Economic stability through narrow measures of inflation

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Version 5.02

May 1, 2017

Abstract

Under the assumption that different measures of inflation draw on the same price data, this paper uses a New Keynesian model to show that targeting a measure which is narrower in scope aides in equilibrium stability. The importance of this result comes into focus when considering a potential increase in the Federal Reserve's targeted rate of inflation, allowing the monetary authority to maintain its current aggressive stance while still maintaining the needed stability.

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1 Introduction

At the undergraduate level, the primary operation of monetary policy boils down to some form of inflation targeting, whether that be the textbook definition or the cloak-and-dagger version of the Federal Reserve System before the financial crisis. However, just as with every aspect of undergraduate economics, the truth is much more nuanced. For instance, while it has been assumed that the Fed was targeting a form of inflation since at least Taylor (1993), the actual measure of inflation has changed from one era to the next. While this change can be viewed as rather slight or uninteresting, this paper shows that it can have significant policy implications. Moving from headline to core measures has been shown to be important, but the change from one type of measure to another can also have an impact.

2 Literature Review

There is a relatively extensive literature that considers the benefits of headline and core inflation targets for monetary policy. Aoki (2001), for example, considers two production sectors which differ only in the fact that one sector operates under a flexible-price regime and the other operates under the assumption of sticky prices. He finds that targeting the sticky prices is optimal, setting up a flexible-price dynamic in relative terms. Benigno (2004) takes this type of model to a common-currency economy, also finding that it is optimal to target the stickiest of prices to bring the economy closer to a flexible price environment. Mankiw and Reis (2003) move away from the theoretical price indexes to create an optimal weighting scheme for policy makers to target. They find that neither of the most popular inflation measures, those derived from the consumer price index (CPI) and the personal consumption expenditures deflator (PCE), are well-suited for policy goals. They find that a significant weight should be placed on nominal wages.

This line of literature, however, typically makes a couple of key assumptions. First, when multiple price indices are modeled, they are derived from sectors that are considered to be mutually exclusive. In reality, when considering the differences between these measures, the term "scope" is appropriate. For instance, a vast majority of the price data considered in the PCE price index is derived from CPI, implying that the measures themselves are not mutually exclusive. Are there differences in what each of these two measure? Yes, but they also consider a lot of the same items. Second, this literature omits fiscal policy. At best, there is a flat tax rate applied to nominal income. The key findings of Leeper (1991) and the literature that follows show that the stance of fiscal policy is vital to the implementation of monetary policy. Additionally, Keinsley (2016) considers a structured fiscal policy in the style of the US federal income tax code. He finds that structured nature of the tax code is important and that even the existence of an inflation index is critical to the existence of a unique rational expectations equilibrium (REE) in these models.

3 N-Sector New Keynesian Model

This section presents a New Keynesian model with multiple production sectors and a structured fiscal policy rule.

3.1 Retailer

This economy also includes a retailer, which simply aggregates the final goods from each sector into a bundle for purchase. It does so with a Cobb-Douglas production function

$$y_t = \prod_{k=1}^{N} y_{k,t}^{\delta_k} \tag{3.1}$$

where N is the number of sectors in the economy. Additionally, we make the constant returns to scale assumptions that $\delta_k > 0$ an $\sum_{k=1}^{N} \delta_k = 1$ for all $k \in [1, N]$. Assuming that this retailer participates in a perfectly competitive environment, this functional form yields the price aggregate

$$p_t = \prod_{k=1}^{N} \left(\frac{p_{k,t}}{\delta_k} \right)^{\delta_k} \tag{3.2}$$

where $p_{k,t}$ is the price index for sector k. This functional form also allows for a tractable mapping to an aggregate inflation rate

$$\pi_t = \prod_{k=1}^{N} \pi_{k,t}^{\delta_k} \tag{3.3}$$

It is assumed that $\pi_{k,t} \equiv {}^{P_{k,t}}/_{P_{k,t-1}}$ for each k. This setup allows for multiple inflation rates in the economy, with π_t representing the broadest measure, while $\pi_{k,t}$ are the sectoral inflation rates. This functional form is similar in setup to a Thörnqvist Index, which is the basis for the chained CPI measure, though the weights used in this setup are considered to be constant.[1]

[1] Being the discrete time analog of a Divisia index, what makes a Thörnqvist index so useful is that it allows for time-varying weights. While the standard setup includes the non-stationary price level, in this model, such a weight could be constructed by simply using real output

$$\delta_j = \frac{1}{2} \left(\frac{y_{j,t-1}}{\sum_k y_{k,t-1}} + \frac{y_{j,t}}{\sum_k y_{k,t}} \right),$$

3.2 Households

A representative household maximizes utility, choosing both an aggregated consumption bundle c_t and labor hours l_t. Its one-period utility function can be expressed as

$$\mathcal{U}_t = \ln c_t - \eta \frac{l_t^{1+\sigma_l}}{1+\sigma_l} \tag{3.4}$$

where $\{\sigma_c, \sigma_l, \eta\} \in \mathbb{R}_+$. Each period, this household earns nominal income via labor supplied to an array of production sectors $\mathbf{W}_t \mathbf{l}_t$ and returns from both debt and equity markets. The vectors $\mathbf{W}_t = [W_{1,t}, W_{2,t}, \dots W_{N,t}]$ and $\mathbf{l}_t = [l_{1,t}, l_{2,t}, \dots l_{N,t}]'$ represent the sectoral nominal wage rates and labor hours, respectively. Aggregate labor hours l_t are expressed in Cobb-Douglas form $l_t = \prod_{k=1}^{N} l_{k,t}^{\delta_k}$. Debt assets are represented by risk-free government bonds B_{t-1}, which pay an interest rate of r_{t-1} each period. Equity assets are represented by ownership D_t of the intermediate goods-producing firms, which will be described later. The household uses this income to purchase its consumption bundle at aggregate price level P_t, invest in new bonds B_t, and pay taxes to the fiscal authority T_t. Thus, the household's budget constraint can be expressed as

$$P_t c_t + B_t + T_t = \mathbf{W}_t \mathbf{l}_t + r_{t-1} B_{t-1} + D_t \tag{3.5}$$

where the tax payments can be expressed as

$$T_t = \tau_t \left[\mathbf{W}_t \mathbf{l}_t + (r_{t-1} - 1) B_{t-1} + D_t \right]. \tag{3.6}$$

The effective tax rate τ_t is set by the fiscal authority and is progressive at level $\gamma \in [0, 1]$

$$\tau_t = 1 - \theta \left[\frac{\chi}{\chi_t} \frac{1}{\pi_t^*} \right]^{\gamma}. \tag{3.7}$$

where $\chi_t = \mathbf{w}_t \mathbf{l}_t + (r_{t-1} - 1) \frac{b_{t-1}}{\pi_t} + d_t$ denotes real taxable income for the household. Here $\mathbf{w}_t = \mathbf{W}_t / P_t$ is the schedule of real wage rates.[2] The parameter $\theta \in (0, 1]$ governs the intended effective tax rate, though imperfect indexation causes a deviation from this. In this model, the tax code consists of a lagged price index π_t^*, which can vary from a broad to a narrow measure.[3] This rate of inflation differs from the aggregate inflation rate in both

though this adds an unneeded level of complexity to the model at this point, allowing a simple aggregation from the price level to the inflation rate.

[2]Note that the real wage rates are derived from dividing the nominal wage rate schedule by the aggregate price level P_t, a scalar.

[3]The federal income tax code is indexed to a price level that is lagged by 16 months, see Keinsley (2016) for more information on the importance of indexing the tax code and for a detailed derivation of this functional form.

level and volatility. The optimizing conditions, therefore, are

$$\frac{1}{c_t} = \lambda_t \tag{3.8}$$

$$\delta_k \eta \frac{l_t^{1+\sigma_l}}{l_{k,t}} = (1-\gamma)(1-\tau_t)\lambda_t w_{k,t} \quad \forall k \tag{3.9}$$

$$1 = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} \left(r_t (1-\tau_{t+1})(1-\gamma) + (1-\gamma)\tau_{t+1} + \gamma \right) \right] \tag{3.10}$$

where λ_t is the shadow price attached to the household's budget constraint.

3.3 Final Goods-Producing Firms

The final goods producing firms act much like the retailer. That is, they aggregate the intermediate goods in their sector, indexed by $i \in [0,1]$. Each firm $k \in [1,N]$ operates in a perfectly competitive environment with production function

$$y_{k,t} = \left[\int_0^1 y_{k,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \quad \forall k. \tag{3.11}$$

Assuming that these firms' only factor input are the intermediate goods, maximizing profits yields the following demand schedule

$$y_{k,t}(i) = \left(\frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\epsilon} y_{k,t} \quad \forall k, i. \tag{3.12}$$

As with the retailer, the use of a CES production function yields the following price dual

$$P_{k,t} = \left[\int_0^1 P_{k,t}(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}} \quad \forall k, i. \tag{3.13}$$

3.4 Intermediate Goods-Producing Firms

Matching the environment of the final goods-producing firms, there are N sectors of intermediate goods-producing firms, indexed by $k \in [1,N]$. Within each sector, there are a continuum of firms indexed by $i \in [0,1]$, which utilize labor hours from the household and a sector-common technology $z_{k,t}$ to produce real output $y_{k,t}(i)$ with linear production function

$$y_{k,t}(i) = z_{k,t} l_{k,t}(i) \tag{3.14}$$

where

$$\ln z_{k,t} = (1-\rho_z)\ln z_k + \rho_z \ln z_{k,t-1} + \zeta\varepsilon_t^{\zeta} + \varepsilon_{k,t}^{z} \tag{3.15}$$

Where ε_t^{ζ} is an aggregate supply shock that impacts all sectors of production and $\varepsilon_{k,t}^{z}$ is an idiosyncratic shock to sector k. These firms maximize their profits subject to their production function, the demand of the respective final good-producing firm, and their own marginal cost of utilizing an additional hour of labor,

$$\varphi_{k,t} = \frac{W_{k,t}}{z_{k,t}} \tag{3.16}$$

Additionally, each period they adjust their price with probability $1-\phi_k$, where $\phi_k \in [0,1]$.

$$\max_{P_{k,t}(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\phi_k^s \left[\left(\frac{P_{k,t}(i)}{P_{k,t+s}}\right)^{1-\epsilon} y_{k,t+s} - \varphi_{k,t+s}P_{k,t+s}^{\epsilon-1}P_{k,t}(i)^{-\epsilon}y_{k,t+s} \right] \tag{3.17}$$

Where

$$\Lambda_{t,t+1} = \beta\frac{\lambda_{1,t+1}}{\lambda_{1,t}} \tag{3.18}$$

is the stochastic discount factor. Solving this problem yields

$$P_{k,t}(*) = \frac{\epsilon}{\epsilon-1}\frac{A_{k,t}}{B_{k,t}} \tag{3.19}$$

where

$$A_{k,t} = \varphi_{k,t}P_{k,t}^{\epsilon-1}y_{k,t} + \phi_k\mathbb{E}_t\left[\Lambda_{t,t+1}A_{k,t+1}\right] \tag{3.20}$$

$$B_{k,t} = P_{k,t}^{\epsilon-1}y_{k,t} + \phi_k\mathbb{E}_t\left[\Lambda_{t,t+1}B_{k,t+1}\right] \tag{3.21}$$

Converting this into a choice about inflation yields the following relationships.

$$\pi_{k,t}(*) = \frac{\epsilon}{\epsilon-1}\frac{a_{k,t}}{b_{k,t}} \tag{3.22}$$

where

$$a_{k,t} = \pi_{k,t}\left(mc_t y_{k,t} + \phi_k\mathbb{E}_t\left[\Lambda_{t,t+1}\pi_{k,t+1}^{\epsilon-1}a_{k,t+1}\right]\right) \tag{3.23}$$

$$b_{k,t} = y_{k,t} + \phi_k\mathbb{E}_t\left[\Lambda_{t,t+1}\pi_{k,t+1}^{\epsilon-1}b_{k,t+1}\right] \tag{3.24}$$

and mc_t is the real marginal cost. It is assumed that the ϕ_k percent of firms that cannot optimize their prices during a given period will simply index their price to the steady state inflation rate in that sector π_k. Thus, in any given period, the inflation rate in sector j can be derived as

$$\pi_{k,t} = \left[(1-\phi_k)\pi_{k,t}(*)^{1-\epsilon} + \phi_k\pi_k^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{3.25}$$

3.5 Monetary and Fiscal Policy

Monetary policy in this model is constructed via a Taylor rule and is concerned only with the broad measure of inflation π_t. That is, while the prices in either sector will fluctuate, the monetary authority is only concerned with the unbiased, broader measure.

$$\ln \frac{r_t}{r} = \rho_r \ln \frac{r_{t-1}}{r} + (1-\rho_r)\left[\rho_\pi \ln \frac{\pi_t}{\pi} + \rho_y \ln \frac{y_t}{y_{t-1}}\right] + \varepsilon_{r,t} \tag{3.26}$$

where $\rho_r \in (0,1)$ denotes the level of interest rate smoothing and $\rho_\pi, \rho_y > 0$ govern the monetary policy maker's responses to inflation and output growth, respectively. The variable $\varepsilon_{r,t}$ is considered to be an i.i.d. shock to the policy rule with standard deviation σ_r.

It is assumed that the fiscal authority purchases goods/services g_t in each period while also fully paying down its debt with interest. To do so, it collects tax revenues from the households and sells debt securities.

$$g_t + \frac{r_{t-1}}{\pi_t} b_{t-1} = t_t + b_t \tag{3.27}$$

Where $t_t = {}^{T_t}/_{P_t}$ and nominal tax revenue T_t follows from expression (3.6). For simplification, government purchases are assumed to be exogenous

$$\ln g_t = (1-\rho_g)\ln g + \rho_g \ln g_{t-1} + \varepsilon_{g,t} \tag{3.28}$$

where $\varepsilon_{g,t}$ is a zero-mean, i.i.d. random variable with standard deviation σ_g.

3.6 Aggregation and Market Clearing

Here are some additional equilibrium conditions regarding aggregation and market clearing.

Labor Market The sector-specific labor hours are divided among the intermediate goods-producing firms. Therefore, aggregation of these hours suggests

$$l_{k,t} = \int_0^1 l_{k,t}(i)di \quad \forall k \tag{3.29}$$

At the macroeconomic level, the labor supplied by the household is demanded by the production sectors, thus

$$l_t = \prod_{k=1}^{N} l_{k,t}^{\delta_k}. \tag{3.30}$$

Goods Market A Calvo sticky-price structure implies price dispersion throughout each of the sectors. Therefore, when aggregating within sector j,

$$y_{k,t} = \frac{z_{k,t} l_{k,t}}{v_{k,t}} \quad \forall k \tag{3.31}$$

where $v_{k,t}$ is the output loss due to this price dispersion in sector k. This can be expressed as

$$v_{k,t} = (1-\phi_k)\left(\frac{\pi_{k,t}(*)}{\pi_{k,t}}\right)^{-\epsilon} + \left(\frac{\pi_k}{\pi_{k,t}}\right)^{-\epsilon} \phi_k v_{k,t-1} \quad \forall k. \tag{3.32}$$

Thus, this term is slightly different from the typical price dispersion terms in this style of model due to the existence of dual sectors. Given this, profits for the intermediate firms can also be aggregated such that

$$d_t = \sum_{k=1}^{N} d_{k,t} \tag{3.33}$$

and

$$d_{k,t} = y_{k,t}\left(1 - mc_{k,t} v_{k,t} \frac{\pi_t}{\pi_{k,t}}\right) \quad \forall k. \tag{3.34}$$

4 An Inflation Portfolio

Considering the price index used in the model,

$$\pi_t = \prod_{k=1}^{N} \pi_{k,t}^{\delta_k}$$

it is easy to see what the monetary authority can expect by adjusting the price index it uses.[4] The inflation index considered in the model is built like a diversified portfolio of N items. For expositional purposes, consider the second-order Taylor-expansion about the steady state.

$$\tilde{\pi}_t \approx \sum_{k=1}^{N} \delta_k \tilde{\pi}_{k,t} + \sum_{k=1}^{N} \frac{\delta_k(\delta_k - 1)}{2} \tilde{\pi}_{k,t}^2 + \sum_{k \neq j} \delta_k \delta_j \tilde{\pi}_{k,t} \tilde{\pi}_{j,t}, \tag{4.1}$$

where $\tilde{x}_t$ denotes the percent deviation of x_t from its steady state. This expression shows that movements in the inflation aggregate are determined by a wighted average of the components' movements. Additionally, with $\delta_k > 0$ and $\sum_k \delta_k = 1$, the second term shows that these deviations decrease as more components are added. Just as in portfolio theory, reducing risk (or volatility) is achieved with portfolio that is diversified with assets that

[4]Note that this assumes that the macroeconomic supply shock ζ_t is at its steady state, for all t. This is only for analysis purposes and does not hold in general.

are negatively correlated.[5] The inflation rates designed in most of the literature are not constructed in this fashion, opting for mutually exclusive, uncorrelated sectors. Here, even if the sectors have covariances near zero, adding more components leads to increased stability in the overall measure.

Constructing a narrow inflation index under this setup will consist of simply aggregating over a subset $M < N$ of these sectors.

$$\pi_t^* = \prod_{k=1}^{M} \pi_{k,t}^{\delta_k^*} \tag{4.2}$$

where it is assumed that

$$\delta_k^* = \frac{\delta_k}{\sum_{k=1}^{M} \delta_k} \quad \forall k \in [1, M],$$

which preserves the weighting restrictions imposed in the original setup. Thus, $\delta_k^* > 0$ and $\sum_{k=1}^{M} \delta_k^* = 1$. Another consequence of this adjustment is that $\delta_k^* > \delta_k$ for all $k \in [1, M]$, which will be discussed further, later.

5 The Two-Sector Case

Though the N-sector setup in the above section allows for excellent insight, a large number of sectors becomes computationally burdensome. To keep the model tractable, let $N = 2$ and $M = 1$, so that the model has the heterogeneity it needs without unneeded complexities. This allows for a similar setup to Aoki (2001), but also considers the fact that pricing data is shared among to the differing inflation measures. To mimic some of what is seen in the data, this paper assumes that one sector is slightly more flexible than the other ($\phi_1 < \phi_2$). For now, it is assumed that the steady state inflation rates of the two sectors are the same ($\pi_1 = \pi_2$), but the model easily allows for deviations from this assumption. It is also assumed that the weight on the more flexible sector sits at $\delta_1 = 0.30$, suggesting that this measure of inflation π_t^* is quite a bit narrower than the broader measure π_t. Under this two-sector case, the result is analogous to simply selecting one of the components out of the broad measure to target.

5.1 Monetary-Fiscal Interaction and the Inflation Target

Sargent and Wallace (1981) show that, in an effort to achieve a unique rational expectations equilibrium (REE) a la Blanchard and Kahn (1980), fiscal policy and monetary policy

[5] If the sectors have covariances of zero, the third term falls out of the expression.

face tradeoffs. Leeper (1991) categorized these tradeoffs as "active" and "passive" regimes, needing a mix of the two to achieve stability. Keinsley (2016) finds that, under a fiscal policy rule structured in the fashion the federal income tax, this tradeoff may be limited. In other words, given a tax code such as those seen in today's economy, monetary authorities can achieve dominance over the existence of a unique REE.

5.2 Results

Below are the calibration and steady state results for a two-sector model. In this baseline case, it is assumed that the sectors only differ in their level of price stickiness. Thus, the steady state inflation rates, technologies, and wages are consistent across the sectors.

5.2.1 Parameter Calibration

The calibrated values are shown below. In order to keep the two sectors identical outside of their price stickiness, the production share parameter δ is set to 0.5. The labor elasticity parameter, σ_l is calibrated to ensure that aggregate labor hours l_t are 0.333 in steady state. The parameters on the tax code are calibrated to match the values found in Keinsley (2016).

	Parameter	Value
Discount Factor	β	0.989
Income Tax: Level	θ	0.901
Income Tax: Progressivity	γ	0.134
Production Share	δ	0.500
Elastisity of Substitution	ε	6.000
Labor Scaling Factor	η	4.000
Labor Elasticity	σ_l	0.453

Table 1: Structural Parameters

5.2.2 Steady State Values

For simplification, aggregate technology is set so as to normalize aggregate output to unity. Government expenditures g_t are fixed to twenty percent of the sum of sectoral output, and the gross nominal interest rate r_t is set to 1.04 in steady state to match the data.

	Variable	Steady State
Aggregate Output	y_t	1.0000
Sector 1 Output	$y_{1,t}$	1.0000
Sector 2 Output	$y_{2,t}$	1.0000
Consumption Expenditures	c_t	1.6000
Government Expenditures	g_t	0.4000
Outstanding Fiscal Debt	b_{t-1}	0.1853
Sector 1 Wage Rate	$w_{1,t}$	2.5000
Sector 2 Wage Rate	$w_{2,t}$	2.5000
Aggregate Labor Hours	l_t	0.3333
Sector 1 Labor Hours	$l_{1,t}$	0.3333
Sector 2 Labor Hours	$l_{2,t}$	0.3333
Gross Nominal Interest Rate	r_t	1.0400
Effective Income Tax Rate	τ_t	0.1014
Gross Aggregate Inflation Rate	π_t	1.0200
Sector 1 Inflation Rate	$\pi_{1,t}$	1.0200
Sector 2 Inflation Rate	$\pi_{2,t}$	1.0200
Taxable Income	χ_t	2.0073

Table 2: Steady State Values

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