Components of Credit Rationing\textsuperscript{1}

Mehdi Beyhaghi\textsuperscript{2}
Federal Reserve Bank of Richmond
mehdi.beyhaghi@rich.frb.org

Fathali Firoozi
University of Texas at San Antonio
fathali.firoozi@utsa.edu

Abol Jalilvand
Loyola University Chicago
ajalilv@luc.edu

June 2019

\textsuperscript{1}The views expressed in this article are solely those of the authors. They do not necessarily reflect the views of the Federal Reserve Bank of Richmond or the Federal Reserve System.

\textsuperscript{2}The corresponding author.
Components of Credit Rationing

Abstract. Existence and implications of credit rationing by lending institutions have been the subject of much research in recent decades. However, little is known about how credit rationing manifests itself in credit markets. This study offers the first known decomposition of credit rationing by characterizing three distinct groups of loan applicants who are fully or partially denied credit. The study first establishes the theoretical basis for the existence of three distinct components of rationing, namely, low-type rationing, market-tightness rationing, and self-imposed rationing. The study then exhibits empirical evidence on the magnitude of each of the three rationing components using semi-annual data from European Central Bank over the period 2009-2018.

Keywords. Asymmetric information; credit risk; incentive compatibility, optimal contracts.

JEL classification: D82, G21, G32.
1 Introduction

In a typical product market, a price below the equilibrium price generates an excess of demand over supply, and thus a form of rationing emerges. A conventional wisdom is that in the absence of regulatory price control, rationing cannot persist and will be resolved by a price increase that expands the quantity of supply and reduces the quantity of demand so that the market equilibrium is restored. However, the credit market exhibits an anomaly because it has been persistently in a state of rationing.

The causes and consequences of credit rationing has been studied extensively in the literature. A class of studies in that literature is focused on explaining why credit rationing exists and whether it is a temporary non-equilibrium or a long-term equilibrium phenomenon. The studies of Keeton (1979) and Stiglitz and Weiss (1981) are some of the seminal studies that present models in which credit rationing coexists with equilibrium, thus coining the phrase “equilibrium credit rationing.” A number of subsequent studies offer conditions that could rule out existence of credit rationing at equilibrium, including Bester (1985), Besanko and Thakor (1987), Arnold and Riley (2009), and Su and Zhang (2017).

A subset of the literature studies the causes of credit rationing and the factors that affect the supply of bank credit. DeYoung, Hunter, Udell (2004) and Frame, Srinivasan, and Woosley (2001), Akhavein, Frame, White (2005), DeYoung, Frame, Glennon, and Nigro (2011) study the effect of deregulation, technological advance,

---


2If the demand for loans is rather fixed or predictable, then the determinants of supply for loans can be regarded as the core causes of credit rationing. However, the demand side of the credit market has its own variability. Dell’Ariccia, Igan, and Laeven (2012) show the role of growth in credit demand.

Another class of studies focuses on the consequences of credit rationing. Blinder and Stiglitz (1983) study the effect of credit rationing on the transmission of monetary policy through so called “credit channels.” McKinnon (1973) studies the impact of credit rationing on the long term growth prospect of economies. A number of other studies focus on the implication of credit rationing for corporate finance, in particular for the firm’s capitalization and capital structure, including Cressy (1996), Calomiris and Wilson (1998), Calomiris and Mason (2003), Colla, Ippolito, and Li (2013), and Graham, Leary, and Roberts (2015).

The existing studies on credit rationing, as summarized above, focus on the two dimensions of causes and consequences of credit rationing. This study adds a new dimension to the literature on credit rationing by offering a decomposition of credit rationing. We rigorously identify the characteristics of three distinct borrower groups who are denied credit at equilibrium. We refer to these three components of credit rationing as: (i) Low-type rationing, (ii) Market-tightness rationing, and (iii) Self-
imposed rationing. Given the stated objective, we first develop a model of credit markets in the presence of heterogeneity in credit contracts, heterogeneity in types of credit applicants, and asymmetric information about the type of applicants where the term “type” generally refers to an applicant’s credit worthiness. Using the model as a rational setting, we then characterize the optimal contracts designed by lenders and demonstrate that three distinct forms of credit rationing can emerge at equilibrium. In the final part of the study, we use a data set from the European Central Bank over the period 2009-2018 to demonstrate an observed magnitude of the three forms of credit rationing suggested by our model.

The characterization of various forms of credit rationing is important for several reasons. It is particularly important for lenders in fine-tuning and targeting their credit contracts. In addition, the knowledge of distinct borrower groups who are denied credit is useful to policy makers who are interested in the macro aspects of credit market functioning. Understanding the different ways credit rationing manifests itself in the bank-based credit market is particularly important considering the widespread use of bank credit as the main source of external financing for almost all types of firms. Robb and Robinson (2012), for example, show that startups rely heavily on various forms of bank financing from the personal bank loans and line of credits to various types of business loans. Cosh, Cumming, and Hughes (2009) find that rejection rates are generally lower in credit markets for new ventures than from other sources of capital, and therefore, many newly founded firms consider bank credit as the first choice of external financing. Berger and Udell (1998), Frame, Srinivasan, and Woosley (2001), DeYoung, Hunter, and Udell (2004), and DeYoung, Goron, Torna, Winton (2015) provide evidence that small businesses rely on loans obtained from
both community and large banks as a crucial source of capital. Moreover, Faulkender and Petersen (2005) argue that even for public firms, bank financing, and not the bond market, is the main choice of debt financing, as only about 19% of public firms have a bond rating in a given year.\footnote{The literature on the reliance of firms on bank credits also includes DeYoung, Goldberg, and White (1999), Berger and DeYoung (2006), Lown and Morgan (2006), Ergungor (2010), and Dell’Ariccia, Igan, and Laeven (2012).} The stated literature has documented extensively the various mechanisms through which the availability of banking products and services affect the economy. However, little is known about the structure of affected loan applicants in the bank credit market. To our knowledge, this is the first attempt to theoretically characterize the components of credit rationing and provide empirical support on their magnitudes.

In Section 2 we present the model. In Section 3 we discuss the optimal credit contracts. In Section 4 we review some of the key parameters that characterize the optimal contracts. In Section 5 we establish that there are three distinct forms of credit rationing that emerge from optimal contracts. In Section 6 we present some empirical support for the three theoretical forms of rationing constructed in Section 5. We conclude in Section 7.

## 2 The model

Credit markets consist of exchanges between lenders and borrowers in the presence of asymmetric information. In a principal-agent context, lenders as principals design credit contracts for various types of borrowers. By signing a credit contract, a borrower becomes an agent committed to delivering the terms of the contract. The literature on modeling of general principal-agent contexts and optimal contracts is
rather extensive. In this section we offer one of the most general models fitted to the credit markets. A central issue in modeling credit markets is the existence of asymmetric information between lenders and borrowers. The asymmetry exists in the sense that a borrower’s type is private information which is unobservable to lenders. The competitive lenders offer various lending contracts targeted towards borrowers of various types where the lenders are uncertain about an applicant’s type.

**Contracts.** A contract is a vector of commonly observable characteristics that specify the terms and obligations of an agreement, such as the interest rate, collateral, and repayment plan. Contracts are designed and posted by the lenders. Different contracts are identified by differences in some of those characteristics. The set of all contracts is denoted by \( X \). For a given contract posted by a lender, several borrowers of various types apply. The “type” is a borrower characteristic that is relevant to the lenders in their lending decisions. We use “type” to reference the true credit worthiness of the applicant. If a borrower’s type is a common knowledge and observable to the lenders, then the type simply becomes another dimension in the contract vectors. However, the focus here is on the case of imperfect information where the borrower’s type is private information to the borrower and unobservable by the lender. The lenders know the finite set of various types but cannot observe

---

4 Reviews of the literature include Bolton and Dewatripont (2004) and Salanie (2017).
5 A report sent by Federal Reserve Board (2017) to Congress in regard to lending to small businesses states: "Lending to small businesses is further complicated by the “informational opacity” of many such firms. Obtaining reliable information on the creditworthiness of a small business is often difficult because little, if any, public information exists about the performance of most small businesses."
6 For the purpose of assigning properties such as compactness and convexity, we assume \( X \) is a subset of the vector space \( \mathbb{R}^n \).
7 We do not explicitly distinguish between the terms "borrowers" and "applicants". However, there will be a clear distinction between borrowers (applicants) who succeed to secure credit and borrowers (applicants) who fail to secure credit.
the type of a borrower. There are $I$ types of borrowers.

For a given contract $x \in X$, the posting lender and an applying borrower “match” if the borrower is approved by the lender and the loan is granted.

**Payoffs.** There is a fixed contract design cost $s > 0$ incurred by each lender. For each contract $x \in X$, the net payoff to a lender who matches with a borrower of type $i \in \{1, \cdots, I\}$ is

$$v_i(x) - s$$

where $v_i : X \to \mathbb{R}$ is the lender’s payoff function when matching with a borrower of type $i$.$^8$ The payoff to a borrower of type $i$ matching with a lender posting the contract $x \in X$ is

$$u_i(x)$$

where $u_i : X \to \mathbb{R}$ is the payoff function of the borrower of type $i \in \{1, \cdots, I\}$. The payoff functions $(v_i, u_i)$ and the contract cost $s$ are assumed given.

The payoff functions $(v_i, u_i)$ will also be interchangeably referred to as “reward”, “utility”, or “preference” functions. They reflect the present value of subjective and objective measures employed by lenders and borrowers in evaluating credit contracts. The features that these functions represent include the sentiments about the state of the economy.

**Market tightness and shares.** For each contract $x \in X$, the market tightness,
or the lenders/borrowers ratio, is defined by the ratio

$$\theta(x) = \frac{\text{Number of lenders posting contract } x}{\text{Number of borrowers applying for contract } x}$$  \hspace{1cm} (3)$$

where $\theta : X \rightarrow [0, \infty]$ is the market tightness (lenders/borrowers ratio) function. A smaller value of $\theta$ (a higher value of $1/\theta$) corresponds to a higher degree of market tightness. When the focus is on contract $x_i$ designed for borrowers of type $i$, the term $\theta_i$ reflects the market tightness faced by borrowers of type $i$.\(^9\) The values ($\theta_i$) will be determined endogenously by the optimality at equilibrium.

For each contract $x \in X$, the share of a borrower of type $i \in \{1, \cdots, I\}$ is defined by

$$\gamma_i(x) = \frac{\text{Number of borrowers of type } i \text{ applying for contract } x}{\text{Number of borrowers applying for contract } x}$$  \hspace{1cm} (4)$$

where $\gamma_i : X \rightarrow [0, 1]$ is the share function of the borrowers of type $i$ satisfying $\sum_{i=1}^{I} \gamma_i(x) = 1$. The aggregate share function is then the vector-valued function $\Gamma : X \rightarrow [0, 1]^I$ where $\Gamma(x)$ is a vector defined by

$$\Gamma(x) = [\gamma_1(x), \cdots, \gamma_I(x)]$$  \hspace{1cm} (5)$$

The share functions $\gamma_i$, thus the aggregate share function $\Gamma$, are unknown and will be endogenously determined by optimality at equilibrium.

**Matching probabilities and expected payoffs.** Both the probability that a lender finds a matching borrower and the probability that a borrower finds a matching lender depend on the market tightness (lenders/borrowers ratio) $\theta$ as defined in

\(^9\)Thus $\theta_i$ is the ratio of number of lenders who offer contract designed for borrowers of type $i$ over the number of borrowers of type $i$.\]
equation (3) above. For a given contract \(x \in X\), the probability that a borrower finds a matching lender is defined by the composite function value

\[ \mu[\theta(x)] \]  

where \(\mu : [0, \infty] \to [0, 1]\) is the borrower’s matching probability function. Because \(\theta\) is the ratio of lenders to borrowers as defined in equation (3), it is rational to assume that a higher ratio leads to a higher probability that borrowers are funded, so we assume that \(\mu\) is increasing with \(\mu(0) = 0\) and \(\mu(\infty) = 1\). Similarly, for a given contract \(x \in X\), the probability that a lender finds a matching borrower is defined by the composite function value

\[ \eta[\theta(x)] \]  

where \(\eta : [0, \infty] \to [0, 1]\) is the lender’s matching probability function. A similar justification shows the rationality of assuming \(\eta\) is a decreasing function with \(\eta(0) = 1\) and \(\eta(\infty) = 0\). The matching probability functions \((\mu, \eta)\) are assumed given.

For a given contract \(x \in X\), the expected payoffs for a posting lender is

\[ \eta[\theta(x)] \sum_{i=1}^{I} \gamma_i(x)v_i(x) - s \]  

and the expected payoff for an applying borrower of type \(i \in \{1, \cdots, I\}\) is

\[ \mu[\theta(x)]u_i(x) \]  

**Ranking of borrower types.** Although lenders cannot observe a borrower’s type, the finite set of types is known to lenders. Lenders rank the types based on their
own type-specific utility functions $v_i$ as defined in equation (1) above. Accordingly, the ranked index set of types is defined by

$$
\Pi = \{1, \cdots, I\}
$$

such that for every contract $x \in X$

$$
v_1(x) \leq v_2(x) \leq \cdots \leq v_I(x)
$$

Therefore, for two type indices $i$ and $j$ with $i < j$, lenders prefer the higher type $j$ over the lower type $i$.

3 Optimal contracts

An equilibrium for the credit market defined in the last section is characterized in this section. For ease of reference, a summary listing of the parameters defined in the last section is provided in Appendix 1. Equilibrium is solution to a problem that consists of a set of interrelated optimization problems faced by lenders in designing contracts. The core assumption by lenders in forming their contract is that every borrower searches all of the posted contracts and applies for one that maximizes the borrower’s expected rewards. The objective of each lender is to design and offer a set of type-specific contracts such that the search by each borrower results in an optimal choice of contract for that borrower consistent with the borrower’s type. The optimization problem thus consists of a set of optimization problems, one for each borrower type $i \in \Pi$ where $\Pi$ is the type-ranked index set defined in equation (10). For each borrower of type $i \in \Pi$, the $i$th optimization problem, denoted $(P_i)$, is
defined by the constrained maximization of the \( i \)th type borrower’s expected payoff as defined in equation (9) over all market tightness and contract vectors:\(^{10}\)

\[
\bar{U}_i = \max_{\theta_i \in [0, \infty], x_i \in X} \mu[\theta_i(x_i)]u_i(x_i)
\]

(12)

where the maximization stated in \{\cdot\} is subject to the following two constraints

\[
(12 - i) \quad \eta[\theta_i(x_i)] \sum_{j=1}^{I} \gamma_j(x_i)v_j(x_i) \leq s.
\]

\[
(12 - ii) \quad \bar{U}_j \leq \bar{U}_i \text{ for all } j \in \Pi, j \neq i.
\]

The constraint stated in (12 – i) reflects the competitive lending market where no lender makes a positive expected payoff at equilibrium. The constraint stated in (12 – ii) ensures that at equilibrium, optimal incentives to borrowers are type-compatible so that, for example, the lower type borrowers have no incentive to apply for a contract that is optimal for the higher type borrowers. As will be discussed further below, the constraint (12 – ii) is the incentive-compatibility constraint that prevents adverse selection by borrowers. The constraints reflect the interrelation feature of the set of optimization problems \((P - i)_{i=1}^{I}\).

**Solution.**\(^{11}\) A solution to the constrained optimization problem in (12) is an equilibrium solution presented by the collection \((\theta^*_i, x^*_i)_{i \in \Pi^*}\) of pairs of market tightness \((\theta^*_i)\) and optimal contracts \((x^*_i)\). In the general case where both of the constraints \((12 - i)\) and \((12 - ii)\) are binding at the solution so that the constraints hold as equality, the Kuhn-Tucker conditions generate the solutions \((\theta^*_i, x^*_i)\) as well as the solutions for the multipliers \(\phi^*_i\) and \(\varphi^*_i\) corresponding to the two constraints. Of course, the

---

\(^{10}\)The optimization in (12) is in fact maximizing rewards for the borrower of type \( i \) but carried out by the lenders. Hence, the parameters used in the optimization, such as the probability assessments \((\mu, \eta)\), reflect the lenders’ beliefs.

\(^{11}\)The discussion of solution and its characteristics in the remainder of this section will be rational and sufficient for our purpose. To the extent possible, we avoid unnecessary formalities.
multipliers turn to zero if the constraints are nonbinding at the solution. Constraint (12 – i) is expected to be binding in reflection of the competitive zero expected net profit for lenders. For the sake of exhibiting our core points regarding credit rationing, we assume that the constrained optimization problem in (12) has a unique separating solution so that each borrower type \( i \) has its own distinct optimal contact \( x^*_i \), with the provision that for some borrower types the optimal contract \( x^*_i \) could be zero. Any rigorous development of conditions for existence and uniqueness of this solution goes beyond the scope and objective of this study. However, on an intuitive ground, the core limiting restriction presented by the incentive-compatibility constraint (12 – ii) deserves an explanation.

**Incentive compatibility.** One of the main issues regarding the existence of a solution is the feasibility of the incentive-compatibility constraint (12 – ii).\(^{12}\) Given the set of contract solutions \( (x^*_1, \ldots, x^*_I) \), the constraint (12 – ii) states that, for a borrower of type \( i \), the condition \( \bar{U}_i(x^*_j) \leq \bar{U}_i(x^*_i) \) holds for all types \( j, j \neq i \), where \( \bar{U}_i(x^*_j) = \mu[\theta_i(x^*_j)]u_i(x^*_j) \). Thus the constraint (12 – ii) performs the act of preventing borrowers from practicing adverse selection, such as, a borrower of type \( i \) choosing a contract \( x^*_j \) designed for a borrower of higher type \( j \) with \( i < j \). As the problem stands, there is no mechanism to prevent adverse selection by the borrowers. Although there are not mechanisms that fully guarantee the holding of the incentive-compatibility constraint, there are a few mechanisms that could increase the likelihood of its holding. For instance, lenders may include in their optimal contracts clauses that are essentially credible threats that could lead the borrowers to avoid adverse selection. As a practical example, most credit cards nowadays have a clause that

\(^{12}\)We assume the other usual conditions for existence hold, such as concavity of utility functions, and compactness and convexity of the contract set \( X \).
effectively states if a borrower type $i$ who may have taken a contract designed for borrowers of higher type $j$ fails a payment due date, the terms of contract become void and the initial rate of 5% will be raised to 30% for the rest of the credit’s life. This threat could be sufficient for a borrower of type $i$ to take a contract designed for type $i$ with an initial interest rate of 15% but allows a couple of late payments per year without a penalty. Implementation of such clauses could in the long run lead to a substantial reduction in adverse selection. We assume in our equilibrium (long-run) setting that contracts have such additional clauses that make the incentive-compatibility constraint $(12 - ii)$ feasible.

**Type sets.** If the number of lenders is fixed, then by the definition in (3), the market tightness value $\theta_i^*$ identifies the number of borrowers that lenders can serve under the contract $x_i^*$. Let the index $\Pi^*$ be a subset of the type index set $\Pi$ and be defined such that for borrowers of type $i \in \Pi^*$ there is a solution vector $(\theta_i^*, x_i^*)$ of optimal tightness and optimal nonzero contract. Thus, $\Pi^*$ is the set of borrower types who find matching contracts at equilibrium. We refer to $\Pi^*$ as the optimal type set. If the optimal set $\Pi^*$ is a proper subset of $\Pi$, then the set difference $\Pi \setminus \Pi^*$ is nonempty and the borrowers of type $i \in \Pi \setminus \Pi^*$ find zero optimal contract, and thus fail to secure credit at equilibrium. In Section 5 we show that the case of nonempty $\Pi \setminus \Pi^*$ is a possible outcome of the equilibrium.

4 Key parameters

The equilibrium solution characterized in the last section depends on a number of parameters that play key roles in setting the constrained optimization problem in

---

13 Given the number of lenders posting a contract, the definition in equation (3) for $\theta_i^*$, and the definition in equation (4) for shares jointly determine the optimal share values $\gamma_i^*$. 

13
equation (12). These parameters include the type-specific lender and borrower reward functions \((v_i, u_i)\) defined in equations (1) and (2), and the matching probabilities \((\mu, \eta)\) defined in equations (6) and (7). In addition, the contract set \(X\) defined in the second paragraph of Section 2 could face regulatory or lender-imposed restrictions such that for a borrower of type \(i \in \Pi\) an optimal nonzero contract \(x^*_i\) may not exist at equilibrium, so the index \(i\) fails to be in the optimal type set \(\Pi^*\). In the extreme case where restrictions on the lender utility \(v_i\) and/or on the contract set \(X\) are overly restrictive, the optimal type set \(\Pi^*\) could be the empty set, reflecting a shut-down of the credit market.

Changes in the outlook of overall economic activity and monetary policy revisions have impact on the solutions of the model mainly through revising the exogenous contract set \(X\) available to the lenders and the exogenous lender and borrower reward functions \((v_i, u_i)\). Following the credit market shock of 2007-2009, regulatory restrictions on banks and lending were introduced by Congress.\(^{14}\) These restrictions amounted to a number of new limitations on the lending contracts set \(X\). Stresses in the banking system and economic outlook also revised the lender reward functions \((v_i)\). These revisions in two of the core parameters of the model limited the contact solutions and raised the optimal market tightness that led to an expansion of credit rationing at equilibrium and further slow down in economic growth. The adverse effects of the rise in credit rationing on the economy were subsequently mitigated by the extensive monetary expansion of the Federal Reserve known as quantitative easing.

\(^{14}\)In response to the credit market shock of 2007-2009, the fiscal policy response was the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010. The Act imposed substantial constraints on the ability of banks to extend credit. Critics argued that the act will ultimately hurt economic growth. In 2018, the Congress passed a bill that will significantly rollback provisions of the Dodd-Frank Act (Investopedia, 2018).
that was effectively initiated in 2010.\textsuperscript{15}

5 Rationing components

As stated in Section 1, lenders and regulators have a natural interest in knowing the composition of borrowers who fail to secure credit. In this section we characterize three distinct groups of borrowers who could fail to secure credit at equilibrium. One of the elementary views on the composition of borrowers who fail to secure credit is that they are low-type (high-risk) borrowers. However, we show in this section that the stated view is too restrictive and could be rationally expanded in a number of directions. We specify three possible subsets of rationing at equilibrium as listed in (A)–(C) below.

(A). \textbf{Low-type rationing.} In the last paragraph of Section 3, we characterized the equilibrium type set $\Pi^*$ of borrowers as a subset of the general set $\Pi$ of all available types defined in (10). The optimal type set $\Pi^*$ contains the types of borrowers who find a nonzero optimal contract at equilibrium, that is, the types that secure credit at equilibrium. When $\Pi^*$ is a strict subset of $\Pi$, the set difference $\Pi \setminus \Pi^*$ is nonempty, and thus the borrowers of type $i \in \Pi \setminus \Pi^*$ do not find a matching contract offered by lenders at equilibrium and will be left out of the credit market. The possibility that the set difference $\Pi \setminus \Pi^*$ is nonempty and consists mainly of low-type borrowers emerges from the equilibrium. The binding form of the constraint $(12 - i)$ reflects the

\textsuperscript{15}According to the Federal Reserve Board (2015), "In December 2008, as evidence of a dramatic slowdown in the U.S. economy mounted, the Federal Reserve reduced its target for the federal funds rate—the interest rate that depository institutions charge each other for borrowing funds overnight—to nearly zero, in order to provide stimulus to household and business spending and so support economic recovery. With short-term interest rates at nearly zero, the Federal Reserve made a series of large-scale asset purchases (LSAPs) between late 2008 and October 2014."
competitive zero expected net profit for lenders defined by

$$\eta[\theta_i(x_i)] \sum_{j=1}^{I} \gamma_j(x_i)v_j(x_i) - s = 0$$

For a borrower of type $i$ and a contract $x_i$, the lender reward function values $v_j(x_i)$, $j = 1, \cdots, I$, are rather low if the borrower $i$ is a low-type borrower relative to the case where the borrower of type $i$ is a high-type borrower. This emerges from the type ranking criterion in (11). Thus, for a borrower of low type $i$, sufficiently low lender utilities could lead to a negative expected net profit, thus a zero optimal contract and, hence, the low-type index $i$ will not be in the optimal set $\Pi^*$ of types that find nonzero matching contacts.\(^{16}\) As elaborated in Section 3, the incentive-compatibility constraint holds in the long run and prevents low-type borrowers who don’t find a matching contract from practicing adverse selection and securing a contract designed for high-type borrowers.\(^{17}\)

(B). Market-tightness rationing. The value $\theta^*_i$ is the optimal market tightness for type $i$ emerging from the optimization in equation (12), thus by its definition in (3), $\theta^*_i$ is the optimal ratio of the number of lenders who offer contract $x^*_i$ over the number of borrowers of type $i$ who apply and match contract $x^*_i$. The optimal market tightness $\theta^*_i$ thus puts its own constraint on the fraction of borrowers who will find a

\(^{16}\)It is notable that the stated low-type rationing is not an outcome of type-screening of individual borrowers by lenders since lenders cannot observe the individual borrower types. It is just an outcome of optimal design of contracts as shown. As shown, a key parameter set that determines the low-type rationing is the set of lenders’ reward functions $(v_j)$.

\(^{17}\)A borrower of low type who does not find a matching contract may, in the short run, apply and secure a contract designed for higher type borrowers. However, in the long run, as the true type gets manifested and recorded, the contract gets cancelled and further applications from the borrower get denied. This is consistent with the discussion and assumption in Section 3 that the incentive compatibility constraint $(12 - ii)$ holds in the long run.
lender with a matching contract. To elaborate on the impact of $\theta^*_i$ on credit rationing, consider the extreme case of $\theta^*_i = 0$. In this case, the lenders at equilibrium do not offer a contract that matches borrowers of type $i$, therefore, the index $i$ will not be in the optimal index set $\Pi^*$. An optimal contract $x^*_i$ may exist in this case, but the high degree of market tightness reflected by the extreme $1/\theta^*_i$ value suggests that there is no lender who offers that contract. Hence, there could be not only low-type borrowers but high-type borrowers who fail to secure credit at equilibrium because of the optimal market tightness conditions. The observed rise in credit rationing as a consequence of the 2007-2009 crash, as briefly highlighted in the last paragraph of Section 4, can be explained mainly in terms of the market tightness rationing.

(C). Self-imposed rationing. A contract is a vector of commonly observable characteristics that specify the terms and obligations of a loan agreement. Although lenders optimally design contracts for borrowers of different types, a borrower's type is a private information to the borrower and unobservable to lenders, thus “type” is not one of the characteristics listed in a contract vector. A borrower of type $i$ may find contract $x^*_i$ most fitting among all contracts listed. However, the class of borrowers of type $i$ could well be individually different in terms of their views on the entire characteristics that are listed in the contract vector $x^*_i$. Accordingly, some borrowers of type $i$ may find some of those listed characteristics unfitting to their desired terms or self-evaluation and refrain from applying. Borrowers in the self-imposed rationing component could be of any type, low or high.
6 Empirical evidence

In this section we first discuss the challenges of performing an empirical examination of the three components of credit rationing that we specified in the last section and then provide some statistics on the relative magnitude of each of the three forms of rationing in the European market over the period 2009-2018. The literature on empirical credit rationing is rather thin, and existing limited studies are focused on either industry-specific rationing or overall rationing. The primary reason for the shortage of empirical studies on credit rationing is insufficiency of available data. There are at least two reasons for the stated insufficiency. First, there are no formal requirements by oversight agencies for lenders to report details on their credit rationing. Second, both lenders and borrowers typically have no individual incentives to publicize the credit rationing that they practiced or experienced. Thus, available data and empirical studies are based on survey questionnaire results gathered from the executives in lending institutions and borrowing firms. For instance, based on survey data collected by the World Bank, Drakos and Giannakopoulos (2011) performed an empirical study of the firms whose loan applications were rejected or discouraged using firm-level rationing data from the transition economies.

Data sources for general rationing are limited and available data sets are survey-based. Data availability is even more problematic for the components of credit rationing, in particular when the components, as we characterized in the last section, are based on the unobservable “types” of applicants. However, we locate a set of authoritative survey data that indicates the existence of the stated three rationing components and their relative magnitudes. The European Central Bank (ECB) collects comprehensive survey data on loan applications comprised of enterprises in the
Euro area. The section of the statistical data warehouse of ECB that is relevant for our purpose is the semiannual survey of loan applications and their outcomes called Survey on Access to Finance of Enterprises (SAFE). We focused on the data over the post-crash period of 2009-2018 regarding bank loan applications and their status for small and medium-sized enterprises. There are various types of financing, but we focus on “bank loan” type of financing. The sample size ranged between nearly 4,000 to 6,000 small and medium size firms (employee size of 250 or less) across Europe’s largest economies.

In terms of outcome of applications, the SAFE reports contain the following four response classes, which we refer to as the “empirical rationing components”: (1) “Applied but was rejected”, (2) “Applied but got only part of it”, (3) “Applied but refused because cost was too high”, and (4) “Did not apply because of possible rejection”. Appendix 2 exhibits the numerical and graphical forms of the four time series collected from Statistical Data Warehouse of ECB. Specifically, the series shown in the Appendix are the SAFE series on the observed semiannual weighted percentages of respondents falling in each of the four empirical rationing components (1)-(4) as defined over the post-crash period of 2009-2018.

We use the following table to associate the “empirical rationing components” (1)-(4) as defined above to the “theoretical rationing components” (A)-(C) defined in the last section.19

---

18 The class "Applied but refused because cost too high" refers to the situation where the bank only partially approved the application in the sense that the bank approved only at a higher interest rate or collateral than one anticipated by the applicant or the bank added additional conditions that were costly to the applicant. The applicant then chose to reject the offer.

19 The exhibited association table is an estimate, thus not exact. For instance, some of the observed "Applied but was rejected" may in fact be due to "Market tightness" rationing. Similarly, some of the observed "Applied but got only part of it" could in fact be a part of "Low-type" rationing. However, we believe that for the most part the exhibited association is more sensible than the alternatives.
### Theoretical Rationing Components

<table>
<thead>
<tr>
<th>Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Low-type</td>
</tr>
<tr>
<td>(B) Market tightness</td>
</tr>
<tr>
<td>(C) Self-imposed</td>
</tr>
</tbody>
</table>

### Empirical Rationing Components

<table>
<thead>
<tr>
<th>Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Applied but was rejected</td>
</tr>
<tr>
<td>(2) Applied but got only part of it</td>
</tr>
<tr>
<td>(3) Applied but refused because cost too high</td>
</tr>
<tr>
<td>(4) Did not apply because of possible rejection</td>
</tr>
</tbody>
</table>

Based on the association table shown above, the four empirical rationing series observed over the period 2009-2018 as exhibited in Appendix 2 offer an empirical indication of the existence of the three theoretical rationing components defined in the last section.

The observed four series shown in Appendix 2 reflect the immediate impact of the credit crunch of the 2007-2009 crash and the effects of the subsequent quantitative easing policies over the period 2010-2018. The “Applied but was rejected” Series 1 in the Appendix has its highest values in the 2009-2010 period with the highest value of 17.30 percent in 2009H2 (second half-yearly of 2009), and then exhibits a downward trend over 2010-2018 to its lowest value of 4.42 in 2017H2. Similarly, the “Applied but got only part of it” Series 2 has its highest values over the 2009-2011 period with the highest value of 19.90 percent in 2011H1 followed by a downward trend reaching the lowest value of 9.91 percent in 2017H2. Assuming that by 2017-2018 the series returned to their pre-crash values, the relative high values of Series 1 and 2 in the periods right after the crash, 2009-2010, reflect the rise in overall credit rationing initiated during the 2007-2009 crash. According to the association table shown above, the rise in rationing during the crash substantially impacted both the low-type and the market-tightness components of overall rationing. The “Applied but
refused because cost too high” Series 3 had its highest value of 4.01 percent in 2009H1 and then exhibited a low-slope downward trend, with high variation, to the lowest value of 0.71 percent in 2017H1. The percentage of respondents suggesting Series 3 were overall markedly lower than that of Series 1 and 2. The “Did not apply because of possible rejection” Series 4 exhibited a pattern different from the other three series. Series 4 appears to be a delayed response to the other three series. It started at one of its low values of 5.05 percent in 2009H1 followed by an upward trend to the its highest value of 8.43 percent in 2014H1, and then followed a downward trend to its lowest value of 4.13 in 2018H2. Overall, the trends in these four series are well consistent with expectations for the post-crash years of 2009-2018, that is, they show a generally downward trend as the credit markets slowly recovered from the crash incident.

We now focus on Series 1, reflecting that a substantial portion of the observed rise in the overall credit rationing brought by the 2007-2009 crash can be attributed to a rise in the theoretical low-type component of credit rationing. As shown in the last section, a key parameter set responsible for the low-type rationing component is the set of lenders’ reward functions \((v_j)\). Therefore, although overall market condition played a role, it appears that a key factor responsible for a substantial portion of the rise in the overall credit rationing during the crash period of 2007-2009 was a sort of lenders’ panic that revised their reward functions with a subsequent tightening impact on their contract offers. This possibility, emerging from the theory and the observed rationing, is consistent with the view that the credit crunch of 2007-2009 was mainly a non-policy, supply-initiated incident.
7 Concluding remarks

Existing literature on credit rationing has focused on the reasons behind the existence of overall credit rationing and the implications of credit rationing for the firm, monetary policy, and economic growth. This study adds a new dimension to the literature by characterizing three distinct forms in which credit rationing is manifested, namely, low-type rationing, market-tightness rationing, and self-imposed rationing. The stated decomposition of overall rationing identifies distinct groups of borrowers who could be primarily impacted by a variation in overall credit rationing. The decomposition emerges from equilibrium in a general model of credit markets under a set of rational conditions. The empirical existence and magnitude of the theoretical rationing components are implied by a set of semi-annual observations collected by the European Central Bank over the period 2009-2018. The results are helpful to lenders in fine-tuning their contracts and to monetary authorities in designing their policy.

One of the implications of this study is that the factors identified in the literature as the drivers of credit rationing could disproportionately affect different components of rationing. For the policy makers interested in the credit market functioning, the knowledge of distinct borrower groups who are denied credit is useful for designing appropriate targets for their policy. The task for a policy maker interested in reducing an observed rise in rationing is to first identify the primary component of rationing that the rise in observed rationing belongs to, and then target the determining parameters for that particular rationing component. A core parameter for the market-tightness rationing component is the set of possible contracts the lenders can offer. Conventional expansionary monetary policy can be effective in relaxing
the limitations on the set of possible contracts the lenders can offer and thus reduce
the market tightness rationing. However, the core parameters for the low-type ra-
tioning component are the lenders’ and borrowers’ rewards functions. These reward
functions are determined primarily by the outlook of lenders and borrowers on the
overall state of the economy and, hence, the monetary authorities have limited tools
to revise them.

The 2007-2009 crash offers a case in point. The initial rise in credit rationing
in the 2007-2009 crash can be characterized to a large extent in terms of a rise in
the low-type rationing component brought by a tightening of lenders’ and borrowers’
rewards that emerged as a consequence of the stresses in the banking system and
the grim economic outlook. The subsequent quantitative easing of the central banks
that took effect in 2009-2010 initially brought a reduction in the market tightness
rationing component. However, it took the subsequent long period of 2010-2018 for
a slow and delayed relaxation of lenders’ and borrowers’ rewards and a reduction in
the low-type rationing component to materialize.

References

novations: An examination of the adoption of small business credit scoring by
large banking organizations. *Journal of Business*, 78(2), 577-596.

2. Arnold, L., and J. Riley (2009). On the possibility of credit rationing in the

Money, Credit and Banking*, 10(2), 170-183.


Appendix 1

Below is a listing of summary definitions for the notation used in Section 3. Detailed definitions are in Section 2.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>Number of borrower types</td>
</tr>
<tr>
<td>$I_i$</td>
<td>$I_i \equiv {1, \cdots, I}$, the set of ranked borrower types</td>
</tr>
<tr>
<td>$\nu_i$</td>
<td>Reward (utility) of lender when lending to a borrower of type $i \in {1, \cdots, I}$</td>
</tr>
<tr>
<td>$u_i$</td>
<td>Reward (utility) of a borrower of type $i$</td>
</tr>
<tr>
<td>$X$</td>
<td>Set of all contracts</td>
</tr>
<tr>
<td>$s$</td>
<td>Fixed cost of designing a contract</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Market tightness for borrowers of type $i$, Number of lenders/Number of borrowers</td>
</tr>
<tr>
<td>$\gamma_i$</td>
<td>Proportion of borrowers of type $i$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Probability that a borrower finds a matching lender</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Probability that a lender finds a matching borrower</td>
</tr>
</tbody>
</table>
Appendix 2

The empirical evidence on rationing components elaborated in Section 6 are presented in this appendix. The data consists of four series collected semi-annually (half-yearly, H) over the period 2009-2018. Each series is shown in a tabular form and then in graphical form to highlight the series’ variation over time.

Series 1: **Applied but was rejected.** Small and medium-sized enterprises in Euro area. Financing applied - Bank loan. Weighted percentage of responses at half-annuals, 2009-2018.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>11.90</td>
<td>17.30</td>
<td>10.04</td>
<td>9.22</td>
<td>8.67</td>
<td>11.95</td>
<td>14.11</td>
<td>9.95</td>
<td>11.23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>2018H1</th>
<th>2018H2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>4.884</td>
<td>5.73</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2009H1</td>
<td>17.13</td>
<td>17.99</td>
<td>17.75</td>
<td>18.15</td>
<td>18.90</td>
<td>16.31</td>
<td>17.94</td>
<td>17.55</td>
<td>17.80</td>
</tr>
<tr>
<td>2010H1</td>
<td>17.75</td>
<td>18.15</td>
<td>18.90</td>
<td>16.31</td>
<td>17.94</td>
<td>17.55</td>
<td>17.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011H1</td>
<td>18.15</td>
<td>18.90</td>
<td>16.31</td>
<td>17.94</td>
<td>17.55</td>
<td>17.80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012H1</td>
<td>18.90</td>
<td>16.31</td>
<td>17.94</td>
<td>17.55</td>
<td>17.80</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013H1</td>
<td>16.31</td>
<td>17.94</td>
<td>17.55</td>
<td>17.80</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2014H1</td>
<td>17.94</td>
<td>17.19</td>
<td>14.45</td>
<td>11.59</td>
<td>12.86</td>
<td>10.19</td>
<td>10.29</td>
<td>10.72</td>
<td>9.91</td>
</tr>
<tr>
<td>2015H1</td>
<td>14.45</td>
<td>11.59</td>
<td>12.86</td>
<td>10.19</td>
<td>10.29</td>
<td>10.72</td>
<td>9.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2015H2</td>
<td>11.59</td>
<td>12.86</td>
<td>10.19</td>
<td>10.29</td>
<td>10.72</td>
<td>9.91</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2016H1</td>
<td>12.86</td>
<td>10.19</td>
<td>10.29</td>
<td>10.72</td>
<td>9.91</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2016H2</td>
<td>10.19</td>
<td>10.29</td>
<td>10.72</td>
<td>9.91</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2017H1</td>
<td>10.19</td>
<td>10.29</td>
<td>10.72</td>
<td>9.91</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2017H2</td>
<td>10.29</td>
<td>10.72</td>
<td>9.91</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2018H1</td>
<td>10.72</td>
<td>9.91</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2018H2</td>
<td>9.91</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2009H1</td>
<td>4.01</td>
<td>2.15</td>
<td>3.17</td>
<td>1.29</td>
<td>2.58</td>
<td>2.85</td>
<td>1.31</td>
<td>2.25</td>
<td>0.84</td>
</tr>
<tr>
<td>2013H2</td>
<td>1.61</td>
<td>3.11</td>
<td>1.68</td>
<td>1.53</td>
<td>1.68</td>
<td>1.55</td>
<td>1.49</td>
<td>0.71</td>
<td>0.97</td>
</tr>
<tr>
<td>2018H1</td>
<td>1.128</td>
<td>0.961</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2009H1</td>
<td>5.05</td>
<td>6.60</td>
<td>6.08</td>
<td>7.46</td>
<td>6.62</td>
<td>7.42</td>
<td>6.20</td>
<td>6.68</td>
<td>6.82</td>
</tr>
<tr>
<td>2013H2</td>
<td>6.29</td>
<td>8.43</td>
<td>7.68</td>
<td>6.94</td>
<td>5.98</td>
<td>6.39</td>
<td>5.76</td>
<td>5.39</td>
<td>4.77</td>
</tr>
<tr>
<td>2018H1</td>
<td>4.564</td>
<td>4.138</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>