Optimal Taxation of Capital Income when Capital Returns are Heterogeneous

Aart Gerritsen, Bas Jacobs, Alexandra V. Rusu and Kevin Spiritus

July 2015

VERY PRELIMINARY VERSION
DO NOT QUOTE / DO NOT SPREAD

Abstract
We derive optimal linear and non-linear taxes on labor and capital income in a two-period life-cycle extension of the Mirrlees (1971) model where individuals are heterogeneous in their ability to earn labor and to earn capital income. Preferences are weakly separable and identical across individuals, so that capital-income taxes are zero if capital returns are identical for all individuals. We demonstrate that capital income should optimally be taxed if capital returns correlate positively with ability. Conditional on labor income, the capital-income tax base then provides information about an individual’s underlying earning ability. The capital-income tax therefore provides distributional benefits over and above the redistributive benefits of the labor-income tax. The optimal capital-income tax strikes a balance between taxing rents in capital returns and distorting savings.

JEL: H21, H24
Keywords: Optimal taxation, capital-income taxation, heterogeneous returns

1 Introduction

Ever since Pigou (1920) it is argued that taxes on capital income tax labor income twice: first when it is earned and second when it is saved. In this view, inequality in capital income is the natural consequence of inequality in labor earnings. Is taxing capital income desirable then? According to Atkinson and Stiglitz (1976) a redistributive government should refrain from taxing capital income, provided individuals have the same and weakly separable preferences between consumption and labor. Since the only source of inequality is the ability to earn labor income,
taxing capital income cannot yield additional redistributional benefits – over and above the redistributional benefits of taxing labor income. But taxes on capital income do cause additional distortions in savings, which can be avoided by not taxing capital income at all. Consequently, capital income should not be taxed for redistributional reasons when all heterogeneity originates from differences in labor earnings. Indeed, the only case for taxing capital income rests on second-best arguments: under non-separable preferences capital taxes may be useful to alleviate labor-tax distortions caused by labor-income taxation, see for example Erosa and Gervais (2002), Conesa, Kitao, and Krueger (2009) and Jacobs and Broadway (2014).

However, in the real world individuals do not only differ in their ability to earn labor income. People differ in many other dimensions that importantly affect the amount of capital individuals accumulate during their life-time: preferences to save, endowments and bequests. These additional sources of inequality, besides inequality in earning ability, are important in explaining inequalities in wealth and capital income (e.g., Piketty, 2014). Banks and Diamond (2010) cite ample empirical evidence that discount rates are correlated with labor earnings ability. If individuals differ in their preference to save, wealth holdings will differ between individuals – conditional on their labor income – and taxes on capital income are socially desirable (Saez, 2002; Diamond and Spinekewijn, 2011). Individuals also differ in their endowments of certain commodities or the inheritance they receive from their parents. Piketty (2014) presents evidence that inheritances are indeed strongly associated with labor earnings. If endowments or bequests correlate positively with labor earnings, taxing endowments and bequests is socially optimal, even while assuming weakly separable preferences (Cremer, Pestieau, and Rochet, 2001; Piketty and Saez, 2013).

There are virtually no studies that analyze the desirability of taxes on capital income when individuals systematically differ in the rates of return on their wealth holdings, besides their ability to earn labor income. To the best of our knowledge, only Stiglitz (1985) considers this issue and conjectures that taxing capital income is socially desirable. In this paper we take up Stiglitz’ conjecture and analyze optimal taxes on capital income when individuals are heterogeneous with respect to the rate of return they earn on their assets.

We develop a simple two-period model of saving, asset determination and labor supply to analyze optimal linear and non-linear taxes on labor and capital income. Like in Mirrlees (1971) individuals differ in their labor earnings ability, which is private information. In addition, individuals own an asset which yields a heterogeneous return. We label these returns as the individual’s capital earnings ability. Capital earnings ability is also private information. We are agnostic about the underlying mechanisms that generate differences in asset returns, but we provide a

---

1 Jacobs and Bovenberg (2010) argue that taxes on capital income are also optimal to alleviate labor-tax distortions on human capital formation. Christiansen and Tuomala (2008) and Reis (2010) show that taxes on capital income are optimal to avoid tax-arbitrage between labor and capital income if entrepreneurs can transform labor income in lower taxed capital income. Non-insurable labor risk results in precautionary saving, which generate wealth effects in labor supply that exacerbate labor-market distortions. By taxing capital income the government reduces these wealth effects and alleviates labor-supply distortions, see also Diamond and Mirrlees (1978), Golosov, Kocherlakota, and Tsyvinski (2003), or Jacobs and Schindler (2012).
number of plausible micro-foundations. Our main assumption is that financial markets are incomplete so that asset returns are not equalized across all individuals. All individuals can also save and borrow within a perfect capital market where an asset with a common return is traded. The government is unable to distinguish between capital returns from the traded asset and the heterogeneous returns from the non-traded asset. Consequently, all capital incomes are taxed under a single, potentially non-linear tax schedule. We assume that individual preferences are weakly separable and satisfy the conditions for the Atkinson and Stiglitz (1976) theorem to hold. Hence, in the absence of heterogeneous asset returns, optimal taxes on capital income would be zero.

We demonstrate that taxes on capital income are optimal when returns to capital are heterogeneous.

In the general case, there is both heterogeneity in labor and capital earnings ability. We analyze the case in which labor earnings ability and capital earnings ability are correlated and both driven by the same underlying ‘general’ earnings ability. Individuals that have higher capital earning ability earn more capital income – conditional on their labor earnings. This of course implies that differences in capital income do not only originate from differences in labor earning ability, but also from differences in capital earning ability. Hence, the tax base for capital income reveals information on capital earning ability. This holds true irrespective of whether capital earnings ability varies independently from labor earnings ability, or is a function of labor earnings ability. The optimal capital income tax is thus positive if capital returns increase in ability. The optimal capital tax trades off the redistributional benefits from taxing the rents in capital income against the distortions in savings behavior.

Empirics...

The setup of this paper is as follows. ...

2 Model

2.1 Individuals

2.1.1 Preferences and budget constraints

We assume there is a unit-mass continuum of individuals who are indexed by \( i \in \mathcal{I} \). Individuals are assumed to live for two periods. In the first period, they supply labor, \( l^i \), and consume \( c^i_1 \). In the second period, they consume \( c^i_2 \). We assume that all individuals have identical utility functions that are separable between consumption and labor effort:

\[
U^i = u(c^i_1, c^i_2) - v(l^i), \quad u^i_1, u^i_2, v^i > 0, \quad -u^i_{11}, -u^i_{22}, v^i_l < 0, \tag{1}
\]

where subscripts denote partial derivatives, such that \( u^i_1 \equiv \partial u(c^i_1, c^i_2) / \partial c^i_1 \), etc. We assume that utility is strictly concave and increasing in consumption, and that disutility of work, \( v(\cdot) \), is
convex and increasing in labor. These preferences ensure that optimal capital-income taxes are zero when returns to capital are homogeneous and non-linear taxes are optimized (cf. Atkinson and Stiglitz, 1976).

Individuals differ with respect to their ability, $n^i$, which drives both their returns to labor and their returns to savings. As in Mirrlees (1971), all workers are assumed to be perfect substitutes in production. This allows us to normalize individual $i$’s labor productivity or ‘labor-earnings ability’ as $n^i$, while we denote his ‘capital-earnings ability’ as $\theta^i \equiv \theta(n^i)$. Both $n^i$ and $\theta^i$ are private information.

Individual $i$’s effective labor supply is given by $l^i$, such that he produces and earns output equal to $z^i \equiv n^i l^i$. All labor income is earned in the first period and spent on labor-income taxes, first-period consumption, and savings. The government levies a potentially non-linear tax on labor income which, for individual $i$, is denoted by $T^i$. This tax is affected both by changes in individual $i$’s labor income and by government reforms. To capture this, we write the tax burden as a function of labor income and a ‘reform parameter’ $\kappa$, which yields $T^i \equiv T(z^i, \kappa)$. We assume that the tax schedule is twice differentiable in labor income. We further specify the tax schedule in later sections when we study tax reforms as changes in the parameter $\kappa$. We denote savings as $a^i$, such that individual $i$’s first-period budget constraint can be written as:

$$a^i = z^i - T^i - c^i_1. \quad (2)$$

Savings are invested into a freely traded asset, which yields a common rate of return of $r$. Moreover, the individual can borrow at a rate $r$ to invest in a closely held asset. Capital-earnings ability $\theta^i$ is defined as the excess return (net of borrowing costs) on closely held assets. Capital income is earned in the second period and given by:

$$y^i = ra^i + \theta^i. \quad (3)$$

All savings and net-of-tax capital income are used for second-period consumption. The government sets a non-linear tax on capital income that is denoted by $\tau^i$. As with the labor-income tax, an individual’s capital-income tax can change either due to a change in his capital income or because of a reform of the tax schedule. For this reason we write $\tau \equiv \tau^i(y^i, \mu)$, with $\mu$ a reform parameter for the capital-income tax. We assume that the tax schedule is twice differentiable in capital income. We further specify the tax schedule below, when we study reforms of the capital-income tax. We can write the second-period budget constraint as:

$$c^i_2 = a^i + y^i - \tau^i. \quad (4)$$

### 2.1.2 Equilibrium behavior

Individuals maximize their utility, given by eq. (1), with respect to consumption, labor supply, and savings, $\{c^i_1, c^i_2, l^i, a^i\}$, subject to the two budget constraints and the definition of $y^i$, given
by equations (2)-(4). This yields the following first-order conditions:

\[
\frac{v_l(z^i/n)}{u_1(c_1^i, c_2^i)} = (1 - T^i_z) n^i. \tag{5}
\]

\[
\frac{u_2(c_1^i, c_2^i)}{u_1(c_1^i, c_2^i)} = \frac{1}{1 + (1 - \tau^i_y) r} \equiv q^i, \tag{6}
\]

where \(T^i_z \equiv T_z(z^i, \kappa)\) and \(\tau^i_y \equiv \tau_y(y^i, \mu)\) are the marginal labor- and capital-income tax rates. Eq. (5) determines individual \(i\)'s equilibrium labor supply, while eq. (6) is the Euler equation that determines the equilibrium time path of consumption. Notice that we defined the relative marginal price of second-period consumption as \(q^i\), which equals the inverse of the net-of-tax gross interest rate.

Notice that eqs. (2)-(4) and (6) allow us to write second period consumption as a function of labor income, ability, and reform parameters: \(c_2^i = c_2(z^i, n^i, \kappa, \mu)\). The same equations allow us to do the same for capital income, \(y^i = y(z^i, n^i, \kappa, \mu)\), and savings, \(a^i = a(z^i, n^i, \kappa, \mu)\). For later reference, observe that eqs. (3) and (4) imply that the partial derivatives with respect to labor income are related as follows:

\[
\frac{\partial c_2}{\partial z^i} = \frac{1}{q^i} \frac{\partial a^i}{\partial z^i} = \frac{1}{r q^i} \frac{\partial y^i}{\partial z^i}. \tag{7}
\]

We assume that both labor income and capital income are increasing functions of ability. It can be shown that this is satisfied as long as excess returns \(\theta^i\) are not too strongly decreasing in ability \(n^i\).

2.2 Government

The government can levy separate non-linear taxes on labor and capital income.\(^2\) The underlying informational assumptions are that the government can only verify individual labor income \(z^i\) and individual capital income \(y^i\). This implies that the government cannot distinguish between the sources of labor income and capital income. That is, the government cannot observe individual labor supply and wage rates separately, nor can it observe individual capital income from traded assets and individual capital income from the closely held assets separately. Hence, the government taxes all sources of capital income symmetrically. If the excess capital income from the closely held asset \(\theta^i\) would be verifiable, the government could fully tax it away without causing any distortions.

Non-verifiability of \(\theta^i\) also implies that the government cannot observe individual savings and consumption decisions. If the government observes both savings and total capital income, it could fully tax away \(\theta^i\) by taxing capital income at a hundred percent, while providing a subsidy on savings at a rate of \(r\). This does not seem realistic and would provoke huge tax shifting if applied in practice. Similarly, if the government observes both consumption and labor income, it

\(^2\)As shown by Renes and Zoutman (2015), when wages and excess returns to savings are functions of the same underlying ability, an optimal allocation can be decentralised by separable tax schedules as long as social preferences respect individual preferences.
could fully tax away $\theta^i$ by raising consumption taxes to infinity while lowering the labor-income tax to minus infinity. This does not seem feasible in practice, due to (international) tax arbitrage (e.g., through smuggling).

A benevolent social planner maximizes social welfare subject to the government’s budget constraint. Social welfare is given by a weighted integral of utility:

$$W \equiv \int_{\mathcal{I}} \gamma^i U^i \, di,$$

(8)

with $\gamma^i$ an individual-specific weight. In the case of utilitarian social preferences, $\gamma^i = \gamma$ for all $i \in \mathcal{I}$.

The government is constrained in its policy by the amount of available resources. Similar to individuals, the government can save and borrow at an interest rate $r$. Total government revenue is given by the integral over the labor- and capital-income tax burdens:

$$B \equiv \int_{\mathcal{I}} \left( T(z^i, \kappa) + \frac{\tau^i(y^i, \mu)}{1 + r} \right) \, di,$$

(9)

which must, in equilibrium, be non-negative. Notice that revenue from the capital-income tax is discounted at a rate $1 + r$ because capital-income taxes are collected in the second period.

3 Optimal taxes – a heuristic derivation

In this section, we heuristically derive the main theoretical result of the paper: a necessary condition for optimal taxes that shows how the total tax burden should be divided between labor- and capital income. The derivation is heuristic in that it describes the relevant social welfare trade-off between labor- and capital-income taxation in an argumentative fashion. In the next section we reproduce the same condition, as well as produce other insightful necessary conditions, in a more mathematically rigorous way.

The key to our heuristic derivation is to realize that the government can raise identical amounts of revenue from the same individuals through either labor- or capital-income taxes. We assumed that both labor income and capital income are monotonically increasing with ability. Thus, to raise revenue from individuals with an ability level above $n^*$, the government can either raise the marginal labor-income tax rate at $z^*$, or it can raise the marginal capital-income tax rate at $y^*$, where $z^*$ and $y^*$ correspond to the same ability level $n^*$. In the optimum, the government must be indifferent between raising revenue through either tax instrument. Thus, we obtain a necessary condition for the optimum by comparing the welfare effects of the capital-income tax hike with those of the labor-income tax hike, and equating them.

So we consider a labor-income tax hike at $z^*$ and a capital-income tax hike at $y^*$ that retrieve equal amounts of revenue from the same individuals, and thus affect those individuals identically. When comparing the welfare effects of both increases in marginal tax rates we can therefore ignore
the effects on the behavior and utility of individuals with ability above $n^*$. This also implies that we can ignore the effects on social welfare as given by eq. (8). Instead, we can solely focus on the substitution effects on the tax bases of individuals with ability level $n^*$, and their consequences for the government’s tax revenue. We first concentrate on the substitution effects of an increase in the marginal labor-income tax rate, before turning our attention to the substitution effects of an increase in the marginal capital-income tax rate.

3.1 Substitution effects of a labor-income tax hike

Consider an increase in the marginal labor-income tax for an infinitesimally small interval $dz^*$ around labor income $z^*$, such that individuals with higher labor income face a unit increase in taxes. This implies an increase in the marginal tax rate of $1/dz^*$. We denote the compensated net-of-tax rate elasticity of labor income for individuals with income level $z^*$ by $e^*_z$. Hence, a unit increase in the marginal tax rate causes them to reduce their labor earnings by $z^*e^*_z/(1 - T^*_z)$. For every unit decrease in labor income, tax revenue declines by the marginal labor-income tax rate $T^*_z$. Moreover, the reduction in labor income causes individuals to save less. Denoting the marginal propensity to save out of net income for individuals who earn $z^*$ by $s^*$, every unit reduction in labor income leads to $(1 - T^*_z)s^*$ less savings and thus to $r(1 - T^*_z)s^*$ less capital income. Every unit decrease in capital income causes a reduction in present-value tax revenue equal to $\tau^*_y/(1 + r)$.

To obtain the total revenue losses, above effects should be added and multiplied by the number of individuals within the interval $dz^*$. Let $H(z)$ be the proportion of people with labor income below $z$, and $h(z) \equiv H_z(z)$ the associated density function. This implies that the number of individuals within the interval $dz^*$ is given by $h(z^*)dz^*$. The revenue losses caused by the tax hike’s substitution effects then equal:

$$h(z^*) \left( \frac{T^*_z}{1 - T^*_z} + s^* \frac{\tau^*_y r}{1 + r} \right) z^*e^*_z.$$  \hspace{1cm} (10)

The term within large brackets can be interpreted as the tax wedge on net labor income. Indeed, a marginal increase in net labor income causes revenue from the labor-income tax to increase by the first term within brackets; it causes revenue from the capital-income tax to increase by the second term within brackets.

3.2 Substitution effects of a capital-income tax hike

Notice that individuals can exchange first- and second-period resources at a rate $1/q^i$. Individuals with ability above $n^*$ are thus indifferent between a unit increase in the labor-income tax or a $1/q^i$ increase in the capital-income tax. In this subsection we focus on a capital-income tax hike that raises taxes by $1/q^i$ for all individuals with capital income above $y^*$. This implies a $1/(q^*dy^*)$ increase of the marginal capital-income tax rate over an infinitesimally small interval.
dy* around capital income y*. We consider the substitution effects of this marginal tax change.\footnote{Two related things are worth mentioning here. First, recall that the government’s discount rate equals \(1 + r > 1/q^*\). At first glance, a \(1/q^*\) increase in capital-income taxes therefore seems to raise less discounted tax revenue than a unit increase in labor-income taxes. However, consumption smoothing implies that the capital-income tax hike leads to a larger savings response, which exactly makes up for the difference in mechanical revenue gains. Second, with nonlinear capital taxes, \(q^i\) varies across people. Thus, the proposed reform also alters marginal tax rates for individuals with capital income above \(y^*\). We can nevertheless focus solely on the substitution effects at \(y^*\). That is, not only are individuals with ability above \(n^*\) indifferent between the two different tax hikes, but the income current substitution effects of the capital-income tax hike are exactly identical to the income effects of the labor-income tax hike. The next section formally shows the validity of both points.}

An increase in the marginal tax rate of \(1/q^*\) implies a decrease in the net-of-tax gross interest rate of \(r/q^*\). This distorts labor income as it causes its purchasing power to decline. On top of this distortion, a capital-income tax hike also distorts the intertemporal consumption decision as it raises the relative price of second-period consumption. We first consider the intertemporal consumption distortion. Let \(e_{y|z}^*\) be the labor-income conditional gross interest-rate elasticity of capital income for an individual with capital income \(y^*\). Thus, conditional on labor income, a unit reduction of the gross interest rate causes capital income to decline by \(q^*y^*e_{y|z}^*\). A unit reduction in capital income causes discounted tax revenue to decline by \(\tau_y^*/(1 + r)\). The capital-income tax hike thus yields a distortion of consumption that, for every individual within the interval \(dy^*\), causes tax revenue to decline by:

\[
\frac{1}{dy^*}\frac{\tau_y^*r}{1 + r} y^* e_{y|z}^*.
\]  

Second, consider the distortion of labor-income. For this, notice that the price of second-period consumption equals \(q^*\), which is the inverse of the net-of-tax gross interest rate. Lowering the gross interest rate by \(r/q^*\) therefore implies raising the price of second-period consumption by \(r/q^*\).\footnote{That is, \(dy^* = -(q^*)^2d(1/q^*)\). Substituting for \(d(1/q^*) = r/q^*\) yields \(dy^* = rq^*\).} To the extent that marginal labor income is spent on second-period consumption, this price increase affects labor income in the same way as a compensated reduction in net wage rates. That is, a unit increase in second-period prices reduces labor income by \((\partial c^*_z/\partial z^*)z^*e_{z|z}^*/(1 - T^*_z)\). Recall from eq. \((7)\) that \(rq^*\partial c^*_z/\partial z^* = \partial y^*/\partial z^*\). A unit decline in labor income reduces labor-income taxes by \(T^*_z\) and – via lower savings – reduces discounted capital-income taxes by \((1 - T^*_z)rs^*\tau_y^*/(1 + r)\). The capital-income tax hike thus yields a distortion of labor income that, for every individual within the interval \(dy^*\), causes tax revenue to decline by:

\[
\frac{1}{dy^*}\left(\frac{T^*_z}{1 - T^*_z} + s^*\frac{\tau_y^*r}{1 + r}\right)\frac{\partial y^*/\partial z^*}{\partial z^*} z^* e_{z|z}^*.
\]

Recall that the term within large brackets denotes the tax wedge on net labor income.

To obtain the total revenue losses of the capital-income tax hike, both distortions should be added and multiplied by the number of individuals within the interval \(dy^*\). Let \(J(y)\) be the share of individuals that earn capital income below \(y\), and \(j(y) \equiv J_y(y)\) the associated density function. This implies that the number of individuals within the interval \(dy^*\) is given by \(j(y^*)dy^*\). The
revenue losses caused by the tax hike’s substitution effects then equal:

\[ j(y^*) \frac{\tau^* y r}{1 + r} y^* e^*_y + j(y^*) \left( \frac{T^* z}{1 - T^* z} + s^* \frac{\tau^* y r}{1 + r} \frac{\partial y^*}{\partial z^*} z^* e^*_z \right). \]  \tag{13}

The first term gives the revenue losses associated with the intertemporal consumption distortion; the second term gives the revenue losses associated with the labor-income distortion.

### 3.3 Optimal taxes – a necessary condition

As discussed, the two tax hikes only differ in the extent to which they distort the tax bases of individuals with ability \( n^* \). If the distortion of labor-income taxation, as given by eq. (10), is larger than the distortion of capital-income taxation, as given by eq. (13), social welfare can be increased by shifting the tax burden from labor to capital income. Conversely, if the distortion of labor-income taxation is lower than the distortion of capital-income taxation, it is desirable to shift the burden of taxation from capital to labor income. In the optimum, both distortions should be of equal size, such that the government is indifferent between raising taxes on labor income or on capital income. Equating the distortions of eqs. (10) and (13) yields, after rearranging, the following optimality condition:

\[ \frac{\tau^* y r}{1 + r} y^* e^*_y = \frac{z^*}{y^*} \left( h(z^*) \frac{\partial y^*}{\partial z^*} \right) \left( \frac{T^* z}{1 - T^* z} + s^* \frac{\tau^* y r}{1 + r} \right) e^*_z. \]  \tag{14}

The left-hand side gives the intertemporal consumption distortion caused by the capital-income tax hike. In the optimum, this must equal the difference between the labor-income distortions caused by the labor-income tax hike and those caused by the capital-income tax hike – as given by the right-hand side.

We can obtain more insight into eq. (14) by rewriting it in terms of elasticities and tax wedges only. For this, notice that \( H(z^*) = J(y^*) \). Taking the derivative with respect to \( n^* \) yields \( h(z^*)/dn^* = j(y^*)dy^*/dn^* \). Furthermore observe that \( dy^*/dn^* = \partial y^*/dn^* + (\partial y^*/\partial z^*) (dz^*/dn^*) \). Substituting this into eq. (14) yields:

\[ \frac{\tau^* y r}{1 + r} e^*_y = \left( \frac{dz^*}{dn^*} \right)^{-1} \frac{\partial y^*}{\partial n^*} \frac{n}{y^*} \left( \frac{T^* z}{1 - T^* z} + s^* \frac{\tau^* y r}{1 + r} \right) e^*_z. \]  \tag{15}

As a final step, we define \( \xi^*_z \) as the ability elasticity of labor income and \( \xi^*_y|z \) as the labor-income conditional ability elasticity of capital income for individuals with ability \( n^* \). This leads us to the main theoretical result of our paper, as summarized by the following Proposition.

1 For the tax system to be optimal, the following condition must hold at every income level:

\[ \frac{\tau^* y r}{1 + r} e^*_y = \left( \frac{\xi^*_y|z}{\xi^*_z} \right) \left( \frac{T^* z}{1 - T^* z} + s^* \frac{\tau^* y r}{1 + r} \right) e^*_z, \]  \tag{16}
Proof. This section provided a heuristic proof; a formal proof is postponed until the next section.

Eq. (16) gives a necessary condition for optimal taxes in terms of elasticities and tax wedges. If eq. (16) does not hold empirically, it is desirable to shift the tax burden between labor and capital income until the condition does hold. Notice that the compensated elasticities $e^*_{y|z}$ and $e^*_{z}$ are both defined to be positive. Moreover, labor income is increasing with ability, such that $\zeta^*_{y|z}$ is positive as well. Given a positive wedge on labor income, this implies that marginal capital-income taxes are optimally positive if and only if $\zeta^*_{y|z} > 0$. That is, optimal capital-income taxes are positive if, conditional on labor income, capital income is increasing in ability.

This echoes earlier results by, for example, Atkinson and Stiglitz (1976), Mirrlees (1976), Christiansen (1984), Saez (2002) and Jacobs and Boadway (2014). These earlier references show that the labor-income conditional correlation between capital income and ability might be nonzero if preferences are nonseparable between consumption and leisure, or if subutility from consumption is a function of ability as well as consumption. As a result, they show that capital-income taxation is desirable if future consumption is complementary with leisure, or if high-ability individuals have a stronger preference for future consumption. In our paper, we ruled out either mechanism by assuming separable and identical preferences. However, the labor-income conditional ability elasticity of capital is nonzero, and thus optimal capital taxes are nonzero, if the excess returns are a function of ability. This is established by the following Corollary.

1 A positive (negative) marginal capital-income tax at income level $y^*$ is optimal if and only if the excess returns are increasing (decreasing) in ability, such that $\theta_n(n^*) > (\leq)0$.

Proof. A formal proof is postponed until the next section. Here we provide a more intuitive proof. Based on Proposition 1, it suffices to show that the labor-income conditional ability elasticity of capital income is positive (negative) if and only if excess returns are increasing (decreasing) in ability. Consider the case of excess returns that are increasing with ability, $\theta_n(n^*) > 0$. Now take two individuals that differ in ability but (are forced to) have the same labor income. They thus earn the same in the first period, but the high-ability individual earns more excess returns in the second period. Consumption smoothing implies that he consumes more in both periods and thus saves less. The second-period budget constraint, eq. (4), then implies that the high-ability individual earns more capital income. The opposite holds if $\theta_n(n^*) < 0$. ■

Discussion of our results...

4 Optimal taxes – a formal derivation

Notice that eq. (16) is an optimality condition that tells us how a given total tax burden should be distributed between capital and labor. In this section, we not only formally reproduce this condition, but also derive the optimal total tax burden. That is, we derive a condition for optimal labor income taxes given any capital-income tax, as well as a condition for optimal capital-income
taxes given any labor-income tax. Together, these two conditions imply the condition given by eq. (16).

Studies in optimal taxation typically solve for optimal taxes by taking the primal approach. They first solve for the optimal allocation, subject to incentive compatibility constraints, and subsequently determine the tax system that decentralizes this allocation. To derive the optimum in terms of measurable elasticities, however, it is more convenient to take the dual approach and directly solve for the optimal tax schedules. This we do by first parameterizing the tax schedule and then maximizing social welfare with respect to this parameter in the tax schedule. An additional benefit of this approach is that it is more intuitive than the mechanism-design heavy primal approach and closer in spirit to the heuristic proof of the previous section. The primal approach has precursors in Christiansen (1981, 1984), and is described in more detail by Gerritsen (2015).  

4.1 Optimal labor-income taxes

4.1.1 The labor-income tax

We first derive the optimal labor-income tax, before proceeding to consider capital-income taxes. The reason for this is that it is easier to illustrate the dual approach by reference to the well-known and relatively simple labor-income tax problem. To derive the optimal tax system, we consider the welfare consequences of marginal changes, or reforms, to the tax schedules. In the optimum, such marginal reforms should not have any net effect on social welfare. To study the effects of a labor-income tax reform, we characterize the labor-income tax schedule as follows:

\[ T^i \equiv T(z^i, \kappa) = \tilde{T}(z^i) + \kappa t(z^i), \]  

(17)

where \( \tilde{T}(\cdot) \) gives the pre-reform tax schedule, and \( t(\cdot) \) any arbitrary ‘reform function.’ The latter function indicates how a specific reform adjusts the tax burden at any income level \( z^i \). The marginal effects of such a nonlinear reform can be studied by considering a change \( d\kappa \), evaluated at \( \kappa = 0 \). For example, the effect of a reform on the tax burden of individual \( i \) is obtained by taking the derivative of eq. (17) with respect to \( \kappa \):

\[ \frac{dT^i}{d\kappa} = t(z^i) + T_z^i \frac{dz^i}{d\kappa}. \]  

(18)

A reform of the tax schedule alters individual \( i \)’s tax burden directly as his burden is increased by \( t(z^i) \), and indirectly as he might adjust his tax base in response. Similarly, the reform affects his marginal tax rate by:

\[ \frac{dT^i_z}{d\kappa} = t_z(z^i) + T_{zz}^i \frac{dz^i}{d\kappa}, \]  

(19)

In an additional Appendix, available on request, we derive the exact same results by applying the primal approach.
which is obtained by first taking the partial derivative of eq. (17) with respect to \( z^i \), and subsequently taking the total derivative with respect to \( \kappa \). Again, individual \( i \)'s marginal taxes are directly affected by the reform, and indirectly as he adjusts his labor earnings, leading to ‘bracket creep’ in the case of a nonlinear tax schedule (\( T^i_{zz} \neq 0 \)).

### 4.1.2 Behavioral effects of a reform

The welfare-relevant behavioral effects of a labor-income tax reform are those on individuals’ labor- and capital-income tax bases. To formalize these effects, it is useful to first define the elasticities of taxable income, as well as the marginal propensity to save. We define the compensated and uncompensated net-of-tax rate elasticities of labor income and the income semi-elasticity of net labor income, respectively, as:

\[
e^i_z \equiv - \frac{1 - T^i_z}{t(z^i)} \frac{dz^i}{d\kappa} \bigg|_{t(z^i)=0} > 0,
\]

\[
e^i_{z,u} \equiv - \frac{1 - T^i_z}{t(z^i)} \frac{dz^i}{d\kappa} \bigg|_{t(z^i)=z^i} = e^i_z - e^i_{z,u} \in (-1, 0).
\]

The compensated net-of-tax rate elasticity of labor income, \( e^i_z \), gives the relative change in labor income due to a relative change in the marginal net-of-tax rate, given a constant absolute level of taxes. As the change in the marginal net-of-tax rate is given by \( -t(z^i)d\kappa \), the elasticity can be written as in eq. (20). Similarly, the uncompensated net-of-tax rate elasticity of labor income, \( e^i_{z,u} \), gives the relative change in labor income due to a relative change in marginal net-of-tax rates, this time for an identical change in the average tax rate. Finally, the income semi-elasticity of net labor income, \( \eta^i \), denotes the change in net labor income in response to a unit increase in disposable income. As shown in the Appendix, this term equals the difference between the compensated and uncompensated elasticities and is negative but larger than minus unity. In the Appendix, we derive above elasticities in terms of the primitives of our model. For the purposes of our analysis, however, the elasticities themselves function as sufficient statistics.

We define the marginal propensity to save out of net-of-tax first-period income as:

\[
\varsigma^i \equiv - \frac{1}{t(z^i)} \frac{da^i}{d\kappa} = \frac{1}{1 - T^i_z} \frac{da^i}{dz}.
\]

Because we assumed that preferences are separable between consumption and leisure, savings behavior is identically affected by either a decrease in the first-period tax burden or an equal increase in net-of-tax labor income.

Armed with the definition in eqs. (20)-(23), we show in the Appendix that we can write the
effects of a labor-income tax reform on individual tax bases as:

\[
\begin{align*}
\frac{dz^i}{d\kappa} &= -\frac{1}{1-T^i_2}z^i e^it^i(z^i) - \frac{1}{1-T^i_2}\eta^i t(z^i), \\
\frac{dy^i}{d\kappa} &= -rs^i \left( t(z^i) - (1 - T^i_2) \frac{dz^i}{d\kappa} \right). 
\end{align*}
\]

Eq. (24) shows that one can distinguish between two effects of a tax reform on the labor-income tax base: one due to a change in marginal taxes and the other due to a change in the absolute tax burden. An increase in the marginal tax rate, \( t^i(z^i) > 0 \), causes the labor-income tax base to decline proportionately to the compensated elasticity. An increase in the absolute level of taxes, \( t(z^i) > 0 \), raises the labor-income tax base proportionately to the income semi-elasticity of net labor income.

Eq. (25) shows that, conditional on labor income, a change in the marginal labor-income tax does not distort the consumption decision and thus does not affect capital income. However, a unit increase in the absolute level of taxes leads to a decline in savings of \( s^i \) and thus to a decline in capital income of \( rs^i \). This is illustrated by the first bracketed term in eq. (25). The same holds for any reform-induced reduction in net labor income, as illustrated by the second bracketed term in eq. (25).

**4.1.3 Direct social-welfare effects of a reform**

The effect of a labor-income tax reform on the utility of individual \( i \) is obtained by taking the derivative of eq. (1), while applying the envelope theorem:

\[
\frac{dU^i}{d\kappa} = -u_1^i t(z^i). \tag{26}
\]

As usual, the reform only affects utility through changes in the absolute level of taxes, not through changes in marginal tax rates. An increase in the tax burden causes utility to drop in proportion to the marginal utility of (first-period) consumption. The effects on social welfare are obtained by taking the derivative of eq. (8), while substituting for eq. (26):

\[
\frac{dW}{d\kappa} = - \int_{I} \gamma^i u_1^i t(z^i) di. \tag{27}
\]

Higher tax burdens lead to lower levels of social welfare, whereas tax cuts lead to social-welfare gains.
4.1.4 Government-revenue effects of a reform

The reform’s effect on government revenue is obtained by taking the derivative of eq. (9), while substituting for the reform’s effect on tax burdens, as given by eq. (18):

\[
\frac{dB}{dk} = \int_{x} \left( t(z') + T_i^z \frac{dz^i}{dk} + \frac{\tau_i y'}{1 + r} \frac{dy'}{dk} \right) \, \text{d}i. \tag{28}
\]

Government revenue is affected mechanically as taxes are raised by \( t(z^h) \) (first term), and by behavioral effects on the labor- and capital-income tax bases (second and third terms). Substituting for eqs. (24) and (25), we obtain:

\[
\frac{dB}{dk} = \hat{I} \left( \left( 1 - \frac{T_i^z}{1 - T_z} \eta' - s^i \frac{\tau_i y'}{1 + r} (1 + \eta') \right) t(z') - \left( \frac{T_i^z}{1 - T_z} + s^i \frac{\tau_i y'}{1 + r} \right) z^i e_z t_z(z') \right) \, \text{d}i. \tag{29}
\]

Again we can distinguish between the effects of changes in absolute tax burdens and changes in marginal taxes. The effect of an increase in the tax burden, \( t(z') > 0 \), is given by the first bracketed term within the integral. It leads to mechanical revenue gains (first term within brackets), to an increase in the labor-income tax base (second term, recall that \( \eta' \in (-1, 0) \)), and to a net decrease in the capital-income tax base (third term). The effect of an increase in the marginal income tax, \( t_z(z') > 0 \), is given by the second bracketed term within the integral. It reduces the labor-income tax base and consequently also the capital-income tax base, thereby leading to a reduction of government revenue as long as marginal taxes are positive.

4.1.5 Optimal taxes

Naturally, higher government revenue indirectly leads to gains in social welfare as it can be used to lower tax burdens. The net social-welfare effect of the tax reform is thus a weighted sum of eqs. (27) and (29). We define the social marginal value of government revenue as \( \lambda \), and individual \( i \)'s social marginal utility of consumption as \( g^i \equiv \gamma^i u_1^i / \lambda \). The total social welfare effect of a tax reform is then given by \( dW/\lambda dk + dB/dk \), which equals:

\[
\int_{x} \left( \left( 1 - \frac{T_i^z}{1 - T_z} \eta' - s^i \frac{\tau_i y'}{1 + r} (1 + \eta') \right) t(z') - \left( \frac{T_i^z}{1 - T_z} + s^i \frac{\tau_i y'}{1 + r} \right) z^i e_z t_z(z') \right) \, \text{d}i. \tag{30}
\]

Labor-income taxes are set optimally if and only if any small change to the tax schedule leaves total social welfare unaffected. That is, in the optimum, eq. (30) must equal zero for any reform function \( t(z') \).

_Raising a lump-sum tax_ – To further characterize optimal labor-income taxes, we consider two specific reforms of the labor-income tax. The first reform raises the tax burden equally for every individual, such that \( t(z') = 1 \) and \( t_z(z') = 0 \) for all \( i \). Lowering private income affects social welfare through a drop in utility levels, but also through income effects on the tax bases.
This is captured by what Diamond (1975) dubs the marginal social utility of income, which is defined by:

$$\alpha^i \equiv g^i + \frac{T^i}{1 - T^i} \eta^i + s^i \frac{\tau^i r}{1 + r} (1 + \eta^i).$$  \hspace{1cm} (31)

As $\alpha^i$ determines the value of redistributing towards individual $i$, we refer to it as his welfare weight. Substituting the reform into eq. (30) and equating to zero yields:

$$\hat{I} \alpha^i d_i = 1.$$  \hspace{1cm} (32)

The left-hand side gives the average marginal social value of resources in the hands of the private sector, whereas the right-hand side gives the social value of resources in the hands of the public sector. Thus, in the optimum, the government is on the margin indifferent between resources in the hands of the private or public sectors (cf., Jacobs, 2013). Moreover, welfare weights equal one on average.

**Raising a marginal tax rate** – The second reform raises the tax burden equally for every individual with labor income above a certain level $z^*$, while keeping taxes constant for every individual with income below $z^*$. This reform is formalized by letting $t(z^i) = 1$ for all $z^i > z^*$ and $t(z^i) = 0$ for all $z^i \leq z^*$. As a result, the marginal tax rate increases for individuals with income $z^*$ but remains unchanged for the rest. To determine the increase in the marginal tax rate, notice that the definition of the derivative implies that $t'(z^*) = \frac{t(z^* + dz^*) - t(z^*)}{dz^*}$. Substituting for $t(z^* + dz^*) = 1$ and $t(z^*) = 0$, we obtain $t'(z^*) = 1/dz^*$. Substituting the reform into eq. (30) and equating to zero yields:

$$\int_{z^i = z^*} \left( \frac{T^i}{1 - T^i} + s^i \frac{\tau^i r}{1 + r} \right) z^i \epsilon^i \frac{dz^i}{dz^*} = \int_{z^i > z^*} (1 - \alpha^i) \frac{dz^i}{dz^*}$$  \hspace{1cm} (33)

Notice that the integral on the left-hand side is taken over all individuals with $z^i = z^*$, whereas the integral on the right-hand side is taken over all individuals with $z^i > z^*$. The left-hand side gives the marginal revenue losses caused by the hike in the marginal tax rate. The right-hand side gives the marginal redistributive gains of raising the tax burden for all individuals with income above $z^*$. More precisely, it gives the difference between the marginal social value of public resources and the marginal social value of private resources in the hands of individuals with income above $z^*$.

We can rewrite eq. (33) in terms of the labor income distribution. We define the cumulative distribution function $H(z) \equiv \int_{z \leq z} d_i$ as the proportion of individuals with income below $z$. This implies that $dH(z) = \int_{z \leq z} d_i$ and $h(z) \equiv H'(z)$ as the associated density function. Finally, notice that all variables on the left-hand side of eq. (33) are constant for a given income level. This yields the following Proposition.

**Proposition 2** For any given capital-income tax, labor-income taxes are set optimally if and only if the
The following condition holds for every income level $z^*$:

$$\frac{T^*_z}{1 - T^*_z} + s^* \frac{r\tau^*_y}{1 + r} = 1 \frac{1 \quad 1 - H(z^*)}{e^*_z} \left( \frac{1 - \int_{z^* \geq z^*} \alpha^i d_i}{1 - H(z^*)} \right).$$  \hspace{1cm} (34)

where asterisks indicate variable values for individuals with income level $z^*$.

**Proof.** The Proposition immediately follows from the above derivations. \qed

The left-hand side of eq. (34) gives the wedge on net labor income at $z^*$. That is, it gives the social marginal benefit of an increase in individuals’ net income. To see this, note that an additional unit of net income implies an increase in gross income of $1/(1 - T^*_z)$, leading to additional labor-income tax revenue of $T^*_z/(1 - T^*_z)$. Furthermore, as individuals save a proportion $s^*$ of their net income, it leads to an additional discounted revenue gain of $s^* r \tau^*_y/(1 + r)$. The right-hand side of eq. (34) is the standard expression of the optimal wedge on labor income (cf. Saez, 2001; Piketty and Saez, 2013). It indicates that the tax wedge is decreasing in the responsiveness of the tax base – as given by the compensated elasticity of labor income – and the density of labor income at $z^*$, and increasing in the redistributive gains associated with raising the tax burden of individuals with income above $z^*$.

As long as welfare weights are decreasing with income, eq. (32) implies that the redistributive gains are strictly positive for all but the lowest level of labor income. Moreover, with a bounded income distribution, $1 - H(z^*)$ is strictly positive for all but the highest level of labor income. As a result, the optimal wedge on labor income is strictly positive for all but the lowest and highest levels of labor income. However, because the capital-income tax enters the wedge on labor, eq. (34) in itself does not imply that optimal marginal taxes on labor income are positive. To determine this, we must first also derive the optimal capital-income tax, which is our focus in the next subsection.

### 4.2 Optimal capital-income taxes

#### 4.2.1 The capital income tax

It is analytically convenient to characterize the tax schedule in such a way that a reform of the capital-income tax can easily be compared to a reform of the labor-income tax. As indicated in the previous section, it is most natural to compare a unit increase in labor-income taxes with a $1/q^i$ increase of capital-income taxes because individuals will be indifferent between either tax hike. For this reason, we specify the capital-income tax schedule as follows:

$$\tau^i \equiv \tilde{\tau}(y^i, \mu) = \tilde{\tau}(y^i) + (1 + (1 - \tilde{\tau}(y^i)) r) \mu \pi(y^i).$$ \hspace{1cm} (35)

Again, $\tilde{\tau}(y^i)$ gives the pre-reform tax burden, and $\pi(y^i)$ any arbitrary nonlinear reform function. The shape of the reform function determines how a reform adjusts the tax burden at any capital-income level $y^i$. A reform of the capital-income tax can be studied by considering a change $d\mu$
evaluated at $\mu = 0$. Thus, a reform’s effect on individual $i$’s tax burden is obtained by taking the derivative of eq. (35), evaluated at $\mu = 0$:

$$\frac{d\tau^i}{d\mu} = \frac{1}{q^i} \pi(y^i) + \tau^i_y \frac{dy^i}{d\mu}. \quad (36)$$

The reform can be seen to affect individual $i$’s tax burden both directly by raising the tax burden at $y^i$ by $\pi(y^i)/q^i$, and indirectly as the individual might adjust his capital income. Similarly, the reform’s effect on the marginal capital-income tax is given by:

$$\tau_y(y^i, \mu) = \frac{1}{q^i} \pi_y(y^i) + \left( \frac{dy^i}{d\mu} - r\pi(y^i) \right) \tau^i_{yy}, \quad (37)$$

which is obtained by taking the partial derivative of eq. (35) with respect to income, and subsequently taking the total derivative with respect to $\mu$. The first term gives the direct effect of the reform that raises the marginal tax rate at $y^i$ by $\pi_y(y^i)/q^i$. The second term indicates that the marginal tax rate is also affected by the induced change in capital income. Because a reform raises the total tax burden by $\pi(y^i)/q^i$, where $q^i$ might be nonlinear, $\pi(y^i)$ also enters eq. (37). Indeed, a change in capital income only affects the marginal tax rate to the extent that it differs from $r\pi(y^i)$, which, for individual $i$, is exactly the amount of additional capital-income needed to restore pre-reform second-period earnings. In effect, because of the way we specified the tax schedule, it is as if the reform taxes away $\pi(y^i)$ resources in the first period, rather than $\pi(y^i)/q^i$ resources in the second period.

4.2.2 Behavioral effects of a reform

Again, the relevant behavioral effects of the capital-income tax reform relate to the individual tax bases. To formalize these effects, it is useful to first define the labor-income conditional compensated net-of-tax gross interest rate elasticity of capital income, $e_{y|z}^i$. In other words, $e_{y|z}^i$ measures the relative change in capital income in response to a relative change in the net-of-tax gross interest rate, for a given level of labor income. Notice that the net-of-tax gross interest is given by $1 + (1 - \tau^i_y) r$. As shown in eq. (37), a reform of the marginal capital-income tax reduces this interest rate by $r\pi_y(y^i)/q^i$. For this reason, we can define the elasticity as:

$$e^i_{y|z} = -\frac{dy^i}{q^i\pi_y(y^i) r d\mu} \left. \frac{1 + (1 - \tau^i_y) r}{y^i} \right|_{\pi(y^i) = d_z = 0}. \quad (38)$$

For the purposes of our analysis, $e^i_{y|z}$ functions as a sufficient statistic. Nevertheless, in the Appendix we derive it in terms of the model’s primitives.

Given the definition of this labor-income conditional compensated elasticity, we show in the
Appendix that the effects of a capital-income tax reform on individual tax bases are given by:

\[
\begin{align*}
\frac{dz^i}{d\mu} &= -\frac{1}{1-T_2^i} \frac{\partial y^i}{\partial z^i} e^i z^i \pi_y(y') - \frac{1}{1-T_2^i} \eta^i \pi(y'), \\
\frac{dy^i}{d\mu} &= -y^i e^i \pi_y(y') r + r (1-s^i) \pi(y') + rs^i (1-T_2^i) \frac{dz^i}{d\mu}.
\end{align*}
\] (39) (40)

Eq. (39) shows that an increase in the marginal capital-income tax rate, \( \pi_y(y') > 0 \), leads to a decline in labor income as consumption becomes more expensive relative to leisure. This effect depends on the degree to which higher labor income on the margin translates into higher capital income, \( \partial y^i / \partial z^i \). But otherwise the effect is identical to that of an increase in the marginal labor-income tax rate, as can be seen from eq. (24). Similarly identical to the effects of a labor-income tax rate, an increase in the absolute level of capital-income taxes, \( \pi(y') > 0 \), causes individuals to increase their labor income proportional to \( \eta^i \).

Eq. (40) shows that, conditional on labor income, the reform’s potential effects on capital income are twofold. An increase in the marginal capital-income tax rate reduces capital income proportional to the (labor-income conditional) compensated elasticity. Furthermore, an increase in the absolute level of capital taxes causes individuals to increase their capital income proportional to the marginal propensity to consume out of first-period income, \( (1-s^i) \). Intuitively, as the capital income tax is levied in the second period, part of the burden is smoothed out to the first period by saving more and thus earning more capital income. Finally, any reform-induced unit increase in net labor income raises savings by \( s^i \) and thus capital income by \( rs^i \).

4.2.3 Direct social-welfare effects of a reform

The effect of a capital-income tax reform on the utility of individual \( i \) is obtained by taking the derivative of eq. (1), while applying the envelope theorem:

\[
\frac{dU^i}{d\mu} = -u^i_1 \pi(y').
\] (41)

As the second-period tax burden of individual \( i \) is increased by \( \pi(y') / q^i \), it is as if his first-period tax burden is increased by \( \pi(y^i) \). The effects on social welfare are obtained by taking the derivative of eq. (8), while substituting for (41):

\[
\frac{dW}{d\mu} = -\int_\mathcal{X} \gamma^i u^i_1 \pi(y^i) di.
\] (42)

Higher tax burdens lead to lower levels of social welfare, whereas tax cuts lead to welfare gains.
4.2.4 Government revenue effects of a reform

The reform’s effect on the discounted government budget is obtained by taking the derivative of eq. (9), while substituting for the reform’s effect on the tax burden, as given by eq. (36):

\[
\frac{dB}{d\mu} \equiv \hat{\mathcal{I}} \left( \frac{1 + (1 - \tau_i^y)y}{1 + r} \pi(y') + T_i z_1 \frac{dy'}{d\mu} + \tau_i^y \left( \frac{dy'}{d\mu} - r\pi(y') \right) \right) \, \mathrm{d}i. \tag{43}
\]

Government revenue is affected by a combination of a mechanical effect and behavioral effects on either tax base. As the mechanical increase in second-period government revenue is given by \(\pi(y')/q^i\), whereas the government’s discount rate is \(1 + r > 1/q^i\), the discounted mechanical effect of a reform is less than \(\pi(y')\). However, we also know from eq. (40) that because the tax is levied in the second period rather than the first, there will be larger income effect on the capital-income tax base. To capture this, we can rearrange eq. (43) to obtain:

\[
\frac{dB}{d\mu} \equiv \int_I \left( \pi(y') + T_i z_1 \frac{dy'}{d\mu} + \tau_i^y \left( \frac{dy'}{d\mu} - r\pi(y') \right) \right) \, \mathrm{d}i. \tag{44}
\]

The third term within brackets now captures the revenue effects of a change in the capital-income tax base, net of the change in capital income that would restore the individual’s second-period earnings. Substituting for eqs. (39) and (40) yields:

\[
\frac{dB}{d\mu} \equiv \int_I \left( 1 - \frac{T_i z_1}{1 - T_i} \eta^i - s_i \left( \frac{\tau_i^y y'}{1 + r} (1 + \eta^i) \right) \right) \pi(y') \, \mathrm{d}i
\]

\[
- \int_I \left( \left( \frac{T_i z_1}{1 - T_i} + s_i \left( \frac{\tau_i^y y'}{1 + r} \right) \frac{\partial y'}{\partial z_i} z_i e_i + \frac{\tau_i^y y'}{1 + r} y' e_i \right) \right) \pi(y') \, \mathrm{d}i. \tag{45}
\]

Again we can distinguish between the effects of changes in the absolute tax burdens and changes in marginal taxes. The effect of an increase in tax burdens, \(\pi(y') > 0\), is given by the first integral. It leads to mechanical revenue gains (first term within brackets), to an increase in the labor-income tax base (second term, recall that \(\eta^i \in (-1,0)\)), and to a decrease in the capital-income tax base (net of the change in capital income that would restore the individual’s second-period earnings, third term).

The effect of an increase in the marginal capital-income tax, \(\pi_y(y') > 0\), is given by the second integral. It reduces the labor-income tax base and consequently also the capital-income tax base, thereby leading to a reduction of government revenue as long as marginal taxes are positive (first term within outer brackets). It also leads to a labor-income conditional reduction of the capital-income tax base as marginal taxes distort the intertemporal allocation of consumption (second term within outer brackets).
4.2.5 Optimal taxes

The total social-welfare effect of a capital-income tax reform is given by $dW/\lambda d\mu + dB/d\mu$. Combining eqs. (42) and (45), this is given by:

$$
\int_I (1 - \alpha^i)\pi(y')di - \int_I \left( \frac{\tau_i^r}{1 + r} y^i e_{y|z}^i + \left( \frac{T_z^i}{1 - T_z^i} + s^i \frac{\tau_i^r}{1 + r} \right) \frac{\partial y^i}{\partial z} z^i e_z^i \right) \pi_y(y')di, \tag{46}
$$

where we substituted for individuals' welfare weights $\alpha^i$ from eq. (31). Capital-income taxes are set optimally if and only if any marginal reform leaves total social welfare unaffected. That is, in the optimum, eq. (46) must equal zero for any reform function $\pi(y')$.

To obtain more insight into optimal capital-income taxes, we consider a specific reform that raises the tax burden equally for everyone with capital income above $y^*$ and keeps it constant for the rest. We formalize this reform by setting $\pi(y) = 1$ for all $y > y^*$ and $\pi(z) = 0$ for all $y \leq y^*$. As a result, the marginal tax rate increases for individuals with capital income $y^*$ but remains unchanged for the rest. The definition of the derivative implies that the change in the marginal tax rate at $y^*$ is given by $\pi_z(y^*) = 1/dy^*$. Substituting the reform into eq. (46) and equating to zero yields:

$$
\int_{I,y^i=y^*} \left( \frac{\tau_i^r}{1 + r} y^i e_{y|z}^i + \left( \frac{T_z^i}{1 - T_z^i} + s^i \frac{\tau_i^r}{1 + r} \right) \frac{\partial y^i}{\partial z} z^i e_z^i \right) \frac{di}{dy^*} = \int_{I,y^i>y^*} (1 - \alpha^i)di. \tag{47}
$$

Notice that the integral on the left-hand side is taken over all individuals with $y^i = y^*$, whereas the integral on the right-hand side is taken over all individuals with $y^i > y^*$. The left-hand side gives the revenue losses of the reform, whereas the right-hand side gives the redistributive gains.

Analogous to the case of the optimal labor-income tax, we can rewrite eq. (47) in terms of the capital-income distribution. For this, we define $J(y) \equiv \int_{I,y^i<y} di$ as the proportion of individuals with capital income below $y$, such that $dJ(y) \equiv \int_{I,y^i=y} di$. We furthermore denote the associated density function as $j(y) \equiv J_y(y)$. Finally, notice that all variables on the left-hand side of eq. (47) are constant for a given income level. This brings us to the following Proposition.

3 For any given labor-income tax, capital-income taxes are set optimally if and only if the following condition holds for every income level $y^*$:

$$
y^* j(y^*) \left( \frac{\tau_i^r}{1 + r} e_{y|z}^i + \left( \frac{T_z^i}{1 - T_z^i} + s^i \frac{\tau_i^r}{1 + r} \right) \frac{\partial y^i}{\partial z} z^i e_z^i \right) = \int_{I,y^i>y^*} (1 - \alpha^i)di, \tag{48}
$$

where asterisks indicate variable values for individuals with income level $y^*$.

Proof. The Proposition immediately follows from the above derivations. ■

The right-hand side of eq. (48) gives the redistributive gains of raising the marginal capital-income tax at $y^*$. If welfare weights are decreasing in income, this term is strictly positive for all but the lowest and highest levels of $y^*$. The left-hand side gives the distortive costs of raising
the marginal capital-income tax at \( y^* \). These costs are proportional to the capital income at that level, \( y^* j(y^*) \), and represent both the distortion of the intertemporal consumption decision (first term within outer brackets) and the distortion of labor income (second term within outer brackets). As in the case of optimal-labor income taxes, eq. (48) does not tell us whether capital-income taxes are positive in the optimum. For this, we need to combine it with the optimality condition for the labor-income tax.

### 4.3 Capital-income vs labor-income taxes

Combining eq. (34) and eq. (48) yields:

\[
\frac{\tau^*_y r}{1 + r} c^*_y = \frac{z^*}{y^*} \left( \frac{h(z^*)}{j(y^*)} \right) \left( \frac{T^*_z}{1 - T^*_z} + s^* \frac{v^*}{1 + r} \right) e^*_z, \tag{49}
\]

which corroborates eq. (14). Define the ability elasticity of labor income, as well as the labor-income conditional ability elasticity of capital income as:

\[
\zeta^*_z \equiv \frac{d z^*_i}{d n^*_i} \frac{n^*_i}{z^*_i}, \tag{50}
\]

\[
\zeta^*_y | z \equiv \frac{\partial y^*_i}{\partial n^*_i} \frac{n^*_i}{y^*_i} = \left( \frac{d y^*_i}{d n^*_i} - \frac{\partial y^*_i}{\partial z^*_i} \frac{d z^*_i}{d n^*_i} \right) \frac{n^*_i}{y^*_i}, \tag{51}
\]

Now substitute once more for \( h(z^*)dz^*/dn^* = j(y^*)dy^*/dn^* \), as well as the ability elasticities, into eq. (49) to obtain:

\[
\frac{\tau^*_y r}{1 + r} c^*_y = \left( \zeta^*_y | z \right) \left( \frac{T^*_z}{1 - T^*_z} + s^* \frac{v^*}{1 + r} \right) e^*_z, \tag{52}
\]

which corroborates eq. (16). Optimal capital-income taxes are positive if and only if \( \zeta^*_y | z > 0 \).

As shown in the Appendix, this holds if and only if \( \theta_n(n^*) > 0 \).

### 5 Empirics

### 6 Conclusion

### Appendix

#### A1 Total derivatives of the key equations

Along with the definition of capital income, \( y^i \equiv ra^i + \theta^i \), the equations that determine the equilibrium are given by the two temporal budget constraints and the two individual first-order
conditions, given by eqs. (2) and (4)-(5). Taking total derivatives of these equations yield:

\[
dc_1^i = (1 - T_z^i)dz^i - da^i - \frac{\partial T(z^i, \kappa)}{\partial \kappa}d\kappa,
\]

\[
dc_2^i = \frac{1}{q^i}da^i - \frac{\partial \tau(y^i, \mu)}{\partial \mu}d\mu + (1 - \tau_y^i)d\theta^i,
\]

\[
\left( \frac{u_{12}^i}{u_1^i} - \frac{u_{22}^i}{u_2^i} \right)dc_2^i = \left( \frac{u_{12}^i}{u_1^i} - \frac{u_{11}^i}{u_1^i} \right)dc_1^i - \frac{\tau_{yy}^i}{1 + (1 - \tau_y^i)r}da^i
\]

\[
- \frac{r}{1 + (1 - \tau_y^i)r} \frac{\partial \tau_y(y^i, \mu)}{\partial \mu} d\mu - \frac{\tau_{yy}^i}{1 + (1 - \tau_y^i)r}d\theta^i,
\]

\[
\left( \frac{v_l^i}{v_i^i n^i} + \frac{T_z^i}{1 - T_z^i} \right)dz^i = \frac{u_{11}^i}{u_1^i}dc_1^i + \frac{u_{12}^i}{u_1^i}dc_2^i - \frac{1}{1 - T_z^i} \frac{\partial T_z(z^i, \kappa)}{\partial \kappa} d\kappa + \left( 1 + \frac{t^i v_l^i}{v_i^i} \right) \frac{dn^i}{n^i},
\]

where we substituted for the definition of capital income, and suppressed function arguments for the derivatives of the utility function. Substituting for \(dc_1^i\) and \(dc_2^i\) yields:

\[
\left( \frac{\tau_{yy}^i}{1 + (1 - \tau_y^i)r} + \frac{u_{12}^i}{u_2^i} - \frac{u_{11}^i}{u_1^i} \right) + \frac{1}{q^i} \left( \frac{u_{12}^i}{u_1^i} - \frac{u_{22}^i}{u_2^i} \right) \] \(da^i = (1 - T_z^i) \left( \frac{u_{12}^i}{u_2^i} - \frac{u_{11}^i}{u_1^i} \right) dz^i
\]

\[
- \frac{r}{1 + (1 - \tau_y^i)r} \left( \frac{u_{12}^i}{u_1^i} - \frac{u_{22}^i}{u_2^i} \right) \frac{\partial \tau(y^i, \mu)}{\partial \mu} d\mu
\]

\[
- \left( \frac{u_{12}^i}{u_2^i} \frac{u_{11}^i}{u_1^i} \right) \frac{\partial T(z^i, \kappa)}{\partial \kappa} d\kappa - \left( \frac{\tau_{yy}^i}{1 + (1 - \tau_y^i)r} + \frac{u_{12}^i}{u_1^i} - \frac{u_{22}^i}{u_2^i} \right) \frac{1}{1 - \tau_y^i} d\theta^i
\]

\[
\left( \frac{v_l^i}{v_i^i n^i} + \frac{T_z^i}{1 - T_z^i} \right) = \left( \frac{u_{12}^i}{u_2^i} - \frac{u_{11}^i}{u_1^i} \right) \frac{\partial \tau(y^i, \mu)}{\partial \mu} d\mu
\]

These two equations govern the derivations of the elasticities below.

**A2 Behavioral effects of a labor-income tax reform**

We first consider the behavioral effects of a labor-income tax reform, and therefore set \(d\mu = d\theta^i = dn^i = 0\). Furthermore, recall from eqs. (18)-(19) that \(\partial T(z^i, \kappa)/\partial \kappa = t(z^i)\) and \(\partial T_z(z^i, \kappa)/\partial \kappa = t_z(z^i)\). The marginal propensity to save is obtained from eq. (57) and given by:

\[
s^i \equiv - \frac{1}{t(z^i)} \frac{\partial a^i}{\partial \kappa} = \frac{1}{1 - T_z^i} \frac{\partial a^i}{\partial z^i} = \frac{\tau_{yy}^i}{1 + (1 - \tau_y^i)r} + \left( \frac{u_{12}^i}{u_2^i} - \frac{u_{11}^i}{u_1^i} \right) + \frac{1}{q^i} \left( \frac{u_{12}^i}{u_2^i} - \frac{u_{11}^i}{u_1^i} \right).
\]
Substituting this into eqs. (57)-(58) yields:

\[
\frac{da^i}{dk} = (1 - T_z^i) s^i \frac{dz^i}{dk} - s^i t(z^i),
\]

(60)

\[
\left( \frac{v_{ii}}{v_i^0 n^i} + \frac{T_z^i}{1 - T_z^i} - (1 - T_z^i) \frac{u_{i1}}{u_1^i} \right) \frac{dz^i}{dk} = \left( \frac{u_{i2}}{u_2^i} - \frac{u_{i1}}{u_1^i} \right) \frac{da^i}{dk} - \frac{1}{1 - T_z^i} t(z^i) - \frac{u_{i1}}{u_1^i} t(z^i).
\]

(61)

Eq. (60) reproduces eq. (25) in the main text.

Solving for \( dz^i / dk \) yields:

\[
\left( \frac{v_{ii}}{v_i^0 n^i} + \frac{T_z^i}{1 - T_z^i} - (1 - T_z^i) \left( s^i \frac{u_{i2}}{u_2^i} + (1 - s^i) \frac{u_{i1}}{u_1^i} \right) \right) \frac{dz^i}{dk} = -\frac{t_z(z^i)}{1 - T_z^i} - \left( s^i \frac{u_{i2}}{u_2^i} + (1 - s^i) \frac{u_{i1}}{u_1^i} \right) t(z^i).
\]

(62)

Eq. (62) allows us to write the compensated and uncompensated net-of-tax rate elasticities of labor earnings, as well as the income term, in terms of the underlying primitives of the model:

\[
e_i^z \equiv -\frac{1 - T_z^i}{z^i} \frac{dz^i}{dz^i} \bigg|_{t(z^i)=0} = \left( \frac{v_{ii}}{v_i^0 n^i} + \frac{T_z^i}{1 - T_z^i} - (1 - T_z^i) \left( s^i \frac{u_{i2}}{u_2^i} + (1 - s^i) \frac{u_{i1}}{u_1^i} \right) \right)^{-1}
\]

(63)

\[
e_{i,u}^z \equiv -\frac{1 - T_z^i}{z^i} \frac{dz^i}{dz^i} \bigg|_{t(z^i)=t_s(z^i)} = e_i^z + (1 - T_z^i) \left( s^i \frac{u_{i2}}{u_2^i} + (1 - s^i) \frac{u_{i1}}{u_1^i} \right) z^i e_i^h,
\]

(64)

\[
\eta_i \equiv e_{i,u}^z - e_i^z = (1 - T_z^i) \left( s^i \frac{u_{i2}}{u_2^i} + (1 - s^i) \frac{u_{i1}}{u_1^i} \right) z^i e_i^h \in (-1,0).
\]

(65)

Substituting these equations, along with \( dy^i = rda^i \), into eqs. (60) and (62) yields:

\[
\frac{dy^i}{dk} = -rs^i \left( z^i e_i^z t_z(z^i) - (1 + \eta^i)t(z^i) \right),
\]

(66)

\[
\frac{dz^i}{dk} = -\frac{1}{1 - T_z^i} z^i e_i^z t_z(z^i) - \frac{1}{1 - T_z^i} \eta^i t(z^i).
\]

(67)

Eq. (67) reproduces eq. (24) in the main text.

**A3 Behavioral effects of a capital-income tax reform**

To study the behavioral effects of a capital-income tax reform we set \( dk = d\theta^i = dn^i = 0 \), and consider \( d\mu > 0 \). Recall from eqs. (36)-(37) that \( \partial r(y^i, \mu) / \partial \mu = \pi(y^i) / q^i \) and \( \partial \tau(y^i, \mu) / \partial \mu = \pi(y^i) / q^i - \tau_q^i \pi(y^i) \). Substituting into eqs. (57)-(58) and rearranging yields:

\[
\frac{da^i}{d\mu} = (1 - T_z^i) s^i \frac{dz^i}{d\mu} + (1 - s^i) \pi(y^i) - rs^i \left( \frac{u_{i2}}{u_2^i} - \frac{u_{i1}}{u_1^i} \right) \pi(y^i),
\]

(68)

\[
\left( \frac{v_{ii}}{v_i^0 n^i} + \frac{T_z^i}{1 - T_z^i} - (1 - T_z^i) \frac{u_{i1}}{u_1^i} \right) \frac{dz^i}{d\mu} = \left( \frac{u_{i2}}{u_2^i} - \frac{u_{i1}}{u_1^i} \right) \frac{da^i}{d\mu} - \frac{u_{i2}}{u_2^i} \pi(y^i).
\]

(69)
Solving for $dz^i/d\mu$ yields:

$$
\frac{1}{z^i} \frac{dz^i}{d\mu} = -rs^i \pi_y(y') - \left( s^i \frac{u_{12}^i}{u_2^i} + (1 - s^i) \frac{u_{11}^i}{u_1^i} \right) \pi(y'),
$$

(70)

where we substituted for $e_i^z$ from eq. (63). Furthermore substituting for $\eta^i$ from eq. (65) and for $rs^i = \frac{1}{1 - T^s} \frac{\partial \eta^i}{\partial z^s} = \frac{1}{1 - T^s} \frac{\partial \eta^i}{\partial z^s}$ yields:

$$
\frac{dz^i}{d\mu} = -\frac{1}{1 - T^s} \frac{\partial \eta^i}{\partial z^s} \pi_y(y') z^i e_i^z = \frac{1}{1 - T^s} \eta^i \pi(y'),
$$

(71)

which corroborates eq. (39) from the main text.

Now substitute for $dy^i = rda^i$ into eq. (68) to obtain:

$$
\frac{dy^i}{d\mu} = (1 - T^s)rs^i \frac{dz^i}{d\mu} + r(1 - s^i) \pi(y') - rs^i \left( \frac{u_{12}^i}{u_2^i} - \frac{u_{11}^i}{u_1^i} \right) \pi(y') r.
$$

(72)

This allows us to write the labor-income conditional compensated gross interest rate elasticity of capital income as:

$$
e^i_{y/z} = -\left. \frac{dy^i}{d\mu} \frac{1}{1 - T^s} \frac{\partial \eta^i}{\partial z^s} \pi_y(y') \right|_{\pi(y')=dz^i=0} = \frac{1}{1 - T^s} \eta^i \pi(y') r.
$$

(73)

Substituting this back into eq. (72) yields:

$$
\frac{dy^i}{d\mu} = rs^i(1 - T^s) \frac{dz^i}{d\mu} + r(1 - s^i) \pi(y') - y^i e^i_{y/z} \pi_y(y') r,
$$

(74)

which corroborates eq. (40) from the main text.

A4 The labor-income conditional ability elasticity of capital income

The ability-elasticity of capital income, conditional on labor earnings, is defined as:

$$
\zeta^i_{y/z} = \frac{\partial y^i}{\partial n^i} \frac{1}{\pi' y} = \left( \frac{dy^i}{dn^i} - \frac{\partial y^i}{\partial z^i} \frac{dz^i}{dn^i} \right) \frac{n^i}{\pi'}.
$$

(75)

Notice that $dy^i = rda^i + d\theta^i$, where $d\theta^i = (\partial \theta^i / \partial n^i) d\mu$. Substituting for $da^i$ from eq. (57), we obtain:

$$
\zeta^i_{y/z} = \left( \frac{\frac{u_{12}^i}{u_2^i} - \frac{u_{11}^i}{u_1^i} + \frac{u_{12}^i}{u_2^i} - \frac{u_{11}^i}{u_1^i}}{1 + \frac{1}{1 - T^s}} \frac{T^s \eta^i \pi_y(y')}{\pi' y} \right) \frac{\partial \theta^i}{\partial n^i} \frac{n^i}{\pi'}.
$$

(76)

which has the same sign as $\theta_n(n^i) \equiv \partial \theta^i / \partial n^i$. This proves Corollary 1.
References


Gerritsen, Aart. 2015. “Optimal nonlinear taxation: the dual approach.” Mimeo. 4


Jacobs, Bas. 2013. “The Marginal Cost of Public Funds is One at the Optimal Tax System.” mimeo: Erasmus University Rotterdam. 4.1.5


