New evidence on the synchronisation between the US business and financial cycles

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September 26, 2018

Abstract

The aim of this paper is to investigate the synchronisation of US financial (FC) and business cycles (BC) via Cross-Recurrence Quantification Analysis (CRQA). Both cycles are extracted using recent Markov switching techniques, where turning points are identified based on the estimated smoothed probabilities of the models. In the effort to explore dynamic changes occurred in the financial world, we also report results from the sub-couples between business cycle and the single financial cycle components. Besides the combined methodological approach we propose, the contribution of the paper relies upon the use of smoothed probabilities as input variables for the estimation of the cross-RQA measures. The empirical findings on long US data indicate a leading behaviour of BC during the Great Moderation era until the end of the 90s and a progressive leading of the FC evolution over the BC in the aftermath of deregulation. The implementation of the cross-Recurrence Rate shows the presence of diverse state of co-visitation between the triplet of credit, S&P500 and residential property and the BC. Worth noticing the similarity of the cross-Recurrence Rate for the couples of BC-FC and BC-S&P500.

Keywords — Business Cycles, Financial Cycles, Synchronisation, Markov-Switching, Recurrence Quantification Analysis

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1 Introduction

In recent years, the subject of cross-country and cross-market linkages has gained increased interest from policymakers seeking to accurately evaluate systemic risk, design better policy provisions and safeguard welfare and financial stability. In particular, the connectivity between the financial world and the real economy constitute a long-lasting puzzle. Recent evidence showed that this relationship is fairly dynamic and volatile over the years. The 2007-2009 financial crisis prompted an enormous debate on the spillovers transmitted from the financial sector to the real economy. Ultimately, both researchers and decision-makers have turned their focus on this major issue since the consequences of this event affected substantially real economic activity (Reinhart and Rogoff, 2009). This recent episode of financial distress and the subsequent deterioration of economic performance led both academia and decision-makers to revise their understanding of how economies grow, which factors cause instability and which are the necessary means and resources able to facilitate a proactive and discretionary for mitigating observed imbalances.

To further investigate this interesting bipolar between financial performance and real activity, we suggest the combined application of the Markov Switching (MS) modelling and the Cross Recurrence Plots Analysis (CRQA) to a long set of US quarterly data for the real GDP representing the evolution of the business cycle and the triplet of credit-to-GDP, residential property and stock exchange prices approximating the financial cycle. Markov switching models have emerged as a leading approach for the detection and dating of business cycle turning points. Since the seminal papers by Hamilton (1989, 1990), they have been used actively in econometrics to model various economic and financial time series. To cite some, MS models have been applied to exchange rate swings (Engel and Hamilton, 1990; Engel, 1994), stock market returns (Pagan and Schwert, 1990; Hamilton and Lin, 1996), interest rates (Garcia and Perron, 1996; Dahlquist and Gray, 2000; Ang and Bekaert, 2002), asset returns (Kim et al., 1998), and asymmetries over the business cycle (Krolzig, 1997; Chauvet and Hamilton, 2006; Billio and Cavicchioli, 2014). There are few published papers that apply recurrence plot techniques to economic issues, the notable exceptions being Zbilut (2005); Kyrtou and Vorlow (2005); Crowley (2008); Crowley and Schultz (2010); Karagianni and Kyrtou (2011) and Addo et al. (2013). Recurrence plots are not affected by non-stationarity, recognise remarkably path-dependent dynamics and turning points, while they provide measures for phase transition quantification. All the above together with the fact that RQA is a model-free tool and does not depend on the distributional characteristics of the data explain why it looks like a promising approach for analysing economic time series.

In the first step of analysis, through the application of the MS approach the phases of the cycles are determined in a data-driven manner. To do so, we exploit recent results for estimation and filtering of multivariate MS VAR models as proposed in Cavicchioli (2014b,a, 2017). The advantages of using model-based analysis to identify regime shifts
of the stochastic processes and extract the cycles with respect to the commonly used non-parametric filters are that (1) we do not need a priori assumptions on the lengths of the cycle (2) they rely on diagnostic statistics and tests to evaluate accuracy and reliability of the model. CRQA measures will then be evaluated on the resulting smoothed probabilities providing information about the possibility of getting lagging, leading or synchronised behaviour between the business and the financial cycles. In the attempt to explore the evolution occurred in the financial world and capture its most ”susceptible” parts that may be of great importance for the stability of the system, we also report results from the sub-couples between the business cycle and the financial cycle components.

To the best of our knowledge this is the first study in the business cycle literature trying to combine mainstream econometric tools and recurrence analytical measures. The suggested scheme presents advantages over simple model-driven methods, requiring that strict assumptions on the input data must be met, and enhances our understanding on the nonlinear, time-varying, dynamic and complex nature of the underlying series and their interrelationships.

2 Financial performance and real activity: a new world of interactions

As most authors suggest, financial booms and busts are not that frequent, but their magnitude is instantly transmitted into real activity. Early discussion of this issue can be found in Borio et al. (2001); Hamilton and Lin (1996); Kiyotaki and Moore (1997). Several studies using long historical data and focusing on the most active components of the financial cycle, argue over the financial deregulation and the link between the ”floating” leverage/credit and the actual output level.

Geanakoplos (2010b,a) highlights the importance of the leverage cycle as a driving factor of the financial sector. He points to the collateral rates attached to business or household loans as guarantee of repayment and relates them not only to the normative supply-demand forces, but also to the animal spirits prevailing the credit market transactions. In this line, when greed and irrationality enter the credit cycle, collateral rates are underestimated, and only specific policy interventions, aiming at better monitoring and controlling the leverage in the economy, can protect the system from collapsing.

Schularick and Taylor (2012), support that credit is a triggering factor, and not only an amplifier of modern financial crises. Using data from 1870 to 2008 regarding 14 developed countries, conclusions are drawn on the evolution of main financial variables and their impact into real economy. The WW2 is identified as a structural point that alters significantly their dynamics. Although prior to WW2 money, credit, loans and assets remained stable, a persistent upward trend dominates the postwar era, except the monetary aggregates. In terms of how those variables contributed to the unfolding of financial crisis they suggest that excessive credit boom in the form of increased and leveraged aggregate
bank loans burst the risk of financial turmoil. The observed interactions have found fruitful ground within the large and complex financial systems that in turns led to a more vulnerable economic environment.

Gourinchas and Obstfeld (2012) claim that high levels of leverage and currency appreciation were the main factors of the financial crises occurred during 1973 - 2010. To reach this conclusion, they perform an event-study analysis of past currency, banking, sovereign-debt crises, and the 2008 global financial crisis, for both emerging and advanced economies. Their results suggest that for all types of crises, advanced economies tend to be more resilient and recover faster than the emerging countries, except for the banking and the 2008 crisis where advanced economies were mostly hurt. They built up an early-warning indicator framework assigning probabilities of default relative to the performance of some variables from the preliminary event-study analysis. Their results demonstrate that, mainly, domestic credit, exchange rate and fiscal variables can be considered as serious precursors of major events.

Jorda et al. (2015) emphasise the role of credit growth as a detrimental part of financial stability together with the co-existence of high leverage and asset price bubbles. On the basis of a sample of 17 advanced countries from 1870 to 2013, it is shown that recessions caused by a mixture of asset-driven bubbles and credit expansion tend to be more severe and last longer. In search of the roots of recessions, they found that after WW2 financial crises occurred mostly due to bubbles in equities and housing prices, while prior to WW2 other assets were also involved. When normal recessions are taken into account, it seems that pre-WW2 incidences were not related to asset bubbles, whereas half of post-WW2 events were led by equity bubbles. As it emerged, housing prices bubbles mostly contributed to severe episodes of economic distress. The role of housing sector (both in prices and debt) in the burst of financial distortions has been pointed out by Ryoo (2016) as well, who inspired by Minsky’s financial instability hypothesis that justifies bubble explosion to the excessive prosperity causing significant economic costs.

Aikman et al. (2015) point out the importance of the credit cycle, and its relation to economic output, and draw some interesting policy guidelines to achieve a better macro-prudential oversight of the system. First, they empirically validate, that credit (expressed as the ratio of bank lending to GDP) is the main driver of banking crises for UK and US, and usually credit booms are followed by deeper and longer recessions. Then, they adopt a game-theoretic approach and develop different mechanisms that fuel financial cycles run up.

Albuquerque et al. (2016) explore how the dynamics of money and credit affect the performance of US economic output. Starting with a mainstream methodological scheme and different forecasting techniques, they examine the predictive ability of a broad list of credit variables, money spreads and aggregates, over the real GDP, by controlling at the same time for the effect of the 2007-2009 financial crisis. Their results lead to the conclusion that prior to the financial crisis term spread had a dominant predictive
content for the GDP growth. In the post-crisis regime this link is altered, while alternative variables such as credit and money seem to gain the game of predictability of the US economic activity.

Looking at the specific properties of financial cycles, a significant number of studies can be reported. *Claessens et al.* (2011), using the *Burns and Mitchell* (1947) dating technique for business cycles, construct financial cycles from variables capturing the effect of financial intermediation (i.e. credit claims, housing and equity prices), and analyse their characteristics during the recovery and contraction phases (frequency, duration, amplitude, and slope). They investigate both the cross-cycle and the cross-country synchronisation of these financial indicators, for 21 advanced OECD economies from 1960 Q1 to 2007 Q4. Some of their key results suggest that: i) expansions last longer than contractions, ii) there exist a direct linkage between the credit and the housing cycles for most countries, iii) the most synchronised cycles, at a cross-country level, are those of credit and equity.

In contrast to other studies that analyse financial cycles relying on a single variable, *Ng* (2011) compares the performance of three financial cycle measures: i) a financial conditions index, ii) a financial cycle measure, and iii) the credit-to-GDP gap. The first two measures are constructed using factor model techniques for a wide range of variables, while the last one extracts the cycle of the credit-to-GDP ratio using the Hodrik-Prescott filter. Comparative analysis demonstrates that, for the short-term, all three measures, more or less, contain the same predictive content over output performance, while for longer horizons (more than one year) the only indicator than exert a slight predictive power is the cyclical component of the credit-to-GDP.

*Borio* (2014), provides a detailed analysis of the stylised facts of the financial cycles envisaged globally. *Miranda-Agrippino and Rey* (2015), exploiting methods from principal component analysis, try to identify a common factor across different asset categories and geographical regions (i.e. global or regional). The estimated common financial factor is then introduced into a Bayesian VAR setting that examines the impact of a US policy rate shock into various macroeconomic, financial and banking indicators, both in the US and abroad. They conclude that an important part of the variability of risky assets is affected by this single global common factor, acting as an aggregate volatility and risk appetite index, while monetary policy in the US is still a driving force for other economies. In the same spirit, *Rey* (2015) discusses the relationship between a global financial cycle with other measures of market volatility, leverage and capital mobility.

In the aftermath of the recent financial crisis, much attention has been paid on the regulation of the banks’ balance-sheets, not only in the micro-prudential side but also in a system-wide perspective (i.e. macro-prudential). This same event also revised the notion of business cycle and more generally the way that post-2007 macroeconomic sustainability was evaluated, after a long period where emphasis has been given to the stabilisation of output and inflation and the control of labor market frictions.
3 Methodology

Spurred on the studies of Gourinchas and Obstfeld (2012); Schularick and Taylor (2012); Borio (2014), we built up a methodological scheme that combines i) a novel dating technique of the financial cycle and ii) a pattern recognition tool (namely Recurrence Plot) to evaluate the convolution between various synthetic financial measures and macroeconomic indicators. Both financial and business cycles are extracted using recent MS techniques, where turning points are identified based on the estimated smoothed probabilities of the models. We will then explore the leading or lagging behaviours of the business and financial cycles as well as their degree of synchronisation via the cross recurrence quantification analysis toolkit.

3.1 Markov switching VAR: model and estimation

Markov-switching techniques have been applied in the study of financial and business cycles: Hamilton (1989); Durland and McCurdy (1994); Diebold and Rudebusch (1996); Hamilton and Lin (1996); Kim and Nelson (1998); Filardo and Gordon (1998); Kim and Nelson (1999); Harding and Pagan (2002); Artis et al. (2004); Jiménez-Rodríguez et al. (2013); Skare and Stjepanovic (2016); Duprey and Klaus (2017)

Let $y_t$ be an $(K \times 1)$ vector of observed endogenous variables and let $Y_t$ denote a vector containing all observations obtained through date $t$, that is, $Y_t = (y'_t, y'_{t-1}, \ldots)'$. To take in account changes in the process $(y_t)$, we assume that it is governed by an unobserved random variable, called state or regime, which is discrete-valued. The simplest time series model for a discrete-valued random variable is a Markov chain.

Let $(s_t)_{t \geq 0}$ be an $M$-state, homogeneous, irreducible and ergodic Markov chain. Irreducibility means that all states have a nonzero probability of occurring in the steady state, that is, the unconditional probabilities $\pi_i = Pr(s_t = i)$ are positive for $i = 1, \ldots, M$. Set $\pi = (\pi_1, \ldots, \pi_M)'$. Let $P = (p_{ij})_{i,j=1,\ldots,M}$ denote the transition probability matrix of the chain, where $p_{ij}$ gives the probability that the state $s_{t-1} = i$ will be followed by the state $s_t = j$. As usual, suppose that the probability that $s_t$ equals some particular value depends on the past only through the most recent value $s_{t-1}$. Ergodicity of the process $(s_t)$ means that exactly one of the eigenvalues of $P$ is unity and all other eigenvalues are inside the unit circle.

From Hamilton (1994), a useful representation for a Markov chain is obtained by letting $\xi_t$ denote a random $(M \times 1)$ vector whose $j$th element is equal to unity if $s_t = j$ and zero otherwise. We see that the conditional expectation of $\xi_{t+1}$ satisfies the property $E(\xi_{t+1} | \xi_t, \xi_{t-1}, \ldots) = E(\xi_{t+1} | \xi_t) = P'\xi_t$. This implies that it is possible to express a Markov chain in a stable VAR(1) form

$$\xi_{t+1} = P'\xi_t + v_{t+1}$$

where the innovation $v_{t+1} = \xi_{t+1} - E(\xi_{t+1} | \xi_t)$ is a zero mean martingale difference sequence. The vector $\pi$ of ergodic probabilities can be described as the unconditional
expectation of $\xi_t$, that is, $\pi = E(\xi_t)$.

As shown in Hamilton (1994) and Krolzig (1997) (see also Theorem 2.1 in Cavicchioli (2014a)), an optimal inference and forecast for each date $t$ in the sample can be found by iterating on the following pair of recursive formulae

\[
\begin{align*}
\hat{\xi}_{t|t} &= \frac{\hat{\xi}_{t|t-1} \odot \eta_t}{i_M(\hat{\xi}_{t|t-1} \odot \eta_t)} & \hat{\xi}_{t+1|t} &= P\hat{\xi}_{t|t}
\end{align*}
\]

where the symbol $\odot$ denotes the element-by-element multiplication. Smoothed inferences can be calculated using an algorithm which can be written, in vector form, as

\[
\hat{\xi}_{t|T} = \hat{\xi}_{t|t} \odot \{P[\hat{\xi}_{t+1|T}(\div)\hat{\xi}_{t+1|t}]\}
\]

where the symbol $(\div)$ denotes element-by-element division.

Let now $y_t$ be a $(K \times 1)$ random vector which follows an $M$-regime Markov-switching (MS) VAR($p$) process, with $p > 0$:

\[
y_t + \sum_{i=1}^{p} \Phi_{s,i} y_{t-i} = \nu_s + \Sigma_s u_t
\]

where $u_t \sim \text{NID}(0, I_K)$, and $\Phi_{m,i}$ is a $(K \times K)$ matrix for every $m = 1, \ldots, M$.

Define the following matrices:

\[
\Phi_m = (\Phi_{m,1} \cdots \Phi_{m,p}) \quad A_m = (A_m(i, j)) \quad B_m = (B_m(i))
\]

\[
C_m = (C_m(i)) \quad S_m = (\sum_{t=1}^{T} \hat{\xi}_{mt|T}) \quad T_m = (-\sum_{t=1}^{T} y_t \hat{\xi}_{mt|T})
\]

where

\[
A_m(i, j) = \sum_{t=1}^{T} y_{t-i}y'_{t-j}\hat{\xi}_{mt|T} \quad B_m(i) = \sum_{t=1}^{T} y'_{t-i}\hat{\xi}_{mt|T}
\]

and

\[
C_m(i) = -\sum_{t=1}^{T} y_t y'_{t-i}\hat{\xi}_{mt|T}
\]

for $m = 1, \ldots, M$ and $i = 1, \ldots, p$. Let us consider the matrices

\[
X_m = B'_{m}B_m - S_mA_m \quad W_m = T_mB_m - S_mC_m
\]

for every $m = 1, \ldots, M$. Then $X_m = (X_m(i, j))$ and $W_m = (W_m(i))$, where

\[
X_m(i, j) = \left[\sum_{t=1}^{T} y_{t-i}\hat{\xi}_{mt|T}\right] \left[\sum_{t=1}^{T} y'_{t-j}\hat{\xi}_{mt|T}\right] - \left[\sum_{t=1}^{T} \hat{\xi}_{mt|T}\right] \left[\sum_{t=1}^{T} y_{t-i}y'_{t-j}\hat{\xi}_{mt|T}\right]
\]

\[
W_m(i) = -\left[\sum_{t=1}^{T} y_t \hat{\xi}_{mt|T}\right] \left[\sum_{t=1}^{T} y'_{t-i}\hat{\xi}_{mt|T}\right] + \left[\sum_{t=1}^{T} \hat{\xi}_{mt|T}\right] \left[\sum_{t=1}^{T} y_t y'_{t-i}\hat{\xi}_{mt|T}\right]
\]

for every $m = 1, \ldots, M$ and $i, j = 1, \ldots, p$. 

7
As shown in Cavicchioli (2014a), Theorem 4.1, the maximum likelihood estimates of the unknown parameters from the above MS VAR\((p)\) model, \(p > 0\), are given by

\[
\hat{\nu}_m = S_m^{-1}(W_m X_m^{-1}B_m - T_m)
\]

(1.4)  

\[
\hat{\Phi}_m = W_m X_m^{-1}
\]

\[
\hat{\Omega}_m = S_m^{-1} \left[ \sum_{t=1}^{T} (y_{t} - \hat{\nu}_m) + \sum_{i=1}^{p} \hat{\Phi}_{m,i} y_{t-i} \right] (y_{t} - \hat{\nu}_m + \sum_{i=1}^{p} \hat{\Phi}_{m,i} y_{t-i}) \hat{\xi}_{mt[T]}
\]

and

\[
\hat{p}_{ij} = \left[ \sum_{t=1}^{T} \hat{\xi}_{i,t-1[T]} \right]^{-1} \left[ \sum_{t=1}^{T} \hat{\xi}_{ij,t[T]} \right]
\]

where \(\hat{\xi}_{ij,t[T]} = Pr(s_t = j, s_{t-1} = i|Y_T; \hat{\lambda})\).

The Algorithm. We use a new iterative algorithm for getting ML estimates of model parameters, say \(\theta\), as shown in Cavicchioli (2014a). The implementation of this algorithm does not require the numerical maximization of the log-likelihood function of the model. So it improves the EM algorithm. Start the procedure with an arbitrary guess for the parameter vector, say \(\theta^{(0)}\). For this guess, we calculate the smoothed probabilities \(\xi_{i,t[T]}^{(0)}\) from (1.1) and (1.2). Then one can calculate the magnitudes on the right sides of (1.4) with \(\theta^{(0)}\) in place of \(\hat{\theta}\). The left sides of (1.4) then produce new estimates \(\theta^{(1)}\) of \(\theta\). This estimate can be used to compute the smoothed probabilities \(\xi_{i,t[T]}^{(1)}\) and re-calculate the expressions on the right sides of (1.4). One goes on like this until the change between \(\theta^{(l+1)}\) and \(\theta^{(l)}\) is smaller than some specified convergence criterion. This fixed-point algorithm has typically linear convergence.

3.2 Cross Recurrence Analysis

Analysing complex systems with unknown equations of motions and governed by non-linear and non-stationary dynamics is always a demanding task. In this spirit, recurrence analysis is a robust method that can provide useful insights into the evolution of such systems and more recently has found considerable appeal in social research, notably in economics (Kyrtou and Vorlow, 2005; Zbilut, 2005; Strozzi et al., 2007; Crowley, 2008; Karagianni and Kyrtou, 2011; Goswami et al., 2012; Addo et al., 2013). Recurrence Quantification Analysis (RQA) is an exploratory data framework that evaluates the existence of repeating patterns (i.e. recurrences) and is based on the computation of an \(N \times M\) recurrence matrix \((CR_{i,j})\) between each point of the trajectories \(x_i\) and \(y_j\) with \((i, j = 1, \ldots, N)\):

\[
CR_{i,j} = \Theta(\varepsilon - \|x_i - y_j\|)
\]

where \(\varepsilon\) is the threshold of distance, \(\Theta(\cdot)\) is the Heaviside step function and \(\|\cdot\|\) is the norm (e.g. Euclidean norm). Whenever this distance is less than or equal to the pre-determined cutoff threshold \(\varepsilon\), a local (cross-) recurrence occurs and is represented graphically through
the (Cross-) Recurrence Plot (CRP or RP); a tool developed by Eckmann et al. (1987). Both in the univariate (auto-recurrences) and the bivariate (cross-recurrences) case, this matrix contains ones (black dots in the plots), when the $i$ and $j$ points are close to each other, and zeros (white areas in the plots) otherwise. The resulting typology of the CRPs reveals the shared activity of the underlying signals by evaluating how their dynamics unfold over time in terms of periodicity, stochasticity, homogeneity and stationarity. For random systems, recurrences occur by chance and thus the CRP consist of several dots that are not forming any strong diagonal or vertical pattern. On the other hand, when two systems share a high degree of recurrent structures, the CRP can clearly depict their dynamic properties. More specifically, when the systems are closely coupled, meaning that they follow similar paths, the CRP will be filled with long diagonal lines. Additionally, strong vertical lines will be apparent when this recurrent behaviour is persistent over time.

The first step to design and quantify the patterns of the RPs is to reconstruct the phase-space using delay coordinates and embed the time-series so as to recreate the full information of the underlying system that is compressed into a single observable. Popular techniques to determine the optimal values of time delay ($\tau$) and embedding dimension ($m$) is the ACF or mutual information and false nearest neighbours, respectively. According to Takens (1981), this leads to the reconstruction of the full trajectories of the system. Taking into account the specific characteristics of our application and since the original series are not used as inputs in the recurrence plot analysis, the goal of the reconstruction is not met. In this line, the embedding parameters $\tau$ and $m$ are set to 1.

Apart from $\tau$ and $m$, another crucial RQA parameter is the threshold ($\epsilon$). Although, several rules-of-thumb have been proposed, the selection of threshold strongly depends on the nature of data. In our case, indication about the optimal value of $\epsilon = 0.1$ is provided by plotting the histogram of distances between probabilities in the cross recurrence plot.

Once the CRP is obtained, various measures can be derived to quantify the shared dynamics of the time series. Since, our application concentrates on the interrelationship between the phases of financial and business cycles, as represented by the estimated probabilities (for recession or growth) via Markov-Switching models, we are focusing on two features that can be extracted from the CRPs: i) the LOS, which identifies time-shifts and ii) the cross-Recurrence Rate, which quantifies the probability of similar dynamics. The cross-RR represents the density of recurrent points in the CRP, at $t$, and is defined as

$$RR(t) = \frac{1}{N-t} \sum_{l=1}^{N-t} lP_l(l)$$

where $P_l(l)$ is the distribution of the diagonal lines of length $l$ and is estimated in a rolling-windows setting. High values of $RR(t)$ indicate high probability of recurrent states between both systems. In our application, each window is designed to have a width of 20 observations (5 years of data) and is moved by a 1-point shift.

Moreover, the CRP contains information about the synchronisation of the two corresponding time series through the fitting of the non-parametric function of the Line of
Synchronisation (LOS\textsuperscript{1}). When the dynamics between the trajectories are fairly synchronised then the LOS forms a straight line in the direction of the main diagonal of the CRP, while for systems that their co-evolution is characterised by leading/lagging dynamics the LOS is a distorted, discontinuous or bowed to the one or the other side of the CRP.

The CRQA plots, parameters and measures have been computed using MATLAB routines from the Cross Recurrence Plot Toolbox, Version 5.15 (R28.4).

4 Data and Results

This study aims to analyse the nexus between financial, and economic outcomes in the US economy since 1975. To achieve this goal, we use a long historical dataset that spans from Q1:1975 to Q3:2017 (168 observations). Following the rationale of Drehmann et al. (2012), we use credit-to-GDP, residential property prices and S&P500 variables for the financial cycle, and real GDP as a proxy for the business cycle\textsuperscript{2}. Moreover, we are focusing on cross-country comparisons to get a more complete picture of how the differences in between various economic settings may or may not influence our results.

As discussed in the previous sections, business and financial cycles constitute an intriguing topic for academics and practitioners. Concerning the construction/inference of the cyclical components for both business and financial cycles, previous research has highlighted the need of specialised methods. Additionally, as reported, the dynamic nature of their interrelationship requires sophisticated tools able to capture specific temporal characteristics that are not evident using only linear methods.

4.1 Markov switching

In particular, we extract business cycle from quarterly time series of real GDP (in log difference) and financial cycle from the system including log returns (percentage) of credit-to-GDP, residential property prices and S&P500 index. First we extract the business and financial cycles from the univariate and trivariate system, respectively, then we verify whether the cycles extracted from single financial variables share common characteristics with the one extracted from the trivariate model.

Before performing estimation, we test for simultaneous selection of number of regimes and autoregressive order using techniques based on stable linear VARMA representation of the MS VAR model (see Cavicchioli (2014b)). It turns out that for the considered US time series two regimes are required together with 1 lag in the autoregressive coefficient matrices. This is in line with other empirical studies on US data (e.g., Billio and Cavicchioli (2014)).

\textsuperscript{1}Detailed description of the algorithm that computes LOS can be found in Marwan et al. (2002).

\textsuperscript{2}The real GDP and the S&P500 data were obtained from FRED, the St. Louis Fed's economic data website. The Credit-to-GDP ratio and Residential Property prices are based on Bank of International Settlements (BIS) statistics. All series are seasonally adjusted.
Then maximum likelihood estimation is conducted extracting first filtered and smoothed probabilities as in (1.1) and (1.2), respectively, and then using closed-form formulae for estimating parameters as in (1.4) of Section 3.1. The parameters $\nu_{st}$, $\Phi_{st}$ and $\Sigma_{st}$ are defined in the MS VAR specification in Equation (1.3).

Parameter estimates of the Markov switching VAR model where the vector $y$ is equal to credit-to-GDP, residential property prices and S&P500 (taken in this order) in percentage log-returns are reported in Table 1. Parameter estimates of the Markov switching AR models in the single variables: credit-to-GDP, residential property prices, S&P500 (taken in percentage log-returns) and real GDP (in log difference) are reported in Table 2.

<table>
<thead>
<tr>
<th>Trivariate system</th>
<th>1st regime</th>
<th>2nd regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_{st}$</td>
<td>$0.0520^*$</td>
<td>$0.0325^*$</td>
</tr>
<tr>
<td></td>
<td>$(.0024)$</td>
<td>$(.0003)$</td>
</tr>
<tr>
<td></td>
<td>$0.1944^*$</td>
<td>$0.0047^*$</td>
</tr>
<tr>
<td></td>
<td>$(.0036)$</td>
<td>$(.0006)$</td>
</tr>
<tr>
<td></td>
<td>$-3.3715^*$</td>
<td>$4.4637^*$</td>
</tr>
<tr>
<td></td>
<td>$(.1178)$</td>
<td>$(.0251)$</td>
</tr>
<tr>
<td>$\Phi_{st}$</td>
<td>$[-0.5509^* 0.0258^* -0.0265^*]$</td>
<td>$[-0.4404^* -0.1305^* 0.0149^*]$</td>
</tr>
<tr>
<td></td>
<td>$(.0048)  (.0049)  (.0012)$</td>
<td>$(.0011)  (.0011)  (.0003)$</td>
</tr>
<tr>
<td></td>
<td>$0.0485^* -0.8879^* 0.0235^*$</td>
<td>$0.0006  -0.9663^*  -0.0157^*$</td>
</tr>
<tr>
<td></td>
<td>$(.0074)  (.0076)  (.0019)$</td>
<td>$(.0019)  (.0020)  (.0005)$</td>
</tr>
<tr>
<td></td>
<td>$3.7002^* -0.3547 -0.2328^*$</td>
<td>$-1.4676^*  0.3906^*  0.0609^*$</td>
</tr>
<tr>
<td></td>
<td>$(.2395)  (.2457)  (.0622)$</td>
<td>$(.0834)  (.0856)  (.0217)$</td>
</tr>
<tr>
<td>$\Sigma_{st}$</td>
<td>$[0.0003^* 0.0003^* 0.0122^*]$</td>
<td>$[0.0000  0.0000  0.0003]$</td>
</tr>
<tr>
<td></td>
<td>$(.0001)  (.0001)  (.0047)$</td>
<td>$(.0001)  (.0001)  (.0006)$</td>
</tr>
<tr>
<td></td>
<td>$0.0003^* 0.0006^* 0.0172^*$</td>
<td>$0.0000  0.0000  0.0017$</td>
</tr>
<tr>
<td></td>
<td>$(.0001)  (.0002)  (.0072)$</td>
<td>$(.0001)  (.0001)  (.0011)$</td>
</tr>
<tr>
<td></td>
<td>$0.0122^* 0.0172^* 0.6317^*$</td>
<td>$0.0003  0.0017  0.0766$</td>
</tr>
<tr>
<td></td>
<td>$(.0047)  (.0072)  (.2333)$</td>
<td>$(.0006)  (.0011)  (.0462)$</td>
</tr>
</tbody>
</table>

Table 1: Maximum likelihood estimates of the Markov switching VAR model where the vector $y$ is equal to Credit-to-GDP, Residential property prices and S&P500 (taken in this order) in percentage log-returns. Standard errors are shown in parentheses. The parameters $\nu_{st}$, $\Phi_{st}$ and $\Sigma_{st}$ are defined in the MS VAR specification in Equation (1.3). The symbol “∗” indicates 95% significance level.

From Table 1, it is worth noticing that first regime corresponds to periods of crisis where the returns are negative and volatility is high and significant, while in the second regime average returns are positive, especially for stock prices, and volatility estimates are not significant indicating mild times. The same general conclusion can be obtained from
the estimation of MS autoregressive models on the individual series which are reported in Table 2. The first regime corresponds to recessionary periods where intercepts are lower than in the second regimes and volatilities are significantly greater.

<table>
<thead>
<tr>
<th>Variables</th>
<th>1st regime</th>
<th>2nd regime</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Credit-to-GDP</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_{st}$</td>
<td>0.0675</td>
<td>0.2740</td>
</tr>
<tr>
<td></td>
<td>(1.0151)</td>
<td>(1.1579)</td>
</tr>
<tr>
<td>$\Phi_{st}$</td>
<td>-0.2739*</td>
<td>1.0106*</td>
</tr>
<tr>
<td></td>
<td>(.0523)</td>
<td>(.0610)</td>
</tr>
<tr>
<td>$\Sigma_{st}$</td>
<td>0.6703*</td>
<td>0.5152*</td>
</tr>
<tr>
<td></td>
<td>(.3352)</td>
<td>(.2576)</td>
</tr>
</tbody>
</table>

| Residential property prices      |            |            |
| $\nu_{st}$                       | 0.0979     | 0.2608     |
|                                  | (1.4084)   | (.7787)    |
| $\Phi_{st}$                      | 0.9870*    | 0.7479*    |
|                                  | (.0342)    | (.0188)    |
| $\Sigma_{st}$                    | 0.9918*    | 0.3032     |
|                                  | (.1516)    | (.4959)    |

| S&P500                           |            |            |
| $\nu_{st}$                       | -0.6968    | 4.3827*    |
|                                  | (1.8215)   | (1.3749)   |
| $\Phi_{st}$                      | -0.1801*   | 0.3232*    |
|                                  | (.0262)    | (.0301)    |
| $\Sigma_{st}$                    | 2.3021*    | 0.5698     |
|                                  | (1.1510)   | (1.7849)   |

| Real GDP                         |            |            |
| $\nu_{st}$                       | 0.0041*    | 0.0077*    |
|                                  | (.0008)    | (.0017)    |
| $\Phi_{st}$                      | -0.0905*   | 1.0878*    |
|                                  | (.0062)    | (.0156)    |
| $\Sigma_{st}$                    | 0.0126*    | 0.0044*    |
|                                  | (.0058)    | (.0024)    |

Table 2: Maximum likelihood estimates of the Markov switching AR models in the single variables: Credit-to-GDP, Residential property prices, S&P500 (taken in percentage log-returns) and real GDP (in log difference). Standard errors are shown in parentheses. The parameters $\nu_{st}$, $\Phi_{st}$ and $\Sigma_{st}$ are defined in the MS VAR specification in Equation (1.3). The symbol * indicates 95% significance level.

Finally we plot estimated smoothed probabilities of being in the first regimes in Figure 1; red lines are the smoothed probabilities and grey areas are the corresponding time span identifying crisis periods. Those new series will be used to test synchronisation between cycles using CRQA.
Figure 1: Estimated smoothed probabilities (in red) and corresponding shaded areas of the first regimes (corresponding to crisis). From the top, the order is: credit-to-GDP, residential property prices, S&P500, trivariate financial system, real GDP.
4.2 Recurrence Quantification Analysis

In Figures 2 to 5, we present the cross-recurrence plots (CRPs) and the respective lines of synchronisation LOS (solid red line) and main diagonal (dashed blue line) of the four main couples of smoothed probabilities (by order of appearance: BC-FC, BC-S&P500, BC-Credit-to-GDP, BC-Residential Property) to get a first visualisation of the resulting interrelationships. The investigation of the recurrent properties between the BC and the individual series that were used to construct the FC is performed in order to assess the dominant components of the FC. For all couples, the embedding parameters ($\tau$ and $m$) are set to 1 while threshold ($\varepsilon$) equals 10%.

As it can be seen, the denser CRP is that of BC and FC in Figure 2, where several and persistent diagonal and vertical lines are apparent signifying that the common structure is rather complex. We also observe bands of white space, indicating changes in the shared dynamics, followed by changes in the density of the RP (i.e. change of dynamical regime). Concerning the LOS, until the end of ’90 (observation 100, Q4:2000) it is evident the leading of BC transition probabilities. After this area, a shift occurs making the relationship lagging, where the financial cycle becomes the dominant variable.
Comparing Figure 3 to Figures 4 and 5, one can see that the CRP between BC and S&P500 presents more dense structure, meaning that the recurrent shared structure between BC and S&P500 is more persistent. The LOS for the BC-S&P500 couple moves close to the main diagonal for the first 40 observations (10 years of data), i.e. around the 1st quarter of 1985, and then starts deviating rapidly to the left confirming the leading role of S&P500 over the cycle.
Regarding the BC-Credit and BC-Residence couples the CRPs indicates a sparse structure. The first relationship does not present distinguishable patterns. The LOS fluctuates randomly around the main diagonal obeying no-persistent deviations able to describe clear leading or lagging behaviour. The latter couple shows signs of co-evolution like the ones observed in Figure 2 where BC and FC are compared. Observation 100 seems to be again a tipping point affecting the shape of LOS. During the short region posterior to this point, the observed offsetting indicates the driving of the residential property transition probabilities.
Figure 4: Cross Recurrence Plot between BC and Credit-to-GDP (in %)

Cross Recurrence Plot between BC and Credit-to-GDP (in %)
Dimension: 1, Delay: 1
Threshold: 0.1 (fixed distance euclidean norm)
Finally, in order to quantify the information depicted in the CRPs, we estimate the Recurrence Rate (RR) for all couples in sliding windows (window size = 20 and step = 1), using identical input parameters for comparison reasons ($\tau = m = 1$, and $\varepsilon = 0.1$).

Figure 5: Cross Recurrence Plot between BC and Residential Property (in %)
Figure 6 validates the visual patterns observed in the CRPs. Indeed the most coupled system in terms of state recurrences is the one between the BC and FC which after observation 140 (Q4:2010) increases rapidly signifying that both cycles tend to demonstrate the same probability of recession or growth. When it comes to the individual series, the most pronounced coupling is that between the BC and the S&P500 which, after around the 140 point, starts to deteriorate. Interestingly, the BC-Credit couple gets its highest similarity during the 120-140 (Q1:2006-Q4:2010) time period. A worth noticeable fact is that a few steps posterior to observation 60 (beginning of 90’s), the shape of RR for all couples shows strong similarity.

Combining the information provided by the LOSs and the RRs, we conclude that the empirical results revealed that the BC and FC couple tend to share common recurrent states, most of them are persistent during the 70-100 (Q2:1993 - Q4:2000) period and BC is mostly leading the FC up until around observation 100, after this point the relationship reverses. Concerning the remaining couples, the mostly recurrent state structure is identified between BC and S&P500 which are mainly synchronised at the beginning of the sample (until observation 40, Q4:1985) and after that there exists a clear offset of the LOS making the S&P500 leading. Finally, the CRPs for BC-Credit and BC-Residence are sparser meaning that the recurrent dynamics become less frequent, while the findings of the respective LOSs imply that either this common structure is synchronised for BC-Credit or that BC mostly leads regarding the BC-Residence case.

5 Conclusions

Our paper presents an innovative view of the cross-dynamics between business and financial cycles. Their relationship has been quantified so far using methodologies built
on linear aggregate measures, i.e. correlation or frequency-domain techniques that seem to be sensitive to the non-normal distributional characteristics of real data. However, the interactive and dynamic nature of business and financial cycles requires the adoption of techniques able to capture their intrinsic complex structures. In this spirit, we suggest the joint application of econometric models and recurrence technique and thus i) identify first cyclical movements, and then ii) analyse their inter temporal characteristics. The implementation of measures from the cross-RQA methodology provides interesting insights and uncover the phases where the smoothed probabilities of the MS-VAR models are recurring, i.e. being on the same state as well as the timing over which these recurrent structures occurs.

More specifically, at first stage smoothed probabilities are estimated using Markov-Switching models on US quarterly data spanning from Q1-1975 to Q3-2017 for credit-to-GDP, residential property prices and S&P500 variables for the financial cycle, and real GDP as a proxy for the business cycle. Next, we proceed with the examination of the cross-recurrent patterns created by their interaction. The empirical findings on long US data indicate a leading behaviour of BC during the Great Moderation era until the end of the 90s. However, after the beginning of deregulation, a significant off-set of Line of Synchronisation (LOS) in favour of the FC as driving force is detected, suggesting a progressive leading evolution over the BC. Except S&P500 and residential property prices that less or more lead BC after the end of the 90s, the LOS of credit-to-GDP shows reverting evolution to the main diagonal of the cross-RP. Quantifying the dynamics of the cross-RPs provides rolling estimates for the cross-Recurrence Rate, a measure that illustrates the probability for two signals to visit the same state. More specifically, the couples of smoothed probabilities BC-FC and BC-S&P500 share common recurrent states until the last quarter of 2010. The sub-sample Q1:2006-Q4:2010 is also marked by a significant jump in the cross-Recurrence Rate of the couple BC-residential property. After 2010, both credit-to-GDP and stock exchange contribute significantly to the dynamics of recurrence of the FC. Over the same time-period, where the FC leads the BC, their cross-Recurrence Rate increases due to high co-visitation.
References


