Dynamic Tax Evasion with Habit Formation

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Tax evasion is a world-wide problem

- US: not reported 18-19% of total income leading to a tax gap of about 500 billion dollars

- Europe: not reported about 20% of GDP, accounting for potential loss of about 1 trillion euros each year

- Widely studied problem, no effective solutions found
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Our contribution – Capital Accumulation (1/2)

- With a Wiener process, the capital $k_t$ evolves according to

$$dk_t = ((1 - \tau + \tau e_t) y_t (k_t) - c_t) \, dt + \sigma \tau e_t y_t (k_t) \, dW_t$$

- This approach suffers of some problems:
  - (i) only small (infinitesimal) changes are allowed
  - (ii) both positive and negative changes may occur ($dW_t$ is Gaussian)
  - (iii) on average the fee does not affect $dk_t$ ($\mathbb{E}_t [dW_t] = 0$)
  - (iv) the fee is strictly proportional the evaded taxes
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Our proposal: use Poisson jumps ($d\Pi_t$):

$$dk_t = ((1 - \tau + \tau e_t)y_t(k_t) - c_t) dt - \eta(\tau)e_t y_t(k_t) d\Pi_t$$

- The fee is a function of $\tau$: $\eta(\tau) = \eta_0 + \eta_1 \tau$
- Only negative jumps (when audited) are allowed:
  $$d\Pi_t = \begin{cases} 
1, & \lambda dt \\
0, & 1 - \lambda dt 
\end{cases}$$
- Finite jumps are allowed
- On average the fee affects $dk_t$:

$$\mathbb{E}_t \left[-\eta(\tau)e_t y_t(k_t) d\Pi_t\right] = -\eta(\tau)e_t y_t(k_t) \lambda dt$$
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Preferences usually belong to the CRRA family ($\delta = 1 \Rightarrow \ln c_t$):

$$U(c_t) = \frac{c_t^{1-\delta} - 1}{1 - \delta}$$

- We have used HARA preferences:

$$U(c_t) = \frac{(c_t - c_0)^{1-\delta}}{1 - \delta}$$

- (i) take into account a subsistence consumption
- (ii) richer model (two parameters instead of one)
- (iii) optimal consumption as an affine transformation of GDP
- (iv) time dependent risk aversion: $RRA = \frac{\delta}{c_t - c_0}$
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In this contribution:

\[ U(c_t) = \frac{(c_t - h_t)^{1-\delta}}{1-\delta} \]

where \( h_t \) is habit:

\[ h_t = h_0 e^{-\beta t} + \alpha \int_0^t c_s e^{-\beta(t-s)} ds \]

or, in differential form \( dh_t = (\alpha c_t - \beta h_t) dt \).

\( \alpha \): importance of consumption in adjusting the habit

\( \beta \): importance of the past (the higher \( \beta \) the less important the past)

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The Problem

- We solve the following problem

\[
\max_{e_t, c_t} \mathbb{E} \left[ \int_{0}^{\infty} \frac{(c_t - h_t)^{1-\delta}}{1-\delta} e^{-\rho t} dt \right]
\]

- Under the dynamic constraint

\[
dk_t = \left( \frac{(1 - \tau + \tau e_t) Ak_t - c_t}{\sqrt{y_t}} \right) dt - \eta(\tau) e_t Ak_t d\Pi_t
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<table>
<thead>
<tr>
<th></th>
<th>Evasion $e_t^*$</th>
<th>Consumption $c_t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA</td>
<td>$E$</td>
<td>$k_t F$</td>
</tr>
<tr>
<td>HARA</td>
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\[
E \equiv \frac{1}{\eta A} \left(1 - \left(\frac{\lambda \eta}{\tau}\right)^{\frac{1}{\delta}}\right) \quad F \equiv \frac{\rho + \frac{\lambda - \frac{\tau}{\eta} + (\delta - 1)(1-\tau)A}{\delta}}{\delta} + A \tau E
\]
Comparative statics: tax rate

- with CRRA preferences (and $\eta = \eta_0 + \eta_1 \tau$):

$$\frac{\partial e_t^*}{\partial \tau} \geq 0 \iff \frac{\eta_0}{\tau \eta_1} \geq \delta \frac{1 - \left(\frac{\lambda \eta}{\tau}\right)^{\frac{1}{\delta}}}{\left(\frac{\lambda \eta}{\tau}\right)^{\frac{1}{\delta}}}$$

- with HARA preferences

$$\frac{\partial e_t^*}{\partial \tau} \geq 0 \iff c_0 \geq \text{threshold}$$

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Comparative statics: audit parameters

- w.r.t. both \( \eta_0 \) and \( \eta_1 \):
  \[
  \frac{\partial e_t^* \eta_0}{\partial \eta_0 e_t^*} > \frac{\partial e_t^* \eta_1}{\partial \eta_1 e_t^*} \iff \eta_0 > \eta_1 \tau
  \]

- w.r.t. both \( \eta_0 \) and \( \lambda \)
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  \frac{\partial e_t^* \eta_0}{\partial \eta_0 e_t^*} > \frac{\partial e_t^* \lambda}{\partial \lambda e_t^*} \iff \frac{\eta_0}{\eta_1 \tau} > \frac{1}{\delta} \frac{\left(\frac{\lambda \eta}{\tau}\right)^{\frac{1}{\delta}}}{1 - \left(\frac{\lambda \eta}{\tau}\right)^{\frac{1}{\delta}}}
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- When either \( \eta = \eta_0 \) or \( \eta = \eta_1 \tau \):
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$$\frac{\partial e_t^* \eta_1}{\partial \eta_1 e_t^*} > \frac{\partial e_t^* \lambda}{\partial \lambda e_t^*} \iff \frac{\eta_1 \tau}{\eta_0} > \frac{1}{\delta} \frac{\left(\frac{\lambda \eta}{\tau}\right)^{\frac{1}{\delta}}}{1 - \left(\frac{\lambda \eta}{\tau}\right)^{\frac{1}{\delta}}}$$

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Simulations: base scenario

- $A = 0.8$
- $\tau = 0.4$
- $\lambda = 0.25$
- $\rho = 0.05$
- $\alpha = \frac{1}{3}$
- $\beta = 0.05$
- $\delta = 2.5$
- $k_0 = 100$
- $h_0 = 10$
- $\eta_0 = 0.1$
- $\eta_1 = 1.2$
Simulations: base scenario

Consumption as % of yield

Habit as % of yield

Evasion

Consumption as % of habit

Dynamic Tax Evasion with Habit Formation
Simulations: higher tax ($\tau = 0.5$)
Simulations: short memory ($\alpha = 0.8$, $\beta = 1$)

- Consumption as % of yield
- Habit as % of yield
- Evasion
- Consumption as % of habit