The (un)Demand for Cash in Canada

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Abstract

We exploit a novel dataset from the Bank of Canada to estimate the deposit functions for banknotes in Canada for three denominations, \$1,000, \$100 and \$50. The broad flavour of our empirical findings is that denominations are different monies and we recover structural estimates of the underlying sources of the non-neutrality. We find evidence of large and significant deposit costs for the highest value denomination, the \$1,000 banknote, and insignificant costs for the \$100 and \$50 denominations. Our results imply that the interest rate elasticity of deposit is positive for the \$1,000, negative for the \$100 and insignificantly different from zero for the \$50. Third, we find that between 5 per cent of the \$1,000, 29 per cent of the \$50 and 30 per cent of the \$100 ever-issued by the Bank of Canada, and remaining outside of financial institutions for the given denomination, do not circulate (in Canada). In short, the composition of monetary balances matters.

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1 Introduction

One difficulty with the empirical analysis of models of money demand is that the researcher typically requires individual- or household-level survey data. Survey responses are generally received from a sub-set of individuals or households invited to participate in the survey and those responses are, in most cases, self-reported. Since one salient feature of money is anonymity, it is unclear whether individuals who respond to surveys are representative of typical money users and/or whether the responses they give are accurate or truthful. Using aggregate money data is no less problematic since one cannot easily identify money demand from 'flow' data on withdrawals and deposits and 'stock' data on notes in circulation which is the data commonly reported by central banks. In this paper, we use unique deposit data from the Bank of Canada to study the complement of money demand: given outstanding monetary balances, what factors lead to their deposit at financial institutions? Succinctly, we study the undemand for money.

To study the undemand for money we exploit episodes of active currency management by the Bank of Canada which allow us to measure cash deposits at financial institutions in Canada (in this paper we will interchangeably use money, currency and cash to refer to banknotes issued by the Bank of Canada). On several occasions since 2000, the Bank of Canada has, with the voluntary participation of the financial institutions in the Bank Note Distribution System (BNDS), accelerated the withdrawal of certain series of banknotes from circulation in the economy.[1] Accelerated withdrawal programs require financial institutions to pass any banknotes of a targeted series deposited, or held, at their branches (including ATMs) directly to the Bank of Canada. Banknotes targeted by such accelerated withdrawal programs are labelled unfit.[2] The Bank of Canada redeems these unfit notes and may replace them with a new series of banknotes of the same denomination. Unfit banknote deposits are observed by the Bank of Canada since no unfit banknote is re-issued by a financial institution. Thus, the Bank of Canada observes the *deposit* of every banknote targeted in an accelerated withdrawal program.

As one example of such an accelerated withdrawal program, when the Bank of Canada introduced polymer \$100 banknotes, it simultaneously declared all remaining, ever-issued, paper \$100

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[1] There are nine financial institutions that comprise the BNDS. A brief discussion of the BNDS is presented in Section 2.

[2] As we discuss in Section 2, banknotes may also be labelled unfit if they fail to pass a quality screening by banknote sorting machines at Bank of Canada processing centres.

banknotes unfit. At the time of the accelerated withdrawal program for the \$100, there were roughly 312 million \$100 banknotes held outside the Bank of Canada. Using data from the Bank of Canada's currency processing centres, we construct a novel dataset of cash deposits for three episodes of accelerated withdrawal programs: from 2000-2015 for the \$1,000 banknote, from 2011-2015 for the \$100 banknote and from 2012-2015 for the \$50 banknote. These three accelerated withdrawal programs are similar to the extent that all existing paper banknotes series of these denominations were declared unfit. One notable difference is that the \$1,000 banknote was declared unfit and no replacement series of this denomination was issued whereas polymer banknotes series were issued for the \$100 and \$50 denominations. The rationales for an accelerated withdrawal program are varied, but these programs are generally undertaken to ensure public confidence in banknotes remains high. For instance, at the time of the \$1,000 accelerated withdrawal program, there were concerns raised by law-enforcement that the \$1,000 denomination was overwhelmingly used for criminal transactions.

We show how to use a standard inventory-theoretic choice framework to exploit our dataset to recover estimates of the fractions of outstanding, ever-issued, banknotes of a denomination that circulate, and their probability of being deposited as functions of the opportunity costs. We define circulation broadly to include any banknote that, *ex-ante* has a non-zero probability of being deposited. Thus, our definition of circulation does not include banknotes that are hoarded, have been lost, are held outside of Canada or that may exclusively circulate outside of the legitimate (formal) economy. One nice feature of our identification scheme is that we do not require individual data from depositors. We show that the choice, for an individual or merchant, of whether to deposit or hold a banknote of a given denomination depends only on a wedge driven by the opportunity costs incurred from not depositing and that this wedge is the same for all individuals and merchants. Our choice framework yields a standard logit specification for banknote deposits.

The dataset we construct, using Bank of Canada currency processing data, contains all banknote deposits geographically distributed across Canada (roughly according to the provincial boundaries) for three denominations: \$1,000, \$100 and \$50. Our estimates suggest, for these denominations, that between 5 and 30 per cent of ever-issued notes, unredeemed by the Bank of Canada, do not actively circulate through financial institutions (in Canada). As a fraction of the \$70 billion currency liabilities of the Bank of Canada as of 2015, this represents approximately 18% of the total value. Perhaps importantly, we find that the factors which affect the probability of deposit for a \$1,000 banknote are very different than those for a \$100 or \$50 banknotes. Thus, while monetary value is divisible by construction, the functions of money appear denomination specific. We also find that the interest rate elasticity of deposit is denomination specific in *sign* – it is positive for the \$1,000, negative for the \$100 and is not statistically different from zero for the \$50. Changes in the price level matter for the \$1,000 in half of the regions (accounting, however, for a supermajority of the Canadian population) but are insignificant for the \$100 and \$50. The broad flavour of our empirical findings is that denominations are different monies and we find evidence in favor of the non-neutrality of monetary denominations.

One concern with the comparison between denominations is that the \$1,000 denomination may suffer from survivorship bias as, once deposited, these notes cannot be re-obtained from a financial institution. In contrast, both the \$100 and \$50 denominations continue to be issued by the Bank of Canada, albeit as polymer notes. We conduct a robustness exercise which accounts for survivorship bias in the distribution of note holdings for the \$1,000 and find no difference to our conclusion that denominations are different monies.

We also find that the probability of deposit is falling in the face value of the note: \$50 notes have a deposit probability of roughly 10% per month, \$100 notes of roughly 7% and \$1,000 of roughly 2%. We argue that these deposit probabilities pose something of a puzzle, since the opportunity cost in terms of consumption of holding notes is rising in the denomination. Thus, deposit probabilities are inversely related to their opportunity costs which appears to stand in contrast to typical predictions from most theoretical monetary models.

The sample period for the \$1,000 denomination encompasses a period of legislative changes to financial regulations governing reporting requirements for large-value cash transactions and also the Financial Crisis of 2008/09. The financial tracking changes required regulated institutions (*e.g.* banks, investment brokers, accountants, money services businesses) to report the identities of individuals making large-value cash deposits to the authorities (Fintrac). Thus while depositing large cash sums remained a legal activity, such transactions were no longer anonymous. We show that our choice framework can identify the costs associated with the heightened reporting requirements and loss of anonymity. We find that, for the \$1,000 note, changes to financial tracking laws for large value cash deposits had no significant effect for the probability of deposit at a financial institution for any of the ten regions., In contrast, our results for the Financial Crisis, which we proxy by the Lehman bankruptcy, suggest that the crisis led to a 2-3 times increase in the deposit probabilities in five of the ten regions. These regions are also, in almost all cases, the regions in which the inflation elasticitiy of deposit for the $1,000 is significantly different from zero.

The demand for money has a long, and voluminous, history in economics. We contribute to three strands of this research. First, our results appear to be informative for the recently revived debate regarding measurement of monetary aggregates. Lucas and Nicolini (2015), Belongia and Ireland (2015), Hendrickson (2014) and Barnett *et al.* (2013) examine the empirical content of US monetary aggregates for economic activity and propose alternative measures of money supply to the simple-sum aggregates typically reported (*e.g.* M1, M2, etc). Lucas and Nicolini (2015) extend a the theoretical model of Prescott (1987) and Freeman and Kydland (2002) to define a new monetary aggregate, NewM1, using currency, reserves and commercial bank deposits as distinct elements. Belongia and Ireland (2015), Hendrickson (2014) and Barnett *et al.* examine the recent empirical performance of Divisia monetary aggregates introduced in Barnett (1980). While the results in this paper are silent as to the relative merits of Divisia monetary aggregates or NewM1, our findings suggest that even simple-sums for currency are misleading measures of currency liquidity, at least for Canada.

Our paper also provides estimates of regional patterns of money circulation in Canada. The estimation of regional circulation is typically challenging because econometricians rarely observe both withdrawals and deposits for a given banknote. Thus, an econometrician is unable to determine where a note circulates between these two events. Our model, as a by-product of the choice-theoretical framework, provides an estimate of the stock of existing notes in circulation by the regional distribution of Bank of Canada processing centres at the time of an accelerated withdrawal program implementation. We find some evidence of regional differences in per-capita money holdings. There appears to be no previous research measuring bank note circulation in domestic regions in Canada (or indeed any other nation). The lack of research does not appear due to a lack of interest. Dow (1982) argues that understanding regional money-multipliers is useful for (New) Keynesian models because such models are often very applicable to regional government spending and because aggregate dynamics may depend on the aggregation of regional multipliers. Mulligan and Sala-i-Martin (1992) emphasis the importance of regional (disaggregated) data for the estimation of money demand. Both of these papers focus on non-cash measures of money such as demand deposits.

In a partly-related, empirical, literature, Bartzsch and Seitz (2015), Bartzsch *et al.* (2011a,b), provide estimates for the shares of Euros circulating outside of the Eurozone and Doyle (2000) and Porter and Judson (1996) provide estimates of the foreign demand for US dollars. Our identification approach, based upon a choice-theoretic framework rather than time-series regressions using monetary aggregates, is fundamentally different to these existing approaches. One potential limitation of our approach is that we cannot separately identify foreign demand from other non-circulating notes, *e.g.* hoarded notes, although in our theoretical model such a distinction is immaterial.[3] However, the time series approach is also limited in that it typically assumes that all domestic demand in the proxy countries used for identification is in fact domestic, and circulating. At least for one proxy country often used in this literature, Canada, our results suggest this is untrue. The time series approach is also, typically, atheoretic in terms of the specification of money demand whereas our model is structural.

A final literature to which our results may apply is the study of the denomination structure of money. Most attention has focussed on the optimal divisibility of monetary instruments, typically in models in which money is principally introduced as a means of payment. For example, Telser (1995), Van Hove and Heyndels (1996), Lee *et al.* (2005), Lee (2010) and Bouhdaoui *et al.* (2011) consider the optimal denomination structures of monetary instruments as payment instruments. Our results would appear to cast some doubt on some conclusions from this literature, particularly for the higher denomination notes which we study, because these conclusions are based on environments in which monetary objects circulate for transactional purposes. Our results suggest that monetary units are not equally likely to circulate and that monetary objects with different nominal face-values are unique monies. An alternative theoretical view from Wallace and Zhu (2004) is that money is a commodity good. The results we present in this paper are not a formal test of the models proposed in Wallace and Zhu, however the non-neutrality results we find for banknotes appear consistent with the random-matching model of non-divisible money proposed in that paper.

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[3] In our model, notes either have a zero probability of deposit in the initial period or a positive probability. Foreign held notes and hoarded notes should, almost certainly, have a zero probability of deposit since both are held, presumably, for future consumption. In this sense, hoarded notes and foreign held notes are one in the same and the geographic distinction simply partitions the set of such notes. While there may be policy questions related to the geography of note holdings our model is silent on such issues.

Our estimates are informative for both policy analysis and operational planning by central banks. Calza and Zaghini (2011) stress the importance of circulating bank notes for the calculation of the welfare effects of inflation and Judson (2011) stresses the importance of measuring the quantity of US bank notes circulating abroad for seigniorage. Rogoff (2014) emphasizes the importance of circulating currency for the zero-lower bound on the nominal interest rate. Our results also suggest that the zero-lower bound on the nominal interest rate may be particularly important in practical terms given the proportions of high denomination notes that appear to be non-circulating.

Our results are also informative for the question of whether electronic currencies, or other alternative payment instruments, may replace physical specie. It would appear that the answers to such questions are almost certainly denomination specific. One may also wonder how feasible it would be, or how long it would take, to replace circulating physical specie with electronic currencies (e-monies) without some measure of demonetization, given the large proportion of notes that do not circulate into financial institutions and the relatively low deposit rates for those that do. Indeed, in terms of operational planning, estimates of the deposit probabilities of actively circulating notes are useful for planning future accelerated withdrawal programs.

2 The Bank Note Distribution System

Together with nine private financial institutions, the Bank of Canada distributes banknotes across Canada via the Bank Note Distribution System (BNDS). The BNDS was reformed in 1996 as part of a large-scale overhaul of the payment system in Canada.[4] The BNDS owns two currency processing centres, in Toronto and Montréal, from which it distributes banknotes to one of 44 regional distribution centres. Each regional distribution centre is owned by one of the nine financial institutions which are part of the BNDS.[5] The regional distribution centres receive (distribute) banknotes from (to) bank branches. The regional distribution centres are associated with 10 regional distribution points (roughly mapped to the provinces of Canada) and at these points the financial institutions exchange fit banknotes from their distribution centres using the BNDS. Fit banknotes are notes which pass a quality-index threshold for redistribution to the economy. The quality score of a note is determined by factors such as tears, graffiti, folds, stains, color degradation and other

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[4] Bilkes (1997) provides a comprehensive overview of the changes.

[5] The nine financial institutions that are members of the BNDS are: Banque Nationale, Desjardins, Banque Laurentienne, Scotia Bank, RBC, TD, CIBC, Alberta Treasury Board and the Credit Union Central of Alberta.

machine-readable quality measures. Stock and flow data is transmitted to the Bank of Canada through a digital inventory-management program, which records note counts by denomination.

Financial institutions deposit unfit banknotes received by their regional distribution centres to one of the ten regional distribution points where these deposits are processed by the Bank of Canada and sent to one of the two processing centres. During accelerated withdrawal programs, all notes of the chosen denomination received by a regional distribution centre are deposited though the BNDS system to one of the processing centres. The data we report in this paper are the deposit data at the processing centres (which are dated to their arrival at the regional distribution centre). Figures 1, 2 and 3 in the Appendix show the deposits by region for the \$50, \$100 and \$1,000 denominations respectively. One feature of the data, which is expected, is the decay process in banknote deposits. This pattern is largely true for all denominations in all regions. However, for the \$50 denomination, there is a spike in each region for the initial period. This is due to the start date of the accelerated withdrawal process, March 26th, which means that the initial period is roughly 1/6 of a month, unlike for the other denominations. As a result, for our estimations for the \$50 denominations we merge the data for the first two months, March and April, of the accelerated withdrawal program. We perform a robustness check to this change of data by defining deposits per business day and find no difference to our results.

3 A Model of Deposits

In this Section we propose a simple model which shares the inventory perspective of the classic Baumol-Tobin cash-demand model but with a twist: we focus on cash deposits not withdrawals. Our model starts *in media res* – agents in the economy hold notes which they have already withdrawn from the central bank. Our focus is to examine the deposit of these notes to financial institutions and to uncover the key factors, if any, driving deposit differences over time and across regions.

We assume that any agent, who can be either a merchant or an individual, deposits a note from their inventory of holdings whenever it is preferable for them to do so. Consumption theory implies, for time-separable preferences and linear budget constraints, that the Euler equation describing the trade-off between consumption across periods can be written as:

$$u'(c_t) = \mathbb{E}_t[\beta(1+r)u'(c_{t+1})],$$

where c_t and c_{t+1} are consumption at time t and $t+1$ respectively, β is a time discount factor, $1+r_t = (1+\iota_t)/(1+\pi_{t+1})$ is the gross real interest rate (with ι the nominal interest rate and π the inflation rate) and $\mathbb{E}$ is the expectation operator over any relevant processes. (We do not include time subscripts on r, ι and π for notational convenience unless not doing so would be confusing.) We assume also that an agent's decision to deposit a note does not affect his or her current, or expected future, income. Thus, our interest is in the agent's decision to continue to hold cash rather than depositing the cash in a financial institution.

Since we will assume that depositing cash is costly, the relevant trade-off for an agent holding cash is whether the present-value cost of depositing a note is greater than the present-value expected benefit. (We stress that although we have written our payoff functions as utility functions defined over consumption, our approach allows a more general interpretation of $u(c)$ as simply the payoff to spending cash held by the agent.) We assume that $x_{\mathsf{d}} \geq 1$ is the fixed gross per-dollar transaction fee for making a deposit of a note at a financial institution and for acquiring a new payment instrument of that value in period $t+1$ so that $1/x_{\mathsf{d}}$ is the fraction remaining after fees in period $t+1$. The expected benefit (revenue) of depositing a note of denomination d is, for $\mathsf{d}(r)$ sufficiently small, simply $\mathbb{E}[\beta(\frac{\mathsf{d}(1+r_t)}{x_{\mathsf{d}}})u'(c_{t+1})]$. In contrast, the expected cost of depositing is $u'(c_t)$. An agent will deposit a note **iff**:

$$u'(c_t) < \mathbb{E}_t[\frac{\beta\mathsf{d}(1+r_t)}{x_{\mathsf{d}}}u'(c_{t+1})].$$

We note that the Euler equation for an agent not depositing (or spending) a note implies that:

$$u'(c_t) < \mathbb{E}_t[\frac{\beta\mathsf{d}}{1+\pi_{t+1}}u'(c_{t+1})],$$

where π_{t+1} is the inflation rate in period $t+1$. By induction, comparing the benefits of depositing to those of not depositing, an agent will deposit a note **iff**:

$$1 < \mathbb{E}_t[(1+\iota_t)/(x_{\mathsf{d}})].$$

Iterating on expectations and taking logarithms, we write:

$$U^D < -\ln(x_{\mathsf{d}}) + \ln(1 + \mathbb{E}_t[\iota])], \tag{1}$$

where U^D is the utility threshold for depositing a note (normalized to 0). Although the assumption that x_{d} is fixed may be innocuous in a two-period setting, in our empirical work which follows we exploit time series data which implicitly implies a stronger assumption that x_{d} is fixed over all periods. Given technological progress in the financial sector and also general increases in the price level this may not be reasonable. To relax this assumption for our empirical work, we allow for change over time in the transactions fees such that the fee in period t is $x_{\mathsf{d},t} = x_{\mathsf{d}}(1 + \Pi_t)$, where Π_t is the change in the price level at time t compared to $t = 0$. In this interpretation, transactions fees rise inline with nominal costs which implies that eventually the costs of deposit may exceed the benefits for a particular denomination which would appear a reasonable representation of reality.

Equation (1) implies that the decision to deposit a note or not rests on the same factors for any agent. However, the assumption that all agents face the same transaction fees is likely overly strong. We assume that agents face idiosyncratic deposit costs (such as weather shocks, traffic delays, illness) for notes held in their possession and that, for each note, a deposit cost shock e is drawn independently from a Type 1 extreme-value distribution. In this formulation, e captures note-specific fixed costs of deposit. We acknowledge that assuming draws of e are independent across notes for an agent is possibly overly strong. As a result, we assume that agents consider each note individually and do not simply make a decision of whether to deposit every note in their possession or not. This assumption does however, have substantive benefits. First, this formulation permits non-binary deposit decisions by agents, *i.e.* deposit decisions are not all or nothing decisions in terms of notes in possession. Second, because the periodicity of our data is monthly, it captures an environment in which an agent faces the decision to deposit bills at different times within the month (when correlation in draws of e are likely weaker). Third, we find no evidence that relaxing this assumption would change our results as our model appears to capture the variance in the data well (as measured by R^2).

Given e and assuming ι small, we approximate the decision to deposit a bill or not as:

$$U^D = -\ln(x_{\mathsf{d}}) + \mathbb{E}_t[\iota_t] - \ln \Pi_t + e$$

In our empirical analysis, we assume that agents have rational expectations over ι and that transactions fees are known.[6] The first assumption might appear innocuous because the nominal interest offered at the time of deposit is typically advertised (although one concern is that agents face interest rates different from those we assume). The second assumption regarding transaction fees may appear innocuous but our assumption that fees are indexed to the general price level may imply a potential mismeasurement. We address the concerns over the potential endogeneity of both ι and Π using a control function approach which we outline in our empirical work below.

Under the assumption that e is distributed Type 1 extreme-value, we can write the probability that a note is deposited at a financial institution in region i in period t, $p_{i,t}$, as:

$$p_{i,t} = \frac{\exp^{-\ln(x_{\mathsf{d}})+\iota_t-\ln(\Pi_t)}}{1+\exp^{-\ln(x_{\mathsf{d}})+\iota_t-\ln(\Pi_t})} \tag{2}$$

where we allow the deposit probability to vary across regions and over time.

3.1 Notes in Circulation

One outstanding question is the identification of the outstanding stock of notes which circulate in region i in period t which we define as $M_{i,t}$. Since we have defined the probability of deposit, $p_{i,t}$, then it must be that the number of notes deposited per period per region is $p_{i,t}M_{i,t}$. Obviously, an equilibrium model of cash demand and supply would require that bank note withdrawals minus bank note deposits would equal the growth in the notes in circulation. Unfortunately, data on the number of notes in circulation deposited at financial institutions is not generally available, particularly by denominations or units. Thus, while data on cash withdrawals by financial institutions are available, one cannot, in general, infer anything about the level of deposits of those notes.

Fortunately, there exist episodes in the issuance of bank notes in which it is possible to measure the level of deposits. In particular, the Bank of Canada has, on occasion, with the support of financial institutions declared certain denominations unfit so that *every* existing note of the given denomination is directly deposited with the Bank of Canada when received by a financial institution. Thus, for these notes, deposits at the Bank of Canada are the sum of all deposits of that denomination at a financial institution at that time. This process is known as an accelerated withdrawal. When an accelerated withdrawal process is used, all current circulating notes of the denomination are simultaneously declared unfit. More pertinently, during an accelerated withdrawal,

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[6] We note that these are contemporaneous expectations over the current period realizations that an agent faces once arrived at a financial institution and not expectations over the future evolution of these prices.

withdrawals of existing notes of that denomination are 0 even though withdrawals of new series are positive.

We define unfit note deposits at the Bank of Canada in region i in period t as $UF_{i,t}$ and define $M_{i,0}$ as the stock of initial currency circulating in region i at the time of the unfit policy change. Thus, in the initial period $t = 0$,

$$UF_{i,0} = M_{i,0} p_{i,0} \tag{3}$$

which allows us to pin down the initial circulation of notes in region i. In subsequent periods, $t \geq 1$, expected unfit note deposits are given by:

$$UF_{i,t} = p_{i,t}(M_{i,0} - \sum_{m=0}^{t-1} UF_{i,m}), \tag{4}$$

where the term in brackets is the cumulative stock of notes remaining in circulation. In theory, we are free to impose the restriction that $M_{i,0} = UF_{i,0}/p_{i,0}$, however in our empirical application below, we estimate $M_{i,0}$ as a fixed effect for each region i and sum the total across the regions i because imposing the restriction directly is problematic when we control for endogeneity of ι and Π. (However, we can report that there is no significant difference (and typically only a very small quantitative difference) when we impose the restriction in our models without an endogeneity correction.) We then compare this sum to the Bank of Canada's record of the number of notes in circulation in Canada. As we document, our estimates of the total stock of notes in circulation are typically lower than the number of notes in circulation. Given our definition of circulation, we conclude that these missing notes: have been destroyed or lost; are hoarded; are held internationally or; circulate in a sector of the economy which does not deposit notes at financial institutions.

Our first application is to estimate $p_{i,t}$ and $M_{i,0}$ using data for the \$1,000 bank note which was declared unfit on May 12, 2000. The BNDS data from the Bank of Canada provides the number of unfit deposits for these notes for each region in Canada. The use of the \$1,000 banknote is particularly useful because the period of the policy change also includes a period when financial tracking rules for large value cash deposits changed and also encompasses the financial crisis. The financial tracking rule change arguably imposed a higher cost for depositing notes by requiring large value cash depositors to provide personal information. We are able to quantify how the change in policy affected the probability of deposit for the \$1,000 note by region and find that it had no statistically significant effect. However, as we show below, the financial crisis affected deposit probabilities in roughly half of the regions.

3.2 $50 and $100 Notes

As a second application, we estimate the deposit probabilities and the stocks of $50 and $100 notes in circulation using data from the introduction of the polymer notes for these denominations. The Bank of Canada introduced polymer $100 bills on Nov 14, 2011 and $50 bills on March 26, 2012 and simultaneously declared existing paper notes of these denominations to be unfit on the same day (though, unlike the $1,000 note the Bank of Canada issued polymer notes of these denominations as replacements).[7] Unfortunately, for the $50 and $100 notes, the Bank of Canada's unfit data do not distinguish between unfit paper notes and polymer notes that have become unfit because of some type of physical deformation (*e.g.* creased through use, defaced, torn, etc.). Thus, the unfit series for these notes are a mixture of paper note deposits and polymer notes and so, by extension, our empirical identification is less clean.

The unfit $50 and $100 notes deposited to the Bank of Canada in the initial period of polymer introduction will be *all* notes currently deposited at a financial institution in region i, so we can continue to define the initial stock of paper notes in circulation as:

$$UF_{i,0} = p_{i,0} M_{i,0}.$$

Recall that the accelerated withdrawal program for the $100 and $50 denominations was simultaneous to the introduction of new polymer banknotes of the same denomination. Thus, in period 1, the number of unfit $50 and $100 notes received by the Bank of Canada is equal to the number of paper notes received by financial institutions plus the number of new polymer notes that were introduced in period $t = 0$ which became unfit. Thus,

$$UF_{i,1} = p_{i,1}(M_{i,0} - UF_{i,0} + \ddot{q}_{i,0}),$$

where the first element of the RHS is the number of newly deposited paper notes from the total pool of such notes and the second term, $\ddot{q}_{i,t}$, is the number of unfit polymer notes from the number

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[7] Although the Bank of Canada subsequently introduced polymer notes of other denominations, they did not simultaneously declare existing paper notes unfit for those denominations. Accelerated withdrawal programs require the voluntary, active, involvement and participation of all financial institutions that operate regional distribution centres and can be logistically challenging, particularly for financial institutions.

of polymer notes introduced in period $t = 0$. We define $\ddot{q}_{i,t}$:

$$\ddot{q}_{i,t} = \begin{cases} 0 & \text{if } t = 0 \\ \sum_{i=1}^{t-1} \alpha^j (1-\alpha^j)^{t-(i+1)} Q_{i,t} & \text{if } t \geq 1; \end{cases} \tag{5}$$

where α is the probability that a polymer note has become unfit in a given period (we ignore differences across regions in unfitness rates) and $Q_{i,t}$ is the number of polymer notes withdrawn in region i in period t. This specification assigns the same **per-period** probability that a polymer note becomes unfit to the population of polymer notes in circulation. Thus $\ddot{q}_{i,t}$ is the expected number of unfit polymer notes deposited with the Bank of Canada given the existing age distribution of polymer notes in circulation.

We choose to estimate α using auxiliary data from circulation trials conducted by the Currency Department of the Bank of Canada which are designed precisely for this purpose. The circulation trials follow roughly 20 million polymer notes (for both the \$50 and \$100 denominations) and use optical scanners to assess the fitness of these notes as they circulate through the Bank of Canada's Agency Operation Centres. Our estimates of α for the \$50 and \$100 denominations are from these trials. The Circulation Trials report cumulative unfit percentages for each denomination. We assume a constant per-month unfitness rate, α, and a continuous-time environment to estimate α from the cumulative unfit percentages. We estimate $\alpha = 0.00009$ for the \$100 denomination and $\alpha = 0.00022$ for the \$50. These estimates are precise as there appears to be little time-series variation in the unfitness rates for the polymer notes. [8] We experiment with using a range of α around our estimated values for both the \$100 and \$50 and do not qualitatively or quantitatively find different results from those we report in the paper.[9] Nevertheless, it is probable that our estimates of $\ddot{q}_{i,t}$ are mismeasured which could imply that our estimates of $p_{i,t}$ are biased if the measurement errors are correlated with the deposit probabilities. We again use a control function approach to account for such endogeneity.

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[8] We do not report the full estimation results but these are available upon request to the authors, subject to the data availability policies of the Bank of Canada.

[9] An alternative approach would be to estimate α directly in our main regression however we do not choose to do this because identification of α in our main equation, while theoretically possible, relies on exploiting a difference between a polynomial $\ddot{q}$ and the sum of unfit notes. Given our relatively short time series, our estimates of α in this approach are generally not very robust.

4 Results

In this section we present our main results for the \$1,000 denomination and also our results for the \$50 and \$100 denominations. One advantage of our data is that we are able to use regional-level unfit deposit data from the 10 Bank of Canada regional distribution points: Newfoundland; Halifax; Quebec City; Montreal; Ottawa; Toronto; Winnipeg; Regina, Calgary and Vancouver. Moreover, given the parsimony of our model, we estimate for these regions separately (we do not impose any cross-equation restrictions). Our sample sizes are balanced across regions and are 180, 42 and 38 for each of the 10 regions for the \$1,000, \$100 and \$50 respectively.[10]

We adopt the following baseline econometric specification of the deposit probability for each denomination,

$$p_{i,t} = \frac{\exp^{\alpha_{1,i}+\alpha_{2,i}\iota_t-\alpha_{3,i}\ln(\Pi_{t,i})+\gamma_i S_t}}{1+\exp^{\alpha_{1,i}+\alpha_{2,i}\iota_t-\alpha_{3,i}\ln(\Pi_{t,i})+\gamma_i S_t}}, \tag{6}$$

with $\alpha_{1,i} \equiv -\ln(x_{\mathsf{d}})$ for region i and $\alpha_{2,i}$ and $\alpha_{3,i}$ being the relative contributions of the nominal interest rate and the non-seasonally adjusted price level for deposits in region i, respectively, and $\gamma_i S_t$ being a vector of quarterly dummy variables to account for possible seasonal patterns. Thus, our estimating equation for the \$1,000 is:

$$UF_{i,t} = p_{i,t}(M_{i,0} - \sum_{m=0}^{t-1} UF_{i,m}) + u_{i,t}, \tag{7}$$

and our estimating equations for the \$100 and \$50 are:

$$UF_{i,t} = p_{i,t}(M_{i,0} - \sum_{m=0}^{t-1} UF_{i,m} + \ddot{q}_{i,m}) + \tilde{u}_{i,t}, \tag{8}$$

where $u_{i,t}$ and $\tilde{u}_{i,t}$ are the regression residuals. We estimate our models using non-linear least squares. In our regressions where we use control functions to adjust for the possible endogeneity of the nominal interest rate or CPI inflation, we augment our specification of the deposit probability to include the residuals from the first stage regressions for these variables. Given our use of control functions, we use a bootstrap procedure to estimate the standard errors. We bootstrap with 199

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[10] Clearly imposing cross-equation restrictions and estimating a non-linear seeming-unrelated regression, for example, would increase our apparent sample sizes by a factor of 10. We chose not to present our results in this way so that differences in the estimated parameters and standard errors act roughly as a cross-validation exercise for our model specification. Since we find reasonably stable parameter estimates across all specifications and our method is a conservative approach in terms of standard errors, we choose to present our results from separate regressions. Results from NLSUR specifications are available upon request.

resamples of our data for every model we present. Arguably, 199 is on the low side but we do not observe much difference in our estimates for trials with 999 resamples.

Our model identifies two variables of interest, the stock of notes in circulation at $t = 0$, $M_{i,0}$ and the deposit probabilities, $p_{i,t}$, which allow us to estimate the interest elasticity of bank note deposit. First, our model identifies the stock of notes in circulation in each region (recall we define circulation as having a non-zero probability of being deposited at $t = 0$) and so summing across regions provides an estimate of the number of notes in circulation at the time such notes were declared unfit. Comparing these estimates with the number of notes ever-issued of that denomination at that time provides an estimate of the number of destroyed, irretrievably lost notes or notes that circulate outside of the financial system. Our model also isolates the effects of the price level and nominal interest rates separately for the deposits of bank notes. While this is not direct evidence of the effect of these factors for 'new' money demand (i.e. withdrawals from financial institutions) it is evidence of the effect of these factors for the demand to hold money.

Because the Bank of Canada regions do not exactly correspond to the Canadian provinces, we match the regions with the nearest provincial level data on prices which is obtained from Statistics Canada. We use the 30 day treasury bill rate as the nominal interest rate to correspond with the one-month periodicity of our data. We are naturally concerned that the price level and/or the treasury bill rate is (are) either mismeasured (perhaps because of errors in expectations by agents which we do not observe or because our geographies do not exactly overlap) or determined by an omitted variable contemporaneously with unfit deposit levels. We control for endogeneity using a control function approach where we follow Ng and Bai (2009) and use two-period lagged values of the first four principal components (two for the \$100 and \$50 denominations because of the shorter sample period) from interest rate data, asset market data and foreign exchange rate data (we present this data in more detail in the Appendix) and also two-period lagged values of gasoline and food prices from the CPI subindices. These instruments will be valid as long as innovations to our treasury bill and CPI inflation data are not predictable at more than a one month frequency.

In almost all regions and denominations, our control function approach does not suggest any evidence of endogeneity. However, for the \$1,000 denomination for the Toronto and Vancouver regions, we find some evidence of endogeneity although the bias appear very small by comparison with the non-instrumented results. Thus, for Toronto and Vancouver, we report the summary results from the regressions including the control functions when we presented aggregated tables. We stress though, that adjusting for endogeneity appears to make very little difference to our estimates.

4.1 Notes in Circulation: Estimates

We first present a summary of our estimates of notes in circulation for the \$1,000, \$100 and \$50 denominations in the regions we observe.[11] Although we estimate both including and excluding control functions, in the interests of brevity, we summarize our results below using the estimates from the specification which appears most suitable. Thus, if our regression estimates show evidence of endogeneity we present our estimates in italics from the control function specification but if we find no evidence of endogeneity we present our baseline results without italics. We define evidence of endogeneity to be either a p-value less than 0.05 on an individual control function parameter estimate or a p-value on joint signficance of the control function parameter estimates less than 0.05. We acknowledge that these criteria are arbitrary but our mean estimates of notes in circulation are not sensitive to any conventional threshold we might choose (the standard errors do change meaningfully in a few cases). In the Appendix, we report the individual regression results by region.[12]

Perhaps not surprisingly, there is a strong correlation between market size and the location of notes with the three largest regions (Toronto, Montreal and Vancouver) accounting for a supermajority of outstanding notes. We use our 'mean' estimates to calculate the total outstanding stock of unfit notes of each denomination which can be accounted for by our model. We include our estimates of the proportion of notes not in circulation and label this region "Unknown" as it's location is uncertain. [13] To put these estimates into context, we calculate the value of notes not in circulation using our baseline estimates as \$12.2 billion which, to put in perspective, is roughly 18% of the currency liabilities of the Bank of Canada as of November 2015. One possibility is that these notes have been destroyed or irretrievably lost although, given the large-value denominations

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[11] The detailed regression results for each region are reported in the Appendix.

[12] We consider only the two specifications of our regression equations, *i.e.* with and without the control functions, and do not test down our models to more parsimonious specifications as doing so would require arbitrary decisions over sequences and p-value thresholds.

[13] Our estimates are compatible with the findings of Hogg, Liang and Stuber (2012) who examined the deposits to-date of notes in accelerated withdrawal programs. Our estimates of the circulating specie are higher than the actual receipts observed by those authors.

involved, this would seem unlikely to explain the entire discrepancy. Nevertheless, despite being unobserved, the Unknown Region is the largest region in terms of the circulation of $100 notes, fully 7 percentage points larger than Toronto. It is the second largest region in terms of notes for the $50 denomination, but rather unremarkable for the $1,000 denomination. The geographic concentrations of bank notes largely follow the regional populations with Vancouver having slightly more $100 notes per-capita than other regions overall and Toronto and Montreal having much more $1,000 per-capita.[14] The concentration of $1,000 notes in Toronto and Montreal accounts for roughly 60% of the initial outstanding circulation of $1,000 notes. One possibility is that this concentration of notes is a legacy from the initial introduction of such notes as a settlement vehicle for financial institutions. However, our estimates of the initial stock of circulating notes is imprecise in one region for both the $50 and $1000 denominations although it is precise for all regions for the $100. Thus, we cannot statistically reject that, for the $50 and $1000, the Unknown Region is Winnipeg and Winnipeg and Vancouver respectively.

A final observation is that the proportion of $1000 notes in the Unknown Region is signfiicantly smaller than for either the $100 or $50 denominations. On the one hand, this result appears intuitive as the opportunity cost of this denomination is, ignoring deposit costs, 10 and 20 times larger than for the $100 and $50 denominations, respectively. On the other hand, these results suggest that the reason for cash holding in the Unknown Region is unlikely to be for savings since the physical storage costs for the $1,000 note is, for a given nominal amount, the lowest of the three denominations studied. To the extent that the Unknown Region is similar across denominations, we conclude that these results are indicative that the Unknown Region either represents an (sub) economy with circulating physical specie or else irrational hoarding.

4.2 Deposit Probabilities

We next present our estimates of the average deposit probabilities of the unfit banknotes. To clarify, these deposit probabilities are the average conditional probability that an undeposited and unfit

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[14] The reader may wonder why we do not consider a specification with a time trend for $M_{i,0}$. There are two reasons. The first is simply that allowing growth over time complicates the identification strategy since we cannot separate growth in initial currency stocks from trend growth in deposit probabilities. The second complication is that we would also need to consider the effects of flows across regions which is beyond the scope of our data. By focussing our analysis on initial stocks, there are no dynamic spillover considerations since these are already implicit in the data. If we had initial withdrawal locations for notes we could try to examine regional shifts, however such data is unavailable.

Table 1: Circulation by Region

Circulation by Region

		$1,000			$100			$50	
Region	Estimate	Std Err	Share	Estimate	Std Err	Share	Estimate	Std Err	Share
Calgary	**138235**	22292.19	4%	**1.72E+07**	3.15E+05	6%	**1.09E+07**	1.12E+06	6%
Halifax	**145334**	7224.873	4%	**9552655**	2607087	3%	**4943648**	553376.8	3%
Montreal	**1094830**	40580.76	31%	**5.83E+07**	3.87E+06	19%	**3.09E+07**	2.56E+06	16%
Newfoundland	**18881**	1081.797	1%	**2327261**	215534.8	1%	**1542480**	153238.9	1%
Ottawa	**69329**	7449.631	2%	**3705896**	136397.3	1%	**3432704**	320158.4	2%
Quebec City	**226085**	12682.06	6%	**7630105**	580253.2	2%	**3575339**	515178	2%
Regina	**44941**	7182.756	1%	**3382492**	464734.5	1%	**2976548**	302239.3	2%
Winnipeg	89657	3304120	3%	**5144663**	291795.9	2%	3796396	3.27E+08	2%
Toronto	**1238588**	**93263**	35%	**7.93E+07**	9.01E+06	25%	**5.65E+07**	3.34E+06	29%
Vancouver	*299497*	*8400207*	8%	***3.33E+07***	*4.20E+05*	11%	**1.81E+07**	9.33E+05	9%
Unknown	182803		5%	9.20E+07		30%	5.62E+07		29%

Notes: Italicized estimates are from the control function specification. Bold indicates the estimates have a p-value of less than 0.01. Share refers to the estimate as a proportion of ever-issued notes outstanding from the Bank of Canada at the time of the unfit announcement.

note is deposited in a financial institution.[15] There are at least two reasons why a note will not be deposited: (1) the note is held for anticipated exchange, or (2) the note is being held as savings because the cost of deposit to a financial institution is greater than the potential gain.[16] If (1) is correct we might expect deposit rates to rise in the face-value of the denomination since these notes are less typically used in transactions, see Henry, Huynh and Shen (2015). If (2) is correct, we should observe deposit probabilities that are increasing in the value of the denomination assuming that the gain from depositing is simply interest earnings. As Table 2 suggests, the pattern we observe is inconsistent with our expectations from (1) and (2). Of course, one possibility is that the deposit probabilities differ only because the sample periods do not coincide so that the conditioning variables are of different magnitudes. We explore and reject this conjecture.

The average estimated deposit probabilities imply that the probability of deposit is inversely related to the value of the denomination. In all regions, the $50 denomination has the highest average estimated deposit probability, followed by the $100 while the $1,000 denomination has the lowest deposit probability. The number of observations for each denomination is different because the unfit episodes have different calendar timing but the overall picture of the deposit probabilities for the $1,000 denomination does not appear to depend on changes in the underlying interest rates for the sample period as the standard errors of the estimate are tight. The estimates we have chosen to present in Table 2 summarize the estimates across various specifications which we

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[15] These deposit probabilities are conditional on the nominal interest rate and the price level for the sample period.

[16] We acknowledge that these reasons, while exahustive in terms of motivation, do not separately capture notions of hoarding which, we believe, could plausibly be interpreted as either reason.

consider. In the Table, the estimated deposit probabilities for Vancouver for the \$1,000 and the \$100 are those calculated from the control function regressions, but these estimates are not sensitive to the endogeneity correction. In the Appendix, we present detailed estimates from additional specifications which confirm that our main estimates are essentially unchanged across specifications. We also note that the deposit probabilities are, by and large, fairly homogeneous across regions.

Table 2: Average Estimated Deposit Probabilities by Region (in Percent)

	\$1,000		\$100		\$50	
Region	Average	Std Err	Average	Std Err	Average	Std Err
Calgary	1.21%	0.71%	9.59%	6.04%	5.50%	6.47%
Halifax	0.92%	0.33%	4.58%	1.40%	9.50%	9.67%
Montreal	1.05%	0.69%	4.34%	0.93%	7.08%	2.10%
Newfoundland	1.40%	0.39%	4.76%	1.25%	8.50%	7.31%
Ottawa	0.75%	0.75%	6.43%	2.81%	6.19%	2.26%
Quebec City	0.79%	0.58%	3.87%	0.76%	6.65%	3.06%
Regina	0.66%	0.64%	6.03%	2.18%	14.76%	10.10%
Winnipeg	0.43%	0.38%	6.33%	2.65%	5.82%	5.73%
Toronto	0.78%	0.81%	7.06%	2.43%	8.04%	4.18%
Vancouver	*1.51%*	*0.62%*	*7.81%*	*2.60%*	12.22%	8.80%

Notes: Italicized estimates are from the control function specification.

Our individual regression results, reported in the Appendix, suggest that the key driver of the deposit probability is the nominal interest rate for the \$1,000 and \$100 denominations, though of opposite sign, while the price level changes are not statistically significant. For the \$50 denomination, we observe no statistically significant effect for either the price level or the nominal interest rate. For the \$1,000 and \$100 we do not observe any seasonal differences in the deposit probabilities but for the \$50 we do observe some seasonal patterns in roughly half of the regions. We next calculate the average interest elasticity of unfit note deposit for our data to gauge how responsive unfit note deposits are to interest rate changes. We do so for the \$1,000 and \$100 denominations since the nominal interest rate is significant for these denominations for notes that circulate (obviously the interest rate elasticity is zero for notes in the Unknown Region). Our measure of the sample average interest rate elasticity for region i is:

$$\varepsilon_i(\iota, UF) = \frac{1}{T}\sum_{j=1}^{t} \alpha_2 p_{i,j}(1-p_{i,j})\frac{\iota_j}{UF_{i,j}}(M_{i,0} - \sum_{m=0}^{j-1} UF_{i,m})$$

Our estimates are presented in Table 3. We observe two significant findings for the sample interest rate elasticity of unfit deposits: for the $1,000 it is roughly 0.35 while for the $100 it is roughly -2. Since Canadian treasury bill rates were unusually low by historical standards during the $100 sample period (ranging between 0.5 and 1%), one possibility is that the sample interest rate elasticity is non-linear in the interest rate even if the model does not suggest such a channel. It is also possible that the introduction of the $100 polymer bill led to a change in usage of all high denomination bills which implied that all such notes changed their interest elasticity.

To investigate this hypothesis, we introduced a dummy variable for the $100 sample period and also interacted this dummy variable with the interest rate in the regressions for the $1,000 note. The coefficient estimates reject non-linearity in the interest rate elasticity in all regions but provide some support for the hypothesis that the introduction of the polymer $100 changed fixed cost of depositing the $1000 note in two regions (Ottawa and Vancouver). In these regions, the introduction of the $100 polymer lowered the fixed cost associated with the deposit of the $1000. One narrative consistent with this finding is that the increased security provided by the new polymer security features and durability led to switch by non-depositers from paper $1,000 to polymer $100.[17] However, in general, the introduction of the $100 polymer note had little impact on the deposits of the $1,000 unfit paper notes.

Thus, our estimates suggest that deposits of the $1,000 denomination increase with rises in the nominal interest rate. In contrast, deposits of the circulating $100 decrease by a factor of 2 with rises in the nominal interest rate (the same is true for our estimates for the $50 but these are not statistically significant). The results for the $1,000 are suggestive of an interest rate opportunity cost of holding money as in standard monetary theory. Surprisingly those for the $100 are suggestive of the opposite, although we remind the reader that this estimate is only for notes in circulation.

While these numbers are not directly comparable to the interest rate elasticity of money *demand* because they measure the *undemand* of money balances, they do suggest that the distribution of bank note denominations plays a crucial role in the evaluation of such elasticities. We conclude that the composition of money balances should not be overlooked.

In a similar manner, we calculate the (log) price-level elasticity of deposit since the change in the log price level is significant for some regions for the $1,000 note. We find an elasticity of deposit of roughly -1.5 for the regions for which log price level is a significant covariate. We

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[17] The detailed results of these regressions are available upon request.

interpret this finding as evidence that rising nominal prices reduces the benefits of deposit for the high denomination bills in this regions.

Table 3: Average Sample Interest Rate Elasticity

	Interest Rate Elasticity		Price Level Elasticity	
Region	\$1000	\$100	\$1000	\$100
Calgary	**0.43**	**-2.65**	-1.24	0.02
Halifax	**0.30**	-1.25	**-0.67**	-0.38
Montreal	**0.27**	**-1.61**	**-1.17**	-0.14
Newfoundland	**0.47**	-1.46	-0.26	**-0.28**
Ottawa	**0.25**	**-2.40**	**-1.74**	0.08
Quebec City	**0.27**	**-1.55**	**-1.40**	-0.19
Regina	0.14	**-2.17**	-2.16	-0.34
Winnipeg	**0.44**	**-2.80**	**-1.87**	-0.19
Toronto	**0.21**	**-1.84**	**-1.78**	-0.04
Vancouver	***0.47***	***-1.75***	-0.45	**0.11**

Notes: Italicized estimates are from the control function specification. Bold indicates the estimates have a p-value of less than 0.01.

4.3 Survivorship

Our estimates of the deposit probabilities and elasticities reported thus far rely on a stationary distribution for the unobserved deposit costs, e, at least conditioning on the price level. This restriction seems palatable for the \$100 and \$50 denominations since the introduction of polymer notes at the time the existing notes were declared unfit means that the ability of an agent to obtain such a denomination was unaffected. However, for the \$1,000 note, the declaration of unfit status was not accompanied by the introduction of a new series. Thus, \$1,000 notes became harder to obtain. This suggests that there may be a survivorship bias evident in the distribution of e over time.

To examine the robustness of our results to survivorship, we adapt our model to reflect increasing (mean) costs over time. We write the deposit decision for an agent as:

$$U_t^D = (\ln(x_{\mathsf{d}}) + \ln(1 + \bar{x}^t)) + \mathbb{E}_t[\iota_t - \ln(\Pi_t)] + e$$

where $\ln(1+\bar{x}^t)$ is the increasing cost to the surviving agents for depositing a note ($\bar{x}$ is the growth rate in mean costs). This specification implies that notes may be increasingly held by agents who find depositing the note to be more costly than agents in a previous period, if $\bar{x} > 0$, which reflects the survivorship bias. We note that the inclusion of $\bar{x}$ may also help to explain the deposit elasticity for the \$1,000 denomination since interest rates were mostly declining over our sample period. If the survivorship bias leads to fewer deposits, then this may weaken the observed relationship between the interest rate and the deposits.

We estimate this version of our model using a nonlinear seeming-unrelated regression, imposing the constraint that $\alpha_{k,i} = \alpha_k$. We impose the constraint to aid identification of $\bar{x}$ different from the time path of ι because the ι is common across regions. However, given our earlier results for the baseline specification without $\bar{x}$ suggest some differences in $\alpha_{k,i}$ across regions, this restriction may increase the variance in our estimates of α_k while aiding identification of $\bar{x}$ (survivorship bias is likely to be more common across regions). Our results suggest that the survivorship bias does not play an important role (see Table 14). Our estimates for $\bar{x}$ are insignificantly different from zero in the baseline specification and we find no evidence that the control function specification is preferable.[18] Our remaining point estimates are likewise essentially unaffected and we observe that the variance of α_k is actually smaller than in the individual regressions which suggests that this restriction does not impose undue structure on the data. We conclude that our evidence for the deposit rates and interest elasticities reported above is unchanged by survivorship.

Comparing all the results reported thus far for the \$1,000, \$100 and \$50 denominations, and the detailed estimates presented in Tables 8 to 14, there appears to be little similarity in the deposit probabilities and the fractions of notes in circulation. We find evidence that the \$1,000 denomination has statistically significant fixed costs and that its deposit rates rise as nominal interest rate rise. We find no evidence of fixed costs for the \$100 denomination and find statistically significant evidence that its deposit rates fall as the nominal interest rate rise. Finally, we find no evidence of statistically significant fixed costs for the \$50 and no evidence that deposit rates depend on the nominal interest rate. The only similarity across denominations is that the deposit rates for all denominations are unaffected by changes in the inflation rate. We acknowledge that our assumption that e is uncorrelated across individual notes may be overly strong. One may question the merit of imposing the conditional choice structure as we do to obtain estimates of 'structural' parameters. Regardless, our model structure does not change the underlying difference between denominations in the correlation of deposit rates to the nominal interest rate or inflation.

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[18] For these regressions we report jackknife standard errors since bootstrapping our data is complicated by the inclusion of $\ln(1 + \bar{x}^t)$.

We emphasize that our results are robust across regions, without imposing any cross-equation restrictions beyond the model structure. Indeed, we interpret the similarity of our results, for each denomination, across regions as validation for our theoretical model. If our theoretical choice model were grossly mis-specified, we would expect to see results vary across regions for each denomination, which we do not.

4.4 Regulatory Change and the Financial Crisis

The sample period from May 2000 until April 2015 for the \$1,000 unfit notes encompasses a financial regulatory change for cash deposits that was ushered in after September 11th, 2001. In Canada, the government created Fintrac, an agency with oversight over financial transactions deemed suspicious (thresholds for such are legally defined). One regulatory change, enforced as of November 30, 2002, required financial institutions to report large cash deposits in excess of \$10,000 and to require depositors to identify themselves and the source of the funds being deposited. This reporting requirement may have imposed a cost on deposits of bank notes, likely particularly so for large denominations such as the \$1,000. Individuals processing large-value cash transactions regularly would be most likely to trigger the reporting requirements. To examine the impact of the regulatory change, we include a dummy variable in our deposit probability specification.[19]

A second, arguably unexpected, change which may have affected bank note deposits was the financial crisis arising from the Lehman bankruptcy.[20] One plausible mechanism is that bank deposits became less desirable because of depositor concerns over the soundness of the financial system, although we note that no Canadian financial insititution did fail during this period. We introduce a second dummy variable dated from September 2009 to account for the effect of the Lehman bankruptcy for the deposit probability of the \$1,000 notes.

We present the estimated coefficients for the Fintrac dummy variable and the Lehman bankruptcy dummy variable in Table 4. Our results suggest no evidence that the Fintrac legislation affected the costs of deposit of the \$1,000 banknote. In contrast, the Lehman bankruptcy had a significant effect and reduced the costs of deposit in roughly half of the regions. In the final column of the table, we calculate the Lehman effect as a fraction of the expected fixed effect for deposit costs on deposits, $e^{-\ln(x_{\mathsf{d}})+q}/e^{-\ln(x_{\mathsf{d}})}$, where q is the estimated coefficient on the Lehman dummy. To a

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[19] Unfortunately the sample periods for the \$100 and \$50 are too short (or too late) for a similar analysis.

[20] The exact timing when the financial crisis became a salient effect is, of course, unclear. However, we do not find much difference to alternative datings, such as the Bear Stearns bankruptcy.

first-order approximation, this fraction represents the unconditional increase in deposit rates due to the Lehman effect. For the regions in which the Lehman Bankruptcy dummy variable is significant, the estimated fraction of the fixed effect is roughly 200-300%. Of particular interest is that the regions in which the Lehman dummy is significant are also the regions in which the price level coefficient is estimated to be significant. One narrative is that depositors may have expected that the response of monetary authorities to the financial crisis was likely to lead to inflation and so their response was to deposit notes. We also note that the regions in which the Lehman dummy was significant constitute roughly four fifths of the Canadian population.

Table 4: Marginal Effect on Deposits of Fintrac Legislation

	Fintrac Legislation			Lehman Bankruptcy		
Region	Estimate	Std Err	FE Share	Estimate	Std Err	FE Share
Calgary	0.26	0.32	129%	1.29	0.51	363%
Halifax	-0.09	0.12	91%	**0.57**	**0.14**	176%
Montreal	0.09	0.11	109%	**0.66**	**0.14**	193%
Newfoundland	0.03	0.21	103%	0.62	15.07	185%
Ottawa	0.27	0.24	130%	**1.20**	**0.43**	331%
Quebec City	0.11	0.14	112%	**0.81**	**0.21**	225%
Regina	0.40	0.53	150%	1.83	1.29	624%
Winnipeg	0.32	0.19	138%	0.89	0.39	243%
Toronto	*0.34*	*0.18*	140%	**1.25**	**0.32**	349%
Vancouver	*-0.01*	*0.16*	144%	***0.72***	***0.18***	297%

Notes: Italicized estimates are from the control function specification. Bold estimates are statistically significantly different from 0 at the 5% level. FE Share refers to the percentage of the deposit costs of the estimated dummy parameter.

5 Discussion

We argue that the deposit probabilities and the proportion of notes held in the Unknown Regions constitute something of a puzzle for existing models of money demand. We briefly consider how the predictions of two canonical models, the inventory method pioneered by Baumol (1952) and Tobin (1956) and the search friction approach pioneered by Kiyotaki and Wright (1989), relate to our results. Both approaches suggest that the opportunity cost of holding cash is the foregone nominal interest. If true, one would expect the deposit probability to increase in the denomination since higher denomination notes are more costly to hold in terms of foregone interest on the marginal dollar and their velocity is likely relatively low – most cash transactions in Canada are far below the high denominations nominal values, see Henry, Huynh and Shen (2015). We find the opposite.

A second alternative is that an agent's cash balances reflect anticipated consumption and that higher denomination notes are more likely to be held to ensure that consumption is unconstrained. This alternative hypothesis suggests that some agents (sellers) should hold cash balances less than the highest denomination in order to 'make' change. In such a world, the deposit probability of the seller is higher for large denomination notes than it is for smaller denomination notes. To see why, consider an agent in a money-search model where agents are uncertain whether they will be buyers or sellers (possibly because of matching frictions or uncertainty about type). In such an environment, the seller should have maintained high enough cash balances to ensure consumption if he or she were a buyer but in denominations small enough to ensure that he or she can 'make change' in the case that he or she is a seller. Thus, sellers should not wish to carry large denomination cash balances across periods. From the buyers' side, a similar argument suggests that buyers should maintain sufficiently divisible cash balances to facilitate trade, though as Lee *et al.* (2005) note too small of a denomination is not welfare improving either. Indeed, Telser (1995), Van Hove and Heyndels (1996) and Lee (2010) all consider the question of optimal denomination structures and, in general, their results suggest that the \$1,000 has very limited value as a means to facilitate exchange. In an inventory-theoretic model, the argument is less direct because such models do not typically model sellers' cash inventory behaviour and would typically require an adjustment to the model environment. Nevertheless, in both the inventory and search-theoretic environments, it would seem logical that high denomination notes would be held as a savings vehicle, rather than for next-period consumption. Our results regarding the non-neutrality of denominations appear consistent with the commodity-money, random matching environment proposed in Wallace and Zhu (2004). Our results are, however, not a formal test of that environment and we are sanguine whether alternative model environments might also be consistent with our empirical results.

Another observation is that our model estimates imply no statistically significant fixed costs for deposits of the \$50 and \$100 denominations but does imply a statistically significant fixed cost for the \$1,000. For the \$1,000, our estimates of α_1 in all regions is roughly -4.5 which implies $x_{\mathsf{d}} \approx 0.01$. This implies that current period costs for depositing and then withdrawing the \$1,000 note are much larger (approximately 100 times larger) than simply holding the note for the future period. This suggests that the costs of deposit for the \$1,000 note are larger than simply the foregone interest from our discussion above and also suggests that the costs of deposit may reflect more than simply a monetary calculation. Although our model cannot distinguish between the costs of acceptance at merchants and the costs of depositing directly at financial institutions, it does suggest that the \$1,000 note possesses characteristics that may make it costly as a medium of exchange. Returning to the \$50 and \$100 denominations, our estimates do not reject that $x_{\mathsf{d}} = 1$ for these denominations.

Our final observation is that our estimates of the size of the Unknown Region appear consistent with our evidence on the deposit costs of denominations. Our estimates suggest that the Unknown Region for the \$1,000 is proportionately the smallest of the three denominations we study. Our evidence also suggests that the \$1,000 note has the highest cost of exchange of the denominations. Thus, the \$1,000 note would seem to be principally a means to save, and saving in the Unknown Region may be costly in terms of foregone interest payments. Certainly, this narrative is consistent with our finding that the Lehman Bankruptcy and associated policy responses increased deposits for the \$1,000 note. It appears, therefore, that the holders of the \$1,000 may wish to transfer these balances to financial institutions but find it costly to do so, although not prohibitively so. Thus, if the Unknown Region features active markets with monetary exchange, one might expect the Unknown Region for the \$1,000 to be smaller. For the \$50 and \$100 notes, the size of the Unknown Regions appear consistent with usage in unmeasured parts of the economy. These notes do not appear to be costly to circulate in exchange. The deposit probability for the \$100 denomination, in particular, is negatively related to the nominal interest rate which suggests that its usage, in terms of notes circulating through financial institutions, increases in economic expansions (given that the short-term nominal interest rate is generally pro-cyclical). Thus, our estimates of the Unknown Regions for the \$50 and \$100 would appear indicative of expected usage by agents who do not circulate notes through financial institutions. Why the \$100 Unknown Region is proportionately the largest cannot be answered in our model environment but would appear worthy of further study.

6 Conclusion

We have used a distinct change in the treatment of circulating paper notes that occurred as the Bank of Canada phased in polymer notes to estimate the number of \$50, \$100 and \$1,000 notes in circulation through financial institutions. Our baseline results suggest that roughly 29% of the unredeemed \$50 notes ever issued by the Bank of Canada, 30% of the unredeemed \$100 notes ever issued and 5% of the unredeemed $1,000 notes ever issued do not circulate to Canadian financial institutions. We find evidence that the deposit probability of $1,000 notes increases as nominal interest rates rise, while the opposite is true for the $100. We also find evidence that the financial crisis increased the deposit probability of the $1,000 by roughly 200-300% for several regions.

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7 Appendix

Figure 1: $50 Unfit Note Deposits by Region

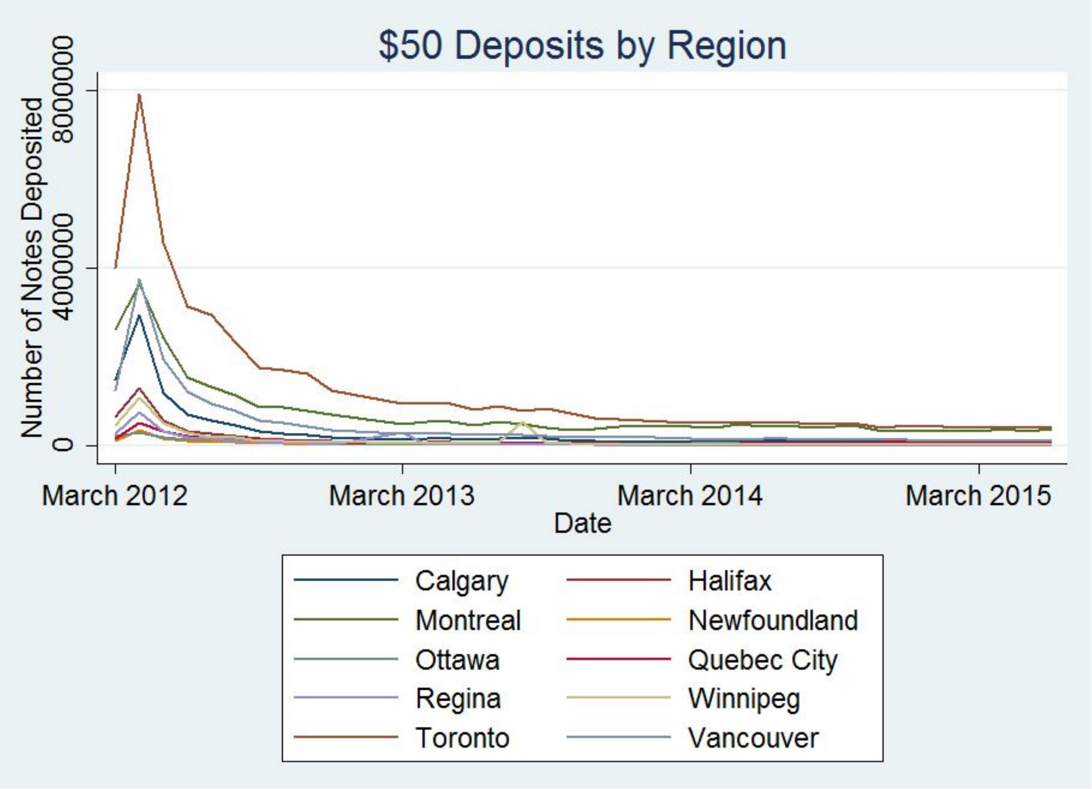

Figure 2: $100 Unfit Note Deposits by Region

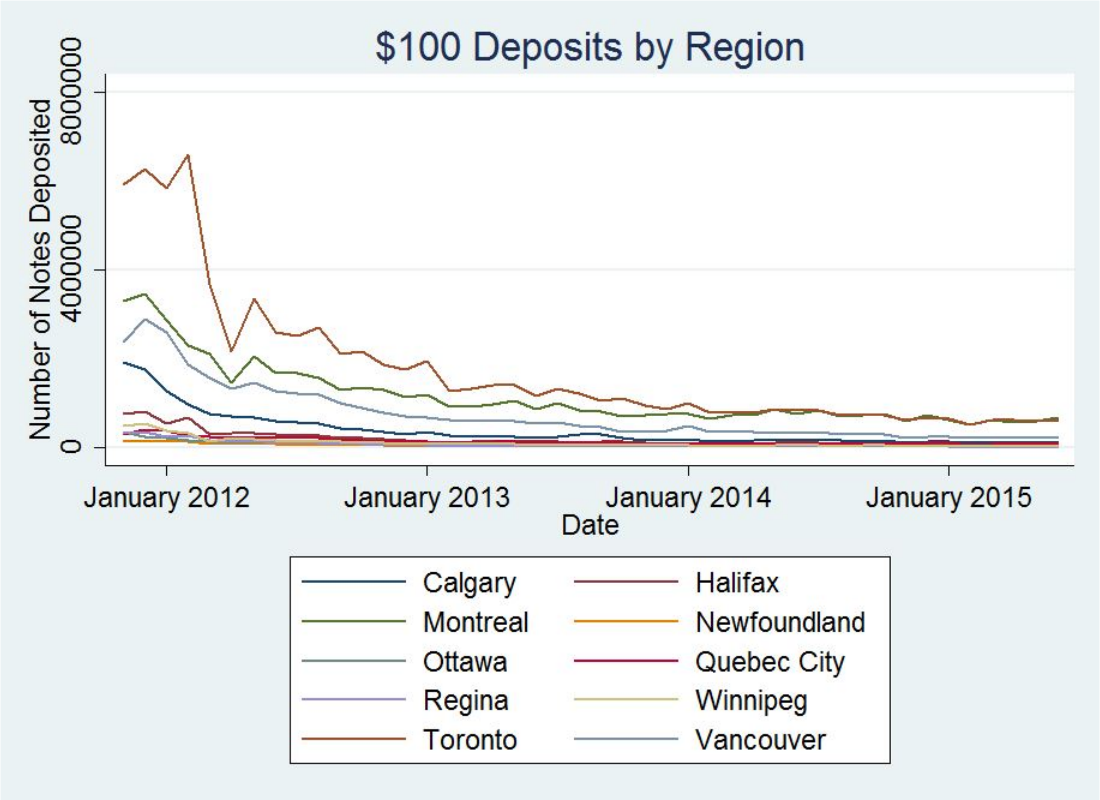

Figure 3: $1,000 Unfit Note Deposits by Region

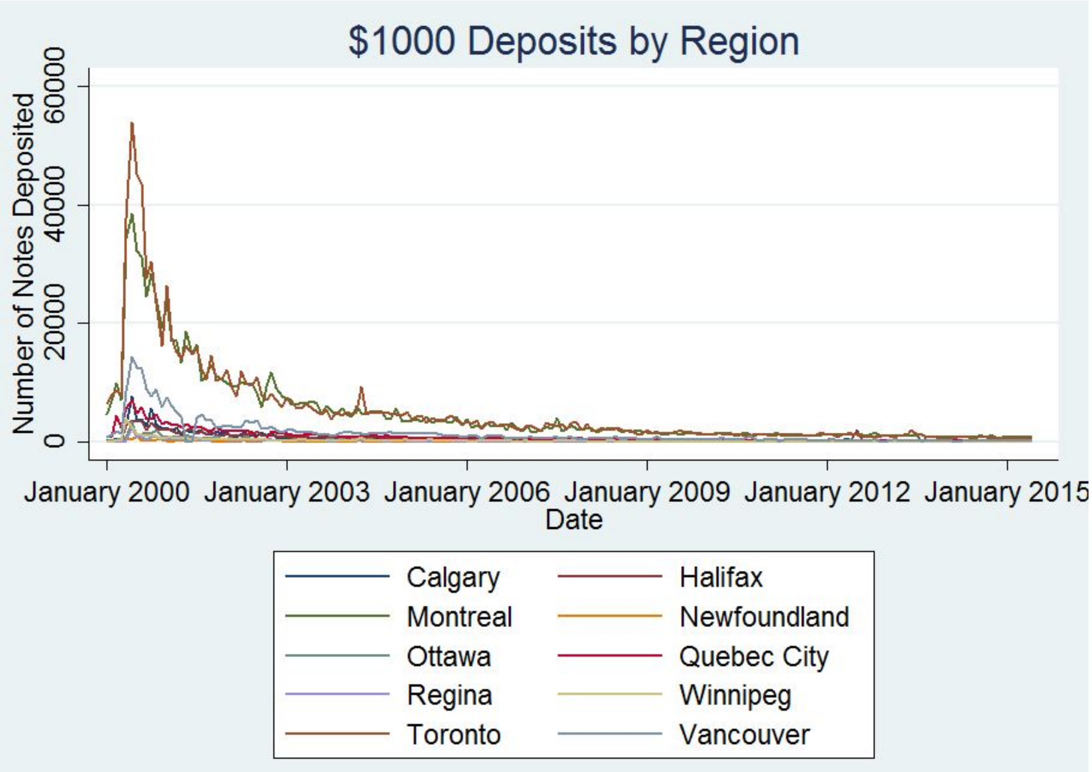

Table 5: Control Function Interest Rate Variable List

Bank rate, last Tuesday or last Thursday
Bank rate
Chartered bank administered interest rates - prime business
Chartered bank - consumer loan rate
Forward premium or discount (-), United States dollar in Canada: 1 month
Forward premium or discount (-), United States dollar in Canada: 3 month
Prime corporate paper rate: 1 month
Prime corporate paper rate: 2 month
Prime corporate paper rate: 3 month
Bankers' acceptances: 1 month
Bankers' acceptances: 2 month
Bankers' acceptances: 3 month
Overnight money market financing, 7 day average
Selected Government of Canada benchmark bond yields: 2 year
Selected Government of Canada benchmark bond yields: 3 year
Selected Government of Canada benchmark bond yields: 5 year
Selected Government of Canada benchmark bond yields: 7 year
Selected Government of Canada benchmark bond yields: 10 years
Selected Government of Canada benchmark bond yields: long term
Government of Canada marketable bonds, average yield: 1-3 year
Government of Canada marketable bonds, average yield: 3-5 year
Government of Canada marketable bonds, average yield: 5-10 year
Government of Canada marketable bonds, average yield: over 10 years
Chartered bank - 90 day term deposits
Chartered bank - conventional mortgage: 1 year
Chartered bank - conventional mortgage: 3 year
Chartered bank - conventional mortgage: 5 year
Chartered bank - 5 year personal fixed term
Chartered bank - daily interest savings (balance over $100,000)
Chartered bank - non-chequable savings deposits
Chartered bank - Guaranteed Investment Certificates: 1 year
Chartered bank - Guaranteed Investment Certificates: 3 year
Chartered bank - Guaranteed Investment Certificates: 5 year
Treasury bill auction - average yields: 3 month
Treasury bill auction - average yields: 3 month, average at values
Treasury bill auction - average yields: 6 month
Treasury bill auction - average yields: 1 year
Treasury bill auction - amount auctioned: 3 month (dollars x 1,000,000)
Treasury bill auction - amount auctioned: 6 month (dollars x 1,000,000)
Treasury bill auction - amount auctioned: 1 year (dollars x 1,000,000)
Treasury bill auction - amount maturing (dollars x 1,000,000)
Treasury bills: 1 month
Treasury bills: 2 month
Treasury bills: 3 month
Treasury bills: 6 month
Treasury bills: 1 year
Real return bonds, long-term
Government of Canada marketable bonds, average yield, average of Wednesdays: 1-3
Government of Canada marketable bonds, average yield, average of Wednesdays: 3-5
Government of Canada marketable bonds, average yield, average of Wednesdays: 5-10
Government of Canada marketable bonds, average yield, average of Wednesdays: over 10
Average residential mortgage lending rate: 5 year
Chartered bank - chequable personal savings deposit rate
First coupon of Canada Savings Bonds

Table 6: Control Function Asset Market Variable List

Standard and Poor's/Toronto Stock Exchange Composite Index, high
Standard and Poor's/Toronto Stock Exchange Composite Index, low
Standard and Poor's/Toronto Stock Exchange Composite Index, close
Toronto Stock Exchange, stock dividend yields (composite), closing quotations
Standard and Poor's/Toronto Stock Exchange 60 Index
Standard and Poor's/Toronto Stock Exchange Canadian Consumer Discretionary Index
Standard and Poor's/Toronto Stock Exchange Canadian Consumer Staples Index
Standard and Poor's/Toronto Stock Exchange Canadian Energy Index
Standard and Poor's/Toronto Stock Exchange Canadian Financial Index
Standard and Poor's/Toronto Stock Exchange Canadian Gold Index
Standard and Poor's/Toronto Stock Exchange Canadian Industrial Index
Standard and Poor's/Toronto Stock Exchange Canadian Information Technology Index
Standard and Poor's/Toronto Stock Exchange Canadian Materials Index
Standard and Poor's/Toronto Stock Exchange Canadian Diversified Metals and Mining
Standard and Poor's/Toronto Stock Exchange Canadian Telecommunication Service
Standard and Poor's/Toronto Stock Exchange Canadian Utilities Index

Table 7: Control Function Foreign Exchange Market Variable List

United States dollar, noon spot rate, average
United States dollar, 90-day forward noon rate
Danish krone, noon spot rate, average
Japanese yen, noon spot rate, average
Norwegian krone, noon spot rate, average
Swedish krona, noon spot rate, average
Swiss franc, noon spot rate, average
United Kingdom pound sterling, noon spot rate, average
United Kingdom pound sterling, 90-day forward noon rate
United States dollar, closing spot rate
United States dollar, highest spot rate
United States dollar, lowest spot rate
United States dollar, 90-day forward closing rate
United States dollar, 30-day forward closing rate
United States dollar, 60-day forward closingrate
United States dollar, 180-day forward closing rate
United States dollar, 1-year forward closing rate
Australian dollar, noon spot rate, average
Hong Kong dollar, noon spot rate, average
New Zealand dollar, noon spot rate, average
Mexican pesos, noon spot rate, average
Canadian dollar effective exchange rate index (CERI) (1992=100)
European euro, noon spot rate, average

Table 8: Estimates by Region, $1,000 Denomination

Region	1		2		3		4		5	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
T Bill	255.36	75.74	157.96	33.21	158.02	22.56	235.31	81.53	154.40	72.61
CPI	6.83	3.51	3.51	1.27	9.09	1.90	1.47	2.22	14.01	4.94
x_{d}	-4.12	0.28	-4.51	0.14	-3.92	0.10	-4.81	0.26	-3.60	0.37
Q3	-0.34	0.26	-0.02	0.11	-0.10	0.09	0.32	0.19	-0.65	0.22
Q4	-0.16	0.28	-0.08	0.10	-0.12	0.10	0.02	0.15	-0.37	0.23
Q1	-0.50	0.29	-0.19	0.10	-0.16	0.09	-0.17	0.19	-0.62	0.19
Fintrac1	0.26	0.32	-0.09	0.12	0.09	0.11	0.03	0.21	0.27	0.24
Lehman	1.29	5.12E-01	0.57	0.14	0.66	0.14	0.62	15.07	1.20	0.43
M_0	138235.20	22292.19	155406.30	9724.87	1094830.00	40580.76	18880.57	1081.80	69329.22	7449.63
R^2	327.64		242.68		1074.47		55.44		130.60	
Nobs	180		180		180		180		180	

Region	6		7		8		9		10	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
T Bill	160.14	36.61	83.03	173.70	241.60	66.00	145.11	44.74	311.66	61.08
CPI	11.16	2.37	15.49	10.93	11.92	3.53	15.84	3.03	8.15	5.50
x_{d}	-4.05	0.16	-3.48	0.94	-4.76	0.67	-3.63	0.30	-4.53	0.37
Q3	-0.19	0.11	-0.63	0.34	-0.25	0.21	-0.10	0.15	0.20	0.31
Q4	-0.22	0.12	-0.42	0.31	-0.46	0.21	-0.33	0.15	-0.03	0.32
Q1	-0.23	0.10	-0.60	0.31	-0.38	0.19	-0.25	0.14	-0.05	0.34
Fintrac1	0.22	0.14	0.40	5.31E-01	0.32	0.19	0.34	0.18	0.25	0.33
Lehman	0.81	0.21	1.83	1.29	0.89	0.39	1.25	3.17E-01	1.26	4.60E-01
M_0	226085	12682	44941	7183	89657	3304120	1238588	93263	299121	9604
R^2	260.52		123.15		125.37		1690.95		710.47	
Nobs	180		180		180		180		180	

Table 9: Estimates by Region Control Function Correction, $1,000 Denomination

Region	1		2		3		4		5
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.
Treasury Bill	182.83	64.04	140.69	39.65	148.12	25.60	146.59	49.08	148.82
CPI	4.27	1.92	2.77	1.36	7.54	1.70	-0.15	1.58	7.29
x_d	-4.43	1.03	-4.54	0.16	-4.04	0.11	-4.61	0.20	-3.95
Q3	0.02	0.14	0.01	0.11	-0.02	0.09	0.14	0.13	-0.25
Q4	0.15	0.15	-0.05	0.09	-0.03	0.08	0.05	0.15	-0.07
Q1	-0.03	0.12	-0.12	0.09	-0.05	0.07	-0.12	0.18	-0.34
CF1	-329.94	411.60	-295.84	180.07	-187.77	140.19	-206.85	274.77	-118.10
CF2	7.50	5.72	3.78	4.69	4.84	5.41	3.37	6.08	0.96
Fintrac1	-0.05	0.15	-0.14	0.12	-0.01	0.09	-0.09	0.14	0.02
Lehman	0.52	0.24	0.42	0.14	0.45	0.11	0.26	0.15	0.57
M_0	150402	11700000	156804	12483	1107789	72123	18991	4011	64471
$R\hat{2}$	240.10		231.41		937.84		40.31		105.71
Nobs	180		180		180		180		180

Region	6		7		8		9		10
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.
Treasury Bill	155.45	46.95	66.87	55.56	196.10	38.41	205.73	43.09	281.74
CPI	9.02	1.80	4.64	1.97	4.36	2.38	9.35	2.27	3.69
x_d	-4.23	0.15	-4.23	0.23	-4.67	0.25	-4.28	0.16	-4.90
Q3	-0.06	0.11	-0.11	0.14	0.00	0.12	0.02	0.11	0.55
Q4	-0.08	0.09	0.05	0.18	-0.16	0.13	-0.10	0.11	0.34
Q1	-0.10	0.08	-0.12	0.15	-0.09	0.12	-0.07	0.10	0.43
CF1	-154.22	202.10	-139.12	344.68	-403.73	233.42	-376.40	212.19	-465.86
CF2	-1.30	7.48	1.00	9.10	-5.97	6.33	-8.57	9.69	5.19
Fintrac1	0.11	0.14	-0.18	0.18	0.13	0.15	0.05	0.12	-0.01
Lehman	0.56	0.18	0.31	0.21	0.24	0.15	0.56	0.15	0.72
M_0	227924	14231	45595	5912	64390	14204	1288871	147450	299497
$R\hat{2}$	234.86		91.81		98.47		1300.70		549.61
Nobs	180		180		180		180		180

CF1 and CF2 refer to control functions for the nominal interest rate and inflation respectively.

Table 10: Estimates by Region, $100 Denomination

Region	1		2		3		4		5
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.
Treasury Bill	-5164.68	1217.93	-1850.88	1679.86	-2333.53	916.95	-2164.44	1142.56	-3523.74
CPI	-3.99	4.18	19.78	12.29	6.87	9.24	12.72	6.35	-4.13
x_{d}	1.35	0.98	-1.22	1.30	-1.23	0.70	-1.04	0.96	-0.30
Q3	0.10	0.13	-0.08	0.13	0.03	0.09	-0.08	0.12	0.16
Q4	-0.11	0.13	-0.12	0.16	-0.06	0.08	-0.16	0.12	-0.02
Q1	-0.06	0.13	-0.05	0.22	-0.05	0.09	-0.11	0.12	0.03
M_0	1.67E+07	3.15E+05	9.55E+06	2.61E+06	5.83E+07	3.87E+06	2.33E+06	2.16E+05	3.71E+06
R^2	0.99		0.96		0.99		0.98		0.98
Nobs	42		42		42		42		42

Region	6		7		8		9		10
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.
Treasury Bill	-2199.04	600.64	-3417.71	1186.15	-4272.23	1635.00	-2790.59	1086.81	-3265.00
CPI	9.13	7.50	19.41	11.21	8.57	6.28	2.52	8.85	0.12
x_{d}	-1.38	0.50	0.25	0.95	0.64	1.32	-0.62	0.85	-0.09
Q3	0.07	0.08	-0.08	0.12	0.04	0.15	0.11	0.13	0.07
Q4	-0.13	0.09	-0.24	0.15	-0.08	0.16	0.00	0.14	-0.23
Q1	-0.07	0.08	-0.13	0.14	-0.10	0.18	0.24	0.18	-0.08
M_0	7630105	580253	3382492	464735	5144663	291796	7.93E+07	9009425	3.36E+07
R^2	0.98		0.99		0.97		0.97		0.99
Nobs	42		42		42		42		42

Table 11: Estimates by Region Control Function Correction, $100 Denomination

Region	1		2		3		4		5	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	-4022.48	1585.27	-5456.78	3169.10	-2911.57	1357.59	-1710.04	1388.75	-2502.37	1085.71
CPI	-0.83	6.81	-0.33	17.32	-13.07	9.96	10.23	8.73	-9.42	5.72
x_{d}	0.47	1.34	1.59	2.60	-0.96	1.03	-1.42	1.25	-1.18	0.88
Q3	0.07	0.09	0.02	0.13	0.10	0.07	-0.08	0.10	0.16	0.07
Q4	-0.26	0.12	-0.27	0.18	0.00	0.09	-0.26	0.13	-0.07	0.10
Q1	-0.25	0.15	-0.81	0.27	-0.09	0.12	-0.14	0.15	-0.03	0.12
CF1	3917.31	1870.07	8011.50	4695.74	2097.47	1603.21	2519.62	2719.76	2231.44	1671.82
CF2	-5.18	13.23	-52.51	34.30	-20.21	17.01	-12.82	15.76	-3.24	15.22
CF3	-3.67E+03	4.56E+03	-9.52E+02	1.76E+05	-4.93E+03	9.99E+03	-5.16E+03	1.86E+04	-4.78E+03	5.13E+03
M_0	1.72E+07	4.34E+06	8.62E+06	9.43E+06	5.33E+07	3.56E+06	2.35E+06	2.12E+06	3.75E+06	1.86E+05
R^2	0.99		0.97		0.99		0.98		0.98	
Nobs	42		42		42		42		42	

Region	6		7		8		9		10	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	-2336.23	1029.18	-4135.39	2256.84	-1392.62	1216.50	-3397.57	2635.08	-2613.23	875.10
CPI	-4.26	9.39	13.88	17.22	9.98	7.06	1.79	15.48	-21.93	10.14
x_{d}	-1.39	0.80	0.82	1.97	-1.74	1.31	-0.13	2.08	-0.75	0.72
Q3	0.11	0.06	-0.03	0.12	-0.03	0.10	0.13	0.15	0.10	0.05
Q4	-0.05	0.09	-0.22	0.20	-0.18	0.14	0.06	0.20	0.02	0.11
Q1	-0.14	0.09	-0.22	0.23	-0.13	0.18	0.21	0.24	-0.01	0.11
CF1	1739.19	1246.52	2202.62	3634.54	846.71	2563.97	2091.57	4047.25	857.42	1287.94
CF2	-6.65	14.71	-17.77	25.15	-12.24	29.18	-23.56	33.61	-23.56	12.27
CF3	-8217.09	6216.39	-6249.62	4332111.00	-8502.15	4826084.00	-6985.95	48004.14	-5066.81	3526.53
M_0	7.11E+06	6.22E+05	3.32E+06	1.63E+08	5.64E+06	3.09E+08	7.85E+07	2.77E+07	3.33E+07	4.20E+05
R^2	0.99		0.98		0.97		0.97		1.00	
Nobs	42		42		42		42		42	

CF1, CF2 and CF3 refer to control functions for the nominal interest rate, inflation and polymer unfit notes, respectively.

Table 12: Estimates by Region, $50 Denomination

Region	1		2		3		4		5	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	15657.92	11189.29	15747.37	21324.54	2715.16	6489.16	11879.15	12242.38	-517.46	5152.67
CPI	119.30	49.94	33.46	298.00	-4.13	24.03	9.96	20.91	-8.56	18.88
x_{d}	-12.94	8.17	-13.44	16.49	-4.44	4.77	-10.78	9.25	-2.42	3.77
M_0	1.09E+07	1.12E+06	4.94E+06	5.53E+05	3.09E+07	2.56E+06	1.54E+06	1.53E+05	3.43E+06	3.20E+05
R^2	0.96		0.89		0.86		0.89		0.85	
Nobs	38		38		38		38		38	

Region	6		7		8		9		10	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	6072.07	7658.00	12704.54	15186.73	6622.87	15806.50	5858.81	7166.28	10162.13	11978.59
CPI	7.85	24.21	8.45	110.19	127.21	109.55	0.51	18.75	-113.26	79.14
x_{d}	-6.89	5.59	-11.32	11.59	-4.99	13.01	-6.49	5.32	-9.85	9.22
M_0	3.58E+06	5.15E+05	2.98E+06	3.02E+05	3.80E+06	3.27E+08	5.65E+07	3.34E+06	1.81E+07	9.33E+05
R^2	0.83		0.84		0.79		0.91		0.92	
Nobs	38		38		38		38		38	

Table 13: Estimates by Region Control Function Correction, $50 Denomination

Region	1		2		3		4		5	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	-2520.81	2224.48	-2055.21	2229.96	-2196.45	1454.35	-1148.02	1608.68	-2535.04	2538.56
CPI	33.88	16.15	38.23	17.30	1.86	12.45	31.31	10.90	-5.39	10.09
x_d	-0.58	1.93	-0.98	1.85	-1.22	1.21	-1.69	1.33	-1.22	2.26
CF1	387.73	3984.07	169.12	4138.86	89.90	2248.70	307.48	2674.35	2372.91	2710.35
CF2	32.25	20.29	7.73	35.75	-9.96	28.60	17.76	20.87	13.57	23.07
CF3	-5420.96	1977192.00	0.00	1.00	-3880646.00	3869804.00	0.00	cf3	-2348.87	4568.05
M_0	1.52E+07	7.23E+08	6.35E+06	9.61E+05	3.50E+07	1.86E+06	2.03E+06	1.61E+05	3.71E+06	3.12E+08
R^2	0.95		0.95		0.96		0.97		0.90	
Nobs	38		38		38		38		38	

Region	6		7		8		9		10	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	-3851.82	2172.48	9890.11	326695.80	38128.88	447122.60	-2426.12	1450.70	-4495.41	2225.10
CPI	13.78	14.90	-37.67	1174.61	-39.92	410.63	15.01	10.18	-41.45	21.15
x_d	0.02	2.01	-10.00	242.36	-33.32	350.88	-0.69	1.21	1.13	1.77
CF1	2381.25	2642.60	-11209.25	507886.40	7051.93	13600000.00	216.97	2207.44	480.07	2646.32
CF2	19.38	33.48	-19.27	1570.38	#REF!	#REF!	8.20	19.46	-15.48	27.77
M_0	-5.10E-01	6.07E-01	-1.13E+04	6.60E+03	0.00E+00	cf3	-1.79E+02	1.96E+03	-9.00E-02	9.27E-01
R^2	4531414.00	468000000.00	3102698.00	689000000.00	#REF!	#REF!	63500000.00	2296586.00	19200000.00	364317.60
Nobs	0.9221		0.7321		85613.7769		213800		80474.932	
	38		38		38		38		38	

CF1, CF2 and CF3 refer to control functions for the nominal interest rate, inflation and polymer unfit notes, respectively.

Table 14: Depositor Heterogeneity \$1,000 Denomination: NLSUR

	NLSUR		CF NLSUR	
	Est.	Std Err.	Est.	Std Err.
Treasury Bill	197.375	32.66484	114.7382	393.8377
CPI	9.518971	3.331117	5.96606	12.10477
x_{d}	-0.065	0.182885	-4.13632	980946.9
$\bar{x}$	0.027497	0.128873	0.819291	0.372474
Q3	-0.065	0.182885	-0.03113	0.250461
Q4	-0.14904	0.189218	-0.06698	0.374522
Q1	-0.1602	0.157171	-0.04228	0.283553
Fintrac1	7.24E-05	0.112453	-0.08597	980945
Lehman	0.386423	0.158838	0.068536	12.99774
CF1			-246.622	377.9773
CF2			1.957839	17.54737
$M_{0,\text{Calgary}}$	151790.4	28141.99	147109.6	172292.1
$M_{0,\text{Halifax}}$	125146.4	17361.2	129626.6	500937.1
$M_{0,\text{Montreal}}$	1094248	185514.3	1066351	572176.5
$M_{0,\text{Newfoundland}}$	17476.43	1869.902	18397.43	73276.98
$M_{0,\text{Ottawa}}$	63306.99	12457.12	61078.53	60700.49
$M_{0,\text{Quebec City}}$	190094.7	31571.51	186433.6	199776.4
$M_{0,\text{Regina}}$	36257.92	5958.578	36421.12	144342.7
$M_{0,\text{Winnipeg}}$	58190.18	9174.65	53073.56	174896.2
$M_{0,\text{Toronto}}$	1256367	236323.6	1191436	1311358
$M_{0,\text{Vancouver}}$	326647.7	51520.16	325302.6	176088.2

CF1 and CF2 refer to control functions for the nominal interest rate and inflation respectively.