Okun's Law and the Business Cycle*

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Abstract

We develop and estimate a business cycle model with indivisible employment, endogenous participation and involuntary unemployment. To overcome indivisibilities, optimal allocations involve lotteries over labor force participation instead of employment/unemployment outcomes, as in Hansen (1985) and Rogerson (1988); conditional on participation, households search for jobs. Shifts in involuntary unemployment are due to search externalities as in Diamond (1982), and satisfy Okun's Law. Self-fulfilling fluctuations emerge if search externalities are strong enough, as households expectations of high unemployment lower participation and are consistent with lower output through Okun's Law, in turn validating those expectations. The mechanism generates a positive correlation in forecastable movements in consumption and employment, an empirical feature of the business cycle that endogenous business cycle models do not usually match. Finally, on the policy side, we demonstrate that unemployment insurance reduces the dimensions of indeterminacy and, therefore, is a powerful automatic stabilizer.

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1 Introduction

To understand how unemployment evolves over the business cycle Okun's law (Okun, 1962) has long been the first port of call, with its result that a one-percentage point reduction in unemployment should be associated with approximately 2% more output. Thus, if fluctuations in unemployment are not offset by adjustments of the labor inputs along other margins (hours and participation), Okun's relationship implies that labor productivity must be procyclical. For this reason, the real business cycle framework requires large procyclical productivity shocks to match the observed comovement of employment and output, implying that the real wage should be strongly procyclical. Instead, other types of economic disturbances that are believed to be important to explain the business cycle phenomena, such as shocks to the relative price of investment, counterfactually generate a negative conditional correlation between employment and consumption, a point famously made by Barro and King (1984).

A prominent alternative theory of the business cycle is that fluctuations are endogenous, caused by beliefs shocks that lead to self-fulfilling expectations, as consequence of empirically plausible levels of increasing return to scale à la Benhabib and Farmer (1996). However, as shown by Schmitt-Grohé (2000), this class of endogenous business cycle models also implies a negative correlation between consumption and employment, conditional on beliefs shocks. Although the connection between both results seems to have remained unnoticed, the Barro and King (1984) and the Schmitt-Grohé (2000) critiques of, respectively, investment shocks and beliefs shocks as sources of business cycle fluctuations bear an eerie similarity. They both follow from the intra-temporal substitution between consumption and leisure and the fact that in the absence of productivity driven shifts in labor demand, leisure and consumption must be positively correlated.

In this paper we propose a theory of self-fulfilling unemployment fluctuations. We develop a business cycle model that includes indivisible labor, endogenous labor force participation and involuntary unemployment. The model builds on a relationship between unemployment and the output gap consistent with Okun's law. This relationship is incorporated in an otherwise standard equilibrium model of the business cycle, in which optimal allocations involve lotteries over participation in the labor force, instead of over employment outcomes as in the approach of Hansen (1985) and Rogerson (1988). In turn, involuntary unemployment exists because of search frictions and there are externalities in search.

We find that self-fulfilling business cycle fluctuations may emerge whereby, if individuals expect unemployment to be high, then labor force participation and output are low, which in turn validates the high unemployment expectations because of Okun's law. The origins of this local indeterminacy can be attributed to an externality in the labor market which is akin to the coordination failure in Diamond (1982) coconut model.

The possibility of indeterminacy in our model arises because of involuntary unemployment. We provide necessary conditions for local indeterminacy that offer an intuitive explanation for the emergence of self-fulfilling fluctuations. The crucial insight is that, for a given real wage, the existence of unemployment introduces a countercyclical wedge between the marginal utility of consumption and the marginal utility of leisure. Thus, increasing returns to scale are not required for indeterminacy and, contrary to other endogenous business cycle models, we do not require the correlation of consumption and employment conditional on beliefs shocks to be negative (the critique of Schmitt-Grohé, 2000). Thus, the model overcomes two important empirical shortcomings of the canonical endogenous business cycle model.

By conducting a Bayesian estimation of the model for the US economy, we test empirically if externalities in the labor market are sufficiently strong to generate local indeterminacy. This is done using methods proposed in Lubik and Schorfheide (2004) and Farmer et al. (2015) which can handle the likelihood function of models with local indeterminacies. Two versions of the model are considered. The first version restricts the parameter space to the region where local indeterminacy emerges, while the second restricts the admissible parameter space to guarantee a unique equilibrium. This allows us to test in a Bayesian framework which version of the model is a better description of the US data. The estimated model includes several stochastic shocks found to be important to explain the US macroeconomic time series, including a stochastic trend in labor productivity, a transitory neutral technology shock and an investment efficiency shock (as in Fisher, 2006). In addition, the model with local indeterminacy adds an additional shock revisions in expectations (sunspot shocks). It is found that the model estimated in the parameter region with local indeterminacy compares well in terms of Bayesian posterior odds to the alternative model exhibiting local determinacy.

This paper also contributes to the important literature on automatic stabilizers, understood broadly as features of the tax and transfer system that respond automatically to current conditions in the economy, thereby lowering business cycle volatility. The stabilizing effect of automatic stabilizers, in particular unemployment insurance, is traditionally thought to be most effective in environments featuring incomplete opportunities for private insurance (McKay and Reis, 2016). For example, an unemployment insurance may dampen fluctuations in disposable income and thus stabilize the business cycle in environments with nominal rigidities and market incompleteness (Brown, 1955). Similarly, unemployment insurance may redistribute income across individuals that have different marginal propensity to spend and, thus, contribute to aggregate demand stabilization when markets are incomplete (Blinder, 1975). Instead, we use the model developed in this paper to study the role of unemployment insurance in a setting with perfect private insurance markets and, consequently, no motivation for redistribution. Unemployment insurance is shown to make local indeterminacy less likely and, therefore, is a powerful automatic stabilizer. This result is noticeable because it offers a justification for the existence of government sponsored unemployment insurance, even in an environment with perfect private insurance markets. This is because the government insurance dampens the volatility of the wedge between the marginal utility of consumption and the marginality utility of leisure and, therefore, curtails the mechanism responsible for the self-fulfilling fluctuations. This result resonates well with the empirical finding that more generous unemployment insurance lowers business cycle volatility (see, for example, Di Maggio and Kermani, 2016).

Although there exist other models in which automatic stabilizers reduce business cycle volatility despite the existence of perfect private insurance markets (for example, by lowering the aggregate labor supply elasticity, as in Janiak and Santos Monteiro, 2016), this is in general not efficient. Indeed, it is generally argued that for automatic stabilizers to improve welfare, agents must be unable to enter private insurance contracts. In contrast, in the model with search externalities that we propose, unemployment insurance may both lower aggregate volatility and improve welfare in a set-up with perfect private insurance market, by preventing inefficient beliefs-driven business cycle fluctuations. We use the estimated model to conduct specific and implementable policy experiments. In particular, we consider the model estimated in the region of local indeterminacy and calculate the required increase in the unemployment insurance replacement ratio, starting from the current level in the US, which would move the economy away from the indeterminacy region and, therefore, eliminate inefficient fluctuations.

The rest of the paper is organized as follows. Section 2 introduces the search externality which is central to our model. Section 3 embeds this externality in a complete general equilibrium model. Section 4 looks at the properties of the model in steady state. Section 5 looks at the model's dynamics and, in particular, derives necessary conditions for multiplicity of equilibrium to arise, and Section 6 studies the role of unemployment insurance. Section 7 considers an estimated version of the model. Finally, Section 8 concludes.

2 Search Externalities and Okun's Law

The purpose of this section is to establish a simple relationship between output and unemployment, which is consistent with Okun (1962) formulation, and has sound theoretical foundations. The theory that we propose is based on the search model of Diamond (1982), which emphasizes the importance of search externalities. We define $\widetilde{Y}_t$ to be aggregate output (in deviation from a trend component which is defined in the following section). At the start of date t, there is a continuum of individuals of mass $\pi_t \in (0,1)$ searching for work, corresponding to the size of the labor force, and a continuum of recruiters in the unit interval. Each recruiter posts a single vacancy (at zero cost), and each worker is matched with a recruiter with probability

$$\mathscr{P}\left(\widetilde{Y}_t\right) = \left(1 + \mu \widetilde{Y}_t^{-\eta}\right)^{-1}. \tag{1}$$

with $\mu > 0$ and $\eta > 0$.

Thus as in Diamond (1982), there is a search externality, as workers are only able to sell their output (labor) with a given probability, $\mathscr{P} \in (0,1)$, which is an increasing function of aggregate output. The choice of functional form for $\mathscr{P}$ is made for its tractability and is not essential for our results, and the parameter η controls the output elasticity of the matching probability. If a match is formed, an employed individual produces one unit of intermediate output (labor services), which is sold by the recruiters to the the final good producers at price w_t. In the Appendix C we show that because recruiters are allowed to create vacancies at zero cost and individuals have access to perfect insurance markets, employed workers earn $w_t = \varpi_t h_0$ (where ϖ_t denotes the "hourly" wage rate).

Since an individual whose search fails remains unemployed, we obtain the following relationship between unemployment and output

$$1 - u_t = \left(1 + \mu \widetilde{Y}_t^{-\eta}\right)^{-1}, \tag{2}$$

where u_t corresponds to the unemployment rate. Although, as we show in Section 4, the natural rate of unemployment is generically indeterminate, we define $u^\star$ to be it. Then, after taking the log-linear approximation of equation (2), we obtain the gap formulation of Okun's Law, given by

$$u_t - u^\star = -\theta \ln\left(\widetilde{Y}_t / \underline{Y}\right), \tag{3}$$

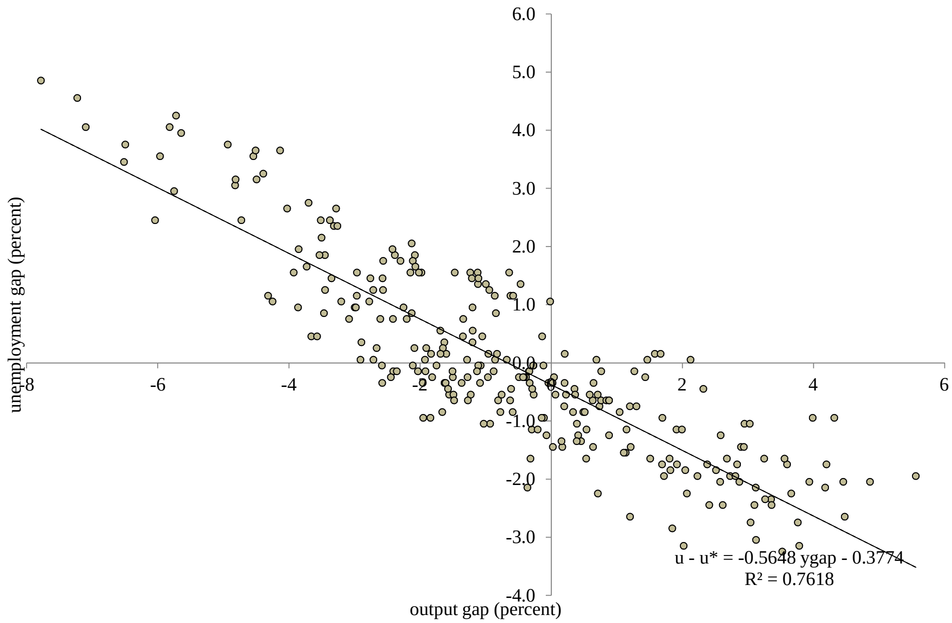

Figure 1: Okun's Law

with $\underline{Y}$ that denotes the steady state level of $\widetilde{Y}_t$, and where

$$\theta = \eta\,(1 - u^{\star})\left(1 + \mu\underline{Y}^{-\eta}\right)^{-1}\mu\underline{Y}^{-\eta}, \tag{4}$$

is the gradient of the Okun's relationship. The empirical relevance of this relationship is illustrated in Figure 1, which is based on the output gap implied by the Congressional Budget Office's historical estimates of potential GDP, and using quarterly data for the period 1949—2016.

In the sequel, we consider equation (3) as a structural relationship in an equilibrium business cycle model. By doing so, we provide a theory of fluctuations in involuntary unemployment driven by search frictions and externalities in the labor market, in a way that is consistent with the empirical evidence on the comovement of unemployment and the output gap.[1]

3 Equilibrium Model

We consider an indivisible labor economy in which labor market adjustment occurs entirely along the extensive margin and there are three possible labor market states: employment, unemployment

[1]For a recent appraisal of the Okun's Law stability over time see Ball et al. (2013).

and non-participation. The formulation of the problem assumes that an individual who is part of the labor force (either employed or unemployed) uses her endowment of time instead of enjoying leisure. In particular, we consider the Hansen (1985) and Rogerson (1988) economy, but with individuals playing lotteries over labor market participation. Thus, the opportunity cost of employment is the same as that of unemployment, with the upshot that any equilibrium with unemployment in this economy is not Pareto efficient. The problem solved by the stand-in agent in the household sector is given by

$$\max_{c_t,\pi_t} \mathbf{V} = \sum_{t=0}^{\infty} \beta^t \Big[\ln(c_t) + \psi \pi_t \ln(1-h_0) \Big], \tag{5}$$

subject to the constraints

$$c_t + i_t = \pi_t (1-u_t) w_t + r_t k_t, \tag{6}$$

$$k_{t+1} = e^{\epsilon_t^i} i_t + (1-\delta) k_t, \tag{7}$$

with $\psi > 0$ and $h_0 \in (0,1)$, and where ϵ_t^i is an exogenous stochastic shock to the efficiency of investment.[2] In particular, the individual's problem follows the formulation in Hansen (1985) involving lotteries, but with lotteries played over the labor force participation/non-participation outcomes, instead of over the employment/unemployment outcomes.[3] Hence, $\pi_t \in [0,1]$ denotes the probability of the individual being part of the labor force.

Since consumption and leisure are separable in the utility function and their are complete insurance markets, individuals participating in the labor force (either employed or unemployed) enjoy the same level of consumption as the individuals who do not participate, denoted $c_t > 0$. Individuals in the labor force are either employed or unemployed and the unemployment rate is denoted $u_t \in (0,1)$. The rental rate of capital is r_t and is set competitively, while w_t is the wage rate and corresponds to the surplus generated by each job match, as explained above. Finally, x_t denotes investment and k_{t+1} the end of period capital stock holdings of the stand-in household.

The first-order conditions solving the stand-in household's problem are given by

$$1 = \frac{(1-u_t) w_t}{\phi c_t}, \tag{8}$$

$$\frac{\mathcal{Q}_t}{c_t} = \beta \left[\left(\frac{1-\delta+r_{t+1}}{c_{t+1}} \right) \mathcal{Q}_{t+1} \right], \tag{9}$$

with $\phi = -\psi \ln(1-h_0) > 0$, and where $\mathcal{Q}_t = e^{-\epsilon_t^i}$ is the relative price of investment. These

[2] We follow the formulations in Fisher (2006), with the only difference that the stochastic process for ϵ_t^i is stationary.

[3] See Appendix A for details.

conditions are standard, except for the presence of the unemployment rate in (8).

Indeed, the distinct feature of this economy is the existence of involuntary unemployment. In particular, in the neighborhood of the steady state, the rate of unemployment satisfies Okun's Law given by equation (3) and which we repeat here for convenience

$$u_t - u^\star = -\theta \ln\left(\widetilde{Y}_t/\underline{Y}\right). \tag{3'}$$

Equation (3′) is a structural feature of the economy and is, therefore, taken as given by agents. Taken together with (5), it leads to coordination problems in trade similar to those in Diamond (1982). In his framework, agents are randomly presented with production opportunities and, once they produce a fixed quantity of output, they must search for a buyer and cannot undertake production if they have unsold output. Our framework offers similar opportunities and constraints. In particular, individuals receive production opportunities (in our case, the ability to search for work) with a given probability π_t and, conditional on participation, they must find a buyer for their fixed supply of labor h_0, subject to the search frictions described in Section 2.[4]

Final output is produced by competitive firms combining capital K_t and intermediate output (labor services) N_t , through the following Cobb-Douglas technology

$$Y_t = e^{\epsilon_t^z} K_t^\alpha \left(A_t N_t\right)^{1-\alpha}, \tag{10}$$

with $\alpha \in (0,1)$, and where ϵ_t^z is a transitory productivity shock following an autoregressive process, and A_t is a labor-augmenting technological shift which, in logs, is assumed to follow a random walk process with drift, as follows

$$\ln A_t = \ln \mathcal{G} + \ln A_{t-1} + \nu_t^a, \tag{11}$$

with ν_t^a a standard gaussian innovation, and $\mathcal{G}$ is the gross growth rate of the economy along the deterministic balanced growth path. Equilibrium factor prices are given by

$$w_t = (1-\alpha)\left(Y_t/N_t\right), \tag{12}$$

$$r_t = \alpha\left(Y_t/K_t\right). \tag{13}$$

Finally, if we let Π_t denote the labor force participation rate and use capital letters to denote

[4] In Diamond (1982), the arrival rate of production opportunities is exogenous and individuals must choose if they pursue production if given the opportunity. Instead, in our model all individuals who participate (receive a production opportunity) also search for work, but the probability of participation π_t is chosen endogenously. The opportunity cost of participation is h_0 units of leisure. Of course, this is only a small difference in the protocol, that does not change the fundamental coordination problem that emerges when there are search externalities.

aggregate variables, the market clearing conditions are given by

$$\widetilde{c}_t = \widetilde{C}_t, \tag{14}$$

$$\widetilde{i}_t = \widetilde{I}_t, \tag{15}$$

$$\widetilde{k}_t = \widetilde{K}_t, \tag{16}$$

$$\pi_t = \Pi_t, \tag{17}$$

$$N_t = \Pi_t \left(1 - u_t\right). \tag{18}$$

where the notation $\widetilde{X}_t$ denotes the stationary version of X_t, given by (X_t/A_t).[5] Combining the market clearing conditions with the efficiency conditions (8) and (9), production function (10), the stochastic process (11), the factor prices (12) and (13), and Okun's equation (3′), yields the following equilibrium conditions

$$\phi \Pi_t \widetilde{C}_t = \left(1 - \alpha\right) \widetilde{Y}_t, \tag{19}$$

$$\frac{\mathcal{Q}_t}{\widetilde{C}_t} = \left(\beta/\mathcal{G}\right) E_t \left[\left(\frac{1 - \delta + \alpha \mathcal{G} e^{\nu^a_{t+1}} \widetilde{Y}_{t+1}/\widetilde{K}_{t+1}}{\widetilde{C}_{t+1}} \right) \mathcal{Q}_{t+1} e^{-\nu^a_{t+1}} \right], \tag{20}$$

$$\widetilde{Y}_t = e^{\epsilon^z_t - \alpha \nu^a_t} \left(\widetilde{K}_t/\mathcal{G} \right)^{\alpha} \left(\Pi_t \left(1 - u_t\right)\right)^{1-\alpha}, \tag{21}$$

$$\widetilde{C}_t + \widetilde{I}_t = \widetilde{Y}_t, \tag{22}$$

$$\widetilde{K}_{t+1} = \left(\widetilde{I}_t/\mathcal{Q}_t \right) + \frac{(1-\delta)}{\mathcal{G}} \widetilde{K}_t e^{-\nu^a_t}, \tag{23}$$

$$\mathcal{Q}_t = e^{-\epsilon^i_t}, \tag{24}$$

$$u_t - u^{\star} = -\theta \hat{y}_t, \tag{25}$$

with $\phi = \psi\left(1 - h_0\right) > 0$, and where $\hat{y}_t = \ln\left(\widetilde{Y}_t/\underline{Y}\right)$ denotes the final aggregate output in log-deviation from its steady state (output gap).

The shocks ϵ^i_t and ϵ^z_t are both assumed to follow independent stationary autoregressive processes, as follows

$$\epsilon^i_t = \rho^i \epsilon^i_{t-1} + \nu^i_t, \tag{26}$$

$$\epsilon^z_t = \rho^z \epsilon^z_{t-1} + \nu^z_t. \tag{27}$$

[5]The only exception is $\widetilde{K}_t$ which is defined as (K_t/A_{t-1}), since K_t is determined at date $t-1$.

4 Steady State

In what follows, we look at the properties of the steady state and next characterize the equilibrium dynamics of the model in the neighborhood of its deterministic steady-state. One interesting feature of this model economy is that it is only possible to define an unique steady state up to a choice for the steady state unemployment rate (henceforth, the natural rate). In turn, the natural rate $u^\star$ may not be uniquely defined, depending on the form of the function $\mathscr{P}$. Thus, if we do not specify a natural rate the model may exhibits many steady state equilibria. For a given choice of the natural rate, the deterministic steady state of this economy corresponds exactly to the steady state of the neoclassical growth model, as shown in Appendix D.

A second curious feature of the steady state, is that the participation rate $\underline{\Pi}$, which is given by equation (D.4), is independent of the natural unemployment rate. This result follows from the fact that the preferences exhibit unit elasticity of substitution between consumption and leisure: an increase in the natural unemployment rate lowers the opportunity cost of non-participation (substitution effect), but this effect is exactly offset by the negative income effect implied by the lower expected labor income.

Proposition 1 *The economy may exhibit several steady state equilibria, indexed by the natural rate* $u^\star \in (0,1)$*, depending on the shape of the function* $\mathscr{P}$*. However, the steady state participation rate* $\underline{\Pi}$ *is independent of the natural rate. Finally, for a given natural rate, the steady state of this economy corresponds exactly to that of the neoclassical growth model.*

To show that the steady state equilibria are indexed by the natural unemployment rate, is suffices to note that in steady state the capital-output ratio is independent of $u^\star$, given by

$$(\underline{K}/\underline{Y}) = \left[\frac{\alpha}{1/\beta - (1-\delta)/\mathcal{G}}\right]. \tag{28}$$

In turn, as shown in Appendix D, the steady state capital stock is proportional to $(1-u^\star)$. All other variables of interest are proportional to the capital-output ratio as in the standard neoclassical growth model. Thus, the steady state is fully determined, conditional on the level of $u^\star$, the natural rate of unemployment. The economy may exhibit a large number of natural unemployment rates and, hence, steady state equilibria, depending on the form of the function $\mathscr{P}$. These equilibria are Pareto ranked and, as in Diamond (1982), each steady state equilibrium is locally inefficient.[6] In this paper we do not focus on this global indeterminacy and, instead, consider a given $u^\star$.

[6]Given the functional form chosen for $\mathscr{P}$, there exist at most two natural unemployment rates.

5 Search Externalities and Local Indeterminacy

In what follows, we show that because of the existence of search externalities, there will be parameter regions for which the steady state is locally indeterminate. In doing this analysis, we follow the method of Wen (2001) who obtains necessary and sufficient conditions for local indeterminacy in RBC models. The local indeterminacy of the perfect-foresight equilibrium implies the existence of stationary sunspot equilibria. In particular, our focus in this section is to provide necessary conditions for local indeterminacy that offer an intuitive explanation for the possible emergence of self-fulfilling fluctuations.

Consider the following proposition:

Proposition 2 *A necessary condition for indeterminacy is*

$$\xi\left(\underline{R}, \underline{\Pi}\right) < \xi\left(1-u^{\star}, \underline{\Pi}\right) + \xi\left(\underline{w}, \underline{\Pi}\right), \tag{29}$$

where $\xi\left(R, \Pi\right)$ *denotes the elasticity of the gross rate of capital return to participation,* $\xi\left(1-u^{\star}, \Pi\right)$ *denotes the elasticity of the employment rate to participation, and* $\xi\left(w, \Pi\right)$ *the elasticity of wages to participation. Condition* (29) *imposes restrictions on* η*, that is,*

$$\eta u^{\star} \in \left[\frac{1}{1-\alpha} - (1-\delta)\left(\beta/\mathcal{G}\right), \frac{1}{1-\alpha}\right]. \tag{30}$$

Proof. See Appendix F ■

It is possible to give concrete economic intuitions for the necessary condition (30). In particular, in Appendix I, we show that in the neighborhood of the steady state the total elasticity of output to changes in participation is given by

$$\xi\left(\bar{Y}, \bar{\Pi}\right) = \frac{dY}{d\Pi}\frac{\Pi}{Y} = \left[\frac{(1-\alpha)}{1-\eta\left(1-\alpha\right)u^{\star}}\right]. \tag{31}$$

This elasticity is positive as long as $\eta u^{\star} < 1/\left(1-\alpha\right)$, which corresponds to the upper bound in condition (30). Thus, a necessary condition for indeterminacy is that the elasticity of output to changes in participation is positive, $\xi\left(\bar{Y}, \bar{\Pi}\right) > 0$. If this condition is satisfied, an increase in participation leads to an increase in output, which in turn may generate a multiplier effect if search externalities are sufficiently strong. This is the case if the "effective wage", defined as $\left(1-u_t\right)w_t$, increases sufficiently following a rise in participation. For this to be the case, the participation elasticity of the "effective wage", given by $\xi\left(1-u^{\star}, \underline{\Pi}\right) + \xi\left(\underline{w}, \underline{\Pi}\right)$, must exceed the participation elasticity of the return to capital, $\xi\left(\underline{R},\underline{\Pi}\right)$. The "effective wage", as we just defined, is the relevant measure of the return to labor in an economy with involuntary unemployment because, conditional on labor market participation, only the fraction $(1-u_t)$ of labor market participants earns a wage.

The relevant elasticities are given by

$$\xi\left(\underline{R},\underline{\Pi}\right)=\left[\frac{(1-\alpha)\left(1-(1-\delta)\left(\beta/\mathcal{G}\right)\right)}{1-\eta\left(1-\alpha\right)u^{\star}}\right], \tag{32}$$

$$\xi\left(1-u^{\star},\underline{\Pi}\right)=\left[\frac{\eta\left(1-\alpha\right)u^{\star}}{1-\eta\left(1-\alpha\right)u^{\star}}\right], \tag{33}$$

$$\xi\left(\underline{w},\underline{\Pi}\right)=-\left[\frac{\alpha}{1-\eta\left(1-\alpha\right)u^{\star}}\right], \tag{34}$$

Thus, notice that when $\eta u^{\star}<1/\left(1-\alpha\right)$, the participation elasticity of the wage rate, $\xi\left(\underline{w},\underline{\Pi}\right)$, is negative. Indeed, as there are diminishing returns to labor, the slope of the equilibrium wage-employment loci is negative. Moreover, labor supply in our model is standard and, in particular, preferences feature a unit elasticity of substitution between consumption and leisure. However, there may still be local indeterminacies if the search externalities are sufficiently strong, so that the employment rate $(1-u^{\star})$ increases sufficiently following a rise in participation.[7]

To see how self-fulfilling equilibria may emerge in our economy if search externalities are sufficiently strong, consider the following argument. Suppose the economy is on a given equilibrium path, and there is a shock to agents' beliefs about the shadow price of capital. For example, suppose agents believe that the shadow price of capital has declined and, thus, increase consumption. From condition (19), the increase in consumption has to be associated with an increase in the "effective wage rate", $(1-u_t)\,w_t$. If there are no search externalities and, thus, the unemployment rate stays at its natural level, the participation rate must fall to raise wages. This, in turn, implies a decline in output and in the capital stock, raising the return to capital in the next period. This implies further declines in the shadow price of capital to support the initial change in beliefs. But these dynamics would violate boundary conditions and, thus, are not an equilibrium.

If, instead, search externalities are sufficiently strong, an increase in the "effective wage rate" is possible without a decline in the participation rate, if the unemployment rate falls sufficiently. Thus, consumption and participation may both increase. In turn, the increase in participation raises output and further lower unemployment, allowing the return to capital and investment to increase initially. However, the increase in the capital stock eventually leads to a decline in the

[7]This is in contrast to the Benhabib and Farmer (1994) seminal model, that requires the slope of the labor demand curve to be sufficiently steeper than that of the labor supply curve, to obtain a sufficiently positive labor elasticity of wages.

return to capital, which leads to an appreciation of the price of capital, and declines in consumption and participation, as the economy returns to its balanced growth path.[8] Because it is the "effective wage" which matters in our economy, employment and consumption may both increase, conditional on an extrinsic shock. Instead, in endogenous business cycles models based on increasing return to scale à la Benhabib and Farmer (1994), an extrinsic shock that raises consumption is associated with an increase in wages and, hence, a decline in employment. This counterfactual prediction about the conditional correlation of consumption and employment is singled out by Schmitt-Grohé (2000) as an important shortcoming of the canonical endogenous business cycle model. Search externalities in the labor market, as proposed in this paper, overcomes this problem.

5.1 Diagrammatic exposition

The preceding analysis provided necessary conditions offering an intuition behind the mechanism that generates self-fulfilling fluctuations. Next, we show that equilibrium can be presented in a two-dimensional graph as intersection of two schedules: Okun's Law and equilibrium in the final producers sector. This graph can be used to analyse the effects of both fundamental shocks and non-fundamental (sunspot) shocks.

From section 2, the log-linear form of Okun's relation is given by

$$\hat{y}_t = -\left[\frac{1}{\eta u^*(1-u^*)}\right](u_t - u^*).$$

This relation describes the matching process between recruiters and labor market participants; thus, it is as an equilibrium condition in the recruiters market and we denote it as recruiters equilibrium and represent it with the RE locus in Figure 2.

In turn, recruiters sell labor to final good producers. The latter combine aggregate labor services and capital services to produce output. The final producers equilibrium (FPE) condition is given by

$$\widehat{y}_t = -\frac{1-\alpha}{\alpha(1-u^*)}(u_t - u^*) + \widehat{k}_t - \frac{1-\alpha}{\alpha}\widehat{c}_t + \frac{\epsilon_t^z}{\alpha},$$

which is obtained from combining the production function, the demand equations for labor and capital services, and the intra-temporal first order condition of households.

[8] We have shown that if search externalities are sufficiently large, for any initial equilibrium path it is possible to construct an alternative equilibrium path supported by a change in beliefs (sunspots). Of course, if at least two equilibria can be obtained, then the set of equilibria under indeterminacy is a continuum.

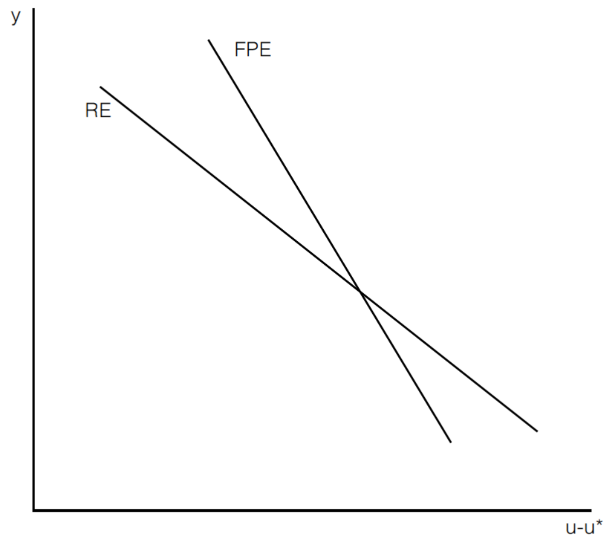

Figure 2: FPE and RE schedules

The FPE and RE schedules are drawn in the $(\widehat{y}, u - u^*)$-space in Figure 2, holding consumption and the productivity shock fixed (as the capital stock is predetermined). The following Lemma compares the slopes of FPE and RE schedules:

Lemma 1 *If the necessary condition for indeterminacy is satisfied, then the slope of the FPE schedule is steeper than the slope of the RE schedule.*

Proof. The proof is straightforward. The slope of FPE is steeper than the slope of RE if and only if

$$\frac{1-\alpha}{\alpha} > \frac{1}{\eta u^*}.$$

The latter is always satisfied if ηu^* is restricted as in Proposition 2 ■

Figure 2 is useful to understand the effects of sunspot shocks and, also, fundamental shocks. Let us start with sunspot shocks. Following the argument of the previous section, suppose households hold beliefs that justify higher consumption than the current equilibrium path (suppose initially the economy is at the steady state, that is, FPE and RE intersect at zero). The FPE schedule moves to the left since $\widehat{c} > 0$. At the new intersection, output increases and unemployment falls. Lower unemployment implies that the participation rate increases. Moreover, households increase investment since the return on capital is higher. As more capital is accumulated, the marginal product falls and dynamics are reversed back to the steady state. Since consumption falls, the FPE schedule moves back to the right until it crosses RE at zero.

The upshot of the previous analysis is that sunspot shocks, by changing consumption, change the position of the FPE schedule relative to the RE schedule and affect the real allocation of resources. The next experiment considers the effects of a transitory productivity shock. Starting from the steady state, we uncover the following surprising result. A transitory expansionary shocks to technology implies, on impact, a contraction of output, investment and consumption, an increase of unemployment, and a decrease of participation. This result is in sharp contrast with the benchmark RBC model (without search externalities), in which expansionary technology shocks generate a boom. However, it is consistent with the findings in Gali (1999) and Basu et al. (2006), who found that aggregate technology shocks in the U.S. economy lower employment, investment, and the real interest rate in the short run.

The intuition can be explained with the help of Figure 2. Consider a positive shock to technology, so that $\epsilon^z > 0$. Holding consumption fixed at the steady state, $\widehat{c} = 0$, the FPE schedule moves to the right. At the new point of intersection, and since the FPE is steeper than the RE schedule, output falls and unemployment increases. Higher unemployment rates lower participation by households since search in the labor market becomes less attractive. In turn, the lower employment lowers the marginal product of capital and investment.

6 Unemployment Insurance

Suppose the government provides unemployment insurance (UI). In particular, it replaces a fraction γ of the wage income of households that participate in the labor market but are unemployed. The government finances this policy by taxing all households with lump-sum taxes T_t. The period t budget constraint of the stand-in agent modifies as[9]

$$c_t + i_t = \pi_t(1 - u_t)w_t h_0 + \gamma\pi_t u_t w_t h_0 + r_t k_t - T_t, \tag{35}$$

where $\gamma \in [0, 1]$ is the replacement ratio. A policy of $\gamma = 1$ is defined as full replacement policy. Moreover, the government follows a balance-budget policy, requiring

$$T_t = \gamma u_t w_t h_0 \Pi_t. \tag{36}$$

[9] Although the economy already offers perfect private insurance opportunities, the introduction of the government sponsored UI program affects the budget constraint of the stand-in agent, as it does not have to be purchased and, thus, crowds-out some of the private insurance. See Appendix B for details.

The equilibrium conditions for the economy with UI are the same as those for the baseline economy without UI, except for the intra-temporal equilibrium condition (19) which is replaced by

$$\phi \Pi_t \widetilde{C}_t = \left[\frac{1 - (1-\gamma) u_t}{1 - u_t} \right] (1-\alpha) \widetilde{Y}_t. \tag{19'}$$

Of course, the economy with $\gamma = 0$ is identical to the baseline economy, since then (19) and (19′) are the same. However, with a positive UI replacement ratio the mechanism which could generate equilibrium indeterminacy in the baseline economy are weaker. In the baseline economy individuals would participate less in the labor market when high unemployment was expected and this could lead to self-fulfilling high unemployment. However, in an economy with UI, expectations of high unemployment have a less detrimental effect on participation as as unemployed workers still receives the government transfer. Thus, the self-fulfilling multiplier effect is less salient.

More formally, we establish the following proposition:

Proposition 3 *Consider a baseline economy without UI, and suppose the baseline economy is parameterized such that there are multiple equilibrium paths converging to the same BGP (local indeterminacy). It is always possible to transform the baseline economy by introducing an UI policy in order to guarantee a unique equilibrium path around the BGP. In particular, the full replacement UI policy always guarantees uniqueness. Moreover, there exists a range of replacement ratios* $\gamma \in \left(\underline{\gamma}, 1\right]$ *for which there is a unique saddle path stable equilibrium.*

Proof. See Appendix G ■

Thus, UI is a powerful automatic stabilizer in the sense that its presence may allow for the existence of a unique saddle path equilibrium, and prevent self-fulfilling business cycle fluctuations driven by sunspots. In the following section we explore the positive and normative implications of this model empirically, using the US economy as a laboratory.

7 Estimated Model for the US Economy

In this section we evaluate empirically if search externalities in the labor market and self-fulfilling fluctuations are important in the U.S. business cycle. In particular, using an approach similar to that proposed in Lubik and Schorfheide (2004) and Farmer et al. (2015) we conduct a Bayesian estimation of the model developed in this paper restricting the admissible parameter space to the region featuring local indeterminacy around the deterministic steady state.

To explain how this is implemented consider log-linear model, and to simplify the illustration suppose the permanent productivity innovation ν_t^a is the only exogenous shock. We obtain

$$\begin{bmatrix} \hat{k}_{t+1} \\ E_t\left(\hat{c}_{t+1}\right) \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} + \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} \nu_t^a = \Gamma \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} + \Psi \nu_t^a. \tag{37}$$

Following Lubik and Schorfheide (2004), define the conditional expectation of consumption, ξ_t, and the associated rational expectation's forecast error, η_t^c, as follows

$$\xi_t = E_t\left(\hat{c}_{t+1}\right), \tag{38}$$

$$\vartheta_t = \hat{c}_t - \xi_{t-1}. \tag{39}$$

Thus, combining with (37) we obtain

$$\xi_t = \Gamma_{21}\hat{k}_t + \Gamma_{22}\xi_{t-1} + \Gamma_{22}\vartheta_t^c + \Psi_2\nu_t^a. \tag{40}$$

Whether the model is locally indeterminate or, instead, has a unique solution depends on the eigenvalues of the matrix Γ. As shown in the Appendix E, this matrix can be shown to have at least one eigenvalue inside the unit circle. When the second eigenvalue of Γ is outside the unit circle, the model has a unique solution with the property that, conditional on $\nu_t^a = 0$, the rational expectations's forecast error is given by $\vartheta_t = 0$. Instead, when both eigenvalues of Γ are less than one in absolute value, the model is locally indeterminate and any ϑ_t is admissible. Thus, the likelihood function of the model is not defined when the model is locally indeterminate.

To circumvent this problem and evaluate the likelihood function of the local indeterminate model, we follow the approach in Farmer et al. (2015). This consists of postulating a serially uncorrelated exogenous stochastic process for the rational expectation's forecast error, that acts as an equilibrium selection mechanism (sunspot shock). Thus, ϑ_t is now taken to be a fundamental shock and the equilibrium conditions are represented as follows

$$\begin{bmatrix} \hat{k}_{t+1} \\ \xi_t \\ c_t \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & 0 \\ \Gamma_{21} & \Gamma_{22} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \xi_{t-1} \\ c_{t-1} \end{bmatrix} + \begin{bmatrix} \Psi_1 & \Gamma_{12} \\ \Psi_2 & \Gamma_{22} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \nu_t^a \\ \vartheta_t \end{bmatrix}, \tag{41}$$

with ϑ_t an exogenous gaussian innovation. This system has one unstable root and, therefore, a unique solution given initial conditions $\hat{k}_0$ and ξ_{-1}. Thus, we may evaluate the likelihood function of this model using the Kalman filter and estimate the model using Bayesian methods.

Table 1: Calibrated Parameters (time unit of model: quarterly)

Ξ	Value	Target (and source)
α	0.400	Capital's income share of 40% (Cooley and Prescott, 1995);
$\mathcal{G}$	1.004	Annualized quarterly GDP per capita growth of 1.771% (BEA, 1948:q2–2016:q2);
δ	0.018	Annual investment/capital ratio of 7.6% (Cooley and Prescott, 1995);
β	0.992	Annual capital/output ratio of 3.32 (Cooley and Prescott, 1995);
$u^\star$	0.058	Average civilian unemployment rate of 5.8% (BLS, 1948:q2–2016:q2);
γ	0.120	Average U.S. unemployment insurance gross replacement ratio of 12% (OECD);
		Implied set for $\theta = \eta u^\star (1 - u^\star)$ for which there is local indeterminacy
		$\theta \in \left[0.782, 1.077\right]$

7.1 Calibrated Parameters and the Indeterminacy Region

Not all parameters of the model are to be estimated. In particular, we follow the standard practice of calibrating some of the parameters to match selected secular features of the macroeconomic time series (as in, for example, Christiano et al., 2014). The calibration procedure is useful as it allows us to restrict more easily the domain of the model to include only the indeterminacy region. In particular, the eigenvalues of the matrix Γ are functions of the structural parameters $\Xi = (\alpha, \beta, \delta, u^\star, \mathcal{G}, \gamma)$ and the output-elasticity of unemployment, η, in the Okun's relationship. Thus, in order to have a clear characterization of the indeterminacy region, the strategy that we follow is to calibrate the elements of the parameter vector Ξ according to the secular features of the US macroeconomic time-series. The calibrated parameters and the targets informing this calibration are reported in Table 1. The upshot is that local indeterminacy hinges on the value of the Okun's coefficient $\theta = \eta u^\star (1 - u^\star)$. Therefore, we are able to restrict the support of the prior distribution of θ, to ensure that we estimate the model over the indeterminacy domain. This implied admissible set for is also reported in Table 1.

7.2 Data, Priors and Posteriors

The model is estimated for the U.S. economy using quarterly data over the period 1948:q2–2016:q2, using 5 macroeconomic time-series: GDP, consumption, investment, the real wage, and the rate of unemployment (a full description of the data set is provided in Appendix H). Based on these

Table 2: Prior and Posterior Distributions

Parameter	Prior distribution (mean; std. deviation)	Posterior mode	Posterior std. dev
η	normal distribution (17.0147;0.1);	17.5180	0.0775
ρ^z	Beta distribution (0.5; 0.2);	0.8784	0.0450
ρ^i	Beta distribution (0.5; 0.2);	0.2818	0.1490
$\sigma(\vartheta)$	Inverse-gamma (0.003; 0.001);	0.0028	0.9414‰
$\sigma(\nu^a)$	Inverse-gamma (0.003; 0.001);	0.0078	0.3997‰
$\sigma(\nu^z)$	Inverse-gamma (0.003; 0.001);	0.0037	0.1638‰
$\sigma(\nu^i)$	Inverse-gamma (0.003; 0.001);	0.0035	0.9168‰
$\sigma(\zeta^y)$	Inverse-gamma (1.000; 1.000);	3.5693	0.0346
$\sigma(\zeta^c)$	Inverse-gamma (1.000; 1.000);	4.5382	0.0368
$\sigma(\zeta^i)$	Inverse-gamma (2.000; 1.000);	5.9986	0.1900

time-series, the measurement equations are given by

$$\begin{bmatrix} 100\times\Delta\ln(\text{GDP})_t \\ 100\times\Delta\ln(\text{consumption})_t \\ 100\times\Delta\ln(\text{investment})_t \\ 100\times\Delta\ln(\text{real wage})_t \\ 100\times\Delta(\text{unemployment rate})_t \end{bmatrix} = \begin{bmatrix} 100\left(\mathcal{G}^4-1\right) \\ 100\left(\mathcal{G}^4-1\right) \\ 100\left(\mathcal{G}^4-1\right) \\ 100\left(\mathcal{G}^4-1\right) \\ 0 \end{bmatrix} + \begin{bmatrix} 400\left(\hat{y}_t-\hat{y}_{t-1}+\nu_t^a\right) \\ 400\left(\hat{c}_t-\hat{c}_{t-1}+\nu_t^a\right) \\ 400\left(\hat{i}_t-\hat{i}_{t-1}+\nu_t^a\right) \\ 400\left(\hat{w}_t-\hat{w}_{t-1}+\nu_t^a\right) \\ 100\left(u_t-u_{t-1}\right) \end{bmatrix} + \begin{bmatrix} \zeta_t^y \\ \zeta_t^c \\ \zeta_t^i \\ 0 \\ 0 \end{bmatrix}. \quad (42)$$

The theoretical economy includes 4 structural shocks: the permanent productivity shock, ν_t^a, the transitory productivity shock, ν_t^z, the transitory investment shock, ν_t^i, and the sunspot shock, ϑ_t. As this is one fewer than the number of time series used for the estimation, we need to assume that at least one variable is measured with error. Thus, we include the measurement errors ζ_t^y, ζ_t^c, ζ_t^i in the measurement equations for output, consumption and investment, respectively. The unemployment rate and the real wage are assumed to measured without error. There are 10 parameters that need to be estimated. They are η (the coefficient controlling the slope of the Okun curve), the autoregressive coefficients ρ^i and ρ^z, the standard deviations of the 4 structural shocks, and the standard deviations of the 3 measurement errors. As is standard, the mode of the posterior distribution is obtained by maximizing the log posterior function, and the Metropolis-Hastings algorithm is used to obtain the structural parameters' posterior marginal density functions, and the relevant impulse response functions.

As explained above, the support of the prior distribution for the parameter η is restricted so that $\theta\in[0.782,1.077]$, corresponding to the interval for which there is local indeterminacy, given the

Table 3: Variance Decomposition (in %)

Variable	ν^a	ν^z	ν^i	ϑ
$\Delta \ln Y$	42.00	37.70	9.35	10.95
$\Delta \ln C$	67.75	2.95	15.25	14.05
$\Delta \ln I$	36.30	45.54	8.27	9.89
$\Delta \ln W$	18.85	61.45	8.92	10.77
u	22.13	28.92	30.16	18.80

calibrated parameters. Thus, the prior density function for η is chosen to be a uniform distribution with support

$$\eta \in \left[\frac{0.782}{(1-u^\star)\, u^\star}, \frac{1.077}{(1-u^\star)\, u^\star} \right],$$

and mode in the mid-point of this interval. The choice of priors on the stochastic processes follows usual practice and is summarized in Table 2. In particular, the priors on the standard deviations of the structural shocks are set to Inverse-gamma distributions with mean 0.1% and standard deviation 0.1%, and similar distributions are assumed for the priors on the standard deviations of the measurement errors, with mean and standard deviation reported in Table 2. Finally, the prior for the persistence of the autoregressive processes, ρ^z and ρ^i, is the Beta distribution with mean 0.5 and standard deviation 0.2.

Table 2 also reports the posterior mode and standard deviation of the estimated parameters marginal posterior distribution. The fact that the standard deviations are substantially smaller than the prior standard deviation indicates that the likelihood function contributes substantially for the identification of the structural parameters. The posterior mode of the parameter η is found to be 17.6099 and there seems to be very little uncertainty concerning its value. This in turn implies a slope of the Okun relation, $-\theta = -\eta u^\star (1 - u^\star)$, given by 0.96 which is somewhat higher than the regression coefficient of -0.5 often reported in the literature for the Okun relation, although within the range found in empirical studies of the Okun's Law (for example in Ball et al., 2013).

7.3 Impulse Response Functions and Variance Decompositions

In Table 3 we show the contribution of each shock to the variance of the growth rate of output, consumption, investment and the real wage, and the level of unemployment. Both the permanent productivity shock and the transitory productivity shock are found to explain a substantial fraction of the overall volatility. Taken together, they explain more than 2/3 of the overall volatility. But, the sunspot shock explain almost 20% of the variance of unemployment and consumption, suggesting

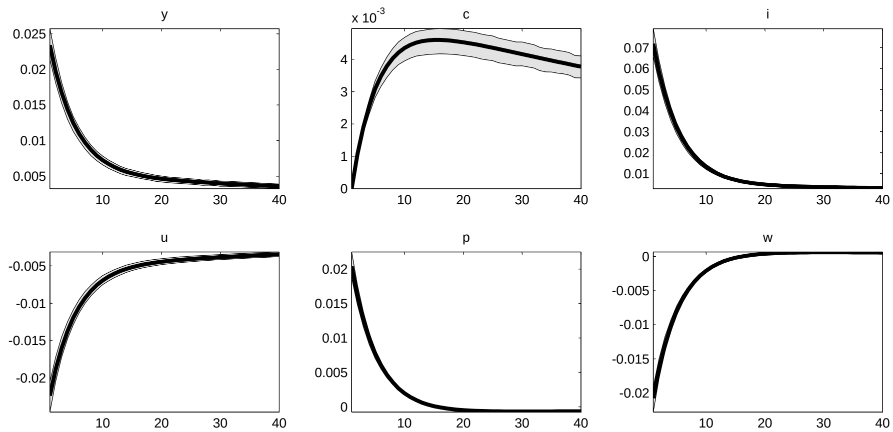

Figure 3: impulse response functions: permanent productivity shock, ν^a

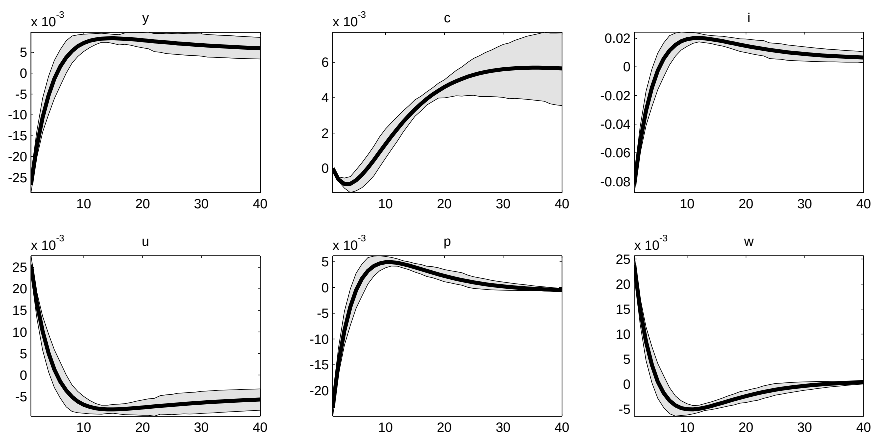

Figure 4: impulse response functions: transitory productivity shock, ν^z

that inefficient fluctuations driven by extrinsic uncertainty may have a substantial detrimental impact on welfare. In addition, the sunspot shock turns out to be more important to explain business cycle fluctuations than the investment shock.

Figures 3 and 4 show the impulse response functions to, respectively, permanent and transitory productivity shocks. Although the two productivity shocks generate responses which are similar in magnitude, they are qualitatively very different. While permanent productivity shocks are found to be expansionary in the short-run, a transitory productivity shock is found upon impact to lower output, investment and participation, and to raise unemployment, following from the discussion in Section 5. This finding, which is sharp contrast with the canonical RBC model is, instead,

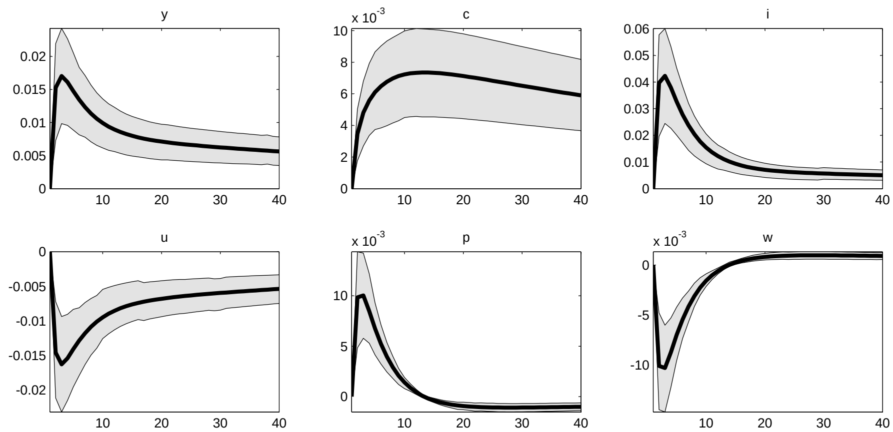

Figure 5: impulse response functions: investment shock, ν^i

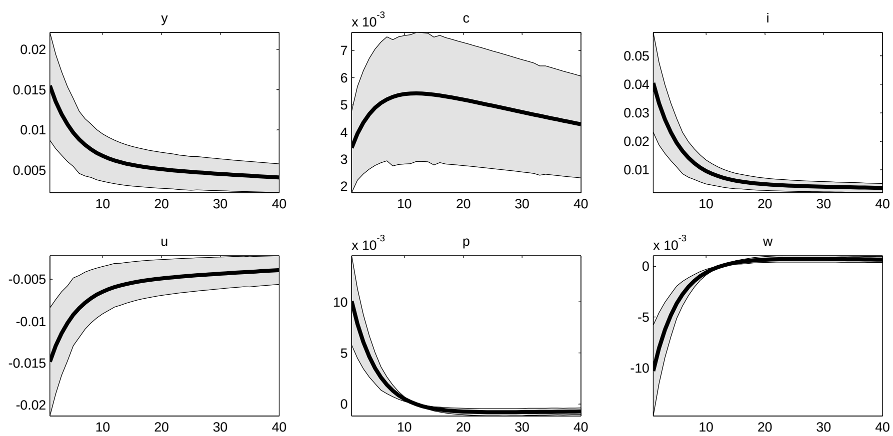

Figure 6: impulse response functions: sunspot shock, ϑ

consistent with the empirical findings by Gali (1999), using structural VAR, and Basu et al. (2006), using a “purified” Solow residual to measure TFP. Thus, our model seems to avoid an important shortcoming of standard RBC models, regarding the role of technology shocks.

Next, Figures 5 and 6 show the impulse response functions to, respectively, an investment and a sunspot shocks. It is found that the two shocks generate responses of consumption and of investment which are both qualitatively and quantitatively very similar. Thus, although the sunspot shock is found to explain a greater share of the variability in the data, this finding suggests that it may be difficult to identify the separate contributions of each shock. Indeed, with the kind of externalities in search that this paper considers, sunspot shocks and investment shocks play very similar roles. Importantly, our model generates a positive correlation in forcastable movements of consumption and employment conditional on a sunspot shock and is, therefore, immune to the Schmitt-Grohé (2000) critique of endogenous business cycle models. Similarly, our model generates positive comovement between consumption and employment conditional on an investment shock and, therefore, overcomes the Barro and King (1984) challenge.

8 Conclusion

We develop and estimate a business cycle model that successfully captures many salient features of the U.S. business cycle, including Okun's Law, and overcomes many difficulties faced by the canonical real business cycle model. Our framework includes involuntary unemployment due to search frictions, subject to externalities as in Diamond (1982). Self-fulfilling fluctuations emerge if search externalities are strong enough, as households expectations of high unemployment lower participation and are consistent with lower output through Okun's Law, in turn validating those expectations. We provide intuitive necessary conditions for self-fulfilling fluctuations to emerge and show how unemployment insurance is an important automatic stabilizer, even in an economy with perfect private insurance. Finally, we estimate the model using Bayesian methods and show that extrinsic uncertainty (sunspots) contribute to almost 20% of the volatility of consumption and unemployment and, therefore, have a substantially detrimental effect on welfare.

Appendix

A Decentralized Equilibrium and the Lottery Mechanism

This appendix explains in greater detail the institutional and market arrangements used to support the competitive equilibrium. The set-up follows the that of Hansen (1985) and also Andolfatto (1996), but extended to allow for three states $s \in \{1, 2, 3\}$ in the labor market: corresponding to employment, unemployment and non-participation, respectively. As in Andolfatto (1996), although individuals experience different employment histories, the existence of perfect insurance markets guarantee that labor income (net of insurance premia) will be independent of the employment history. Therefore, we may describe the problem of the stand-in agent. This appendix shows this formally.

Individuals choose a probability of participation each period, and conditional on participation face a probability of unemployment u, taken as given. As in Andolfatto (1996), individuals who participate in the labor market are exogenously shuffled around the available jobs regardless of employment history and, thus, u also corresponds to the unemployment rate. Those who participate in the labor market (either in employment or unemployed) give up $h_0 \in (0, 1)$ units of time. Finally, individuals have access to competitive insurance markets which implement complete markets allocations. In particular, at the start of each period, individuals may purchase the quantity of insurance y_2 and y_3, at price $p_2(\pi, u)$ and $p_3(\pi, u)$, where y_2 and y_3 are the units of consumption to be received in case of, respectively, unemployment and non-participation.

The problem solved by the stand-in agent can be represented as follows

$$\begin{aligned} \max_{\pi,c,y,k'} \mathbf{V}(k; u, K) = {} & \pi(1-u)\left[\ln(c_1) + \psi \ln(1-h_0) + \beta \mathbf{V}\left(k_1'; u', K'\right)\right] \\ & + \pi u \left[\ln(c_2) + \psi \ln(1-h_0) + \beta \mathbf{V}\left(k_2'; u', K'\right)\right] \\ & + (1-\pi)\left[\ln(c_3) + \beta \mathbf{V}\left(k_3'; u', K'\right)\right], \end{aligned} \tag{A.1}$$

subject to

$$c_1 + i_1 = w(u, K) + r(u, K)k - p_2(\pi, u)y_2 - p_3(\pi, u)y_3, \tag{A.2}$$

$$c_2 + i_2 = y_2 + r(u, K)k - p_2(\pi, u)y_2 - p_3(\pi, u)y_3, \tag{A.3}$$

$$c_3 + i_3 = y_3 + r(u, K)k - p_2(\pi, u)y_2 - p_3(\pi, u)y_3, \tag{A.4}$$

$$k_s' = (1-\delta)k + i_s, \quad \text{for} \quad s = 1, 2, 3, \tag{A.5}$$

where for simplicity, but without loss of generality, we consider a stationary economy with $\mathcal{G} = 1$ and without investment shocks, and where c_s, i_s and y_s are the allocations chosen contingent on the realization of state $s \in \{1, 2, 3\}$, that are employment, unemployment and non-participation, respectively.

The insurance company maximizes expected profits, given by

$$\wp = p_2(\pi, u)\, y_2 + p_3(\pi, u)\, y_3 - \pi u y_2 - (1 - \pi) y_3, \tag{A.6}$$

and, with competitive insurance markets, we have that $p_2(\pi, u) = \pi u$, and $p_3(\pi, u) = (1 - \pi)$.

Turning again to the stand-in agent's problem, we denote λ_1, λ_2 and λ_3 the multipliers of (A.2), (A.3) and (A.4). The first order condition with respect to y_2 and y_3 are

$$\lambda_1 = \lambda_2 \left(\frac{1 - \pi u}{\pi u} \right) - \lambda_3, \tag{A.7}$$

$$\lambda_1 = \lambda_3 \left(\frac{\pi}{1 - \pi} \right) - \lambda_2. \tag{A.8}$$

Combining these two necessary conditions obtains

$$\frac{\lambda_1}{\lambda_2} = \frac{1 - u}{u}, \quad \frac{\lambda_2}{\lambda_3} = \left(\frac{\pi}{1 - \pi} \right) u. \tag{A.9}$$

The first order conditions with respect to c_s, for $s \in \{1, 2, 3\}$, are

$$\pi(1 - u) = \lambda_1 c_1, \quad \pi u = \lambda_2 c_2, \quad (1 - \pi) = \lambda_3 c_3. \tag{A.10}$$

Combining (A.9) and (A.10) yields $c_1 = c_2 = c_3 = c$. From the envelope condition, we have that $1/c = \beta d\mathbf{V}(k_s; u, K)/dk_s = \mu$. Thus, this implies $k'_s = k'$ for all $s \in \{1, 2, 3\}$ and, given (A.5), $i_s = i$ for all $s \in \{1, 2, 3\}$. Finally, it follows from the budget constraints (A.2), (A.3) and (A.4) that $y_2 = y_3 = w$. Thus, the existence of perfect insurance markets guarantee that labor income (net of insurance premia) will be independent of the employment history.

It follows that the stand-in agent's problem can be equivalently written as follows

$$\max_{c_t, \pi_t} \mathbf{V}(k; u, K) = \max_{c, \pi, k'} \ln(c) + \psi \pi \ln(1 - h_0) + \beta \mathbf{V}(k'; u', K'), \tag{A.11}$$

with $c + k' - (1 - \delta) k = \pi (1 - u) w + rk$.

B Unemployment Insurance and the Lottery Mechanism

In addition to private insurance markets, unemployed households also benefit from government sponsored unemployment insurance. Specifically, the government replaces a fraction γ of wage income of the individuals who are unemployed. The government finances this policy by taxing everyone lump-sum, that is, subject to the following balanced-budget rule

$$T = \pi u \gamma w. \tag{B.1}$$

Compared to Appendix A, the stationary individual problem modifies as

$$\begin{aligned}\max_{\pi,c,y,k'} V(k;u,K) = \pi(1-u)\left[\ln(c_1) + \psi \ln(1-h_0) + \beta V\left(k_1';u',K'\right)\right] \\ + \pi u\left[\ln(c_2) + \psi \ln(1-h_0) + \beta V\left(k_2';u',K'\right)\right] \\ + (1-\pi)\left[\ln(c_3) + \beta V\left(k_3';u',K'\right)\right],\end{aligned} \tag{B.2}$$

subject to

$$c_1 + i_1 = w\,(u,K) + r\,(u,K)\,k - p_2\,(\pi,u)\,y_2 - p_3\,(\pi,u)\,y_3 - T(\pi,u,K), \tag{B.3}$$

$$c_2 + i_2 = y_2 + \gamma w(u,K) + r\,(u,K)\,k - p_2\,(\pi,u)\,y_2 - p_3\,(\pi,u)\,y_3 - T(\pi,u,K), \tag{B.4}$$

$$c_3 + i_3 = y_3 + r\,(u,K)\,k - p_2\,(\pi,u)\,y_2 - p_3\,(\pi,u)\,y_3 - T(\pi,u,K), \tag{B.5}$$

$$k_s' = (1-\delta)k + i_s, \quad \text{for} \quad s = 1,2,3. \tag{B.6}$$

As in Appendix A, $c_1 = c_2 = c_3 = c$, $k_1' = k_2' = k_3' = k'$, and $i_1 = i_2 = i_3 = i$. Making use of the above budget constraints, the upshot is $y_2 = w(1-\gamma)$ and $y_3 = w$. Thus, the previous problem can be written as

$$\max_{c_t,\pi_t} \mathbf{V}\,(k;u,K) = \max_{c,\pi,k'} \ln(c) + \psi\pi \ln(1-h_0) + \beta \mathbf{V}\left(k';u',K'\right), \tag{B.7}$$

with $c + k' - (1-\delta)\,k + T = \pi\,(1-u)\,w + \pi u \gamma w + rk$.

C Wage Determination

In this appendix we describe in greater detail the bargaining solution used to determine the wage received by workers that are successfully matched with a recruiter.

Each recruiter is matched to a single employee who supplies h_0 units of labor. In turn, each match produces one unit of the intermediate good (labor services), which is sold to final good producers at price w. Let ϖ denote the "hourly" wage received by the worker. We assume the Nash bargaining protocol, implying the following bargaining problem solved by each employer-employee pair

$$\max_{\varpi} \mathcal{H}(u, K)^{\chi} \mathcal{J}(u, K)^{1-\chi} \tag{C.1}$$

where $\mathcal{H}$ and $\mathcal{J}$ are the match surplus of, respectively, the employee and the employer, and with χ the employee's bargaining power.

The match surplus earned by the worker is

$$\mathcal{H}(u, K) = \mu(\varpi h_0 - (c_1 - c_2) - y_2) + \ln(c_1) - \ln(c_2), \tag{C.2}$$

where, following the notation in Appendix A, c_1 and c_2 are the consumption levels contingent on employment and unemployment, y_2 is the payment received by the private insurance in case of unemployment and μ is the marginal utility of wealth for the stand-in agent.

Next, the match surplus earned by the recruiter is

$$\mathcal{J}(u, K) = \mu(w - \varpi h_0) + \rho\beta\mu' \mathcal{J}(u', K'), \tag{C.3}$$

where ρ is the probability that the match is pursued the following period and x' denotes the continuation value of variable x.

The solution to the Nash bargaining problem must satisfy the necessary condition

$$\chi \mathcal{J}(u, K)\left(\frac{\partial \mathcal{H}}{\partial \varpi}\right) + (1-\chi)\mathcal{H}(u, K)\left(\frac{\partial \mathcal{J}}{\partial \varpi}\right) = 0. \tag{C.4}$$

We conjecture that an equilibrium solution to the bargaining problem of the is given by $\varpi h_0 = w$, no matter the bargaining power distribution, and proceed to verify this conjecture. First, notice that, as shown in Appendix A, in equilibrium $y_2 = w$ and $c_1 = c_2$. Thus, if $\varpi h_0 = w$, we have that $\mathcal{H}(u, K) = 0$, and condition (C.4) may be written as

$$\chi \mathcal{J}(u, K)\left(\frac{\partial \mathcal{H}}{\partial \varpi}\right) = 0. \tag{C.5}$$

In turn, if $\varpi h_0 = w$ the functional equation (C.3) becomes

$$\mathcal{J}(u, K) = \rho\beta\mu'\mathcal{J}\left(u', K'\right), \tag{C.6}$$

which has solution $\mathcal{J}(u, K) = 0$ for all (u, K). This solution is an admissible equilibrium solution, as it satisfies the free entry condition. In particular, since there are no costs of creating a vacancy, firms must have zero capital value. Finally, notice that with $\mathcal{J}(u, K) = 0$ condition (C.5) is satisfied and, therefore, the conjectured solution is an equilibrium solution, and the equilibrium wage rate is given by

$$\varpi = w/h_0. \tag{C.7}$$

Thus, although we allow for a bargaining protocol, the equilibrium outcome coincides with the competitive equilibrium in which workers are paid their marginal product. This is the upshot of two features of the economy: the zero cost of creating vacancies which forces the capital value of a job to zero, and the perfect insurance markets.

D Steady State

Let $\underline{X}$ denote the steady state of $\widetilde{X}$. For a given natural rate of unemployment $u^\star \in (0, 1)$, the following equations uniquely characterize the steady state of the model

$$(\underline{K}/\underline{Y}) = \left[\frac{\alpha}{1/\beta - (1-\delta)/\mathcal{G}}\right], \tag{D.1}$$

$$(\underline{C}/\underline{K}) = \left[\frac{1/\beta - (1-\delta)/\mathcal{G}}{\alpha} + \frac{1-\delta}{\mathcal{G}} - 1\right], \tag{D.2}$$

$$(\underline{I}/\underline{K}) = \left[1 - \frac{(1-\delta)}{\mathcal{G}}\right], \tag{D.3}$$

$$\underline{\Pi} = \left[\frac{(1-\alpha)(\underline{Y}/\underline{C})}{\phi}\right], \tag{D.4}$$

$$\underline{K} = \left[\frac{(\underline{K}/\underline{Y})}{\mathcal{G}^\alpha}\right]^{1/(1-\alpha)} \underline{\Pi}(1-u^\star). \tag{D.5}$$

Thus, if we do not specify a natural rate the model exhibits a multiplicity of steady state equilibria, for each possible $u^\star$. In turn, from (2) the natural rate of unemployment $u^\star$ is given by

$$u^\star = \frac{\mu\underline{Y}^{-\eta}}{1 + \mu\underline{Y}^{-\eta}}, \tag{D.6}$$

This equation has a unique interior solution for $\eta \in (0,1)$ and has at most two solutions.

E Log-linear Equilibrium Conditions

Let $\hat{x} \equiv \ln\left(\widetilde{X}/\underline{X}\right)$ denote the variable $\widetilde{X}$ in log-deviation from steady state. The log-linearized equilibrium conditions (around the deterministic steady state) are given by

$$\hat{\pi}_t + \hat{c}_t = \hat{y}_t, \tag{E.1}$$

$$E_t\left(\hat{c}_{t+1} - \hat{c}_t\right) = \alpha\beta\left(\underline{Y}/\underline{K}\right) E_t\left(\hat{y}_{t+1} - \hat{k}_{t+1}\right) + \left(1-\rho^i\right)\epsilon_t^i, \tag{E.2}$$

$$\left(1-u^\star\right)\hat{y}_t = \alpha\left(1-u^\star\right)\hat{k}_t + \left(1-\alpha\right)\left(1-u^\star\right)\hat{\pi}_t - \left(1-\alpha\right)\left(u_t - u^\star\right) + \left(1-u^\star\right)\epsilon_t^z, \tag{E.3}$$

$$\left(\underline{C}/\underline{K}\right)\hat{c}_t + \left(\underline{I}/\underline{K}\right)\hat{i}_t = \left(\underline{Y}/\underline{K}\right)\hat{y}_t, \tag{E.4}$$

$$\hat{k}_{t+1} = \left(\underline{I}/\underline{K}\right)\left(\epsilon_t^i + \hat{i}_t\right) + \frac{(1-\delta)}{\mathcal{G}}\left(\hat{k}_t - \nu_t^a\right), \tag{E.5}$$

$$u_t - u^\star = -\theta\hat{y}_t, \tag{E.6}$$

$$\epsilon_t^i = \rho^i\epsilon_{t-1}^i + \nu_t^i, \tag{E.7}$$

$$\epsilon_t^z = \rho^z\epsilon_{t-1}^z + \nu_t^z. \tag{E.8}$$

with the unemployment gap, $(u - u^\star)$, included in levels instead of logs.

Thus, the deterministic version of the model in log-linear form can be written as

$$\begin{bmatrix} \hat{k}_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} = \Gamma \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} \tag{E.9}$$

with

$$\Gamma_{11} = \left[\frac{\alpha\left(\underline{Y}/\underline{K}\right)}{\alpha - (1-\alpha)\sigma} + \frac{1-\delta}{\mathcal{G}}\right],$$

$$\Gamma_{12} = -\left[\frac{(1-\alpha)\left(\underline{Y}/\underline{K}\right)}{\alpha - (1-\alpha)\sigma} + \left(\underline{C}/\underline{K}\right)\right],$$

$$\Gamma_{21} = \left[\frac{\alpha(1-\alpha)\beta\left(\underline{Y}/\underline{K}\right)\sigma\Gamma_{11}}{\alpha - (1-\alpha)\sigma + \alpha(1-\alpha)\beta\left(\underline{Y}/\underline{K}\right)}\right],$$

$$\Gamma_{22} = \left[\frac{\alpha - (1-\alpha)\,\sigma + \alpha\,(1-\alpha)\,\beta\,(\underline{Y}/\underline{K})\,\sigma\Gamma_{12}}{\alpha - (1-\alpha)\,\sigma + \alpha\,(1-\alpha)\,\beta\,(\underline{Y}/\underline{K})} \right].$$

and where $\sigma = \eta u^{\star}$. The eigenvalues of the matrix Γ are

$$\begin{bmatrix} \kappa_1 \\ \kappa_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \Gamma_{11} + \Gamma_{22} - \sqrt{(\Gamma_{11}+\Gamma_{22})^2 - 4\,(\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21})} \\ \Gamma_{11} + \Gamma_{22} + \sqrt{(\Gamma_{11}+\Gamma_{22})^2 - 4\,(\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21})} \end{bmatrix}. \tag{E.10}$$

Indeterminacy arises if the real part of both eigenvalues are less than one in absolute value.

F Proof of Proposition 2

Following the approach of Wen (2001), consider the consumption of the stand-in household at date t

$$c_t = (1-\beta)\left[R_t k_t + w_t\,(1-u_t)\,\pi_t + \sum_{j=1}^{\infty}\left(\prod_{i=1}^{j} R_{t+i}^{-1}\right) w_{t+j}\,(1-u_{t+j})\,\pi_{t+j}\right], \tag{F.1}$$

where $R_t = r_t + (1-\delta)$ is the gross rate of return on capital. Making use of the intratemporal condition (8) and the market clearing conditions, we obtain

$$\widetilde{C}_t = (1-\beta)\left[(R_t/\mathcal{G})\,\widetilde{K}_t + \phi\widetilde{C}_t\Pi_t + \phi\sum_{j=1}^{\infty}\left(\prod_{i=1}^{j} R_{t+i}^{-1}\right)\mathcal{G}^j\widetilde{C}_{t+j}\Pi_{t+j}\right]. \tag{F.2}$$

In turn, solving forward the aggregate Euler equation yields

$$\left(\prod_{i=1}^{j} R_{t+j}^{-1}\right)\mathcal{G}^j\widetilde{C}_{t+j} = \beta^j\widetilde{C}_t. \tag{F.3}$$

Substituting the latter into the previous relationship, we obtain

$$\widetilde{C}_t = (R_t/\mathcal{G})\,\widetilde{K}_t\left[\sum_{j=0}^{\infty}\beta^j\,(1-\phi\Pi_{t+j})\right]^{-1}. \tag{F.4}$$

Finally, substituting for consumption using the intra-temporal first order condition and market clearing conditions, yields

$$\mathcal{G}\sum_{j=0}^{\infty}\beta^{j}\left(1-\phi\Pi_{t+j}\right)=\frac{\phi R\left(u_{t},\Pi_{t}\right)\widetilde{K}_{t}}{(1-u_{t})\widetilde{w}\left(u_{t},\Pi_{t}\right)}, \tag{F.5}$$

with $R\left(u_{t},\Pi_{t}\right)=(1-\delta)+\alpha\left[\frac{(1-u_{t})\mathcal{G}\Pi_{t}}{\underline{K}}\right]^{1-\alpha}$, and $\widetilde{w}\left(u_{t},\Pi_{t}\right)=(1-\alpha)\left[\frac{\underline{K}}{\mathcal{G}(1-u_{t})\Pi_{t}}\right]^{\alpha}$.

We want to obtain necessary conditions for local indeterminacy. Thus, the next step is to log-linearise the difference equation (F.5) around the steady state given the initial condition $\widetilde{K}_{t}=\underline{K}$. The log-linear version of (F.5) is

$$\frac{-\left(1-\beta\right)\phi\underline{\Pi}}{1-\phi\underline{\Pi}}\sum_{j=0}^{\infty}\beta^{j}\hat{\pi}_{t+j}=\left(\xi\left(\underline{R},\underline{\Pi}\right)-\xi\left(1-u^{\star},\underline{\Pi}\right)-\xi\left(\underline{w},\underline{\Pi}\right)\right)\hat{\pi}_{t}, \tag{F.6}$$

where $\xi\left(x,y\right)=\frac{dx}{dy}\left(y/x\right)$ denotes the total elasticity.

The relevant elasticities are given by

$$\xi\left(\underline{R},\underline{\Pi}\right)=\left[\frac{\left(1-\alpha\right)\left(1-\left(1-\delta\right)\left(\beta/\mathcal{G}\right)\right)}{1-\eta(1-\alpha)u^{\star}}\right], \tag{F.7}$$

$$\xi\left(1-u^{\star},\underline{\Pi}\right)=\left[\frac{\eta\left(1-\alpha\right)u^{\star}}{1-\eta(1-\alpha)u^{\star}}\right], \tag{F.8}$$

$$\xi\left(\underline{w},\underline{\Pi}\right)=-\left[\frac{\alpha}{1-\eta(1-\alpha)u^{\star}}\right], \tag{F.9}$$

where (F.8), in particular, is obtained from the total differentiation of (2).

Starting from an equilibrium path, a small deviation of Π_{t} from this path will not violate (F.6) if and only if the change in both sides of this equation caused by the change in Π_{t} are exactly the same. Since the elasticity of the left-hand side of equation (F.6) is negative, a necessary condition for local indeterminacy is that

$$\xi\left(\underline{R},\underline{\Pi}\right)-\xi\left(1-u^{\star},\underline{\Pi}\right)-\xi\left(\underline{w},\underline{\Pi}\right)\leq 0, \tag{F.10}$$

and condition (F.10) is equivalent to

$$\left[\frac{(1-\alpha)(1-\delta)\left(\beta/\mathcal{G}\right)}{1-\eta(1-\alpha)u^{\star}}\right]\geq 1, \tag{F.11}$$

This condition is satisfied if

$$\eta u^{\star} \in \left[\frac{1}{1-\alpha} - (1-\delta)\left(\beta/\mathcal{G}\right), \frac{1}{1-\alpha}\right]. \tag{F.12}$$

G Proof of Proposition 3

First, we compute the new steady state with UI and then we verify the claim in Proposition 3. The new steady state modifies as

$$(\underline{K}/\underline{Y}) = \left[\frac{\alpha}{1/\beta - (1-\delta)/\mathcal{G}}\right], \tag{G.1}$$

$$\underline{\Pi} = \left[\frac{(1-\alpha)\left(\underline{Y}/\underline{C}\right)}{\phi}\right]\frac{1-u^{\star}(1-\gamma)}{1-u^{\star}}, \tag{G.2}$$

$$\underline{K} = \left[\frac{(\underline{K}/\underline{Y})}{\mathcal{G}^{\alpha}}\right]^{1/(1-\alpha)}\underline{\Pi}\left(1-u^{\star}\right). \tag{G.3}$$

As in the model without UI, steady state unemployment is given by

$$u^{\star} = \frac{\mu\underline{Y}^{-\eta}}{1+\mu\underline{Y}^{-\eta}}, \tag{G.4}$$

The log-linear equilibrium conditions are identical to those for the economy without UI, except for the intratemporal condition governing consumption and leisure choices, which is now given by

$$\widehat{c}_t + \widehat{\pi}_t = \widehat{y}_t + \left[\frac{\gamma}{(1-u^{\star})\left(1-u^{\star}\left(1-\gamma\right)\right)}\right]\left(u_t - u^{\star}\right). \tag{G.5}$$

Thus, the system of equilibrium conditions is still given by (E.9), with the only difference that the coefficient σ is now given by

$$\sigma = \left[\frac{\theta\left(1-\gamma\right)}{1-u^{\star}\left(1-\gamma\right)}\right].$$

We now turn to Proposition 3. Consider a baseline economy without UI, which is obtained when $\gamma = 0$, and let the parametrization of this economy be such that both eigenvalues κ_1 and κ_2 (defined in Appendix E) are inside the unit circle. Thus, in the baseline economy there is local indeterminacy of equilibrium around the BGP. Next, consider an otherwise identical economy, but for which there is UI with full replacement ratio, so that $\gamma = 1$. For $\gamma = 1$, we obtain $\sigma = 0$ and the eigenvalues κ_1

and κ_2 simplify as

$$\begin{aligned} \kappa_1 &= \Gamma_{11} > 1, \\ \kappa_2 &= \Gamma_{22} \in (0,1)\,, \end{aligned} \tag{G.6}$$

given $(\beta/\mathcal{G}) \in (0,1)$. Therefore, the economy with full UI replacement ratio ($\gamma = 1$) must be locally determinate (saddle path stable).

Finally, since the eigenvalues are continuous functions of γ, it follows that, in the neighborhood of the economy with full UI replacement ratio, there exists a range of $\gamma \in \left(\underline{\gamma}, 1\right]$ for which the equilibrium is saddle path stable.

H Data

To be completed.

I Additional derivations and results

Consider the following total derivative

$$\begin{aligned} \frac{dY}{d\Pi} &= (1-\alpha)\, K^{\alpha} (1-u)^{-\alpha}\, \Pi^{-\alpha} \left[1 - u + \frac{d\,(1-u)}{d\Pi}\Pi\right], \\ &= (1-\alpha)\, Y \left[\frac{1}{\Pi} + \frac{\theta}{Y}\left(\frac{dY}{d\Pi}\right)\frac{1}{1-u}\right]. \end{aligned} \tag{I.1}$$

Next, from (2), we obtain

$$\begin{aligned} \frac{d\,(1-u)}{d\Pi} &= \frac{\eta Y^{-\eta-1}}{(1+Y^{-\eta})^2}\left(\frac{dY}{d\Pi}\right), \\ &= \frac{\eta}{1+Y^{-\eta}}\frac{Y^{-\eta-1}}{1+Y^{-\eta}}\left(\frac{dY}{d\Pi}\right), \\ &= \theta\left(\frac{dY}{d\Pi}\right)Y^{-1} = \theta\left(\frac{dY}{d\Pi}\frac{\Pi}{Y}\right)\Pi^{-1}. \end{aligned} \tag{I.2}$$

Combining equations (I.1) and (I.2) yields

$$\frac{dY}{d\Pi}\frac{\Pi}{Y} = \left[\frac{(1-\alpha)\,(1-u)}{1-u-\theta\,(1-\alpha)}\right], \tag{I.3}$$

the total elasticity of output to changes in participation.

Making use of the above result, we obtain

$$\begin{aligned}\frac{dR}{d\Pi} &= \alpha \left(\frac{dY}{d\Pi}\right) K^{-1}, \\ &= \alpha \left[\frac{(1-\alpha)(1-u)(Y/K)}{1-u-\theta(1-\alpha)}\right] \Pi^{-1},\end{aligned} \tag{I.4}$$

and, using the fact that $R = 1 - \delta + \alpha (Y/K)$, yields

$$\frac{dR}{d\Pi}\frac{\Pi}{R} = \left[\frac{(1-\alpha)(1-u)}{1-u-\theta(1-\alpha)}\right]\left[\frac{\alpha (Y/K)}{1-\delta+\alpha (Y/K)}\right]. \tag{I.5}$$

Similarly, making use of the fact that $w = (1-\alpha)(Y/\Pi)/(1-u)$, we obtain

$$\begin{aligned}\frac{dw}{d\Pi} &= \frac{(1-\alpha)}{(1-u)\Pi}\left(\frac{dY}{d\Pi}\right) - \frac{(1-\alpha)Y}{(1-u)^2\Pi^2}\left[1-u+\frac{d(1-u)}{d\Pi}\Pi\right], \\ &= \frac{w}{(1-u)\Pi}\left[\frac{dY}{d\Pi}\frac{\Pi}{Y}(1-u-\theta)-(1-u)\right],\end{aligned} \tag{I.6}$$

which yields the total elasticity

$$\begin{aligned}\frac{dw}{d\Pi}\frac{\Pi}{w} &= \frac{dY}{d\Pi}\frac{\Pi}{Y}\frac{1-u-\theta}{1-u} - 1, \\ &= \left[\frac{(1-\alpha)(1-u-\theta)}{1-u-\theta(1-\alpha)}\right] - 1, \\ &= -\left[\frac{\alpha(1-u)}{1-u-\theta(1-\alpha)}\right].\end{aligned} \tag{I.7}$$

Finally, notice that

$$\begin{aligned}\frac{du}{d\Pi}\frac{\Pi}{u} &= -\frac{d(1-u)}{d\Pi}\frac{\Pi}{1-u}, \\ &= -\theta\left(\frac{dY}{d\Pi}\frac{\Pi}{Y}\right)\frac{1}{1-u}, \\ &= -\left[\frac{\theta(1-\alpha)}{1-u-\theta(1-\alpha)}\right].\end{aligned} \tag{I.8}$$

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