A Monetary Theory of Observational Equivalence with the Taylor Principle*

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Abstract

The paper uses a monetary economy with banking producing an exchange substitute to money. This endogenizes velocity with a unique resulting feature of the Euler equation for interest rates. The Euler equation of the economy is a generalization of a broad class of "Taylor rules", with the key parameter on the inflation term being always greater or equal to one as in the Taylor Principle, despite the central bank's role of long run inflation rate targeting through its supply of money. Unlike previous studies that used monetary economies to compare to the Taylor rule and then find an inflation coefficient of one, as in cash-only cash-in-advance economies, this paper instead has a closed form solution for the parameter on the inflation rate term that exceeds one for all interest rates above zero because this causes velocity to rise above one. A Lucas critique emerges in that as the stationary long run money supply growth rate increases, the "behavioral parameter" of the coefficient on the inflation rate also rises, showing that such Taylor rule coefficients are not invariant to monetary policy changes such as the higher rate of money supply growth after the Great Recession. The paper verifies the validity of the model's Taylor equation from its Euler equilibrium condition by estimating the interest rate rule from simulated data. When expected inflation equals actual inflation, the interest rate equation reduces to a Fisher form in the long term.

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1 Introduction

The lack of a unique identification of the Taylor rule through empirical estimation is well-known (Cochrane, 2011). This paper presents an economy however that derives an exact Euler condition that includes an endogenous coefficient on the inflation rate term that is always greater than or equal to one. This coefficient depends on the underlieing utility and technology parameters of the model, plus one policy parameter: the stationary money supply growth rate. When the stationary money supply growth rate is set at the Friedman (1969) optimum, such that the nominal interest rate is zero, then money velocity is one and the inflation coefficient is one. When the stationary money supply growth rate is higher than the optimum, such that the nominal interest rate is greater than zero, then money velocity is greater than one and so is the inflation rate coefficient of the Euler asset pricing equation.

Estimation of the model's Euler condition should thereby pass the first test on what is deemed to be required in order to be competitive with a type of observational equivalence to what is deemed to be estimated in a Taylor equation estimation, in which the Taylor principle of an inflation rate coefficient greater than one is found. The difference in the Taylor rule estimation and the interest rate estimation in this model economy is that rather than the estimation resulting from an active central bank trade-off in deciding monetary policy, here the economy's equilibrium condition is a result of money simply being supplied as an exogenous shock around a stationary growth rate as the result of balancing the federal budget.

The Taylor-like feature of the economy results from the particular fashion in which velocity is made endogenous. It cannot be reproduced in a standard cash-in-advance economy with only money as a means of exchange. It can be reproduced in an cash-in-advance economy with a transactions cost function added onto it that represents the cost of using an alternative means of exchange. But the problem with just any amalgous transactions cost function is that these are typically backed out so as to yield a Baumol-Tobin or Cagan money demand function. And this guarantees nothing about the particular Euler equation of the economy, such as the ingenious general representation found in Bansal and Coleman (1996).

In contrast, the transaction cost of the economy is the one found in actual markets: the cost of the financial intermediation industry for which there is a broad micro-based literature. Using the advance by Clark (1984) and Hancock (1985), the financial services are produced using labor, capital and financial deposits that the bank for example collects. This route then gives a transactions cost to exchange credit equal to the average cost of exchange per dollar as produced by the financial intermediary that is supplying the exchange credit. And because this average cost of exchange per unit is less that the nominal interest rate, the exchange credit is used up to the point at which the marginal cost of the production of exchange credit per unit just equals the marginal cost of exchange from using money, which is the nominal interest foregone. Since the average cost of exchange equals a weighted average of the cost of using money and the cost of using exchange credit, and average exchange cost using exchange credit is less than the nominal interest rate, the average exchange cost is less than the nominal interest rate.

The nominal interest rate is the exchange cost per unit in a cash-only cash-in-advance economy, but with exchange credit, there is a second interest rate that represents this weighted average of the cost of the two alternative means of exchange and this average is less than the nominal interest rate. As a result, both the expected next period nominal rate and the expected average cost of exchange using the two means of exchange enter the Euler equation, and do so as a ratio. Normally, in a cash-only version of the model, the next period expected interest rate would drop out. But here, the ratio of the next period nominal interest rate to the next period average cost of exchange using the two means of exchange enters the Euler equation of the asset pricing of the nominal interest rate on the government bond. Now that there are two interest rates, one the usual nominal interest rate and the second the rate that is a weighted average of the two exchange costs, the weights used in the latter weighted average also enter the Euler equation.

These weights are simply the fraction of exchange done with cash versus the fraction of the exchange done using credit, with the fractions adding to one. But of course the fraction of consumption bought with cash is the inverse of the consumption velocity of money, while the other fraction of that done with credit could be called the inverse of the consumption velocity of credit. Through these weights the velocity of money enters the Euler equation. If the velocity is one, the weights go to unity for money and zero for credit, and the Euler equation is the standard one with a coefficient of unity on the inflation rate term. But if the weights put some usage on exchange credit, then the velocity enters such that the coefficient on inflation rises above one.

The reason for the coefficient on inflation being greater than one is that the consumer decides to evade the inflation tax by using some cheaper on average exchange credit. The evasion of the inflation tax means that the interest rate is a weighted average of the current expected interest rate and the future expected interest rate with the current growth rate of velocity and the expected future growth rate of velocity being the weighting terms. As long as the expected future interest rate and the current interest rate differ, the inflation rate affects the nominal interest rate by a factor that rises above one the higher is the stationary money supply growth rate. And with the expected future interest rate different from the actual interest rate, the interest rate equation is a forward-looking one that depends as well on the future expected interest rate.

When the stationary money supply growth rate is the Friedman optimum, the inflation factor on interest rates drops to one, even when the expected future interest rate is different from the current interest rate. However, once the expected future interest rate equals the actual interest rate, the inflation factor also drops to one regardless of the rate of the stationary money supply growth. In this convergence of expected and actual interest rates, the equilibrium interest rate loses its the forward looking term. This nice equilibrium sees the case of the Fisher equation holding, with a nominal interest rate that depends only upon the expected inflation rate, with a factor of one on this term, since the velocity effect that raises the inflation rate term gets canceled out by having the same velocity effect on the now single, current period, interest rate term. The expected growth rate of consumption, factored by its utility elasticity of substitution, then comprises the real interest rate part of the Fisher type interest rate equation, and the only other part if also the equilibrium is a balanced growth path equilibrium.

The simulated data of this economy provides a data base from which the Euler equation can be estimated. The result that the estimation from the simulated data yields an equation that is observationally equivalent to various versions of Taylor rule estimations suggests that economists estimating such equations could indeed simply be showing how the market formulates the interest rate asset pricing condition in an economy in which the central bank provides money to balance the budget.

Interpreting interest rate estimations may help in enabling central banks to continue to successfully target inflation without undue macroprudential policy distortion. As Mervyn King (2016) writes, "timeless" interest rate rules have a "fundamental flaw" "since our understanding of the economy is incomplete and constantly evolving" (p.169). The contributions of this paper on how interest rates may be formed in the actual economy can improve the ability of "the central bank to justify its behaviour in terms of presenting convincing economic arguments and evidence for them" (King, p. 169).[1]

2 Related Literature

The paper uses a monetary economy with a Cobb-Douglas banking function to provide an exchange credit service and examines its Euler interest rate equation over the business cycle and lower frequencies. The bank production function is founded in the microeconomic banking literature's estimation of its form, which includes deposited funds as a "third" factor of production. In general equilibrium, this allows a unique equilibrium between money and exchange credit; it solves the King and Plosser (1984) dilemma of how to include the production of exchange credit within a monetary business cycle economy. The Euler equation is identical in form to that of Taylor equations that are commonly estimated in

[1] For example, rising inflation and rising economic growth can happen in what Stanley Fischer is quoted as a Phillips curve behavior (Wall Street Journal, November 15, 2016, "Fed's Fischer, Yellen, Have Faith in the Phillips Curve"), but here in this economy, an inflation and employment rate positive correlation would result because the inflation tax may lessen output growth (Gillman and Kejak, 2005a,b, 2011) while not keeping output growth from advancing during a real business cycle expansion.

the literature, the inference being that the model's Euler-Taylor equation is plausible for real data estimation. The difference is that, in this endogenous velocity model, the inflation coefficient at the heart of the Taylor rule has a close formed solution as a function of the economies balanced growth path equilibrium and that increases when the monetary policy parameter of the central bank's long run money supply growth increases. By estimating the model's Euler equation of interest rates using simulated data, we also provide support for Fama's (2012) perspective that estimation of interest rates may reflect the economy's fundamental features rather than representing a given central bank reaction function per se. This demonstration provides a different angle than Canzoneri et al. (2007) by using simulated data to show that the theoretical Euler equation is not rejected while they used actual data for their estimation.

The economy with "banking time" rather than shopping time is one which produces a realistic money demand without reverse engineering the utility function or the transaction cost function (eg. Lucas, 2000) in order to yield either a Baumol (1952)-Tobin (1956) or Cagan (1956) function. Rather the Cobb-Douglas function of the financial intermediation microfoundations yields a velocity function that Benk et al. (2010) show explains fluctuations in US velocity over an 86 year period, as an extension of the stable money demand literature examined by for example Kuttner and Friedman (1992), Hoffman, Rasche, and Tieslau (1995) and Lucas and Nicolini (2015). Extending this Benk et al. economy by decentralizing the bank sector as in Gillman and Kejak (2011), the paper's overriding contribution is in its meeting the Canzoneri et al. (2007) challenge for monetary Euler equations. By using a model that reliably explains velocity and money demand, as based in banking microfoundations, it makes the Euler equation investigation one of an economy that successfully explains key monetary factors as based on the microfoundations of the financial intermediation sector of Berger and Humphrey (1997). A well-founded velocity weighs heavily in the plausibility of the analysis since the economy's Euler equation includes the balanced growth path (BGP) velocity as the key determinate the effect of expected inflation on the interest rate, as well as a main factor similarly impacting the other variables of the Euler equation.

In qualification, the paper uses the "price-theoretic" microfoundations of production functions and utility parameters, so that the money demand velocity is derived from the "Baumol" equation that sets the price of exchange credit to its marginal cost which in turn equals its marginal factor cost divided by its marginal factor product, as well as equaling the marginal cost of money. This is used rather than the "deeper" search-theoretic foundations of a rapidly growing literature of New Monetarism, as perhaps best reviewed by Lagos, Rocheteau and Wright (2016). They offer a comparison view of the cash-in-advance paradigm which still seemingly may arise in the New Monetarist models according to Lagos et al., but as a result of a different motivation than the exchange motive of Lucas (1980). Instead, here the Lucas (1980, 1990), Lucas and Stokey (1987), Bansil and Coleman (1996) and Alvarez, Lucas and Weber (2001) focus on the implied interest rate Euler equation of the "old monetarist" monetary economy is extended to when the consumer is assumed to use exchange credit to optimally avoid the inflation tax. An inflation tax view is also taken by Lagos et al. who characterize "the effects of inflation — a tax on the use of money". Or in Lagos et al and Hicks (1935) terms, we "model the friction" posed by exchange through the resources used up in banking in order to avoid the inflation tax.[2] This makes the paper a contribution to the "old monetarism" such that it remains consistent with the New monetarism by its use of avoidance of the inflation tax using a "segmented market" for its exchange credit production by banks and in its recent focus on policy especially as that occurring during the recent low interest rate regime.

Section 3 describes the extended economy; Section 4 derives the model's Euler equation and Section 5 provides the calibration. Section 6 describes the econometric methodology applied to model-simulated data and presents the estimation results. Section 7 provides discussion of other approaches and Section 8 Concludes.

3 Stochastic Endogenous Growth with Banking

The representative agent economy is modified from Benk et. al (2008) by decentralizing the bank sector that produces the exchange credit as a means of avoiding the inflation tax. Velocity is thereby endogenous according to the use of money versus exchange credit. By combining the business cycle with endogenous growth, stationary inflation lowers the output growth rate as supported empirically in Gillman et al. (2004) and Fountas et al. (2006), for example. Further, money supply shocks can cause inflation at low frequencies, as in Haug and Dewald (2012) and as supported by Sargent and Surico (2008, 2011), which can lead to output growth effects if the shocks are persistent and repeated. This allows shocks over the business cycle to cause changes in growth rates and in stationary ratios. The shocks to the goods sector productivity and the money supply growth rate are standard, while the third shock to credit sector productivity exists by virtue of the model's endogenous money velocity. Exchange credit is produced via a functional form used extensively in the financial intermediation microeconomics literature starting with Clark (1984) and promulgated by Berger and Humphrey (1997) and Inklaar and Wang (2013), for example.

The shocks occur at the beginning of the period, are observed by the consumer before the decision making process commences, and follow a vector first-order autoregressive process. For goods sector productivity, z_t, the money supply growth rate, u_t, and bank

[2] Hicks (1935) states that to model the friction, '. my suggestion can be expressed by saymg that we ought to regard every individual in the community as being, on a small scale, a bank. Monetary theory becomes a sort of generalization of banking theory.'

sector productivity, v_t:

$$Z_t = \Phi_Z Z_{t-1} + \varepsilon_{Zt}, \tag{1}$$

where the shocks are $Z_t = [z_t \ u_t \ v_t]'$, the autocorrelation matrix is $\Phi_Z = diag\{\varphi_z, \varphi_u, \varphi_v\}$ and $\varphi_z, \varphi_u, \varphi_v \in (0,1)$ are autocorrelation parameters, and the shock innovations are $\varepsilon_{Zt} = [\epsilon_{zt} \ \epsilon_{ut} \ \epsilon_{vt}]' \sim N(0, \Sigma)$. The general structure of the second-order moments is assumed to be given by the variance-covariance matrix Σ.

A representative consumer has expected lifetime utility from consumption of goods, c_t, and leisure, x_t; with $\beta \in (0,1)$, $\psi > 0$ and $\theta > 0$, this is given by

$$U = E_0 \sum_{t=0}^{\infty} \beta \frac{(c_t x_t^{\psi})^{1-\theta}}{1-\theta}. \tag{2}$$

Output of goods, y_t, and increases in human capital, are produced with physical capital and effective labor each in Cobb-Douglas fashion; the bank sector produces exchange credit using labor and deposits as inputs. Let s_{Gt} and s_{Ht} denote the fractions of physical capital that the agent uses in goods production (G) and human capital investment (H), whereby:

$$s_{Gt} + s_{Ht} = 1. \tag{3}$$

The agent allocates a time endowment of one between leisure, x_t, labor in goods production, l_{Gt}, time spent investing in the stock of human capital, l_{Ht}, and time spent working in the bank sector (F subscripts for Finance), denoted by l_{Ft}:

$$l_{Gt} + l_{Ht} + l_{Ft} + x_t = 1. \tag{4}$$

Output of goods can be converted into physical capital, k_t, without cost and is thus divided between consumption goods and investment, denoted by i_t, net of capital depreciation. The capital stock used for production in the next period is therefore given by:

$$k_{t+1} = (1-\delta_k)k_t + i_t = (1-\delta_k)k_t + y_t - c_t. \tag{5}$$

The human capital investment is produced using capital $s_{Ht}k_t$ and effective labor $l_{Ht}h_t$, with $A_H > 0$ and $\eta \in [0,1]$, such that the human capital flow constraint is

$$h_{t+1} = (1-\delta_h)h_t + A_H(s_{Ht}k_t)^{1-\eta}(l_{Ht}h_t)^{\eta}. \tag{6}$$

With w_t and r_t denoting the real wage and real interest rate, the consumer receives nominal income of wages and rents, $P_t w_t (l_{Gt} + l_{Ft}) h_t$ and $P_t r_t s_{Gt} k_t$, a nominal transfer from the government, T_t, and dividends from the bank. The consumer buys shares in the bank by making deposits of income at the bank. Each dollar deposited buys one share at a fixed price of one, and the consumer receives the residual profit of the bank as dividend income in proportion to the number of shares (deposits) owned. Denoting the real quantity of deposits by d_t, and the dividend per unit of deposits as R_{Ft}, the consumer receives a nominal dividend income of $P_t R_{Ft} d_t$. The consumer also pays to the bank a fee for credit services, whereby one unit of credit service is required for each unit of credit that the bank supplies the consumer for use in buying goods. With P_{Ft} denoting the nominal price of each unit of credit, and q_t the real quantity of credit that the consumer can use in exchange, the consumer pays $P_{Ft}q_t$ in credit fees.

With other expenditures on goods, of $P_t c_t$, and physical capital investment, $P_t k_{t+1} - P_t(1-\delta_k)k_t$, and on investment in cash for purchases, of $M_{t+1} - M_t$, and in nominal bonds $B_{t+1} - B_t(1+R_t)$, where R_t is the net nominal interest rate, the consumer's budget constraint is:

$$P_t w_t (l_{Gt} + l_{Ft}) h_t + P_t r_t s_{Gt} k_t + P_t R_{Ft} d_t + T_t \geq P_{Ft} q_t + P_t c_t + P_t k_{t+1} - P_t(1-\delta_k)k_t + M_{t+1} - M_t + B_{t+1} - B_t(1+R_t). \tag{7}$$

The consumer can purchase goods by using either money M_t or credit services. With the lump sum transfer of cash T_t coming from the government at the beginning of the period, and with money and credit equally usable to buy goods, the consumer's exchange technology is:

$$M_t + T_t + P_t q_t \geq P_t c_t. \tag{8}$$

Since all cash comes out of deposits at the bank and credit purchases are paid off at the end of the period out of the same deposits, total deposits are equal to consumption. This gives the constraint that:

$$d_t = c_t. \tag{9}$$

Given k_0, h_0, and the evolution of M_t $(t \geq 0)$ as given by the exogenous monetary policy in equation (17) below, the consumer maximizes utility subject to the budget, exchange and deposit constraints (**??**)-(9).

The bank produces credit that is available for exchange at the point of purchase. The bank determines the amount of such credit by maximizing its dividend profit subject to the labor and deposit costs of producing the credit. The production of credit uses a constant returns to scale technology with effective labor and deposited funds as inputs. In particular, with $A_F > 0$ and $\gamma \in (0, 1)$:

$$q_t = A_F e^{v_t} \left(l_{Ft} h_t\right)^{\gamma} d_t^{1-\gamma}, \tag{10}$$

where $A_F e^{v_t}$ is the stochastic factor productivity.

Subject to the production function in equation (10), the bank maximizes profit Π_{Ft} with respect to the labor l_{Ft} and deposits d_t:

$$\Pi_{Ft} = P_{Ft} q_t - P_t w_t l_{Ft} h_t - P_t R_{dt} d_t. \tag{11}$$

Equilibrium implies that:

$$\left(\frac{P_{Ft}}{P_t}\right) \gamma A_F e^{v_t} \left(\frac{l_{Ft} h_t}{d_t}\right)^{\gamma-1} = w_t; \tag{12}$$

$$\left(\frac{P_{Ft}}{P_t}\right) (1-\gamma) A_F e^{v_t} \left(\frac{l_{Ft} h_t}{d_t}\right)^{\gamma} = R_{dt}. \tag{13}$$

These indicate that the marginal cost of credit, $\left(\frac{P_{Ft}}{P_t}\right)$, is equal to the marginal factor price divided by the marginal factor product, or $\frac{w_t}{\gamma A_F e^{v_t} \left(\frac{l_{Ft} h_t}{d_t}\right)^{\gamma-1}}$, and that the zero profit dividend yield paid on deposits is equal to the fraction of the marginal cost given by

$$\left(\frac{P_{Ft}}{P_t}\right) (1-\gamma) \left(\frac{q_t}{d_t}\right) = R_{dt}.$$

In turn the consumer problem implies that $\frac{P_{Ft}}{P_t} = R_t$, and so

$$R_t \geq R_t \left(1-\gamma\right) \frac{q_t}{d_t} = R_{dt}.$$

The firm maximizes profit given by $y_t - w_t l_{Gt} h_t - r_t s_{Gt} k_t$, subject to a standard Cobb-Douglas production function in effective labor and capital:

$$y_t = A_G e^{z_t} (s_{Gt} k_t)^{1-\alpha} (l_{Gt} h_t)^{\alpha}. \tag{14}$$

The first order conditions for the firm's problem yield the standard expressions for the wage rate and the rental rate of capital:

$$w_t = \alpha A_G e^{z_t} \left(\frac{s_{Gt} k_t}{l_{Gt} h_t} \right)^{1-\alpha}, \tag{15}$$

$$r_t = (1-\alpha) A_G e^{z_t} \left(\frac{s_{Gt} k_t}{l_{Gt} h_t} \right)^{-\alpha}. \tag{16}$$

It is assumed that government policy includes sequences of nominal transfers as given by:

$$T_t = \Theta_t M_t = (\Theta^* + e^{u_t} - 1) M_t, \qquad \Theta_t = [M_t - M_{t-1}]/M_{t-1}, \tag{17}$$

where Θ_t is the growth rate of money and Θ^* is the stationary gross growth rate of money. Definition of equilibrium and first-order conditions are provided in Appendix A.

4 General Equilibrium Taylor Condition

The Euler equation of interest rates follows as

$$1 = \beta E_t \left\{ \frac{c_{t+1}^{-\theta} x_{t+1}^{\psi(1-\theta)}}{c_t^{-\theta} x_t^{\psi(1-\theta)}} \frac{\tilde{R}_t}{\tilde{R}_{t+1}} \frac{R_{t+1}}{\pi_{t+1}} \right\}, \tag{18}$$

where R and π are gross rates of nominal interest and inflation, respectively. The term $\tilde{R}_t$ represents (one plus) a 'weighted average cost of exchange' as follows:

$$\tilde{R}_t = 1 + \frac{m_t}{c_t}(R_t - 1) + \gamma\left(1 - \frac{m_t}{c_t}\right)(R_t - 1),$$

where a weight of $\frac{m}{c}$ is attached to the opportunity cost of money $(R_t - 1)$ and a weight of $(1 - \frac{m}{c})$ is attached to the average cost of credit, $\gamma(R_t - 1)$, and $\frac{m_t}{c_t}$ is the real consumption normalised demand for money (i.e. the inverse of the consumption velocity of money). In effect, equation (18) augments a standard consumption Euler equation with the (growth rate of) the weighted average cost of exchange. If all goods purchases are conducted using money ($m_t/c_t = 1$) then equation (18) reverts back to the familiar consumption Euler equation which would constitute an equilibrium condition of a standard, unit velocity cash-in-advance model without a money alternative.

For any variable z_t, define $\widehat{z}_t \equiv \ln z_t - \ln z$, where the absence of a time subscript denotes a *BGP* stationary value, and define $\widehat{g}_{z,t+1} \equiv \ln z_{t+1} - \ln z_t$, which approximates the growth rate at time $t+1$ for sufficiently small z_t. Consider a log-linear approximation of (18) evaluated around the *BGP*:

$$0 = E_t\left\{\theta\widehat{g}_{c,t+1} - \psi(1-\theta)\widehat{g}_{x,t+1} + \widehat{g}_{\tilde{R},t+1} - \widehat{R}_{t+1} + \widehat{\pi}_{t+1}\right\}.$$

Rearranging this in terms of $\widehat{R}_t$ gives the Taylor condition expressed in log-deviations from the *BGP* equilibrium:

$$\widehat{R}_t = \Omega E_t\left\{\begin{array}{c}\widehat{\pi}_{t+1} + \theta\widehat{g}_{c,t+1} - \psi(1-\theta)\widehat{g}_{x,t+1} \\ +\left(\frac{\Omega-1}{\Omega}\right)\left[(R-1)\left(\frac{\frac{m}{c}}{1-\frac{m}{c}}\right)\widehat{g}_{\frac{m}{c},t+1} - \widehat{R}_{t+1}\right]\end{array}\right\} \tag{19}$$

where

$$\Omega \equiv 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \geq 1. \tag{20}$$

The Taylor condition (19) can now be expressed in net rates (denoted by over-barred terms) and absolute deviations from the *BGP* equilibrium, as demonstrated by the following proposition.

Proposition 1 *An equilibrium condition of the economy takes the form of a Taylor Rule which sets deviations of the short-term nominal interest rate from some baseline path in proportion to deviations of variables from their targets:*

$$\overline{R}_t - \overline{R} = \Omega E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t \left(\overline{g}_{c,t+1} - \overline{g}\right) - \Omega\psi\left(1-\theta\right) E_t \overline{g}_{x,t+1} + \frac{\left(\Omega - 1\right)\overline{R}\frac{m}{c}}{1-\frac{m}{c}} E_t \overline{g}_{\frac{m}{c},t+1} - \left(\Omega - 1\right) E_t \left(\overline{R}_{t+1} - \overline{R}\right). \tag{21}$$

where $\Omega \geq 1$, and for a given w, then $\frac{\partial\Omega}{\partial R} > 0$ and $\frac{\partial\Omega}{\partial A_F} > 0$, and the target values are equal to the balanced growth path equilibrium values.[3]

Proof. Since the *BGP* solution for normalized money demand is:

$$0 \leq \frac{m}{c} = 1 - A_F \left(\frac{\overline{R}\gamma A_F}{w}\right)^{\frac{\gamma}{1-\gamma}} \leq 1, \tag{22}$$

then $\Omega \equiv 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{\left(1+\overline{R}\right)\left[\gamma+\frac{m}{c}(1-\gamma)\right]} \geq 1$ and, given w, $\frac{\partial\Omega}{\partial R} \geq 0$ and $\frac{\partial\Omega}{\partial A_F} \geq 0$. ■

Corollary 2 *Lucas Critique: An increase in the BGP money supply growth rate Θ^*, for the log-utility case of $\theta = 1$, cause the BGP nominal interest rate $\overline{R}$ to increase, and so causes Ω to increase.*

Proof. From the cash-in-advance constraint (8), and equations (17), (18) and (22) for $\theta = 1$, the money supply growth rate after the lump sum transfer equals the growth rate of the nominal price level plus the rate of time preference: $\overline{R} = \Theta^* + \rho$. If Θ^* increases, then $\overline{R}$ increases, and so by the preceding proposition Ω increases. ■

For a linear production function of goods w is the constant marginal product of labor but more generally w is endogenous and will change; however this change in w is quantitatively small compared to changes in R and A_F, so that the derivatives above almost always hold true. Note that for a unitary consumption velocity of money, the velocity growth and forward interest terms drop out of equation (21)

The term $\bar{\pi}$ in equation (21) can be compared to the inflation target that features in many interest rate rules (e.g. Taylor, 1993; Clarida et al., 2000). This is usually set as an exogenous constant in a conventional rule but represents the *BGP* rate of inflation in the Taylor condition.[4] The term in consumption growth is similar, but not identical to, the first difference of the output gap that features in the so-called 'speed limit' rule (Walsh, 2003). Alternatively, the term in the growth rate of leisure time can be compared to the unemployment rate which sometimes features in conventional interest rate rules in place of the output gap.[5]

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[3] This is the the Brookings project form of the Taylor rule as described in Orphanides (2008).

[4] Although see Ireland (2007) for an example of a conventional interest rate rule with a time-varying inflation target.

[5] For example, Mankiw (2001) includes the unemployment rate in an interest rate rule and Rudebusch (2009) includes the 'unemployment gap'.

Equation (21) also contains two terms which are not usually found in standard monetary policy reaction functions. Firstly, there is a term in the growth rate of the real (consumption normalized) demand for money. Conventional interest rate rules are usually considered in the context of models which omit monetary relationships and thus money demand does not feature directly in the model.[6] Secondly, the Taylor condition contains a term in the expected future nominal interest rate. This contrasts with the lagged nominal interest term which is often used to capture 'interest rate smoothing' in a conventional rule (e.g. Clarida et al., 2000). Other interpretations in terms of a backwards looking rule and a credit interpretation are given in Appendix B.

In general, the coefficient on inflation in equation (21) exceeds unity ($\Omega > 1$). This replicates the 'Taylor principle' whereby the nominal interest rate responds more than one-for-one to (expected future) inflation deviations from 'target'. However, the inflation coefficient in the Taylor condition does not reflect policy-makers' preferences. Rather, it is a function of the BGP nominal interest rate (R), the consumption normalized demand for real money balances (m/c) and the efficiency with which the banking sector transforms units of deposits into units of the credit service, as reflected by the magnitude of ($1-\gamma$). Furthermore, higher productivity in the banking sector (A_F) causes a higher velocity and implies a larger inflation coefficient in the Taylor condition. The magnitude of Ω clearly does not reflect a response to inflation in the conventional 'reaction function' sense.[7]

Equation (21) can alternatively be rewritten in terms of the consumption velocity of money, $V_t \equiv \frac{c_t}{m_t}$, and the productive time, or 'employment', growth rate ($l \equiv l_G + l_H + l_F = 1 - x$). Using the fact that $\widehat{x}_t = -\frac{1-x}{x}\widehat{l}_t$:

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} \\ &\quad -\Omega_V E_t\overline{g}_{V,t+1} - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{23}$$

Where over-barred terms again denote net rates and:

$$\Omega_V \equiv \frac{\left(\overline{R}\right)}{1+\overline{R}}\left(\frac{(1-\gamma)\frac{m}{c}}{\gamma + (1-\gamma)\frac{m}{c}}\right).$$

[6] Specifically, shifts in the demand for money are perfectly accommodated by adjustments to the money supply in order to maintain the rule-implied nominal interest rate. This, it is claimed, renders the evolution of the money supply an operational detail which need not be modelled directly (e.g. Woodford, 2008).

[7] Unlike Sørensen and Whitta-Jacobsen's (2005, pp.502-505) quantity theory based equilibrium condition, the inflation coefficient in (21) exceeds unity for any (admissible) interest elasticity of money demand. In their expression, the inflation coefficient falls below unity if the interest (semi) elasticity of money demand exceeds one in absolute value. In the Benk et al. (2010) model, the coefficient on inflation would exceed unity even in this case but the central bank would not wish to increase the money supply growth rate to this extent because seigniorage revenues would begin to recede as the elasticity increases beyond this point.

Proposition 3 *For the Taylor condition of equation (23), it is always true that* $0 \leq \Omega_V \leq 1 \leq \Omega$.

Proof.

$$\begin{aligned}
\Omega &\equiv 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{\left(1+\overline{R}\right)\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \geq 1;\ \frac{m}{c} = 1 - A_F^{\frac{1}{1-\gamma}}\left[\frac{\overline{R}\gamma}{w}\right]^{\frac{\gamma}{1-\gamma}} \leq 1; \\
1 &\geq (1-\gamma)\left(1-\frac{m}{c}\right) \geq 0;\ \Rightarrow 0 \leq \Omega_V \equiv \frac{\overline{R}\,(1-\gamma)}{1+\overline{R}}\left(\frac{\frac{m}{c}}{1-(1-\gamma)\left(1-\frac{m}{c}\right)}\right) \leq 1; \\
&\Rightarrow 0 \leq \Omega_V \leq 1 \leq \Omega.
\end{aligned}$$

■

At the Friedman (1969) optimum ($\overline{R} = 0$), $\frac{m}{c} = 1$, $\omega = 0$, and the velocity coefficient (Ω_V) takes a value of zero. The velocity growth term only enters the Taylor condition when the nominal interest rate differs from the Friedman (1969) optimum and fluctuates. In turn, this has implications for $\Omega = 1 + \left(\frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{\left(1+\overline{R}\right)\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\right)$,since when $\overline{R} = 0$, $(1-\gamma)\left(1-\frac{m}{c}\right) = 0$, and $\Omega = 1$. For $\frac{m}{c}$ below one (velocity above one), which is true for most practical experience, the model's equivalent of the 'Taylor principle' ($\Omega > 1$) holds.

Corollary 4 *Given* w, *then* $\frac{\partial \Omega}{\partial \overline{R}} \geq 0$, $\frac{\partial \Omega_V}{\partial \overline{R}} \geq 0$, $\frac{\partial \Omega}{\partial A_F} \geq 0$, $\frac{\partial \Omega_V}{\partial A_F} \leq 0$.

Proof. This comes directly from the definitions of parameters above. ■

A higher target nominal interest rate can be accomplished only by a higher *BGP* money supply growth rate. This would in turn make the inflation and consumption growth coefficients larger and the forward interest rate and velocity coefficients would become more negative. A higher credit productivity factor A_F, and so a higher velocity, leads to a higher inflation coefficient and a more negative response to the forward-looking interest term but a less negative coefficient on the velocity growth term.

The Taylor condition above would look identical with exogenous growth. However, under exogenous growth the targeted inflation rate and growth rate of the economy are unrelated and exogenously specified. Under endogenous growth the targets are instead the endogenously determined *BGP* values for inflation, the growth rate, and the nominal interest rate and each of these are determined, in part, by the long run stationary money supply growth rate Θ^*, which is exogenously given. In turn, Θ^* translates directly into a long run inflation target accepted by the central bank, such as the two percent target often incorporated into conventional interest rate rules (for example, Taylor, 1993). So the model assumes only a long run money supply growth target, or alternatively, a long run inflation rate target.

4.1 Long Run when Expected Equals Actual Inflation

Once inflation expectations settle down and the current expected inflation rate at time t equals the actual inflation rate at time t, the Taylor equivalence of the rule disappears and instead a Fisher form emerges in its place. Consider that this accurate expectations occurs such that $\overline{R}_t = E_t\left(\overline{R}_{t+1}\right)$. Then equation 23 becomes

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} \\ &\quad -\Omega_V E_t\overline{g}_{V,t+1} - \left(\Omega - 1\right)\left(\overline{R}_t - \overline{R}\right). \qquad (24)\end{aligned}$$

With $\overline{R}_t$ now on both sides of the equation, an interest rate forward-looking "smoothing" equation, with two time periods for the interest rate no longer results. The interest rate reduces down to

$$\begin{aligned}\overline{R}_t - \overline{R} + \left(\Omega - 1\right)\left(\overline{R}_t - \overline{R}\right) &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) \\ &\quad +\Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \Omega_V E_t\overline{g}_{V,t+1}. \qquad (25)\end{aligned}$$

This means the Ω term is now divided throughout the equation to solve for $\overline{R}_t$ and get that

$$\begin{aligned}\overline{R}_t - \overline{R} &= E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) \\ &\quad +\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \frac{\Omega_V}{\Omega}E_t\overline{g}_{V,t+1}, \qquad (26)\end{aligned}$$

Suppose that the employment term drops out in the long run, such as along the BGP equilibrium, as the growth in labor time does not change over time, then $E_t\overline{g}_{l,t+1} = 0$. The remaining equation looks exactly like a Fisher equation with a coefficient of one on the inflation rate, and of θ from the CES utility function on the real interest rate term from the Euler equation, that is for the $E_t\left(\overline{g}_{c,t+1} - \overline{g}\right)$ term. If velocity is also stationary in the long run, then $E_t\overline{g}_{V,t+1} = 0$, and the equation is indeed the Fisher equation of interest rates where $\theta\overline{g}_{c,t+1}$ is the real interest rate.

4.2 Euler Condition Restated with Output Growth

It is not surprising to find that the growth rate of consumption appears in equation (23) rather than the output growth rate given that the derivation of the Taylor condition begins from the consumption Euler equation (18). However, the Taylor condition can be rewritten to include an output growth term and hence correspond more closely to standard Taylor rule specifications, in particular the 'speed limit' rule considered by Walsh (2003). To derive this alternative rule, consider that the identity $y_t = c_t + i_t$ implies that $\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t$, where $\widehat{i}_t = \frac{k}{i}\left[\widehat{k}_t - (1-\delta)\widehat{k}_{t-1}\right]$. The growth rate of investment can be understood as the acceleration of the growth of capital gross of depreciation. The Taylor condition can be rewritten as:

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[\frac{y}{c}E_t\left(\overline{g}_{y,t+1} - \overline{g}\right) - \frac{i}{c}E_t\left(\overline{g}_{i,t+1} - \overline{g}\right)\right] \\ &\quad + \Omega\psi(1-\theta)\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \Omega_V E_t\overline{g}_{V,t+1} - (\Omega-1)E_t\left(\overline{R}_{t+1} - \overline{R}\right).\end{aligned} \tag{27}$$

A term in investment growth does not appear in standard Taylor rules but plays a role as part of what is interpreted as the output gap growth rate in this modified Taylor condition. Equation (27) forms the basis for the two misspecified estimating equations considered in Section 5. The first misspecified estimating equation simply replaces the consumption growth term in equation (23) with an output growth term as follows:

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[E_t\left(\overline{g}_{y,t+1} - \overline{g}\right)\right] \\ &\quad + \Omega\psi(1-\theta)\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \Omega_V E_t\overline{g}_{V,t+1} - (\Omega-1)E_t\left(\overline{R}_{t+1} - \overline{R}\right).\end{aligned} \tag{28}$$

Comparing equation (27) and equation (28) shows that the latter erroneously overlooks the weighting on the output growth rate ($\frac{y}{c}$) and omits the term in the investment growth rate. Replacing consumption growth with output growth without the additional term in investment therefore misrepresents the structure of the underlying Benk et al. (2010) model and as such equation (28) is misspecified. Note that with no physical capital in the economy, equation (28) would be a valid equilibrium condition of the economy.

4.3 Misspecified Standard Taylor Rule

The second misspecified model erroneously imposes the same restrictions used to arrive at equation (28) but also drops the terms in productive time and velocity, giving:

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[E_t\left(\overline{g}_{y,t+1} - \overline{g}\right)\right] \\ &\quad - (\Omega-1)E_t\left(\overline{R}_{t+1} - \overline{R}\right).\end{aligned} \tag{29}$$

This can be interpreted as a conventional interest rate rule with a forward-looking 'interest rate smoothing' term; the additional restriction that $\Omega = 1$ would replicate a standard interest rate rule without interest rate smoothing. Once again, equation (29) does not

Preferences		
θ	1	Relative risk aversion parameter
ψ	1.84	Leisure weight
β	0.96	Discount factor
Goods Production		
α	0.64	Labor share in goods production
δ_k	0.031	Depreciation rate of goods sector
A_G	1	Goods productivity parameter
Human Capital Production		
ε	0.83	Labor share in human capital production
δ_h	0.025	Depreciation rate of human capital sector
A_H	0.21	Human capital productivity parameter
Banking Sector		
γ	0.11	Labor share in credit production
A_F	1.1	Banking productivity parameter
Government		
σ	0.05	Money growth rate

Table 1: Parameters

accurately represent an equilibrium condition of the Benk et al. (2010) economy and is therefore misspecified. Equation (29) with $\Omega = 1$ would be the correct equilibrium condition if the economy featured neither physical capital nor exchange credit.

5 Calibration of Inflation Rate Coefficient

We follow Benk et al. (2010) in using postwar U.S. data to calibrate the model (presented here in Table 1) and calculate a series of 'target values' consistent with this calibration (presented here as Table 2); see Benk et al. for the shock process calibration.

Subject to this calibration, we derive a set of theoretical 'predictions' for the coefficients of the Taylor condition (23). These values will subsequently be compared to the coefficients estimated from artificial data simulated from the model. Consider first the inflation coefficient (Ω). According to the calibration and target values presented in tables 1 and 2, its theoretical value is

$$\Omega = 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} = 1 + \frac{(1-0.11)(1-0.38)}{1.0944\,(1-(1-0.11)(1-0.38))} = 2.125$$

For $R = 1$, only cash is used so that $\frac{m}{c} = 1$ and Ω reverts to its lower bound of 1. This also happens with zero credit productivity ($A_F = 0$), in which case only cash is used in

g	0.024	Avg. annual output growth rate
π	0.026	Avg. annual inflation rate
R	0.0944	Nominal interest rate
l_G	0.248	Labor used in goods sector
l_H	0.20	Labor used in human capital sector
l_F	0.0018	Labor used in banking sector
i/y	0.238	Investment-output ratio in goods sector
m/c	0.38	Share of money transactions
x	0.55	Leisure time
$l \equiv 1 - x$	0.45	Productive time

Table 2: Target Values

exchange.

The remaining coefficients, except for velocity, are simple functions of the inflation coefficient. The consumption growth coefficient is $\Omega\theta$, which with $\theta = 1$ for log-utility should simply take the same magnitude as the coefficient on inflation ($\theta\Omega = 2.125$). The coefficient on the productive time growth rate should take a value of zero with log utility. However with leisure preference calibrated at 1.84, and productive time ($1 - x \equiv l$) equal to equal to 0.45 along the BGP, the estimated value of the productive time coefficient can be interpreted as implying a certain θ factored by $\Omega\psi\frac{l}{1-l} = (2.125)\,(1.84)\,\frac{0.45}{0.55} = 3.199$. Given the magnitude of the inflation coefficient, the coefficient on the forward interest term is simply $-(\Omega - 1) = -1.125$. The velocity coefficient ($-\Omega_V$) is -0.065 using:

$$-\frac{(R-1)}{R}\left(\frac{(1-\gamma)\,\frac{m}{c}}{\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\right) = -\frac{(1.0944-1)}{1.0944}\left(\frac{(1-0.11)\,0.38}{(1-(1-0.11)\,(1-0.38))}\right).$$

At the Friedman (1969) optimum ($R = 1$), $\Omega_V = 0$. In this case the omission of the term in velocity growth in the estimation exercises that follow would be innocuous but this is not true in general.

6 Artificial Data Generation and Estimation

The Benk et al. (2010) model presented in Section 2 is simulated using the calibration provided in Table 1 in order to generate 1000 alternative 'joint histories' for each of the variables in equation (23), where each history is 100 periods in length. To do so, 100 random sequences for the shock vector innovations are generated and control functions of the log-linearized model are used to compute sequences for each variable. Each observation within a given history may be thought of as an annual period given the frequency considered by the Benk et al. (2010) model. The data set used to estimate the coefficients of the Taylor condition can therefore be viewed as comprising of 1000, '100-year', samples of artificial data.

6.1 Estimation Methodology

This section presents the results of estimating a 'correctly specified' estimating equation based upon the true theoretical relationship (23) against artificial data generated from the Benk et al. (2010) model.[8] In a similar manner, two alternative estimating equations are evaluated using the same data set. Since these alternative estimating equations differ from the expression based upon the true theoretical relationship, they necessarily constitute misspecified empirical models.[9]

Prior to estimation, the simulated data is filtered by either 1) a Hodrick-Prescott (HP) filter with a smoothing parameter selected in accordance with Ravn and Uhlig (2002); 2) a $3-8$ period ("year") Christiano and Fitzgerald (2003) band pass filter for 'business cycle frequencies'; or 3) a $2-15$ year Christiano and Fitzgerald (2003) band pass filter which retains more of the lower frequency trends in the data than the $3-8$ year filter, in the spirit of Comin and Gertler's (2006) 'medium-term cycle'.[10] *A priori*, the $2-15$ band pass filter might be regarded as the 'most relevant' to the underlying theoretical model because shocks in the model can cause low frequency events during the business cycle, such as a change in the permanent income level without a reversion to its previous level.[11]

The first estimation technique considered is OLS, as used by Taylor (1999) in the context of a contemporaneous interest rate rule. However, if expected future variables are correlated with the error term then a suitable set of instruments are required to proxy for these forward-looking terms.[12] Two instrumental variables (IV) techniques are considered and each differs by the instrument set employed. The first is a two stage least squares (2SLS) estimator under which the first lags of inflation, consumption growth, productive time growth and velocity growth and the second lag of the nominal interest rate are used as instruments. Adding a constant term to the instrument set provides a 'just identified' 2SLS estimator. In using lagged variables as instruments we exploit the fact that such terms are pre-determined

[8] The exercise conducted here is similar to those conducted by Fève and Auray (2002), for a standard CIA model, and Salyer and Van Gaasbeck (2007), for a 'limited participation' model.

[9] We acknowledge that in a full information maximum likelihood estimation that uses all of the equilibrium conditions of the economy we may be able to recover the theoretical coefficients of the Taylor condition almost exactly; we leave that exercise as an important part of future research that encompasses the entire alternative model; and then we could also compare it to the standard three equation central bank policy model.

[10] However, Comin and Gertler's 'medium-term cycle' is defined using a wider 2-200 quarter filter. However, the 2-15 filter will still retain periodicities that the HP and 3-8 filters consign to the 'trend'.

[11] In principle, the filtering procedure takes account of the Siklos and Wohar (2005) critique of empirical Taylor rule studies which do not address the non-stationarity of the data. However, standard ADF and KPSS tests suggest that the simulated data is stationary prior to filtering (results not reported). Accordingly, the filters do not implement a de-trending procedure.

[12] Empirical studies usually deal with expected future terms either byreplacing them with realised future values and appealing to rationalexpectations for the resulting conditional forecast errors (e.g. Clarida et al., 1998, 2000) or by using private sector or central bank forecasts as empirical proxies (e.g. Orphanides, 2001; Siklos and Wohar, 2005).

and thus not susceptible to the simultaneity problem which motivates the use of IV techniques. The 2SLS procedure applies a Newey-West adjustment for heteroskedasticity and autocorrelation (HAC) to the coefficient covariance matrix.

The second IV procedure is a generalized method of moments (GMM) estimator under which three additional lags of inflation, consumption growth, productive time growth and velocity growth and two further lags of the nominal interest rate are added to the instrument set.[13] Expanding the instrument set in this manner reduces the sample size available for each of the 1000 simulated sample periods but the over-identifying restrictions can now be used to test the validity of the instrument set using the Hansen J-test. The GMM estimator used iterates on the weighting matrix in two steps and applies a HAC adjustment to the weighting matrix using a Bartlett kernel with a Newey-West fixed bandwidth.[14] A similar HAC adjustment is also applied to the covariance weighting matrix.

The results are presented in three sets of tables, one set for each estimating equation, and are further subdivided according to the statistical filter applied to the simulated data. Alongside the estimates obtained from an 'unrestricted' estimating equation, each table also reports estimates derived from a 'restricted' estimating equation which arbitrarily omits the forward interest rate term ($\beta_5 = 0$). This arbitrary restriction demonstrates the importance of the dynamic term in equation (23). Each table of results reports mean coefficient estimates along with the standard error of these estimates (as opposed to the mean standard error). The figures in square brackets report the number of coefficients estimated to be statistically different from zero at the 5% level of significance and this count is used as an indication of the 'precision' of the estimates. An 'adjusted mean' figure is also reported for each coefficient; this is obtained by setting non statistically significant coefficient estimates to zero when calculating the averages. The tables also report mean R-square and mean adjusted R-square statistics along with the mean P-value for the F-statistic for overall significance (these cannot be computed for the GMM estimator), the mean P-value for the Hansen J-statistic which tests the validity of the instrument set (these can only be calculated in the presence of over-identifying restrictions), and the mean Durbin-Watson (D-W) statistic which tests for autocorrelation. The number of estimations for which the null hypothesis of the J-statistic is *not* rejected - i.e. the instrument set is not found to be invalid - is reported alongside

[13] Carare and Tchaidze (2005, p.15) note that the four-lags-as-instruments specification is the standard approach in the interest rate rule literature (e.g. Orphanides, 2001). Although the GMM procedure in general corrects for autocorrelation and heteroskedasticity, in estimating with simulated data we use lags as 'valid' instruments for pre-determined variables. These instruments might prove to be 'relevant' because the data is serially correlated but no further lags are needed for the estimating equation itself. For actual data, Clarida et al. (QJE, 2000, p.153) use a GMM estimator "with an optimal weighting matrix that accounts for possible serial correlation in [the error term]" but they also add two lags of the dependent variable to their estimating equation on the basis that this "seemed to be sufficient to eliminate any serial correlation in the error term." (p.157), implying that the GMM correction was insufficient for this purpose.

[14] Jondeau et al. (2004, p.227) state that: "To our knowledge, all estimations of the forward-looking reaction function based on GMM have so far relied on the two-step estimator." They proceed to consider more sophisticated GMM estimators but nevertheless identify advantages to the "simple approach" (p.238) adopted in the literature.

its mean P-value and the number of simulated series for which the D-W statistic exceeds its upper critical value - i.e. the null hypothesis that the residuals are serially uncorrelated cannot be rejected - is reported alongside the mean D-W statistic.[15]

6.2 General Interest Rate Equation

Tables 3-6 present estimates obtained from the following 'correctly specified' estimating equation:

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{c,t+1} + \beta_3 E_t g_{l,t+1} + \beta_4 E_t g_{V,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \quad (30)$$

Expected future variables on the right hand side are obtained directly from the model simulation procedure and are instrumented for as described above.

The key result is that using a $2-15$ year window Table 3 consistently reports an inflation coefficient which exceeds unity for the estimating equation which accurately reflects the underlying theoretical model. This result is found to be robust to the statistical filter applied to the data and to the estimator employed, subject to the estimator providing a 'precise' set of estimates. See Appendix B.3, Tables B-1 and B-2, which report similar results using instead HP filtered and a $3-8$ year business cycle window. The forward interest rate term is also found to be important in terms of generating a coefficient on inflation consistent with the underlying Benk et al. (2010) model. Arbitrarily omitting this dynamic term yields much smaller estimates for the inflation coefficient to the extent that the mean estimate often falls below unity.

In terms of the general features of the results obtained from the unrestricted specification, the OLS and GMM procedures tend to generate a greater number of statistically significant estimates than the 2SLS estimator. Table 3 shows that the 2SLS estimator provides a statistically significant estimate for the inflation coefficient for only 580 of the 1000 simulated histories while the OLS and GMM estimators both return 1000 statistically significant estimates. The OLS and GMM procedures generate reasonably large R-square and adjusted R-square statistics, whereas negative R-square statistics are obtained from the simple 2SLS estimator. Expanding the instrument set in order to implement the GMM procedure leads to 1000 rejections of the J-test for instrument validity across all three filters. One might also be wary of the high number of D-W null hypothesis rejections produced by the OLS estimator, although the mean D-W statistic remains 'reasonably large' in each case; 1.56 for the $2-15$ filter, for example.[16]

[15] The D-W count excludes cases for which the test statistic falls in the inconclusive region of the test's critical values.

[16] The results for the $3-8$ band pass filter in Table B-2 are unusual in this sense in that all three estimation procedures produce a high number of D-W test rejections. For the other two filters, this undesirable result

Table 3 reports that the mean estimate for the inflation coefficient is 2.179 using the OLS estimator and 2.306 using the GMM estimator.[17] These estimates compare favorably to the theoretical prediction of $\Omega = 2.125$. The right hand side of Table 3 shows that the mean estimate of the inflation coefficient falls below unity for the OLS and GMM estimators when the forward interest rate term is arbitrarily omitted from the estimating equation. A precise mean estimate of 0.614 is obtained from the OLS estimator and a similarly precise mean estimate of 0.964 is obtained from the GMM procedure. Similar OLS and GMM estimates are obtained for the inflation coefficient under the two alternative filters in Appendix B, Tables B-1 and B-2, both in terms of the mean coefficient estimates for the unrestricted specification and in terms of the decline in magnitude induced by the arbitrary restriction.

In contrast to the estimated inflation coefficients, the estimated coefficients for consumption growth and productive time growth diverge from their theoretical predictions for the 'unrestricted' estimating equation. Under log utility ($\theta = 1$), the former should take the same magnitude as the coefficient on inflation and the latter should take a value of zero. The coefficient estimates can be used to 'back-out' an estimate of the coefficient of relative risk aversion (θ). Firstly, using the mean GMM estimate for the coefficient on consumption growth of 0.302 (Table 3) and the corresponding estimate of Ω, an implied estimate of θ can be calculated as $\frac{\beta_2}{\beta_1} = \frac{0.302}{2.306} = 0.131$, which is substantially smaller than the baseline calibration of $\theta = 1$. Alternatively, the relationship $\beta_3 = \beta_1 \psi (1 - \theta) l/(1 - l)$, which is obtained from equation (23) with Ω replaced by its estimate β_1, can also be used to obtain an implied estimate of θ. Using the estimates presented in Table 3, the implied estimate would be $\theta = 1.103$, which is much closer to the calibrated value.

Table 3 also reports that both the OLS and GMM procedures generate 1000 statistically significant estimates for the coefficient on velocity growth under the unrestricted estimating equation and that the mean estimate is correctly signed for both estimators. The mean coefficient estimates are reported as -0.196 and -0.269 for OLS and GMM estimators respectively; these estimates are somewhat smaller than the theoretical prediction of -0.065. Similar estimates are obtained under the HP and $3-8$ filters. Finally, Table 3 reports mean estimates of -1.761 (OLS) and -1.729 (GMM) for the forward interest rate coefficient compared to a theoretical prediction of -1.125. The mean estimates are therefore correctly signed but, again, smaller than the theoretical prediction.

In a standard interest rate rule an inflation coefficient in excess of unity is interpreted to reflect policy-maker's dislike of inflation deviations from target. However, this interpretation is not applicable to the Taylor condition. The result that the coefficient on inflation exceeds unity is a consequence of a money growth rule not an interest rate rule. Similarly, the break-

is confined to the OLS estimator.

[17] The discussion focuses on the OLS and GMM estimators because they produce more 'precise' estimates and also because the OLS estimator tends to reject the null hypothesis of the F-statistic more frequently than the 2SLS estimator (1000 vs. 907 rejections in Table 3, for example). The OLS regressions are possibly afflicted by autocorrelation however, as discussed above, thus one might favor the GMM estimates.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
2-15 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-2.26E-06 [0]	4.22E-05 [0]	2.54E-07 [22]	-1.82E-06 [0]	-2.46E-06 [0]	3.44E-06 [21]
Standard error	4.01E-05	0.001	4.82E-05	3.27E-05	4.83E-05	6.26E-05
Adjusted mean	-	-	4.25E-07	-	-	9.03E-07
$E_t\pi_{t+1}$	2.179 [1000]	3.816 [580]	2.306 [1000]	0.614 [999]	1.127 [763]	0.964 [999]
Standard error	0.195	51.040	0.272	0.108	0.640	0.169
Adjusted mean	2.179	1.402	2.306	0.614	0.936	0.963
$E_t g_{c,t+1}$	0.277 [1000]	0.570 [730]	0.302 [1000]	0.170 [1000]	0.262 [851]	0.207 [1000]
Standard error	0.016	5.546	0.025	0.017	0.103	0.026
Adjusted mean	0.277	0.265	0.302	0.170	0.230	0.207
$E_t g_{l,t+1}$	-0.295 [997]	-0.737 [526]	-0.359 [999]	-0.210 [732]	-0.263 [405]	-0.277 [935]
Standard error	0.067	8.208	0.085	0.088	0.203	0.111
Adjusted mean	-0.294	-0.242	-0.359	-0.182	-0.152	-0.272
$E_t g_{V,t+1}$	-0.196 [1000]	-0.347 [807]	-0.269 [1000]	-0.158 [998]	-0.307 [944]	-0.236 [1000]
Standard error	0.024	0.271	0.031	0.032	0.077	0.042
Adjusted mean	-0.196	-0.273	-0.269	-0.158	-0.292	-0.236
$E_t R_{t+1}$	-1.761 [1000]	-5.586 [335]	-1.729 [1000]			
Standard error	0.201	114.905	0.322			
Adjusted mean	-1.761	-0.712	-1.729			
$Mean;$						
R-square	0.830	<0	0.782	0.625	<0	0.522
Adjusted R-square	0.821	<0	0.770	0.609	<0	0.501
Pr(F-statistic)	2.50E-24 (1000)	0.051 (907)	N/A	5.04E-10 (1000)	0.003 (985)	N/A
Pr(J-statistic)	N/A	N/A	0.315 {1000}	N/A	0.298 {757}	0.298 {1000}
Durbin-Watson	1.558 <141>	2.059 <972>	2.040 <881>	1.954 <864>	2.052 <998>	2.205 <977>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.

· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and <> the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table 3: Taylor Condition Estimation, Band Pass Filtered Data (2-15 years), 100 Years Simulated, 1000 Estimations Average.

down of the Taylor principle under the 'restricted' estimating equation ($\beta_5 = 0$) cannot be interpreted as a softening of policy-makers' attitude towards inflation; this result simply emanates from model misspecification.

6.3 Interest Relation with Output Growth

The same estimation procedure is now applied to an estimating equation which replaces consumption growth in equation (30) with output growth as follows:

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{y,t+1} + \beta_3 E_t g_{l,t+1} + \beta_4 E_t g_{V,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \quad (31)$$

The simulated data remains unchanged, therefore equation (31) represents a misspecified version of the 'correct' estimating equation, which continues to be equation (30).[18] In particular, equation (31) can be seen to correspond to the misspecified Taylor condition, equation (28).

The results are similar across the HP, the $3-8$ band pass and the $2-15$ band pass filters but as the latter filter gives the most statistically significant coefficient estimates only estimates from the $2-15$ filter are presented in Table 4. Comparing the general features of the results to those presented in Table 3, there has been a decline in the precision with which the coefficients are estimated, a decline in the magnitude of the R-square and adjusted R-square statistics and a decline in the number of rejections of the null hypothesis of the F-statistic for joint significance. This is not surprising given that an element of misspecification has been introduced into the estimating equation. The number of rejections of the null hypothesis of the D-W test statistic also tends to increase although the GMM procedure applied to $2-15$ filtered data still fails to reject the null for 94.5% of the simulated samples.

The estimated inflation coefficients are now found to be substantially greater than the coefficients obtained from the 'correctly specified' estimating equation (30) and hence substantially greater than the predicted value. For instance, the GMM estimate for the unrestricted estimating equation rises from 2.306 in Table 3 to 5.274 in Table 4 (or 5.235 according to the adjusted mean). Similarly, the OLS estimate increases from 2.179 to 4.219 (or 4.185 adjusted).[19]. The estimates clearly diverge further from the theoretical prediction of $\Omega = 2.125$ under this particular form of misspecification.

The incorrectly specified estimating equation also induces a substantial decrease in the estimated coefficients for the productive time growth rate and the forward nominal interest rate. The estimated coefficient on the productive time growth rate decreases from -0.294 to

[18] The instrument sets used for the 2SLS and GMM estimators are modified by replacing consumption growth with output growth but remains unchanged in terms of the number of lags included.

[19] Corresponding upward shifts in the estimated inflation coefficient are found for the $3-8$ band pass filter results (results not reported) and even larger increases are found for the HP filtered data (results not reported).

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
2-15 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-3.66E-06 [0]	0.001 [0]	3.07E-06 [16]	-9.34E-07 [0]	-4.46E-06 [0]	2.80E-06 [5]
Standard error	6.19E-05	0.031	8.58E-05	2.43E-05	9.65E-05	6.42E-05
Adjusted mean	-	-	-9.38E-07	-	-	4.23E-07
$E_t\pi_{t+1}$	4.219 [971]	-25.003 [189]	5.274 [961]	0.541 [941]	2.338 [882]	1.101 [990]
Standard error	1.715	1290.303	2.582	0.170	1.151	0.264
Adjusted mean	4.185	2.522	5.235	0.532	2.010	1.100
$E_t g_{y,t+1}$	0.303 [967]	-2.019 [206]	0.406 [940]	0.038 [563]	0.211 [262]	0.082 [956]
Standard error	0.125	122.643	0.204	0.020	0.304	0.029
Adjusted mean	0.300	0.244	0.402	0.029	0.077	0.081
$E_t g_{l,t+1}$	-2.098 [959]	14.698 [211]	-2.815 [954]	-0.284 [424]	-1.462 [353]	-0.610 [941]
Standard error	0.892	825.197	1.417	0.189	1.740	0.232
Adjusted mean	-2.073	-1.672	-2.790	-0.189	-0.655	-0.598
$E_t g_{V,t+1}$	-0.118 [884]	0.069 [310]	-0.191 [970]	-0.095 [733]	-0.246 [587]	-0.158 [923]
Standard error	0.042	16.979	0.064	0.043	0.164	0.061
Adjusted mean	-0.113	-0.117	-0.190	-0.084	-0.161	-0.156
$E_t R_{t+1}$	-3.878 [907]	29.503 [128]	-4.498 [849]			
Standard error	1.812	1437.910	2.802			
Adjusted mean	-3.767	-1.757	-4.372			
$Mean$;						
R-square	0.361	<0	0.127	0.246	<0	<0
Adjusted R-square	0.327	<0	0.079	0.214	<0	<0
Pr(F-statistic)	0.001 (995)	0.379 (411)	N/A	0.020 (924)	0.055 (829)	N/A
Pr(J-statistic)	N/A	N/A	0.226 {1000}	N/A	0.260 {682}	0.264 {1000}
Durbin-Watson	1.882 <699>	1.982 <868>	2.342 <945>	2.295 <1000>	2.145 <997>	2.728 <999>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.

· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and <> the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table 4: Output Growth instead of Consumption Growth, Band Pass Filtered data (2-15 years), 100 Years Simulated, 1000 Estimations Average.

−2.073 (both adjusted means) between Table 3 and Table 4 according to the OLS estimator and from −0.359 to −2.790 for the GMM estimator, hence the estimates diverge further from their predicted value of $\beta_3 = 0$. The GMM estimates of the forward interest rate term also decrease from −1.729 in Table 3 to −4.372 in Table 4 (adjusted means where appropriate). Again, the estimates diverge further from theoretical prediction of −1.125.

The estimated coefficients for output growth in Table 4 are comparable to those for consumption growth presented in Table 3, despite the effect that the misspecification has on the other estimates. For example, the OLS estimate for β_3 is 0.300 (adjusted mean) in Table 4 compared to the corresponding estimate of 0.277 in Table 3. For the GMM estimator the coefficient on output growth is 0.402 (adjusted mean) in Table 4 compared to the corresponding estimate of 0.302 reported in Table 3.

The velocity growth term is estimated precisely by the GMM estimator even after the modification to the estimating equation. Estimates of β_4 retain the correct sign and are of a similar magnitude as under the correctly specified estimating equation; for example, a GMM estimate of −0.190 in Table 4 compared to a corresponding estimate of −0.269 in Table 3.

For the restricted specification ($\beta_5 = 0$), the estimates undergo similar changes as those obtained from the restricted version of the 'correct' estimating equation (30). The OLS and GMM estimators generate inflation coefficients which often fall below unity in a manner incompatible with the theoretical model from which the Taylor condition is derived, although the GMM estimator provides a notable exception (Table 4).

In short, the results obtained from applying equation (31) to the simulated data show that adapting the estimating equation in a seemingly minor way can have a substantial impact upon the coefficient estimates obtained. The erratic results produced by this misspecified estimating equation provide an illustration of the fundamental difference between the Taylor condition and a conventional interest rate rule. Unlike a Taylor rule, the Taylor condition cannot be modified in an *ad hoc* manner.[20] In order to make the progression from (30) to (31) in a legitimate manner, one would need to alter the underlying model by excluding physical capital, for example. A new set of artificial data would then need to be simulated from this alternative model prior to re-estimation.

[20] In contrast, conventional interest rate rules are exogenously specified and thus amenable to arbitrary modifications. Clarida et al. (1998), for example, add the exchange rate to the standard Taylor rule and Cecchetti et al. (2000) and Bernanke and Gertler (2001) consider whether policymakers should react to asset prices.

6.4 A Common Interest Rate Equation

The estimation procedure is now re-applied to the following estimating equation:

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{y,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \tag{32}$$

This estimating equation corresponds to the misspecified representation of the Taylor condition with output growth plus further restrictions on the terms in productive time and the velocity of money; see equation (29). Equation (32) can be interpreted as a 'dynamic forward-looking Taylor rule' for $\beta_5 \neq 0$ or a 'static forward-looking Taylor rule' under the restriction $\beta_5 = 0$. Notably, the term in velocity growth is absent from this expression. This omission might be expected to have a bearing on the estimates because equations (30) and (31) produced a large number of statistically significant estimates for the velocity growth coefficient.

The results are again similar across the HP and band pass filters so only the $2-15$ band pass results are presented in Table 5.[21] The estimates are generally found to be poor in terms of the number of statistically significant estimates produced and in terms of mean R-square and adjusted R-square statistics. This is not surprising given that yet another source of misspecification has been added to the estimating equation.

The mean coefficient on inflation does not exceed unity for any of the three estimators considered. The results are also comparatively weak in terms of the frequency with which the null hypothesis of the F-statistic is rejected and in terms of the number of non-rejections of the null hypothesis of the Hansen J-test. The latter finding calls into question the validity of the instrument set used for the GMM estimator for equation (32).

The inflation coefficients are estimated surprisingly precisely under the restriction on β_5. However, these estimates differ quite substantially between estimating procedures for the $2-15$ filter; 0.317 (adjusted mean) for OLS compared to 0.892 for GMM (adjusted mean).

In short, imposing a 'conventional Taylor rule' restricts the true estimating equation to such an extent that the theoretical prediction that the coefficient on expected inflation exceeds unity is not verified. An estimated inflation coefficient of this magnitude might erroneously be interpreted to signify that the Taylor principle is violated but this result is simply a product of a misspecified estimating equation in the present context. Only if the model excluded physical capital and set velocity to one, by excluding exchange credit for example, would such an estimating equation be appropriate.

[21] The instrument set now comprises of four lags of expected future inflation, four lags of expected future output growth, the second, third and fourth lags of the nominal interest rate and a constant term for the GMM estimator or just the shortest lag of each and a constant term for the exactly identified 2SLS estimator.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
2-15 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-6.42E-07 [0]	7.62E-05 [0]	-2.48E-06 [8]	-7.00E-07 [0]	5.16E-06 [0]	-1.85E-07 [12]
Standard error	2.57E-05	0.006	1.15E-04	2.58E-05	2.12E-04	1.13E-04
Adjusted mean	-	-	-1.12E-06	-	-	4.92E-07
$E_t\pi_{t+1}$	0.310 [239]	0.671 [22]	0.132 [344]	0.326 [926]	1.472 [416]	0.894 [981]
Standard error	0.448	162.564	0.955	0.099	3.431	0.315
Adjusted mean	0.185	0.017	0.092	0.317	0.826	0.892
$E_t g_{y,t+1}$	0.020 [277]	-0.185 [40]	0.019 [237]	0.021 [408]	0.035 [163]	0.031 [460]
Standard error	0.017	16.569	0.027	0.012	0.582	0.027
Adjusted mean	0.011	0.012	0.012	0.013	0.041	0.024
$E_t R_{t+1}$	0.008 [147]	4.225 [27]	0.918 [524]	N/A	N/A	N/A
Standard error	0.475	88.304	1.002			
Adjusted mean	0.012	0.104	0.788			
$Mean;$						
R-square	0.169	<0	<0	0.153	<0	<0
Adjust R-square	0.142	<0	<0	0.136	<0	<0
Pr(F-statistic)	0.029 (887)	0.527 (162)	N/A	0.024 (891)	0.149 (599)	N/A
Pr(J-statistic)	N/A	N/A	0.050 {339}	N/A	0.352 {679}	0.058 {440}
Durbin-Watson	1.828 <850>	2.012 <974>	2.236 <991>	1.817 <783>	2.047 <996>	2.186 <990>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.

· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and <> the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table 5: Output Growth in a Standard Taylor Rule, Band Pass Filtered Data (2-15 years), 100 Years Simulated, 1000 Estimations Average.

7 Discussion

The difference between equation (**??**) and related Taylor rule equations is that the coefficients of the Taylor rule are fixed while in equation (**??**) they depend on the *BGP* money supply growth rate. McCallum (2004) points out that Walters (1971) apparently first makes the argument that the solutions to such equations as (**??**) would have parameters that depend on the money supply process, while distributed lag coefficients with fixed parameters do not, and that this was an application of what became known as the Lucas (1976) critique more generally. In equations (**??**) to (**??**), the solution is consistent with Walters in that $\lambda = \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{\left(1+\overline{R}\right)\left[\gamma+\frac{m}{c}(1-\gamma)\right]} = \frac{(1-\gamma)A_F\left(\frac{\overline{R}\gamma A_F}{w}\right)^{\frac{\gamma}{1-\gamma}}}{\left(1+\overline{R}\right)\left(\gamma+\left[1-A_F\left(\frac{\overline{R}\gamma A_F}{w}\right)^{\frac{\gamma}{1-\gamma}}\right](1-\gamma)\right)}$ is an unambiguously positive function of the *BGP* money supply growth rate through the (log-utility) *BGP* nominal interest rate of $\bar{R} = \Theta^* + \rho$; so $\lambda = \lambda\left(\Theta^*\right)$. The coefficient rises as the *BGP* money supply growth rate rises.

Applying Equation (**??**) but without the velocity term or using a standard Taylor (1993) to explain pre 1981 and post 1981 US history of nominal interest rates leads to an underestimate of the nominal interest rate during the late 60s and 1970s and an overestimate of the nominal interest rate in the early 1980s. That is well known. Using Equation (**??**) would ameliorated that outcome. The expected future inflation rate jumped up during (the Vietnam War) period of the 60s and 70s; if this expected rate at any time t have been higher than the actual inflation rate at time t, the predicted nominal interest rate would have been higher and so closer to the actual nominal interest rate. Similarly during the Volcker regime that implemented the Congress mandated deflation in the early 1980s, if the expected deflation was greater at any time t than the actual deflation, then the predicted interest rate at time t would have been lower and so closer to the actual nominal interest rate. Equation (**??**) by the higher expected than actual inflation rate would be able to better explain interest rates in the pre 1981 period, and by a lower expected than actual inflation rate would be able to better explain interest rates in the post 1980 period.

Sørensen and Whitta-Jacobsen (2005, pp.502-505) present a related way in which the coefficients of the 'rule', under the assumption of constant money growth, relate to elasticities of money demand rather than the preferences of policy-makers. Alvarez et al. (2001), Schabert (2003) consider the link between money supply rules and interest rate rules.[22] Instead of an exogenous fraction of agents being able to use bonds as in Alvarez et al. (2001), here the consumer purchases goods with an endogenous fraction of bank-supplied intratemporal credit that avoids the inflation tax on exchange as found in Reynard (2004, 2006) and Benk et al. (2010). Schabert's inflation coefficient in his interest rate equation is unity which we get in the special cash-only case (no credit, velocity fixed at one).

Our paper might be viewed as trying to reconcile Canzoneri et al's. (2007) account of

[22] Many others exist, for example Chowdhury and Schabert (2008).

the empirical shortcomings of standard estimated Euler equations, but by using estimated data instead of actual data, which would be the next step. As a corollary, we pursue Lucas (1976) and Lucas and Sargent's (1981) agenda of postulating "policy functions" with coefficients that depend explicitly upon the economy's underlying utility, technology, and policy parameters. The average growth rate of money supply on the BGP is a "structural" policy parameter that the consumer understands as part of the equilibrium conditions used to determine their behavior and which turn determines the BGP inflation rate which acts as the "target" inflation rate of the model.

8 Conclusion

The paper presents theoretically consistent estimation with simulated data of an interest rate Euler equation. Over the business cycle frequency it can resemble versions of the 'Taylor rule'. As the economy is a monetary business cycle model adjusted with an endogenous explanation of velocity, it suggests how a standard Taylor rule may result as a special case. Once the expected and actual inflation rate converge, the interest rate equation becomes more like a Fisher equation with a coefficient of one on the inflation rate term. The interest rate equation predicts known policy changes in the money supply growth rate, such as how an increase in the stationary money supply growth rate increases the interest rate equation's coefficient on the expected inflation rate term. This is a simple application of the Lucas (1976) critique within the economy, showing how the inflation rate coefficient is a behavioral coefficient subject to change when known policy parameters change.

Have a money supply rule that entails a stationary mean growth rate of the money supply, from which deviations occur due to financing fiscal expenditures, abstracts from more complex fiscal theories that involve the money supply on numerous factors. Here, it is just assumed that some fraction of inflation tax revenue is achieved on average over time, with deviations in this average. This translates the economy into one in which inflation targeting occurs indirectly, and it is manifested at the same time that the asset pricing equation of interest rates appears in form similar to a generalized Taylor rule, albeit without the central bank policy reaction function interpretation.

The paper allows expectations of how the estimated interest rate equations will change as policy change occurs, with the design of aiding policy evaluation for monetary practitioners engaged in inflation rate targeting. As King (2016) notes, "From the outset, inflation targeting was conceived as a means by which central banks could improve the credibility and predictability of monetary policy" (p. 170). If interest rate behavior depends on velocity as suggested in this paper, then markets that predict velocity and its affect on interest rates after certain events occur will lessen uncertainty and increase central bank credibility. In addition, it may be worth taking away the result that as expected interest rates converge to current interest rates, the interest rate equation of a forward-looking Taylor form simplifies to one of a Fisher form with a one to one relation between interest rates and inflation.

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A Definition of Competitive Equilibrium

The representative agent's optimization problem can be written recursively as:

$$V(s) = \max_{c,x,l_G,l_H,l_F,s_G,s_H,q,d,k',h',M'} \{u(c,x) + \beta EV(s')\} \tag{33}$$

subject to the conditions (3) to (9), where the state of the economy is denoted by $s = (k,h,M,B;z,u,v)$ and a prime (') indicates next-period values. A competitive equilibrium consists of a set of policy functions $c(s)$, $x(s)$, $l_G(s)$, $l_H(s)$, $l_F(s)$, $s_G(s)$, $s_H(s)$, $q(s)$, $d(s)$, $k'(s)$, $h'(s)$, $M'(s)$, $B'(s)$ pricing functions $P(s)$, $w(s)$, $r(s)$, $R_F(s)$, $P_F(s)$ and a value function $V(s)$, such that:

i. the consumer maximizes utility, given the pricing functions and the policy functions, so that $V(s)$ solves the functional equation (33);

ii. the goods producer maximizes profit similarly, with the resulting functions for w and r being given by equations (15) and (16);

iii. the bank firm maximizes profit similarly in equation (11) subject to the technology of equation (10)

iv. the goods, money and credit markets clear, in equations (**??**) and (14), and in (8), (17), and (10).

The consumer's first-order conditions follow:

$$\max_{c_t,x_t,l_{Gt},l_{Qt},s_{Ft},q_t,d_t,k_{t+1},h_{t+1},M_{t+1}} \mathcal{V}(k_0,h_0,M_0;z_0,u_0,v_0) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t x_t^{\Psi})^{1-\theta}}{1-\theta} \tag{34}$$

subject to

$$\begin{aligned}
\lambda \quad &: \quad w_t\,(l_{Gt}+l_{Ft})\,h_t + r_t s_{Gt} k_t + R_{Ft} d_t + \frac{T_t}{P_t} && (35)\\
&\geq \quad \frac{P_{Ft}}{P_t} q_t + c_t + k_{t+1} - (1-\delta_K)k_t + \frac{M_{t+1}}{P_t} - \frac{M_t}{P_t} &&\\
\mu \quad &: \quad \frac{M_t}{P_t} + \frac{T_t}{P_t} + q_t \geq c_t && (36)\\
\varepsilon \quad &: \quad c_t = d_t && (37)\\
\psi \quad &: \quad h_{t+1} = (1-\delta_H)h_t + A_H\left[(1-l_{Gt}-l_{Ft}-x_t)\,h_t\right]^{\eta}\left[(1-s_{Gt})\,k_t\right]^{1-\eta}. && (38)
\end{aligned}$$

$$
\begin{aligned}
0 &= (c_t x_t^{\Psi})^{-\theta} x_t^{\Psi} - \lambda_t - \mu_t + \varepsilon_t, \\
0 &= \Psi c_t^{1-\theta} x_t^{\Psi(1-\theta)-1} - \psi_t \eta A_H h_t \left(l_{Ht} h_t\right)^{\eta-1} \left(s_{Ht} k_t\right)^{1-\eta}, \\
0 &= \lambda_t w_t h_t - \psi_t \eta A_H h_t \left(l_{Ht} h_t\right)^{\eta-1} \left(s_{Ht} k_t\right)^{1-\eta}, \\
0 &= \lambda_t w_t h_t - \psi_t \eta A_H h_t \left(l_{Ht} h_t\right)^{\eta-1} \left(s_{Ht} k_t\right)^{1-\eta}, \\
0 &= \lambda_t r_t k_t - \psi_t \left(1-\eta\right) A_H k_t \left(l_{Ht} h_t\right)^{\eta} \left(s_{Ht} k_t\right)^{-\eta}, \\
0 &= -\lambda_t \left(\frac{P_{Ft}}{P_t}\right) + \mu_t, \\
0 &= \lambda_t R_{Ft} - \varepsilon_t, \\
0 &= -\lambda_t + \beta E_t \left\{\lambda_{t+1} \left[1 - \delta_K + r_{t+1} s_{G,t+1}\right]\right\} \\
&\quad + \beta E_t \left\{\psi_{t+1} \left(1-\eta\right) A_H s_{Ht+1} \left(l_{Ht+1} h_{t+1}\right)^{\eta} \left(s_{Ht+1} k_{t+1}\right)^{-\eta}\right\}, \\
0 &= -\psi_t + \beta E_t \left\{\lambda_{t+1} w_{t+1} \left(l_{Gt+1} + l_{Ft+1}\right)\right\} \\
&\quad + \beta E_t \left\{\psi_{t+1} \left[1 - \delta_H + \eta A_H l_{Ht+1} \left(l_{Ht+1} h_{t+1}\right)^{\eta-1} \left(s_{Ht+1} k_{t+1}\right)^{1-\eta}\right]\right\}, \\
0 &= -\frac{\lambda_t}{P_t} + \beta E_t \left\{\frac{\lambda_{t+1} + \mu_{t+1}}{P_{t+1}}\right\}.
\end{aligned}
$$

B Alternative Equilibrium Interpretations of Asset Pricing

B.1 Backward Looking Taylor Condition

Consider two alternative representations of the Taylor condition; a backward-looking version and an alternative version which features credit. First, the Taylor condition can be reformulated to feature a lagged dependent variable on the right hand side instead of the lead dependent variable which appears in equation (23). This yields a similar expression written in terms of R_{t+1} instead of R_t:

$$
\begin{aligned}
\overline{R}_{t+1} - \overline{R} &= \frac{\Omega}{(\Omega-1)} E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \frac{\Omega\theta}{(\Omega-1)} E_t \left(\overline{g}_{c,t+1} - \overline{g}\right) \\
&\quad - \frac{\Omega\psi(\theta-1)\frac{l}{1-l}}{(\Omega-1)} E_t \overline{g}_{l,t+1} - \frac{\Omega_V}{(\Omega-1)} E_t \overline{g}_{V,t} - \frac{1}{(\Omega-1)} \left(\overline{R}_t - \overline{R}\right).
\end{aligned} \tag{39}
$$

While equation (39) compares better to interest rate rules which feature a lagged dependent variable on the right hand side as an 'interest rate smoothing' term, the lead nominal interest rate is now the dependent variable. Such an expression is more akin to a forecasting equation for the nominal interest rate than an interest rate rule. Such a transformation also raises the fundamental issue discussed by McCallum (2010). He argues that the equilibrium conditions of a structural model stipulate whether any given difference equation is forward-looking ("expectational") or backward-looking ("inertial") and that the researcher is not free to alter the direction of causality implied by the model as is convenient. The forward looking representation of the Taylor condition (23) is the long accepted rational expectations version; for example, Lucas (1980) suggests that the forward looking "filters" suit models which feature an optimizing consumer. In fact, we would argue that the timing of the cash-in-advance economy is such that our forward-looking rule in equation (23) is the correct model, while equation (39) is consistent with the alternative "cash-when-I'm-done" timing which we do not employ (see Carlstrom and Fuerst, 2001).

B.2 Credit Interpretation of the Taylor Condition

Christiano et al. (2010) have considered how the growth rate of credit might be included as part of a Taylor rule so that "allowing an independent role for credit growth (beyond its role in constructing the inflation forecast) would reduce the volatility of output and asset prices." The term in velocity growth can be rewritten as the growth rate of credit in the following way: Since $V_t = \frac{c_t}{m_t} = \frac{1}{1-\left(1-\frac{m_t}{c_t}\right)}$, then $V\widehat{V}_t = \left(1-\frac{m}{c}\right)\widehat{\left(1-\frac{m_t}{c_t}\right)}$ and $\overline{g}_{V,t} = \frac{m}{c}\left(1-\frac{m}{c}\right)\overline{g}_{\left(1-\frac{m}{c}\right),t}$ where $\overline{g}_{\left(1-\frac{m}{c}\right),t}$ is the growth rate of normalized credit. The Taylor condition is now rewritten as:

$$\begin{aligned}\overline{R}_t - \overline{R} \quad = \quad & \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} \\ & -\Omega_{\left(1-\frac{m}{c}\right)}E_t\overline{g}_{\left(1-\frac{m}{c}\right),t+1} - \Omega_R E_t\left(\overline{R}_{t+1} - \overline{R}\right). \qquad (40)\end{aligned}$$

The credit coefficient can be derived as $\Omega_{\left(1-\frac{m}{c}\right)} \equiv \frac{(R-1)(1-\gamma)\left(\frac{m_t}{c_t}\right)^2}{(1-\gamma)^2}\frac{(1-\gamma)\left(1-\frac{m_t}{c_t}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m_t}{c_t}\right)\right]} \geq 0.$ A positive expected credit growth rate decreases the current net nominal interest rate $\overline{R}_t$. With velocity set at one as in a standard cash-in-advance economy, neither credit nor velocity would enter the Taylor condition since the credit service does not exist and velocity does not vary over time.

B.3 Results with Alternate Filters for Estimation

Tables B-1 and B-2 show the similar business cycle window results of estimations of Equation (30) using the HP filter (Ravn and Uhlig (2002) Smoothing Parameter) and a 3 to 8 Christiano and Fitzgerald (2004) band pass filter, instead of the 2 to 15 window presented in the main body of the paper.

HP filtered data, where HP $\lambda = 6.25$	Unrestricted			Assumed $\beta_5 = 0$		
	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-9.68E-07 [0]	-2.04E-07 [0]	8.09E-07 [17]	-7.60E-07 [0]	-4.78E-07 [0]	-7.57E-08 [8]
Standard error	2.87E-05	2.15E-05	3.15E-05	2.39E-05	1.67E-05	4.29E-05
Adjusted mean	-	-	-4.27E-08	-	-	2.59E-07
$E_t\pi_{t+1}$	2.019 [1000]	2.309 [691]	2.299 [1000]	0.315 [830]	0.757 [397]	0.621 [925]
Standard error	0.248	1.488	0.268	0.126	0.856	0.265
Adjusted mean	2.019	1.800	2.299	0.293	0.475	0.614
$E_t g_{c,t+1}$	0.251 [1000]	0.336 [959]	0.293 [1000]	0.172 [1000]	0.313 [989]	0.231 [1000]
Standard error	0.024	0.096	0.020	0.020	0.048	0.025
Adjusted mean	0.251	0.324	0.293	0.172	0.311	0.231
$E_t g_{l,t+1}$	-0.243 [890]	-0.536 [774]	-0.374 [997]	-0.281 [864]	-0.530 [774]	-0.427 [996]
Standard error	0.094	0.321	0.079	0.100	0.231	0.111
Adjusted mean	-0.236	-0.448	-0.374	-0.265	-0.453	-0.427
$E_t g_{V,t+1}$	-0.137 [990]	-0.267 [800]	-0.212 [1000]	-0.098 [889]	-0.317 [888]	-0.190 [992]
Standard error	0.031	0.228	0.033	0.036	0.109	0.056
Adjusted mean	-0.137	-0.229	-0.212	-0.093	-0.293	-0.190
$E_t R_{t+1}$	-1.819 [1000]	-2.338 [646]	-2.005 [1000]	N/A	N/A	N/A
Standard error	0.221	2.282	0.277			
Adjusted mean	-1.819	-1.692	-2.005			
$Mean$;						
R-square	0.789	<0	0.796	0.544	<0	0.482
Adjusted R-square	0.778	<0	0.785	0.525	<0	0.459
Pr(F-statistic)	2.35E-15 (1000)	0.015 (974)	N/A	3.93E-09 (1000)	0.003 (992)	N/A
Pr(J-statistic)	N/A	N/A	0.258 {1000}	N/A	0.159 {482}	0.269 {1000}
Durbin-Watson	1.474 <151>	2.243 <1000>	2.194 <970>	1.732 <419>	2.145 <999>	2.047 <882>
Sample size (1000x)	99	98	96	99	98	96

Notes:
· 'Standard error' measures the variation in the coefficient estimates.
· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.
· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).
· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).
· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.
· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and <> the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table B-1: Taylor Condition Estimation, HP Filtered Data, Ravn and Uhlig (2002) Smoothing Parameter, 100 Years Simulated, 1000 Estimations Average.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
3-8 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-7.57E-07 [0]	6.54E-06 [0]	-7.10E-07 [3]	7.73E-07 [0]	-2.86E-06 [0]	-1.51E-06 [1]
Standard error	1.65E-05	3.24E-04	2.09E-05	1.45E-05	4.37E-05	2.68E-05
Adjusted mean	-	-	-1.83E-07	-	-	-9.81E-08
$E_t\pi_{t+1}$	2.166 [998]	2.484 [724]	2.423 [1000]	0.633 [969]	2.417 [965]	0.682 [974]
Standard error	0.391	32.906	0.298	0.195	1.125	0.222
Adjusted mean	2.166	2.141	2.423	0.628	2.291	0.679
$E_tg_{c,t+1}$	0.283 [1000]	0.304 [623]	0.314 [1000]	0.155 [1000]	0.175 [834]	0.168 [1000]
Standard error	0.043	7.324	0.027	0.029	0.074	0.030
Adjusted mean	0.283	0.231	0.314	0.155	0.160	0.168
$E_tg_{l,t+1}$	-0.237 [827]	-0.573 [430]	-0.312 [982]	-0.222 [685]	-0.595 [666]	-0.268 [870]
Standard error	0.131	10.199	0.099	0.133	0.361	0.135
Adjusted mean	-0.229	-0.193	-0.312	-0.195	-0.455	-0.259
$E_tg_{V,t+1}$	-0.152 [982]	-0.453 [351]	-0.174 [998]	-0.174 [976]	-0.604 [973]	-0.194 [984]
Standard error	0.043	15.068	0.039	0.052	0.252	0.057
Adjusted mean	-0.151	-0.128	-0.174	-0.173	-0.578	-0.193
E_tR_{t+1}	-2.026 [994]	-1.532 [424]	-2.289 [1000]			
Standard error	0.435	116.012	0.339			
Adjusted mean	-2.024	-1.211	-2.289			
$Mean;$						
R-square	0.789	<0	0.842	0.576	<0	0.590
Adjusted R-square	0.778	<0	0.833	0.558	<0	0.572
Pr(F-statistic)	4.75E-10 (1000)	0.077 (874)	N/A	2.08E-07 (1000)	0.005 (981)	N/A
Pr(J-statistic)	N/A	N/A	0.213 {1000}	N/A	0.344 {848}	0.249 {1000}
Durbin-Watson	1.568 <54>	1.653 <330>	1.517 <49>	1.728 <333>	1.728 <378>	1.715 <306>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.

· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and <> the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table B-2: Taylor Condition Estimation, Band Pass Filtered Data (3-8 years), 100 Years Simulated, 1000 Estimations Average.