Trade tariff, wage gap and public spending

Michele Giuranno* and Antonella Nocco†

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Abstract

This paper studies the interplay between wage gap and government spending in a small open economy facing a liberalization of commodities trade with the external world. We consider an economy with two sectors: an export sector, which uses capital and unskilled labour, and an import-competing sector, which uses capital and skilled labour. In this specific factor model, the return to capital is the link between the two sectors. We show that, since trade liberalization decreases the skilled-unskilled wage gap, government spending will increase, unless there are some kind of perturbation due to either an unbalanced distribution of bargaining power or tariff revenue co-financing of public spending.

Key words: skilled; unskilled; positive political economy.

JEL Classifications: F16; H5.

1 Introduction

This paper studies the interplay between wage gap and government spending in a small open economy facing a liberalization of commodities trade with the external world.

The aim of this analysis is to explore the link between tariff policies and the size of public spending for public goods. We develop a model

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*University of Salento, Faculty of Economics "A. De Viti De Marco", Lecce, 73100 Italy.
†University of Salento, Faculty of Economics "A. De Viti De Marco", Lecce, 73100 Italy.
were public spending is a result of a collective decision-making mechanism where the equilibrium is a compromise among conflicting collective interests.

We consider an economy with two sectors: an export sector, which uses capital and unskilled labour, and an import-competing sector, which uses capital and skilled labour. In this specific factor model, the return to capital is the link between the two sectors. Trade liberalization decreases the skilled-unskilled wage gap as it decreases the skilled and increases the unskilled wages.

Workers are also voters. We capture the conflicting redistributive interests of skilled and unskilled workers by allowing them to elect a representative, one for each group, who is responsible of negotiating the size of public spending in the government.

2 The Economy

Following Acharyya and Kar (2015 ??), we consider a small open economy producing two goods, an export good $x$ using unskilled labour, $L$, and capital, $K$, and an import-competing good $y$ using skilled labour, $S$, and capital, under constant return to scale technology. The government protects the import-competing sector by imposing an "ad valorem" tariff, denoted by $t \in [0, 1]$. Flexible money wages, denoted by $w$ and $w_s$ for unskilled and skilled workers respectively, and flexible rate of return to capital, $r$, coupled with perfectly competitive markets ensure full employment of labour and capital.

The small open economy cannot influence the world prices of these goods, denoted by $P_W^x$ and $P_W^y$.

We now state the set of equations that describes the economy.

$$ P_W^x = a_{LX} w + a_{KX} r \quad (1) $$

$$ (1 + t) P_Y^w = a_{SY} w_s + a_{KY} r \quad (2) $$

where, $a_{LX}$ ($a_{SY}$) denotes the requirement of unskilled (skilled) labour to produce one unit of the export (import-competing) good $x$ ($y$); $a_{KX}$ ($a_{KY}$) denotes the requirement of capital to produce one unit of good $x$ ($y$). Equations (1) and (2) indicate that the domestic producers earn zero profit due to perfect competition and free entry, which equalise price to unit cost.

The following set of equations gives us the least-cost input choice:

$$ a_{LX} = a_{LX} \left( \frac{w}{r} \right), a_{SY} = a_{SY} \left( \frac{w_s}{r} \right), a_{KX} = a_{KX} \left( \frac{w}{r} \right), a_{KY} = a_{KY} \left( \frac{w_s}{r} \right). \quad (3) $$
The following equations describe the full employment conditions:

\[ S = a_{SY}Y \]  
\[ L = a_{LX}X, \]  
where, \( X \) and \( Y \), denote the output levels of good \( x \) and \( y \) respectively; and the terms \( L, S \) denote fixed endowments of unskilled and skylled labour respectively. For what follows, we also assume \( L \geq S \).

Finally, full capital employment is given by

\[ K = a_{KX}X + a_{KY}Y, \]  
where, \( K \) denotes fixed endowment of capital.

Given the world commodity prices and tariff rate, the above set of nine conditions together determine three factor prices, four input choices, and two output levels. It is evident that the wages consistent with zero-profit and full-employment conditions vary with the trade policy choice as captured here by the tariff rate.\(^1\) Clearly, exogenous changes in the tariff rate would change the equilibrium value of wages. The above set of conditions can be used to derive the precise nature and magnitude of changes in wages due to changes in tariff rates, as described below (the proof is in the Appendix),

\[ \hat{w} = -\frac{\theta_{KY} \alpha Y}{\theta_{LX} \alpha} \hat{T} \]  
for the unskilled wage, and

\[ \hat{w}_S = \frac{\theta_{KY}}{\theta_{SY}} \left( \frac{1 - \theta_{KY} \alpha Y}{\theta_{KY}} \right) \hat{T} \]  
for the skilled wage.

Change in the rate of return to capital is given by:

\[ \hat{r} = \frac{\alpha Y}{\alpha} \hat{T}. \]  

In the above equations, "hat" over a variable denotes its proportional change (e.g., \( \hat{w}_S = \frac{dw_S}{\bar{w}_S} \)); \( T = (1 + t) \) and \( \hat{T} = \frac{t}{1+t} \hat{T} \); \( \theta_{LX} = \frac{a_{LX} \bar{w}}{\bar{P}_X} \) is the share of unskilled labour in unit cost of producing good \( x \); cost shares \( \theta_{KY}, \theta_{KX} \) and \( \theta_{SY} \) are similarly defined; \( \alpha \equiv \frac{\lambda_{KY}}{\theta_{SY}} \sigma_Y + \frac{\lambda_{KX}}{\theta_{LX}} \sigma_X = \alpha_Y + \alpha_X \)

\(^1\)This is a typical specific factor model \( a' la \) Jones (1971).

\(^2\)Note that \( \hat{T} = \frac{dt}{1+t} = \frac{dt}{1+t} \hat{T} = \frac{t}{1+t} \hat{T} \).
where, \( \alpha_Y \equiv \frac{\lambda_{KY}}{\sigma_Y} \) and \( \alpha_X \equiv \frac{\lambda_{KX}}{\sigma_X} \) and \( \lambda_{KY} \equiv \frac{\alpha_{KY}Y}{K} \) and \( \lambda_{KX} \equiv \frac{\alpha_{KX}X}{K} \) are the share of sector \( y \) and \( x \) respectively in total employment of capital and \( \sigma_j \) is the elasticity of factor substitution in sector \( j \) \((j = x, y)\), as described in the Appendix.

It follows that the equilibrium wages would change asymmetrically with a reduction of the tariff rate. We write the functional relationship between wages and tariff as follows:

\[
\frac{\partial w(t)}{\partial t} < 0 \quad \text{and} \quad \frac{\partial w_S(t)}{\partial t} > 0.
\] (10)

The intuition is simple. Consider an initial equilibrium with factor prices and output levels. A tariff reduction contracts the domestic production in the import-competing sector through competition from importers. Firms will exit the market of the import-competing good until the decline in the rate of return to capital enables them to break even. As a result, skill wages decline. Capital released from the import-competing sector creates scope for expansion in the export sector, where production increases. Since the fall in the rate of return to capital in the import sector makes the export production more profitable, the demand for unskilled workers raises the unskilled wage under full employment. Therefore, a reduction in tariff has asymmetric effects on skilled and unskilled wages.

3 Public good provision

The government is compounded by the representatives of the two groups of workers. The representatives have to negotiate over the level of public spending for a public good \( g \), to be financed through both a proportional tax \( \tau \) on wages, with \( \tau \in [0, 1] \), and tariff revenue denoted by \( V = tI_y(t) \). In the latter equation, \( I_y(t) \) represents the value of imports. The government budget constraint is:

\[
g = \tau (\omega L + \omega_s S) + tI_y(t) \Rightarrow \tau = \frac{g - tI_y(t)}{\omega L + \omega_s S}
\] (11)

where, \( g - tI_y(t) > 0 \); while, the internal after-tax wages are \( w = \omega (1 - \tau) \) and \( w_s = \omega_s (1 - \tau) \). Furthermore, according to (10), we get

\[
\frac{\partial w(t)}{\partial t} < 0 \Rightarrow \frac{\partial \omega(t)}{\partial t} < 0 \quad \text{and} \quad \frac{\partial w_S(t)}{\partial t} > 0 \Rightarrow \frac{\partial \omega_S(t)}{\partial t} > 0.
\] (12)

Tax paid by an unskilled individual is \( \tau \omega = \omega \frac{g - tI_y(t)}{\omega L + \omega_s S} \) and tax paid by a skilled individual is \( \tau \omega_S = \omega_s \frac{g - tI_y(t)}{\omega L + \omega_s S} \).

The utility of an unskilled individual is
\[ u = \omega - \frac{\omega}{L\omega + S\omega_s} (g - tI_y(t)) + \ln (g + 1). \]  \hfill (13)

Similarly, the utility of a skilled individual is

\[ u_s = \omega_s - \frac{\omega_s}{L\omega + S\omega_s} (g - tI_y(t)) + \ln (g + 1). \]

In the legislature, the representatives of both groups, skilled and unskilled, must form a government where they can negotiate over the size of public spending \( g \).

We also assume that an agreement is needed in order to approve government budget and implement public policy; that is, in the extreme case of disagreement, wages cannot be taxed and we get \( \tau = g = 0 \). Therefore, even if tariff revenue is positive, it cannot be used for the provision of the public good under government impasse, as the government cannot decide. Hence, the disagreement utility is given by private consumption only:

\[ u^d = \omega \text{ and } u^d_s = \omega_s. \]  \hfill (14)

According to the Nash bargaining axiomatic approach, an agreement will occur if and only if

\[ \phi = u - u^d = -\frac{\omega}{L\omega + S\omega_s} (g - tI_y(t)) + \ln (g + 1) \geq 0 \]

and \( \phi_s = u_s - u^d_s = -\frac{\omega_s}{L\omega + S\omega_s} (g - tI_y(t)) + \ln (g + 1) \geq 0 \).

In order to find the equilibrium policy outcome, we maximise the following Nash bargaining product:

\[ g = \arg\max \left[ -\frac{\omega}{L\omega + S\omega_s} (g - tI_y(t)) + \ln (g + 1) \right] \left[ -\frac{\omega_s}{L\omega + S\omega_s} (g - tI_y(t)) + \ln (g + 1) \right]^{1-\gamma}. \]  \hfill (15)

The first order condition is

\[ \gamma \frac{-\frac{\omega}{L\omega + S\omega_s} (g - tI_y(t)) + \ln (g + 1)}{\frac{\omega}{L\omega + S\omega_s}} + \frac{1}{g+1} + (1 - \gamma) \frac{-\frac{\omega_s}{L\omega + S\omega_s} (g - tI_y(t)) + \ln (g + 1)}{\frac{\omega_s}{L\omega + S\omega_s}} + \frac{1}{g+1} = 0, \]  \hfill (16)

which implies \(-\frac{\omega}{L\omega + S\omega_s} + \frac{1}{g+1} \geq 0 \) and \(-\frac{\omega_s}{L\omega + S\omega_s} + \frac{1}{g+1} \leq 0 \) as \( \omega \leq \omega_s \); that is, in equilibrium, the unskilled voters would like to have higher taxation and public spending than the skilled voters, the reason being that the unskilled workers bear a lower marginal cost; i.e., \( MC = \frac{\omega}{L\omega + S\omega_s} \leq \frac{\omega_s}{L\omega + S\omega_s} = MC_s \).

4 The relation between tariff and public spending

In this section, we study how exogenous changes in the tariff policy \( t \) influences government spending \( g \). Our analysis is based on the following Lemma.
Lemma 1  Tariff policy influences government spending as specified in the following relation:
\[
\frac{dg^*}{dt} \geq 0 \text{ if } \epsilon_s \left[ (g - V) \left( \frac{S}{\phi s} + \frac{L}{\phi_s} \right) + V' \left( \frac{\omega_s}{\phi s} - \frac{\omega_s'}{\phi_s'} \right) \right] \geq (1 - \gamma) \frac{L}{\phi s} - \gamma \frac{S}{\phi} \tag{17}
\]

Proof.  The proof is in the Appendix B. ■

We use the above Lemma to study how changes in the tariff \( t \) affects the equilibrium level \( g^* \) of public spending in the following cases: when public good provision is (or is not) co-financed by the tariff revenue and representatives have the same bargaining power and when public good provision is (or is not) co-financed by the tariff revenue and representatives have different bargaining power.

We start from the case where public good provision is not co-financed by the tariff revenue and representatives have the same bargaining power; that is, \( tI_y(t) = 0 \) and \( \gamma = 1/2 \). As a result, the comparative statics in the above Lemma becomes
\[
\frac{dg^*}{dt} \geq 0 \text{ if } \epsilon_s (g - V) \left( \frac{S}{\phi s} + \frac{L}{\phi_s} \right) \geq \frac{1}{2} \left( \frac{L}{\phi s} - \frac{S}{\phi} \right), \tag{18}
\]
which leads to the following Proposition.

Proposition 1  Assume the representatives of skilled and unskilled workers have the same bargaining power and the public good provision is not co-financed by the tariff revenue, then a higher tariff protection leads to both a larger wage gap and a lower public spending.

Proof.  The proof is a straightforward application of Lemma 1. ■

A higher tariff increases the wage gap between skilled and unskilled workers and this worsens the redistributive conflicts between the two working classes. As a result, government provision of public good declines. The opposite relation is also true. This, in turn, implies that government spending for public good provision increases under free-trade (\( t = 0 \)) as this leads to a lower domestic wage disparity.

Now, we study what happens when tariff revenue co-finances the public good provision, while the two representatives have the same bargaining power; i.e.: \( tI_y(t) > 0 \) and \( \gamma = 1/2 \). In this case, the relation in Lemma 1 becomes
\[
\frac{dg^*}{dt} \geq 0 \text{ if } \epsilon_s \left[ (g - V) \left( \frac{S}{\phi} + \frac{L}{\phi} \right) + \frac{\omega_s}{\phi} \left( \frac{\omega}{\phi_s} - \frac{\omega_s}{\phi_s} \right) L \omega + S \omega_s \right] \geq \frac{1}{2} \left( \frac{L}{\phi_s} - \frac{S}{\phi} \right),
\]

which leads to the following Proposition.

**Proposition 2** Assume the representatives of skilled and unskilled workers have the same bargaining power and the public good provision is co-financed by the tariff revenue, then a higher tariff protection leads to a larger wage gap, while public spending decreases if the tariff revenue declines and is ambiguous otherwise.

Now, what happens when skilled and unskilled workers have different bargaining power in the legislature? The answer is in the following Proposition.

**Proposition 3** Assume \( \gamma < 1/2 \). An increase in \( t \) leads to a decrease in public spending when the marginal tariff revenue is weakly negative or tariff revenue is not used at all, and is ambiguous otherwise. On the contrary, when \( \gamma > 1/2 \) and \( t \) increases, the change in government spending is always ambiguous.

**Proof.** The proof is a straightforward application of Lemma 1.

**5 Efficiency**

In this section we study the efficient policy outcome and compare it with the Nash bargaining solution. After solving the following maximization problem

\[
g = \arg\max (Su + Lu_s) = \arg\max \frac{\gamma S}{\omega S} \left[ \omega - \frac{\omega (g - t I_y (t))}{\omega L + \omega_s S} + \ln (g + 1) \right] + (1 - \gamma) L \left[ \omega_s - \frac{\omega_s (g - t I_y (t))}{\omega L + \omega_s S} + \ln (g + 1) \right],
\]

we find that the efficient policy outcome, \( g^e \), is given by

\[
g^e = \frac{\gamma S + (1 - \gamma) L}{\gamma S \omega + (1 - \gamma) L \omega_s} \left( \omega L + \omega_s S \right) - 1, \tag{21}
\]

which does not depend on the co-financing of tariff revenue.

We find that the efficiency condition and the Nash bargaining condition tend to equalise under free trade \( (t = 0) \) when the two groups have the same bargaining power (see Appendix 3).
6 Concluding remarks

In our framework, trade liberalization reduces the redistributive conflicts between the two working groups. As a result, government spending increases unless there are some kinds of perturbation due to either an unbalanced distribution of bargaining power or tariff revenue co-financing of public spending.

In a world without unbalanced political influence, when the government co-finances the provision of public goods through both a direct wage tax and tariff revenues, an increase in the tariff will decrease public spending when it decreases the value of imports. The reason being that decreasing tariff revenue worsens the internal redistributive conflict. On the contrary, a higher tariff may increase public spending when the worsening in the wage-gap is more then compensated by an increase in tariff revenue.

However, there may be reasons why the two groups of voters have different bargaining leverage in the government. The unskilled workers, for instance, are the largest group in the economy, and this might give them a higher bargaining leverage. On the other hand, the smaller group might have a higher political influence. Clearly, the higher the political influence of the skilled workers, the lower the public good provision, unless tariff revenue are sufficiently high.

Finally, we find that free-trade coupled with a balanced political power increase the efficiency of public spending.

References


7 Appendix A

From the full employment condition for unskilled labour (5), we obtain

dL = daLX X+aLX dX; 0 = daLX XaLX +XaLX dX; 0 = XaLX (aLX + X),
which leads to the following proportional change:

\[ \tilde{a}_{LX} + \tilde{X} = 0. \] (22)

Similarly, from the full employment condition for skilled labour (4), we obtain \( d\tilde{S} = d\tilde{a}_{SY} \tilde{Y} + a_{SY} d\tilde{Y} \); \( 0 = \frac{d\tilde{a}_{SY}}{a_{SY}} \tilde{Y} + a_{SY} \frac{d\tilde{Y}}{\tilde{Y}} \), which leads to

\[ \tilde{a}_{SY} + \tilde{Y} = 0. \] (23)

From the full employment condition for capital (6), we obtain

\[ d\tilde{1} = d\tilde{a}_{KY} \frac{\tilde{X}}{K} + a_{KY} d\tilde{X}; \quad \tilde{a}_{KY} \frac{a_{KY} X}{K} + a_{KY} X \tilde{X} + \tilde{a}_{KY} \frac{a_{KY} Y}{K} + a_{KY} Y \tilde{Y} = 0; \] this leads to

\[ \left( \tilde{a}_{KY} + \tilde{X} \right) \lambda_{KY} + \left( \tilde{a}_{KY} + \tilde{Y} \right) \lambda_{KY} = 0. \] (24)

Substitution of (22) and (23) in (24) yields

\[ (\tilde{a}_{KY} - \tilde{a}_{LX}) \lambda_{KY} + (\tilde{a}_{KY} - \tilde{a}_{SY}) \lambda_{KY} = 0. \]

After using the definition of factor substitution elasticity in sector X and Y, \( \sigma_{X} = \frac{\tilde{a}_{KY} - \tilde{a}_{LX}}{\omega s - r} \) and \( \sigma_{Y} = \frac{\tilde{a}_{KY} - \tilde{a}_{SY}}{\omega s - r} \), the above equation boils down to:

\[ \sigma_{X} (\tilde{w} - \tilde{r}) \lambda_{KY} + \sigma_{Y} (\tilde{w}_{s} - \tilde{r}) \lambda_{KY} = 0. \] (25)

Now, we use the zero profit conditions (2) and (1) in order to obtain the relationship between changes in wages and in the rate of return to capital, as follows:

\[ (1 + t) P_{Y}^{W} = a_{SY} \omega s + a_{KY} r; \quad d(1 + t) P_{Y}^{W} = d\tilde{a}_{SY} \omega s + d\tilde{a}_{KY} r + a_{SY} d\omega s + a_{KY} dr; \] since it must be \( d\tilde{a}_{SY} \omega s + d\tilde{a}_{KY} r = 0 \) (see Krugman, Obstfeld and Meliz, 2015, pp. 449-450), we can write

\[ d(1 + t) = \frac{a_{SY} \omega s}{P_{Y}^{W}} dr; \quad \frac{d(1 + t)}{(1 + t)} = \frac{a_{SY} \omega s}{P_{Y}^{W}} \frac{d\omega s}{\omega s} + \frac{a_{SY} r}{P_{Y}^{W}} \frac{dr}{r}, \]

\[ (1 + t)(1 + t) = \frac{a_{SY} \omega s}{P_{Y}^{W}} \tilde{w}_{s} + \frac{a_{SY} r}{P_{Y}^{W}} \tilde{r}; \]

now, since \( d(1 + t) = dt \Rightarrow \frac{d(1 + t)}{(1 + t)} = \frac{dt}{t} \Rightarrow (1 + t)(1 + t) = \frac{dt}{t} \Rightarrow (1 + t) \)

\[ \tilde{t} \Rightarrow \tilde{t} \]

therefore, we can now write \( \frac{\tilde{t}}{(1 + t)} = \theta_{SY} \tilde{w}_{s} + \theta_{KY} \tilde{r}; \) that is,

\[ \tilde{w}_{s} = \frac{\tilde{T}}{\theta_{SY}} - \frac{\theta_{KY}}{\theta_{SY}} \tilde{r}, \] (26)

with \( T = 1 + t \) and \( \tilde{T} = \frac{\tilde{t}}{1 + t} \).

Furthermore, from the zero profit conditions (1), we obtain
1 = \frac{1}{P_X} (a_{LX}w + a_{KX}r); d1 = (a_{LX}w + a_{KX}r) d\frac{1}{P_X} + \frac{1}{P_X} d(a_{LX}w + a_{KX}r);
\frac{1}{P_X} d(a_{LX}w + a_{KX}r) = 0; \frac{1}{P_X} (da_{LX}w + a_{LX}dw + da_{KX}r + a_{KX}dr) = 0;

\text{since it must be } da_{LX}w + da_{KX}r = 0 \text{ (see Krugman, Obstfeld and Meliz, 2015, pp. 449-450), we can write}
\frac{\alpha_{LX}}{P_X} dw + \frac{\alpha_{KX}}{P_X} dr = 0; \theta_{LX} \hat{w} + \theta_{KX} \hat{r} = 0; \Rightarrow
\hat{w} = -\frac{\theta_{KX}}{\theta_{LX}} \hat{r}.
(27)

Substitution of these values in (25) yields the change in the rate of return to capital (9), as shown below
\sigma_X (\hat{w} - \hat{r}) \lambda_{KX} + \sigma_Y (\hat{w} - \hat{r}) \lambda_{KY} = 0; \sigma_X \left(-\frac{\theta_{KX}}{\theta_{LX}} \hat{r} - \hat{r}\right) \lambda_{KX} + \sigma_Y \left(\frac{\hat{r}}{\theta_{SY}} - \frac{\theta_{KY}}{\theta_{SY}} - \hat{r}\right) \lambda_{KY} = 0;
\hat{r} \sigma_X \lambda_{KX} \left(\frac{\theta_{KX}}{\theta_{LX}} + 1\right) + \sigma_Y \lambda_{KY} - \hat{r} \sigma_Y \lambda_{KY} \left(\frac{\theta_{KY}}{\theta_{SY}} + 1\right) = 0;
\sigma_Y \lambda_{KY} \hat{T} = \hat{r} \left[\sigma_Y \lambda_{KY} \left(\theta_{KY} \hat{T} \left(\theta_{KY} \hat{T} + \theta_{SY}\right) + \frac{\sigma_X \lambda_{KX}}{\theta_{LX}} \left(\theta_{KX} + \theta_{LY}\right)\right)\right];
\text{since } (\theta_{KY} + \theta_{SY}) = 1 \text{ and } (\theta_{KX} + \theta_{LY}) = 1 \text{ (see Krugman, Obstfeld and Meliz, 2015, p. 451), we can write}
\sigma_Y \lambda_{KY} \hat{T} = \hat{r} \left[\sigma_Y \lambda_{KY} + \frac{\sigma_X \lambda_{KX}}{\theta_{LX}}\right];
\text{recalling that } \alpha \equiv \frac{\lambda_{KX}}{\theta_{SY}} \sigma_Y + \frac{\lambda_{KX}}{\theta_{LY}} \sigma_X = \alpha_Y + \alpha_X, \text{ with } \alpha_Y \equiv \frac{\lambda_{KY}}{\theta_{SY}} \sigma_Y \text{ and }
\alpha_X \equiv \frac{\lambda_{KX}}{\theta_{LY}} \sigma_X, \text{ we obtain } \alpha_Y \hat{T} = \hat{r} \alpha, \text{ which leads to equation (9).}

Finally, substitution of (9) in (26) and (27) yields the changes in the domestic wages (8) and (7).

8 Appendix B

Denote by Z the first order condition:
\Gamma = \gamma\left(\frac{\omega}{L_w+S_\omega} + \frac{1}{g+1}\left(g - tI_y(t)\right) + \ln (g + 1) + (1 - \gamma) \right) - \frac{\omega}{L_w+S_\omega} \left(g - tI_y(t)\right) + \ln (g + 1) = 0
(28)

Clearly, the second order condition is negative:
\Gamma_y < 0.

Therefore, the sign of \frac{d\Gamma}{dt} \equiv -\frac{\Gamma_y}{\Phi_y} \text{ depends on the sign of } Z_t.
\begin{align*}
\frac{\partial}{\partial t} \frac{\omega}{L_w+S_\omega} (g - tI_y(t)) &= -S \frac{\omega}{(L_w+S_\omega)} (g - tI_y(t)) - \frac{\omega}{L_w+S_\omega} \left(t'I_y(t) + t'I_y(t)\right) \\
\frac{\partial}{\partial t} \frac{\omega}{L_w+S_\omega} (g - tI_y(t)) &= L \frac{\omega}{(L_w+S_\omega)} (g - tI_y(t)) - \frac{\omega}{L_w+S_\omega} \left(t'I_y(t) + t'I_y(t)\right) \\
\Gamma_t = \gamma - \frac{\Phi_y^2}{(L_w+S_\omega)^2} \left[g - tI_y(t)\right] + \frac{\omega}{L_w+S_\omega} \left(t'I_y(t) + t'I_y(t)\right) \left[\Phi_y^2 - \frac{\Phi_y^2}{(L_w+S_\omega)^2}\right] + \\
+ (1 - \gamma) \frac{-L \frac{\omega}{(L_w+S_\omega)} \Phi_s}{(L_w+S_\omega)^2} + \left[\frac{L \frac{\omega}{(L_w+S_\omega)} (g - tI_y(t)) - \frac{\omega}{L_w+S_\omega} \left(t'I_y(t) + t'I_y(t)\right)}{\Phi_s^2}\right] \Phi_s
\end{align*}
\[ \Gamma_t > 0 \Rightarrow \]

\[
\frac{(\omega \omega'_s - \omega_s \omega')}{(L \omega + S \omega_s)^2} \frac{S \phi}{\phi^2} \left[ S (g - t I_y (t)) + \frac{\omega (L \omega + S \omega_s)}{(\omega \omega'_s - \omega_s \omega')} (t' I_y (t) + t I'_y (t)) \right] \phi' + \\
\frac{\omega \omega'_s - \omega_s \omega'}{(L \omega + S \omega_s)^2} (1 - \gamma) \frac{-L \phi_s + \left[ L (g - t I_y (t)) - \frac{\omega (L \omega + S \omega_s)}{(\omega \omega'_s - \omega_s \omega')} (t' I_y (t) + t I'_y (t)) \right]}{\phi^2} \phi' > 0
\]

\[
\epsilon_s \left\{ \frac{S (g - t I_y (t)) + \omega (L \omega + S \omega_s)}{(\omega \omega'_s - \omega_s \omega')} (t' I_y (t) + t I'_y (t))}{\phi} + \frac{L (g - t I_y (t)) - \omega (L \omega + S \omega_s)}{(\omega \omega'_s - \omega_s \omega')} (t' I_y (t) + t I'_y (t))}{\phi} \right\} > (1 - \gamma) \frac{L \phi - \gamma S}{\phi}
\]

\[
(29)
\]

Now, since \( \epsilon_s \leq 0 \), the left hand side is always weakly negative when \( V' \leq 0 \). Instead, the right hand side is always negative when \( \gamma \leq 1/2 \), as \( L \geq S \) and \( \phi_s \leq \phi \) given \( \omega \leq \omega_s \).

9 Appendix C

\[
u = \omega - \frac{\omega}{\omega L + \omega_s S} (g - t I_y (t)) + \ln (g + 1).
\]

(30)

\[
u_s = \omega_s - \frac{\omega_s}{\omega L + \omega_s S} (g - t I_y (t)) + \ln (g + 1).
\]

(31)

\[
g = \arg\max \ (Su + Lu_s)
\]

\[
= \arg\max \ S \left[ \omega - \frac{\omega}{\omega L + \omega_s S} (g - t I_y (t)) + \ln (g + 1) \right] + L \left[ \omega_s - \frac{\omega_s}{\omega L + \omega_s S} (g - t I_y (t)) + \ln (g + 1) \right]
\]

Foc:

\[
\gamma S \left( -\frac{\omega}{\omega L + \omega_s S} + \frac{1}{g + 1} \right) + (1 - \gamma) L \left( -\frac{\omega_s}{\omega L + \omega_s S} + \frac{1}{g + 1} \right) = 0
\]
\[ g = \frac{\gamma S + (1 - \gamma)L}{\gamma S \omega + (1 - \gamma)L \omega_s} (\omega L + \omega_s S) - 1. \tag{32} \]

We now compare the above equilibrium with the Nash bargaining equilibrium

\[ \gamma \frac{\omega}{L \omega + S \omega_s} - \frac{1}{g+1} \left( g - t I_y(t) \right) + ln(g + 1) + (1 - \gamma) \frac{-\omega}{L \omega + S \omega_s} - \frac{1}{g+1} \left( g - t I_y(t) \right) + ln(g + 1) = 0 \]

Since efficiency does not depend on \( t \), we put \( t = 0 \) \( \Rightarrow \omega = \omega_s \) \( \Rightarrow \)

\[ \gamma \frac{-\omega}{L \omega + S \omega} + \frac{1}{g+1} \left( g - t I_y(t) \right) + ln(g + 1) + (1 - \gamma) \frac{-\omega}{L \omega + S \omega} + \frac{1}{g+1} \left( g - t I_y(t) \right) + ln(g + 1) = 0. \]

\[ (\gamma + 1 - \gamma) \left( -\frac{1}{L + S} + \frac{1}{g+1} \right) = 0 \]

The effect of different bargaining power is rescinded. Thus,

\[ g = L + S - 1 \tag{33} \]

Similarly, if we put \( \omega = \omega_s \) into the efficiency condition, we get:

\[ g = \frac{\gamma S + (1 - \gamma)L}{\gamma S \omega + (1 - \gamma)L \omega} (\omega L + \omega_s S) - 1 \]

with \( \gamma = 1/2 \) \( \Rightarrow \)

\[ g = \frac{S + L}{S \omega + L \omega} (\omega L + \omega_s S) - 1 = S + L - 1 \]

As a result, free trade \( (t = 0) \) leads to the social optimum when the two groups have the same bargaining power.

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\(^3\)Note that, in order to simplify the discussion we are assuming that wages equalise under free-trade.