can observe not only the job search activities on the individuals, but also the type of job the individual searches for. This might be considered too strong an informational assumption. Suppose instead the government can only observe that an individual applied for work, but was unsuccessful in obtaining a job. In this case, \( b_I^i = b_I^I \), \( \forall i \). As noted already, with fixed participation and fixed search effort this would not affect the outcome as the government is already optimally chooses a common value of \( b_I^I \). It will, however, constrain the government when either search or participation is endogenous.

To see this, continue to assume wages are fixed. As shown already, the government only needs one policy instrument to tax/transfer to the employed with fixed wages. Assume it uses \( b_i \). The first-order condition on \( b_i \) will still be given by (61) and that on \( b_I \) will also be unchanged. The first-order condition on \( b_I^I \) becomes:

\[
\sum_i \left[ n_i (1 - s_i f(\theta_i)) u'(b_I^I) - \lambda \left( n_i (1 - s_i f(\theta_i)) - t_{pi} \frac{\partial n_i}{\partial b_I^I} - t_{ei} n_i f(\theta_i) \frac{\partial s_i}{\partial b_I^I} \right) \right] = 0 \quad (67)
\]

We can again determine an expression illustrating the insurance-incentive trade-off the government faces in selecting the transfers to the employed and the involuntary unemployed by using the first-order conditions on \( b_i \) and \( b_I^I \), (61) and (67), respec-
tively.\textsuperscript{25}

\[
\left(\frac{\sum_i n_i(1 - s_i f(\theta_i))}{\sum_i n_i}\right) \sum_i n_i s_i f(\theta_i) \left( \frac{v'_i(w_i + b_i) - u'(b')}{v'_i(w_i + b_i)} \right) = \sum_i t_{ei} n_i f(\theta_i) \frac{\partial s_i}{\partial b'} \tag{68}
\]

When \(b'\) cannot be conditioned on ability-type, a change in \(b'\) affects the search behaviour of all individuals (as represented by the right-hand side expression) as well as the unemployment insurance they receive (as represented by the left-hand side expression) where the first term on the left-hand side is the share of the population that is involuntarily unemployed. The above reduces to (64) if \(b'\) can be conditioned on ability-type.

\section{Two-Period Model}

In this section, we go back to the one-type case. Assume now that there are two periods and individuals can save. We begin with a description of each of the two periods, the timing of decisions and the labour market outcomes. We then outline the individuals’s decisions and characterize the equilibrium. Finally, we determine the government’s optimal policies, highlighting how the results of the static model carry-over to this simple dynamic setting. The formal analysis of this two period model is in Appendix C.

\subsection{Timing of Decisions and Outcomes}

There are two periods. Period 1 is similar to the static model with a single productivity type considered in Section 3. There is a measure 1 of households. They choose

\textsuperscript{25}The following expression is obtained by first substituting (3) and (6) into (61). Then divide each equation in (61) by the relevant marginal utility of consumption, then add them all together, and solve for \(\lambda\). Similarly, another expression for \(\lambda\) can be obtained by substituting (3) and (6) into (67). Divide the result by the relevant marginal utility of consumption and solve the resulting expression for \(\lambda\). Setting these two expressions for \(\lambda\) equal to one another and simplifying yields the expression in the text.
whether to participate in job search, anticipating not only their chances of getting a job but also what happens in period 2. Utility for the various types of households - employed, involuntarily unemployed and voluntarily employed, and the fixed cost of search are identical as in the static model. For simplicity, households only make a participation decision in the first period. The same matching technology applies as above: a proportion \( p = sf(\theta) \) of the \( n \) that search find jobs at the endogenous wage \( w_1 \) where the subscript denotes the equilibrium wage in period 1.\(^{26}\) After wages are paid and transfers received, individuals can choose to save for the next period using their initial asset wealth \( a_0 \), but cannot borrow against their future income (i.e., they are credit constrained to have non-negative financial wealth). Employed persons can save some of their first-period earnings to self-insure denoted by \( a^W \). Unemployed persons in the first period, either voluntary and involuntary, can also choose to save some of their income transfers, denoted by \( a^V \) and \( a^I \), respectively. Let both the interest rate and the endowment of assets, denoted \( a_0 \), be zero for simplicity.

At the beginning of period 2, an exogenous proportion \( \sigma \) of employed workers are separated from employment and become involuntarily unemployed. As mentioned, we assume that no one changes their participation decision at the end of the first period. However, those who are involuntarily unemployed in the first period search for work in the second period.

The firm’s zero-profit condition and proportional bargaining need to take account of period-2 outcomes. We assume that the bargained wage is constant for two periods since workers are risk-averse while firms are risk-neutral. That is, workers employed in period 1 who remain employed in period 2, receive \( w_1 \) in both periods. Thus, firms assume any risk. We denote by \( w_2 \) the equilibrium wage determined in period 2 and that is paid to households who were involuntarily unemployed in period 1 and employed in the second period.

The government implements a linear income tax at rate \( \tau_j \) with a lump-sum trans-

\(^{26}\)Throughout this section, we use a subscript to denote the time period the given variable was determined.
fer to the employed $b_j$ where the subscript $j = 1, 2$ denotes the period in which the wage was determined. As in the previous cases considered, the government is assumed to be able to observe wage rates so the income tax system can be conditioned on the observable wages. In period 2, there will be two different wages being paid - one to workers who have been employed for two periods ($w_1$) and one for workers who have only been employed one period ($w_2$). The government also provides an income transfer to the voluntary unemployed $b^V$. This transfer applies for both periods.\footnote{It is possible the government would want to condition $b^V$ on the period to allow different transfers to the short-term and long-term voluntarily unemployed. For now, we assume away this possibility by having a participation decision only at the beginning of the first period, so individuals will be voluntarily unemployed for both periods.} The involuntary unemployed obtain $b^I$ in the first period of their unemployment and $b^{II}$ if they are unemployed for a second period. In other words, unemployment insurance differs for long-term (periods 1 and 2) versus short-term (period 1 or 2) unemployed: $b^{II}$ versus $b^I$.\footnote{The government is assumed to know whether an individual received an involuntary unemployment transfer in the previous period. As there are two different wages in the second period, with $b^{II}$ going to those who would have received $w_2$ and $b^I$ going to those who would have received $w_1$, the government is effectively conditioning the involuntary unemployed transfer on wages.}

The timing is as follows: At the beginning of period 1, the government sets policies ($\tau_1, b_1, \tau_2, b_2, b^I, b^{II}, b^V$), all individuals make a participation decision and the participants make a search effort decision. Period 1 labour market equilibrium ($\theta_1, w_1$) is then determined and employed individuals paid wages and pay taxes, and voluntary and involuntary unemployed receive income transfers. At the end of period 1, savings decision for individuals made by the voluntary unemployed, the involuntary unemployed and the employed. At the beginning of period 2, search decision by individuals who were involuntarily unemployed in period 1 and exogenous separation of individuals who were employed in period 1. Period 2 labour market equilibrium ($\theta_2, w_2$) is then determined and employed individuals pay wages and pay taxes, and voluntary and involuntary unemployed receive income transfers.
5.2 Individual Decisions

To analyze equilibrium outcomes, begin with period 2 and work backwards.

5.2.1 Second-period search decision

At the beginning of period 2, only the first-period involuntarily unemployed make a decision. They search for a job, where search effort satisfies:

$$\max_{s_2} s_2 f(\theta_2) v(c^{IW}) + (1 - s_2 f(\theta_2)) u(c^{II}) - \phi(s_2)$$

where $c^{IW} = (1 - \tau_2)w_2 + b_2 + a^I$, and $c^{II} = b^{II} + a^I$. The first-order condition is:

$$f(\theta_2)[v((1 - \tau_2)w_2 + b_2 + a^I) - u(b^{II} + a^I)] - \phi'(s_2) = 0.$$  

This yields optimal second-period search $s(\tau_2, b_2, b^{II}; a^I, \theta_2, w_2)$.

5.2.2 Savings decisions

After the employment outcomes are determined in period 1, the voluntary unemployed, the involuntary unemployed and the employed can all make a savings decision. Recall our assumption that $a_0 = 0$ and the interest rate is zero. The voluntary unemployed solve the following problem:

$$\max_{\{s\geq 0\}} \mu(b^V - a^V) + \delta + \mu(b^V + a^V) + \delta$$

which yields $a^V(b^V; \delta) = 0$. With a zero interest rate, the voluntary unemployed will never save. The voluntary unemployed’s value function will be $V^V_V(b^V; \delta)$.

The involuntary unemployed anticipate their optimal search behaviour when they choose how much to save in period one and solve the following problem:

$$\max_{\{a^V \geq 0\}} u(b^I - a^I) + s(\cdot)f(\theta_2)v(c^{IW}) + (1 - s(\cdot)f(\theta_2)) u(c^{II}) - \phi(s(\cdot))$$
The first-order condition for \( a^I \) is:

\[
-u'(b^I - a^I) + s(\tau_2, b_2, b^{II}; a^I, \theta_2, w_2)f(\theta_2)v'(1 - \tau_2)w_2 + b_2 + a^I + (1 - s(\tau_2, b_2, b^{II}; a^I, \theta_2, w_2))u'(b^{II} + a^I) = 0
\]

which yields \( a^I(\tau_2, b_2, b^I, b^{II}; \theta_2, w_2) \) and by substituting back into the objective, the involuntary unemployed value function \( V^I(\tau_2, b_2, b^I, b^{II}; \theta_2, w_2) \).

The employed solve the following problem:

\[
\max_{\{a^W \geq 0\}} v((1 - \tau_1)w_1 + b_1 - a^W) + (1 - \sigma)v((1 - \tau_1)w_1 + b_1 + a^W) + \sigma u(b^I + a^W)
\]

The first-order condition is:

\[
-v'(c^W) + (1 - \sigma)v'(c^{WW}) + \sigma u'(c^{WI}) = 0
\]

where \( c^W = (1 - \tau_1)w_1 + b_1 - a^W \) and \( c^{WW} = (1 - \tau_1)w_1 + b_1 + a^W \) which yields \( a^W(\tau_1, b_1, b^I; w_1) \) and the value function for the employed persons \( V^W(\tau_1, b_1, b^I; w_1) \).

### 5.2.3 First-period search and participation decision

Next, search effort for those who search at the beginning of period 1 satisfies:

\[
\max_{\{s_1\}} s_1f(\theta_1)V^W_1(\tau_1, b_1, b^I; w_1) + (1 - s_1f(\theta_1))V^I_1(\tau_2, b_2, b^I, b^{II}; \theta_2, w_2) - \phi(s_1)
\]

where \( \theta_1, w_1, \theta_2 \) and \( w_2 \) as well as government policies are all taken as given by the individual worker. The first-order condition is

\[
f(\theta_1)[V^W_1(\tau_1, b_1, b^I; w_1) - V^I_1(\tau_2, b_2, b^I, b^{II}; \theta_2, w_2)] - \phi'(s_1) = 0
\]

which yields \( s_1(\tau_1, b_1, b_2, b^I, b^{II}; \theta_1, w_1, \theta_2, w_2) \).

The participation condition can then be written as:

\[
s_1(\cdot)f(\theta_1)V^W_1(\tau_1, b_1, b^I; w_1) + (1 - s_1(\cdot)f(\theta_1))V^I_1(\tau_2, b_2, b^I, b^{II}; \theta_2, w_2) - \phi(s_1(\cdot)) \geq V^V_1(b^V; \delta)
\]

which binds at \( \delta(\tau_1, b_1, \tau_2, b_2, b^I, b^{II}, b^V; \theta_1, w_1, \theta_2, w_2) \) and those individuals with \( \delta \leq \delta(\cdot) \) participate in the labour market and those with \( \delta > \delta(\cdot) \) choose to be non-participants.
5.3 Labour Market Equilibria

5.3.1 Period 2

The firm’s zero-profit condition for a vacancy yields $q(\theta_2) = k/(y - w_2) = f(\theta_2)/\theta_2$ and $\theta_2(w_2)$ with $\theta'_2(w_2) < 0$. The wage $w_2$ satisfies

$$\max_{w_2} (\Delta v_2)^\beta (y - w_2)^{1-\beta}$$

where

$$\Delta v_2 = v((1 - \tau_2)w_2 + b_2 + a^I) - u(b^{II} + a^I) = v(c^W) - u(c^{II})$$

and the first-order condition yields

$$w_2 = y - \frac{1 - \beta}{\beta} \frac{\Delta v_2}{(1 - \tau_2)v'(c^W)}.$$  

At the start of period 2, the saving of the involuntary unemployed has already been determined, as shown above, and is given by $a^I(\tau_2, b_2, b^I, b^{II}; \theta_2, w_2)$. Using this, the above yields $w_2(\tau_2, b_2, b^I, b^{II})$.

5.3.2 Period 1

The firm’s zero-profit condition for a vacancy in period 1 is:

$$q(\theta_1)(y - w_1 + (1 - \sigma)(y - w_1)) = \frac{f(\theta_1)}{\theta_1}(2 - \sigma)(y - w_1) = k$$

which yields $\theta_1(w_1)$ with $\theta'_1(w_1) < 0$. The wage $w_1$ satisfies

$$\max_{w_1} (\Delta V)^\beta ((2 - \sigma)(y - w_1))^{1-\beta}$$

where

$$\Delta V = V_1^W(\tau_1, b_1, b^I; w_1) - V_1^I(\tau_2, b_2, b^I, b^{II}; \theta_2, w_2)$$

and the first-order condition is

$$w_1 = y - \frac{1 - \beta}{\beta} \frac{\Delta V}{(1 - \tau_1)(v'(c^W) + (1 - \sigma)v'(c^{WW}))}.$$  

Using $a^W(\tau_1, b_1, b^I; w_1)$, the above condition yields $w_1(\tau_1, b_1, \tau_2, b_2, b^I, b^{II})$.  

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5.4 Equilibrium

Given the equilibrium wages, \( w_1(\tau_1, b_1, \tau_2, b_2, b^I, b^{II}) \) and \( w_2(\tau_2, b_2, b^I, b^{II}) \), and the equilibrium tightness of the labour market \( \theta_1(w_1) \) and \( \theta_2(w_2) \), given above, we can write (again abusing notation), the optimal search effort in both the first and second period, the utility of leisure cut-off, \( \bar{\delta} \), and the individual’s indirect utility function if working or involuntary unemployed in the first period as functions solely of the policy parameters, that is,

\[
s_1(\tau_1, b_1, \tau_2, b_2, b^I, b^{II}) = s_1(\tau_1, b_1, \tau_2, b_2, b^I, b^{II}; \theta_1(w_1(\cdot)), w_1(\cdot), \theta_2(w_2(\cdot)), w_2(\cdot))
\]

\[
s_2(\tau_2, b_2, b^I, b^{II}) = s_2(\tau_2, b_2, b^{II}; a^I(\tau_2, b_2, b^I, b^{II}; \theta_2(w_2(\tau_2, b_2, b^I, b^{II}))), w_2(\cdot)), \theta_2(w_2(\cdot)), w_2(\cdot))
\]

\[
n(\tau_1, b_1, \tau_2, b_2, b^I, b^{II}, b^V) = G(\bar{\delta}(\tau_1, b_1, \tau_2, b_2, b^I, b^{II}, b^V; \theta_1(w_1(\cdot)), w_1(\cdot), \theta_2(w_2(\cdot)), w_2(\cdot)))
\]

\[
V_1^W(\tau_1, b_1, \tau_2, b_2, b^I, b^{II}) = V_1^W(\tau_1, b_1, b^I; w_1(\cdot))
\]

\[
V_1^I(\tau_2, b_2, b^I, b^{II}) = V_1^I(\tau_2, b_2, b^I, b^{II}; \theta_2(w_2(\cdot)), w_2(\cdot))
\]

As in the static model, the transfer to the voluntary unemployed only affects the number of labour market participants and not the labour market outcomes.

5.5 The Government’s Problem

Adopting a utilitarian objective function, the government’s problem is as follows:

\[
\max_{\{\tau_1, b_1, \tau_2, b_2, b^I, b^{II}, b^V\}} \left[ s_1(\cdot)f(\theta_1(w_1(\cdot)))V_1^W(\tau_1, b_1, \tau_2, b_2, b^I, b^{II}) \\
+ (1 - s_1(\cdot)f(\theta_1(w_1(\cdot))))V_1^I(\tau_2, b_2, b^I, b^{II}) - \phi(s_1(\cdot)) \right] n(\cdot) + \int_{\delta(\cdot)}^{\delta_{\text{max}}} V_1^V(b^V; \delta)g(\delta)d\delta
\]

subject to

\[
n(\cdot)[s_1(\cdot)f(\theta_1(w_1(\cdot)))(2 - \sigma)]b_1 + n(\cdot)[(1 - s_1(\cdot)f(\theta_1(w_1(\cdot))))s_2(\cdot)f(\theta_2(w_2(\cdot)))b_2
\]

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\[
+n(\cdot)\left[s_1(\cdot)f(\theta_1(w_1(\cdot)))\sigma + (1 - s_1(\cdot)f(\theta_1(w_1(\cdot))))\right]b^I
+n(\cdot)(1 - s_1(\cdot)f(\theta_1(w_1(\cdot))))(1 - s_2(\cdot)f(\theta_2(w_2(\cdot))))b^{II} + 2\left(1 - n(\cdot)\right)b^V
\]

\[
= n(\cdot)s_1(\cdot)f(\theta_1(w_1(\cdot)))\left(2 - \sigma\right)w_1(\cdot)\tau_1 + n(\cdot)\left[(1 - s_1(\cdot)f(\theta_1(w_1(\cdot))))s_2(\cdot)f(\theta_2(w_2(\cdot)))\right]w_2(\cdot)\tau_2
\]

As in the previous sections, we begin with considering some special cases. In the first case, individuals only make a savings decision and we consider both fixed and endogenous wages. In the second case, wages are assumed fixed and individuals make savings, search effort and participation decisions.

### 5.5.1 Fixed participation and search effort

For ease of notation, assume that \( n = G(\delta) = 1 \) so all individuals participate and normalize \( s_1 = s_2 = 1 \).\(^{29}\) With fixed wages, it is again the case that the \( \tau_i \) and \( b_i \) are policy substitutes and Result 1 holds. As in the static case, without moral hazard the government can ensure that individuals are fully insured against involuntary unemployment where involuntary unemployment in this two period model can raise both from not being able to find work and from being involuntarily separated from employment. Full unemployment insurance is obtained whether or not individuals are able to save. With endogenous wages, the proportional wage taxes can again be used to ensure the equilibrium wages determined in each period are optimal, that is, maximize the government’s Lagrangian. As in the static case, the proportional wage tax has a differential effect on the equilibrium wage relative to the lump-sum wage tax in any given period, but unlike the static case the wage taxes chosen in period 1 affect the second period equilibrium wage. The government still has sufficient instruments, however, to ensure the derivative of its Lagrangian with respect to the wages determined in each period are both equal to zero as shown in Appendix C and Result 2 holds. Consequently, the first-order conditions on the transfers to the employed, the short-term involuntary unemployed, and the long-term involuntary unemployed

\(^{29}\)The government’s problem and first-order conditions are given in Appendix C.
are the same as when wages are fixed and we have the following result (which extends Result 5 to a simple dynamic setting):

Result 7 With fixed participation and search effort, the government can achieve the first-best with full equality of the marginal utility of consumption across time and employment states.

5.5.2 Endogenous participation and search effort with fixed wages

As noted above, with fixed wages it is again the case that $\tau_i$ and $b_i$ are policy substitutes and we focus on the use of $b_i$. With individuals making participation and search effort decisions, the government is faced with an incentive-insurance trade-off. To characterize this trade-off in the two-period model, we first define the following participation tax:

$$t_{p1} = -\left[s_1f(\theta_1)(2 - \sigma)b_1 + (1 - s_1f(\theta_1))s_2f(\theta_2)b_2 + (s_1f(\theta_1)\sigma + 1 - s_1f(\theta_1))b^I + (1 - s_1f(\theta_1))(1 - s_2f(\theta_2))b^{II} - 2b^V\right]$$

which represents the change in government revenue if one more individual chooses to participate in the labour market in the first period. Using this expression, we can rewrite the government’s budget constraint as $t_{p1} = 2b^V/n$ which implies that $t_{p1} > 0$ if $b^V > 0$. We also define the first period employment tax

$$t_{e1} = -\left[(2 - \sigma)b_1 - s_2f(\theta_2)b_2 + (\sigma - 1)b^I - (1 - s_2f(\theta_2))b^{II}\right]$$

which represents the change in the government revenue when an individual moves from involuntary unemployment to employment in the first period and the second period employment tax

$$t_{e2} = -(1 - s_1f(\theta_1))\left(b_2 - b^{II}\right)$$

$^{30}$The government’s problem and first-order conditions are given in Appendix C.
which represents the change in government revenue when an individual moves from involuntary unemployment to employment in the second period.

For ease of comparison to the static case, suppose first there was no search effort decision. Given no moral hazard in search effort, the government can ensure full equality of the marginal utilities of consumption for all participants across time and employment states, but given an endogenous participation decision the marginal utility of consumption of the voluntary will be greater than for the participants.

Next, suppose participation is fixed and individuals only make a search effort decision. Without the ability to save, the incentive-insurance trade-off in choosing between \( b_1 \) and \( b' \) and between \( b_2 \) and \( b'' \) depends only on the employment tax and the effect of the unemployment transfer on search effort in the relevant period, i.e., period 1 and period 2, respectively. With the ability to save, the transfer the involuntary unemployed receive in period 1, the tax they pay if employed in period 2, and the transfer they receive if involuntarily unemployed in period 2, all affect how much savings they undertake at the end of period 1. Their savings, in turn, affects their search effort decision at the beginning of period 2. The effect of these three policies \( \{ b', b_2, b'' \} \) on second period search effort must be taken into account in choosing the optimal unemployment insurance and will affect the incentive-insurance trade-off both between \( b_1 \) and \( b' \) and between \( b_2 \) and \( b'' \). In the former case, the second employment tax becomes relevant as opposed to the case without savings. This logic continues to hold when individuals also make a participation decision. As in the static case, the participation tax does not affect the incentive-insurance trade-off (Result 3) and endogenous search effort decision tends to reduce the optimal positive participation tax (Result 4) given positive employment taxes.
6 Conclusion

We have explored the joint design of unemployment insurance and redistributive taxes and transfers in a setting with individuals of different skills who choose both whether to participate in job search and how intensively to search. The existing literature on optimal unemployment insurance sets aside redistributive considerations and focuses on the insurance-moral hazard trade-off in choosing an efficient unemployment insurance system. On the other hand, optimal redistribution models with involuntary unemployment takes search effort as fixed and focuses on the participation decision as in extensive-margin approaches to optimal income taxation. No distinction is made between transfers paid to the voluntary and involuntary unemployed. Our analysis combined these two approaches.

We considered how search decisions influence optimal participation taxes on the one hand, and how participation choices affect unemployment insurance. We find that second-best optimal policy requires using a proportional wage tax to address macro effects arising from changes in wage bargaining outcomes, leaving participation taxes and unemployment insurance to address redistributive and insurance objectives without concern for their impact on wage setting. While optimal participation taxes are moderated by search effort, unemployment insurance is not influenced by participation decisions.

We have adopted a number of simplifying assumptions to facilitate our analysis. We abstract from intensive-margin labour decisions by assuming that work effort is fixed in employment. We have assumed that the government can observe who is involuntarily unemployed so that workers cannot refuse employment or quit jobs to take advantage of unemployment insurance. We have assumed that workers direct their search only to labour markets catering to their skills and we adopt Nash bargaining to wage-setting with risk-averse workers. We have chosen to use proportional wage taxes to correct for the macro effects of redistribution and unemployment insurance policies, rather than say employment subsidies. These assumptions are largely con-
sistent with those that have been made in the related literature, and they enable us to obtain relatively clear and intuitive results. Relaxing them would take us too far afield for the scope of this paper.
References


A Optimality of Search with Risk Aversion

Let the number of job offers $o$, search effort $s$ and participation, or the number of job-seekers $n$, be choice variables. The matching function is $m(s, o)$ or $m(1, o/(sn))$ equivalent to $f(o/(sn))$ where $ns$ is aggregate search effort. Therefore, $nsf(o/(sn))$ is the expected number of matches. Then, $\theta = o/(ns)$ is labour market tightness, and depends on all three endogenous variables. Consider first the market solution and then the optimal one, and abstract from taxes and transfers, so $\tau = b = b^f = b^v = 0$. Worker utility is $v(w)$ if employed, $u$ if involuntarily unemployed and $\mu$ if voluntarily unemployed. Where necessary, we normalize utility using consumption if employed as numeraire, so money-metric utility is $v(w)/v'(w)$, $u/v'(w)$ and $\mu/v'(w)$. We take $u$ and $\mu$ as given in what follows, and we assume for simplicity that the elasticity of $f(o/(sn))$ is constant, so $\eta$ is constant.

Labour market equilibrium

Given free entry of firms, the zero-profit condition applies:

$$nsf(o/(ns))(y - w) = ok \tag{A.1}$$

where $y$ is output per worker and $w$ is the wage rate. The latter is determined by the solution to the Nash bargaining problem between the firm and the worker when a match is made:

$$\max_{\{w\}} (v(w) - u)^\beta (y - w)^{1-\beta} = (\Delta v(w))^\beta (y - w)^{1-\beta}$$

The first-order condition is:

$$\beta(\Delta v(w))^{\beta-1}(y - w)^{1-\beta}v'(w) - (1 - \beta)(\Delta v(w))^\beta(y - w)^{-\beta} = 0$$

which can be written:

$$y - w = (1 - \beta)\left(y - w + \frac{\Delta v(w)}{v'(w)}\right) \tag{A.2}$$

This determines $w$. Substituting it into the zero-profit condition (A.1) gives:

$$\frac{f(o/(ns))}{o/(ns)}(1 - \beta)\left(y - w + \frac{\Delta v(w)}{v'(w)}\right) = k$$
Using \( f'(o/(ns))(o/(ns))/f(o/(ns)) = 1 - \eta \), this can be written:

\[
f'(o/(ns)) \frac{1 - \beta}{1 - \eta} \left( y - w + \frac{\Delta v(w)}{v'(w)} \right) = k \tag{A.3}
\]

Equilibrium search effort satisfies (2), or in the absence of taxes and transfers:

\[
f(o/(ns))\Delta v(w) = \phi'(s) \tag{A.4}
\]

Eq. (A.2) from the Nash bargaining problem can be written

\[
\frac{\Delta v(w)}{v'(w)} = \beta \left( y - w + \frac{\Delta v(w)}{v'(w)} \right) \tag{A.5}
\]

Substituting this in (A.4) gives:

\[
f(o/(ns))\beta \left( y - w + \frac{\Delta v(w)}{v'(w)} \right) = \frac{\phi'(s)}{v'(w)} \tag{A.6}
\]

The participation condition (4) in the absence of taxes and transfers becomes:

\[
sf(o/(ns))v(w) + (1 - sf(o/(ns)))u - \phi(s) \geq \mu + \bar{\delta}
\]

It is binding at \( \bar{\delta} \) where:

\[
sf(o/(ns))\Delta v(w) + u - \phi(s) - \mu - \bar{\delta} = 0
\]

Using (A.5), this can be written:

\[
sf(o/(ns))\beta \left( y - w + \frac{\Delta v(w)}{v'(w)} \right) + \frac{u - \phi(s) - \mu - \bar{\delta}}{v'(w)} = 0 \tag{A.7}
\]

Denote the market equilibrium outcomes as \( w^m, o^m, s^m, \bar{\delta}^m \) and \( n^m = G(\bar{\delta}^m) \). By (A.2), (A.3), (A.6) and (A.7) they satisfy

\[
y - w^m = (1 - \beta) \left( y - w^m + \frac{\Delta v(w^m)}{v'(w^m)} \right) \tag{A.2'}
\]

\[
f'(o^m/(n^m s^m)) \frac{1 - \beta}{1 - \eta} \left( y - w^m + \frac{\Delta v(w^m)}{v'(w^m)} \right) = k \tag{A.3'}
\]

\[
f(o^m/(n^m s^m))\beta \left( y - w^m + \frac{\Delta v(w^m)}{v'(w^m)} \right) = \frac{\phi'(s^m)}{v'(w^m)} \tag{A.6'}
\]
Planning equilibrium

We characterize the planning equilibrium for a given \( w \), thereby focusing on the optimality of the search process. Since \( w \) is given, so is \( v(w) \) and we can drop the argument \( w \) in what follows. The planner chooses \( o, s \) and \( \bar{\delta} \) to maximize

\[
s^m f(o^m/(n^m s^m)) \beta \left( y - w^m + \frac{\Delta v(w^m)}{v'(w^m)} \right) + \frac{u - \phi(s^m) - \mu - \bar{\delta}^m}{v'(w^m)} = 0 \quad (A.7')
\]

where

\[
s_n \left( y - w + \frac{\Delta v}{v'} \right) + n \left( 1 - s f(o/(sn)) \right) \frac{u}{v'} + \left( 1 - n \right) \frac{\mu}{v'} + \int_{\bar{\delta}}^1 \frac{\delta}{v'} dG(\delta) - ok - n \frac{\phi(s)}{v'}
\]

We observe immediately that if \( \beta = \eta \) (the Hosios condition) is satisfied, eqs. (A.3), (A.6') and (A.7') are equivalent to (A.8), (A.9) and (A.10) respectively, and market outcomes are socially optimal.

Suppose the Hosios condition is not satisfied. Then for a given wage rate the market outcome will not be socially optimal. To see this, let \( \theta^m = o^m/(s^m n^m) \). From (A.3) and (A.8),

\[
\frac{F'(\theta^m)}{f'(\theta^m)} = \frac{1 - \eta}{1 - \beta} \iff \eta \leq \beta \iff f(\theta^m) \leq f(\theta^*)
\]

Then, from (A.6) and (A.9),

\[
\frac{f(\theta^m) \beta}{f(\theta^m) \eta} = \frac{\phi'(s^m)}{\phi'(s^*)}
\]
Since $\eta \geq \beta$ iff $f(\theta^m) \geq f(\theta^s)$, we have $\beta f(\theta^m) \geq \eta f(\theta^s)$ regardless of relative sign of $\eta$ and $\beta$, so $s^m \geq s^s$.

Finally, from (A.7) and (A.10), we have:

$$\frac{s^m f(\theta^m)\beta}{s^s f(\theta^s)\eta} = \frac{\delta^m + \phi(s^m) + \mu - u}{\delta^s + \phi(s^s) + \mu - u}$$

so apparently $\bar{\delta}^m \geq \bar{\delta}^s$.

The social optimum can be achieved by applying three lump-sum taxes: $t_o$ on offers, $t_p$ on participation and $t_s$ on search effort, where the signs of each depend on whether $\eta \geq \beta$. To see this, consider first $t_o$. Condition (A.3′) becomes

$$f'(\theta^m/\n^m s^m) \frac{1 - \beta}{1 - \eta} \left( y - w^m + \frac{\Delta v(w^m)}{v'(w^m)} \right) = k + t_o$$

Combining this with (A.8), yields:

$$\frac{f'(\theta^m)}{f'(\theta^s)} = \frac{1 - \eta k + t_o}{1 - \beta t_o}$$

Then, $\theta^m = \theta^s$ if

$$\frac{t_o}{k + t_o} = \frac{1 - \eta}{1 - \beta}$$

So, if $\eta > \beta$, then $t_o > 0$.

Next, suppose $\theta^m = \theta^s$ and consider $t_s$. Condition (A.6′) becomes

$$f(\theta^s)\beta \left( y - w + \frac{\Delta v(w)}{v'(w)} \right) = \frac{\phi'(s^m) + t_s}{v'(w)}$$

Combining this with (A.9), we obtain

$$\frac{\beta}{\eta} = \frac{\phi'(s^m) + t_s}{\phi'(s^s)}$$

For $\eta > \beta$, to make $s^m = s^s$, we require $t_s < 0$.

Finally suppose $s^m = s^s$ and $\theta^m = \theta^s$ and consider $t_p$. Condition (A.7′) becomes

$$s^m f(\theta^s)\beta \left( y - w^m + \frac{\Delta v(w^m)}{v'(w^m)} \right) + u - \phi(s^m) - \mu - \bar{\delta}^m - t_p = 0$$
Combining this with (A.10), we obtain:

\[
\frac{\beta}{\eta} = \frac{\delta^m + t_p + \phi'(s^s) + \mu - u}{\delta^s + \phi'(s^s) + \mu - u}
\]

For \( \eta > \beta \), to make \( \bar{\delta}^m = \bar{\delta}^s \), we require \( t_p < 0 \).

So, to obtain the social optimum in a market equilibrium, if \( \eta > \beta \) we must tax vacancies, \( t_o > 0 \), and subsidize search and participation, \( t_s < 0, t_p < 0 \), and vice versa.

**B One Productivity Ability Type with No Search or Participation Decisions**

To highlight the separate role of the proportional wage tax relative to the lump-sum wage tax, assume both participation and search effort are fixed so \( s = n = 1 \). The first-order conditions of the government’s problem become:

\[
-f(\theta)w(v'(\cdot) - \lambda) + \left( A + \lambda \left[ t_e f'(\theta)\theta'(w) + f(\theta)\tau \right] \right) \frac{\partial w}{\partial \tau} = 0 \tag{B.1}
\]

\[
f(\theta)(v'(\cdot) - \lambda) + \left( A + \lambda \left[ t_e f'(\theta)\theta'(w) + f(\theta)\tau \right] \right) \frac{\partial w}{\partial b} = 0 \tag{B.2}
\]

\[
(1 - f(\theta))(u'(\cdot) - \lambda) + \left( A + \lambda \left[ t_e f'(\theta)\theta'(w) + f(\theta)\tau \right] \right) \frac{\partial w}{\partial b^I} = 0 \tag{B.3}
\]

Suppose the government doesn’t use the proportional wage tax, so \( \tau = 0 \). From the budget constraint, \( f(\theta(w(b, b^I)))b + (1 - f(\theta(w(b, b^I))))b^I = 0 \). An increase in \( b \) has the opposite effect on the wage rate than an increase in \( b^I \). Therefore, we have either \( v' > \lambda > u' \) and \( A + \lambda \left[ t_e f'(\theta)\theta'(w) + f(\theta)\tau \right] < 0 \), \( v' < \lambda < u' \) and \( A + \lambda \left[ t_e f'(\theta)\theta'(w) + f(\theta)\tau \right] > 0 \) or \( v' = \lambda = u' \) and \( A + \lambda \left[ t_e f'(\theta)\theta'(w) + f(\theta)\tau \right] = 0 \).

Now allow for the proportional wage tax. Using the binding FOC on \( b \) and the
following relationship
\[
\frac{\partial w}{\partial \tau} = \beta (y - w) v' - w \frac{\partial w}{\partial b} < -w \frac{\partial w}{\partial b}
\]
we can rewrite the left-hand side of the FOC on \(\tau\) as
\[
\left( A + \lambda \left[ t_e f'(\theta) \theta'(w) + f(\theta) \right] \right) \frac{\beta (y - w) v'}{D}.
\] (B.4)
For an interior solution for \(\tau\), the above expression is equal to zero. Plugging this back into (B.2) yields, \(u' = \lambda\) and would yield \(v' = \lambda\) if we plugged it in back into the (B.3). In this case, we can satisfy the condition that \(v'(w(b, b^l, \tau) + b) = u'(b^l)\) as well as the binding budget constraint since we now have an additional instrument. Whereas with just two instruments, the budget constraint is satisfied and it is possible that \(v'(w(b, b^l) + b) = u'(b^l)\), but it is not guaranteed. It may be that \(v'\) is greater or less than \(u'\) depending on \(w(b, b^l), \theta(w)\) and \(f(\theta)\).

Evaluating (B.4) at \(\tau = 0\) yields
\[
\left( A + \lambda t_e f'(\theta) \theta'(w) \right) \frac{\beta (y - w) v'}{D}.
\] (B.5)
The sign of \(A\) can be positive or negative and the sign of the second term in the brackets is negative given \(t_e = -(b - b^l) = b^l/f\) where the second equality comes from the government’s budget constraint and is positive given \(b^l > 0\). It is possible that the term in the brackets is exactly equal to zero. In which, we happen to obtain full unemployment insurance with no proportional wage tax. If the term is negative, then starting from \(\tau = 0\) the government would want to tax wages and if the term is positive than starting from \(\tau = 0\), the government would want to subsidize wages. As argued above, the wage tax (if not equal to zero) is being used to ensure that \(v'(w(b, b^l, \tau) + b) = u'(b^l)\).

If \(\eta = \beta\), then \(A\) will be zero and (B.5) reduces to
\[
\lambda b^l (1 - \eta) \frac{v'}{v' - \beta (y - w) v''} > 0.
\] (B.6)
and the government would want to increase $\tau$ above zero. Starting from $\tau = 0$, it must be that $b < 0$ if $b' > 0$ and the government would want to increase $\tau$ above 0. A positive $\tau$ has a direct negative effect on the equilibrium wage which in turn increases the tightness of the labour market, $\theta$, and therefore, the number of job finders $f(\theta)$. This increases the wage tax base and reduces the number of unemployment transfer recipients.

A Two Period Model

A.1 Second-period search decision

Optimal second-period search $s(\tau_2, b_2, b^{II}; a^I, \theta_2, w_2)$ where

$$\frac{\partial s(\cdot)}{\partial \tau_2} = -\frac{w_2'v'(c^{IW})f(\theta_2)}{\phi''(s_2)} < 0; \quad \frac{\partial s(\cdot)}{\partial b_2} = \frac{v'(c^{IW})f(\theta_2)}{\phi''(s_2)} > 0;$$

$$\frac{\partial s(\cdot)}{\partial b^{II}} = -\frac{u'(c^{II})f(\theta_2)}{\phi''(s_2)} < 0; \quad \frac{\partial s(\cdot)}{\partial \theta_2} = f'(\theta_2)[v(\theta^{II}) - u(c^{II})] > 0;$$

$$\frac{\partial s(\cdot)}{\partial w_2} = \frac{(1 - \tau_2)v'(c^{IW})f(\theta_2)}{\phi''(s_2)} > 0; \quad \frac{\partial s(\cdot)}{\partial a^I} = \frac{[v'(c^{IW}) - u'(c^{II})]f(\theta_2)}{\phi''(s_2)}.$$ 

A.2 Savings decisions

Optimal savings by involuntary unemployed is $a^I(\tau_2, b_2, b^{II}; \theta_2, w_2)$ where

$$\frac{\partial a^I(\cdot)}{\partial \tau_2} = \frac{s_2f(\theta_2)v''(c^{IW})w_2 - f(\theta_2)\Delta MU \frac{\partial s_2}{\partial \tau_2}}{\Phi_I}; \quad \frac{\partial a^I(\cdot)}{\partial b_2} = \frac{-s_2f(\theta_2)v''(c^{IW}) - f(\theta_2)\Delta MU \frac{\partial s_2}{\partial \theta_2}}{\Phi_I};$$

$$\frac{\partial a^I(\cdot)}{\partial b^{II}} = \frac{u''(c^{II})}{\Phi_I} > 0; \quad \frac{\partial a^I(\cdot)}{\partial \theta_2} = \frac{-s_2\Delta MU f'(\theta_2) - f(\theta_2)\Delta MU \frac{\partial s_2}{\partial \theta_2}}{\Phi_I};$$

$$\frac{\partial a^I(\cdot)}{\partial w_2} = \frac{-s_2f(\theta_2)v''(c^{IW})(1 - \tau_2) - f(\theta_2)\Delta MU \frac{\partial s_2}{\partial w_2}}{\Phi_I}.$$
where

\[ \Phi_I \equiv u''(b^I - a^I) + s_2 f(\theta_2) v''((1 - \tau_2)w_2 + b_2 + a^I) + (1 - s_2 f(\theta_2)) u''(b^{II} + a^I) \]

\[ + f(\theta_2)[v'((1 - \tau_2)w_2 + b_2 + a^I) - u'(b^{II} + a^I)] \frac{\partial s_2}{\partial a^I} \]

and \( \Delta MU = [v'((1 - \tau_2)w_2 + b_2 + a^I) - u'(b^{II} + a^I)] \). It is assumed that \( \Phi_I < 0 \).

Optimal savings by employed persons is \( a^W(\tau_1, b^I, b^{II}; w_1) \) where

\[ \frac{\partial a^W(\cdot)}{\partial \tau_1} = -w_1(v''(c^W) - (1 - \sigma)v''(c^{WW})); \quad \frac{\partial a^W(\cdot)}{\partial b_1} = \frac{\Phi^W}{v''(c^W) - (1 - \sigma)v''(c^{WW})}; \]

\[ \frac{\partial a^W(\cdot)}{\partial w_1} = (1 - \tau_1)(v''(c^W) - (1 - \sigma)v''(c^{WW})); \quad \frac{\partial a^W(\cdot)}{\partial b^I} = \frac{\sigma u'(c^{WI})}{\Phi^W} < 0 \]

where

\[ \Phi_{IW} = v''(c^W) + (1 - \sigma)v''(c^{WW}) + \sigma u''(c^{WI}) < 0. \]

### A.3 Value functions

The involuntary unemployed's value function is \( V^I(\tau_2, b^I, b^{II}; \theta_2, w_2) \) where

\[ \frac{\partial V^I}{\partial \tau_2} = -w_2 v'(c^{IW}) s_2 f(\theta_2) < 0; \quad \frac{\partial V^I}{\partial \theta_2} = v'(c^{IW}) s_2 f(\theta_2) > 0; \]

\[ \frac{\partial V^I}{\partial b^I} = u'(c^I) > 0; \quad \frac{\partial V^I}{\partial b^{II}} = (1 - s_2 f(\theta_2)) u'(c^{II}) > 0 \]

\[ \frac{\partial V^I}{\partial \theta_2} = s_2 f'(\theta_2) [v'(c^{IW}) - u'(c^{II})] > 0; \quad \frac{\partial V^I}{\partial w_2} = (1 - \tau_2) v'(c^{IW}) s_2 f(\theta_2) > 0. \]

The value function for the employed persons is \( V^W(\tau_1, b^I, b^{II}; w_1) \), where

\[ \frac{\partial V^W}{\partial \tau_1} = -v'(c^W) w_1 - (1 - \sigma) v'(c^{WW}) w_1 < 0; \quad \frac{\partial V^W}{\partial b_1} = v'(c^W) + (1 - \sigma) v'(c^{WW}) > 0; \]

\[ \frac{\partial V^W}{\partial b^I} = \sigma u'(c^{WI}) > 0; \quad \frac{\partial V^W}{\partial w_1} = (1 - \tau_1) (v'(c^W) + (1 - \sigma) v'(c^{WW})) > 0. \]
A.4 First-period search and participation decision

Optimal first-period search \( s_1(\tau_1, b_1, \tau_2, b_2, b^I, b^I^H; \theta_1, w_1, \theta_2, w_2) \) where

\[
\frac{\partial s_1(\cdot)}{\partial \tau_1} = \frac{f(\theta_1) \frac{\partial V^w}{\partial \tau_1}}{\phi''(s_1)} < 0; \\
\frac{\partial s_1(\cdot)}{\partial \tau_2} = \frac{-f(\theta_1) \frac{\partial V^I}{\partial \tau_2}}{\phi''(s_1)} > 0; \\
\frac{\partial s_1(\cdot)}{\partial b^I} = \frac{f(\theta_1) \left[ \frac{\partial V^w}{\partial b} - \frac{\partial V^I}{\partial \theta} \right]}{\phi''(s_1)}; \\
\frac{\partial s_1(\cdot)}{\partial \theta_1} = \frac{f'(\theta_1) [V^w - V^I]}{\phi''(s_1)} > 0; \\
\frac{\partial s_1(\cdot)}{\partial \theta_2} = \frac{-f(\theta_1) \frac{\partial V^I}{\partial \theta_2}}{\phi''(s_1)} < 0; \\
\frac{\partial s_1(\cdot)}{\partial b^I} = \frac{-f(\theta_1) \frac{\partial V^I}{\partial b^I}}{\phi''(s_1)} < 0; \\
\frac{\partial s_1(\cdot)}{\partial w_1} = \frac{-f(\theta_1) \frac{\partial V^I}{\partial w_1}}{\phi''(s_1)} < 0.
\]

Optimal first-period taste of leisure cut-off \( \bar{\sigma}(\tau_1, b_1, \tau_2, b_2, b^I, b^I^H; \theta_1, w_1, \theta_2, w_2) \) where

\[
\frac{\partial \bar{\sigma}(\cdot)}{\partial \tau_1} = s_1 f(\theta_1) \frac{\partial V^w_1}{\partial \tau_1} < 0; \\
\frac{\partial \bar{\sigma}(\cdot)}{\partial \tau_2} = (1 - s_1 f(\theta_1)) \frac{\partial V^I_1}{\partial \tau_2} < 0; \\
\frac{\partial \bar{\sigma}(\cdot)}{\partial b^I} = s_1 f(\theta_1) \frac{\partial V^w_1}{\partial b^I} + (1 - s_1 f(\theta_1)) \frac{\partial V^I_1}{\partial b^I} > 0; \\
\frac{\partial \bar{\sigma}(\cdot)}{\partial \theta_1} = s_1 f'(\theta_1) (V^w_1 - V^I_1) > 0; \\
\frac{\partial \bar{\sigma}(\cdot)}{\partial \theta_2} = (1 - s_1 f(\theta_1)) \frac{\partial V^I_1}{\partial \theta_2} > 0; \\
\frac{\partial \bar{\sigma}(\cdot)}{\partial b^V} = -\frac{\partial V^I_1}{\partial b^V} < 0; \\
\frac{\partial \bar{\sigma}(\cdot)}{\partial b^I^H} = s_1 f(\theta_1) \frac{\partial V^w_1}{\partial b^I^H} > 0; \\
\frac{\partial \bar{\sigma}(\cdot)}{\partial w_1} = (1 - s_1 f(\theta_1)) \frac{\partial V^I_1}{\partial w_1} > 0; \\
\frac{\partial \bar{\sigma}(\cdot)}{\partial w_2} = (1 - s_1 f(\theta_1)) \frac{\partial V^I_1}{\partial w_2} > 0.
\]
A.5 Labour Market Equilibria

A.5.1 Period 2

At the start of period 2, the saving of the involuntary unemployed has already been determined, as shown above, and is given by \( a^I(\tau_2, b_2, b^I, b^{II}; \theta_2, w_2) \). Using \( \theta_2(w_2) \) we can then rewrite the equilibrium wage condition as

\[
w_2 = y - F(\tau_2, b_2, b^{II}w_2, a^I(\tau_2, b_2, b^I, b^{II}; \theta_2(w_2), w_2))
\]

where

\[
F(\tau_2, b_2, b^{II}, w_2, a^I(\tau_2, b_2, b^I, b^{II}; \theta_2(w_2), w_2)) = \frac{1 - \beta}{\beta} \frac{v((1 - \tau_2)w_2 + b_2 + a^I(\cdot)) - \theta_2(w_2) - u(b^{II} + a^I(\cdot))}{(1 - \tau_2)v'((1 - \tau_2)w_2 + b_2 + a^I(\cdot))}.
\]

Assuming \( F_{w_2} + F_{a^I} \frac{da^I}{dw_2} \geq 0 \), the above condition yields \( w_2(\tau_2, b_2, b^I, b^{II}) \) where

\[
\begin{align*}
\frac{\partial w_2(\cdot)}{\partial \tau_2} &= -F^\tau - F_{\tau} \frac{\partial a^I}{\partial \tau}, \\
\frac{\partial w_2(\cdot)}{\partial b^I} &= -F^b - F_{b} \frac{\partial a^I}{\partial b}, \\
\frac{\partial w_2(\cdot)}{\partial b^{II}} &= -F_{b^{II}} - F_{b^{II}} \frac{\partial a^I}{\partial b^{II}}.
\end{align*}
\]

Note the following relationships:

\[
F^\tau = -w_2 F_{b_2} + \frac{F}{1 - \tau_2} > -w_2 F_{b_2} \quad \text{and} \quad \frac{\partial a^I}{\partial \tau} = -w_2 \frac{\partial a^I}{\partial b_2}
\] (C.1)

which together imply that

\[
\frac{\partial w_2(\cdot)}{\partial \tau_2} = -w_2 \frac{\partial w_2(\cdot)}{\partial b_2} - \frac{F/(1 - \tau_2)}{1 + F_{w_2} + F_{a^I} \frac{da^I}{dw_2}} < -w_2 \frac{\partial w_2(\cdot)}{\partial b_2}
\] (C.2)

A.5.2 Period 1

Using \( a^W(\tau_1, b_1, b^I; w_1) \) and \( w_2(\tau_2, b_2, b^I, b^{II}) \) as determine above. We can rewrite the equilibrium wage condition as:

\[
w_1 = y - H(\tau_1, b_1, \tau_2, b_2, b^I, b^{II}, w_1, \theta_2(w_2(\tau_2, b_2, b^I, b^{II})), w_2(\tau_2, b_2, b^I, b^{II}), a^W(\tau_1, b_1, b^I; w_1))
\]
where

\[
H(\tau_1, b_1, \tau_2, b_2, b^I, b^{II}, w_1, \theta_2(w_2(\tau_2, b_2, b^I, b^{II})), w_2(\tau_2, b_2, b^I, b^{II})), a^W(\tau_1, b_1, b^I; w_1)) = 1 - \beta \frac{V^W_1(\tau_1, b_1, b^I; w_1) - V^I(\tau_2, b_2, b^I, b^{II}; \theta_2(w_2(\cdot), w_2(\cdot)))}{(1 - \tau_1)(\nu'((1 - \tau_1)w_1 + b_1 - a^W(\cdot)) + (1 - \sigma)\nu'((1 - \tau_1)w_1 + b_1 + a^W(\cdot)))}.
\]

Assuming \( y - H(\cdot) > 0 \) when evaluated at \( w_1 = 0 \) and \( H_{w_1} + a^w \frac{\partial a^I}{\partial w_1} \geq 0 \), the above condition yields \( w_1(\tau_1, b_1, \tau_2, b_2, b^I, b^{II}) \) where

\[
\frac{\partial w_1(\cdot)}{\partial \tau_1} = \frac{-H_{\tau_1} - H_a \frac{\partial a^w}{\partial \tau_1}}{1 + H_{w_1} + H_a \frac{\partial a^w}{\partial w_1}}, \quad \frac{\partial w_1(\cdot)}{\partial \tau_2} = \frac{-H_{\tau_2} - (dH/dw_2) \frac{\partial w_2}{\partial \tau_2}}{1 + H_{w_1} + H_a \frac{\partial a^w}{\partial w_1}}, \quad \frac{\partial w_1(\cdot)}{\partial b^I} = \frac{-H_{b^I} - (dH/dw_2) \frac{\partial w_2}{\partial b^I} - H_a \frac{\partial a^w}{\partial w_1}}{1 + H_{w_1} + H_a \frac{\partial a^w}{\partial w_1}}, \quad \frac{\partial w_1(\cdot)}{\partial b^{II}} = \frac{-H_{b^{II}} - (dH/dw_2) \frac{\partial w_2}{\partial b^{II}}}{1 + H_{w_1} + H_a \frac{\partial a^w}{\partial w_1}}.
\]

Note the following relationships:

\[
H_{\tau_1} = -w_1 H_{b_1} + \frac{H}{1 - \tau_1} > -w_1 H_{b_1}; \quad \frac{\partial a^w}{\partial \tau_1} = -w_1 \frac{\partial a^w}{\partial b_1}; \quad \frac{\partial V^W_1}{\partial \tau_1} = -w_1 \frac{\partial V^W_1}{\partial b_1} \quad (C.3)
\]

which together imply that

\[
\frac{\partial w_1(\cdot)}{\partial \tau_1} = -w_1 \frac{\partial w_1(\cdot)}{\partial b_1} - \frac{H/(1 - \tau_1)}{1 + H_{w_1} + H_a \frac{\partial a^w}{\partial w_1}} < -w_1 \frac{\partial w_1(\cdot)}{\partial b_1}. \quad (C.4)
\]

In addition,

\[
H_{\tau_2} = -w_2 H_{b_2}; \quad \text{since} \quad \frac{\partial V^I_1}{\partial \tau_2} = -w_2 \frac{\partial V^I_1}{\partial b_2}
\]

which implies using (A.5.1) that

\[
\frac{\partial w_1(\cdot)}{\partial \tau_2} = -w_2 \frac{\partial w_1(\cdot)}{\partial b_2} - (dH/dw_2) \frac{F/(1 - \tau_2)}{1 + F_{w_2} + F_a \frac{\partial a^I}{\partial w_2}} \frac{\partial a^I}{\partial b_2}
\]

where

\[
\frac{dH}{dw_2} = -\frac{1 - \beta}{\beta} \frac{dV^I_1/dw_2}{\beta (1 - \tau_1)(\nu'(c^W) + (1 - \sigma)\nu'(c^{WW})} = \frac{1 - \beta}{\beta} \frac{s_2 f(\theta_2) (1 - \tau_2) \nu'(c^{IWW}) \left[ 1 - \frac{(1 - \tau_2) \beta}{\eta_2 (1 - \beta)} \right]}{(1 - \tau_1)(\nu'(c^W) + (1 - \sigma)\nu'(c^{WW}))}
\]

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A.6 Fixed participation and search effort

The government’s problem can be written as

$$\max_{\{\tau_1, b_1, \tau_2, b_2, b', b'^I\}} f(\theta_1(w_1(\cdot)))V_1^W(\tau_1, b_1, \tau_2, b_2, b', b'^I) + (1 - f(\theta_1(w_1(\cdot))))V_1^I(\tau_2, b_2, b', b'^I) - \phi(1)$$

subject to

$$\left[ f(\theta_1(w_1(\cdot))(2 - \sigma) \right] b_1 + \left[ (1 - f(\theta_1(w_1(\cdot))) f(\theta_2(w_2(\cdot))) \right] b_2$$

$$+ \left[ f(\theta_1(w_1(\cdot))) \sigma + (1 - f(\theta_1(w_1(\cdot)))) \right] b' + (1 - f(\theta_1(w_1(\cdot)))(1 - f(\theta_2(w_2(\cdot)))) b'^I$$

$$= f(\theta_1(w_1(\cdot))(2 - \sigma) w_1(\cdot) \tau_1 + \left[ (1 - f(\theta_1(w_1(\cdot))) f(\theta_2(w_2(\cdot))) \right] w_2(\cdot) \tau_2$$

where $w_1(\tau_1, b_1, \tau_2, b_2, b', b'^I)$ and $w_2(\tau_2, b_2, b', b'^I)$.

The first-order conditions are as follows:

$$f(\theta_1) \frac{\partial V_1^W}{\partial \tau_1} + \left[ f(\theta_1) \frac{\partial V_1^W}{\partial w_1} + \Delta V f'(\theta_1) \theta'_1(w_1) \right] \frac{\partial w_1}{\partial \tau_1}$$

$$- \lambda \left[ - f(\theta_1)(2 - \sigma) w_1 + \left[ - t_{e_1} f'(\theta_1) \theta'_1(w_1) - f(\theta_1)(2 - \sigma) \tau_1 \right] \frac{\partial w_1}{\partial \tau_1} \right] = 0$$

$$f(\theta_1) \frac{\partial V_1^W}{\partial b_1} + \left[ f(\theta_1) \frac{\partial V_1^W}{\partial w_1} + \Delta V f'(\theta_1) \theta'_1(w_1) \right] \frac{\partial w_1}{\partial b_1}$$

$$- \lambda \left[ f(\theta_1)(2 - \sigma) + \left[ - t_{e_1} f'(\theta_1) \theta'_1(w_1) - f(\theta_1)(2 - \sigma) \tau_1 \right] \frac{\partial w_1}{\partial b_1} \right] = 0$$

$$(1 - f(\theta_1)) \frac{\partial V_1^I}{\partial \tau_2} + \left[ (1 - f(\theta_1)) \frac{\partial V_1^I}{\partial w_2} + \Delta v_2 f'(\theta_2) \theta'_2(w_2) \right] \frac{\partial w_2}{\partial \tau_2} + \left[ f(\theta_1) \frac{\partial V_1^W}{\partial w_1} + \Delta V f'(\theta_1) \theta'_1(w_1) \right] \frac{\partial w_1}{\partial \tau_2}$$

$$- \lambda \left[ - (1 - f(\theta_1)) f(\theta_2) w_2 + \left[ - t_{e_2} f'(\theta_2) \theta'_2(w_2) - (1 - f(\theta_1)) f(\theta_2) \tau_2 \right] \frac{\partial w_2}{\partial \tau_2}$$

$$+ \left[ - t_{e_1} f'(\theta_1) \theta'_1(w_1) - f(\theta_1)(2 - \sigma) \tau_1 \right] \frac{\partial w_1}{\partial \tau_2} \right] = 0$$
\[
(1 - f(\theta_1)) \frac{\partial V^I}{\partial b_2} + \left[ (1 - f(\theta_1)) \frac{\partial V^I}{\partial w_2} + \Delta v_2 f'(\theta_2) \theta_2'(w_2) \right] \frac{\partial w_2}{\partial b_2} + \left[ f(\theta_1) \frac{\partial V^W}{\partial w_1} + \Delta V f'(\theta_1) \theta'_1(w_1) \right] \frac{\partial w_1}{\partial b_2} \\
- \lambda \left[ (1 - f(\theta_1)) f(\theta_2) + \left[ -t_{e2} f'(\theta_2) \theta_2'(w_2) - (1 - f(\theta_1)) f(\theta_2) \tau_2 \right] \frac{\partial w_2}{\partial b_2} \right] + \left[ -t_{e1} f'(\theta_1) \theta'_1(w_1) - f(\theta_1) (2 - \sigma) \tau_1 \right] \frac{\partial w_1}{\partial b_2} = 0
\]

\[
f(\theta_1) \frac{\partial V^W}{\partial b^I} + (1 - f(\theta_1)) \frac{\partial V^I}{\partial b^I} + \left[ (1 - f(\theta_1)) \frac{\partial V^I}{\partial w_2} + \Delta v_2 f'(\theta_2) \theta_2'(w_2) \right] \frac{\partial w_2}{\partial b^I} + \left[ f(\theta_1) \frac{\partial V^W}{\partial w_1} + \Delta V f'(\theta_1) \theta'_1(w_1) \right] \frac{\partial w_1}{\partial b^I} \\
- \lambda \left[ (f(\theta_1) (2 - \sigma)) + \left[ -t_{e2} f'(\theta_2) \theta_2'(w_2) - (1 - f(\theta_1)) f(\theta_2) \tau_2 \right] \frac{\partial w_2}{\partial b^I} \right] + \left[ -t_{e1} f'(\theta_1) \theta'_1(w_1) - f(\theta_1) (2 - \sigma) \tau_1 \right] \frac{\partial w_1}{\partial b^I} = 0
\]

\[
(1 - f(\theta_1)) \frac{\partial V^I}{\partial b^{II}} + \left[ (1 - f(\theta_1)) \frac{\partial V^I}{\partial w_2} + \Delta v_2 f'(\theta_2) \theta_2'(w_2) \right] \frac{\partial w_2}{\partial b^{II}} + \left[ f(\theta_1) \frac{\partial V^W}{\partial w_1} + \Delta V f'(\theta_1) \theta'_1(w_1) \right] \frac{\partial w_1}{\partial b^{II}} \\
- \lambda \left[ (1 - f(\theta_1))(1 - f(\theta_2)) + \left[ -t_{e2} f'(\theta_2) \theta_2'(w_2) - (1 - f(\theta_1)) f(\theta_2) \tau_2 \right] \frac{\partial w_2}{\partial b^{II}} \right] + \left[ -t_{e1} f'(\theta_1) \theta'_1(w_1) - f(\theta_1) (2 - \sigma) \tau_1 \right] \frac{\partial w_1}{\partial b^{II}} = 0
\]

where

\[
t_{e1} = -((2 - \sigma)(b_1 - w_1 \tau_1) - f(\theta_2)(b_2 - w_2 \tau_2) + (\sigma - 1) b^I - (1 - f(\theta_2)) b^{II})
\]

\[
t_{e2} = -(1 - f(\theta_1))(b_2 - w_2 \tau_2 - b^{II})
\]

given \( s_1 = s_2 = 1 \).

To begin assume that wages are also fixed. The first-order conditions become:

\[
f(\theta_1) \frac{\partial V^W}{\partial \tau_1} + \lambda f(\theta_1) (2 - \sigma) w_1 = 0 \quad (\tau_1)
\]

\[
f(\theta_1) \frac{\partial V^W}{\partial b_1} - \lambda f(\theta_1) (2 - \sigma) = 0 \quad (b_1)
\]

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\[(1 - f(\theta_1)) \frac{\partial V_I}{\partial \tau_2} + \lambda (1 - f(\theta_1)) f(\theta_2) w_2 = 0 \quad (\tau_2)\]

\[(1 - f(\theta_1)) \frac{\partial V_I}{\partial b_2} - \lambda (1 - f(\theta_1)) f(\theta_2) = 0 \quad (b_2)\]

\[f(\theta_1) \frac{\partial V_I}{\partial b_I} + (1 - f(\theta_1)) \frac{\partial \bar{V}_I}{\partial b_I} - \lambda \left[ 1 - f(\theta_1)(1 - \sigma) \right] = 0 \quad (b')\]

\[(1 - f(\theta_1)) \frac{\partial \bar{V}_I}{\partial b_{II}} - \lambda \left[ (1 - f(\theta_1))(1 - f(\theta_2)) \right] = 0 \quad (b^{II})\]

- The FOC on \(\tau_i\) is equivalent to the FOC on \(b_i\) multiplied by \(-w_i\). Once again with fixed wages - there is no separate role for the income tax.

- The first-order conditions can each be solved for \(\lambda\):

\[\lambda = u'(c^{II}) = v'(c^{IW}) = \frac{v'(c^W) + (1 - \sigma) v'(c^{WW})}{2 - \sigma} = \frac{(1 - f(\theta_1)) u'(c^I) + f(\theta_1) \sigma u'(c^{W_I})}{1 - f(\theta_1) + f(\theta_1) \sigma}\]

which implies the government provides full unemployment insurance in the second period.

- Together with the optimal savings decision rule for the involuntary unemployed

\[u'(c^I) = f(\theta_2) v'(c^{IW}) + (1 - f(\theta_2)) u'(c^{II})\]

implies that \(u'(c^I) = u'(c^{II}) = v'(c^{W})\) which in turn implies that \(u'(c^I) = u'(c^{W_I})\). This, together with the optimal savings decision rule for the involuntary unemployed

\[v'(c^W) = (1 - \sigma) v'(c^{WW}) + \sigma u'(c^{W_I})\]

imply that \(v'(c^{WW}) = u'(c^I) = v'(c^W)\). The government chooses its policies \((b_1, b_2, b', b^{II})\) to ensure full equality of marginal utility of consumption across time and employment state.

- If individuals could not save, the same outcome would be obtained.
Now consider the case with endogenous wages. Multiply the first-order condition on \( b_1 \) by \(-w_1\) and substituting it into the first-order condition on \( \tau_1 \) using the relationships in (C.3) and (C.4) yields the following expression for the first-order condition on \( \tau_1 \):

\[
\left[ f(\theta_1) \frac{\partial V^W}{\partial w_1} + \Delta V f'(\theta_1)\theta'_1(w_1) - \lambda \left[ - t_{e1} f'(\theta_1)\theta'_1(w_1) - f(\theta_1)(2 - \sigma)\tau_1 \right] \right] \left( - \frac{H/(1 - \tau_1)}{1 + H_{w_1} + H_{a}\frac{\partial a}{\partial w_1}} \right) = 0
\]

The expression in the square brackets is the effect of a marginal change of \( w_1 \) on the government’s constrained optimization problem and given the expression in the round brackets is negative, the government through its choice of \( \tau_1 \) will ensure that the expression in the square brackets is zero, that is,

\[
\left[ f(\theta_1) \frac{\partial V^W}{\partial w_1} + \Delta V f'(\theta_1)\theta'_1(w_1) - \lambda \left[ - t_{e1} f'(\theta_1)\theta'_1(w_1) - f(\theta_1)(2 - \sigma)\tau_1 \right] \right] = 0 \quad (C.5)
\]

Following similar steps, we can multiple the first-order condition on \( b_2 \) by \(-w_2\) and substituting it into the first-order condition on \( \tau_2 \) using the relationships in (C.1) and (C.2) yields the following expression for the first-order condition on \( \tau_2 \):

\[
- \left[ f(\theta_1) \frac{\partial V^W}{\partial w_1} + \Delta V f'(\theta_1)\theta'_1(w_1) - \lambda \left[ - t_{e1} f'(\theta_1)\theta'_1(w_1) - f(\theta_1)(2 - \sigma)\tau_1 \right] \right] \left( \frac{\frac{dH}{dw_2} \left( \frac{F/(1 - \tau_2)}{1 - F w_2 - F a \frac{\partial a}{\partial w_2}} \right)}{1 + H_{w_1} + H_{a}\frac{\partial a}{\partial w_1}} \right)
\]

\[
- \left[ (1 - f(\theta_1)) \frac{\partial V^I}{\partial w_2} + \Delta v_2 f'(\theta_2)\theta'_2(w_2) - \lambda \left[ - t_{e2} f'(\theta_2)\theta'_2(w_2) - (1 - f(\theta_1)) f(\theta_2)\tau_2 \right] \right] \left( \frac{\frac{dF}{dw_2} \left( \frac{F/(1 - \tau_2)}{1 - F w_2 - F a \frac{\partial a}{\partial w_2}} \right)}{1 + F_{w_2} + F a \frac{\partial a}{\partial w_2}} \right) = 0
\]

Using (C.5) and noting that the term in the round brackets on the second line is non-zero implies that the first-order condition on \( \tau_2 \) ensures that

\[
\left[ (1 - f(\theta_1)) \frac{\partial V^I}{\partial w_2} + \Delta v_2 f'(\theta_2)\theta'_2(w_2) - \lambda \left[ - t_{e2} f'(\theta_2)\theta'_2(w_2) - (1 - f(\theta_1)) f(\theta_2)\tau_2 \right] \right] = 0 \quad (C.6)
\]

where the left-hand side of condition (C.6) is the marginal effect on of a change in \( w_2 \) on the government’s Lagrangian. Using conditions (C.5) and (C.6), the first-order conditions on \( b_1, b_2, b^I \) and \( b^{II} \) reduce to the same conditions as when wages are
fixed. Consequently, the government chooses its policies to ensure full equality of the marginal utility of consumption across time and employment states. Allowing for endogenous search and participation would not affect this result that \( \tau_1 \) and \( \tau_2 \) are sufficient policy instruments to internalize the macro effects of the wages determined in each period. Therefore, we focus below on the case with fixed wages.

### A.6.1 Fixed wages and endogenous search and participation

The government’s problem is as follows:

\[
\max_{\{\tau_1, b_1, \tau_2, b_2, b^l, b^H, b^V\}} \left[ s_1(\cdot)f(\theta_1)V^W_1(\tau_1, b_1, b^l) + (1-s_1(\cdot)f(\theta_1))V^I_1(\tau_2, b_2, b^H, b^{II}) - \phi(s_1(\cdot)) \right] n_1(\cdot)
\]

subject to

\[
n_1(\cdot)[s_1(\cdot)f(\theta_1)(2-\sigma)b_1 + n_1(\cdot)\left((1-s_1(\cdot)f(\theta_1))s_2(\cdot)f(\theta_2)\right)b_2 + n_1(\cdot)[s_1(\cdot)f(\theta_1)\sigma+(1-s_1(\cdot)f(\theta_1))\right]b^l + n_1(\cdot)[(1-s_1(\cdot)f(\theta_1))(1-s_2(\cdot)f(\theta_2))b^{II} + 2(1-n_1(\cdot)b^V
\]

\[
= n_1(\cdot)s_1(\cdot)f(\theta_1)(2-\sigma)w_1(\cdot)\tau_1 + n_1(\cdot)[(1-s_1(\cdot)f(\theta_1))s_2(\cdot)f(\theta_2)]w_2\tau_2
\]

where \( w_1, \theta_1, w_2, \theta_2 \) are fixed, \( s_1(\tau_1, b_1, \tau_2, b_2, b^H, b^{II}), s_2(\tau_2, b_2, b^{II}; a^I(\tau_2, b_2, b^H, b^V) \)

and \( n_1(\tau_1, b_1, \tau_2, b_2, b^H, b^{II}, b^V) = G(\bar{\delta}(\tau_1, b_1, \tau_2, b_2, b^H, b^V)) \).

Using the Envelope Theorem (given the individuals’ optimal first-period search and participation decisions), the first-order conditions are:

\[
n_1s_1f(\theta_1)\frac{\partial V^W_1}{\partial \tau_1} - \lambda \left[ -n_1s_1f(\theta_1)(2-\sigma)w_1 - t_{e1}n_1f(\theta_1)\frac{\partial s_1}{\partial \tau_1} - t_{p1}\frac{\partial n_1}{\partial \tau_1} \right] = 0
\]

\[
n_1s_1f(\theta_1)\frac{\partial V^W_1}{\partial b_1} - \lambda \left[ n_1s_1f(\theta_1)(2-\sigma) - t_{e1}n_1f(\theta_1)\frac{\partial s_1}{\partial b_1} - t_{p1}\frac{\partial n_1}{\partial b_1} \right] = 0
\]
\[ n_1(1 - s_1 f(\theta_1)) \frac{\partial V^I}{\partial \tau_2} - \lambda \left[ - n_1(1 - s_1 f(\theta_1)) s_2 f(\theta_2) w_2 - t_{e_1} n_1 f(\theta_1) \frac{\partial s_1}{\partial \tau_2} - t_{e_2} n_1 f(\theta_2) \frac{ds_2}{d\tau_2} - t_{p_1} \frac{\partial n_1}{\partial \tau_2} \right] = 0 \]

\[ n_1(1 - s_1 f(\theta_1)) \frac{\partial V^I}{\partial b_2} - \lambda \left[ n_1(1 - s_1 f(\theta_1)) s_2 f(\theta_2) - t_{e_1} n_1 f(\theta_1) \frac{\partial s_1}{\partial b_2} - t_{e_2} n_1 f(\theta_2) \frac{ds_2}{db_2} - t_{p_1} \frac{\partial n_1}{\partial b_2} \right] = 0 \]

\[ n_1 s_1 f(\theta_1) \frac{\partial V^W}{\partial b^I} + n_1(1 - s_1 f(\theta_1)) \frac{\partial V^I}{\partial b^I} - \lambda \left[ n_1(s_1 f(\theta_1) \sigma + 1 - s_1 f(\theta_1)) - t_{e_1} n_1 f(\theta_1) \frac{\partial s_1}{\partial b^I} \right. \]

\[ \left. - t_{e_2} n_1 f(\theta_2) \frac{\partial s_2}{\partial a^I} \frac{\partial a^I}{\partial b^I} - t_{p_1} \frac{\partial n_1}{\partial b^I} \right] = 0 \]

\[ n_1(1 - s_1 f(\theta_1)) \frac{\partial V^I}{\partial b^{II}} - \lambda \left[ n_1(1 - s_1 f(\theta_1))(1 - s_2 f(\theta_2)) - t_{e_1} n_1 f(\theta_1) \frac{\partial s_1}{\partial b^{II}} - t_{e_2} n_1 f(\theta_2) \frac{ds_2}{db^{II}} - t_{p_1} \frac{\partial n_1}{\partial b^{II}} \right] = 0 \]

\[ \int_{\delta(\cdot)}^{\delta_{max}} \frac{\partial V^V}{\partial b^V} g(\delta) d\delta - \lambda \left[ 2(1 - n_1) - t_{p_1} \frac{\partial n_1}{\partial b^V} \right] = 0 \]

- The FOC on \( \tau_i \) is equivalent to the FOC on \( b_i \) multiplied by \(-w_i\). Once again with fixed wages - there is no separate role for the proportional wage tax.

Consequently, we can use the first-order conditions on \( b_i, b^I, b^{II}, \) and \( b^V \) to determine the optimal government policies and the trade-offs the government faces with endogenous search and participation.

Assume first participation is fixed and for ease of notation, set \( s_1 = s_2 = 1 \). The first-order conditions on \( b_1, b_2, b^I, b^{II}, \) and \( b^V \) become:

\[ n_1 f(\theta_1) \frac{\partial V^W}{\partial b_1} - \lambda \left[ n_1 f(\theta_1)(2 - \sigma) - t_{p_1} \frac{\partial n_1}{\partial b_1} \right] = 0 \]

\[ n_1(1 - f(\theta_1)) \frac{\partial V^I}{\partial b_2} - \lambda \left[ n_1(1 - f(\theta_1)) f(\theta_2) - t_{p_1} \frac{\partial n_1}{\partial b_2} \right] = 0 \]
$$n_1 f(\theta_1) \frac{\partial V^W}{\partial b^I} + n_1(1 - f(\theta_1)) \frac{\partial V^I}{\partial b^I} - \lambda \left[ n_1 (f(\theta_1) \sigma + 1 - f(\theta_1)) - t_{p1} \frac{\partial n_1}{\partial b^I} \right] = 0$$

$$n_1 (1 - f(\theta_1)) \frac{\partial V^I}{\partial b^I} - \lambda \left[ n_1 (1 - f(\theta_1))(1 - f(\theta_2)) - t_{p1} \frac{\partial n_1}{\partial b^I} \right] = 0$$

$$\int_{\delta^{\max}} \frac{\partial V^V}{\partial b^V} g(\delta) \delta d\delta - \lambda \left[ 2(1 - n_1) - t_{p1} \frac{\partial n_1}{\partial b^V} \right] = 0$$

- Solving for $t_{p1} > 0$ for each first-order condition on $(b_1, b_2, b^I, b^{II})$ and using the expressions for the various partials defined above and setting the resulting expressions equal to one another yields:

\[
u'(c^{II}) = v'(c^{IW}) = \frac{u'(c^W) + (1 - \sigma)v'(c^{WW})}{2 - \sigma} = \frac{(1 - f(\theta_1))u'(c^I) + f(\theta_1)\sigma u'(c^{WI})}{1 - f(\theta_1) + f(\theta_1)\sigma} < \lambda
\]

which implies the government provides full unemployment insurance in the second period.

- Together with the optimal savings decision rules

\[
u'(c^I) = f(\theta_2)v'(c^{IW}) + (1 - f(\theta_2))u'(c^{II})
\]

\[
v'(c^W) = (1 - \sigma)v'(c^{WW}) + \sigma u'(c^{WI})
\]

imply that the government chooses its policies $(b_1, b_2, b^I, b^{II})$ to ensure full equality of marginal utility of consumption across time and employment state for all participants.

- Solving for $t_{p1}$ from the first-order condition on $b^V$ yields

$$\mu'(b^V) > \lambda$$

which implies the moral hazard arising from the participation decision prevents the government from equating the marginal utility of consumption between participants and the voluntarily unemployed.
Assume next that participation is fixed and for ease of notation, assume \( n = 1 \) so \( b^V \) is no longer a relevant policy instrument. The first-order conditions on \( b_1, b_2, b^I \) and \( b^{II} \) become:

\[
\begin{align*}
    s_1 f(\theta_1) \frac{\partial V^W}{\partial b_1} - \lambda \left[ s_1 f(\theta_1)(2 - \sigma) - te_1 f(\theta_1) \frac{\partial s_1}{\partial b_1} \right] &= 0 \\
(1 - s_1 f(\theta_1)) \frac{\partial V^I}{\partial b_2} - \lambda \left[ (1 - s_1 f(\theta_1))s_2 f(\theta_2) - te_1 f(\theta_1) \frac{\partial s_1}{\partial b_2} - te_2 f(\theta_2) \left( \frac{\partial s_2}{\partial b_2} + \frac{\partial s_2}{\partial a^I} \frac{\partial a^I}{\partial b_2} \right) \right] &= 0 \\
    s_1 f(\theta_1) \frac{\partial V^W}{\partial b^I} + (1 - s_1 f(\theta_1)) \frac{\partial V^I}{\partial b^I} - \lambda \left[ (s_1 f(\theta_1)\sigma + 1 - s_1 f(\theta_1)) \right. \\
    - te_1 f(\theta_1) \frac{\partial s_1}{\partial b^I} - te_2 f(\theta_2) \frac{\partial s_2}{\partial b^I} \frac{\partial a^I}{\partial b^I} = 0 \\
(1 - s_1 f(\theta_1)) \frac{\partial V^I}{\partial b^{II}} - \lambda \left[ (1 - s_1 f(\theta_1))(1 - s_2 f(\theta_2)) - te_1 f(\theta_1) \frac{\partial s_1}{\partial b^{II}} - te_2 f(\theta_2) \left( \frac{\partial s_2}{\partial b^{II}} + \frac{\partial s_2}{\partial a^I} \frac{\partial a^I}{\partial b^{II}} \right) \right] &= 0
\end{align*}
\]

• Suppose individuals are unable to self-insure, so \( a^I = a^W = 0 \). It follows that \( c^W = c^{WW}, c^W = c^I \) and the first-order conditions on \( b_1 \) and \( b^I \) reduce to:

\[
\begin{align*}
    s_1 f(\theta_1) (2 - \sigma) v'(c^W) - \lambda \left[ s_1 f(\theta_1)(2 - \sigma) - te_1 f(\theta_1) \frac{\partial s_1}{\partial b_1} \right] &= 0 \\
    [s_1 f(\theta_1)\sigma + 1 - s_1 f(\theta_1)] u'(c^I) - \lambda \left[ (s_1 f(\theta_1)\sigma + 1 - s_1 f(\theta_1)) - te_1 f(\theta_1) \frac{\partial s_1}{\partial b^I} \right] &= 0
\end{align*}
\]

Solving for \( \lambda \) and equating yields:

\[
    s_1 (1 - \sigma) \left[ s_1 f(\theta_1)\sigma + 1 - s_1 f(\theta_1) \right] \frac{u'(c^I) - v'(c^W)}{v'(c^W)} = -te_1 \frac{\partial s_1}{\partial b^I}
\]

where \( s_1 \) will be decreasing in \( b^I \) if individuals cannot self-insure.
The first-order conditions on \( b_2 \) and \( b^{II} \) reduce to:

\[
(1 - s_1 f(\theta_1)) \frac{\partial V^I}{\partial b_2} - \lambda \left[ (1 - s_1 f(\theta_1)) s_2 f(\theta_2) - t_{e1} f(\theta_1) \frac{\partial s_1}{\partial b_2} - t_{e2} f(\theta_2) \frac{\partial s_2}{\partial b_2} \right] = 0
\]

\[
(1 - s_1 f(\theta_1)) \frac{\partial V^I}{\partial b^{II}} - \lambda \left[ (1 - s_1 f(\theta_1))(1 - s_2 f(\theta_2)) - t_{e1} f(\theta_1) \frac{\partial s_1}{\partial b^{II}} - t_{e2} f(\theta_2) \frac{\partial s_2}{\partial b^{II}} \right] = 0
\]

Solving for \( \lambda \) and equating yields:

\[
s_2(1 - s_1 f(\theta_1))(1 - s_2 f(\theta_2)) \frac{u'(c^{II}) - v'(c^{IW})}{v'(c^{IW})} = -t_{e2} \frac{\partial s_2}{\partial b^{II}}
\]

where \( s_2 \) is decreasing in \( b^{II} \). With endogenous search, it must be that \( v(c^{IW}) > u(c^{II}) \) to induce individuals to search for work in the second period. Assuming either the same utility of consumption function with some positive disutility of work, this implies that \( v(c^{IW}) < u(c^{II}) \) and the employment tax \( t_{e2} \) must be positive. The government cannot fully insure individuals against involuntary unemployment in the second period. Note as well that the effect of these second-period transfers or the incentive-insurance trade-off is not affected by the search effort decision in the first period. This will still be the case with savings.

- Suppose now individuals can save. Combining the first-order conditions on \( b_1 \) and \( b^I \) yields

\[
s_1 \left[ \frac{\partial v^W}{\partial b^I} - \frac{\partial v^I}{\partial b^I} \right] \left[ \frac{\partial V^W}{\partial b_1} \left[ (s_1 f(\theta_1)\sigma + 1 - s_1 f(\theta_1)) \right] - \left( s_1 f(\theta_1) \frac{\partial V^W}{\partial b^I} + (1 - s_1 f(\theta_1)) \frac{\partial V^I}{\partial b^I} \right)(2 - \sigma) \right]
\]

\[
= -t_{e1} \frac{\partial s_1}{\partial b^I} + t_{e2} f(\theta_2) \frac{\partial s_2}{\partial a^I} \frac{\partial a^I}{\partial b^I} s_1 \left[ \frac{\partial v^W}{\partial b^I} - \frac{\partial v^I}{\partial b^I} \right]
\]

The above reduces to the expression above when \( a^I = a^W = 0 \). Unlike that case, the incentive-insurance trade-off in choosing between \( b_1 \) and \( b^I \) is now affected by the second-period employment tax since \( b^I \) affects second-period search behaviour through the savings decision. The one-period unemployment transfer affects how much the involuntary unemployed can save which in turn affects how much they search at the beginning of period 2.
Combining instead the first-order conditions on $b_2$ and $b^{II}$ yields

\[
(1 - s_1 f(\theta_1))(1 - s_2 f(\theta_2)) \frac{\partial V_1^I}{\partial b_2} - (1 - s_1 f(\theta_1)) s_2 f(\theta_2) \frac{\partial V_1^I}{\partial b^{II}}
\]

\[
= t_{e2} f(\theta_2) \left[ v'(c^{IW}) \frac{\partial s_2}{\partial b^{II}} + \frac{\partial s_2}{\partial a^I} \left( \frac{\partial V_1^I}{\partial b_2} \frac{\partial a^I}{\partial b^{II}} - \frac{\partial a^I}{\partial b_2} \frac{\partial V_1^I}{\partial b^{II}} \right) \right]
\]

Again the above reduces to what is above when $a^I = a^W = 0$. As in that case, the incentive-insurance trade-off in choosing between $b_2$ and $b^{II}$ is independent of first-period search.

Incorporating the participation decision would not affect the above two conditions characterizing the incentive-insurance condition between $b_1$ and $b^I$ and between $b_2$ and $b^{II}$, respectively. But, as in the static case the optimal participation tax is affected by the moral hazard as a result of endogenous search effort.