Opening the black box of joint household production: child welfare and parental leisure*

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Abstract

We study a model of joint household production: parental time invested in children produces both welfare for the child and leisure for the caring parent. Process benefits from child care capture precisely this: parents enjoy part of the time they spend with their children. Insight in process benefits is important to obtain an accurate description of parent and child welfare. We apply our model nonparametrically to Dutch consumption and time use data and we find empirical support for process benefits. We also propose a method to quantify the proportion of parental time that is perceived as leisure. We finally demonstrate that individuals who earn more than the median wage enjoy parental time more than individuals who earn less.

Keywords: joint household production, child care, process benefits, leisure, revealed preference, labour supply

JEL codes: C14, D13, J13, J22

1 Introduction

One of the crucial determinants of child welfare is parental time invested in children.¹ The household economics literature models child welfare as the output of a household

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production technology with parental time and money expenditures as inputs. One key element that is usually overlooked in this literature is the possible ‘joint production’ aspect of parental time. Parental time may simultaneously produce welfare for the child and leisure for the caring parents, when parents also enjoy part of the time they spend with children. Juster (1985) introduced the term ‘process benefits’ to capture the latter effect.

Separating the effect of parental time on the caregiver’s leisure (process benefits) from the effect of parental time on child welfare (household production) is non-trivial. Process benefits affect the construction of equivalence scales and the measurement of parental and child welfare. First, applications of the household production model without process benefits may underestimate parental welfare, because parental total leisure is not fully captured. Second, applications without process benefits may also overestimate the marginal productivity of parental time inputs in terms of generating child welfare. This study proposes an elementary model, with joint production and a flexible household production technology, that still allows us to disentangle process benefits from the production of child welfare. We subsequently apply a nonparametric version of this model to Dutch time use data to quantify process benefits from child care. This indicates which proportion of child care is perceived as supplementary leisure time.

Household production of child welfare. The theory of time allocation, initiated by Becker (1965), posits that household members allocate their time to the production of commodities. The author started from the idea that agents purchase market goods and spend personal time to produce ‘domestic’ commodities, which they subsequently consume. More recently, the household economics literature has applied this theory of home production to the development of child welfare in the family. Cherchye, De Rock, and Vermeulen (2012), for instance, implemented a collective labour supply model with household production to assess the impact of male and female wage changes on the domestic production of children’s welfare. Next, Del Boca, Flinn, and Wiswall (2013) found significant effects of parents’ time on the cognitive development of their children, over and beyond the effect of money expenditures. More recently, Chiappori, Salanié, and Weiss (2017) demonstrated that this household production of children’s welfare also depends on the parents’ human capital.

Most empirical analyses of household production of child welfare rely on two assump-
tions. The main assumption is no joint production. This implies that parents value the domestic output (i.e., child welfare) but not the domestic inputs per se (i.e., time spent with children). However, time spent with children may also produce leisure for the caring parent, in the form of process benefits. In this respect, Browning, Chiappori, and Weiss (2014) argued that “a father who spends time with his child contributes to the development of the child and may also enjoy spending time with the child”. Similarly, Pollak and Wachter (1975) claimed that “jointness is pervasive because time spent in many production activities is a direct source of utility as well as an input into a commodity”. The second assumption is that the production function of child welfare takes a Cobb Douglas form. This imposes constant returns to scale and constant elasticity of substitution between time inputs and money expenditures on children.

Process benefits. A slightly different strand of the literature has already adopted the concept of process benefits. Graham and Green (1984), Kerkhofs and Kooreman (2003), Görtz (2011) and Stratton (2012) acknowledged that the time allocated to housework depends not only on productivity (relative to market earnings) but also on preferences. Graham and Green (1984) were the first to estimate process benefits in the household. Kerkhofs and Kooreman (2003) and Görtz (2011) further fine-tuned the analysis, by showing identifiability of the model with process benefits and by looking into convenient strategies for estimation. Both analyses suggested that process benefits from housework are rather weak.

One reason for the insignificant results in Kerkhofs and Kooreman (2003) and Görtz (2011) is that housework aggregates a variety of domestic activities like cooking, cleaning, gardening, do-it-yourself work and parental time. Stratton (2012) argued that estimates of process benefits from ‘total’ housework may suffer from aggregation bias. A main distinguishing feature of the current study is that we focus on one specific type of housework: child care. After all, Hallberg and Klevmarken (2003) found that parental time invested in children ranks consistently among the most enjoyable domestic activities, better than market work and far better than household chores like cooking and cleaning.4 Furthermore, similar to Stratton (2012) but different from Kerkhofs and Kooreman (2003) and Görtz (2011), we also consider money expenditures (and not only parental time inputs) in the home production process.

A second reason is that the analyses of Graham and Green (1984), Kerkhofs and Kooreman (2003) and Görtz (2011) rely on the theory of time allocation by Gronau

4Juster (1985) developed a list of survey questions to rank the process benefits of various daily activities. A subset of these questions was included in the Swedish Panel Study Market and Non-market Activities (HUS) by Hallberg and Klevmarken (2003). We note that market work was considered to be quite enjoyable, but less so than parental time. Our (relative) process benefits should therefore be interpreted as the satisfaction from parental time over and above the satisfaction from paid work (see Kerkhofs and Kooreman (2003) and Görtz (2011)).
The Gronau (1977) model assumes that the output of housework is marketable, i.e. that there exist goods in the market that are substitutes to the domestic output. A distinguishing feature of the current study is that we relax this assumption. First, marketability of the domestic output (in casu child welfare) is highly debatable given our focus on child care. Second, Gertz (2011) suggested that it is generally difficult to separate process benefits from ‘consumption benefits’, which capture the idea that households attribute higher value to home produced commodities. The marketability assumption goes against the idea of consumption benefits and makes it impossible to control for the latter.

Analysis of the joint products of parental time. Household production models without process benefits may (i) understate parental leisure and welfare and (ii) exaggerate the productivity of parental time inputs in terms of generating child welfare. Indeed, process benefits contribute to parental leisure and reduce the cost of producing child welfare per se. The main objective of the current analysis is to capture and quantify these process benefits. Identification is based on the idea that parental time is separable from parental consumption in the standard household production model but not in the household production model with process benefits. Time spent with children is a direct source of (parental) utility, over and beyond its role in producing child welfare.

However, identification of key elements of household production models is challenging. As Kerkhofs and Kooreman (2003) argued, “the implicit need for a detailed description of activities and the introduction of production functions, about the outputs of which typically no information is available, have hampered empirical applications of this framework.” Empirical applications usually rely on detailed time data per production activity as well as specific restrictions on the shape of the household production function. We make a distinction between the following three approaches to implement the concept of household production.

The first, based on Gronau (1977), assumes perfect substitutability between home produced commodities and commodities purchased in the market. As indicated earlier, it is not reasonable to make this assumption in the present context because child welfare cannot be outsourced completely. Furthermore, we take into account that the valuation of child outputs is household-specific.

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5This also explains why Blundell, Pistaferri, and Saporta-Eksten (2018) dropped the distinction between preferences and technology altogether. Instead, they incorporated parental time inputs directly, via non-separabilities, in the utility function. Although such approach in principle admits process benefits, it is less well-suited to recover the process benefits.

6Chiappori (1997) discussed the implications of (non-)marketability of the domestic commodities in household analysis.

7See Chiswick (1982) for more discussion. We take into account that the valuation of domestic outputs is household-specific. In this way, we control for consumption benefits (in the spirit of Gertz (2011)) while quantifying process benefits.
The second approach is to observe the domestic output. However, it is notoriously difficult to quantify child welfare. There exist proxies of child welfare that combine measures of psychological well-being and cognitive skills into a single metric. Nonetheless, it remains difficult to define child welfare because welfare also depends on the perception of the parents.

The third option is to assume no joint production and constant returns to scale. Under these assumptions, one can recover commodity prices (e.g. the ‘price’ of child welfare) and apply standard microeconomic theory to express the demand for commodities as a function of commodity prices. Cherchye, De Rock, and Vermeulen (2012) and Lise and Yamada (2018) followed this approach. However, both assumptions come into conflict with our objective to quantify process benefits (and hence joint production) while also allowing for a flexible production function (that may exhibit non-constant returns to scale).

**Our contributions.** We first propose an elementary model of (joint) household production that allows us to separate the effect of parental time on leisure (in the form of process benefits) from the effect of parental time on child welfare (via the production function). The model admits a fully flexible production function and process benefits. Furthermore, the elasticity of substitution between parental inputs may vary, domestic and market commodities need not be substitutes and the domestic output need not be observed. We impose structure on household preferences only. More specifically, we assume that each household maximizes an aggregate index of parental sub-utility and child welfare. This mimics the structure of the collective labour supply model (Chiappori, 1988, 1992) in which one sub-utility function captures parental utility and another captures child welfare. Furthermore, incorporating process benefits in such setting captures the idea that parental time has a (specific) public good character within the household.8

We then derive consistency conditions from our model. The conditions fit in the ‘empirical’ revealed preference literature starting from Samuelson (1938) and further developed by Afriat (1967), Diewert (1973) and Varian (1982). More recently, Cherchye, De Rock, and Vermeulen (2007, 2009, 2011) characterized the revealed preference conditions for ‘non-unitary’ household models that consist of more than one utility function. Revealed preference tests of this type take the form of systems of linear inequalities, which use prices (including wages) and choices (consumption and time use) as inputs. One important advantage of this approach is that identification is based on a global characterization of process benefits (compared to the local identification results in Kerkhofs and Kooreman (2003) and Görtz (2011)).

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8Note the analogy with Bargain and Donni (2012) and Bargain, Donni, and Kwenda (2014) who investigated the intra-household distribution of resources. The authors claimed that a subset of goods in the household is public and consumed by parents as well as children.
We finally apply our procedure to data from the LISS panel, which contains information on time use (leisure, parental time invested in children, market work and chores) and consumption. We also observe money expenditures related to children. For our sample, we create 46 household ‘types’ based on the number of children, the age of the youngest child and the schooling of mothers. We will test goodness-of-fit and recover process benefits from parental time for each type separately. The degree of substitutability between time inputs and money expenditures can vary over types. Note that this includes the possible scenario that higher-educated mothers are more productive caregivers (due to increased human capital, see Chiappori, Salanié, and Weiss (2017); or Guryan, Hurst, and Kearny (2008)) as well as the scenario in which parental time inputs are substitutes when children are younger but complements when children are older (Milkie, Nomaguchi, and Denny (2015)). While controlling for all this, we quantify the extent to which parents perceive child care as leisure time.

Outline. We formally present our household production model with process benefits (from parental time) in Section 2. We derive revealed preference conditions in Section 3. These conditions will be useful, in a later stage, to compare the goodness-of-fit of the household production model with/without process benefits. Section 4 presents our data sample from the LISS panel and gives summary statistics for all relevant variables, including expenditures, individual wages and time use data. In Section 5, we quantify the process benefits from parental time. We also compare our fully nonparametric bounds with semi-parametric bounds based on a functional specification for process benefits by Graham and Green (1984). We finally investigate individual heterogeneity in process benefits, in function of the wage of the caregiver (in line with, e.g., Kimmel and Connelly (2007)). Section 6 concludes.

2 Model of joint household production

We introduce a model of ‘joint’ household production of child welfare and parental leisure. Parental time produces both welfare for the child (via a flexible production function) and leisure for the caring parent (in the form of process benefits). Process benefits capture the effect of child care on parental leisure/welfare, over and above its effect on child welfare. A distinguishing feature of our model is that the optimal trade-off between time inputs and money expenditures (used to produce child welfare) is no longer independent of parental preferences, including preferences for consumption.

Set-up. Households consist of two parents ($i = 1, 2$) and one or more children. Each parent $i$ faces a time constraint.
with total time equal to $T$. We let $T$ denote the time budget net of sleep and activities other than (pure) leisure $l^i$, parental time invested in children $h^i$ and market work $m^i$.\footnote{This implies that the time budget can vary across individuals ($T^i$); we omit superscript $i$ for simplicity of notation.}

The household’s budget constraint is

$$c^p + c^k \leq w^1 m^1 + w^2 m^2 + y$$

with wage $w^i$ per unit of market work and (residual) non-labour income $y$. Households can use this income to purchase commodities for parental consumption $c^p$ and commodities for consumption by children $c^k$. Combining time and budget constraints gives

$$c^p + c^k + w^1 (h^1 + l^1) + w^2 (h^2 + l^2) \leq y + T \sum_{i=1,2} w^i.$$

**Joint products: child welfare and parental leisure.** We now formally introduce ‘jointness’ in the household production of child care. We explicitly allow that parental time generates both child welfare via a flexible household production function and parental leisure via the process benefits from parental time.

Child welfare is produced domestically by means of parental time $h^i$ and money expenditures $c^k$. Let $q = Q(h^1, h^2, c^k)$ denote the output of domestic production. It is typically assumed that $Q$ exhibits constant returns to scale. We drop this assumption. In a similar vein, we let the elasticity of substitution vary. It is only assumed that $Q$ is continuous, increasing and concave. Furthermore, our empirical analysis will condition on characteristics such as parental schooling, age and number of children in the household. This takes into account that the degree of substitutability between parental inputs depends, for instance, on the age of children and the education of parents.

Parental leisure is the sum of (observed) pure leisure and (unobserved) process benefits from parental time. We adopt the concept of process benefits from Graham and Green (1984), Kerkhofs and Kooreman (2003) and Görtz (2011) and apply it to parental time. Indeed, $h^i$ produces not only child welfare $q = Q(h^1, h^2, c^k)$ but also a supplement $g^i = G(h^i)$ to leisure $l^i$ for the caring parent. Consequently, parental leisure\footnote{The linear specification of $L$ is not strictly needed. However, it improves the tractability of our model. It is also coherent with the interpretation of process benefits from parental time $g^i$ in terms of leisure for the caregiver.} is

$$L(l^i, g^i) = l^i + g^i.$$
a supplement to pure leisure. Graham and Green (1984) and Kerkhofs and Kooreman (2003) proposed that $G$ satisfies the following properties:

$$0 \leq G'(h^i) \leq 1$$
$$G''(h^i) < 0$$

The first property implies that the marginal leisure from a one-unit increase in $h^i$ is bounded between 0 and 1. The second implies that the process benefits function $G(h^i)$ is concave in $h^i$.

Summarizing, we focus on the possibility that parental time $h^i$ produces both child welfare $q = Q(h^1, h^2, c^k)$ and a parental leisure supplement $g^i = G(h^i)$. This is the specification that we bring to the data in Section 5. At this point, we would like to draw the reader’s attention to two further generalizations.

First, $G$ itself could depend on individual household member characteristics, such as gender, education, salary, etc. We discuss this possibility in more detail in Subsection 5.4, where we let $G^i$ depend on the salary of the caregiver. However, we mainly adopt a homogeneous process benefits function in order to make a sharp distinction between the production function of child welfare $Q$ and the extent to which parental time is perceived as leisure $G$.

Second, we focus on process benefits from parental time rather than process benefits from household chores. Addressing both types of process benefits simultaneously may hamper the identification of our main variable of interest: the process benefits from parental time. Of course, we cannot rule out that alternative uses of time generate some leisure. Technically, it is possible to extend our model with process benefits $f^i = F(t^i)$ from chores $t^i$, and subsequently augment leisure in the following way:

$$L^i(l^i, g^i, f^i) = l^i + f^i + g^i.$$  

We address process benefits from household chores in Subsection 5.4. However, it should be noted that process benefits are always relative, in the sense that they capture the satisfaction of home production activities, over and beyond that of paid work (Kerkhofs and Kooreman (2003) and Gertz (2011)). If chores were less enjoyable than market work (which is indeed what Hallberg and Klevmarken (2003) found) then the concept of process benefits would be ill-suited to describe time devoted to chores.

**Household preferences.** Each household has preferences $V$ for child welfare $q$, parental leisure $l^i + g^i$ and parental consumption $c^p$:

$$V(q, l^1 + g^1, l^2 + g^2, c^p);$$  

$$...$$
with $g^i = G(h^i)$ and $q = Q(h^1, h^2, c^k)$. We drop the assumption that $q$ is marketable. The formulation in (2) is therefore also consistent with the idea that households have specific valuations of the domestic output.$^{11}$

We make one final assumption to improve the empirical tractability of the household production model. In particular, we assume that household utility $V$ depends on a parental sub-utility index $U(l^1 + g^1, l^2 + g^2, c^p)$ that aggregates leisure and parental consumption. We impose the following structure on household preferences:

$$V(q, u) = \omega q + (1 - \omega)u; \quad (3)$$

with $u = U(l^1 + g^1, l^2 + g^2, c^p)$. In words, the household maximizes a weighted sum of parental sub-utility $u$ and child welfare $q$, with respective weights $\omega$ (and $(1 - \omega)$). We motivate this structure on several grounds.

First, this formulation is consistent with a social welfare function that takes child welfare $q$ and parental sub-utility $u$ as arguments. In this sense, $\omega$ captures the degree of caring for child welfare relative to parental sub-utility in the household. In addition, $\omega$ depends on a variety of observed and unobserved factors, and will therefore differ over the households in our sample. This allows us to incorporate a specific type of interhousehold preference heterogeneity in our analysis.

Second, the proposed structure on household preferences will improve the empirical tractability of our model, as well as simplify the following revealed preference conditions (in Section 3).$^{12}$ We are now in a position to describe formally the household production model with process benefits from parental time. Each household chooses levels of consumption $c^p, c^k$, leisure $l^1, l^2$, and parental time $h^1, h^2$, that solve Problem 1:

**Problem 1.** Optimizing behaviour. Each household chooses $\{c^p, c^k, l^1, l^2, h^1, h^2\} =$

$$\arg \max_{c^p, c^k, l^1, l^2, h^1, h^2} \omega q + (1 - \omega)U(l^1 + g^1, l^2 + g^2, c^p)$$

$$s.t. \quad c^p + c^k + w^1(h^1 + l^1) + w^2(h^2 + l^2) \leq y + T \sum_{i=1,2} w^i;$$

with $g^i = G(h^i)$ and $q = Q(h^1, h^2, c^k)$.

The optimality conditions associated with $h^1$ and $h^2$ are of particular interest, because

$^{11}$We thereby implicitly control for consumption benefits discussed in Gertz (2011).

$^{12}$We could further generalize our model by allowing that mothers and fathers have different preferences for child welfare, and that bargaining power depends on household characteristics, in line with the (standard) collective labour supply model. However, such generalization is not strictly needed to capture the process benefits from child care. More practically, Gertz (2011) argued that “such extensions seem desirable, but would inevitably enhance the empirical identification problems already present, given the limited information in the data about the value of household production, intermediate inputs and intra-household matters.”

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$h^i$ produces both child welfare and parental leisure.\textsuperscript{13} The first-order conditions imply that:

$$\omega Q_h^i + (1 - \omega)U_L^i \gamma^i = \lambda w^i, \tag{4}$$

where $Q_h^i$, $U_L^i$ and $\gamma^i = G_h^i$ denote the marginal productivity of parental time, the marginal utility of leisure and marginal process benefits, respectively. The marginal utility of income is represented by $\lambda$.

The first term $\omega Q_h^i$ is the product of the household’s relative degree of caring for children ($\omega$) and the marginal productivity, in terms of child welfare, of parental time ($Q_h^i$). The second term $(1 - \omega)U_L^i \gamma^i$ is the product of the household’s degree of caring for parental sub-utility $(1 - \omega)$, the marginal utility of leisure $(U_L^i)$ and the extent to which an additional unit of parental time is a supplement to leisure $(\gamma^i)$. In equilibrium, the sum of both equals the opportunity cost of parental time (i.e. forgone wages).

Moreover, from the first-order conditions associated with pure leisure, it follows that

$$(1 - \omega)U_L^i = (1 - \omega)U_L^i = \lambda w^i. \tag{5}$$

Summarizing, equations (4) and (5) yield:

$$\frac{\omega Q_h^i}{\lambda} = w^i (1 - \gamma^i). \tag{6}$$

This shows that applications of household production models without process benefits may not only underestimate parental total leisure but also overestimate the (perceived) productivity of parental time inputs $Q_h^i$. Marginal process benefits, in the form of $\gamma^i$, affect the equilibrium condition on parental productivity and the opportunity cost of parental time. Suppose that process benefits are relevant, i.e. $\gamma^i > 0$. The productivity of parental time is now $w^i (1 - \gamma^i)$ instead of $w^i$. Suppose now that process benefits are extremely strong, i.e. $\gamma^i \rightarrow 1$. Then parents spend time with children mainly because they like this use of time. Estimates of parental productivity that only take into account opportunity cost $w^i$ (and not process benefits $\gamma^i$) could severely overestimate the true productivity.

3 Identification of process benefits

Identifiability of process benefits stems from the fact that trade-offs between time inputs and money expenditures on children are separable from parental consumption in the ‘standard’ household production model but not in the ‘joint’ household production model (with process benefits). In other words, the optimal choice of $(c_k^k, h_a^1, h_a^2)$ depends not only

\textsuperscript{13}The full list of first-order conditions is given in Appendix A.
on home production function $Q$ but also on parental preferences.

This nonseparability however complicates the practical implementation of our model. In the absence of marketable (as in Gronau (1977)) or observable commodities, identification of key elements of household production models typically relies on the assumptions of no joint production and constant returns to scale. Cherchye, De Rock, and Vermeulen (2012) started from a two-stage representation of the household’s problem. The first stage determines the optimal input bundle needed to produce commodities $q$. This stage also associates a ‘commodity price’ with each unit of $q$, given constant returns to scale. The second stage then determines the optimal bundle of commodities while taking into account this domestic price. The first stage reflects a cost minimization procedure whereas the final stage can be solved along the lines of standard demand theory. With joint production or non-constant returns to scale, Pollak and Wachter (1975) showed that commodity prices depend on the commodity bundle consumed. In other words, the analogy between the second stage of the production function approach and classical demand theory is lost. Indeed, demand theory relies on the idea that the commodity price is independent of the chosen commodity bundle. For our setting, with joint production and non-constant returns to scale, we propose an alternative approach that is robust to the endogeneity issue.

**Revealed preference theory.** We propose to use the revealed preference conditions of the (novel) household production model with process benefits. We start from a data set $S = \left(c_s^p, c_s^k, l_s^i, h_s^i, w_s^i \right)_{i \in \{1,2\}, s \in N}$ that contains the consumption and time use choices, alongside wages, for a group of households $N$. The households in each group are similar in terms of observed characteristics. Each household has a unique index $s \in N$. We observe nor estimate the (endogenous) price of domestic outputs. Similarly, we observe nor specify the functional form of $U$, $Q$ or $G$. Furthermore, we allow the household production function to exhibit non-constant returns to scale and a variable elasticity of substitution. We only assume that $U$, $Q$ and $G$ are well-behaved in the usual sense. Finally, we admit that $\omega_s$ is household-specific. This captures interhousehold variation in the relative degree of caring for child welfare. Like the other functions, $\omega_s$ is unobserved.

In this framework, Definition 1 presents our notion of rationality.

**Definition 1.** Consider a data set $S = \left(c_s^p, c_s^k, l_s^i, h_s^i, w_s^i \right)_{i \in \{1,2\}, s \in N}$, The data set $S$ is consistent with the household production model with process benefits from parental time if and only if there exist functions $U$, $Q$ and $G$, and weights $\omega_s$, such that each $s \in N$ solves Problem 1.

To bring this notion of rationality to the data, we start from the first-order conditions of Problem 1. We combine this with (global) concavity conditions on the functions $U$, $Q$ and $G$. We obtain systems of inequalities that can be used (i) to test consistency of
the data $S$ with Definition 1 and (ii) to recover bounds on process benefits from parental time. Our result is similar to the nonparametric characterization of the collective labour supply model (without household production) developed by Cherchye, De Rock, and Vermeulen (2007, 2009, 2011). A key building block of such characterization is the so-called Generalized Axiom of Revealed Preference.\footnote{In the spirit of Samuelson (1938), Afriat (1967), Diewert (1973) and Varian (1982).} Definition 2 formally presents the GARP.

**Definition 2.** Consider a data set $S = \{ (p_s, q_s) \}_{s \in N}$ with price vectors $p_s$ and quantity vectors $q_s$. The data set $S$ satisfies the Generalized Axiom of Revealed Preference (GARP) if and only if there exist binary relations $R^0_{s,v}$ and $R_{s,v} \in \{0, 1\}$ such that:

1. $p'_s q_s \geq p'_v q_s \Rightarrow R^0_{s,v} = 1$.
2. $R^0_{s,r} = 1, R^0_{r,t} = 1, \ldots, R^0_{z,v} = 1$ for some sequence $(s, r, t, \ldots, z) \Rightarrow R_{s,v} = 1$.
3. $R_{s,v} = 1 \Rightarrow p'_s q_s > p'_v q_v$.

First, the GARP states that a bundle $s$ is directly revealed preferred over a bundle $v$ if bundle $v$ was affordable when $s$ was chosen. The second condition constructs $R^0_s$ transitive closure by imposing transitivity. Finally, for all bundles $s$ revealed preferred over $v$, it should be the case that they were not (strictly) cheaper when $v$ was chosen. Otherwise bundle $v$ is irrational, given that it fails to minimize expenditure among all bundles that are (at least) as good as $v$. Provided that one observes either prices $p$ or quantities $q$, the GARP can be implemented as a linear programming problem with binary variables. These variables capture binary relations $R_{s,v}$.

Let us then use the Lagrange multipliers $\lambda_s$ associated with Problem 1 and the ‘caring’ weights $\omega_s$ to define new multipliers $\eta_s^k = \frac{\lambda_s}{\omega_s}$ and $\eta_s^p = \frac{\lambda_s}{1 - \omega_s}$. We can now formally state the equivalence between consistency with Definition 1 and two sets of revealed preference conditions. Appendix A contains the proof.

**Proposition 1.** For a given data set $S = (c^k_s, c^k, \tau^i_s, h^i_s, w^i_s)_{i \in \{1, 2\}, s \in N}$, the following statements are equivalent:

1. The data set $S$ is consistent with the household production model with process benefits from parental time, in the sense of Definition 1.
2. Given $S$, there exist numbers $g^i_s \in [0, \tau^i_s], \gamma^i_s \in [0, 1]$ with $g^i_s \geq \gamma^i_s h^i_s$; $u_s, q_s \in \mathbb{R}$, and multipliers $\eta^k_s, \eta^p_s \in \mathbb{R}_+$ such that $\forall s, v \in N$ and $\forall i, j \in \{1, 2\}$:
   
   \begin{align*}
   (a) & \quad u_v - u_s \leq \eta^p_s \left[ w^1_s (l^1_v + g^1_v - l^1_s - g^1_s) + w^2_s (l^2_v + g^2_v - l^2_s - g^2_s) + (c^p_v - c^p_s) \right]; \\
   (b) & \quad q_v - q_s \leq \eta^k_s \left[ w^1_s (1 - \gamma^1_s) (h^1_v - h^1_s) + w^2_s (1 - \gamma^2_s) (h^2_v - h^2_s) + (c^k_v - c^k_s) \right];
   \end{align*}


(c) \( g^i_v - g^i_s \leq \gamma^i_s \left[ h^i_v - h^i_s \right] \).

3. Given \( S \), there exist numbers \( g^i_s \in [0, h^i_s], \gamma^i_s \in [0, 1] \) with \( g^i_s \geq \gamma^i_s h^i_s \), such that \( \forall s, v \in N \) and \( \forall i, j \in \{1, 2\} : \)

(a) \( \left\{ \left( 1, w^1_s, w^2_s \right), \left( c^1_s, l^1_s + g^1_s, l^2_s + g^2_s \right) \right\}_{s \in N} \) satisfies GARP;

(b) \( \left\{ \left( 1, w^1_s(1 - \gamma^1_s), w^2_s(1 - \gamma^2_s) \right), \left( c^k_s, h^1_s, h^2_s \right) \right\}_{s \in N} \) satisfies GARP;

(c) \( g^i_v - g^i_s \leq \gamma^i_s \left[ h^i_v - h^i_s \right] \).

Condition (1) contains our definition of the household model with process benefits. Many elements of the model are unobserved (such as \( U, Q, G \) and \( \omega_s \)) which makes it difficult to implement the first condition. Conditions (2) and (3) provide alternatives that are better suited for empirical applications. We will show that condition (3), in particular, can be transformed into a linear programming problem with binary variables.

Condition (2) contains Afriat inequalities associated with parental sub-utility, household production and process benefits. The latter stems from our assumption that \( G(h^i) \) is concave. The inequalities in conditions (2a) and (2b) are nonlinear in variables \( u_s, g_s, \eta^k_s, \eta^i_s, g^1_s, g^2_s, \gamma^1_s \) and \( \gamma^2_s \). As is well-known, finding (optimal) solutions to nonlinear programming problems is complicated due to the existence of local optima.

Condition (3) contains the equivalent ‘GARP’ formulation. Finding solutions to the GARP conditions boils down to solving linear programming problems (with binary variables), provided that either prices or quantities are known. The econometrician observes prices \( \left( 1, w^1_s, w^2_s \right) \) in condition (3a) and quantities \( \left( c^k_s, h^1_s, h^2_s \right) \) in condition (3b), which enables us to ‘linearize’ the testable conditions. Furthermore, condition (3c) is linear in \( g^i_v, g^i_s \) and \( \gamma^i_s \). We can therefore treat \( g^i_v, g^i_s \) and \( \gamma^i_s \) as (unobserved) variables in our program. It is worth noting that different specifications of \( G \) are generally independent, i.e. consistency of the data with strong process benefits is not sufficient for consistency without process benefits, and vice versa. This means that we can in principle recover lower and upper bounds on \( G \). In practice, to recover these bounds on \( g^i_s \) it suffices to add an objective function that minimizes (maximizes) \( \sum_{i \in \{1,2\}, s \in N} g^i_s \) to conditions (3a)-(3c) in Proposition 1. This analysis applies to each group of households \( N \) separately.

Alternatively, one may also use Proposition 1 to test data consistency with prior values of \( g^i_s \). In this respect, Proposition 1 allows us to test the hypothesis of zero process benefits (\( g^i_s = 0 \)) against the alternative hypothesis that process benefits are strictly positive (\( g^i_s > 0 \)).

\(^{15}\) In our application, we control for possible variation in the strength of our conditions by also reporting their ‘discriminatory power’, following a procedure by Bronars (1987) and Beatty and Crawford (2011).
Remarks. Before moving to our empirical application, two final remarks are in order.

First, Proposition 1 suggests a ‘sharp’ test in the sense that the data can either pass or fail the system of inequalities. We will complement this sharp test with a finer measure of goodness-of-fit. Intuitively, we are not only interested in a binary pass/fail outcome but also in the degree to which our model fits each group of households. To this end, we apply the Houtman and Maks (1985) inconsistency index (HMI). This index searches for the minimal number of households that should be removed from a data set, such that the set of remaining households is consistent with the proposed model.

Second, the conditions in Proposition 1 reflect the main differences between household production models without process benefits and flexible household production models with process benefits. The presence of process benefits affects the construction of preference relations for parents (via $g^i_s$) as well as children (via $\gamma^i_s$). Suppose that a group of households violates condition (3a) (with $g^i_s = 0$) because there is a household $s$, characterized by low wages, in which both parents consume less leisure than expected. One explanation is that their total leisure is underestimated. Process benefits $g^i_s > 0$ may remedy the issue in this first example. Alternatively, suppose that a group of households violates condition (3b) (with $\gamma^i_s = 0$) because there is a household $s$, characterized by high wages, in which both parents supply much child care although their opportunity cost is high. One explanation is that forgone wages reflect both the productivity of parental time and the direct utility gains from process benefits. Marginal process benefits $\gamma^i_s > 0$ thus remedy the issue in this second example.

4 Data

We apply our household production model with process benefits to data from the (Dutch) Longitudinal Internet Studies for the Social sciences (LISS). The LISS panel is a representative sample of Dutch individuals and invites participants to answer monthly Internet surveys. This data contains information on household characteristics, labour income, consumption expenditure and expenditure on children, and time spent by household members on leisure, parental time, market work and household chores. The relevant data is organized in different modules. We use data from the Background Variables, Family and Household, Work and Schooling and Time Use and Consumption modules. The latter was specifically added to investigate consumption and time use within households. The information in this module is collected in three waves: 2009, 2010 and 2012. Cherchye, De Rock, and Vermeulen (2012) constructed their sample from the same modules.

\footnote{Recall that we impose concavity on $Q$.}
\footnote{The panel is based on a true probability sample of households drawn from the population register. Households that could not otherwise participate are provided with a computer and Internet connection. The survey covers a large variety of domains including work, education, income, housing, time use, political views, values and personality.}
that wage information is not directly available, we re-construct wages by dividing the monthly net labour income by the (average) number of hours worked.

Sample selection. Let us first explain how we selected the sample. We selected all households with 1, 2 or 3 children. We dropped all households with missing household head and/or wedded or unwedded partner and all households with members other than parents or children. Moreover, observations with missing values for wages, consumption or time use were withdrawn. Households with unreasonably high or low derived wages were removed from the sample, since the wage plays a crucial role to determine the opportunity cost of child care. Our final sample consists of 436 households.

Table 1: Summary statistics of sample of 436 households, with 1, 2 or 3 children

<table>
<thead>
<tr>
<th>Expenditure (EUR per week)</th>
<th>Husband Mean</th>
<th>SD</th>
<th>Wife Mean</th>
<th>SD</th>
<th>Household Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption expenditures</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>626</td>
<td>213,8</td>
</tr>
<tr>
<td>Expenditure on children</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>144,3</td>
<td>123,7</td>
</tr>
<tr>
<td>Time use (hours per week)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leisure</td>
<td>25,5</td>
<td>13,6</td>
<td>23,9</td>
<td>13,3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Child Care</td>
<td>11,7</td>
<td>9,1</td>
<td>18,1</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market work (incl commuting)</td>
<td>46,8</td>
<td>11,4</td>
<td>30,3</td>
<td>11,7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Household work</td>
<td>9,3</td>
<td>7</td>
<td>17,0</td>
<td>9,4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SocioEconomic variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage rate (EUR per hour)</td>
<td>12,5</td>
<td>3,8</td>
<td>11,6</td>
<td>3,7</td>
<td>2</td>
<td>0,7</td>
</tr>
<tr>
<td>Age</td>
<td>42,4</td>
<td>6,5</td>
<td>40,1</td>
<td>6,6</td>
<td>9,3</td>
<td>5,9</td>
</tr>
<tr>
<td>Number of children</td>
<td>2</td>
<td>0,7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean age children</td>
<td>9,3</td>
<td>5,9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age variation children</td>
<td>1,3</td>
<td>1,2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Higher education</td>
<td>0,38</td>
<td>0,49</td>
<td>0,38</td>
<td>0,49</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 reports summary statistics of (nondurable) expenditure in the household, time use and socio-economic background factors. First, consumption expenditure consists of private consumption (eating at home, food and drinks outside the house, cigarettes and other tobacco products, clothing, personal care products and services, medical care and health costs not covered by insurance, leisure time expenditures, (further) schooling, donations and gifts, personal expenses) and public consumption (mortgage costs, rent, general utilities, transport and means of transport, insurances, alimony and financial support, debts and loans and expenditure on cleaning the house or maintaining the garden). Second, expenditure on children is the sum of abovementioned private expenditure –on

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18A small number of households with 4 up to 7 children are present in the LISS data. However, we did not include those in our final sample. After cleaning the data, too few observations remain to do a sound analysis of process benefits for those household types.
behalf of children— and daycare costs. Finally, we normalize all expenditure to a weekly basis.

Parents allocate time to leisure, parental time, market work and household chores. First of all, individual leisure is directly available from the LISS panel. Second, parental time includes bathing, dressing, playing, reading stories, going with the child to the doctor, taking the child to school or hobbies. On average, the fathers in the sample take care of their children 12 hours a week, while the mothers spend 18 hours a week taking care of their children. Next, market work is the sum of actual labour time and time necessary for traveling from and to work. Finally, household chores include cleaning, laundry, shopping, cooking and gardening. We assume that leisure, parental time, market work and household chores are weakly separable from residual time use categories, as is standard in the literature.

**Households’ observed characteristics.** To perform the analysis, we divide the 436 households into 46 household types based on observed characteristics. Similar procedures were followed by Famulari (1995) and Cherchye and Vermeulen (2008). First, we condition on the education level of the mother. Our sample consists of 269 households with less-educated mothers and 167 households with higher-educated mothers.

Furthermore, we condition on the number of children living in the household. The number of children affects the amount of time and money that parents need to invest. To capture possible economies of scale and non-separabilities between time use and fertility choices, we distinguish between households with exactly one child and households with two or three children. Our sample consists of 103 households with 1 child and 333 households with 2 or 3 children.

Finally, the age of the children in the household allows us to further subdivide the sample. We distinguish between households in which there is a child less than 5 years old and households in which there are no children less than 5 years old. Young children require much more basic parental time, such as feeding, changing diapers and so on. The nature of parental activities with children changes drastically as children grow older, and hence also the appreciation of activities with children might change. We use the mean age of the children in the household to further subdivide the sample until we get 46 subsamples of comparable size. Each final subsample consists of between 9 and 10 households, so that $|N| \in \{9, 10\}$.$^{19}$ While larger subsamples may lead to sharper tests, they also impose stronger homogeneity assumptions on the households belonging to one type.

We obtain 29 household types with less-educated mothers and 17 household types

$^{19}$Subsamples of this size are common in the revealed preference literature (see e.g. Cherchye, De Rock, and Vermeulen (2009, 2011) (8 observations), Demuynck and Verriest (2013) (maximum 8 consecutive observations) and Cosaert (2018) (11 observations)).
with higher-educated mothers. Among these two education categories, there is about the same proportion of households with 1, 2 and 3 children. In total, we obtain 11 household types with one child and 35 types with multiple children. Among the latter, the youngest child is less than 5 years old in 12 household types and more than 5 years old in another 23 types.

**Summary statistics.** Tables 2 and 3 report summary statistics of time use and consumption for specific subtypes—in function of household composition and age. The final four rows, associated with higher education, are of particular interest. Clearly, our data reflect Guryan, Hurst, and Kearny (2008)’s finding that higher-educated parents spend more time with children. On one hand, higher-educated mothers spend slightly less time on other home production activities (cooking, cleaning, ...). At the same time, a higher-educated mother spends considerably more time on parental time, regardless of the fact whether her husband is higher-educated (16.5 versus 20.6 hours per week) or less-educated (16.2 versus 21.6 hours per week). This indicates that the household production of child welfare is significantly different from alternative home production activities.

Table 2: Time use by household type (1=husband, 2=wife), with t indicating household chores

<table>
<thead>
<tr>
<th></th>
<th>#</th>
<th>$l^1$</th>
<th>$l^2$</th>
<th>$h^1$</th>
<th>$h^2$</th>
<th>$m^1$</th>
<th>$m^2$</th>
<th>$t^1$</th>
<th>$t^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>436</td>
<td>25.5</td>
<td>23.9</td>
<td>11.7</td>
<td>18.1</td>
<td>46.8</td>
<td>30.3</td>
<td>9.3</td>
<td>17.0</td>
</tr>
<tr>
<td>1 child</td>
<td>103</td>
<td>24.4</td>
<td>24.4</td>
<td>12.8</td>
<td>19.7</td>
<td>46.5</td>
<td>30.5</td>
<td>9.7</td>
<td>15.2</td>
</tr>
<tr>
<td>2 or 3 children</td>
<td>333</td>
<td>25.9</td>
<td>23.8</td>
<td>11.4</td>
<td>17.6</td>
<td>46.9</td>
<td>30.2</td>
<td>9.2</td>
<td>17.6</td>
</tr>
<tr>
<td>Child younger than 5</td>
<td>153</td>
<td>23.8</td>
<td>19.9</td>
<td>17.3</td>
<td>28.3</td>
<td>45.9</td>
<td>30.0</td>
<td>9.3</td>
<td>14.3</td>
</tr>
<tr>
<td>No child younger than 5</td>
<td>283</td>
<td>26.5</td>
<td>26.1</td>
<td>8.7</td>
<td>12.6</td>
<td>47.3</td>
<td>30.4</td>
<td>9.4</td>
<td>18.4</td>
</tr>
<tr>
<td>Both low edu</td>
<td>197</td>
<td>25.1</td>
<td>24.3</td>
<td>10.5</td>
<td>16.2</td>
<td>47.7</td>
<td>29.2</td>
<td>8.4</td>
<td>17.2</td>
</tr>
<tr>
<td>Only father high edu</td>
<td>73</td>
<td>24.9</td>
<td>25.0</td>
<td>10.3</td>
<td>16.5</td>
<td>48.3</td>
<td>26.4</td>
<td>9.3</td>
<td>19.1</td>
</tr>
<tr>
<td>Only mother high edu</td>
<td>72</td>
<td>26.3</td>
<td>22.0</td>
<td>14.1</td>
<td>21.6</td>
<td>45.2</td>
<td>33.7</td>
<td>11.5</td>
<td>15.1</td>
</tr>
<tr>
<td>Both high edu</td>
<td>94</td>
<td>26.3</td>
<td>23.7</td>
<td>13.5</td>
<td>20.6</td>
<td>45.2</td>
<td>32.9</td>
<td>9.6</td>
<td>16.3</td>
</tr>
</tbody>
</table>

Table 3: Consumption by household type (1=husband, 2=wife)

<table>
<thead>
<tr>
<th></th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$C_p$</th>
<th>$C^K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>12.5</td>
<td>11.6</td>
<td>626</td>
<td>144.3</td>
</tr>
<tr>
<td>1 child</td>
<td>12.5</td>
<td>11.7</td>
<td>652.7</td>
<td>101.1</td>
</tr>
<tr>
<td>2 or 3 children</td>
<td>12.6</td>
<td>11.6</td>
<td>617.8</td>
<td>157.6</td>
</tr>
<tr>
<td>Child younger than 5</td>
<td>12.3</td>
<td>11.6</td>
<td>628.3</td>
<td>169.2</td>
</tr>
<tr>
<td>No child younger than 5</td>
<td>12.7</td>
<td>11.5</td>
<td>624.8</td>
<td>130.8</td>
</tr>
<tr>
<td>Both low edu</td>
<td>11.3</td>
<td>10.7</td>
<td>572.7</td>
<td>120.9</td>
</tr>
<tr>
<td>Only father high edu</td>
<td>14.5</td>
<td>11.1</td>
<td>698.1</td>
<td>178.2</td>
</tr>
<tr>
<td>Only mother high edu</td>
<td>12.0</td>
<td>12.2</td>
<td>632.5</td>
<td>136.0</td>
</tr>
<tr>
<td>Both high edu</td>
<td>14.1</td>
<td>13.2</td>
<td>676.6</td>
<td>173.3</td>
</tr>
</tbody>
</table>
We also condition on the number of children and their age. First, mothers and fathers do not spend less time with children in households with only one child, on average. This observation indicates a trade-off between the number of children and child quality. Alternatively, households with two or three children may rely more on external caregivers. These households also spend more money on children, as reflected in Table 3. Second, the younger the children, the more time parents spend with them. For example, in households with children aged less than 5 years, father and mother care on average 17 and 28 hours per week, respectively. This in contrast to households with older children. In these households, the parents spend only 9 and 13 hours with kids, respectively.

5 Empirical application

Process benefits from child care (represented by $G$ in Section 2) capture the extent to which parents enjoy the time they spend with their children, over and above the effect of parental time on child welfare (represented by $Q$ in Section 2). Our empirical application motivates the study of process benefits and subsequently quantifies these on the basis of Dutch consumption and time use data. First, we test the empirical fit of our household production model with/without process benefits. Second, we recover process benefits from child care.

We present two complementary exercises: one with fully nonparametric process benefits (Subsection 5.1) and one with a parametric specification of process benefits (Subsection 5.2). The parametric specification is in line with the literature on process benefits (see e.g. Graham and Green (1984) and Kerkhofs and Kooreman (2003)) and has been adopted to estimate the extent to which parents enjoy total housework time. Interestingly, it provides us with precisely one parameter to capture the intensity of process benefits.

We also address the question whether the household production model with process benefits is sufficiently powerful to discriminate between observed and randomly simulated choices. In other words, does our methodology capture actual process benefits or rather random noise in the data? This is the subject of Subsection 5.3.

Finally, Subsection 5.4 investigates heterogeneity in process benefits (i) in function of the individual caregiver and (ii) in function of the home production activity under consideration. The former exercise extends the idea that process benefits depend on the number and age of children and the schooling of mothers, with the notion that process benefits also depend on the salary of the caregiver. This is in line with Kimmel and Connelly (2007), who found that parental care has a positive wage elasticity. The latter exercise compares the goodness-of-fit of our main specification with process benefits from parental time (Subsection 5.1) to the goodness-of-fit of alternative specifications with process benefits from household chores.
5.1 Nonparametric recovery of process benefits

We first compare the empirical performance of the household production model without process benefits and the household production model with nonparametric process benefits. We apply both specifications to the 46 household types that we identified in Section 4. Recall that each type consists of 9 or 10 households and that the types differ in terms of the number of children, the age of children and the education of the mother.

The goodness-of-fit criterion that we propose is the Houtman and Maks (1985) inconsistency index (HMI). This index searches for the minimal number of households that should be removed from a data set, such that the set of remaining households is consistent with the proposed model. When HMI is equal to 0, the data under consideration is fully consistent with the proposed model.

Table 4 shows the goodness-of-fit results for the model without and with process benefits, respectively. The result of the ‘sharp’ test is striking. Only 22 per cent of the types are fully consistent with the household production model, while up to 65 per cent of the types are consistent with the household production model with process benefits from parental time. Moreover, the number of households that should be removed to obtain consistency with the household production model is on average 1.26 if process benefits are excluded and only 0.39 if process benefits are taken into account. The model with process benefits is clearly supported. Parents effectively value the time they spend with children, over and above its effect on child welfare. By admitting a highly flexible function for the production of child welfare, our nonparametric characterization controls for the latter.

Table 4: Goodness-of-fit of household production models with and without process benefits.

<table>
<thead>
<tr>
<th></th>
<th>HMI</th>
<th>pass rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min</td>
<td>mean</td>
</tr>
<tr>
<td>Without process benefits</td>
<td>0</td>
<td>1.26</td>
</tr>
<tr>
<td>With nonparametric process benefits</td>
<td>0</td>
<td>0.39</td>
</tr>
</tbody>
</table>

We proceed by quantifying these process benefits from child care. In practice, we compute levels of process benefits \( g_s^i = G(h_s^i) \) and marginal process benefits \( \gamma_s^i = \frac{dG(h_s^i)}{dh_s^i} \) (both per household and per parent) in such way that the linear conditions in Proposition 1 are satisfied. More specifically, we minimize and maximize \( \sum_{s \in N} \sum_i g_s^i \) subject to the abovementioned constraints.\(^{20}\) Let us denote lower and upper bounds on \( G(h) \) by \( G^b(h) \) and \( G^{ab}(h) \), respectively. Table 5 reports summary statistics on parental time, process benefits and average process benefits (per unit of parental time) for fathers and mothers in the 436 households.

\(^{20}\)For types that do not pass the sharp conditions, we allow that a number of households is removed from the exercise. This number cannot exceed the (type-specific) Houtman and Maks inconsistency index that we reported in Table 4.
Table 5: Nonparametric bounds on process benefits, for fathers and mothers in 436 households

<table>
<thead>
<tr>
<th>h</th>
<th>$G_l^b(h)$</th>
<th>$G_u^b(h)$</th>
<th>$G_u^b(h) - G_l^b(h)$</th>
<th>$G_l^b(h)$</th>
<th>$G_u^b(h)$</th>
<th>$G_u^b(h) - G_l^b(h)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>0.03</td>
<td>0.00</td>
<td>0.03</td>
<td>0.00</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>1Q</td>
<td>5.50</td>
<td>0.00</td>
<td>5.00</td>
<td>2.41</td>
<td>0.00</td>
<td>0.98</td>
</tr>
<tr>
<td>mean</td>
<td>14.90</td>
<td>3.59</td>
<td>13.63</td>
<td>10.04</td>
<td>0.23</td>
<td>0.94</td>
</tr>
<tr>
<td>median</td>
<td>12.00</td>
<td>0.46</td>
<td>10.00</td>
<td>7.20</td>
<td>0.06</td>
<td>1.00</td>
</tr>
<tr>
<td>3Q</td>
<td>20.00</td>
<td>3.91</td>
<td>20.00</td>
<td>15.00</td>
<td>0.46</td>
<td>1.00</td>
</tr>
<tr>
<td>max</td>
<td>80.00</td>
<td>51.54</td>
<td>64.00</td>
<td>62.38</td>
<td>0.92</td>
<td>1.00</td>
</tr>
</tbody>
</table>

We find substantial process benefits for a majority of households in the sample. First note that the lower bound on process benefits is equal to zero for more than a quarter of the individuals. This is not surprising since the choices of 22 percent of the household types are consistent with the model without process benefits. Although there is no need to include process benefits into the model for those types, the observed behaviour of those types is still consistent with a strictly positive amount of process benefits.

The average individual perceives between 3.59 and 13.63 hours of parental time as leisure. This corresponds to 23 to 94 percentage of parental time invested in children that is on average perceived as leisure. Moreover, the difference between the upper and lower bound of process benefits per unit of parental time is less than 47 percent for at least a quarter of the individuals. These numbers show that we do find informative bounds on process benefits for a substantial number of individuals. Finally, note that the upper bounds on process benefits are quite large and for more than half of the individuals, it might be that each hour of child care time is valued as leisure. However, in the remainder of the paper, we will focus on the lower bounds of process benefits. Our motivation is twofold. First, lower bounds on process benefits have a straightforward interpretation in terms of small but necessary deviations from the traditional household production model. In addition, a situation in which parental time and pure leisure are perfect substitutes ($G(h) = h$) is highly unlikely. The reason is that all parents in our sample report a strictly positive amount of pure leisure. If parental time is strictly more productive than pure leisure (in terms of child welfare) and, in addition, a perfect substitute for pure leisure (in terms of parental welfare), then strictly positive levels of pure leisure are clearly irrational.

5.2 Parametric recovery of process benefits

The results of Subsection 5.1 are based on a flexible nonparametric characterization of process benefits. We will now test a more specific parameterization of process benefits, adopted from Graham and Green (1984). This specification has been used frequently to model process benefits from aggregate housework. Furthermore, it has the advantage that it associates a single parameter with parental appreciation of child care. This parameter
also facilitates interpersonal comparison of process benefits from parental time.\textsuperscript{21} Process benefits now take the following form

\[ G(h) = h - \frac{1}{T^\delta} \frac{(h)^{1+\delta}}{1 + \delta}. \]  

(7)

This function satisfies the basic properties of monotonicity and concavity. It also satisfies two further limiting conditions: \( \lim_{h \to 0} G'(h) = 1 \) and \( \lim_{h \to T} G'(h) = 0 \). In particular, the first hour of parental time is valued as an hour of leisure whereas each additional hour is valued less so. The extent to which this decreases is determined by \( \delta \), which therefore captures the intensity of the process benefits. If, on one hand, \( \delta \) approaches infinity, then each additional hour of parental time is valued as an hour of leisure, almost up to the point where all available time is spent on parental time. If, on the other hand, \( \delta \) equals 0, then no additional hour of parental time can increase leisure, almost up to the point where no time is spent on child care.

Figure 1: Representation of \( G \) for different values of \( \delta \)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\end{figure}

Figure 1 demonstrates the different shapes of \( G \) conditional on \( \delta \). When \( \delta = 0.05 \), the process benefits derived from parental time time remain close to 0. However, when

\textsuperscript{21}By contrast, the interpersonal comparison of levels \( G(h) \) and average values \( \frac{G(h)}{h} \) is less clear. After all, these process benefits depend on the shape of \( G \) (which may vary over types) and also on the underlying (endogenous) levels of parental time \( h \).
\( \delta = 2 \), the curve associated with \( G(h) \) is quite close to the 45-degree line, indicating that each unit of parental time is (almost) perceived as true leisure.

Technically, we cannot include \( \delta \) as a variable in our conditions in Proposition 1 without making the system of equations highly non-linear. We therefore perform a fine grid search on values for \( \delta \) with step size 0.01.

Although the parametric specification has less flexibility than the nonparametric setting, the parametric specification still fits the data quite well. The pass rate of the sample is 52 percent, indicating that 24 of the 46 household types are fully consistent with the household production model with the abovementioned parametric specification of process benefits. For 39 of the 46 types in our sample, the empirical fit of the parametric specification coincides with the goodness-of-fit of the (more general) nonparametric specification. The average Houtman and Maks inconsistency index is now equal to 0.54, with values ranging from 0 to 2.

Next, we identify the lowest levels of \( \delta \), conditional on the optimal (i.e., minimal) value of the Houtman and Maks inconsistency index. We obtain a lower bound \( \delta^{lb} \) per type. The results are in Table 6. The results are characterized by a large dispersion, as \( \delta^{lb} \) ranges from 0 to 3.59.

Table 6: Lower bounds on parametric process benefits parameter and hours of process benefits for reference male (\( h = 11.7 \) hours) and female (\( h = 18.1 \) hours), for the 46 household types

<table>
<thead>
<tr>
<th></th>
<th>( \delta^{lb} )</th>
<th>( G_{\delta^{lb}}(11.7) )</th>
<th>( G_{\delta^{lb}}(18.1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1Q</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>mean</td>
<td>0.51</td>
<td>3.72</td>
<td>5.58</td>
</tr>
<tr>
<td>median</td>
<td>0.03</td>
<td>1.02</td>
<td>1.40</td>
</tr>
<tr>
<td>3Q</td>
<td>0.33</td>
<td>8.03</td>
<td>11.55</td>
</tr>
<tr>
<td>max</td>
<td>3.59</td>
<td>11.70</td>
<td>18.10</td>
</tr>
</tbody>
</table>

The average value of \( \delta^{lb} \) is equal to 0.51. In addition, Table 6 reports the distribution of process benefits for a male and female with a reference level of parental time. The reference level is the average hours per week that fathers and mothers in the sample spend caring for children, respectively 11.7 and 18.1 hours. Correspondingly, an average male in the sample perceives 3.72 (out of 11.7) hours of child care as leisure while an average female in the sample perceives 5.58 (out of 18.1) hours of parental time as leisure.

5.3 Discriminatory power

It is well-known that revealed preference tests sometimes lack ‘discriminatory power’, i.e. that they are unable to reject the consistency of totally random choices. This may occur when there is insufficient price variation in a data set, or the underlying model is
too weak. In our setting we fully exploit interhousehold heterogeneity in wages, which provides considerable price variation. However, the question remains whether allowing for process benefits does not weaken our testable implications. In other words, we examine whether the model with process benefits is still powerful enough to discriminate between observed choices and randomly simulated choices. Note that different specifications of our model are not necessarily nested. Allowing for higher levels of process benefits can either weaken or strengthen our testable conditions.

To quantify the discriminatory power of our test, we simulate 1,000 random data sets per household type. One such data set consists of random consumption and time use choices for all households of a given type. We simulate random choices by drawing budget shares and time allocations from a uniform distribution on the unit interval. This is similar to Bronars (1987)’s power index, which is based on Becker (1962)’s notion of irrational behaviour. Ideally, the goodness-of-fit of observed choices is much higher than the fit of randomly simulated choices.

| Table 7: Goodness-of-fit of observed versus random choices |
|---------------------------------|-----------------|-----------------|
| Without process benefits        | Pass rate | mean HMI |
| Fit observed data                | 0,22     | 1,26 |
| Fit random data                  | 0,14     | 1,51 |
| Nonparametric process benefits  | Pass rate | mean HMI |
| Fit observed data                | 0,65     | 0,39 |
| Fit random data                  | 0,56     | 0,51 |

In Table 7 we compare the pass rate of the observed types with the average pass rate of the random data. Let us first investigate the performance of the household production model without process benefits. The pass rate of the observed data is 22 percent, as compared to a pass rate of 14 percent for the random data.

We then turn to the performance of the household production model with process benefits. The pass rate of the observed data is now 65 percent, as compared to a pass rate of 56 percent for the random data. This indicates that the ‘predictive success’ (the difference in fit between observed and random data, see e.g. Beatty and Crawford (2011) and Selten (1991)) of the model with nonparametric process benefits is at least as good as the predictive success of the model without process benefits. These results confirm that our extension is reasonable and that our method has ‘empirical bite’, even when preferences, production technologies and process benefits remain unspecified.

### 5.4 Heterogeneity in process benefits

We found empirical support for the hypothesis that parents value directly (as leisure) the time they spend with children, over and above its effect on child welfare. We did the analysis for each type separately, thereby controlling for variation in the age of children,
the number of children and the schooling of mothers. Consequently, each type is characterized by a unique function $G$ that captures the process benefits of households that belong to this type.

We now also admit heterogeneity in process benefits between two households of the same type. Moreover, we allow for heterogeneity in process benefits between two individuals of the same household. Concretely, we take into account that individuals who earn more (resp. less) than the median wage might value parental time differently.\(^{22}\)

At the end of this subsection, we also discuss the possibility that households experience process benefits from chores. We show that process benefits from parental time have separate testable implications compared to process benefits from chores.

**Process benefits depend on the wage of the caregiver.** We first extend our analysis with individual heterogeneity in process benefits. In particular, we let process benefits depend on the wage of individuals. This captures the possibility that parents who earn more than the median wage also enjoy child care more than parents who earn less than the median wage. The rationale behind such extension stems from studies by Kimmel and Connelly (2007) and Guryan, Hurst, and Kearny (2008). These authors used data from the American Time Use Survey (ATUS) to investigate couples’ time allocation to market work, leisure, household chores and parental time. First, Kimmel and Connelly (2007) found a significant and positive wage elasticity for parental time. In addition, Guryan, Hurst, and Kearny (2008) revealed a positive education gradient for parental time. The results suggest that higher-educated individuals\(^{23}\) with higher wages spend more time with children than less-educated with lower wages. In other words, parents with higher opportunity costs allocate more time to child care. One possible explanation of this finding is that higher-educated parents have stronger preferences for spending time with children, over and above their preferences for instantaneous child welfare.\(^{24}\)

Following the nonparametric approach, we analyze each type separately. It is implicitly assumed that there is a (single) household production function for all households of a type. Given that human capital is an important determinant of a parent’s capability to produce child welfare (Chiappori, Salanié, and Weiss (2017)) and hence an important determinant of the parent’s productivity, we control for education. Specifically, all mothers belonging to the same type have the same degree of schooling. An inconvenient feature of

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\(^{22}\)We have also examined the relationship between process benefits and gender of the caregiver. We found no significant effects.

\(^{23}\)The fact that higher-educated parents are also more likely to form couples (Chiappori, Salanié, and Weiss (2017)) further magnifies the discrepancies in child development between children with higher-educated and less-educated parents.

\(^{24}\)Guryan, Hurst, and Kearny (2008) suggested a number of explanations for the positive education gradient: process benefits, but also (i) the possibility that parental time is a luxury good and (ii) the possibility that returns to investment are higher in wealthier households.
this classification is that we cannot exploit variation in schooling per type, to investigate whether process benefits depend on education. By contrast, we can still exploit variation in wages per type. We thus test the hypothesis that parents who earn more (resp. less) than the median wage experience different process benefits. In this way, we use wage variation as a proxy for (remaining) variation in parental income and education that is not captured by our three observed characteristics.

Our following analysis makes a formal distinction between two types of process benefits: function $F(h)$ for individuals who earn more than the median wage and function $G(h)$ for individuals who earn less than the median wage. A priori, we impose no dominance of $F(h)$ over $G(h)$, or vice versa. First, we compute the pass rate and Houtman and Maks inconsistency index. The results are in Table 8. The pass rate, which indicates consistency with the ‘sharp’ test, increases from 65 to 87 per cent by introducing wage based process benefits, successfully describing most types under consideration.

Table 8: Goodness-of-fit of model with nonparametric process benefits, conditional on wage

<table>
<thead>
<tr>
<th>Wage based process benefits</th>
<th>Pass rate</th>
<th>mean HMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fit observed data</td>
<td>0.87</td>
<td>0.13</td>
</tr>
<tr>
<td>Fit random data</td>
<td>0.73</td>
<td>0.29</td>
</tr>
</tbody>
</table>

This increase is not surprising. Adding a second function to capture process benefits (for low wage parents) increases the number of degrees of freedom in our linear programming problems. However, the pass rate of the random data goes up from 56 to 73 percent, which is a smaller increase than for the observed data. This confirms that the model with heterogeneous process benefits is a reasonable extension. An interesting question pertains to the type of heterogeneity: does $G(h)$ effectively dominate $F(h)$? To investigate this, we once more recover bounds on process benefits. Visual inspection shows that the lower bounds on nonparametric process benefits of high wage parents are systematically higher (i.e. not overlapping with the process benefits of low wage parents) in 25 out of 46 types, while the reverse is true in only 5 out of 46 types. These results go in the expected direction.

To further investigate this finding, we also estimate parametric process benefits. This allows us to statistically quantify the difference in process benefits between high and low wage parents. To do so, we perform a two-dimensional grid search on parameter values $\delta_{high}$ for high wage parents and $\delta_{low}$ for low wage parents. This parametric version of the model has a pass rate of 63 percent. The average of $\delta_{high}$ over all types is equal to 0.33, the average $\delta_{low}$ is equal to 0.21. A one sided t-test confirms that the average $\delta_{high}$

Note that there might be multiple combinations of $\delta_{high}$ and $\delta_{low}$ in the grid that have the same Houtman and Maks inconsistency index. Out of those combinations, we have selected the combination that leads to the minimal average process benefits within the household type.
is higher than the average \( \delta_{low} \) at a 10 percent significance level. These findings suggest that high wage parents indeed value parental time more than low wage parents.\(^{26}\)

**Process benefits depend on the type of activity.** The effect on goodness-of-fit is in principle two-fold: \((i)\) process benefits increase the amount of leisure available to the caring parent but also \((ii)\) process benefits generate a gap between the opportunity cost of parental time (i.e. the wage) and its productivity (i.e. its effect on child welfare).

One concern is that the former effect could also be captured by alternative process benefits like cooking and cleaning. Parental time is systematically ranked among the most enjoyable home production activities (Hallberg and Klevmarken (2003)), but previous studies have focused on the process benefits from chores like cooking (rated 6 on a scale of 10) and cleaning (rated 4 on a scale of 10, see Hallberg and Klevmarken (2003)). As a final exercise, we investigate whether these process benefits from other sources provide a satisfactory alternative to the process benefits from child care. Let \( F(t) \) capture process benefits from household chores \( t \) and let \( G(h) \) capture process benefits from parental time \( h \). Given our definition of total leisure \( L^i = l^i + F(t^i) + G(h^i) \), this implies that both \( F(t^i) \) and \( G(h^i) \) are substitutes for leisure.

We obtain the time allocated to household chores (including cooking and cleaning) from the LISS panel. We subsequently compare the goodness-of-fit of the household production model with process benefits from parental time \( (L^i = l^i + G(h^i)) \) and the household production model with process benefits from chores \( (L^i = l^i + F(t^i)) \). The results are in Table 9.

**Table 9: Goodness-of-fit of household production models with and without process benefits.**

<table>
<thead>
<tr>
<th></th>
<th>HMI</th>
<th>pass rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without process benefits</td>
<td>min 0</td>
<td>mean 1,26</td>
</tr>
<tr>
<td>With nonparametric process benefits (parental time)</td>
<td>0,39</td>
<td>0</td>
</tr>
<tr>
<td>With nonparametric process benefits (chores)</td>
<td>0,89</td>
<td>1</td>
</tr>
</tbody>
</table>

Only 32 per cent of the household types are consistent with the idea that individuals may enjoy the time they spend on chores. By contrast, up to 65 per cent of the types are consistent with the idea that individuals may enjoy the time they spend with children. The empirical distribution of HMI for process benefits from chores stochastically dominates the distribution of HMI for process benefits from parental time. This confirms

\(^{26}\text{We used wage variation as a proxy for the remaining variation in parental income and education, over and beyond the variation that we captured by means of age of children, number of children and education of mothers. One may expect the difference in process benefits to be even more outspoken if households with completely different levels of schooling belong to the same type, and hence to the same analysis.}\)
that inconsistencies with the latter model are systematically smaller. This motivates our focus on process benefits from child care once more.

6 Conclusion

We propose and implement empirically a household model with joint production: parental time produces both welfare for the child and leisure for the caring parent. Juster (1985) used the term ‘process benefits’ to capture the leisure component implicit in household production activities.

Our model builds the bridge between two largely separate literatures: (i) the recent literature on home production of child welfare (Cherchye, De Rock, and Vermeulen(2012), Del Boca, Flinn, and Wiswall(2013) and Chiappori, Salanié, and Weiss(2017)) and (ii) the literature on process benefits from housework (Graham and Green(1984), Kerkhofs and Kooreman(2003) and Görtz(2011)). The literature on home production of child welfare typically abstracts from joint production and process benefits. Nonetheless, Browning, Chiappori, and Weiss(2014) stated that parents may also enjoy the time they spend with their children. Our concept of process benefits captures precisely this: the proportion of parental time that is also enjoyable for the caregiver. Household production models without process benefits may understate parental leisure (and welfare) and exaggerate the productivity of parental time inputs. We therefore argue that quantifying process benefits is an important exercise.

To this end, we build a model with flexible household technology (for the production of child welfare) and flexible process benefits from child care. Different from the literature on process benefits from housework, we drop the assumption that the domestic output is marketable. We formulate household preferences as a weighted sum of parental subutility and child welfare. This improves the model’s empirical tractability, while still allowing for interhousehold heterogeneity in the degree of caring for child welfare. We subsequently derive revealed preference conditions to bring this model to the data. The revealed preference approach does not require us to specify the implicit price of child welfare. This price may be endogenous and depend on household preferences, due to non-constant returns to scale in household production and/or the joint production aspect.

In our empirical application, we use data from the Dutch Longitudinal Internet Studies for the Social sciences, which contain information on the household’s consumption, market work, parental time and leisure. We split the sample in 46 cells containing households with similar observed characteristics (in terms of age of children, number of children and schooling of parents) and applied our model to each household type separately. First, we find that household production models that take process benefits from parental time into account outperform similar models without process benefits, even if we control for the discriminatory power of our tests. Second, starting from our model with a fully flexible
household production function, we compute the minimum level of process benefits needed to achieve the best possible goodness-of-fit. We find informative bounds on process benefits for a substantial number of household types. The fully flexible production function without process benefits thus provides an incomplete description of household behaviour. In addition, we find empirical support for the parametric specification of process benefits that was proposed by Graham and Green (1984) and adopted by Kerkhofs and Kooreman (2003) and Görtz (2011). We finally let process benefits vary in function of the wages of individuals. Wages vary between households (including households belonging to the same type) and even within households. We find mild support for the hypothesis that individuals with a higher wage also experience stronger process benefits. This provides a partial explanation for the positive wage elasticity of parental time in Kimmel and Connelly (2007).

We see different avenues for future research.

First, we implemented an elementary model of joint household production. The model is empirically tractable and testable using nonparametric techniques. This simplicity comes at a cost: we excluded for instance the collective nature of decision-making (Cherchye, De Rock, and Vermeulen (2012)), the dynamic aspect of child development (Del Boca, Flinn, and Wiswall (2013)) and the endogenous nature of household formation (Chiappori, Salanié, and Weiss (2017)). None of these extensions poses a fundamental problem to the analysis of joint household production; however, identification of process benefits becomes considerably more complicated when some/all of these aspects are taken into account.

Second, we opened the black box of joint household production by separating the impact of parental time on a child welfare index (in the form of a production function) from the impact of parental time on leisure (in the form of process benefits). More specifically, we included the possibility that parents have direct preferences for particular inputs –over and beyond their role in producing child welfare– and attempted to isolate these preferences. We did not explicitly study the reasons why parents prefer particular inputs over others (in casu: why they value child care time more than money expenditures on children). In fact, there are many reasons why parents may prefer parental time: (i) parents may simply enjoy spending time with children, (ii) parents may internalize social norms regarding the time that they should spend with their children and finally (iii) parents may expect that child care time establishes a stronger relationship and better emotional connection with their kids (thereby improving parental welfare in the long run). We leave the explicit distinction between such explanations for future research. However, it is worth noting that our formulation of process benefits focuses on the first explanation in particular. Indeed, we modeled process benefits from parental time as a substitute for leisure.
References


A Proofs

We prove that the second set of conditions in Proposition 1 is necessary and sufficient for consistency with the household production model with process benefits. Equivalence of the second set of conditions (based on Afriat inequalities) and the third set of conditions (based on ‘cyclical consistency’ or the GARP) follows from Varian (1982), Fostel, Scarf, and Todd (2004) and Ekeland and Galichon (2012). The authors showed that for any finite number of choices, consistency with GARP is equivalent to consistency with the Afriat inequalities.

1. Necessity. We have that each observed \((c_{k_s}^p, c_{k_s}^k, l_{1_s}^1, l_{1_s}^2, h_{h_s}^1, h_{h_s}^2) = \)
arg \max_{c_p, c_h, l_1, l_2, h_1, h_2} \omega_s q + (1 - \omega_s) U(l_1^g, l_2^g, c_p)
{s.t.}
c_p + c_h + w_1^s(h_1^l + l_1^l) + w_2^s(h_2^l + l_2^l) \leq y_s + T \sum_{i=1,2} w_i^s;
\text{with } g^i = G(h^i) \text{ and } q = Q(h_1^1, h_2^2, c_p).

Given concavity, the functions $U$, $Q$ and $G$ are superdifferentiable. Consequently, the objective function is superdifferentiable. Define $\lambda_s$ as the Lagrange multiplier associated with the budget constraint. An optimal interior solution for the above maximization problem must satisfy the following first order conditions:

\begin{align}
(1 - \omega_s) U_{c_p}^v &= \lambda_s; \\
\omega_s Q_{c_h}^v &= \lambda_s; \\
(1 - \omega_s) U_{l_1^l}^v &= \lambda_s w_i^v; \\
\omega_s Q_{h_1}^v + (1 - \omega_s) U_{l_2^l}^v \gamma^s_s &= \lambda_s w_i^v;
\end{align}

for $z_s$, a superderivative of the concave function $z$ defined for $x$ and evaluated at $x_s$. Here is $z$ equal to $U$ or $Q$ and $x_s$ equal to $c^p_s, c_h^s, h_1^v$ or $L_2^v$. The final two equations imply that

$$\omega_s Q_{h_1}^v = \lambda_s w_i^v (1 - \gamma^s_s).$$

Let us denote $U(l_1^l + g_s^1, l_2^l + g_s^2, c_p^v) = u_s$ for each $s$ in $N$. Then concavity of the function $U$ implies

$$u_v - u_s \leq \sum_{i=1,2} U_{L_1^l}^v (l_i^v + g_i^v - l_i^s - g_i^s) + U_{c_p^v} (c_p^v - c_p^s)$$

for $v, s$ in $N$. Combining equations (8) and (10) with the above inequality, we obtain

$$u_v - u_s \leq \frac{\lambda_s}{(1 - \omega_s)} \left[ \sum_{i=1,2} w_i^v (l_i^v + g_i^v - l_i^s - g_i^s) + (c_p^v - c_p^s) \right].$$

Concavity of the function $Q$ implies

$$q_v - q_s \leq \sum_{i=1,2} Q_{h_1}^v (h_i^v - h_i^s) + Q_{c_h^v} (c_h^v - c_h^s)$$

Applying equations (9) and (12) results into the condition

$$q_v - q_s \leq \frac{\lambda_s}{\omega_s} \left[ \sum_{i=1,2} w_i^v (1 - \gamma^s_s) (h_i^v - h_i^s) + (c_h^v - c_h^s) \right].$$

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Renaming \( \eta_s^p = \frac{\lambda_s}{1 - \omega_s} \) and \( \eta_s^k = \frac{\lambda_s}{\omega_s} \), we obtain conditions (a) and (b) in Proposition 1. Finally condition (c) follows from concavity of \( G \) and the assumption that \( G(0) = 0 \).

2. **Sufficiency.** Suppose that the revealed preference conditions hold. One can construct piecewise linear utility functions as follows:

\[
U(l^1 + g^1_s, l^2 + g^2_s, c^p) = \min_{v \in [1, \ldots, N]} \left[ u_v + \eta_s^p \left( \sum_{i=1,2} w_v^i (l^i_s + g^i_s - l^i_v - g^i_v) + (c^p - c^p_v) \right) \right],
\]

\[
Q(h^1_s, h^2_s, c^k) = \min_{v \in [1, \ldots, N]} \left[ q_v + \eta_s^k \left( \sum_{i=1,2} z_v^i (h^i_s - h^i_v) + (c^k - c^k_v) \right) \right],
\]

with

\[
z_v^i = w_v^i (1 - \gamma_v^i).
\]

Moreover, construct a piecewise linear process benefits function:

\[
G(h^i_s) = \min_{j \in \{1,2\}; v \in [1, \ldots, N]} \left[ g_v^j + \gamma_v^j (h^i_s - h^i_v) \right].
\]

Finally, choose \( (1 - \omega_s) = 1/\eta_s^p \) and \( \omega_s = 1/\eta_s^k \).

It is easily verified that the functions \( U \) and \( Q \) are monotone and concave in \( L^1, L^2, c^p \) and \( h^1_s, h^2_s, c^k \), respectively. Similarly, \( G \) is a monotone and concave function in \( h^i_s \). By construction (see Varian (1982) for argumentation), \( U(l^1_s + g^1_s, l^2_s + g^2_s, c^p_s) = u_s, \ Q(h^1_s, h^2_s, c^k_s) = q_s \) and \( G(h^i_s) = g^i_s \).

Consider a combination \((c^p, l^1, l^2, c^k, h^1, h^2)\) such that

\[
(c^p - c^p_s) + (c^k - c^k_s) + \sum_{i=1,2} w^i_s (l^i + h^i - l^i_v - h^i_v) \leq 0.
\]

We now show that

\[
\omega_s Q(h^1, h^2, c^k) + (1 - \omega_s) u(l^1 + G(h^1), l^2 + G(h^2), c^p) \\
\leq \omega_s Q(h^1_s, h^2_s, c^k_s) + (1 - \omega_s) u(l^1_s + G(h^1_s), l^2_s + G(h^2_s), c^p_s)
\]
by means of the following derivation:

$$\omega_s Q(h^1, h^2, c^k) + (1 - \omega_s) U(l^1 + G(h^1), l^2 + G(h^2), c^p)$$

$$\leq \omega_s \left[ q_s + \eta^k_s \left( \sum_{i=1,2} z^i_s (h^i - h^i_s) + (c^k - c^k_s) \right) \right] + (1 - \omega_s) \left[ u_s + \eta^p_s \left( \sum_{i=1,2} w^i_s (l^i + g^i_s + \gamma^i_s (h^i - h^i_s) - l^i_s - g^i_s) + (c^p - c^p_s) \right) \right]$$

$$\leq \omega_s q_s + (1 - \omega_s) u_s + (c^p - c^p_s) + (c^k - c^k_s) + \sum_{i=1,2} w^i_s (l^i - l^i_s + \gamma^i_s (h^i - h^i_s)) + \sum_{i=1,2} z^i_s (h^i - h^i_s)$$

The first inequality follows by definition of $U$, $Q$ and $G$, the second by choosing $(1 - \omega_s) = 1/\eta^p_s$ and $\omega_s = 1/\eta^k_s$.

$$\leq \omega_s q_s + (1 - \omega_s) u_s.$$ 

The final inequality stems from the fact that

$$(c^p - c^p_s) + (c^k - c^k_s) + \sum_{i=1,2} w^i_s (l^i + h^i - l^i_s - h^i_s) \leq 0.$$ 

All of this shows that $(c^p, c^k, l^1_s, l^2_s, h^1_s, h^2_s)$ maximizes

$$\omega_s Q(h^1, h^2, c^k) + (1 - \omega_s) U(l^1 + G(h^1), l^2 + G(h^2), c^p)$$

under the given constraints. Consequently, we constructed functions $U, Q$ and $G$ that satisfy the requested properties. Therefore, data set $S$ is consistent with the household production model with process benefits.

B Houtman and Maks inconsistency index

Consider a data set $S = (c^p, c^k, l^i_s, h^i_s, w^i_s)_{i \in \{1,2\}, s \in N}$. The Houtman and Maks inconsistency (HMI) index computes the minimum number of households to be removed from
S such that $S$ is consistent with the household production model with process benefits from parental time.

Concretely,

$$HMI = \min \sum_{s \in N} b_s$$

s.t.

$$(c^p_s - c^p_v) + \sum_{i=1,2} w^i_s (l^i_s + g^i_s - l^i_v - g^i_v) - M_s b_s \leq M_s x_{sv};$$

$$x_{st} + x_{tv} \leq 1 + x_{sv};$$

$$(c^p_v - c^p_s) + \sum_{i=1,2} w^i_v (l^i_v + g^i_v - l^i_s - g^i_s) - M_v b_v \leq M_v (1 - x_{sv});$$

$$(c^k_s - c^k_v) + \sum_{i=1,2} w^i_s (1 - \gamma^i_s)(h^i_s - h^i_v) - N_s b_s \leq N_s y_{sv};$$

$$y_{st} + y_{tv} \leq 1 + y_{sv};$$

$$(c^k_v - c^k_s) + \sum_{i=1,2} w^i_v (1 - \gamma^i_v)(h^i_v - h^i_s) - N_v b_v \leq N_v (1 - y_{sv});$$

$$g^i_v - g^i_s \leq \gamma^i_s (h^i_v - h^i_s);$$

$$g^i_s \geq \gamma^i_s h^i_s;$$

with $M_s$ and $N_s$ arbitrary numbers such that $M_s, N_s \geq y_{sv} + T \sum_{i=1,2} w^i_s$, and with the following variables (for all $s, t, v \in N$ and $i, j = 1, 2$):

- binary preference relations $x_{sv} \in \{0, 1\}$ and $y_{sv} \in \{0, 1\}$;
- binary variables $b_s \in \{0, 1\}$, with $b_s = 1$ if household $s$ must be eliminated from $S$ and $b_s = 0$ otherwise,
- process benefits $g^i_s$,
- marginal process benefits $\gamma^i_s$. 

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