Banking Crises as Self-Defeating Prophecies*

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Abstract

Banking crises are modeled as fire sales in a calibrated endowment economy. Rare and slowly evolving disasters, together with heterogeneous risk aversion, explain the existence of high banking leverage in normal times, simultaneous with a high risk premium, while accounting for a counter-cyclical Sharpe ratio. In the model, bankers are distinguished from depositors and real estate speculators only by their intermediate level of risk aversion. Contrary to the view that banking crises are the result of self-fulfilling prophecies, it is argued that they are partially due to self-defeating prophecies. It is shown that if agents underestimate the potential for a drop in real estate prices, the realized drop will be larger than if their estimate had been accurate. This dynamic is due to the increase in ex-ante leverage and the resulting increase in ex-post fire sales. Increased leverage can account for up to two-thirds of the realized error. Moreover, the increase in leverage generates losses on mortgages that were considered, and would otherwise be, perfectly safe.

1 Introduction

An endogenous choice of high leverage is informative about prevailing perceptions of downside risk. It reveals that lenders and borrowers believe the price of the collateral asset will not decline significantly over the life of the contract. However, if it is believed that the downward potential of the collateral asset's price is limited in the short run, then a high risk premium for holding that asset during the same period is puzzling, and cannot be explained by a fear of an immanent crash.

In this model of banking, the collateral asset is real estate, which is a risky asset because in equilibrium rent closely tracks the uncertain labor endowment. Agents have Epstein-Zin utility over a Cobb-Douglas in consumption and real estate services. There

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are three types of agents, differentiated only by their level of risk aversion. I refer to the least risk-averse type as speculators, to the most risk-averse type as depositors, and to the in-between type as bankers (because in equilibrium the in-between type lends to the least risk averse and borrows from the most risk averse).

Stocks are not included in the model, but if they had been, then real estate and stocks would share the same risk-reward trade-off as a result of the binomial tree state structure. Lustig and Verdelhan (2012) report that U.S. stocks provide a more favorable risk-return trade-off during recessions than during expansions. Specifically, post-WWII recessions exhibited a Sharpe ratio of 0.82 four quarters into the recession while expansions exhibited a Sharpe ratio of 0.14 three quarters into the expansion. In this paper, I suggest that these Sharpe ratio dynamics are driven by risk aversion differentials between agents. In other words, risk-aversion differentials naturally give rise to counter-cyclical Sharpe ratio dynamics that are governed by bankers' and speculators' wealth dynamics, and their ability to obtain leverage.

To explain the size of the risk premium in normal times, Nakamura, Steinsson, Barro, and Ursúa (2013) use rare disasters, estimated from the data, which, once they hit, are immediately recognizable to agents. In their setting, while a disaster takes several years to unfold, once started it continues with high probability. In anticipation of future declines in endowment, asset prices collapse on impact. The fear of price crashes helps to explain a high ex-ante risk premium because these crashes happen in sync with large declines in expected utility due to the impending consumption contraction. However, anticipated price crashes cannot be squared with a simultaneous availability of high leverage.

In order to account for high leverage in the model, disasters unfold as a sequence of independent negative endowment shocks, each with a probability of 18.5%. Baseline growth is 3.4%. Each negative shock causes a permanent loss of growth of -6.5% on top of the baseline - yielding negative growth of -3.1% in a standard recession. The probability of negative shocks and the size of the endowment declines are set to match the models' probability of a two-period or longer recession, i.e. $(1 - 0.185) \cdot 0.185^2 = 2.8\%$, and the average lost growth given two consecutive negative shocks, i.e. $(2 + 0.185)(-0.065) = 14.2\%$, to the findings of Nakamura et al. (2013). Since news of the disaster arrives piecemeal, the price of the collateral asset falls only gradually. However, the fall in expected utility caused by losses on leveraged real estate increases the value of funds following negative shocks, which helps justify a high risk premium. At the same time, the gradual fall in the price of the collateral asset allows the model to yield banking leverage of 22-to-1 during expansions.

I then use the calibrated rational expectations model to explore the consequences of mistakes in expectations. I demonstrate that in a calibrated model of banking the mechanics of leverage generate the amplification of mistakes. As illustrated in figure 1, when borrowers and lenders err by setting leverage too high, excessive leverage leads to a fire sale dynamic which amplifies the fall in the price of real estate after a negative shock hits the economy.

Contrary to the view that financial crises are the result of self-fulfilling prophecies, as presented in the seminal work of Diamond and Dybvig (1983), I argue that crises typically involve an element of self-defeating prophecies. As expectations shift from

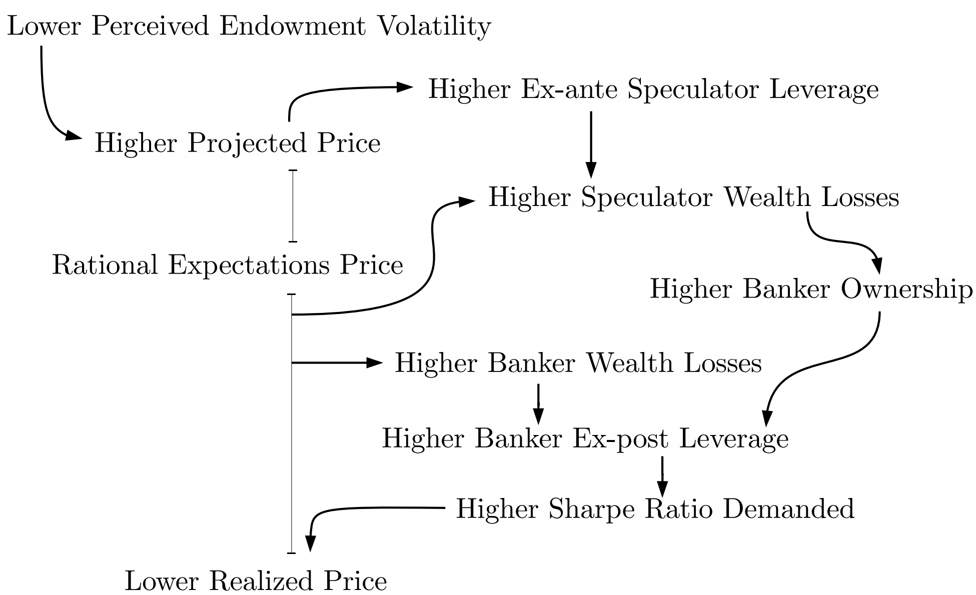

Figure 1: The mechanism generating self-defeating prophecies in phase I (while speculators retain enough equity to avoid default). The lower perceived volatility of the endowment causes an increase in the projected price for the down state, which makes speculators increase their leverage. This increases the speculator wealth loss after the true negative shock materializes. At this point the shock enters the primary feedback loop. Higher losses force the constrained speculators to sell more real estate to bankers, which forces banker to increase their leverage, making bankers demand a higher risk adjusted return, pushing the price of real estate lower and triggering more losses for speculators. The lower realized price also triggers the secondary feedback loop when it generates higher losses for banks, forcing them to increase their leverage, again pushing price lower.

the truth, the future realization changes as well since the change in expectations alters portfolio choices, which in turn change the future state of the economy. While the original mistake consists of only the deviation of expectations from the original equilibrium, the realized mistake adds a shift of the future realization that is due to the change in choices. I view a self-defeating prophecy as a condition in which the future realization shifts away from the original equilibrium in the opposite direction from that of expectations. I show that if all agents underestimate the potential for a decline in the labor endowment, then following a negative shock the market-clearing price of real estate falls further than it would in the absence of mistakes. In fact, up to two-thirds of the difference between the predicted and realized price can be ascribed to the endogenous increase in risk-taking.

The paper can be viewed as an extension of Nakamura, Steinsson, Barro, and Ursúa (2013) which aims to account not only for the risk premium but also for leverage and Sharpe ratio dynamics. It can also be viewed as an application of Fostel and Geanakoplos (2008) and Fostel and Geanakoplos (2012) to banking, where risk aversion heterogeneity replaces belief and endowment heterogeneity as the reason for leverage, and the aggregate risk has been calibrated to data. As in Greenwood, Hanson, and Jin (2016) and Bordalo, Gennaioli, and Shleifer (2016), the paper examines credit cycles under mistakes, but without specifying how mistakes are formed.

Alternatively, the paper can be viewed as a response to Gertler and Kiyotaki (2015), who explored how multiple Nash equilibria arise in a calibrated financial accelerator setting, and who argued that banking crises originate from sun-spot coordination. I consider the existence of such coordination devices to be less likely than small parameter mistakes, and offer a calibrated exploration of the possibility self-defeating, rather than self-fulfilling, prophecies play a significant role in generating banking crises.

2 The Model

2.1 The Environment

Time is discrete and infinite and states of the world are nodes in an unending binary tree. The set of all states of the world is denoted as S. The root of the tree is denoted as 0. All other states are denoted by strings of u and d corresponding to the moves up or down needed to reach the node from the root of the tree. Given a state s, let s_{-1} denote the previous state. Denote the set containing the two subsequent future states as $0_{+1} \equiv \{u, d\}$ for the root node and $s_{+1} \equiv \{su, sd\}$ for all other nodes. Let $t(0) \equiv 0$ and $t(s) = t(s_{-1}) + 1$.

The labor endowment, $w(s)$, varies exogenously. The model period is calibrated to annual frequency, growth has a constant base rate of $g_u = 3.4\%$ a year. Upward moves occur with probability $(1-\gamma) = 0.815$ and are equal to the base rate. Downward moves occur with probability $\gamma = 0.185$ and involve an additional contraction of $g_d = -6.5\%$.

For simplicity, growth becomes deterministic after period T at its expected rate:

$$\begin{aligned}
\Pr(sd|s) &= \gamma && \forall s \in S, \\
\ln w(s) &= \ln w(s_{-1}) + g_u && \forall s \in \{s|s = s_{-1}u, t(s) \le T\}, \\
\ln w(s) &= \ln w(s_{-1}) + g_u + g_d && \forall s \in \{s|s = s_{-1}d, t(s) \le T\}, \\
\ln w(s) &= \ln w(s_{-1}) + g_u + \gamma g_d && \forall s \in \{s|t(s) > T\}.
\end{aligned}$$

This process was chosen in order to reflect the movements of potential consumption, as measured by Nakamura, Steinsson, Barro, and Ursúa (2013) (see section 3 for details).

There are three kinds of agents: real estate speculators, bankers and depositors, with speculators being the least risk averse, depositors the most risk averse, and bankers in between. Variables associated with real estate speculators, bankers and depositors are denoted by r, b and d subscripts, respectively. All agents have Epstein-Zin preferences, and they share the same rate of intertemporal substitution:

$$\begin{aligned}
U_i(s) &\equiv \left[(1-\beta)u_i(s)^{1-\eta} + \beta\left(\mathbb{E}_{s' \in s_{+1}} U_i(s')^{1-\alpha_i}\right)^{\frac{1-\eta}{1-\alpha_i}}\right]^{\frac{1}{1-\eta}} && \forall i \in \{r, b, d\}, \\
\alpha_d &> \alpha_b > \alpha_r,
\end{aligned}$$

where the per-period utility for all agents is Cobb-Douglas in consumption, $c_i(s)$, and rented housing, $h_i(s)$:

$$u_i(s) \equiv c_i(s)^{1-\rho} h_i(s)^{\rho}.$$

The budget constraint for all agents, $i \in \{r, b, d\}$, is

$$c_i(s) + p_h(s)H_i(s) + R(s)h_i(s) \le W_i w(s) + (p_h(s) + R(s))H_i(s_{-1}) + B_i(s) - d_i(s)B_i(s_{-1})$$

where $c_i(s)$ is consumption, $p_h(s)$ is the price of housing, $H_i(s)$ is the stock of housing owned at the end of the period, $R(s)$ is the rental rate on housing, $h_i(s)$ is the size of the house the agent lives in, W_i is the labor share of the agent, $\sum_{i \in \{r,b,d\}} W_i = 1$, $w(s)$ is the aggregate labor endowment, $B_i(s)$ is the amount borrowed by the agent, and $d_i(s)$ is a state dependent delivery rate on past debt which we use to express all financial contracting. Without loss of generality, we assume speculators and depositors cannot trade in financial contracts directly. Define

$$B(s) \equiv \sum_{i \in \{r,b,d\}} B_i(s)$$

The financial market clearing thus states that

$$\begin{aligned}
B(s) &= 0 && \forall s \in S, \\
\sum_{i \in \{r,b,d\}} d_i(s)B_i(s_{-1}) &= 0 && \forall s \in S.
\end{aligned}$$

The only financial contracts we allow are one-period collateralized debt contracts. Each pair of the form (promise, collateral) is a distinct product, freely bought and sold in a contract-specific centralized market, and we assume all such markets are open. There is no cost to default other than handing over the collateral, and contracts can only be issued or held, but not shorted.

Since the states of the world are nodes in a binomial tree, any equilibrium allocation can be obtained using only the contract that promises to pay exactly the minimum value of the collateral. This contract is safe, as it provides the borrower with no incentive to default. However, following a downward move the borrower is left with nothing. Because this special contract generates a safe asset for the lender and a claim that pays only on upward moves to the borrower, and any collateralized debt contract is equivalent to some mix of these two, every feasible portfolio can be mimicked using only this contract.

In order to solve the model we focus only on this contract. For the most part, this is just a matter of the representation of portfolios. As long as lenders choose to hold the collateral asset directly, alongside safe debt, the model is silent on whether debt is actually safe or not, because that question does not really matter. Lenders are indifferent to replacing some of their holdings of the risky collateral asset and safe debt with an equivalent risky debt. In that case our choice to use just safe debt is a matter of one representation among many. However, when lenders choose not to hold the collateral asset directly, such freedom of interpretation is not available. In this case safe debt is a result of the model.

In the calibrated equilibrium banks hold real-estate in most states of the world, and so in most states their portfolio can be interpreted in terms of only risky mortgages, with no direct ownership of real-estate. The banks' real estate holdings reported in the paper can be thought of as a measure of the degree of risk embedded in their mortgage portfolios. In contrast, depositors choose to hold real-estate in only few states of the world. Most of the time they hold no real-estate and so their lending to banks provides no freedom of interpretation. In these states of the world, the equilibrium yields that the contract depositors sign with banks is structured as a safe deposit - fully collateralized by the banks' asset portfolio.

The special contract implies that delivery does not depend on the state, and therefore is just an interest payment:

$$d_i(su) = s_i(sd) \equiv 1 + r_i(s) \qquad \forall s \in S.$$

In addition, it implies that the amount of debt that can be issued against holdings of $H_i(s)$ is limited by the lowest future value of this stock:

$$(1 + r_i(s))B_i(s) \leq (p_h(sd) + R(sd))H_i(s) \qquad \forall s \in S.$$

Each type of agent gets a fixed share of the aggregate labor endowment, with depositors getting the lion's share. Define

$$H(s) \equiv \sum_{i \in \{r,b,d\}} H_i(s), \qquad h(s) \equiv \sum_{i \in \{r,b,d\}} h_i(s), \qquad c(s) \equiv \sum_{i \in \{r,b,d\}} c_i(s).$$

The stock of housing is fixed and the initial stock is set to one

$$H(0_{-1}) = 1.$$

Market clearing in the housing rental market is expressed by:

$$H(s_{-1}) = h(s) \qquad \forall s \in S.$$

Market clearing in the housing ownership market is expressed by:

$$H(s_{-1}) = H(s) \qquad \forall s \in S.$$

Market clearing in the goods market is expressed by:

$$c(s) = w(s) \qquad \forall s \in S.$$

2.2 First-Order Conditions

Denoting the Lagrange multiplier on agent i's budget constraint as $\Pr(s)\lambda_i(s)$, the first-order conditions for consumption and housing are:

$$c_i(s): \qquad (1-\rho)(1-\beta)U_i(s)^\eta \frac{u_i(s)^{1-\eta}}{c_i(s)} = \lambda_i(s)$$

$$h_i(s): \qquad \rho(1-\beta)U_i(s)^\eta \frac{u_i(s)^{1-\eta}}{h_i(s)} = R(s)\lambda_i(s)$$

Taken together they imply:

$$\frac{R(s)h_i(s)}{\rho} = \frac{c_i(s)}{1-\rho} \qquad \forall i \in \{r, b, d\},\ \forall s \in S$$

Summing over agents, we obtain

$$R(s) = \frac{\rho}{1-\rho}\frac{c(s)}{h(s)} = \frac{\rho}{1-\rho}w(s) \qquad \forall s \in S.$$

Plugging back into the definition of $u_i(s)$ yields:

$$u_i(s) = h_i(s)w(s)^{1-\rho}.$$

The marginal value of goods is:

$$\lambda_i(s) = (1-\rho)(1-\beta)\left(\frac{U_i(s)}{h_i(s)}\right)^\eta w(s)^{(1-\rho)(1-\eta)-1}.$$

Define the stochastic discount factor as

$$M_i(s) = \beta U_i(s)^{-\alpha_i}\frac{\lambda_i(s)}{\lambda_i(s_{-1})}U_i(s_{-1})^\eta \left(\mathbb{E}_{s' \in s_{-1+1}}\left[U_i(s')^{1-\alpha_i}\right]\right)^{\frac{\alpha_i-\eta}{1-\alpha_i}}$$

Denoting the multipliers on the agents' collateral constraint as $\Pr(s)\lambda_i(s)\mu_i(s)$, their first-order conditions for owned housing and debt are given by:

$$H_i(s): \qquad p_h(s) = \mathbb{E}_{s' \in s_{+1}}\left[M_i(s')\left(p_h(s') + R(s')\right)\right] + \mu_i(s)(p_h(sd) + R(sd)),$$

$$B_i(s): \qquad 1 = (1 + r_i(s))\left(\mathbb{E}_{s' \in s_{+1}}\left[M_i(s')\right] + \mu_i(s)\right).$$

The bankers need to decide how to allocate loans between real estate speculators and depositors, and therefore their first-order conditions on the two forms of debt are given by:

$$B_j(s): \qquad 1 = (1 + r_j(s)) \left(\mathbb{E}_{s' \in s_{+1}} \left[M_b(s')\right] + \mu_b(s)\right) \qquad \forall j \in \{s, d\},$$

which reveals that there is no intermediation spread, i.e.

$$r_i(s) = r(s) \qquad \forall i \in \{r, b, d\}.$$

In addition to these conditions, solving the model numerically requires obtaining an equation set for each time T state that does not depend on future values. The derivation is relegated to Appendix A.

2.3 Collateralized Lending Terms

To intuitively understand the model, it is useful to frame it in the conventional terms used to describe collateralized lending. Define Loan-to-Value as:

$$LTV(s) \equiv \frac{p_h(sd) + R(sd)}{(1 + r(s))p_h(s)},$$

Define Excess Collateral as the slack in the collateral constraint:

$$EC_i(s) \equiv LTV(s)p_h(s)H_i(s) - B_i(s).$$

Define Cash-On-Hand as:

$$\begin{aligned} COH_i(s) \equiv\ & (p_h(s) + R(s) - (1 + r(s_{-1}))LTV(s_{-1})p_h(s_{-1}))H_i(s_{-1}) \\ & + W_i w(s) + (1 + r(s_{-1}))EC_i(s_{-1}). \end{aligned}$$

Substituting $c_i(s) = \frac{1-\rho}{\rho}R(s)h_i(s)$, the budget constraint can be rewritten as:

$$\frac{1}{\rho}R(s)h_i(s) + (1 - LTV(s))p_h(s)H_i(s) + EC_i(s) \leq COH_i(s),$$

and the collateral constraint as:

$$EC_i(s) \geq 0.$$

Define $EC(s) = \sum_{i \in \{r,b,d\}} EC_i(s)$. The market-clearing condition for debt becomes

$$EC(s) = LTV(s)p_h(s)H(s).$$

Note that this statement of budget constraints relies on the prediction of prices being correct, so that there are no losses associated with debt that was set up to be safe. When we consider arbitrary initial endowments there can be losses on such debts and budget constraints need to account for them. The statement of the required adjustments is relegated to Appendix B.

Leverage is defined as:

$$LEV_i(s) = \frac{p_h(s)H_i(s)}{p_h(s)H_i(s)(1 - LTV(s)) + EC_i(s)} \qquad \forall i \in \{r, b, d\}.$$

For banks observable leverage includes also the safe loans taken by speculators as assets. It is defined as:

$$LEV_{b+r}(s) = \frac{p_h(s)(H_b(s) + LTV(s)H_r(s)) - EC_r(s)}{p_h(s)H_b(s)(1 - LTV(s)) + EC_b(s)}.$$

The Risk Premium is defined as:

$$RP(s) = \mathbb{E}_{s' \in s_{+1}} \frac{p_h(s') + R(s')}{p_h(s)} - r(s).$$

And finally, the Sharpe Ratio is defined as:

$$SR(s) = \frac{RP(s)}{\left(\mathbb{E}_{s' \in s_{+1}} \left[\frac{p_h(s')+R(s')}{p_h(s)} - r(s) - RP(s)\right]^2\right)^{\frac{1}{2}}}.$$

3 Calibration

The challenge of Mehra and Prescott (1985)'s risk premium puzzle can be formulated in terms of a growth process and a risk aversion parameter: given an endowment growth process consistent with the data, how unreasonable must the risk aversion be in order to match the observed average excess returns on risky assets? I follow a similar approach in accounting for the cyclical variation in Sharpe ratios observed by Lustig and Verdelhan (2012). Sharpe ratios, defined as expected excess returns over their standard deviation, are a better target for asset pricing than the risk premium since they account for objective differences in risk exposure across assets and time periods. The challenge is to match Sharpe ratios in both expansions and contractions with reasonable assumptions about the wealth distribution across agents with different levels of risk aversion.

Table 1 presents the parameters of the model which are calibrated directly from the data. The growth process is set to match the findings of Nakamura et al. (2013). In particular, the probability of each negative shock is set to $\gamma = 18.5\%$, so that the probability of two or more consecutive negative shocks is $(1 - \gamma)\gamma^2 = 2.8\%$.[1] The lost growth due to each negative shock is set to $g_d = -6.5\%$, so that the expected lost growth given two negative shocks is -14%. I limit the number of consecutive negative shocks to $T = 3$ since a fourth consecutive shock would occur only once every 1047 years, and so cannot be verified using data. The expected growth rate is set to $g = 2.2\%$ in order to match the post-1973 average growth rate of potential consumption. Together these imply $g_u = g - \gamma g_d = 3.4\%$.

The inverse of the elasticity of intertemporal substitution is set to $\eta = 0.1$ in order to prevent the interest rate from responding too strongly to changes in the level of

[1] For details, see Nakamura et al. (2013) footnote 21.

Parameter	Value	Role	Target	Source
T	3	# of periods with growth uncertainty.	T=4 cannot be observed in the data.	Assumed.
γ	18.5%	Probability of a negative shock.	$(1-\gamma)\gamma^2 = 2.8\%$	Nakamura et al. (2013).
g_d	-6.5%	Growth loss on each negative shock.	$(2+\gamma)g_d = -14\%$	Nakamura et al. (2013).
g	2.2%	Baseline labor endowment growth rate.	2.2%	Nakamura et al. (2013).
ρ	17%	Share of rent in expenditures.	17%	NIPA Table 2.3.5 line 15/ line 1
η	0.1	Controls the elasticity of intertemporal substitution	Small interest rate responses	Assumed.
β	0.928	Time discount, controls the ratio of the price of real estate to the rent under no uncertainty.	$\frac{p_h}{R} = 16.7$	Z.1 Table B.1 line 3 + line 8 / NIPA Table 2.3.5 line 1

Table 1: Calibration of parameters with direct data counterparts.

aggregate uncertainty. The time discount factor, $\beta = 0.928$, is calibrated so that when there is no uncertainty p_h/R will match the average ratio of the value of all real estate held by households and non-corporate enterprises to the aggregate rent in the U.S. economy from WWII to 2015. The share of rent in expenditures is calibrated to $\rho = 17\%$, in order to match the average ratio of expenditure on housing and utilities to personal consumption expenditures during the same period.

The premise of the model is that bankers have an intermediate level of risk aversion. Therefore, I set the risk aversion of speculators to be very low ($\alpha_r = 0.1$), and the risk aversion of depositors to be very high ($\alpha_d = 50$).

Further, I assume that speculators and bankers have more financial wealth than labor wealth. In particular, the amount of financial wealth they start with allows them to buy $H_b(0) = H_r(0) = 0.04$ without using leverage. However, their labor endowment shares are $W_b(0) = W_r(0) = 0.02$. Using leverage these levels of wealth allow banks and speculators to hold all the real estate, both in the root node ($s = 0$) and after one negative shock ($s = d$). This is required if prices are to be sensitive to the wealth distribution, because when depositors are in the market for real estate they stabilize prices to a great extent.

Lustig and Verdelhan (2012) report that post-WWII U.S. stocks display a Sharpe ratio of 0.66 during recessions, and 0.38 during expansions. Moreover, they report that the Sharpe ratio hits a maximum value of 0.82 four quarters into a recession and a minimum of 0.14 three quarters into an expansion. While the model does not include stocks, if it did they would be priced according to the same Sharpe ratio as real estate. The aim is to match the value of 0.38 observed during expansions. The

Parameter	Value	Role	Target	Source
α_d	50	Depositor relative risk aversion.	$SR(dd) > 1$	Assumed.
α_r	30	Speculator relative risk aversion.	$EC_r(d) = 0$	Assumed.
α_b	3	Banker relative risk aversion.	$SR(0) = 0.38$	Lustig and Verdelhan (2012).
W_d	0.96	Depositor labor endowment share.	$H_d(d) = 0 < H_d(dd)$	Assumed.
W_b, W_r	0.02	Banker and speculator labor endowment share.	$\frac{1}{2}(1 - W_d)$	Assumed.
$H_b(0_{-1}), H_r(0_{-1})$	0.04	Banker and speculator initial housing endowment share.	$H_r(0_{-1}) = 2W_r(0_{-1})$, $H_b(0_{-1}) = 2W_b(0_{-1})$	Assumed.
$H_d(0_{-1})$	0.92	Depositor initial housing endowment share.	$1 - H_r(0_{-1}) - H_b(0_{-1})$	Assumed.

Table 2: Calibration of parameters implied by the equilibrium.

spread between 0.82 and 0.14 is interpreted as an appropriate lower bound for the spread the model should generate between $SR(u)$ and $SR(d)$. The reason behind this is that the difficulties in pinpointing the onset of expansions and contractions lead to a measurement bias which reduces the measured spread relative to the true spread.

With the assumptions in place concerning the distribution of wealth and the levels of risk aversion among speculators and depositors, I find that a relative risk aversion of $\alpha_b = 3$ for a banker matches the Sharpe ratio in the root node to the one observed by Lustig and Verdelhan (2012) in expansions, i.e. $SR(0) = 0.38$.

4 Results

4.1 Results under Rational Expectations

Table 3 presents the dynamics of Sharpe ratios over the first two periods of the model. It can be seen how the risk-return trade-off shifts as a result of shifts in the composition of real estate ownership. The initial endowment of the economy is set to 10 to mirror the 10 trillion dollars of personal consumption expenditures in 2008, just prior to the financial crisis.

The model generates a drop of -6% in the price of real estate following the first negative shock, followed by an additional 3.3% drop following the second shock. Together they are comparable to the 7.7% drop observed in the value of real estate according to the Flow of Funds data for 2008-2010, but fall short of the 21% drop observed in the Case-Shiller 20-City Composite Home Price Index over this period. This could suggest that the model misses a long run effect on the risk premium, but it can also

s	$w(s)$	$p_h(s)$	$SR(s)$	$H_r(s)$	$H_b(s)$	$LEV_r(s)$	$LEV_b(s)$	$LEV_{b+r}(s)$
0	10.0	35.0	0.382	66%	34%	14.2	7.7	21.8
u	10.3	36.3	0.093	90%	10%	13.7	1.9	18.5
d	9.7	32.9	1.027	30%	70%	27.9*	22.9	32.2
uu	10.7	37.1	0.022	100%	0%	13.0	0.0	18.6
ud	10.0	34.6	0.358	54%	46%	23.0*	9.5	20.5
du	10.0	34.7	0.259	63%	37%	21.8*	5.9	15.6
dd	9.4	31.8	1.275	28%	57%	47.6*	47.6*	70.8

Table 3: Wealth and leverage driven Sharpe ratio dynamics. $_{-1}$ - denote a binding collateral constraint, so the agent would have preferred a higher level of leverage (at the safe interest rate).

suggest that the crisis of 2008-2010 was an occurrence around the rational expectations equilibrium, rather than on it.

In the rational expectations model, a drop in real estate prices is not merely a reflexion of the decline in the aggregate endowment, but is also driven by Sharpe ratio dynamics. In terms of amplification the model produces a price in $s = d$ that is 9.8% below the price in $s = u$ even thou the rent is lower by 6.5%. This movement in the price to rent ratio is coupled with the movement in Sharpe ratios, which implies that excess returns are predictable.

In the root node, speculators hold 66% of the stock of housing using leverage of 14.2. Bankers hold the other 34% using leverage of 7.7. When the first negative shock hits, speculators become constrained and they must sell 36% of the housing stock to bankers. Because banks need to absorb this stock while also incurring losses, the leverage at which they hold real estate must increase to 22.9, which makes them demand higher expected returns. However, such increases in leverage can only go so far. Following a second negative shock banks also hit their collateral constraints and depositors must absorb 15% of the housing stock. The response of speculators and banks to negative shocks has some intuitive appeal, in that they try to hold on to losing assets for as long as they can.

In the model speculators and banks hold all the real estate until the second negative shock, so most of the time the entire stock of housing is held at high leverage. In reality, far from all real estate is held at high leverage. In the 2013 American Housing Survey, 76% of owner-occupied homes were reported to be leveraged less than 5-to-1. The fact that in reality many owners are willing to hold on to their homes even when prices are high, and do not seek to extend their holdings when prices are low, suggests that highly risk averse individuals may also be motivated by benefits associated with owning the house they occupy. My model should thus be interpreted as describing only leftover real estate, i.e. those units not occupied by their owner.

Tables 4 and 5 attempt to answer, from the depositors' and bankers' point of view, respectively, the question of how a high risk premium can be reconciled with small declines in prices. The risk premium at $s = 0$ originates from the fact that the subjective discount factor of both real estate speculators and bankers includes a state-

s	$RP(s)$	$U_r(s)^{-\alpha_r}\lambda_r(s)$	$U_r(s)$	$\lambda_r(s)$	$COH_r(s)$	$\mu_r(s)$
0	1.51%	0.057	0.223	0.049	1.68	0.00
u	0.27%	0.048	0.256	0.042	2.73	0.00
d	3.66%	0.105	0.167	0.088	0.35	0.34
uu	0.06%	0.046	0.280	0.040	3.31	0.00
ud	0.95%	0.058	0.180	0.048	0.87	0.15
du	0.68%	0.054	0.188	0.046	1.09	0.11
dd	3.45%	0.125	0.150	0.103	0.19	0.57

Table 4: real estate pricing from the speculators' point of view.

s	$RP(s)$	$U_b(s)^{-\alpha_b}\lambda_b(s)$	$U_b(s)$	$\lambda_b(s)$	$COH_b(s)$	$\mu_b(s)$
0	1.51%	4.43	0.216	0.044	1.68	0.00
u	0.27%	3.19	0.233	0.040	2.24	0.00
d	3.66%	7.03	0.203	0.059	1.02	0.00
uu	0.06%	3.07	0.236	0.040	2.22	0.00
ud	0.95%	3.84	0.221	0.041	1.95	0.00
du	0.68%	2.88	0.243	0.041	2.51	0.00
dd	3.45%	17.82	0.164	0.079	0.38	0.44

Table 5: real estate pricing from the bankers' point of view.

dependent factor of $U_i(s)^{-\alpha_i}\lambda_i(s)$, which places more than double the weight to $s = d$ over $s = u$. This explains why the price is 1.51% below the expected discounted value, which takes only probabilities as weights.

At the root node both agents are unconstrained, $\mu_r(0) = 0$ and $\mu_b(0) = 0$, so both agents are pricing the asset, and we could explain the risk premium from both points of view. From the speculator point of view changes in $U_r(s)$ are not important because $\alpha_r = 0.1$. What matters are changes in the future marginal value of funds, $\lambda_r(s')$, as they are driven by changes in the future expected return. The doubling in the stochastic discount factor of the real estate speculator between $s = u$ and $s = d$ is driven by the variability of the risk premium between these two states. That implies that the main reason real estate speculators demand such a high risk premium in $s = 0$ is that they foresee an even higher risk premium in $s = d$. But since speculators do not price the asset in $s = d$ (because $\mu_r(d) > 0$), we cannot use their pricing equation to explain the high risk premium at $s = d$.

From the point of view of bankers at $s = 0$ changes in the future marginal value of funds across future states only tell half of the story, with changes in expected utility contributing the other half. Since expected utility discounts future utility, to understand the expected utility losses in $s = d$ we must look into $s = du$ and $s = dd$. As we can see, $s = du$ vs. $s = dd$ features a 6-fold variation in the stochastic discount factor, mostly driven by expected utility changes. Furthermore, it is the fear of $U_b(dd)$ that depresses $U_b(d)$. So directly through discounting and indirectly through the high risk premium it generates in $s = d$, the expected utility of bankers in $s = dd$ explains almost all of the risk premium in $s = 0$.

Recall that $s = dd$ is a state where bankers and speculators are both constrained and some of the asset must be sold to depositors. It is in that sense that $s = dd$ is a state of financial system dysfunction. What scares bankers into demanding a high risk premium in normal times, namely $s = 0$, is the fear of utility losses, caused by binding collateral constraints, in the remote event of financial system dysfunction.

4.2 Results around Rational Expectations

The exercise we use to investigate the mistake amplification properties of the model starts by assuming that all agents share a mistaken perception of g_d. Importantly, agents don't just assign zero probability to the true value, they must fail to consider the true value altogether. This is required in order to generate state-by-state mistakes in asset pricing. Our interest is in the amplification of mistakes, but we cannot study this issue without an original mistake to amplify. And if agents' were allowed to consider the true future states, they would have no problem in predicting correct prices state-by-state. Because the agents have full rationality, to generate even the smallest original mistake we must distract agents from even thinking about the true states. We furthermore need to identify states not by their full list of values for exogenous variables, but by their name. For agents to be wrong about what happens in $s = d$, we must maintain that $s = d$ is still the same state, in some sense, under the mistaken perception of parameters, even though it is a different state in the traditional sense that views a state as just the list of values for the exogenous variables.

Consider then a mistake in the perception of g_d which is shared by all agents. Let the true value, taken to be the calibrated value, be denoted by $g_d^{\odot}$. Let any mistaken perception be denoted as $g_d^{\sim}$. Denote the states in the true model as $s = s(g_d^{\odot})$ and the states generated by the modification of beliefs as $s = s(g_d^{\sim})$. Thus $s = 0(g_d^{\sim})$ is the root node in the belief modified equilibrium and $s = d(g_d^{\sim})$ denotes the state predicted to occur after a negative shock under the modified beliefs.

Holding everything else fixed,[2] the model provides a mapping f between different beliefs about g_d and agent's portfolio choices in the altered root state such that, [3]

$$f(g_d^{\sim}) = (H_i(0(g_d^{\sim})), EC_i(0(g_d^{\sim})), LTV(0(g_d^{\sim})), r(0(g_d^{\sim}))_{i\in\{r,b,d\}}.$$

The future effects of these mistakes in perception can be examined by taking them to be the starting portfolios for the original $s = d$ state. In particular, let state $s = d(g_d^{!})$ be the root node of an equilibrium with $T = 2$, a shared belief that g_d is at its true value, i.e. $g_d^{!} = g_d^{\odot}$, and a labor endowment that equals the true $s = d$ labor endowment, i.e. $w(d(g_d^{!})) = w(d(g_d^{\odot}))$, though with initial asset endowments set to

$$(H_i(d(g_d^{!})_{-1}), EC_i(d(g_d^{!})_{-1}), LTV(d(g_d^{!})_{-1}), r(d(g_d^{!})_{-1}))_{i\in\{r,b,d\}} = f(g_d^{\sim}).$$

Because these initial portfolios differ from those that would prevail under correct expectations, the equilibrium that arises under mistakes in g_d, i.e. the one arising in

[2] All parameters other than g_d are held fixed, with the exception of g_u, which maintains $g_u = g - \gamma g_d$.

[3] To ensure that no large wealth effects result from these changes in perceptions, f was generated around the no-trade equilibrium that yields the same allocations as the calibrated equilibrium.

state $s = d(g_d^!)$, differs from the one that arises under correct expectations, i.e. in state $s = d(g_d^{\odot})$.

For any equilibrium variable $x(d)$, the Prediction Error Amplification (PEA) generated by the mistaken belief $g_d^{\sim}$ is:

$$PEA(x(d), g_d^{\sim}) = \frac{x(d(g_d^{\sim})) - x(d(g_d^!))}{x(d(g_d^{\sim})) - x(d(g_d^{\odot}))},$$

and the share of the observed mistake that is due to self-defeating prophecies is $1 - 1/PEA$.

When $PEA > 1$, prophecies are self-defeating and the environment is *hostile* in that it makes agents appear to be less informed than they really are. When $PEA \in [0, 1)$ prophecies are self-fulfilling and the environment is *friendly* in that it makes agents appear to be more informed than they really are.

The left panel of Figure 2 contrasts the predicted price of housing in $s = d(g_d^{\sim})$ with the resulting realization in $s = d(g_d^!)$. The true value of g_d is taken to be -6.5%, so that as we move from left to right in each panel of Figure 2 we are considering the effects of increasingly larger underestimates of the risk in $s = d$.

The main result of this section is that realized real estate prices, $p_h(d(g_d^!))$, are declining in $g_d^{\sim}$ rather than remaining fixed or increasing. This can be seen in the dashed red line in the left panel of Figure 2. Thus, mistakes in expectations in $s = 0(g_d^{\sim})$ affect the price of real estate in a way that increases the distance between predictions and realizations. Prophecies are therefore self-defeating in the sense that part of the reason for the lower realization is the higher prediction. Incorrect predictions thus shift reality away from the predicted value, so that mistakes appear larger than they really are.

The right panel of Figure 2 provides a quantification of the Prediction Error Amplification for the price of housing in $s = d$. We see that for small mistakes PEA can exceed three. That is, up to two-thirds of the observed mistake in the prediction of real estate prices is created endogenously in the model.

To gain an understanding of this result it is useful to think in terms of Sharpe ratios, the distribution of real estate ownership, and cash-on-hand. Figure 3 presents the predicted vs. realized values of these key variables in $s = d$. As can be seen, mistakes in the prediction of real estate prices are driven by similar failures in the prediction of Sharpe ratios, rather than the prediction of interest rates.

In turn, the increase in the Sharpe ratio originates from changes in real estate ownership. As can be seen in the three lower panels of Figure 3, as we move from left to right the composition of real estate ownership initially shifts sharply towards the banks. The larger the mistake, the more real estate ends up being held by the banks, which demand a higher Sharpe ratio than speculators in order to increase their holdings. This is the dynamic described in figure 1.

But the dynamic changes as we consider larger mistakes. Around $g_d^{\sim} = -0.064$ speculators run out of equity and start defaulting on their mortgages. This implies that they do not suffer additional losses as $g_d^{\sim}$ increases and therefore their ownership share stabilizes. The driver for lower prices switches to increased bank leverage due to increased bank losses. This dynamic is illustrated in figure 4.

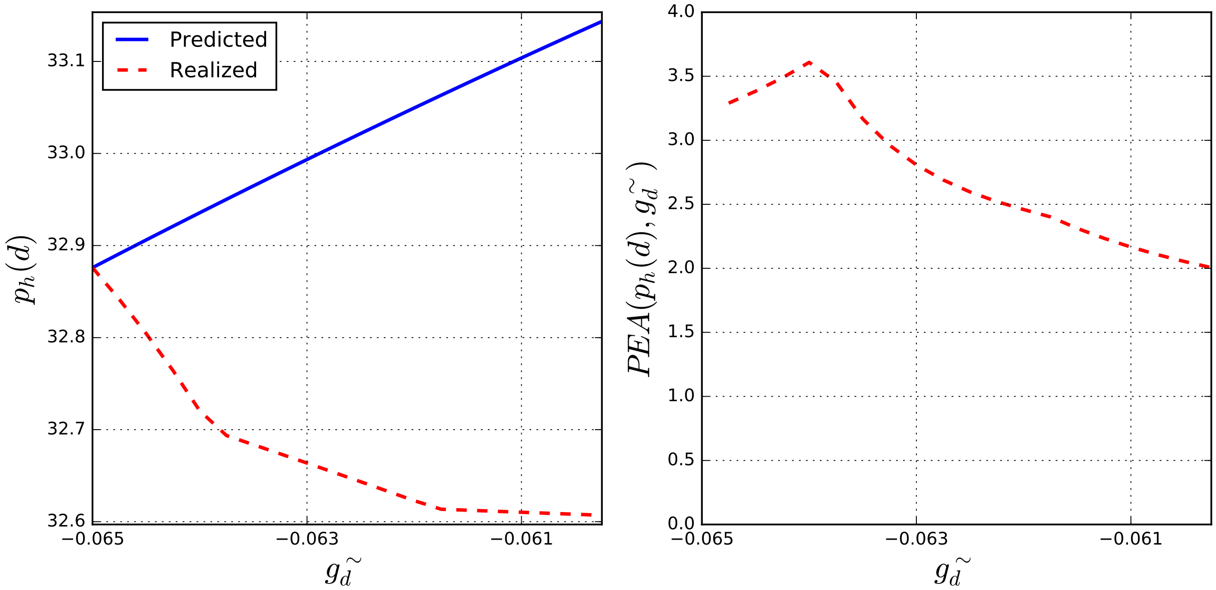

Figure 2: Left panel: Predicted and realized price of housing in state $s = d$ to mistakes made in $s = 0$ as to the value of g_d. On the x axis are the different values for $g_d = g_d^{\sim}$ that agents believe ($g_d^{\sim} = -6.5\%$ is assumed to be the truth). Solid blue presents the implied prediction made in $s = 0$, namely $p_h(d(g_d^{\sim}))$. Dashed red presents the realized market-clearing outcome once $s = d$ is realized, namely $p_h(d(g_d^{!}))$. Right panel: Prediction Error Amplification for the price of housing in $s = d$ as a function of the $g_d^{\sim}$.

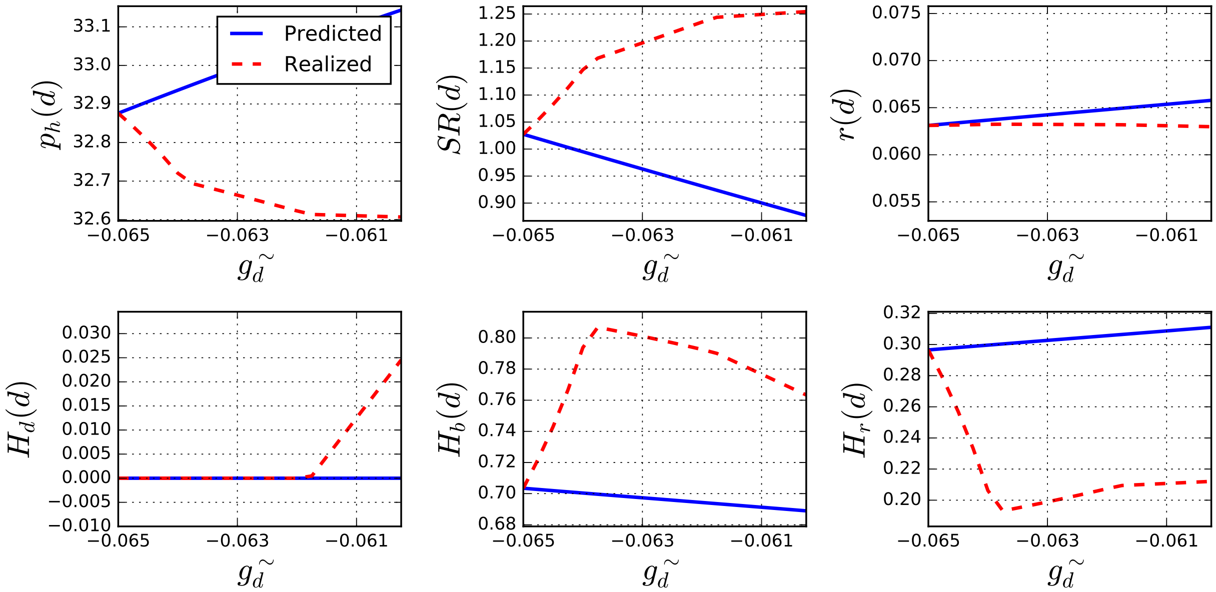

Figure 3: Predicted and realized responses of key $s = d$ variables, showing how prediction error amplification is driven by Sharpe ratio dynamics, which in turn is driven by a shift in the ownership of real estate.

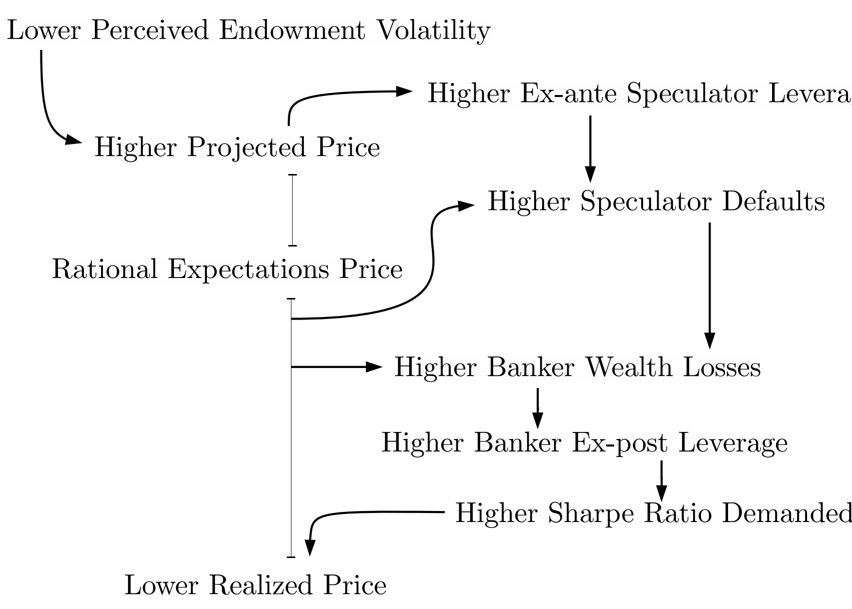

Figure 4: The mechanism generating self-defeating prophecies in phase II (while speculators exhausted their equity but depositors are not yet buying). Higher losses forces bankers to increase their leverage, making bankers demand a higher risk adjusted return, pushing the price of real estate lower and triggering more losses for banks.

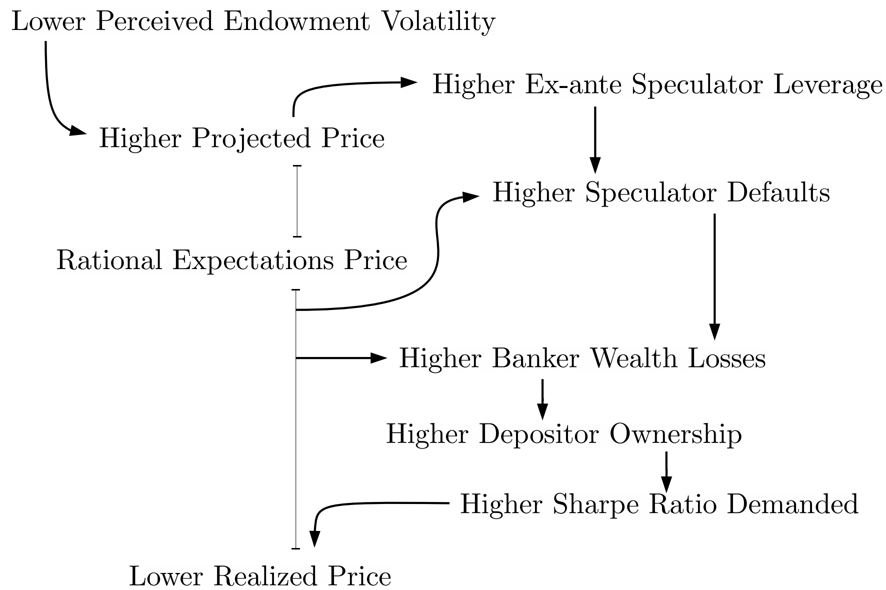

Figure 5: The mechanism generating self-defeating prophecies in phase III (depositors buying). Higher losses forces banker to sell more of the asset pool to depositors, which demand a lower price.

Around $g^{\sim}_d = -0.062$ this dynamic changes again when the Sharpe ratio hits a level high enough to induce depositors to step in and start buying real estate. This last dynamic is illustrated in figure 5. At this point while prices continue to drop qualitatively, their decline is very mild. This is because once depositors become willing buyers, they largely stabilize prices. In this model, this is the end of the fire sale. Even though banks continue to dump real estate, because prices are fairly stable there is not much of a feedback loop generating further losses for the banks. From that point onward, the Prediction Error Amplification declines to below 2.5.

Figure 6 shows the changes in leverage in $s = 0(g^{\sim}_d)$ which lead to the changes in realizations in $s = d$. We see that speculators increase their leverage in response to the underestimate of endowment risk while banks decrease theirs. However, the joint effect is to increase the measured leverage of the banking sector, indicating that, taken as a whole, the financial system becomes more fragile.

The changes in leverage in $s = 0$ drive the sharp decline in the speculators' cash-on-hand, seen in right panel of figure 7, which is responsible for generating Prediction Error Amplification exceeding 3. The sharp drop in $COH_r(d(g^{!}_d))$ is then halted by the exhaustion of speculators' excess collateral. Any further declines in real estate prices do not lead to any more losses to speculators since they are able to walk away from underwater mortgages without incurring any cost. However, as the banks' losses continue to accumulate they are driven to demand an even higher Sharpe ratio in order to compensate for being forced to increase the leverage of their remaining wealth. This dynamic is nonetheless strong enough to keep the Prediction Error Amplification above 2.5.

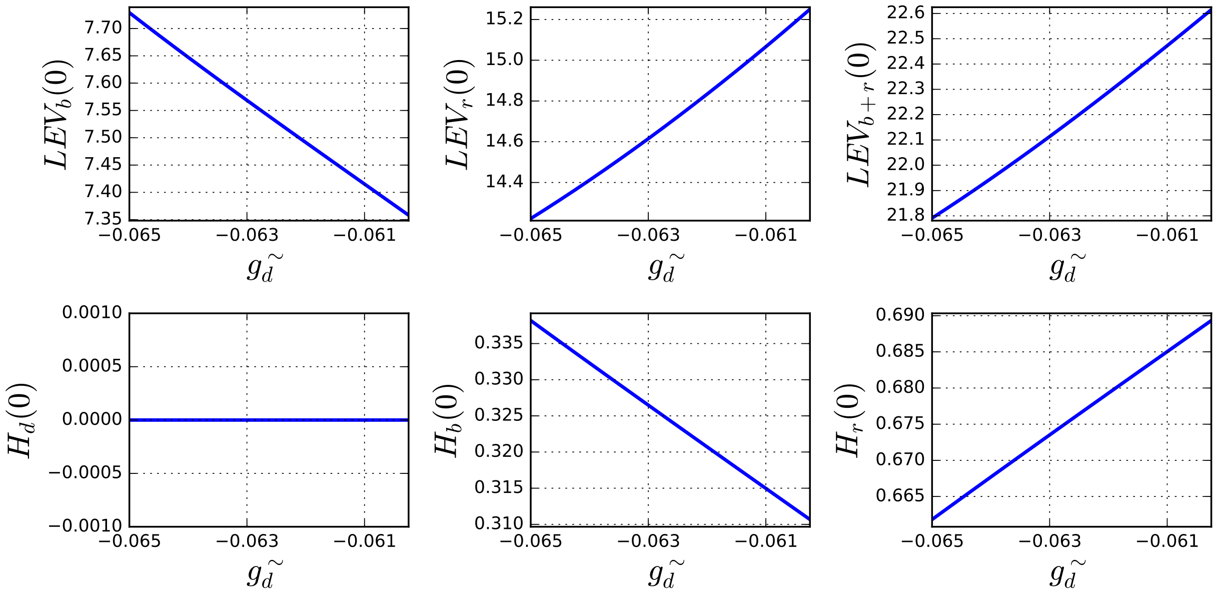

Figure 6: The response of agents' state $s = 0$ leverage and real estate holdings to mistakes in g_d. $LEV_b(0)$ is banking leverage when only risky assets are counted. $LEV_r(0)$ is speculators' leverage. LEV_{b+r} is a measure of banking leverage that also counts safe loans to speculators as assets.

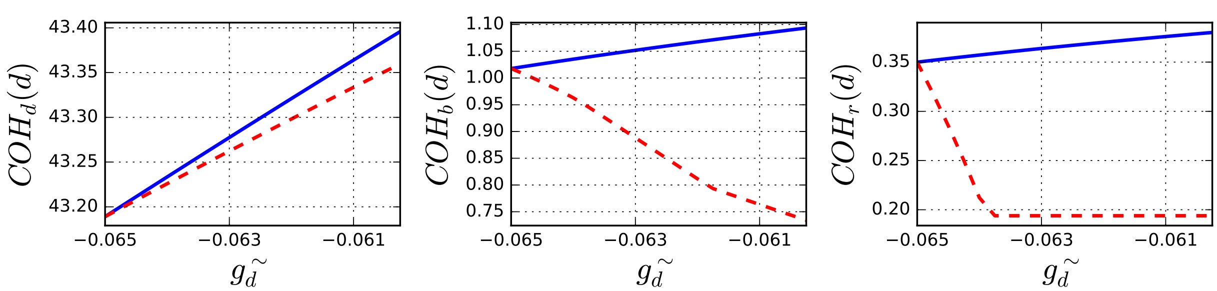

Figure 7: The response of agents' state $s = d$ Cash On Hand. The leverage shift implies more of the initial losses accrue to real estate speculators, but banks balance sheets also suffer from the increased fire sales. Losses on mortgages that were perceived as safe visible as a flattening of the $COH_r(d)$ curve around $g_d^{\sim} = -6.4\%$.

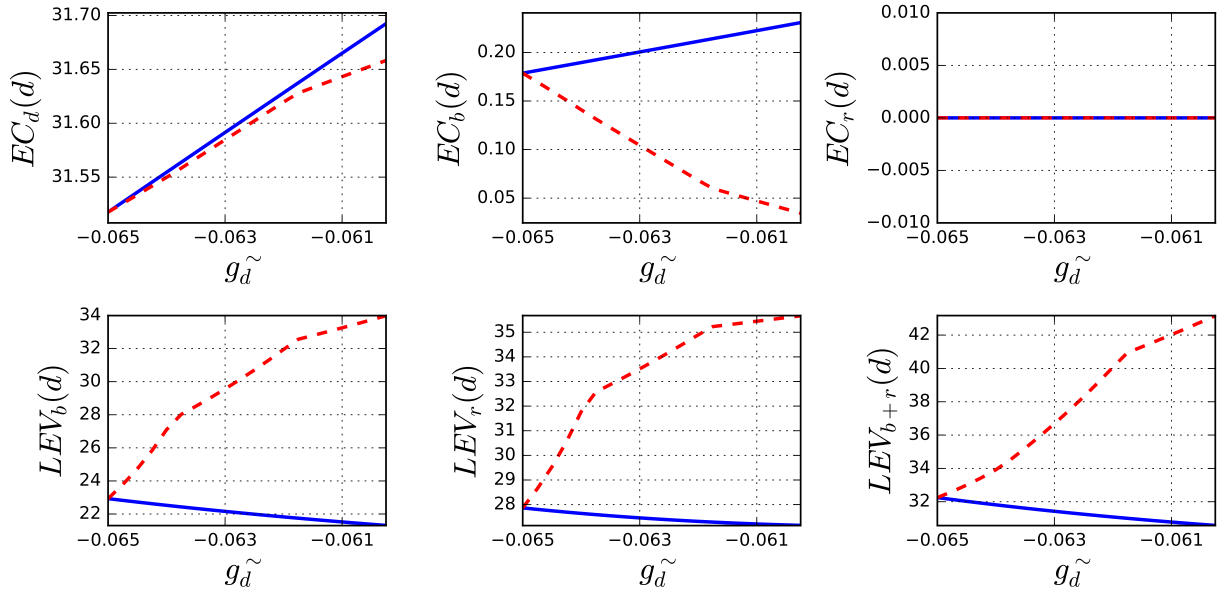

Figure 8: The response of agents' state $s = d$ leverage. Speculator leverage is constrained ($EC_r(d) = 0$) and it increases as a result of higher $LTV(d)$, in turn driven by the lack of strong effects on $p_h(dd)$. Banker leverage increases as a response to (and justification of) higher Sharpe ratios.

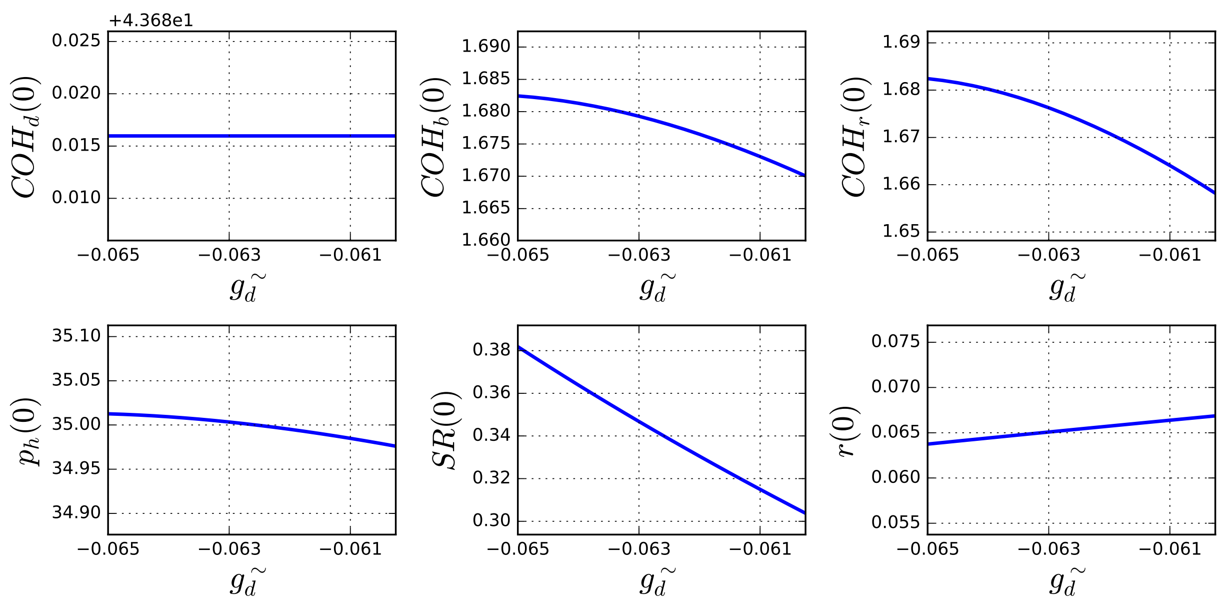

Figure 9: Wealth effects in $s = 0$ are a distraction. They are kept small by assuming initial asset endowments from assets are such that no trade is needed in $s = 0$

Figure 8 presents the response of leverage in $s = d$. Bankers and real estate speculators respond to the lower prices by seeking to increase leverage. Their loss of wealth also forces them to increase leverage in order to maintain the same holdings. But speculators are already at their collateral constraint, as evident by the zero excess collateral in the upper right panel. Their increase in leverage is cause by an increase in the available Loan To Value, which is in turn caused by the relative stability of $p_h(dd(g^!_d))$ in the face of drops in $p_h(d(g^!_d))$.

Figure 9 presents the response of COH in $s = 0$. For the logic of self-defeating prophecies to be visible it is important to control the size of wealth effects at the root node. As perceived uncertainty changes it causes immediate changes in asset prices, which cause wealth to shift between agents. In particular, since real estate speculators are able and willing to hold more of the stock of real estate there is a commensurate drop in the Sharpe ratio pushes the price of real estate higher. But the lower left panel shows that the price of real estate falls by just a bit. This is due to the interest rate effect being stronger than the risk premium effect, even thou the elasticity of intertemporal substitution was set to 10. The mild change in the price implies that there are only mild losses to the speculators and bankers hold the real estate as their initial endowment at $s = 0$. The impact of these losses is further reduced by using initial endowments that require no trade at the calibrated rational expectations model. This means that the relative wealth of speculators and bankers is only affected by the small increase in speculator holdings of real estate, now bought at a slightly lower price.

5 Discussion

Even staunch defenders of rational expectations concede that agents make small mistakes around the rational expectations equilibrium. I examine the dynamics generated by such mistakes in the context of banking and show that they are more problematic than elsewhere. When errors involve underestimation of aggregate risk, real estate speculators will reposition themselves to take on more of the aggregate risk, exposing the banks that lend to them to higher defaults.

While the argument is simple enough, introducing it into a model requires starting from a rational expectations benchmark that features an agent who looks like a bank. Moreover, the benchmark must generate a strong link between wealth distribution and asset pricing. And since banks must borrow from one agent and lend to another, at least three types of agents are required. With a state space that includes the wealth and leverage distribution over three types of agents, it is difficult to solve for a fully recursive equilibrium. However, much of the insight is available from a model in which uncertainty eventually dies out, such as the one we have examined.

A limitation of the model is that while it can generate the level of variation in the Sharpe ratio that we see in the data, it fails to translate that into the implied asset price variation, since the latter involves not only the current Sharpe ratio but also its future path. Once we account for changes in interest rates and volatility expectations, aggregate asset price variation can be thought of as driven by Sharpe ratio variation and its intertemporal correlation.

One of the reasons for the attraction of heterogeneous risk aversion is that it can, in theory, provide the basis for modeling both Sharpe ratio variation and its intertemporal correlation. This is because the wealth lost by less risk averse agents can lead to a prolonged period of high Sharpe ratios, while they rebuild their wealth. The challenge however remains that in order to study this link a recursive equilibrium approach appears to be unavoidable.

Despite its limitations, the calibrated banking model makes it possible to examine the hypothesis that much of the observed mistakes in predicting the downside risk of assets originates from fire sales that are absent in the rational expectations equilibrium. The model supports the hypothesis for small mistakes. In particular, such small mistakes help explain default on mortgages that were set up to be perfectly safe.

However, this logic arrives at its limit for big mistakes due to the limitation of fire sale dynamics. Fire sale dynamics can only last as long as there is no willing buyer, and a big enough discount will draw even the most highly risk-averse agent to start buying the asset, and thereby stabilize prices.

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A Tail values

With no uncertainty the stochastic discount factor reduces to:

$$M_i(s) = \beta \frac{\lambda_i(s)}{\lambda_i(s_{-1})} \left(\frac{U_i(s_{-1})}{U_i(s)} \right)^{\eta}$$

Plugging the marginal value of goods into the stochastic discount factor, we obtain:

$$M_i(s) = \beta \left(\frac{h_i(s)}{h_i(s_{-1})} \right)^{-\eta} \left(\frac{w(s)}{w(s_{-1})} \right)^{(1-\rho)(1-\eta)-1}$$

Since there is no cause for lending, collateral constraints do not bind. The first order conditions for housing stock and debt become:

$$\begin{aligned} H_i(s):& \qquad p_h(s) = M_i(su)\left(p_h(su) + R(su)\right), \\ B_i(s):& \qquad 1 = (1+r(s))M_i(su). \end{aligned}$$

Plugging the discount factor into the first-order condition on debt we have:

$$h_i(s) = (1+r(s))^{-1/\eta} \beta^{-1/\eta} h_i(su) \left(\frac{w(su)}{w(s)} \right)^{((1-\rho)(1-\eta)-1)(-1/\eta)}.$$

Aggregating across agents yields:

$$1 + r(s) = \frac{e^{g(1-(1-\rho)(1-\eta))}}{\beta}.$$

Substituting the interest rate into the first order condition for house ownership we get:

$$p_h(s) = \frac{p_h(su)}{1+r(s)} + \beta e^{g(1-\rho)(1-\eta)} R(s) \qquad \forall s \in \{s | t(s) \geq T\}.$$

Rolling forward we obtain:

$$p_h(s) = \frac{\beta e^{g(1-\rho)(1-\eta)}}{1 - \beta e^{g(1-\rho)(1-\eta)}} R(s) \qquad \forall s \in \{s | t(s) \geq T\}.$$

When there is no uncertainty it does not matter what state follows another. We can therefore define a '+' operator that takes a state and a number and applies the S operator that many times on the state, picking one of the two equal branches each time. By the definition of utility when there is no uncertainty, we obtain:

$$U_i(s) = \left[(1-\beta) u_i(s)^{1-\eta} + \beta U_i(s+1)^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

Thus,

$$U_i(s) = [(1-\beta) \sum_{t=0}^{\infty} \beta^t u_i(s+t)^{1-\eta}]^{\frac{1}{1-\eta}} \qquad \forall \{s | t(s) \geq T\}.$$

and therefore:

$$
\begin{aligned}
U_i(s) &= [(1-\beta)\sum_{t=0}^{\infty}\beta^t(h_i(s+t)w(s+t)^{1-\rho})^{1-\eta}]^{\frac{1}{1-\eta}} \\
&= [(1-\beta)\sum_{t=0}^{\infty}\beta^t(h_i(s)(e^{gt}w(s))^{1-\rho})^{1-\eta}]^{\frac{1}{1-\eta}} \\
&= h_i(s)[(1-\beta)\sum_{t=0}^{\infty}\left(\beta e^{g(1-\rho)(1-\eta)}\right)^t w(s)^{(1-\rho)(1-\eta)}]^{\frac{1}{1-\eta}} \\
&= h_i(s)w(s)^{1-\rho}\left(\frac{1-\beta}{1-\beta e^{g(1-\rho)(1-\eta)}}\right)^{\frac{1}{1-\eta}}
\end{aligned}
$$

Since $LTV(s) = 1$ and the yield on housing equals the interest rate, the budget constraint reduces to:

$$
\begin{aligned}
\frac{1}{\rho}R(s)h_i(s) &= COH_i(s) - EC_i(s), && \forall s \in \{s|t(s) = T\}, \\
EC_i(s) &= \frac{\frac{1}{\rho}e^g R(s)h_i(s) - W_i e^g w(s) + EC_i(su)}{1+r(s)} && \forall s \in \{s|t(s) \geq T\}.
\end{aligned}
$$

Rolling forward the budget constraint yields:

$$
\frac{1}{\rho}\frac{1}{1-\beta e^{g(1-\rho)(1-\eta)}}R(s)h_i(s) = \frac{\beta e^{g(1-\rho)(1-\eta)}}{1-\beta e^{g(1-\rho)(1-\eta)}}W_i w(s) + COH_i(s) \quad \forall s \in \{s|t(s) \geq T\}.
$$

B Off-equilibrium payments

The fact that off-equilibrium payments are bound by collateral requires us to restate the COH equations in order to account for unforeseen losses.

$$
\begin{aligned}
COH_r(s) \equiv\ & W_r w(s) + \max(0,(p_h(s) + R(s) - (1+r(s_{-1}))LTV(s_{-1})p_h(s_{-1}))H_r(s_{-1}) && \forall s \in S, \\
& \qquad + (1+r(s_{-1}))EC_r(s_{-1})) \\
COH_b(s) \equiv\ & + W_b w(s) \\
& + \max(0,\ (p_h(s) + R(s) - (1+r(s_{-1}))LTV(s_{-1})p_h(s_{-1}))H_b(s_{-1}) && \forall s \in S, \\
& \quad + (1+r(s_{-1}))EC_b(s_{-1}) \\
& \quad + \min(0, p_h(s) + R(s) - (1+r(s_{-1}))LTV(s_{-1})p_h(s_{-1}))H_r(s_{-1}) \\
& \qquad + (1+r(s_{-1}))EC_r(s_{-1}))) \\
COH_d(s) \equiv\ & W_d w(s) + (p_h(s) + R(s) - (1+r(s_{-1}))LTV(s_{-1})p_h(s_{-1})H_d(s_{-1}) && \forall s \in S. \\
& + (1+r(s_{-1}))EC_d(s_{-1}) \\
& + \min(0,\ (p_h(s) + R(s) - (1+r(s_{-1}))LTV(s_{-1})p_h(s_{-1}))H_b(s_{-1}) \\
& \quad + (1+r(s_{-1}))EC_b(s_{-1}) \\
& \quad + \min(0, p_h(s) + R(s) - (1+r(s_{-1}))LTV(s_{-1})p_h(s_{-1}))H_r(s_{-1}) \\
& \qquad + (1+r(s_{-1}))EC_r(s_{-1})))
\end{aligned}
$$