# Estimating monetary policy rules from forward guidance*

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February 10, 2017

**Abstract**

Has the Federal Open Market Committee's policy rule changed in recent years? This is difficult to answer given the zero lower bound environment for the federal funds rate throughout late 2008 to 2015. This paper addresses the problem using policymakers' projections for the near horizon from 2012 to 2016, which were often greater than the bound. Projections indicate inconsistent but overall diminishing responsiveness to inflation, a relatively stable and strong response to economic activity, and increasing short-run responsiveness to financial risk. Professional forecasters perceived the size of the response to economic activity, but not inflation, perhaps due to its relative volatility.

JEL Classifications: C24, C32, E58

Keywords: Monetary Policy Rule, Forward Guidance, Zero Lower Bound.

*Replication materials and dataset: https://goo.gl/NHGkaS. Neil Fuller and May Xing provided excellent research assistance.

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# 1 Introduction

Has the Federal Open Market Committee (FOMC)'s policy rule changed in recent years? This is amongst the most important questions to market participants, and a large literature considers the estimation of rules prior to December 2008.[1] However, starting at this point, the federal funds rate remained at the zero lower bound (ZLB) continuously through 2015. Identifying changes in the rule on the basis of the censored rate alone is impossible.

Kim and Pruitt (2017) sidestep this problem and answer the question indirectly; they estimate the market-perceived rule over 1986-2014 using professional forecasts at the one-year-ahead horizon. Unlike the contemporaneous rate, federal funds forecasts were largely uncensored throughout the ZLB period.[2] Forecasters perceived the FOMC placed no statistically significant weight towards inflation, and a greater weight towards economic activity, after the inception of the ZLB in 2008 through 2014. They also perceived a significant response to financial risk, as measured by the 10 year - 2 year Treasury spread.

This paper identifies changes in the policy rule using information straight from the horse's mouth: forward guidance. Like professional forecasts, individual projections from the FOMC's quarterly Statement of Economic Projections (SEP) often went uncensored during the ZLB period.[3] Combined with the SEP's simultaneous projections for inflation, output, and unemployment, this dataset allows one to estimate the implicitly communicated rule, and conduct standard tests of specification and breakpoints. The SEP implies a response to economic activity which was on par with forecasters'

[1] For example, see Bernanke and Mihov (1998) and many others.

[2] Hamilton et al. (2011) previously estimated the market-perceived rule using macroeconomic news over 1994-2007, not including the ZLB period.

[3] These projections are referred to in the press as the Federal Reserve's "dot plot."

perceptions throughout 2012-2014. But in contrast with perceptions, the communicated policy rule was also responsive to inflation. The FOMC did begin communicating less responsiveness to inflation, but not consistently until after 2014. At the same time, the short run response to financial risk began increasing, a tradeoff which continued through 2016. All the while, the communicated response to activity remained relatively constant, and well-perceived.

In sum, the change in policymaking over 2012-2016 may be characterized as substituting responsiveness to financial risk for inflation. Forecasters perceived communication about the response to economic activity, which was relatively constant. However, they did not internalize communication about the inflation response, which was continually changing, and perhaps less credible.

Why might economic projections be particularly well-suited to estimating the policy rule? As Woodford (2012) (p. 10) underscores, announcements by policymakers may have two effects: to influence expectations about future actions, or *reactions*. Consequently, as Campbell et al. (2012) discuss, there are two corresponding types of forward guidance. In one type, the monetary authority commits to a course of action. Date-based guidance is an example. In the other type, the policymaker merely states projections for macroeconomic variables, and the conditional reaction of policy, jointly. The SEP falls under this umbrella. Rudebusch and Williams (2008) argue that such projections are welfare-improving in their ability to implicitly communicate the policy rule. Hubert (2015) provides evidence that inflation forecasts by the Riksbank, Bank of England, Bank of Canada, Swiss National Bank, and Bank of Japan influence private market inflation expectations, but not necessarily because they are accurate. It is specifically because of the policy approach the forecasts implicitly convey. It was well-known even before the beginning of the ZLB environment that economic projections are related to eventual policymaking (Orphanides and Wieland, 2008). In fact, projections from individual FOMC members over 1992-2000 are well-described by a simple Taylor rule (Fendel and Rülke, 2012). This paper merely formalizes the known role of projections as an implicit communicator of policy reaction into an empirical framework, and uses it to infer the otherwise obscured policy rule during the ZLB period.

In comparison with previous work, Morris (2015) first proposed using the SEP to infer policy rules from forward guidance. These were estimated on the basis of the mean, and other quantiles of projections across FOMC participants, due to their anonymity. Herrador and Marquez (2016) conduct a similar exercise which does not account for censoring at the ZLB. In contrast, this paper develops an econometric methodology for obtaining estimates of the policy rule using individual-level participants' projections. It also deals with censoring at the ZLB directly in a Tobit framework. The additional degrees of freedom afforded by individual projections allows for tests of breaks and subsample estimates throughout 2012-2016. This is critical, as the results show that assuming an invariant rule over this period yields misleading conclusions. Moreover, the Tobit framework is necessary to ensure that policy rule estimates are not biased by censored federal funds rate projections.

This paper is structured as follows: Section 2 presents the dataset and its characteristics. Section 3 introduces the model and methodology, dealing with censoring at the ZLB and the anonymity of the SEP. Section 4 presents results from estimating the model over the full sample, and with breaks, showing that assuming an invariant rule yields misleading conclusions. Section 5 considers when and perhaps why forecasters' perceptions of the policy rule deviated from communication, along with illustrating the changes in policymaking over 2012-2016. Section 6 discusses limitations of the study and concludes.

# 2 Dataset

FOMC economic projections, known as the SEP, first began including federal funds rate projections following the January 24-25, 2012 meeting of the FOMC.[4] Ultimately, this evolved into a quarterly routine of March, June, September, and December post-meeting distributions. Each document projects year-end values of the federal funds and unemployment rates over the next several years, along with each respective year's total inflation and output growth. March and June projections encapsulate the concurrent and following two years, while September and December projections extend the horizon to three.

Importantly, these documents present a distribution of projections, rather than a single figure. The range is the span of voting FOMC participants. There is no pretense of group consensus. Moreover, projections should not necessarily be interpreted as efficient forecasts; footnotes to the SEP include the statement, "*Each participant's projections are based on his or her assessment of appropriate monetary policy.*"[5] Disagreement in projections for any statistic may not only be due to economic uncertainty, but also different philosophies about policy stance. Finally, projections are entirely anonymous. There is no way to match projections across horizons, meetings, or variables by-participant.

## 2.1 Federal funds rate projections

Projections for the federal funds rate are released in the form of anonymous dot plots of each voting FOMC member's year-end projections for the next short horizon of years. In any given meeting there are 16-19 such participants operating in shifts drawn from a

[4]The SEP is publicly available at `https://www.federalreserve.gov/monetarypolicy/fomccalendars.htm`. Projections for macroeconomic variables alone began earlier, in 2009.

[5]Arai (2016) finds inflation projections from the SEP may generally be regarded as efficient forecasts, but that efficiency is often rejected for output and unemployment.

full set of 28 sometime participants.[6] Following their first release, Bernanke (2012) (p. 17) suggested in the Chairman's news conference that the most representative statistic from this range of dots is the median, in that it best approximates the result of the democratic process by which the FOMC arrives at a single target. In part for this reason, and in part due to the anonymity of projections, the median is a useful starting point from which to understand this dataset.

Table 1 presents 73 historical meeting-horizon medians across participants computed from 1,269 total individual projections for,

$$r = \text{horizon year-end's nominal federal funds rate.}$$

There are at least three features of immediate note. First, median projections are censored by the effective ZLB of 0.125% for most projections at the 2012-2014 horizon. However, this is not the case for all individual projections at this horizon. At the 16th meeting of September 16-17, 2015, all censoring evidently ceases, as one participant produces negative projections for 2015. Second, revisions (in columns, top to bottom) are primarily downward, indicating early projections tended to be too optimistic, or otherwise above the ZLB. Third, the furthest right column in this table is a "Longer run" projection, which may be interpreted to be analogous to the meeting contemporaneous estimate of,

$$r^* = \text{horizon year-end's equilibrium nominal federal funds rate.}$$

This figure falls over the sample by 1.25%, reflecting discussions regarding a possibly falling equilibrium real interest rate (Hamilton et al., 2016; Laubach and Williams, 2016).

[6] These participants are catalogued historically in the dataset.

Table 1: Median projected federal funds rate, 2012-2016 (%).

| | | | Horizon | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Meeting | Date | Participants | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | Longer run |
| 1 | Jan 24-25, 2012 | 17 | 0.13 | 0.13 | 0.63 | | | | | | 4.13 |
| 2 | Apr 24-25, 2012 | 17 | 0.13 | 0.13 | 0.88 | | | | | | 4.13 |
| 3 | Jun 19-20, 2012 | 19 | 0.13 | 0.13 | 0.38 | | | | | | 4.13 |
| 4 | Sep 12-13, 2012 | 19 | 0.13 | 0.13 | 0.13 | 0.88 | | | | | 3.88 |
| 5 | Dec 11-12, 2012 | 19 | 0.13 | 0.13 | 0.13 | 0.88 | | | | | 3.88 |
| 6 | Mar 19-20, 2013 | 19 | | 0.13 | 0.13 | 0.88 | | | | | 3.88 |
| 7 | Jun 18-19, 2013 | 19 | | 0.13 | 0.13 | 0.88 | | | | | 3.88 |
| 8 | Sep 17-18, 2013 | 17 | | 0.13 | 0.13 | 0.88 | 1.88 | | | | 3.88 |
| 9 | Dec 17-18, 2013 | 17 | | 0.13 | 0.13 | 0.63 | 1.63 | | | | 3.88 |
| 10 | Mar 18-19, 2014 | 16 | | | 0.13 | 0.88 | 2.13 | | | | 3.88 |
| 11 | Jun 17-18, 2014 | 16 | | | 0.13 | 1.00 | 2.38 | | | | 3.63 |
| 12 | Sep 16-17, 2014 | 17 | | | 0.13 | 1.38 | 2.88 | 3.63 | | | 3.63 |
| 13 | Dec 16-17, 2014 | 17 | | | 0.13 | 1.13 | 2.38 | 3.63 | | | 3.63 |
| 14 | Mar 17-18, 2015 | 17 | | | | 0.63 | 1.88 | 3.13 | | | 3.63 |
| 15 | Jun 16-17, 2015 | 17 | | | | 0.63 | 1.63 | 2.88 | | | 3.63 |
| 16 | Sep 16-17, 2015 | 17 | | | | 0.38 | 1.38 | 2.63 | 3.38 | | 3.38 |
| 17 | Dec 15-16, 2015 | 17 | | | | 0.38 | 1.38 | 2.38 | 3.13 | | 3.38 |
| 18 | Mar 15-16, 2016 | 17 | | | | | 0.88 | 1.88 | 2.88 | | 3.13 |
| 19 | Jun 14-15, 2016 | 17 | | | | | 0.88 | 1.63 | 2.38 | | 2.88 |
| 20 | Sep 20-21, 2016 | 17 | | | | | 0.63 | 1.13 | 1.88 | 2.63 | 2.75 |
| 21 | Dec 13-14, 2016 | 17 | | | | | 0.63 | 1.38 | 2.13 | 2.88 | 2.88 |

*Notes:* Projections for horizon year's-end. Medians, not including Longer run = 73. Total individual projections = $1,269$. Federal funds rate projections are publicly reported in quarter-percentage intervals. Before 16th meeting of September 16-17, 2015, data is recorded as maximum of bottom of interval, or effective ZLB of 0.125%. Beginning at 16th meeting, all censoring ceases, as one participant produces negative projections for 2015. *Source: Federal Reserve, author's calculations.*

## 2.2 Macroeconomic projections

Along with projections for the federal funds rate, FOMC participants are asked to provide anonymous projections for other important macroeconomic variables. These are released in *Minutes* at a one month lag and include,

$$\pi = \text{horizon year's total PCE inflation}$$

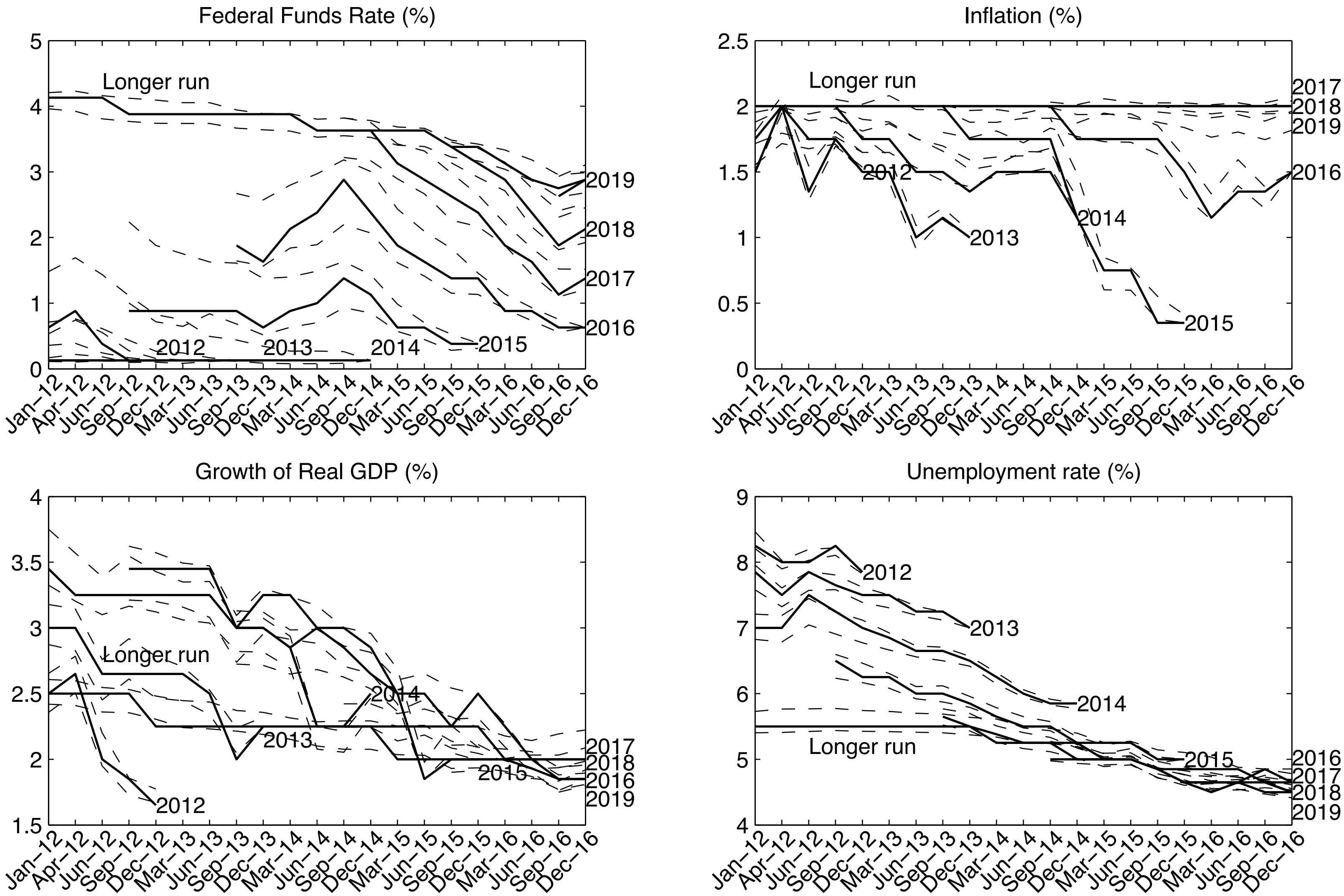
$$\%\ \text{chg.}\ y = \text{horizon year's total percentage change in real GDP}$$

$$u = \text{horizon year-end's unemployment rate}$$

Figure 1 depicts median projections for each of these variables across participants, along with the federal funds rate medians in Table 1. Mirroring the downward revisions in federal funds rate projections over time, inflation and real GDP projections tended to be too optimistic. However, the unemployment rate tended to consistently outperform early projections.

Figure 1 also depicts conventional 95% confidence intervals across participants centered at the mean. Standard errors have unclear interpretation for the federal funds rate given censoring, but are sensible for all other variables, including the equilibrium rate. For each of these, the median falls near the center of the interval, reflecting the fact that projections tend to be symmetrically distributed. Moreover, confidence intervals tend to have the same magnitude across horizons, and over meetings, by-variable. Given these observations, it is fair to assume that participants' projections for each of these variables are drawn from an approximately normal distribution with means that vary by meeting and horizon, but with in-common covariance. Reasonably, this would be the outcome of participants forming projections in part using impulse-responses generated

Figure 1: Median and 95% confidence interval for projected variables.




*Notes:* Solid line: Median projection across participants by horizon year, over meetings. Dashed: 95% confidence interval across participants centered at mean. All figures are annualized. *Source: Federal Reserve, author's calculations.*

from standard econometric models.[7]

## 2.3 The relationship between projections

Jointly, federal funds rate and macroeconomic projections say something about the rules policymakers may have in mind, or which at the very least characterize their behavior. In making this comparison, there are three statistics of interest from macroeconomic projections. First, since "Longer run" inflation is interpretable as target inflation, one may compute participants' projections for excess inflation, written,

$$\text{Excess inflation:} \quad \Delta\pi = \text{PCE inflation } \pi \ - \ \text{target inflation } \pi^*$$

Target inflation is 2% across all participants and meetings. Since median inflation projections never exceed this figure, all median excess inflation projections are less than or equal to zero.

Second, we wish to compute a measure of the output gap,

$$\text{Output gap:} \quad \Delta y = \text{Percentage difference of real GDP } y \text{ from potential real GDP } y^*$$

There are two obstacles involved in inferring projections for the output gap from FOMC projections. First, Federal Reserve staff projections of potential output are only available at a 5 year lag, in the Greenbook. Instead, meeting real-time forecasts of potential from the Congressional Budget Office (CBO) are utilized as a next-best guess of these projections. Second, in order to compute projections for the gap, observable projections for the growth rate of real GDP, % chg. $y$, must be transformed into projections for the

[7] These include dynamic stochastic general equilibrium, or vector autoregressive models, with normally distributed and heteroscedastic errors, for example. Both are known to be used by Federal Reserve staff.

level, $y$. The problem is that projections for the rate at each horizon are anonymous, meaning the growth path for levels imagined by each individual participant is not observable. Nonetheless, the probability mass function (PMF) for levels at each horizon is observable, since the universe of categorical projected growth rates across participants is observable. Together, these points allow us to assemble moments of the distribution for the output gap, such as the median and mean.

Finally, we wish to infer projections for cyclical unemployment,

$$\text{Cyclical unemployment:} \quad \Delta u = \text{unemp. rate } u \; - \; \text{natural rate of unemp. } u^*$$

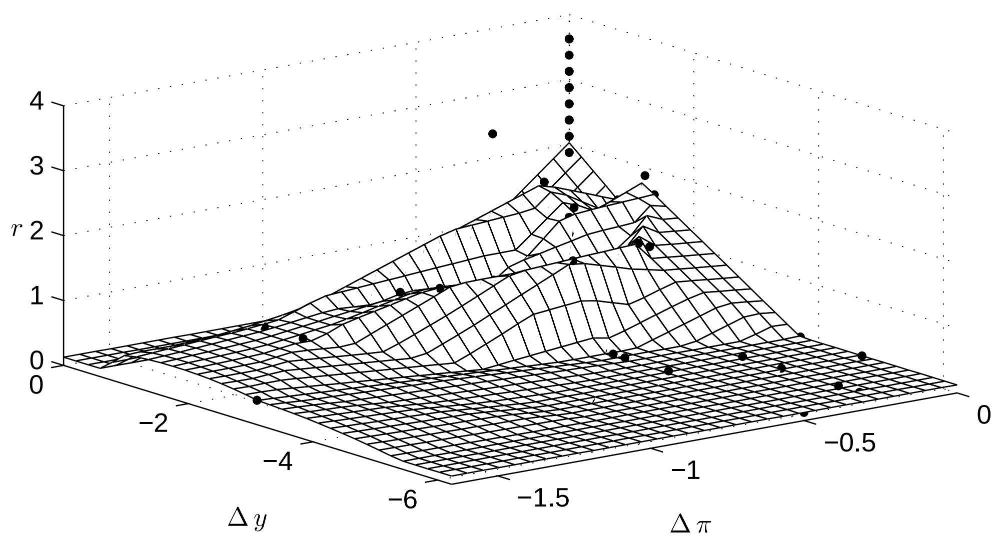
Real-time forecasts are again utilized from the CBO for $u^*$.

Figure 2 plots the 73 median federal funds rate projections tabulated in Table 1 as a function of macroeconomic projections. Panel A depicts the output gap and excess inflation, while Panel B depicts scaled cyclical unemployment and excess inflation. Cyclical unemployment is scaled by a factor of $-2$ for apples-to-apples comparability with the output gap by Okun's law. Clearly, there is a systematic and intuitive relationship between macroeconomic and policy projections, possibly indicative of an underlying policy rule.

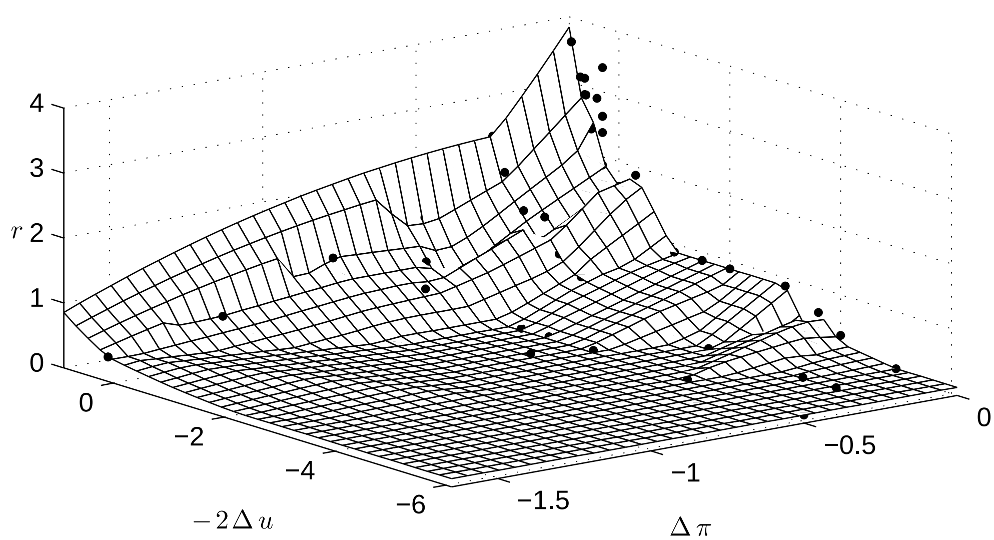
Yet, estimating such a rule is met with two complications. First, Figure 2 depicts large flat regions of censoring at the ZLB. So, a purely linear model will not do. Second, the relationship between medians does not necessarily say anything about policy rules participants have in mind. What we are really interested is individual participants' projections. However, projections are anonymous and may not be matched by-participant at any horizon. The next section presents a framework for estimation suited to these concerns.

Figure 2: Median projected federal funds rate vs. median macroeconomic projections.

(A) Output and inflation




(B) Unemployment and inflation




*Notes*: Surface is interpolated from 73 horizons (scatter plot, duplicate points are averaged). Cyclical unemployment is scaled by a factor of $-2$ for apples-to-apples comparability with output gap by Okun's law. Lower bound at effective ZLB of $r = 0.125\%$. *Source: Federal Reserve, Congressional Budget Office, author's calculations.*

# 3 Model

Consider monetary policy rules of the form,

$$\underline{r}_i = r_i^* + \mathbf{x}_i'\mathbf{b}_i \tag{1}$$

$\underline{r}_i$ is the censored (shadow) federal funds rate projection (Wu and Xia, 2016). In the data, we observe $r_i = \max\{\text{ZLB}, \underline{r}_i\}$ for ZLB the zero lower bound. The Taylor (1993) rule has covariates,

$$\underset{(2\times 1)}{\mathbf{x}_i} = (\Delta\pi_i,\ \Delta y_i)' \tag{2}$$

Unlike the Taylor rule, the coefficients $\mathbf{b}_i$ are also subscripted. $i$ is not a time index, but rather a triplet. This index denotes how FOMC *participants* generate economic projections at various *horizons* during FOMC *meetings*, when a rule of the form of (1) is utilized.

$$i = p,\ h,\ m$$

$$p = 1, \ldots, 28 \text{ Participant.} \quad h = 2012, \ldots, 2019 \text{ Horizon.} \quad m = 1, \ldots, 21 \text{ Meeting.}$$

Inasmuch, participants may not only generate different projections, but reasonably have different rules in mind altogether.

Projections are anonymous. So, for each meeting-horizon combination, $r_i$ may not be matched to $r_i^*$, $\mathbf{x}_i$, and $\mathbf{b}_i$ by-individual. As a result, (1) may not be estimated directly. The suggestion of this paper is to instead estimate the reduced form model,

$$\underline{r}_i = \overline{r}^*_{h,m} + \overline{\mathbf{x}}'_{h,m}\mathbf{b} + u_i \tag{3}$$

$\overline{r}^*_{h,m}$ and $\overline{\mathbf{x}}_{h,m}$ are the average of $r_i$ and $\mathbf{x}_i$ across participants for each horizon-month combination. Both of these are observable (Section 2.3). Appendix A shows that $u_i \sim N(0, \sigma^2)$ and the static coefficient $\mathbf{b}$ is interpretable as the average policy parameter across participants. This is the parameter of interest to us, since ultimately only one policy response is acted upon, and that response is decided upon by deliberate consensus-making.[8] In other words, $\mathbf{b}$ would be what we could estimate given uncensored historical data.

Given the censoring of $\underline{r}_i$, the log-likelihood of the reduced form (3) has Tobit form,

$$\ell = \sum_{i=1}^{N} \left\{ \mathbb{1}_i \ln \left[ \frac{1}{\sigma} \phi \left( \frac{r_i - \overline{r}^*_{h,m} - \overline{\mathbf{x}}'_{h,m}\mathbf{b}}{\sigma} \right) \right] \right.$$
$$\left. + (1 - \mathbb{1}_i) \ln \left[ \Phi \left( \frac{\text{ZLB}_m - \overline{r}^*_{h,m} - \overline{\mathbf{x}}'_{h,m}\mathbf{b}}{\sigma} \right) \right] \right\} \quad (4)$$

$$\mathbb{1}_i = \begin{cases} 1 & \text{if } r_i > \text{ZLB}_m \\ 0 & \text{if } r_i = \text{ZLB}_m \end{cases}$$

$\phi$ is the standard normal PDF and $\Phi$ is the standard normal CDF. The ZLB is dependent on the meeting $m$, since no bound is enforced in later meetings. The maximum likelihood estimator (MLE) for the parameters $\mathbf{b}$ and $\sigma$ is described in Appendix B.

# 4 Results

## 4.1 Full sample

Table 2 presents full sample, January 2012 to December 2016, MLE estimates. These estimates are for the average policy rule across participants, the reduced from parameters

[8]Both former Chair Bernanke and present Chair Yellen have been noted to seek consensus in deliberations (e.g. Hilsenrath (2014)). This is in contrast to majority rule over extreme alternatives, such as polar "hawk" and "dove" policy rule coefficients, wherein the mean would not be representative of policymaking. See Appendix A for discussion.

**b** in (3). In total there are 1,269-404=865 uncensored observations in the sample. Specifications (1) - (3) use excess inflation $\Delta\pi$ and either the output gap $\Delta y$, scaled cyclical unemployment $-2\Delta u$, or both, as covariates. Of these three, specification (3) including both output and unemployment has the best overall explanatory power. However, an immediate concern is that output and scaled unemployment are positively correlated, with a Pearson coefficient of $\rho = 0.72$. The effect of multicollinearity when two covariates are positively correlated is to bias one estimate up, and one estimate down. Therefore, the coefficients in (3) are difficult to interpret.

Instead, specification (4) utilizes the simple average of $\Delta y$ and $-2\Delta u$ as an index of economic activity.[9] This specification has the same overall explanatory power as including output and unemployment individually, but has meaningful coefficients. On the basis of these estimates, the communicated inflation response during 2012-2016 was 1.90. This is above Taylor (1993)'s suggested parameterization of 1.5 and Kim and Pruitt (2017)'s forecaster-perceived pre-ZLB estimate of 1.56 (SE = 0.35).[10] With respect to economic activity, Taylor (1993) suggests a response of 0.5, while Taylor (1999) suggests 1. The forecaster-perceived pre-ZLB estimated output gap response was 0.39 (SE=0.03).[11] The communicated response is again stronger than any of these at 1.01.

Kim and Pruitt find that after the beginning of the ZLB environment in December 2008 through 2014, forecasters perceived the inflation response fell to -0.02 (SE=0.58) and the output gap response rose to 0.64 (SE=0.04). Projections from 2012-2016 also indicate a strengthened response to output after 2008. But more so than perceived.

[9]The "Activity" index remains apples-to-apples with the output gap via Okun's law.

[10]SE is standard error. Pre-ZLB=1986-2008. See Kim and Pruitt Table 1, Panel C, "Tobit, Only Forecast Data."

[11]Kim and Pruitt use only cyclical unemployment in their analysis. Their estimates are scaled by -1/2 to be comparable with the output gap and estimates in this paper via Okun's law. Standard errors are scaled by 1/4 using the delta method.

Table 2: Policy rule, 2012 - 2016.

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| Inflation | 1.09 | 3.08 | 1.87 | 1.90 | 0.55 |
| | (0.12)** | (0.09)** | (0.12)** | (0.09)** | (0.11)** |
| Output | 0.93 | | 0.52 | | |
| | (0.03)** | | (0.03)** | | |
| $-2\times$Unemployment | | 0.82 | 0.49 | | |
| | | (0.02)** | (0.03)** | | |
| Activity | | | | 1.01 | 0.61 |
| | | | | (0.03)** | (0.02)** |
| Spread, Horizon +0 | | | | | -1.16 |
| | | | | | (0.08)** |
| Spread, Horizon +1 | | | | | -0.84 |
| | | | | | (0.04)** |
| Spread, Horizon +2 | | | | | -0.76 |
| | | | | | (0.03)** |
| Spread, Horizon +3 | | | | | -0.53 |
| | | | | | (0.04)** |
| Sample | 1,269 | 1,269 | 1,269 | 1,269 | 1,269 |
| Censored | 404 | 404 | 404 | 404 | 404 |
| Adj. $R^2$ | 0.38 | 0.39 | 0.44 | 0.44 | 0.65 |
| Exclusion ($p$ value) | 0.00 | 0.00 | 0.00 | 0.00 | |

*Notes:* */** denotes significance at 10/1 percent level. "Inflation" is excess inflation, "Output" is the output gap, and "Unemployment" is cyclical unemployment. "Activity" is the average of the output gap and $-2\times$cyclical unemployment Standard errors in parentheses. Reported Adjusted $R^2$ is McFadden (1973)'s adjusted pseudo R-squared $= 1 - (\widehat{\ell} - k)/\widehat{\ell_0}$ where $k$ is the size of $\mathbf{b}$, $\widehat{\ell}$ is the unrestricted maximized likelihood, and $\widehat{\ell_0}$ is the restricted maximized likelihood with $\mathbf{b} = \mathbf{0}_{k\times 1}$. "Exclusion" is likelihood ratio test of exclusion restrictions.

Moreover, projections indicate the inflation response actually increased. What accounts for these discrepancies?

Kim and Pruitt also find that forecasters perceived a significant response to financial risk, as measured by the 10 year - 2 year Treasury spread. Table 2 specification (5) includes the meeting-contemporaneous spread as a horizon fixed effect.[12] The impact of risk is significant, and intuitively negative and tapered by-horizon.[13] Given this control, the communicated activity response of 0.61 is indistinguishable from the perceived ZLB response of 0.65 (SE=0.04). The inflation response of 0.55 is indistinguishable from the perceived ZLB response of -0.02 (SE=0.60). $R$-squared increases by including risk, and exclusion restrictions on (1)-(4) are each rejected at the 1% level, suggesting this is the correct specification.

These results indicate that perceptions do match communication, controlling for risk. However, while the perceived and communicated inflation responses are indistinguishable, the former is insignificant, and the latter is not. This makes the conclusion that the two are the same tenuous. Moreover, the perceived sample of 2008-2014 and communicated subsample of 2012-2016, while overlapping, are not equivalent. An important robustness check is that equivalence between perception and communication holds in-sample previous to 2015. This need not be the case if the communicated rule changed over time.

## 4.2 Breaks

Kim and Pruitt (2017) consider forecasters' perceptions during the ZLB period of 2008-

[12]Coefficients are allowed to differ based on what horizon the projection corresponds to, since current risk should reasonably have a relatively lesser impact on more distant horizons.

[13]The one-year-ahead projection, "Spread, Horizon +1," is analogous to Kim and Pruitt's one-year-ahead forecast spread. The point estimate of -0.84 is stronger than the perceived response of -0.24 (SE=0.07). However, the perceived response is fixed over all of 1986-2014. Therefore, these estimates are not entirely comparable.

2014, whereas the full sample ranges from 2012-2016. As a robustness check of the full sample result that perceptions match communication, we first consider the hypothesis that there is a break in the communicated rule after December 2014.[14] In Table 3, specifications (3) and (4), pre-2015 "in-sample" and post-December 2014 "out-of-sample" estimates are reported.[15] The hypothesis of no break is rejected at the 1% level. In-sample, the activity response of 0.64 remains indistinguishable from perception. But the in-sample communicated inflation response of 1.96 is greater than perception with significance, invalidating the full sample conclusion. It is not until after 2014 that the communicated inflation response becomes indistinguishable from zero. The full-sample inflation response is merely a weighted average of these two substantively different sub-sample responses.

Why is there such a dramatic shift in communicated policy at the end of December 2014, a date which does not seem particularly economically meaningful? To answer this question we also consider two nearby, more economically meaningful breaks.[16] Table 3 specifications (1) and (2) considers an earlier break, after December 2013, when Chair Bernanke's tenure ended. During the Bernanke period, inflation and activity responses are similar to a Taylor (1993) rule. But during the Yellen period, both the inflation and activity responses fell, while remaining significant. At the same time, short run responsiveness to risk increased.[17] So, one possibility is that the overall decrease in responsiveness to inflation may be due to a shift in regime. Table 3 also considers the

[14]December 2014 is also the data-preferred break in the full sample of 2012-2016, in that it results in the largest likelihood improvement. In order to identify the data-preferred break, the full sample was broken into two parts after each meeting, and re-estimated for each break.

[15]Table 3 and subsequent results refer to the index of economic activity rather than output and unemployment individually for space considerations, and the multicollinearity concern of including both previously described. All results presented here were also validated using output and unemployment individually. These estimates are available in the replication materials.

[16]The null hypothesis that there is not a break at either of these dates is rejected at the 1% level.

[17]The coefficients on the spread at horizons +0 and +1 increase, while they decrease for horizons +2 and +3.

Table 3: Policy rule: Other breaks.

| | Chair break *Dec 2013* | | Kim and Pruitt (2017) break *Dec 2014* | | ZLB break *Sep 2015* | |
|---|---|---|---|---|---|---|
| | Bernanke | Yellen | In-sample | Out-of-sample | ZLB | Post-ZLB |
| | 1/12 - 12/13 | 3/14 - 12/16 | 1/12 - 12/14 | 3/15 - 12/16 | 1/12 - 9/15 | 12/15 - 12/16 |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Inflation | 1.36 | 0.31 | 1.96 | 0.21 | 0.59 | 0.09 |
| | (0.52)** | (0.09)** | (0.36)** | (0.15) | (0.15)** | (0.23) |
| Activity | 0.60 | 0.34 | 0.64 | 0.38 | 0.65 | 0.60 |
| | (0.06)** | (0.08)** | (0.04)** | (0.11)** | (0.03)** | (0.24)* |
| Spread, Horizon +0 | -1.05 | -1.57 | -1.03 | -1.58 | -1.13 | -1.10 |
| | (0.24)** | (0.10)** | (0.13)** | (0.12)** | (0.10)** | (0.08)** |
| Spread, Horizon +1 | -0.87 | -1.01 | -0.58 | -1.09 | -0.79 | -0.66 |
| | (0.16)** | (0.05)** | (0.09)** | (0.05)** | (0.06)** | (0.07)** |
| Spread, Horizon +2 | -0.95 | -0.56 | -0.73 | -0.52 | -0.77 | -0.22 |
| | (0.08)** | (0.03)** | (0.05)** | (0.04)** | (0.04)** | (0.07)** |
| Spread, Horizon +3 | -0.76 | -0.15 | -0.58 | -0.14 | -0.58 | -0.22 |
| | (0.07)** | (0.04)** | (0.06)** | (0.05)** | (0.05)** | (0.07)** |
| Sample | 561 | 708 | 793 | 476 | 963 | 306 |
| Censored | 328 | 76 | 400 | 4 | 404 | 0 |
| Adj. $R^2$ | 0.83 | 0.68 | 0.73 | 0.74 | 0.67 | 0.79 |
| No spread ($p$ value) | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| No break ($p$ value) | | 0.00 | | 0.00 | | 0.00 |

*Notes:* */** denotes significance at 10/1 percent level. "Inflation" is excess inflation and "Activity" is average of output gap and $-2\times$cyclical unemployment. Standard errors in parentheses. Reported $R^2$ is McFadden (1973)'s adjusted pseudo R-squared $= 1 - (\widehat{\ell} - k)/\widehat{\ell}_0$ where $k$ is the size of $\mathbf{b}$, $\widehat{\ell}$ is the unrestricted maximized likelihood, and $\widehat{\ell}_0$ is the restricted maximized likelihood with $\mathbf{b} = \mathbf{0}_{k\times 1}$. "No spread" is LR test of null that spread coefficients are zeros. "No break" is LR test of null that there is no structural break.

antithesis to Kim and Pruitt's research question, a break from the ZLB period to the post-ZLB period, in specifications (5) and (6). After the ZLB beginning in December 2015, the inflation response becomes insignificant. So, another possibility is that the shift in responsiveness is due to a change in paradigm, as also may have occurred when the ZLB began.

On the basis of the full sample, perceptions about the policy rule match communication. But breaking this sample up into parts reveals that the communicated policy response changed. In-sample previous to 2015, the perceived inflation response was meaningfully lesser than the communicated inflation response, but activity was equivalent. Why?

# 5 Discussion

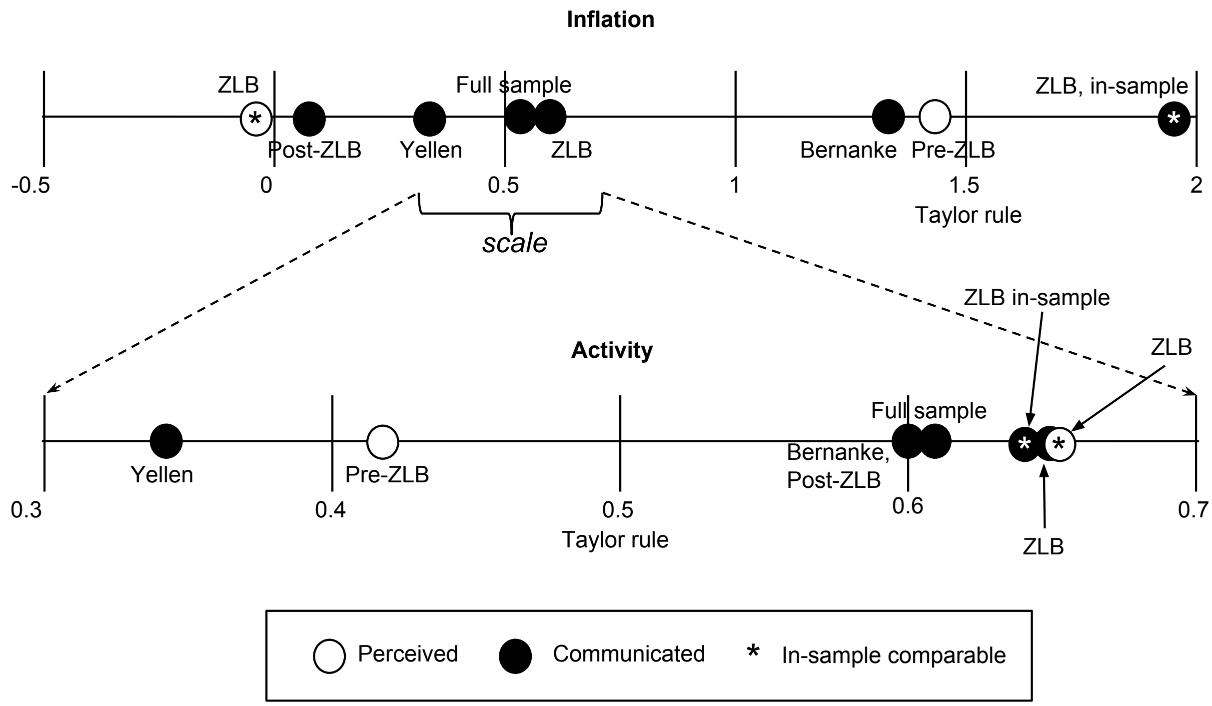
## 5.1 Why wouldn't forecasters believe the FOMC?

Why did forecasters' perceptions of the inflation response differ from communication, whereas perceptions of the activity response were nearly equivalent? A possibility is that communication about the response to activity was more consistent and credible. Figure 3 depicts all previously discussed estimates in a diagram. With respect to the activity response, most communicated responses pile up tightly in the range of 0.6-0.65. The in-sample communicated activity response of 0.64 is indistinguishable from the perceived response of 0.65. But with respect to inflation, communicated responses vary more widely from near 0, where the perceived in-sample response lies, to near 2, where the communicated response lies.[18]

In particular, did communication about the inflation response change even more

[18] Note the difference in scale between the inflation and activity axes in Figure 3.

Figure 3: Perceived vs. communicated policy rule.




*Notes:* Risk-adjusted policy rule coefficients. "Full sample" is 2012-2016. "ZLB, In-sample" is 2012-2014, comparable with the perceived "ZLB" estimates from 2008-2014. Communicated "ZLB" is 2012-2015 and "Post-ZLB" is 2015-2016. "Bernanke" is 2012-2013 and "Yellen" is 2014-2016. "Pre-ZLB" is 1986-2008.

frequently, such as within the in-sample 2012-2014 period? A possibility is that rules continually evolved, akin to the discretion participants are free to utilize. Figure 4 depicts rolling nine-meeting point estimates and standard errors for the communicated responses to inflation, activity, and risk. These estimates therefore begin at the ninth meeting in the sample, December 2013, when Chair Bernanke's tenure ended.[19] Figure 4 shows that the communicated inflation response throughout the in-sample period of 2012-2014 is noisy in comparison to the later part of the sample, moving full percentage points meeting-to-meeting. On the other hand, the activity response shifted upward only conservatively meeting-to-meeting. Forecasters were able to understand, or at least willing to place stake in, communication about the response that was consistent from meeting-to-meeting. But they were not able to do the same for inconsistent communication.

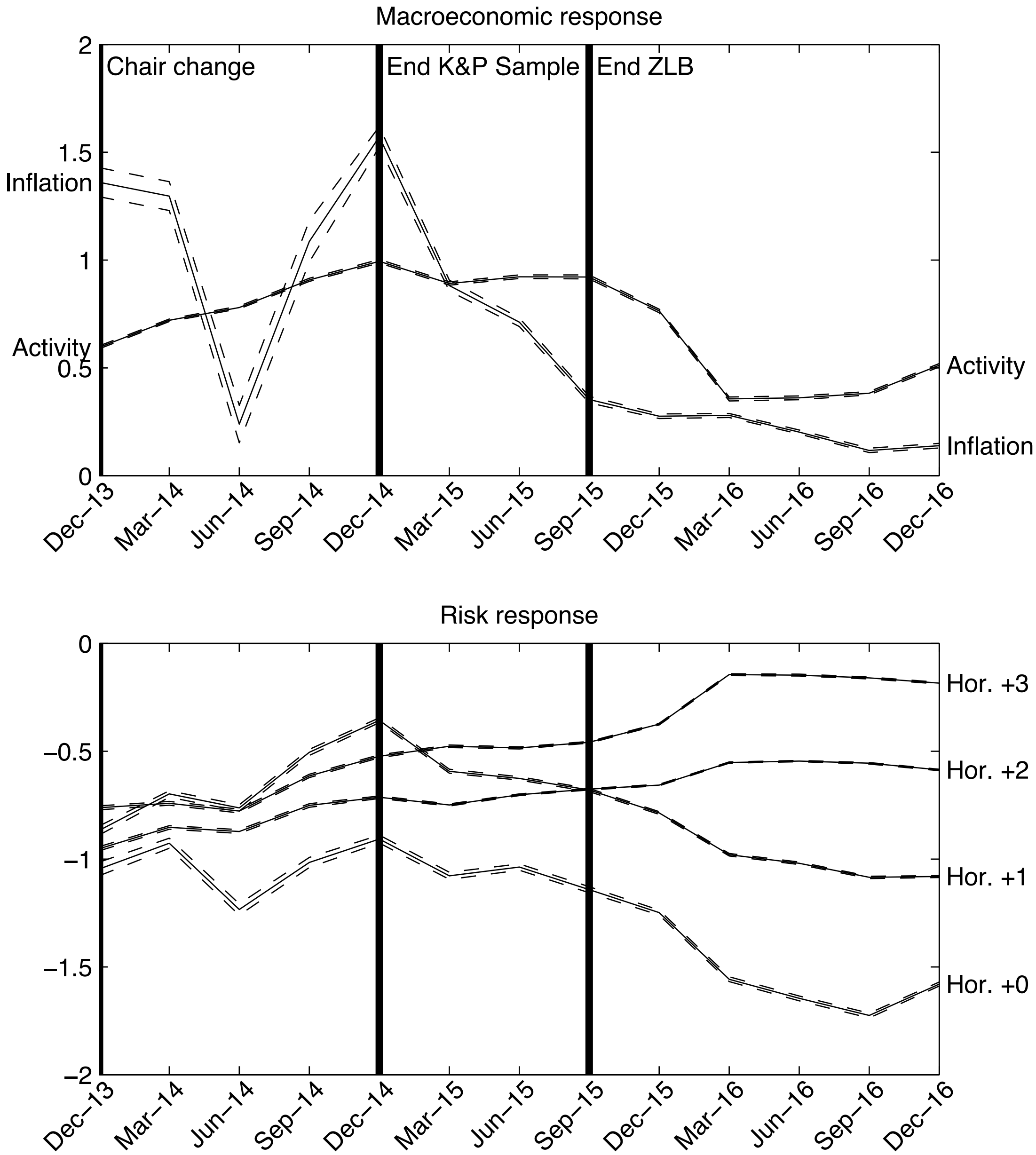
## 5.2 How did policymaking change over 2012-2016?

What do projections say about the change in policy over the sample as a whole? Table 3 seemed to suggest that not only the inflation response, but also activity, fell in the later Yellen and out-of-sample periods (specifications (2) and (4)). Is this borne out in smaller windows?

Figure 4 shows that after December 2014, the inflation response began to steadily fall, as forecasters had previously expected, along with the response to activity. The inflation response would fall to near zero, but the activity response would eventually come back to near its level in 2012. Meanwhile, the response to risk in the shorter-run increased as the response to risk in the longer run decreased. In summary, projections indicate inconsistent but overall diminishing responsiveness to inflation, a relatively stable and

[19] The minimum number of uncensored federal funds rate projections in any of these samples is 233.

Figure 4: Policy rule: Rolling estimates.




*Notes:* Rolling 9-meeting parameter estimates. Minimum uncensored observations = 233. Black vertical lines are previously studied breakpoints, beginning with Chair change from Bernanke to Yellen. Other breakpoints are end of Kim and Pruitt (2017)'s sample (also the single break which maximizes likelihood), and end of ZLB environment. Top panel is response of federal funds rate to macroeconomic variables. "Inflation" is difference of inflation from target and "Activity" is average of output gap and $-2\times$cyclical unemployment. Bottom panel is response of federal funds rate to meeting-concurrent 10 year - 2 year Treasury spread. "Hor. +0" (Horizon +0) is response at meeting-concurrent year end. "Hor $+i$" is response $i$ years into the future.

strong response to economic activity, and an increasing short-run response to financial risk. Overall, the change in policymaking over this period may be characterized as substituting responsiveness to financial risk for inflation.

Finally, it is notable that the communicated rule by 2016 finally looks like perceptions, albeit at a lag. The post-ZLB communicated policy rule in Table 3 specification (6) is statistically indistinguishable to perception through December 2014. Thus, communication about both inflation and activity would eventually match perception, but not until a year later. The implications of this observation are discussed in the conclusion.

# 6 Conclusion

This paper has provided estimates of FOMC policy rules during 2012-2016 using the implicit informational content of forward guidance. One substantive implication of this analysis is that forecaster-perceived and policymaker-communicated rules during this period were not always the same. This distinction was primarily with respect to the inflation response, which forecasters perceived to be lesser than policymakers communicated during 2012-2014. A reasonable explanation for this finding is that the communication of the inflation response was considerably noisier than the response to economic activity.

Eventually, the communicated response to both inflation and activity did match perception, but at about a year's lag. Did the tail wag the dog? One possibility is that professional forecasters also anticipate changes in the rule; one year is also the horizon of their forecasts. Another possibility is that forecasters perceived shifts in policymaking before policymakers were ready to accept them. In either case, this timing may simply be a red herring. This paper is limited to conclusions regarding contemporaneous communication and perception, which were not in-line through 2014. At the very least, inconsistency in the implicit inflation response does not facilitate its communication. Policymakers could improve information sharing about the policy rule by taking more stake in, and being more consistent about, the implicit policy response in their projections.

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# A Reduced form

As noted in Section 2.2, it is reasonable to assume that $r_i^*$ and $\mathbf{x}_i$ are normally distributed across participants, with horizon and meeting-dependent means, but common covariances.

$$\begin{bmatrix} r_i^* \\ \mathbf{x}_i \end{bmatrix} \sim \mathrm{N}\left( \begin{bmatrix} \overline{r}_{h,m}^* \\ \overline{\mathbf{x}}_{h,m} \end{bmatrix}, \begin{bmatrix} \sigma_r^2 & \cdot \\ \boldsymbol{\sigma}_{xr} & \boldsymbol{\sigma}_{xx} \end{bmatrix} \right) \tag{A.1}$$

Moreover, from (1) and (3) we have the identity,

$$u_i = (r_i^* - \overline{r}_{h,m}^*) + (\mathbf{x}_i'\mathbf{b}_i - \overline{\mathbf{x}}_{h,m}'\mathbf{b}) \tag{A.2}$$

Different participants likely have different preferences for the policy parameters $\mathbf{b}_i$. One common way of thinking about this is that policymakers may be characterized as inflation hawks (larger inflation response) or doves (smaller). In this case the PMF for $\mathbf{b}_i$ is Bernoulli,

$$p(\mathbf{b}_i) = \begin{cases} \pi_H \text{ if } \mathbf{b}_i = \mathbf{b}_H \\ 1 - \pi_H \text{ if } \mathbf{b}_i = \mathbf{b}_D \end{cases} \tag{A.3}$$

and so, $\mathbf{x}_i'\mathbf{b}_i$ is a mixture of normals, with PDF

$$p(\mathbf{x}_i'\mathbf{b}_i) \sim N\left(\overline{\mathbf{x}}_{h,m}'\mathbf{b}, \mathbf{b}'\boldsymbol{\sigma}_{xx}\mathbf{b}\right) \tag{A.4}$$

$$\mathbf{b} = \pi_H\mathbf{b}_H + (1 - \pi_H)\mathbf{b}_D \tag{A.5}$$

Therefore, the reduced form errors $u_i$ are normally distributed with mean zero and variance

$$\sigma^2 = \sigma_r^2 + \mathbf{b}'\boldsymbol{\sigma}_{xx}\mathbf{b} + 2\boldsymbol{\sigma}_{xr}'\mathbf{b} \tag{A.6}$$

Furthermore, $\mathbf{b}$ is interpretable from (A .5) as the average monetary policy response across participants. Generally for $R \geq 2$ types of responses, we would have the same conclusions, using the respective categorical probabilities to compute $\mathbf{b} = \sum_{r=1}^{R} \pi_r \mathbf{b}_r$.

# B Estimator

Consider the log-likelihood function $\ell$ (4). The parameters are $\boldsymbol{\theta}' = (\mathbf{b}',\ \sigma)$. Defining $\omega = 1/\sigma$, $\boldsymbol{\psi} = \omega\mathbf{b}$, and $\boldsymbol{\mu}' = (\boldsymbol{\psi}',\ \omega)$, we may also write the log-likelihood as

$$\ell = \sum_{i=1}^{N} \left\{ \mathbb{1}_i \left[ -\frac{1}{2}\ln 2\pi + \ln\omega - \frac{1}{2}\left(\omega(r_i - \overline{r}^*_{h,m}) - \overline{\mathbf{x}}'_{h,m}\boldsymbol{\psi}\right)^2 \right] + (1 - \mathbb{1}_i)\ln\Phi\left(\omega(\mathrm{ZLB}_m - \overline{r}^*_{h,m}) - \overline{\mathbf{x}}'_{h,m}\boldsymbol{\psi}\right) \right\} \quad \text{(B.1)}$$

This is known as Olsen (1978)'s transformation (see Greene (2005)). $\ell$ may be maximized with respect to the $(3 \times 1)$ vector $\boldsymbol{\mu}$ to obtain the MLE $\widehat{\boldsymbol{\mu}}$. The normal equations are,

$$\left.\frac{\partial \ell}{\partial \boldsymbol{\psi}'}\right|_{\boldsymbol{\mu}=\widehat{\boldsymbol{\mu}}} = \left.\left( \sum_{i=1}^{N} \mathbb{1}_i \left[ \left(\omega(r_i - \overline{r}^*_{h,m}) - \overline{\mathbf{x}}'_{h,m}\boldsymbol{\psi}\right)\overline{\mathbf{x}}'_{h,m} \right] - (1 - \mathbb{1}_i)\lambda_{h,m}\overline{\mathbf{x}}'_{h,m} \right)\right|_{\boldsymbol{\mu}=\widehat{\boldsymbol{\mu}}} = \mathbf{0}_{1\times 2} \quad \text{(B.2)}$$

$$\left.\frac{\partial \ell}{\partial \omega}\right|_{\boldsymbol{\mu}=\widehat{\boldsymbol{\mu}}} = \left.\left( \sum_{i=1}^{N} \mathbb{1}_i \left[ 1/\omega - (\omega(r_i - \overline{r}^*_{h,m}) - \overline{\mathbf{x}}'_{h,m}\boldsymbol{\psi})(r_i - \overline{r}^*_{h,m}) \right] + (1 - \mathbb{1}_i)\lambda_{h,m}(\mathrm{ZLB}_m - \overline{r}^*_{h,m}) \right)\right|_{\boldsymbol{\mu}=\widehat{\boldsymbol{\mu}}} = 0 \quad \text{(B.3)}$$

$\lambda_{h,m}$ is the inverse Mills ratio (see Heckman (1979)). Its functional form is,

$$\lambda_{h,m} = \frac{\partial \ln\Phi(z_{h,m})}{\partial z_{h,m}} = \frac{\phi(z_{h,m})}{\Phi(z_{h,m})} \qquad \text{for} \qquad z_{h,m} = \omega(\mathrm{ZLB}_m - \overline{r}^*_{h,m}) - \overline{\mathbf{x}}'_{h,m}\boldsymbol{\psi} \quad \text{(B.4)}$$

One advantage of the Olsen transformation is that second derivatives are easily derived.

$$\frac{\partial \ell}{\partial \boldsymbol{\psi}\partial \boldsymbol{\psi}'} = \sum_{i=1}^{N} - \left[\mathbb{1}_i - (1 - \mathbb{1}_i)\delta_{h,m}\right]\overline{\mathbf{x}}_{h,m}\overline{\mathbf{x}}'_{h,m} \quad \text{(B.5)}$$

for $-1 < \delta_{h,m} < 0$ (See Maddala (1983)).

$$\delta_{h,m} = \frac{\partial \lambda_{h,m}}{\partial z_{h,m}} = -z_{h,m}\lambda_{h,m} - \lambda_{h,m}^2 \tag{B.6}$$

And,

$$\frac{\partial \ell}{\partial \boldsymbol{\psi} \partial \omega} = \sum_{i=1}^{N} \left[ \mathbb{1}_i (r_i - \overline{r}_{h,m}^*) - (1 - \mathbb{1}_i)\delta_{h,m}(\text{ZLB}_m - \overline{r}_{h,m}^*) \right] \overline{\mathbf{x}}_{h,m} \tag{B.7}$$

$$\frac{\partial \ell}{\partial \omega^2} = \sum_{i=1}^{N} \left\{ \mathbb{1}_i \left[ -1/\omega^2 - (r_i - \overline{r}_{h,m}^*)^2 \right] + (1 - \mathbb{1}_i)\delta_{h,m}(\text{ZLB}_m - \overline{r}_{h,m}^*)^2 \right\} \tag{B.8}$$

Because $-1 < \delta_{h,m} < 0$, the Hessian is always negative definite. So, another advantage of the Olsen transformation is that numerical likelihood maximization can be achieved by Newton's method. A good starting point for maximization is the scaled least squares estimator $\boldsymbol{\mu}'_{\texttt{start}} = (\boldsymbol{\psi}'_{\texttt{start}},\ \omega_{\texttt{start}})$ for $\omega_{\texttt{start}} = 1/\sigma_{\texttt{start}}$ and $\boldsymbol{\psi}_{\texttt{start}} = \omega_{\texttt{start}} \mathbf{b}_{\texttt{start}}$ where,

$$\widehat{\mathbf{b}}_{\texttt{start}} = \left( N \bigg/ \sum_{i=1}^{N} \mathbb{1}_i \right) \left[ \frac{1}{N} \sum_{i=1}^{N} \overline{\mathbf{x}}_{h,m} \overline{\mathbf{x}}'_{h,m} \right]^{-1} \frac{1}{N} \sum_{i=1}^{N} \overline{\mathbf{x}}_{h,m}(r_i - \overline{r}_{h,m}^*)$$

$$\widehat{\sigma}_{\texttt{start}} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (r_i - \overline{r}_{h,m}^* - \overline{\mathbf{x}}'_{h,m} \widehat{\mathbf{b}}_{\texttt{start}})^2}$$

The MLE for the parameters of interest is $\widehat{\boldsymbol{\theta}}' = (\widehat{\mathbf{b}}',\ \widehat{\sigma})$ for $\widehat{\mathbf{b}} = (1/\widehat{\omega})\widehat{\boldsymbol{\psi}}$ and $\widehat{\sigma} = 1/\widehat{\omega}$. By the delta method, a consistent estimator for the asymptotic variance of this estimator is

$$\underset{(3\times 3)}{\widehat{\text{Avar}}(\widehat{\boldsymbol{\theta}})} = \left.\frac{\partial \boldsymbol{\theta}}{\partial \boldsymbol{\mu}'}\right|_{\boldsymbol{\mu}=\widehat{\boldsymbol{\mu}}} \widehat{\text{Avar}}(\widehat{\boldsymbol{\mu}}) \left( \left.\frac{\partial \boldsymbol{\theta}}{\partial \boldsymbol{\mu}'}\right|_{\boldsymbol{\mu}=\widehat{\boldsymbol{\mu}}} \right)' \tag{B.9}$$

$$\underset{(3\times 3)}{\widehat{\text{Avar}}(\widehat{\boldsymbol{\mu}})} = \left( - \begin{bmatrix} \left.\frac{\partial \ell}{\partial \boldsymbol{\psi} \partial \boldsymbol{\psi}'}\right|_{\boldsymbol{\mu}=\widehat{\boldsymbol{\mu}}} & \cdot \\ \left.\frac{\partial \ell}{\partial \boldsymbol{\psi} \partial \omega'}\right|_{\boldsymbol{\mu}=\widehat{\boldsymbol{\mu}}} & \left.\frac{\partial \ell}{\partial \omega^2}\right|_{\boldsymbol{\mu}=\widehat{\boldsymbol{\mu}}} \end{bmatrix} \right)^{-1} \qquad \underset{(3\times 3)}{\left.\frac{\partial \boldsymbol{\theta}}{\partial \boldsymbol{\mu}'}\right|_{\boldsymbol{\mu}=\widehat{\boldsymbol{\mu}}}} = \begin{bmatrix} \mathbf{I}_2 \otimes 1/\widehat{\omega} & -(1/\omega^2)\widehat{\mathbf{b}} \\ \mathbf{0}_{1\times 2} & -1/\widehat{\omega}^2 \end{bmatrix}$$