What Does Investor Sentiment Reflect: Animal Spirits or Risks?

Sung Bin Sohn*

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Abstract

The role of investor sentiment in the stock market has attracted attentions of many economists. Previous papers show that investor sentiment has return predictability and it is more pronounced among stocks that are more difficult to value and/or to arbitrage, and emphasize the behavioral role of investor sentiment. However, it still remains unclear whether this predictability is actually due to a causal effect of autonomous animal spirits or not. Alternatively, investor sentiment may simply reflect systematic risks, which would affect stock returns. In this alternative case, the predictability would be mere coincidence, not causation. In this paper, I try to understand the meaning of innovations in investor sentiment. I use the investor sentiment index constructed by Baker and Wurgler (2006). I set up a structural model in which sentiment innovations arise from both animal spirit shocks and several risk shocks, and animal spirit shocks could affect stock returns. By matching impulse response functions from data simulated by the theoretical model to those from the actual US data, I estimate parameters in the model. The estimated model moderately replicates the historical data in the actual stock market. The estimation results show that a substantial amount of variation in investor sentiment is explained by systematic risk shocks as well as by animal spirit shocks, and that animal spirit shocks can have significant effects on stock returns. The findings suggest that investor sentiment is a noisy proxy of animal spirits and autonomous animal spirits are at least in part responsible for the apparent return predictability of investor sentiment.

*Peking University HSBC Business School; Address: C324 Peking University, University Town, Nanshan District, Shenzhen, 518055, China; Email: sungbin.sohn@phbs.pku.edu.cn; Tel: +86-755-2603-5324; Fax: +86-755-2603-5344; I am grateful to Professor Ulrike Malmendier, Professor Adam Szeidl, Professor Yuriy Gorodnichenko, and Professor Martin Lettau for their generous guidance.
1 Introduction

In classical finance theory, there is no room for investor sentiment to play a role. Stock prices are determined as the expected present value of future cash flows and any influence from wrong expectations or irrational behaviors will be offset by the actions of rational arbitrageurs. However, many economists have argued that some financial phenomena can plausibly be understood within the framework in which some agents are not fully rational. Campbell and Kyle (1993) argue that irrationality can affect stock prices by showing that fundamental risk deters rational investors from aggressively betting against noise traders. De Long, Shleifer, Summers, and Waldmann (1990) shows that even without the fundamental risk, stock prices can depart from fundamental values due to the uncertainty about noise traders’ future bullishness. Shleifer and Vishny (1997) provide a theoretical framework in which arbitrage does not always perfectly work. Hirshleifer (2001) and Barberis and Thaler (2003) survey theories and evidence that there are limits to arbitrage and investor psychology is a determinant of asset prices.

Meanwhile, the role of investor sentiment has attracted attention of economists. Baker and Wurgler (2006, 2007) construct a measure of investor sentiment using six proxy variables and show that the investor sentiment index can predict returns for some groups of stock portfolios. Kumar and Lee (2006) show evidence that supports a noise trader model in which investor sentiment play an important role in the stock market. Brown and Cliff (2005) use survey data on investor sentiment and provide evidence that investor sentiment affects asset valuations. Qiu and Welch (2004) and Neal and Wheatley (1998) show that some investor sentiment measures can explain the small-firm return spread and the return spread between stocks held disproportionately by retail investors and those held by institutional investors. Tetlock (2007) shows that high media pessimism is followed by a temporary downward pressure on market prices and argues that his findings are not consistent with the view that media contents work as new shocks about fundamental asset values or market volatility. Baker, Wurgler, and Yuan (2011) study the time-series and cross-sectional return predictability of investor sentiment in an international context. These papers have the common conclusion that investor sentiment predicts stock returns in the cross-section or in the aggregate, and the predictability is more pronounced among stocks that are more difficult to value and to arbitrage.\footnote{Some studies present empirical results that show little predictive power of investor sentiment for near-term future stock returns (Brown and Cliff, 2004). The underlying reasoning is simple as follows. If some stocks have significant arbitrage constraints and valuation of them is challenging, high investor sentiment could bid up their stock prices for a while, which accounts for cross-sectional return comovements. As sentiment disappears, the prices eventually revert to their fundamental values, and this explains that high investor sentiment could predict low subsequent returns for those stocks. These papers argue that macroeconomic information, systematic risks, or analyst earnings forecast revisions are not likely to explain these results, which suggests that...
investor sentiment has an independent causal effect on stock returns. In this paper, this causal channel of investor sentiment to returns is referred to as the animal spirit view.\textsuperscript{2}

Despite the empirical evidence, however, it still remains unclear whether the return predictability of investor sentiment is indeed due to a causal effect of autonomous animal spirits. Alternatively, investor sentiment may reflect systematic risks that could affect the future expected returns of some stocks, and the predictability could be mere coincidence that simply reflect a complex pattern of compensation for risks, not causation. For example, if reduced risks lower the expected future discount rates and boost up investor sentiment, then it will produce a current high return and subsequent low returns. In this case, the apparent return predictability of investor sentiment can be observed, but investor sentiment does not have any causal effect. I refer to this coincident relationship between returns and sentiment as the systematic risks view. In the previous papers, authors try to isolate the pure effect of autonomous animal spirits from investor sentiment by orthogonalizing components that may reflect the current economic conditions, but the attempts are still far from perfection. For example, Baker and Wurgler (2006, 2007) construct the sentiment index based on proxies that have been orthogonalized to some macroeconomic variables. However, this treatment is not enough because current macroeconomic variables may fail to incorporate some news shocks as argued in Beaudry and Portier (2006) and Barsky and Sims (2011). Kumar and Lee (2006) and Brown and Cliff (2005) respectively use trading activities of retail investors and the number of bullish, bearish, or neutral market newsletters as a measure of investor sentiment, which are likely to reflect systemic risks and information as well as autonomous animal spirits.

In this paper, I try to understand the meaning of innovations in investor sentiment and their role in the stock market. Unlike previous papers that survey the effects of investor sentiment based on regressions of reduced-form specifications, I introduce a structural model and this enables us to extract the pure effect of investor sentiment through autonomous animal spirits as opposed to that through systematic risks. For this purpose, Section 2 starts with the reduced-form vector autoregressive (VAR) analyses that show the empirical relationship between investor sentiment and returns. The VAR estimation results are consistent with previous studies and show that there exists apparent return predictability in innovations in investor sentiment for some group of stocks. The reduced-form VARs are unguided by a model and seemingly irrelevant to a structural study, but they work as important statistics in estimating model parameters as detailed later.

Since it is hard to interpret the empirical evidence without a structure, in Section 3, I modify the standard long-run risks model (Bansal and Yaron, 2004) in such a way that investor sentiment could play a role in stock returns. The model enables us to measure the extent to which the apparent empirical return predictability of investor sentiments reflects animal spirits that have causal effects on economic activities.

\textsuperscript{2}I use this terminology following Barsky and Sims (2012). They study whether consumer confidence reflects animal spirits that have causal effects on economic activities.
sentiment is attributed to animal spirits shocks and systematic risk shocks, respectively. In the model, I allow the possibility that investors could have erroneous perception of the expected growth rate in the economy. This setup is in line with De Long, Shleifer, Summers, and Waldmann (1990), in which noise traders misperceive the expected price of the risky asset. I interpret this erroneous perception as the animal spirits and assume that innovations in investor sentiment arise from both this animal spirit shock and systematic risk shocks. Depending on parameter values, this model can either be reduced to a classical asset pricing model in which animal spirit shocks play no role, or describe a behavioral finance theory in which investor sentiment not only reflects systemic risks but also has a causal effect on the stock market.

In Section 4, I estimate parameters in the model by minimizing the generalized distance between impulse response functions from data simulated according to the model in Section 3 to those from actual data in Section 2. The estimated model moderately replicates the historical data in the actual stock market such as average returns, return volatilities, dividend growth, price-dividend ratios, etc. Using the estimated parameters, I construct the theoretical impulse response functions to each structural shock to see the pure effect of animal spirits shocks as opposed to other systematic risk shocks. I also decompose the forecast error variances of returns and examine how much of return variations are attributed to animal spirits shocks and risk shocks, respectively. The estimation results show that a substantial amount of variation in investor sentiment is explained by systematic risk shocks as well as by animal spirit shocks, and that animal spirit shocks can have significant effects on stock returns. The findings suggest that investor sentiment is a noisy proxy of animal spirits and that autonomous animal spirits can at least in part account for the apparent return predictability of investor sentiment.

Finally, Section 5 concludes the paper.

2 Investor Sentiment and Stock Returns

Does investor sentiment have any effects on stock returns? The simplest and the most direct way to answer this question is to run regressions of stock returns on a measure of investor sentiment and other control variables. Indeed, Baker and Wurgler (2006) follow this strategy. They first construct a measure of investor sentiment by extracting a common factor from six proxies of investor sentiment. Then, they show that the sentiment index has return predictability. Specifically, they run the following time-series regressions for several long-short portfolios and find that the sentiment index has significant effects on returns,

\[ R_{it} = c + d \text{SENTIMENT}_{t-1} + u_{it}, \]

where \( R_{it} \) is the return of long-short portfolio \( i \) at time \( t \) (e.g., \( i \in \{ \text{Small-minus-Big, Young-minus-Old, etc.} \} \)), and \( \text{SENTIMENT}_t \) is the measure of investor sentiment at
time $t$. They conclude that the several aspects of the results are inconsistent with the classical explanation that the results reflect compensation for systematic risks.

The univariate regressions that Baker and Wurgler (2006) survey are the most straightforward approach and indeed they successfully reveal the relationship between investor sentiment and returns. However, they do not incorporate the interdependencies between sentiment, returns, and other control variables. According to Brown and Cliff (2004), past returns are an important determinant of investor sentiment. Their studies also lack the dynamic effects of an innovation in investor sentiment on returns.

The VAR analysis covers the shortcomings of univariate regressions. The VAR analysis captures the interdependencies among considered variables and enables us to construct impulse response functions to an unexpected change in a variable. Note that, of course, we cannot use the result of a reduced-form VAR to interpret the effects of structural shocks. To identify the pure effect of a structural shock, we need to set up a model and specify a structural VAR based on the model. Although the reduced-form VAR analysis neither derives from a model nor provides structural interpretations, it is still informative in that the results from the reduced-form VAR provide guidance for estimating parameters in the structural model I will introduce in the following section. For this reason, this section presents the results of reduced-form VAR regressions with investor sentiment, the small-cap portfolio return, the large-cap portfolio return and other control variables, and compares them with Baker and Wurgler (2006).

2.1 Data

I use the sentiment index constructed by Baker and Wurgler (2006) as the measure of investor sentiment, which is denoted by Sent in this paper. It is constructed as a linear function of the six proxy variables: the closed-end fund discount, the log turnover ratio of NYSE stocks, the number of IPOs, the average first-day returns of IPOs, the share of total equity issues in total equity and debt issues, and the premium of dividend-paying stocks as opposed to non-paying stocks. The weights are obtained from the principal component analysis, and the resulting index is standardized to have a mean of zero and a unit variance. This measure has been widely used by many researchers in various contexts as in Kurov (2010), Massa and Yadav (2010), Cen, Lu, and Yang (2011) and McLean and Zhao (2011) among many others. The sample is the annual investor sentiment index that spans from 1934 to 2005. The data are downloaded from Jeffrey Wurgler’s homepage.\(^3\)

To examine its return predictability, I consider the returns on a small-cap stock portfolio and a large-cap stock portfolio. Arguably, small-cap stocks bear more valuation difficulty and arbitrage for these stocks is more limited, so that they are more subject to investor sentiment than large-cap stocks. Indeed, many previous papers consider small-cap stocks to test return predictability of investor sentiment (Neal and Wheatley, \(^3\)http://people.stern.nyu.edu/jwurgler)
The small-cap (large-cap) stock portfolio is constructed at the end of each June based on the June market capitalization and associated NYSE breakpoints. Specifically, stocks in all NYSE, AMEX and NASDAQ whose market capitalization is smaller (larger) than the bottom (top) 30% stock in NYSE are included in the portfolio. The small-cap and large-cap returns are the value-weighted average returns of these portfolios, respectively. The data are downloaded from Kenneth French’s Data Library.\footnote{http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html}

For return predictors, I consider the consumption growth and the log price-dividend ratios of the portfolios. Asset pricing models based on consumption-related utility maximization imply that consumption growth is a determinant of asset returns (Campbell, 2003). And, the log-linearization of a return as in Campbell and Shiller (1988) and the variance decomposition as in Cochrane (1991) imply that the log price-dividend ratios could have return predictability. For these reasons, I include the consumption growth and the log price-dividend ratios in the VAR system. The consumption growth is the log difference of the seasonally adjusted per-capita real consumption available on the Bureau of Economic Analysis (BEA). The log price-dividend ratios of the portfolios are derived from the portfolio returns with and without dividends.\footnote{The exact procedure of the data generation is as follows. Without loss of generality, assume that the market value of the portfolio is 1 at time 0 \((P_0 = 1)\). The subsequent prices \((P_{t+1})\) and dividends \((D_{t+1})\) are determined such that \(P_{t+1} = P_t \times R_{t+1}^d\) and \(D_{t+1} = P_t \times (R_{t+1}^d - R_{t+1}^x)\), where \(R_{t+1}^d\) and \(R_{t+1}^x\) are the gross returns at time \(t + 1\) with and without dividends, respectively.}

\section{2.2 VAR Regressions}

In this subsection, I empirically show the relationship between investor sentiment and the small-cap return by estimating a couple of reduced-form VAR systems. The Bayesian information criterion (BIC) suggests the lag length of one in each VAR estimation. Hence \(VAR(1)\) is the default setup throughout the paper. However, the main results are robust to changes in the lag length. Figure 1 depicts impulse response functions from the VAR in which investor sentiment \((Sent)\), consumption growth \((\Delta c)\), log price-dividend ratios \(((p-d)_j)\), and log returns \((r_j)\) are included in the stated order, where \(j\) is either small-cap stock portfolio (S) or large-cap stock portfolio (L). This figure clearly shows negative subsequent returns of the small-cap portfolio after an unexpected positive change in investor sentiment. In contrast, the large-cap return does not show as significant responses to such a sentiment innovation as the small-cap return does. Investor sentiment is decaying naturally. These results are consistent with the finding in Baker and Wurgler (2006): negative return on the small-minus-large portfolio subsequent to high sentiment.
Note that VAR estimation results are sometime sensitive to the ordering of the variables in the system. To check the robustness of the empirical relationship, I re-estimate the system with investor sentiment ordered last and compare the results (Figure 2). When investor sentiment is ordered last, it implicitly imposes restrictions that a sentiment innovation does not have any contemporaneous effects on other variables. For this reason, returns have zero responses at the impact but the subsequent responses are almost identical to the previous case, which shows that the empirical relationship between investor sentiment and small-cap returns is robust.

When only sentiment and returns are included in the VAR, the empirical relationship is more clearly revealed. The small-cap returns are more significantly negative in subsequent years and the response of the large-cap return remains less significant (not reported). These results imply that the empirical return predictability of investor sentiment is not primarily driven by interactions of returns and other variables such as consumption growths and log price-dividend ratios.

These findings confirm that there exists an empirical relationship between investor sentiment and returns of some portfolios. However, it still remains unclear whether this predictability is indeed due to a causal effect of autonomous animal spirits as suggested by previous literature. Since it is hard to interpret the empirical evidence without a structure, the next section introduces a simple model.

3 The Long-Run Risks Model with Investor Sentiment

In this section, I modify the long-run risks model (Bansal and Yaron, 2004; Bansal, Kiku, and Yaron, 2012) in such a way that the role of investor sentiment in stock returns is explicitly taken into account. This modification is slight but meaningful in that it provides a theoretical framework through which we can analyze the structural effects of investor sentiment.

In this model, there is a representative agent who has the recursive type of preferences as in Epstein and Zin (1989),

\[
V_t = \left[ (1 - \delta) C_t^{1-\gamma} + \delta (E_t[V_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}},
\]

where \( C_t \) is real consumption at time \( t \), \( \delta \in (0, 1) \) indicates the agent’s time preference, \( \gamma \) is the coefficient of relative risk aversion, \( \theta = \frac{1-\gamma}{1-1/\psi} \), and \( \psi \) is the elasticity of inter-temporal substitution. The agent maximizes the life-time utility \( V_t \) subject to the

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\( ^6 \)I want to emphasize that I do not mean that the VAR specifications I examine here are necessarily correct and/or the IRFs have structural interpretations. However, the VAR analyses are still meaningful as detailed in Section 4.
following budget constraint,

\[ W_{t+1} = R_{C,t+1}(W_t - C_t), \tag{2} \]

where \( W_t \) is the level of wealth at time \( t \) and \( R_{C,t} \) is the gross return on aggregate wealth.

The time series of consumption and dividends are determined by the following dynamics,

\[
\begin{align*}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t \eta_{t+1} \tag{3} \\
x_{t+1} &= \rho x_t + \varphi_c \sigma_t e_{t+1} \tag{4} \\
\sigma^2_{t+1} &= \nu \sigma^2_t + (1 - \nu) \sigma^2 + \sigma_w w_{t+1} \tag{5} \\
\Delta d_{j,t+1} &= \mu_j + \phi_j x_t + \pi_j \sigma_t \eta_{t+1} + \varphi_j \sigma_t u_{d,t+1}, \tag{6}
\end{align*}
\]

where \( \Delta c_t \) and \( \Delta d_{j,t} \) are the first difference of log consumption and log dividends of asset \( j \), respectively. \( x_t \) and \( \sigma^2_t \) are time-varying components of the expected growth rate and the conditional variance of consumption, respectively, and they follow the first-order autoregressive processes. \( \eta_t \) is the level shock that has an immediate but one-time effect on consumption, \( e_t \) is the news shock that has lagged but long-lasting effects, \( w_t \) is the volatility shock, and \( u_{d,t} \) is the market-wide dividend shock. All shocks follow the jointly independent standard normal distribution. The above equations indicate that consumption and dividends are growing with time-varying drift and volatility.

Now I add the role of investor sentiment into the model. Investor sentiment is assumed to follow the first-order autoregressive (\( AR(1) \)) process. This specification is justified by the fact that the measure of investor sentiment used in this paper is indeed well fitted by \( AR(1) \). To allow the possibility that investor sentiment can reflect both animal spirits and systematic risks, sentiment innovations are assume to arise from an animal spirit shock as well as other systematic risk shocks. To define the animal spirit shock, I assume that the representative agent receives a noisy signal of the time-varying expected consumption growth \( (x_t) \). Formally,

\[ s_{x,t} = x_t + \sigma_a a_t, \tag{7} \]

where \( s_{x,t} \) is noisy signal of the expected consumption growth, \( \sigma_a \) is the volatility of the noise errors, and \( a_t \) follows the \( i.i.d. \) standard normal distribution. Following a positive \( a_t \), the agent would erroneously perceive a higher expected growth rate than it actually is, and vice versa. For this reason, I interpret \( a_t \) as the animal spirit shock.\(^7\)

I assume that the agent estimates the unobserved expected growth rate from the noisy signal \( (s_{x,t}) \), the consumption growth \( (\Delta c_t) \) and the market dividend growth \( (\Delta d_{M,t}) \) using the Kalman filter.\(^8\) Based on the estimated expected growth rate (denoted

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\(^7\)Barsky and Sims (2012) also interpret the measurement error of a noisy signal as the animal spirit shock for the same logic.

\(^8\)See the Appendix I for the detailed procedure.
by \( \tilde{x} \), the agent consequently forms perceptions for the structural shocks (denoted by \( \tilde{\eta}, \tilde{e} \) and \( \tilde{u}_d \)) according to Equation (3), (4), and (6). Formally,

\[
\tilde{\eta}_{t+1} = \eta_{t+1} + \frac{1}{\sigma_t} (x_t - \tilde{x}_t) \\
\tilde{e}_{t+1} = \frac{1}{\varphi_e \sigma_t} (\tilde{x}_{t+1} - \rho \tilde{x}_t) \\
\tilde{u}_{d,t+1} = u_{d,t+1} + \frac{1}{\varphi_{M} \sigma_t} (\phi_M - \pi_M)(x_t - \tilde{x}_t).
\]

Figure 4 shows an illustrative example of this process. For a given time series of unobserved state variable \( \{x_t\}_{t=1}^T \), the representative agent observe the time series of \( \{s_{x,t}, \Delta c_t, \Delta d_{M,t}\}_{t=1}^T \). Each time she gets a new set of observations, she rationally and recursively updates her expectation on the unobserved state variable. In this illustration (and in the estimated model), the consumption growth and the market dividend growth are much noisier than the signal \( (s_x) \), and the agent put much larger weight on the signal than on the consumption growth and the dividend growth when she estimates the expected growth rate. Consequently, the Kalman estimates of the expected growth rate \( (\tilde{x}) \) significantly resembles the signal.

Note that depending on the realization of the animal spirit shock, perceived structural shocks can be different from the actual ones. However, the agent is assumed to still believe that her perceived structural shocks follow the i.i.d. standard normal distribution. This setup is in line with De Long, Shleifer, Summers, and Waldmann (1990) in that the agent has a biased perception about the expected value but correctly perceives the variance.

\[ \text{Animal spirit shocks may materialize through the volatility channel, but in this paper, the perception of the volatility shock (} w_t \text{) is assumed to be unaffected by the animal spirit shocks.} \]

\[ \text{These theoretical impulse response functions will be discussed in more detail in Section 4.2.} \]
where $u_{m,t+1}$ captures all other factors that affect investor sentiment and is assumed to follow the standard normal distribution. It could also be interpreted as the measurement error in the sentiment data. The resulting time series of investor sentiment is expressed as follows,

$$\text{Sent}_{t+1} = \rho_s \text{Sent}_t + \epsilon_{t+1},$$

(12)

where $\text{Sent}_t$ is a measure of investor sentiment at time $t$ and $\rho_s$ is the coefficient of persistence.

With the erroneous perceptions of expected growth rates and structural shocks, the representative agent maximizes her expected utility subject to the budget constraint. Note that the agent’s problem yields the following inter-temporal Euler equation for any asset $j$ (Epstein and Zin, 1989),

$$E_t \left[ \exp \left( \theta \log \delta - \frac{\theta}{\psi} \Delta_c_{t+1} + (\theta - 1) r_{C,t+1} + r_{j,t+1} \right) \right] = 1,$$

(13)

where $r_{j,t}$ is the log gross return of asset $j$ at time $t$. The solving strategy is the same as Bansal and Yaron (2004) except that perceived variables, $\{\tilde{x}, \tilde{\eta}, \tilde{e}, \tilde{u}_d\}$, are substituted for the actual variables, $\{x, \eta, e, u_d\}$. First, solve for the log return on the consumption claim $r_{C,t+1}$ (Campbell and Shiller, 1988), and then assume that the log wealth-consumption ratio ($z_t$) is linear in the two state variables: perceived expected consumption growth ($\tilde{x}_t$) and the conditional variance ($\sigma^2_t$). Formally,

$$r_{C,t+1} = k_0 + k_1 z_{t+1} - z_t + \Delta_c_{t+1}$$

(14)

$$z_t = A_0 + A_1 \tilde{x}_t + A_2 \sigma^2_t,$$

(15)

where $k_1 = \exp(\bar{z}) / (1 + \exp(\bar{z}))$, $k_0 = \log(1 + \exp(\bar{z})) - k_1 \bar{z}$, and $\bar{z} = A_0 + A_2 \sigma^2$. By substituting (14) and (15) into the Euler equation (13) and putting $j = C$, $\{A_0, A_1, A_2\}$ can be analytically solved for. Similarly, the log return on asset $j$ is expressed as follows,

$$r_{j,t+1} = k_{0,j} + k_{1,j} z_{j,t+1} - z_{j,t} + \Delta d_{j,t+1}$$

(16)

$$z_{j,t} = A_{0,j} + A_{1,j} \tilde{x}_t + A_{2,j} \sigma^2_t,$$

(17)

where $z_{j,t}$ is the log price-dividend ratio of asset $j$. $\{k_{0,j}, k_{1,j}\}$ are defined in the similar way as above, and $\{A_{0,j}, A_{1,j}, A_{2,j}\}$ are calculated using the Euler equation.\textsuperscript{11}

Note that the effect of animal spirit shocks on the return of stock $j$ depends on the parameter values of $A_{1,j}$, $k_{1,j}$ and $\sigma_a$. For example, if $\sigma_a$ is equal to zero, this model is completely reduced to a classical asset pricing model in which asset prices are entirely determined by the expected present value of future cash flows and animal spirit shocks play no role. It is worth noting that even in this case, a measure of investor sentiment can still have return predictability because the investor sentiment may reflect systematic

\textsuperscript{11}See Appendix II for the analytical solutions for $A$’s and $A_j$’s.
risks such as volatility shocks and news shocks. However, this predictability should not be interpreted as evidence of a causal effect of market irrationality. On the other hand, if $\sigma_a$ is significantly greater than zero and $A_{1,j}$ and $k_{1,j}$ are sufficiently large for some $j$, it could be the case that investor sentiment is a noisy measure of animal spirit shocks and some asset prices are substantially affected by not only systematic risks but also animal spirits.

4 Estimation

In this section, I estimate parameters in the model and discuss their meaning. To be comparable to the empirical finding in Section 2, I consider two assets: the small-cap stock portfolio ($S$) and the large-cap stock portfolio ($L$). The model in Section 3 is characterized by 29 parameters which determine investors’ preference ($\delta, \gamma, \psi$), the growth of consumption ($\mu_c, \nu, \bar{\sigma}, \sigma_w$), the growth of dividends on the stock market index ($\mu_M, \phi_M, \pi_M, \varphi_M$), the growth of dividends on the small-cap stock portfolio ($\mu_S, \phi_S, \pi_S, \varphi_S$), the growth of dividends on the large-cap stock portfolio ($\mu_L, \phi_L, \pi_L, \varphi_L$), and the evolution of investor sentiment ($\rho_S, \sigma_a, \zeta_{\eta}, \zeta_e, \zeta_w, \zeta_d, \zeta_a, \zeta_m$). Among these, the first 13 parameters are already introduced in the original version of the long-run risks model (Bansal and Yaron, 2004), and the last 16 parameters are additionally introduced in this paper to take into account the role of investor sentiment. Bansal, Kiku, and Yaron (2012) calibrate the first 13 parameters and successfully describe the stock market phenomena such as risk premium, return volatility, risk-free rate, etc. Since this calibration is quite standard, in this paper I take the calibrated parameter values as given. The annual time series of investor sentiment is standardized to have a mean of zero and a unit variance as in Baker and Wurgler (2006), and the last parameter ($\zeta_m$) is determined by the restriction given all other parameters. Hence, we only estimate the remaining 15 parameters.

4.1 Methodology and Results

I estimate the parameters by minimizing the distance between the impulse response functions from the model-simulated data and those from the actual data. This “impulse response functions matching” approach has several advantages. First, the empirical impulse response functions should not necessarily be computed from a structural VAR, nor the reduced-form VAR should be correctly specified. Although the empirical impulse response functions do not have structural interpretations, they still work as important

\footnote{Campbell and Vuolteenaho (2004) decompose the unexpected return into the component attributed to cash-flow news and that attributed to discount-rate news. In the long-run risks model where there are four structural shocks ($\eta, e, w, and u_d$), it turns out that volatility shocks ($w$) and a part of news shocks ($e$) are classified as discount-rate news, whereas other shocks are cash-flow news. The discount-rate news has return predictability by its nature.}
statistics that a well-estimated model should be able to match. Second, although it utilizes limited information, this approach lets the estimation focus on the fact a researcher put the most emphasis on: the relationship between investor sentiment innovations and subsequent returns.\footnote{This estimation approach is implemented by Rotemberg and Woodford (1997), Christiano, Eichenbaum, and Evans (2005), and Barsky and Sims (2012), among others.} For the estimation, I focus on only impulse response functions to an orthogonalized innovation in investor sentiment because impulse responses to other innovations are not of primary interest in this research.

Let \(\hat{\Psi}_{i,h}\) be the empirical impulse response function of variable \(i\) at horizon \(h\) from a sentiment innovation, and \(\hat{\Psi}\) be the stacked vector of them. In addition to impulse response functions, I consider six statistics that are important in characterizing the portfolios: \(E[r_S], E[r_L], E[\Delta d_S], E[\Delta d_L], E[(p-d)_S]\) and \(E[(p-d)_L]\). These additional moments help the estimated model to better match the real historical data, but do not significantly affect the estimation result. Since I focus on six variables (\(i = \text{Sent}, \Delta c, (p-d)_S, (p-d)_L, r_S, r_L\) up to two years from an impact \((h = 0, 1, 2)\), the resulting \(\hat{\Psi}\) has 24 (= 6 \times 3 + 6) moments in total.

I simulate a dataset for a given set of parameters, \(\Theta\). Following the standard in literature, I assume that investors make monthly decisions. To be comparable with the actual data with annual frequency, I appropriately aggregate the monthly observations to the annual values. The resulting simulated data have the same sample size as the actual data. Impulse response functions are computed from the simulated dataset, and the similarly stacked impulse response functions and additional moments constitute \(\Psi(\Theta)\).

The estimates are obtained by solving the following minimization problem,

\[
\hat{\Theta} = \arg\min_{\Theta} J(\Theta) \equiv \left( \hat{\Psi} - \Psi(\Theta) \right)' W^{-1} \left( \hat{\Psi} - \Psi(\Theta) \right)
\]

where \(W\) is a diagonal matrix whose diagonal elements are the empirical variance of \(\hat{\Psi}\). The empirical variances of impulse response functions are computed based on 1,000 Bootstrap simulations. The estimated parameters and their standard errors are tabulated in Table 1 together with calibrated parameters.

Figure 3 depicts the impulse response functions generated from the data simulated given the estimated set of parameters, \(\hat{\Theta}\). Note that the simulated impulse response functions moderately reside within the confidence bands of the empirical impulse responses. The estimated model also modestly replicate the historical data in the stock market such as average returns, return volatility, dividend growth, price-dividend ratios, etc. In Table 2, the Data column shows the moments computed from the actual data, while the Model and the Population columns show the moments from the simulated data. The Model moments are calculated from 1,000 sets of simulated data that has the same sample size as the actual data, while the Population moments are from a very long sample period of simulated data. Note that the stock market characteristics are
reasonably matched by the estimated model, which supports the validity of the model setup and the estimation.

4.2 Discussion

In this subsection, I explain the meaning of the estimation result and discuss the relationship between investor sentiment and stock returns.

The estimated parameters on dividends shed light on distinctive characteristics between small-cap and large-cap stocks. Note that $\phi_j$, $\pi_j$ and $\varphi_j$ indicate the sensitivity of the dividend growth to news shocks, consumption level shocks and market-wide dividend shocks, respectively. Table 1 shows that the dividend growth of small-cap stocks is more strongly responding to news shocks and market-wide dividend shocks than that of large-cap stocks is, while consumption level shocks have stronger impact on large-cap stocks than on small-cap stocks. In addition, small-cap stocks’ dividend growth rate is greater than that of large-cap stocks ($\mu_S > \mu_L$), which is consistent with intuition. The significant loadings ($\zeta$’s) of structural shocks on innovations of investor sentiment show that the innovation in investor sentiment arises not only from animal spirits but also from systematic risks. This result implies that investor sentiment can be a noisy measure of irrationality but it also reflects some systematic risks.

The estimated parameters provide insightful information about investor sentiment and return characteristics, but the more important implication is that we can construct the theoretical impulse response functions of each variable to each structural shock from the estimated model, which enables us to evaluate their pure effects. Figure 5 shows how investors’ perception is influenced by the existence of animal spirit shocks, and Figure 6 shows how the returns of small-cap stocks and large-cap stocks are affected by each structural shock.\footnote{Refer to Appendix II to see how the theoretical impulse response functions of returns are derived.}

As for Figure 5, first note that each structural shock is independent so that an actual shock is not responding to any shocks other than itself (denoted in dotted lines). Also note that the actual expected consumption growth rate ($x_t$) is responding only to the news shock ($e_t$). Due to the animal spirit shock and the consequent investors’ imperfect observation, however, the perceived expected growth rate ($\tilde{x}_t$) can have non-zero impulse response functions to all structural shocks. Consequently, so do perceived structural shocks.

Specifically, the perceived expected growth rate (the 4th row in Figure 5) has the following impulse responses. A positive animal spirit shock ($a_t$) leads investors to observe a higher signal for the expected growth rate than it actually is. Even in response to a positive level shock ($\eta_t$), the perceived expected growth rate increases because investors consider the possibility that the current increase in consumption growth is due to the high expected growth rate in the previous period (Recall $\Delta c_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1}$).
In response to a positive news shock, the perceived expected growth rate increases but not by as much as the actual one does because investors attribute some effects from the news shock to other shocks. As time passes, investors adjust their perception and the perceived expected growth rate converge to its actual trajectories.

The impulse responses of a perceived news shock (the 2nd row in Figure 5) are explained as follows. As explained above, after a positive animal spirit shock, investors observe an erroneously high expected growth rate and interpret it as a positive news shock. But after one period, investors observe a “disappointing” signal for the expected growth rate and believe a negative news shock happens. Even after a real news shock, due to the imperfect observation, investors don’t fully appreciate the news shock.

The perceived level shock (the 1nd row in Figure 5) are also worth noting. In response to a positive level shock, the perceived impulse response and the actual one are almost identical. After a positive news shock, the expected growth rate increases and it subsequently affects the consumption growth. Observing this, investors erroneously believe that a positive level shock occurs. Hence, the perceived level shock has a positive response although the actual level shock is not affected by a news shock. A positive animal spirit shock makes investors overestimate the expected growth rate. Hence, when investors observe “disappointing” consumption growth, they erroneously believe that a negative level shock occurs.

The impulse response functions of returns in Figure 6 are naturally derived from Figure 5. Note that the persistence and the sign of the subsequent effects vary across shocks. While a dividend shock has a large but essentially only one-time effect on returns, other shocks have effects that last for more than one period, which implies that these shocks have return predictability. Specifically, a consumption level shock and an animal spirit shock have a positive contemporaneous impact on returns but subsequently have negative subsequent return responses. Recall that investors perceive negative news shocks after a positive level shock, which in turn leads to negative realization of returns. A positive animal spirit shock makes investors perceive not only negative news shocks but also negative level shocks. Therefore, the animal spirit shock produces more pronounced return fluctuation. Although not clear in the figure, a negative volatility shock also yields a positive contemporaneous return but negative subsequent returns. In the long-run risks model, risk premiums vary over time due to persistently fluctuating uncertainty and a volatility shock is the one that perturbs uncertainty. Therefore, a volatility shock can be interpreted as a discount rate news in Campbell and Vuolteenaho (2004) and induces a predictable return trajectory.

Not only do the theoretical impulse response functions indicate the qualitative movement of returns, but they also enable us to compare their quantitative magnitude between small-cap and large cap stocks. Figure 6 shows that structural shocks have differential effects between small-cap stocks and large-cap stocks. Note that the return of small-cap stocks is more strongly responding to all shocks except for a level shock. For example, a positive news about the future growth rate raises the small-cap return by
about 2.5%, but the large-cap return reacts by less than 1%. A reduced uncertainty
(or a favorable dividend news) induces the small-cap return to jump up about 3% (or
6%), but the large-cap portfolio gains less than 1.5% (or 4%). In contrast, the large-cap
return is more sensitive to a consumption level shock than the small-cap return. The esti-
mation result shows that an animal spirit shock also has substantial effects on returns.
Interestingly, the small-cap returns are more affected by an animal spirit shock than
the large-cap return, which is exactly aligned with the findings of Baker and Wurgler

Based on the estimated model, I decompose the one-period ahead forecast error vari-
arces and compute how much of them is attributed to each shock. The decomposition
result is tabulated in Table 3. Consistent with the implications from the theoretical
impulse responses, the unexpected variation in investor sentiment mostly comes from
animal spirit shocks (66.78%). However, systematic risk shocks also explain a substan-
tial amount of sentiment variation. Specifically, volatility shocks and dividend shocks
account for 22.21% and 11.00% of the total sentiment variation, respectively. As for
returns, although structural shocks account for most of the forecast error variances,
animal spirit shocks still play a non-trivial role. About 8% of the total unexpected
variation of small-cap returns is attributed to the animal spirit shocks. When it comes
to the large-cap return, the animal spirit is less responsible for unexpected return vari-
ation (4.52%). The decomposition result also states that small-cap stocks are relatively
more affected by news shocks and dividend shocks, while about the half of the large-cap
return variations are explained by consumption level shocks.

The theoretical impulse responses and the variance decomposition provide support
for both the animal spirit view and the systematic risks view. In other words, investor
sentiment reflects animal spirits of investors, and the apparent relationship between
investor sentiment and returns is at least in part attributed to the causal effect of
autonomous animal spirits. However, systematic risks in the stock market also explains
a substantial variation in investor sentiment and not all apparent return predictability
is attributed to animal spirit shocks. Volatility shocks, which also have the implication
of return predictability, explain substantial amount of return variations (25.16% for
small-cap returns and 20.13% for large-cap returns).

5 Conclusion

The role of investor sentiment in the stock market has attracted attentions of economists,
but there has been little attempt to analyze it in a structural framework. In this paper,
I try to understand the meaning of innovations in investor sentiment and their role in
the stock market using a structural model. I modify the long-run risks model and this
enables us to evaluate the pure effect of investor sentiment through autonomous animal
spirits as opposed to that through systematic risks. In the model, I allow the possibility
that investors could have erroneous perception of the growth rate in the economy. I
interpret this erroneous perception as the animal spirits and assume that innovations in investor sentiment arise from both animal spirit shocks and several systematic risk shocks. Depending on parameter values, this model can either be reduced to a classical asset pricing model in which animal spirit shocks play no role, or describe a behavioral finance theory in which investor sentiment not only reflects systemic risks but also has a causal effect on the stock market.

By matching impulse responses from model-simulated data to those from actual data, I estimate parameters in the model. The estimated model moderately replicates the historical data in the actual stock market. The estimation results show that a substantial amount of variation in investor sentiment is explained by animal spirit shocks, and that animal spirit shocks can have significant effects on stock returns. The findings suggest that investor sentiment is a noisy proxy of animal spirits and that autonomous animal spirits are at least in part responsible for the apparent return predictability of investor sentiment. However, given that systematic risk shocks also account for a non-trivial amount of investor sentiment variation, further research is necessary for a clearer understanding of investor sentiment.
Appendix I

This appendix section explains the estimation procedure of $\tilde{x}$ using the Kalman filter. Note that the relationship of observed variables ($s_{x,t}$, $\Delta c_t$ and $\Delta d_{M,t}$) and unobserved expected consumption growth ($x_t$) is represented by a state space model.

\begin{align}
Z_t &= \mu + HX_t + Q_{t-1}e_{x,t} \\
X_t &= B X_{t-1} + W_{t-1} e_{x,t},
\end{align}

where $Z_t$ is the vector of observed variables, $X_t$ is the vector of unobserved variables, $\mu$ is a constant term, $B$ is the state transition matrix, and $H$ is the output matrix. $e_{x,t}$ and $e_{x,t}$ have jointly independent and identical standard normal distributions, and $Q_t$ and $W_t$ capture the covariance structures of errors. Formally,

\begin{align*}
Z_t &= \begin{pmatrix} s_{x,t} \\ \Delta c_t \\ \Delta d_{M,t} \end{pmatrix}, \\
X_t &= \begin{pmatrix} x_t \\ x_{t-1} \end{pmatrix}, \\
H &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & \phi_M \end{pmatrix}, \\
B &= \begin{pmatrix} \rho & 0 \\ 1 & 0 \end{pmatrix},
\end{align*}

\begin{align*}
Q_t &= \begin{pmatrix} \sigma_a & 0 & 0 \\ 0 & \sigma_t & 0 \\ 0 & \pi_M \sigma_t & \varphi_M \sigma_t \end{pmatrix}, \\
W_t &= \begin{pmatrix} \varphi e_{\sigma_t} \\ 0 \end{pmatrix}, \\
e_{z,t} &= \begin{pmatrix} a_t \\ \eta_t \\ u_{t,t} \end{pmatrix}, \\
e_{x,t} &= e_t,
\end{align*}

and $\mu = (0, \mu_c, \mu_M)'$.

The Kalman filter provides an easy way to recursively compute the conditional mean and variance of a state vector given observations. We start with arbitrary initial values of a state vector and its covariance matrix $(X_{0|0}, \Sigma^X_{0|0})$. Since the initial values are arbitrary, we assign large variances to it to limit its effect. The influence of this arbitrary initial value becomes less significant as the Kalman filter keeps correcting the next state values in a deliberate way described below. With the initial values of state variables, the next state variables and the observed variables are predicted based on Equation (20) and (19). Formally,

\begin{align*}
X_{t|t-1} &= BX_{t-1|t-1}, \\
Z_{t|t-1} &= \mu + HX_{t|t-1}, \\
\Sigma^X_{t|t-1} &= B \Sigma^X_{t-1|t-1} B' + W_{t-1} W_{t-1}', \\
\Sigma^Z_{t|t-1} &= H \Sigma^X_{t|t-1} H' + Q_{t-1} Q_{t-1}'
\end{align*}

for $t \geq 1$.

where $X_{t|t-1}$ and $Z_{t|t-1}$ are the expected values of $X$ and $Z$ given information at $t-1$, and $\Sigma^X_{t|t-1}$ and $\Sigma^Z_{t|t-1}$ are the conditional variances of $X$ and $Z$, respectively. The difference between the actually observed value ($Z_t$) and the predicted value ($Z_{t|t-1}$) is taken into account to correct the state variables in next period. Formally,

\begin{align*}
P_t &= \Sigma^X_{t|t-1} H' (\Sigma^Z_{t|t-1})^{-1}, \\
X_{t|t} &= X_{t|t-1} + P_t (Z_t - Z_{t|t-1}), \\
\Sigma^X_{t|t} &= \Sigma^X_{t|t-1} - P_t \Sigma^Z_{t|t-1} P_t' \quad \text{for} \ t \geq 1.
\end{align*}
By iteration, the unobserved state variables are estimated:

$$\tilde{x}_t = x_{t|t} = e_1'X_{t|t},$$

where $e_2 = (1, 0)'$.

**Appendix II**

In this Appendix section, the analytical solutions for $A$’s and $A_m$’s are provided. The wealth-consumption ratio for the consumption claim is $z_t = A_0 + A_1 x_t + A_2 \sigma_t^2$, where

$$A_0 = \frac{1}{1 - k_{1}} \left[ \log \delta + k_{0} + \left( 1 - \frac{1}{\psi} \right) \mu_c + k_1 A_2 (1 - \nu)^2 \sigma^2 + \frac{\theta}{2} (k_1 A_2 \sigma_w)^2 \right]$$

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - k_1 \rho}$$

$$A_2 = \frac{-(\gamma - 1)(1 - \frac{1}{\psi})}{2(1 - k_1 \nu)} \left[ 1 + \left( \frac{k_1 \varphi}{1 - k_1 \rho} \right)^2 \right].$$

Let $\lambda_{\eta} = \gamma$, $\lambda_c = (1 - \theta) k_1 A_1 \varphi$, and $\lambda_w = (1 - \theta) k_1 A_2$. Then, the price-dividend ratio for asset $j$ is $z_{j,t} = A_{0,j} + A_{1,j} x_t + A_{2,j} \sigma_t^2$, where

$$A_{0,j} = \frac{1}{1 - k_{1,j}} \left[ \Gamma_0 + k_{0,j} + \mu_j + k_{1,j} A_{2,j} (1 - \nu)^2 \sigma^2 + \frac{1}{2} (k_{1,j} A_{2,j} - \lambda_w)^2 \sigma_w^2 \right]$$

$$A_{1,j} = \frac{\phi_j - \frac{1}{\psi}}{1 - k_{1,j} \rho}$$

$$A_{2,j} = \frac{1}{1 - k_{1,j} \nu} \left[ \Gamma_2 + \frac{1}{2} \left\{ (\pi_j - \lambda_{\eta})^2 + (k_{1,j} A_{1,j} \varphi - \lambda_c)^2 + \varphi_j^2 \right\} \right],$$

where $\Gamma_0 = \log \delta - \frac{1}{\psi} \mu_c - \frac{1}{2} \theta (\theta - 1) (k_1 A_2 \sigma_w)^2$ and $\Gamma_2 = (\theta - 1) (k_1 \nu - 1) A_2$.

Having solved for $A$’s and $A_j$’s, the log return can be expressed in terms of a constant, state variables ($\tilde{x}_t$, $\sigma_t^2$), and structural shocks ($\tilde{\eta}_t$, $\tilde{e}_t$, $w_t$, and $\tilde{u}_{d,t}$) as follows.

$$r_{j,t+1} = \mu_j + k_{0,j} - A_{0,j} (1 - k_{1,j}) + k_{1,j} A_{2,j} (1 - \nu) \sigma^2$$

$$+ (\phi_j - A_{1,j} (1 - k_{1,j} \rho)) \tilde{x}_t - A_{2,j} (1 - k_{1,j} \nu) \sigma_t^2$$

$$+ \pi_j \sigma \tilde{\eta}_{t+1} + k_{1,j} A_{1,j} \varphi \sigma_t \tilde{e}_{t+1} + k_{1,j} A_{2,j} \sigma_w w_{t+1} + \varphi_{j} \sigma_t \tilde{u}_{d,t+1}.$$
References


Figure 1: Empirical Impulse Responses to Investor Sentiment Innovation (Investor Sentiment Ordered First)

Note: The figures depict the empirical impulse responses to a one standard deviation investor sentiment innovation. Sent is the measure of investor sentiment, dc is the consumption growth (%), $(p - d)_j$ is the log price-dividend ratio of asset $j$, and $r_j$ is the log return on asset $j$ (%), where $j \in \{\text{small-cap stock portfolio (S)}, \text{large-cap stock portfolio (L)}\}$. The red lines indicate the one standard error confidence band. The confidence band is constructed based on the 1,000 Bootstrap simulations. Investor sentiment, consumption growth, log price-dividend ratios, and returns are included in the VAR system, where investor sentiment is ordered first. The sample period is from 1934 to 2005.
Figure 2: Empirical Impulse Responses to Investor Sentiment Innovation (Investor Sentiment Ordered Last)

Note: The figures depict the empirical impulse responses to a one standard deviation investor sentiment innovation. Sent is the measure of investor sentiment, $dc$ is the consumption growth (%), $(p-d)_j$ is the log price-dividend ratio of asset $j$, and $r_j$ is the log return on asset $j$ (%), where $j \in \{\text{small-cap stock portfolio (S), large-cap stock portfolio (L)}\}$. The red lines indicate the one standard error confidence band. The confidence band is constructed based on the 1,000 Bootstrap simulations. Investor sentiment, consumption growth, log price-dividend ratios, and returns are included in the VAR system, where investor sentiment is ordered last. The sample period is from 1934 to 2005.
Figure 3: Simulated Impulse Responses to Investor Sentiment Innovation (Investor Sentiment Ordered First)

Note: The figures depict the simulated impulse responses to a one standard deviation investor sentiment innovation. $Sent$ is the measure of investor sentiment, $dc$ is the consumption growth (%), $(p − d)_j$ is log price-dividends ratio of portfolio $j$, and $r_j$ is the log return of portfolio $j$ (%), where $i \in \{\text{small-cap stock portfolio (S), large-cap stock portfolio (L)}\}$. The red lines indicate the one standard error confidence band. The confidence band is constructed based on the 1,000 Bootstrap simulations. Monthly data are simulated according to the model in Section 3 and aggregated to annual values. Simulated values of investor sentiment, consumption growth, log price-dividend ratios, and returns are included in the VAR system, where investor sentiment is ordered first. The simulated annual sample has the same size as the actual dataset.
Figure 4: Example of the Kalman Estimation of the Expected Consumption Growth Rate ($x_t$)

Note: This figure depicts an illustrative example of the Kalman estimation of the expected consumption growth rate. $x_t$, $s_{x,t}$, $\Delta c_t$ and $\Delta d_{M,t}$ are simulated time series of then expected consumption growth rate, its noisy signal, consumption growth, and the market dividend growth. The simulation is implemented with the set of estimated parameters according to Equation (3)-(eq:sx), and the length of the simulated time series is the same as the actual data period (73 years = 876 months). $x_{t|t}$ is the conditional expectation of the unobserved expected consumption growth given observations at $t$. The conditional expectation is recursively computed by the Kalman filter as described in Appendix I.
Figure 5: Theoretical Impulse Responses of Perceived and Actual Structural Shocks

Note: The figures depict the theoretical impulse responses of selected variables to a one standard deviation structural shock in the estimated model. $\eta$ is a consumption level shock, $e$ is a news shock, $u$ is a market-wide dividend shock, and $a$ is an animal spirit shock. $x$ is the expected consumption growth. The solid lines are the perceived impulse responses, while the dotted lines are the actual impulse responses.
Figure 6: Theoretical Impulse Responses of Returns

Note: The figures depict the theoretical impulse responses of returns to a one standard deviation structural shocks in the estimated model. $\eta$ is a consumption level shock, $e$ is a news shock, $-w$ is a negative volatility shock, $u$ is a dividend shock, and $a$ is an animal spirit shock. $r_j$ is the log return on asset $j$ (\%), where $j \in \{\text{small-cap stock portfolio (S), large-cap stock portfolio (L)}\}$.  

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Table 1: Configuration of Model Parameters

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<th>( \varphi_e )</th>
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<th>( \pi_M )</th>
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<th>( \varphi_M )</th>
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| Estimation | Small-Cap Dividends | \( \mu_S \) | 0.0042 | \( \phi_S \) | 4.38 | \( \pi_S \) | 3.00 | \( \varphi_S \) | 7.55 |
|-------------|---------------------|--------|-------------|-----|-------------|-----|-------------|-----|-------------|------|

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<th>( \zeta_a )</th>
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Note: Table 1 reports the model parameters. The parameters related to preferences, consumption, and market dividends are from Bansal, Kiku, and Yaron (2012). The other parameters are estimated in this paper. Standard errors for the estimated parameters are numerically computed and given in parentheses. All parameters are in monthly terms. \( \delta \) is the discount factor, \( \gamma \) is the relative risk aversion, \( \psi \) is the elasticity of inter-temporal substitution, \( \mu_c \) is the mean growth rate of consumption, \( \rho \) is the persistence of long-run risks, \( \varphi_e \) is the volatility multiple for long-run risks, \( \nu \) is the persistency of uncertainty, \( \bar{\sigma}^2 \) is the mean uncertainty, \( \sigma_w \) is the volatility of uncertainty, \( \mu_j \) is the mean dividend growth of asset \( j \), \( \phi_j \) is the exposure of dividend growth of asset \( j \) to long-run risks, \( \pi_j \) is the impact of a consumption shock on dividend growth of asset \( j \), \( \varphi_j \) is the volatility multiple for dividend growth of asset \( j \), \( \sigma_a \) is the volatility of animal spirit shocks, \( \rho_S \) is the persistence of investor sentiment, and \( \zeta_i \) is the loading of structural shock \( i \) on the innovations in investor sentiment, where \( j \in \{ \text{small-cap stock portfolio (S)}, \text{large-cap stock portfolio (L)}, \text{stock market index (M)} \} \) and \( i \in \{ \eta, e, w, d, a \} \).
Table 2: Selected Moments

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<th>Model</th>
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<td>$E[\Delta c]$</td>
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<td>$\sigma(\Delta c)$</td>
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<tr>
<td>$\sigma((p - d)_L)$</td>
<td>0.47</td>
<td>0.18</td>
<td>0.28</td>
</tr>
<tr>
<td>$E[R_F]$</td>
<td>0.69</td>
<td>1.18</td>
<td>0.96</td>
</tr>
<tr>
<td>$\sigma(R_F)$</td>
<td>4.21</td>
<td>0.95</td>
<td>1.27</td>
</tr>
</tbody>
</table>

Note: Table 2 reports selected moments from the actual data (in the Data column) and the simulated data (in the Model and the Population columns). The model moments are calculated from 1,000 sets of simulated data that has the same sample size as the actual data. The population moments are calculated from a very long sample period (7200 years) of simulated data. The actual data are in real terms, sampled on an annual frequency, and cover the period from 1930 to 2012. $E[\cdot]$, $\sigma(\cdot)$, $Corr(\cdot, \cdot)$, and $AC1(\cdot)$ are mean, standard deviation, correlation coefficient, and $s^{th}$-order autocorrelation coefficient, respectively. $\Delta c$ is the consumption growth, $\Delta d_j$ is the dividend growth of asset $j$, $R_j$ is the percentage net return of asset $j$, and $(p - d)_j$ is the log price-dividend ratio of asset $j$, where $j \in \{\text{small-cap stock portfolio (S), large-cap stock portfolio (L), stock market index (M), risk free asset (F)}\}$. 
Table 3: Forecast Error Variance Decomposition

<table>
<thead>
<tr>
<th>Due to</th>
<th>Variance of</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Sent_{t+1} - E_tSent_{t+1}$</td>
</tr>
<tr>
<td>Consumption Shock ($\eta$)</td>
<td>0.00</td>
</tr>
<tr>
<td>News Shock ($e$)</td>
<td>0.01</td>
</tr>
<tr>
<td>Volatility Shock ($w$)</td>
<td>22.21</td>
</tr>
<tr>
<td>Dividend Shock ($u_d$)</td>
<td>11.00</td>
</tr>
<tr>
<td>Animal Spirit Shock ($a$)</td>
<td>66.78</td>
</tr>
<tr>
<td>Sum</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Note: Table 3 reports the result of the forecast error variance decomposition in the estimated model. Each number indicates the percentage of the 1-period ahead forecast error variance of the selected variable attributed to the corresponding structural shock. $Sent$ is the measure of investor sentiment and $r_j$ is the log return on asset $j$, where $j \in \{\text{small-cap stock portfolio (S), large-cap stock portfolio (L)}\}$. 