Financial Frictions, Capital Misallocation, and Input-Output Linkages

Hsuan-Li Su*

January 30, 2019

Abstract

This paper studies how input-output linkages amplify the aggregate impact of sectoral financial distortions through the lens of a dynamic general equilibrium model with endogenous capital wedges. The aggregate impact of a shock can be decomposed into weighted productivity changes and changes in capital allocative efficiency. Uncertainty shocks, second-moment shocks to Solow-neutral (capital-augmenting) productivity, induce heterogenous responses in sectoral capital wedges, reducing allocative efficiency and aggregate TFP. In the calibrated model to the U.S. data, I show input-output linkages amplify the aggregate effect of financial distortions by two-fold more than an equivalent economy without linkages. Shocks that generates the spike of credit spreads similar to the magnitude during the Great Recession can decrease aggregate TFP by 0.3% and aggregate output by 1.6%. Among all sectors, the model indicates that the Financial sector is most sensitive to changes in its financial constraint and has the largest impact on aggregate output.

JEL: E32 E44 G10

Keywords: Input-Output Linkages, Financial Frictions, Capital Wedges, Uncertainty shocks.

---

*Department of Economics, National Taiwan University (email: shanesu@ntu.edu.tw). This paper is based on my Ph.D. dissertation at the University of Wisconsin-Madison. I am indebted to Enghin Atalay, Dean Corbae, Steven Durlauf, Kenneth West, and Yiting Li for their invaluable advice. In addition, I thank all seminar participants at UW–Madison, the Central European University,the University of Hong Kong, Academia Sinica, National Taiwan University, and the 2016 Asian Meeting of the Econometric Society. Any remaining errors are mine.

1 Introduction

Recessions are often explained by large aggregate shocks. However, one feature of the Great Recession is that small, sectoral shocks that stemmed from the subprime mortgage market propagated to the whole economy, inducing the enormous economic turmoil.[1] In fact, though these shocks were initially constrained to the financial sector, their consequence was far-reaching. Since then, there are upsurges of research interests of two strands of the macroeconomics literature. One is on the relation between financial frictions and business cycles. The other is the resurgence on how small shocks are amplified through a network of input-output linkages.[2]

This paper is at the interplay between the above two strands of literature. More specifically, I seek to answer the following question: how does the network of input-output linkages amplify the aggregate impact from sectoral financial distortions in a dynamic environment? And how large is the amplification magnitude? The idea is simple. Consider firms in a production network that require funds for their factors of production. A tightening of a firm's financial constraint could affect its own production and, hence, its output supplied as intermediate inputs to downstream firms; a decrease in outputs of these firms in turn could affect their connected firms, and so on and so forth, resulting in a multiplier effect. The question is then is this amplification magnitude empirically significant?

The aggregate effect of a financial shock in a network depends on two things: the first is the affected producers' impact on aggregate output, and the second is their sensitivity to financial shocks. Hulten (1978) provides the foundation theorem that states producers' impact on aggregate TFP and output is summarized by sales to GDP share, i.e., Domar weights, in efficient economies. In contrast, in inefficient economies, Domar weights are not sufficient to describe sectoral impact on aggregate variables. One needs to account for producers' heterogeneous responses to financial shocks and changes in factor reallocation. To see these heterogeneous responses, Figure 1 depicts the sectoral corporate bond spreads of selected sectors and the average across all sectors from 2003 to 2014. Among all sectors, the Finance, Insurance, and Real Estate (FIRE) sector has the highest spike during the recession: from a low of 0.8% in 2006 to the high of 5.2% in 2008. At the same time, the

1. Gorton and Ordonez (2014) show that only 4 percent of subprime securities had been impaired and the realized principal losses on the AAA subprime bonds were 17 basis points as of February 2011.

2. See, for example, Acemoglu et al. (2012) and Acemoglu, Akcigit, and Kerr (2015).

Manufacturing sector has the lower-than-average spread. While these two sectors both have the highest Domar weights in the United States, their impacts under financial shocks are much different.

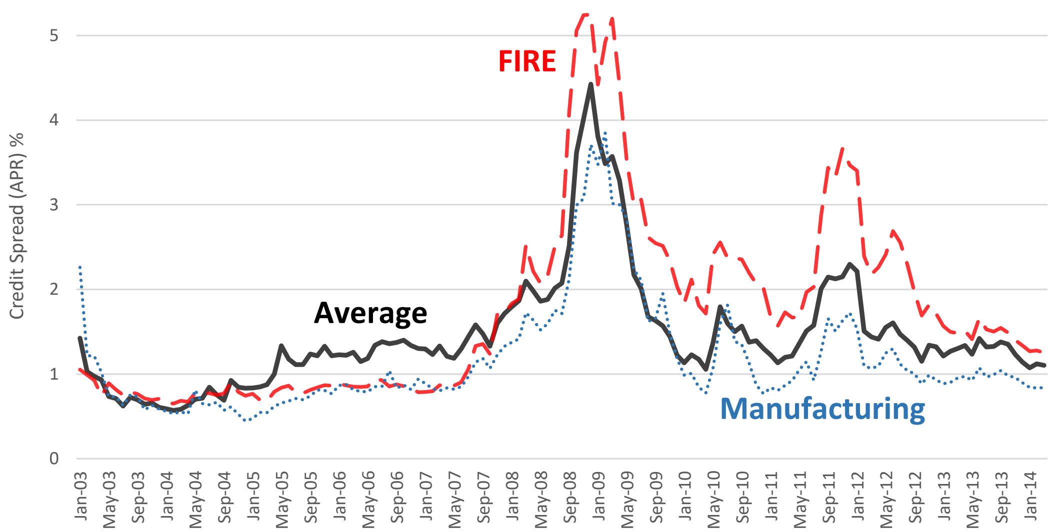

Figure 1: The sectoral corporate bond spreads from 2003 to 2014Q1. The bold black line is the sectoral average. The red long dashed line is the spread of the FIRE sector. The blue short dashed line is the spread of the Manufacturing sector. The data is from the TRACE bond prices database and calculated by the author.

To explore this heterogenous financial response channel in a production network, I develop a dynamic general equilibrium model with financial frictions and input-output linkages. The financial friction here refers to the costly state verification problem proposed by Townsend (1979), which is then combined into the multi-sector real business cycle framework based on Long and Plosser (1983).[3] In the model, entrepreneurs, who manage firms, need to borrow to purchase capital with their net worth. However, idiosyncratic returns on capital are private information, and verifications from lenders are costly. In equilibrium, lenders sign a standard debt contract with borrowers, and the verification cost is interpreted as default cost, which generates a capital wedge (credit spread). Entrepreneurs of different sectors differ in their default costs and volatilities of idiosyncratic returns. Hence, in equilibrium, capital wedges differ across sectors. This allocative inefficiency, i.e., capital misallocation, reduces aggregate TFP. Entrepreneurs are subject to uncertainty shocks, which are innovations to the sectoral

3. My model is built on the works of Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), and Christiano, Motto, and Rostagno (2014). I extend these papers' analysis of production by examining multiple sectors with input-output linkages among them.

volatility of idiosyncratic capital returns. These are second-moment shocks to Solow-neutral (capital-augmenting) productivity, directly affecting credit spreads.

I modify Hulten's theorem in this inefficient dynamic economy with endogenous capital wedges. The main theoretical result is that aggregate TFP change can now be decomposed into two components: changes in productivity weighted by Domar weights and changes in allocative efficiency. In efficient economies, changes in allocative efficiency are always zero, and the aggregate effect falls on productivity changes, as stated in Hulten's theorem. In inefficient economies, changes in allocative efficiency may be nonzero. A sectoral productivity shock can induce both effects. An aggregate productivity shock only has the first effect but not the second effect, since the relative marginal product of capital remains the same across sectors. Uncertainty shocks, however, no matter aggregate or sectoral, always affect capital allocative efficiency and aggregate TFP, because endogenous capital wedges respond differently across sectors. In response to an aggregate uncertainty shock, the relatively more financially constrained producers are more vulnerable, and capital flows from them to less financially constrained producers, reducing allocative efficiency and aggregate TFP.

I then calibrate the model to the U.S. data and estimates shocks processes through the maximum likelihood estimation. There are two quantitative results. First, input-output linkages amplify the impact of an aggregate uncertainty shock by more than two folds. In response to an aggregate uncertainty shock that generates a spike of average credit spread by about 3%, as observed during the recession in Figure 1, aggregate output decreases by 1.6% and aggregate TFP decreases by 0.3%. To quantify the amplification magnitude from linkages, I compare the impulse responses with another multi-sector model economy with the same level of financial frictions, but without linkages. The ratio of aggregate TFP drop between two model economies is 1.7, and the ratio of aggregate output drop between them is 2.3. Both model economies have a similar impulse response of capital misallocation, so the large difference is mainly from the amplification of input-output linkages.

Second, the amplification magnitude of input-output linkages on sectoral uncertainty shocks ranges from 1.2 to 10 (the ratio of aggregate output drop between two model economies). Among all sectors, a sectoral shock to the FIRE sector induces the largest decline in aggregate output. This is because the FIRE sector has the largest default cost, is most sensitive to a change in its financial constraints, and has the second largest Domar weight in the U.S. On the other hand, while the Manufacturing sector has the largest Domar weight, its response to uncertainty shocks is modest.

This paper adds values to the new strand of the literature that consider frictions into production networks.[4] For example, Jones (2011, 2013) are the pioneer works that consider how linkages amplify the effect of misallocation in development accounting. Altinoglu (2016) studies the credit network when intermediate inputs are financed by supplier credit. Baqaee (2018) shows entry and exit provide a new mechanism for propagating shocks, along with input-output networks. Luo (2016) studies the relation between financial frictions and trade. Liu (2018) studies industrial policies in production networks.

This paper is most closely related to Bigio and La'O (2017) and Baqaee and Farhi (2018). Bigio and La'O (2017) ask how the structure of a production network affects the impact of sectoral distortions and quantify the amplification of linkages on distortions. Their result on the amplification magnitude of aggregate financial frictions shocks ranges from 1.7 to 2.4, consistent with this paper. Baqaee and Farhi (2018) provide a general theory and nonparametric formulas for inefficient economies with input-output networks and distortion wedges. All three works share the feature that changes in allocative efficiency reduce aggregate TFP. However, the unique feature of this paper that deviates from theirs is that capital wedges and credit spreads are endogenous in a dynamic environment, while the above two works both take the reduced-form markup and wedge approach via static modeling. Bigio and La'O (2017) focus on labor market distortions and use the sectoral credit spreads constructed by Gilchrist and Zakrajšek (2012) as a proxy to infer sectoral financial distortions, interpreted as working capital constraints on intratemporal variables. However, this approach fails to account for the fact that the majority of funds in the credit market is for intertemporal usage. In contrast, I focus on capital market distortions with endogenous credit spreads, and this allows me to directly identify the degree of financial distortions from data.

The modeling element of financial frictions builds on Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2014). The type of uncertainty shock used here is first proposed by Christiano, Motto, and Rostagno (2014). Instead of considering any nominal rigidity, I focus on multi-sector real economies. This setting allows me to simplify the analysis and focus on the amplification from input-output linkages and enables me to perform sectoral analysis.

4. For the literature on input-output networks in efficient economies, see Atalay (2017), Acemoglu et al. (2012), Dupor (1999), Durlauf (1993), Foerster, Sarte, and Watson (2011), Horvath (1998, 2000), and Shea (2002).

The rest of the paper proceeds as follows. Section 2 starts from a frictionless economy as the benchmark, then develops the dynamic general equilibrium model with frictions, and characterizes the equilibrium. Section 3 explains data, calibrates and estimates the model parameters, discusses meanings of financial parameters. Section 4 shows quantitative analysis. Section 5 concludes. Proofs and other technical details are in the appendix.

2 The General Equilibrium Model

To illustrate the value added of considering frictions within a multi-sector real business cycle model, I first layout a frictionless economy served as the benchmark and show some properties that are known to the literature. I then present the general model with financial frictions and show how adding frictions would modify the result.

2.1 The Frictionless Benchmark

Consider a static variant of the multisector model of Long and Plosser (1983) and Acemoglu et al. (2012). In a perfectly competitive economy, there is a representative consumer endowed with one unit of labor, supplied inelastically, with a Cobb-Douglas preference over N goods.

$$U(C) = C = Y = \prod_{j=1}^{N} c_j^{\beta_j},$$

where c_j is consumption of good j, and $\sum_{j=1}^{N} \beta_j = 1$. Alternatively, one can think there is a final good Y produced by Cobb-Douglas technology. There are N production sectors; each of them produces a good indexed by $j = 1, \dots, N$. Assume there is a representative firm within each sector and it uses a constant return to scale Cobb-Douglas technology. The output of sector j is given by

$$Y_j = A_j l_j^{1-m_j} \prod_{i=1}^{N} M_{ij}^{\gamma_{ij}}, \quad \sum_{i=1}^{N} \gamma_{ij} = m_j,$$

where A_j is Hicks-neutral sectoral productivity of j, l_j is the amount of labor employed by the sector, and M_{ij} is the amount of intermediate input of good i purchased by j. The exponent γ_{ij}, which is the equilibrium intermediate input share of M_{ij}, represents input-output linkages. m_j is the total intermediate input share of sector j, and $(1 - m_j)$ is labor share of j.

Let $\mathbf{\Gamma}$ denote the input-output matrix of N sectors:

$$\mathbf{\Gamma} = \begin{pmatrix} \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{1,N} \\ \gamma_{2,1} & \gamma_{2,2} & \cdots & \gamma_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N,1} & \gamma_{N,2} & \cdots & \gamma_{N,N} \end{pmatrix}.$$

$\mathbf{\Gamma}$ can be treated as an adjacency matrix of the weighted production network, with each element representing the *directed* link from sector i to j.

The profits of a producer in sector j is

$$\pi_j = p_j Y_j - w_j l_j - \sum_{i=1}^{N} p_i M_{ij},$$

where p_j is the price of good j, and w_j is the wage rate at sector j. In equilibrium, $w_j = w, \forall j$.

Finally, to clear the market, the sectoral gross output is equal to its final use and factor use:

$$Y_j = c_j + \sum_{i=1}^{N} M_{ji}, \quad \forall j = 1, \ldots, N.$$

And the labor market clearing condition is

$$\sum_{j=1}^{N} l_j = 1.$$

In this frictionless benchmark economy, due to inelastic labor supply, the aggregate output change is equivalent to aggregate TFP change. The change of aggregate output (TFP) in response to sectoral productivity shocks are stated in the following theorem.

Theorem 1 (Hulten 1978)**.** *The first-order approximation of aggregate output (TFP) change is given by:*

$$\hat{y} \equiv \mathrm{d}\ln Y = \mathrm{d}\ln \tilde{A} = \sum_j v_j \mathrm{d}\ln A_j = \boldsymbol{v}' \mathrm{d}\ln \boldsymbol{A},$$

where $\tilde{A}$ is aggregate TFP, $v_j = \frac{p_j Y_j}{Y}$ is sectoral sales to output ratio, and $\boldsymbol{A} = (A_1, \ldots, A_N)'$ is the vector of sectoral productivity.

Hulten's theorem simply states that, up to a first-order approximation, in a competitive economy with intermediate inputs, sectoral influence on aggregate output and TFP is its productivity change weighted by its equilibrium sales shares, i.e., Domar weights. Given its meaning on the impact to aggregate output, Acemoglu et al. (2012) called $\boldsymbol{v}$ the influence vector. The influence vector describes the aggregate output elasticity of sectoral technological change, since

$$\frac{d\ln Y}{d\ln A_j} = v_j.$$

That is, sectoral size determines its impact on aggregate output.[5]

In addition, the market clearing condition and the first-order conditions on intermediate inputs relate Domar weights into the final use share and the Leontief inverse:

$$\boldsymbol{v} = [I_N - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta},$$

where $\boldsymbol{\beta} = (\beta_1, \dots, \beta_N)$ is the vector of final use share, and $[I_N - \boldsymbol{\Gamma}]^{-1}$ is the Leontief inverse matrix. That is, in this input-output economy, equilibrium sectoral size depends on its final demand from the households and also its intermediate use demand from other producers. Due to the presence of intermediate inputs, the impact of sectoral productivity shocks to aggregate output is amplified.

Definition 1. *The aggregate input-output multiplier of productivity shocks is*

$$\upsilon_A = \sum_j \frac{d\ln Y}{d\ln A_j} = \sum_j v_j > 1.$$

υ_A (upsilon) describes the total output change in response to a systematic uniform technology shock across sectors. That is, if productivity in every sector falls by 1%, then aggregate output would decrease by υ_A%. The last inequality comes from the presence of intermediate use. Intuitively, when a bad shock hits a sector, its sectoral output decreases (price increases), and hence all intermediate inputs supplied to (or purchased from) its direct downstream producers decreases; these downstream producers then decrease their outputs, and so on.[6] The aggregate input-output multiplier is also the ratio of aggregate output

5. For this size argument, see Gabaix (2011, 2016).

6. This effect can be seen directly from the algebra

$$[I_N - \boldsymbol{\Gamma}]^{-1} = I_N + \boldsymbol{\Gamma} + \boldsymbol{\Gamma}^2 + \dots,$$

where the first I_N is the direct effect from the impacted sector, the second term is the effect of its downstream change between the input-output economy and its equivalent multisector economy without input-output linkages. In this way, υ_A describes the amplification magnitude from input-output linkages under productivity shocks.

Next, I will show that how adding financial frictions modifies the result of Hulten's theorem by changing allocative efficiency, and how input-output linkages amplify the impact of allocative inefficiency on reducing aggregate TFP in a dynamic general equilibrium model.

2.2 The Model with Financial Frictions

The general model combines financial frictions through costly state verification with uncertainty shocks, as in Christiano, Motto, and Rostagno (2014), and the multi-sector real business cycle framework with input-output linkages, as in Long and Plosser (1983). Bernanke, Gertler, and Gilchrist (1999) is the first to combine the costly state verification into a business cycle model, and they call it the financial accelerator. Since this modeling technique is well-known and popularly used, I simplify its presentation here and put all technical details in the appendix.

Consider a perfectly competitive economy. There are five types of agents in the model: a representative household (workers and entrepreneurs), N intermediate goods producing sectors, a final goods producer, a capital producer, and a financial intermediary. Here I use the large household assumption as in Christiano, Motto, and Rostagno (2014) to incorporate the costly state verification problem into a standard business cycle model. This large household assumption also follows from Gertler and Karadi (2011) to incorporate financial intermediaries and maintain the tractability of the representative household approach.[7]

2.2.1 Technology

Final Goods

There is a representative, competitive final goods producer that combines intermediate goods, X_{jt}, from N intermediate goods sectors to produce a homogeneous final good, Y_t, which is used for consumption and investment. The price of the final good is normalized to one. The

customers, and the third term is its customers' customers.

7. An alternative way, used by Bernanke, Gertler, and Gilchrist (1999), is to assume entrepreneurs to be another type of households with risk-neutral preferences. They work, consume, and manage firms with an exogenous death rate. The equations that characterize the equilibrium are almost the same in both approaches, and all results remain the same. It is just different ways of model presentation.

firm uses constant return to scale Cobb-Douglas technology. The representative final goods producer maximizes its profits as follows:

$$\max_{X_{jt}} \quad Y_t - \sum_{j=1}^{N} p_{jt} X_{jt}, \tag{2.1}$$

$$s.t. \quad Y_t = \prod_{j=1}^{N} X_{jt}^{\beta_j}, \quad \sum_{j=1}^{N} \beta_j = 1,$$

where β_j represents the final use share of each intermediate input, X_{jt}, and p_{jt} is its price.

Intermediate Goods

There are N intermediate goods producing sectors. Firms in sector j are managed by entrepreneurs of type j. Each firm's capital is owned by its entrepreneur and is purchased one period ahead. Given entrepreneurs' capital, firms need to decide how much labor l_{jt} to hire and how many intermediate inputs M_{ijt} to buy from other sectors. After production and factor payments, entrepreneurs receive the leftovers as capital income, $r^k_{j,t} k_{jt}$. Here, $r^k_{j,t}$ is the return on capital through production (capital rent) in sector j, and k_{jt} is entrepreneurs' capital in sector j. The technology exhibits constant return to scale.

The firm's profit maximization problem is:

$$\max_{l_{jt}, M_{ijt}} \quad r^k_{j,t} k_{jt} = p_{jt} Y_{jt} - w_{jt} l_{jt} - \sum_{i=1}^{N} p_{it} M_{ijt}, \tag{2.2}$$

$$s.t., \quad Y_{jt} = A_{jt} (k_{jt}^{\alpha_j} l_{jt}^{1-\alpha_j})^{1-m_j} \prod_{i=1}^{N} M_{ijt}^{\gamma_{ij}}, \quad \sum_{i=1}^{N} \gamma_{ij} = m_j,$$

where A_{jt} is Hicks-neutral sectoral productivity of j at period t, α_j is capital share, M_{ijt} is intermediate goods produced in sector i, purchased as input by sector j with the price p_{it}, and w_{jt} is the wage rate in sector j. Similarly, denote $\mathbf{\Gamma}$ as the input-output matrix of N sectors.

In equilibrium, first-order conditions imply that

$$r^k_{j,t} = (1 - m_j) \alpha_j p_{jt} Y_{jt} / k_{jt}, \tag{2.3}$$

that is, the capital rent is equal to the sectoral marginal revenue product of capital (MRPK).

Capital Production

Capital is homogeneous across sectors, and this assumption allows capital to be reallocated easily across sectors. After production of all goods in period t, entrepreneurs sell undepreciated capital, $(1-\delta)K_t$, to capital producers in a competitive market, where δ is the depreciation rate.[8] The representative capital producer combines existing capital, K_t, and investment, I_t, to produce new capital, K_{t+1}. This new capital is then repurchased by entrepreneurs in all sectors. The representative capital producer's problem is:

$$\max_{I_t} \quad Q_t K_{t+1} - Q_t(1-\delta)K_t - I_t, \tag{2.4}$$
$$s.t. \quad K_{t+1} = \Phi(I_t/K_t)K_t + (1-\delta)K_t,$$

where Q_t is capital price. The first-order condition implies

$$Q_t = \frac{1}{\Phi'(I_t/K_t)}. \tag{2.5}$$

Let $\Phi(I_t/K_t) = I^\theta K^{-\theta}$. This concave function represents convex capital adjustment cost as in Hayashi (1982), which is used to pin down capital price, Q_t.

2.2.2 Households

There is a representative household with a continuum of members of measure unity. The household consumes, saves, and supplies labor. Within the household there are two types of members: workers and entrepreneurs. Workers supply labor and return the wages to the household, and they consists of every type of differentiated labor supplied to every intermediate goods producing sector. Entrepreneurs manage intermediate goods producing firms and return part of the firms' profits as dividends to the household. Type j workers and entrepreneurs work and manage firms in sector j, respectively.

A type j entrepreneur in this period stays entrepreneur of type j in the next period with probability κ_j, which is exogenously given. The average survival rate for a type j entrepreneur is $1/(1-\kappa_j)$. This finite horizon for entrepreneurs is used to ensure that over time they do not accumulate enough net worth to fund their capital fully. When entrepreneurs exit, they give their retained earnings, their net worth, back to the household as

8. The big letter K represents aggregate capital, while little k_j is used to indicate sectoral capital at sector j.

dividends and become workers. A similar number of workers randomly become entrepreneurs, keeping the relative proportion of each type fixed. The household then gives some startup funds to its new entrepreneurs for operation. Within the family there is perfect consumption insurance.

Let C_t be family consumption and H_t family labor supply. The preference of the representative household is given by:

$$E_t \sum_{t=1}^{\infty} \tilde{\beta}^t \left\{ \frac{C_t^{1-\varrho}}{1-\varrho} - \frac{\epsilon H_t^{1+\epsilon/\epsilon}}{1+\epsilon} \right\}, \tag{2.6}$$

where $\tilde{\beta}$ is the discount factor, ϱ the coefficient of constant relative risk aversion (CRRA), and ϵ the Frisch elasticity of labor supply.

The household saves in a competitive financial intermediary and receives the risk-free rate, R_t, as gross return. The representative household chooses consumption, labor supply, and risk-free deposit (C_t, H_t, D_{t+1}) to maximize expected discount utility, Eq (2.6), subject to the following budget constraint:

$$C_t + D_{t+1} = w_t H_t + R_t D_t + \Pi_t,$$

where w_t is the wage rate and Π_t the net transfer from entrepreneurs (dividends from exiting entrepreneurs minus start up funds for new entrepreneurs). Family labor supply is the sum of total labor supplied in every sector ($H_t = \sum_{i=1}^{N} l_{jt}$). Since the labor market is competitive and there are no frictions, in equilibrium, the wage rate in every sector, w_{jt} in Eq (2.2), is the same across sectors and equal to w_t.

2.2.3 Financial Frictions

At the end of period t, after production and factor payments, an entrepreneur i in sector j sells his or her undepreciated capital to the capital producer and has net worth $n^i_{j,t+1}$. He or she needs to determine how much capital to buy for the next period, $k^i_{j,t+1}$, with the price Q_t. If the value of capital is larger than his or her own net worth, he or she needs to borrow the amount $Q_t k^i_{j,t+1} - n^i_{j,t+1}$ from the financial intermediary. As it turns out, in equilibrium, every entrepreneur's net worth is insufficient to buy capital and therefore he or she will need to borrow.

However, after purchasing capital but before putting it into production at $t+1$, there is an idiosyncratic shock $\omega^i_{j,t+1}$, which converts one unit of purchased raw capital into $\omega^i_{j,t+1}$ units of effective capital, $\omega^i_{j,t+1}k^i_{j,t+1}$. $\omega^i_{j,t+1}$ could be interpreted as a Solow-neutral (capital-augmenting) productivity shock. Assume $\omega^i_{j,t+1}$ is drawn from a unit mean lognormal distribution with standard deviation $\sigma_{j,t+1}$ independently and identically across time. That is, $E[\omega^i_j] = 1, \forall j$. Here, the standard deviation, $\sigma_{j,t+1}$, is assumed to follow a stochastic process, and the innovation of it is called the uncertainty shock, second-moment shocks to Solow-neutral productivity.[9]

While the level of sectoral uncertainty, $\sigma_{j,t+1}$, is observable to everyone, the realization of $\omega^i_{j,t+1}$ is private information. The lender must pay a cost to observe the realized return. As in Bernanke, Gertler, and Gilchrist (1999), the optimal contract is a standard debt contract that specifies the amount the entrepreneur can borrow, $B^i_{j,t+1} = Q_t k^i_{j,t+1} - n^i_{j,t+1}$, and the gross loan rate, $z^i_{j,t+1}$, that the entrepreneur needs to repay. So the specified total loan payment is $z^i_{j,t+1}B^i_{j,t+1}$.

From the entrepreneur's perspective, the total return from capital is

$$R^k_{j,t+1} = \frac{r^k_{j,t+1} + Q_{t+1}(1-\delta)}{Q_t}, \tag{2.7}$$

where $r^k_{j,t+1}$ is capital rent specified in Eq (2.3). And the realized payoff from capital at period $t+1$ is $\omega^i_{j,t+1}R^k_{j,t+1}Q_t k^i_{j,t+1}$. There exists a threshold $\bar{\omega}_{j,t+1}$ for each type j entrepreneurs (in sector j) such that the total payoff is equal to the pre-specified loan payment, that is,

$$\bar{\omega}_{j,t+1}R^k_{j,t+1}Q_t k^i_{j,t+1} = z^i_{j,t+1}B^i_{j,t+1}.$$

Ceteris paribus, specifying the loan interest rate is equivalent to specifying the threshold $\bar{\omega}_{j,t+1}$. For each individual entrepreneur, if realized $\omega^i_{j,t+1} < \bar{\omega}_{j,t+1}$, he or she defaults. In this case, the financial intermediary then seizes the entrepreneur's asset, $\omega^i_{j,t+1}R^k_{j,t+1}Q_t k^i_{j,t+1}$, and pays a fraction μ_j of the asset as verification cost. Since this cost only occurs under default, it is interpreted as default cost here. μ_j is an exogenous parameter governing this default cost, interpreted as the degree of financial frictions. Figure 2 summarizes the environment.

9. Christiano, Motto, and Rostagno (2014) use the exactly the same type of shock and call it the risk shock, the risk faced by entrepreneurs. Since innovations to $\sigma_{j,t+1}$ are actually second-moment shocks to Solow-neutral productivity, I found it is more natural to call it uncertainty shocks as in Bloom (2009) and Bloom et. al. (2018).

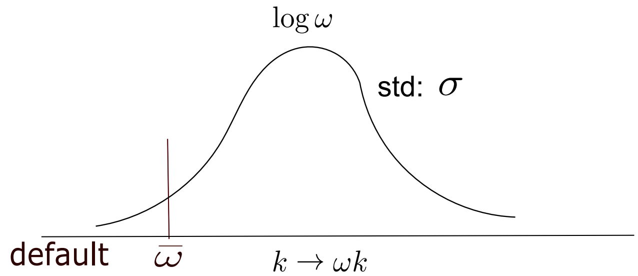

Figure 2: Distribution of ω and the default threshold.

Denote $F(\omega) = Pr[\omega^i < \omega]$ a continuous probability distribution. Then $F(\bar{\omega})$ is the default region in Figure 2.

The total expected profit of sector j is $R^k_{j,t+1} Q_t k_{j,t+1}$ (the sum of all entrepreneurs' expected profits in sector j). Ex ante, the expected loan payment from entrepreneurs in sector j is

$$\underbrace{[\omega^i_{j,t+1} Pr(\omega^i_{j,t+1} < \bar{\omega}_{j,t+1}) + \bar{\omega}_{j,t+1} Pr(\omega^i_{j,t+1} \geq \bar{\omega}_{j,t+1})]}_{\text{denote this fraction as } \Omega_{j,t+1}} R^k_{j,t+1} Q_t k^i_{j,t+1}.$$

That is, ex ante, a fraction $1 - \Omega_{j,t+1}$ of sector j's expected profits goes to entrepreneurs, and the financial intermediary collects the fraction $\Omega_{j,t+1}$ of expected profits and pays the default cost.

I impose the zero-profit condition for the financial intermediary. The sum of expected profits from lending in each sector (after paying expected default cost) is equal to the deposit interest to be paid to the representative household allocated in that sector:

$$[\Omega_{j,t+1} - \mu_j \omega_{j,t+1} Pr(\omega_j < \bar{\omega}_j)] \underbrace{R^k_{j,t+1} Q_t K_{j,t+1}}_{\text{capital payoff}} = \underbrace{R_{t+1} B_{j,t+1}}_{\text{deposit interest}}. \tag{2.8}$$

The optimal contract then maximizes each entrepreneur's expected return, entrepreneurs' expected profits divided by opportunity cost,

$$\max_{L^i_{j,t+1}, \bar{\omega}_{j,t+1}} E_t \frac{[1 - \Omega_{j,t+1}] R^k_{j,t+1} Q_t k^i_{j,t+1}}{R_{t+1} n^i_{j,t+1}}, \tag{2.9}$$

by choosing the leverage and the threshold, $(L^i_{j,t+1}, \bar{\omega}_{j,t+1})$, subject to the zero-profit condition, Eq (2.8). The leverage is defined as $L^i_{j,t+1} = Q_t k^i_{j,t+1} / n^i_{j,t+1}$. After solving the optimal contract in each sector, we can get an equilibrium condition of the spread between the expected sectoral capital return, $E_t R^k_{j,t+1}$, and the risk-free rate, R_{t+1}, as the following:

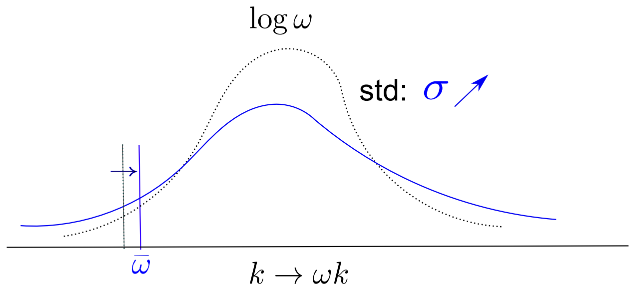

Figure 3: A mean-preserving spread induces higher $\bar{\omega}$ and raises capital wedge.

$$E_t \frac{R^k_{j,t+1}}{R_{t+1}} = \varphi(\bar{\omega}_{j,t+1}, \sigma_{j,t+1}), \quad j = 1, \ldots, N. \tag{2.10}$$

Eq (2.10) says that the spread is a function φ of endogenous threshold $\bar{\omega}_{j,t+1}$ and exogenous sectoral uncertainty $\sigma_{j,t+1}$. And it turns out that sectoral capital only depends on sectoral aggregate net worth, and every entrepreneur ends up with the same leverage within a sector. So we only need to keep track of sectoral net worth, $N_{j,t+1} = \sum_i n^i_{j,t+1}$.[10]

Rewriting Eq (2.10) gives the wedge representation:

$$E_t R^k_{j,t+1} = E_t R_{t+1}(1 + \tau^k_{j,t+1}). \tag{2.11}$$

The endogenous capital wedge, $1 + \tau^k_{j,t+1}$, is determined through the optimal contract at each period. Due to the presence of default cost, $\tau^k_{j,t+1}$ is always positive.

In Eq (2.10), assume that the standard deviation in each sector follows a first-order autoregressive process, AR(1), with innovation $\epsilon^\sigma_{j,t+1}$, which is called the uncertainty shock.[11] $\epsilon^\sigma_{j,t+1}$ affects the endogenous capital wedge through the following mechanism. Consider a mean-preserving spread ($\epsilon^\sigma_{j,t+1} > 0$) in Figure 3. When the standard deviation increases, the original default region (under the black dashed line and the dashed threshold line) becomes larger. The financial intermediary is then paying higher default cost under the original threshold. To make it break-even, the intermediary needs to raise the threshold, i.e., the loan interest rate. Facing a higher borrowing cost, entrepreneurs borrow less. All this results in a higher return spread in Eq (2.10) and higher capital wedge.

10. For more technical details, please refer to the appendix.
11. The detail of this AR(1) process is described in Section 3 where I estimate the shock processes.

2.2.4 Equilibrium Characterizations

The competitive equilibrium in the model is defined as follows:

Definition 2. *A competitive equilibrium consists of a sequence of vectors* $\{l_{jt}, k_{jt}, N_{jt}, Y_{jt}, M_{ijt}, X_{jt}, \bar{\omega}_{jt}, Y_t, C_t, H_t, I_t, D_t\}_{t=0}^{\infty}$ *and a sequence of vectors of prices* $\{p_{jt}, R^k_{j,t}, R_t, w_t, Q_t\}_{t=0}^{\infty}$ *for* $j = 1, \ldots, N$*, such that*

1. Households maximize lifetime utility (Eq (2.6)*).*

2. Entrepreneurs maximize expected profits (Eq (2.9)*).*

3. Firms maximize profits (Eq (2.1)*,* (2.2)*,* (2.4)*).*

4. All markets are clear.

Regarding the market clearing conditions, there are five markets: capital, sectoral goods, final goods, labor, and credits markets.

$$K_t = \sum_{j=1}^{N} k_{jt}, \tag{2.12}$$

$$Y_{jt} = X_{jt} + \sum_{i=1}^{N} M_{jit}, \forall j \tag{2.13}$$

$$Y_t = C_t + I_t + \sum_{j=1}^{N} \mu_j G_{j,t} R^k_{jt} Q_{t-1} k_{jt}, \tag{2.14}$$

$$H_t = \sum_{j=1}^{N} l_{jt}, \tag{2.15}$$

$$D_{t+1} = \sum_{j=1}^{N} (Q_t k_{j,t+1} - N_{j,t+1}). \tag{2.16}$$

The last term in Eq (2.14) is the total amount of default cost across all sectors, where $\mu_j G_{j,t}$ is the share of sector j's profits paid as default cost at period t (defined in the appendix). These disposed default costs across sectors are the source of credit spreads.

To characterize the general equilibrium, it is helpful to define some allocation variables. Let $\boldsymbol{x_t^k}$ and $\boldsymbol{x_t^l}$ be vectors of capital and labor allocation variables, respectively, so that in equilibrium,

$$x^k_{jt} = \frac{k_{jt}}{K_t}, \quad x^l_{jt} = \frac{l_{jt}}{H_t}.$$

Next, define intermediate inputs allocation variables as η_{jit} for sector j, so that

$$M_{jit} = \eta_{jit} Y_{jt}, \quad X_{jt} = (1 - \eta_{jt}) Y_{jt},$$

where $\eta_{jt} = \sum_{i=1}^{N} \eta_{jit}$. Note that η_{ji} is the allocation for commodity j used as intermediate inputs in sector i. $Y_{jt} = X_{jt} + \sum_{i=1}^{N} M_{jit}$. M_{jit} is the goods of sector j purchased as input by sector i, and X_{jt} is final demand of goods j. Finally, define the influence vector $\{v_{jt}\}_{j=1}^{N}$ as:

$$v_{jt} \equiv \frac{p_{jt} Y_{jt}}{Y_t}.$$

Proposition 1. *In the competitive equilibrium, the influence vector* $\boldsymbol{v}$*, the intermediate inputs allocation matrix* $\boldsymbol{\eta}$*, and the labor allocation vector* $\boldsymbol{x^l}$ *are constants across time.*

$$\begin{aligned}
\boldsymbol{v} =& [I_N - \boldsymbol{\Gamma}]^{-1} \boldsymbol{\beta}, \\
\boldsymbol{\eta} =& [\frac{1}{\boldsymbol{v}} * \boldsymbol{v}'] \circ \boldsymbol{\Gamma}, \\
x_j^l =& \frac{\lambda_j^l}{\sum_i \lambda_i^l},
\end{aligned}$$

where $\lambda_j^l = (1 - m_j)(1 - \alpha_j) v_j$ *is the aggregate output elasticity of sectoral labor, and* $\circ$ *is the Hadamard product for matrices.*

The result that the influence vector, the intermediate inputs allocation matrix, and labor allocation vector are constant arises from the assumptions of constant return Cobb-Douglas technology, perfect competition, and frictionless intermediate inputs and labor markets.

The equilibrium sales share, the influence vector, is determined by the product of Leontief inverse and the final use share. Generally speaking, we can extend Hulten's theorem from technology shock to shocks that affect sectoral output, and the influence vector describes the aggregate output elasticity with respect to sectoral output change, that is,

$$\frac{d \ln Y_t}{d \ln Y_{jt}} = v_j.$$

The labor allocation vector simply states that labor should be allocated proportionally to the aggregate output elasticity of sectoral labor λ_j^l, which is the sectoral output elasticity of labor, $(1 - m_j)(1 - \alpha_j)$, times the aggregate output elasticity, v_j. λ_j^l thus describes the degree of aggregate output change with respect to sectoral labor input change. This result can be treated more generally as a first-order approximation of any homogeneous of degree one production functions under similar environment.

Due to frictions in the capital market, however, the optimal capital allocation variable is not a constant. Denote sectoral marginal revenue product of capital as $MRPK_{jt} \equiv \frac{\partial p_{jt} Y_{jt}}{\partial k_{jt}}$. The following proposition solves the capital allocation in equilibrium.

Proposition 2. *With frictions on capital, the capital allocation vector in the competitive equilibrium is:*

$$x_{jt}^k = \frac{\lambda_j^k (MRPK_{jt})^{-1}}{\sum_i \lambda_i^k (MRPK_{it})^{-1}},$$

where $\lambda_j^k = (1 - m_j)\alpha_j v_j$ *is the aggregate output elasticity of sectoral capital.*

Similarly to the labor allocation vector, λ_j^k is the change of aggregate output with respect to sectoral capital input change. In the first best allocation, capital should be allocated proportionally according to its impact on aggregate output, and MRPK is equalized across sectors. When frictions are present, $MRPK_{jt} \propto R_{jt}^k \propto (1+\tau_{jt}^k)$ in Eq (2.11), and this deviates capital allocation from the first best case. The wedge representation form of Proposition 2 gives

$$x_{jt}^k \propto \frac{\lambda_j^k (1 + \tau_{jt}^k)^{-1}}{\sum_i \lambda_i^k (1 + \tau_{it}^k)^{-1}}.$$

Intuitively, Proposition 2 says that capital is allocated proportionally to λ_j^k and the inverse of distortion relative to the weighted harmonic average distortion, $(\sum_i \lambda_i^k (1 + \tau_{it}^k)^{-1})^{-1}$, along the equilibrium path. A larger than average capital wedge implies that this producer purchases too small amount of capital from a social perspective.

Proposition 2 has several implications. First, a sectoral productivity shock could induce capital reallocation in this inefficient economy. In contrast, in efficient economies, productivity shocks—no matter aggregate or sectoral—do not affect capital reallocaion, since sectoral MRPK is always equalized in equilibrium and x_j^k is a contant. Second, an aggregate productivity shock does not affect allocative efficiency, since in this case sectoral MRPK changes by the same amount. Similarly, if one takes the reduced form approach and considers an aggregate wedge shock, then there would be no resource reallocation. The result that aggregate shocks do not affect allocative efficiency is common in the literature.[12] Finally, however, a systematic uncertainty shock here can possibly induce heterogeneous responses in endoge-

12. See Bigio and La'O (2017) and Baqaee and Farhi (2018).

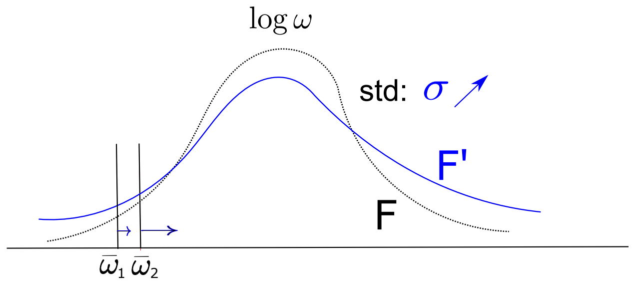

Figure 4: Change of the default region and the thresholds of two sectors when $\mu_2 > \mu_1$.

nous capital wedges across sectors and hence affect allocative efficiency. This happens when sectors have different degrees of default cost, μ_j, and uncertainty levels, σ_j, thus various endogenous threshold, $\bar{\omega}_{jt}$.

In general, in response to the same size of uncertainty shocks, ceteris paribus, sectors with higher degrees of default cost μ_j would have higher increases in capital wedges. This is the key to understanding how an aggregate uncertainty shock affects capital allocative efficiency. To see this point, consider an economy with two sectors, $j = 1, 2$. Both sectors have the same parameters except that $\mu_1 < \mu_2$. In equilibrium, sector 2 bears higher amount of default cost and thus $\bar{\omega}_2 > \bar{\omega}_1, F(\bar{\omega}_2) > F(\bar{\omega}_1)$, and $\tau_2^k > \tau_1^k$. Thus $x_1^k > x_2^k$. Suppose now a systematic uncertainty shock hits the economy such that σ_j in both sectors increase by the same amount. Denote the new cumulative distribution function as $F'(\bar{\omega})$. Then the increase of default region under the original thresholds would be $F'(\bar{\omega}_2) - F(\bar{\omega}_2) > F'(\bar{\omega}_1) - F(\bar{\omega}_1)$. This can be easily seen from Figure 4, where the change of default region is covered among the blue line, the threshold line, and the black dashed line. Thus, to maintain the zero-profit condition of the financial intermediary, $\bar{\omega}_2$ must increase more than $\bar{\omega}_1$ in the new equilibrium, and hence $\Delta\tau_2^k > \Delta\tau_1^k$. Subsequently, capital is reallocated from sector 2 to sector 1.

In words, when a systematic uncertainty shock hits the economy, the relatively more financially constrained sectors (higher capital wedges) are more vulnerable, and capital would flow from these more constrained sectors to less constrained ones, reducing allocative efficiency. The following theorem then solves the aggregate output and TFP in general equilibrium.

Theorem 2. *In the competitive equilibrium with capital misallocation, the solution for the* *total production of aggregate output has the form:*

$$Y_t = \tilde{A}_t K_t^{\tilde{\alpha}} H_t^{1-\tilde{\alpha}},$$

where

$$\ln(\tilde{A}_t) = \boldsymbol{v}' \ln \boldsymbol{A_t} + \boldsymbol{\lambda^{k}}' \ln \boldsymbol{x_t^k} + \textit{constant},$$
$$\tilde{\alpha} = \sum_i \lambda_i^k,$$

$\tilde{A}_t$ is aggregate TFP, and $\boldsymbol{A_t}$ is the vector of sectoral productivity.[13]

The first-order approximation of aggregate TFP change with capital misallocation is

$$\mathrm{d} \ln \tilde{A}_t = \boldsymbol{v}' \mathrm{d} \ln \boldsymbol{A}_t + \boldsymbol{\lambda^{k}}' \mathrm{d} \ln \boldsymbol{x_t^k}. \tag{2.17}$$

Theorem 2 modifies Hulten's theorem under the inefficient economy with capital distortions. Aggregate TFP now depends on the level of misallocation $\boldsymbol{x_t^k}$ and is maxmized when $x_{jt}^k \propto \lambda_j^k$ in the first best allocation.[14] Theorem 2 states that the change of aggregate TFP can be decomposed into two components: the sum of weighted sectoral productivity changes, holding the distribution of capital fixed, and the equilibrium changes of capital reallocation. $\boldsymbol{\lambda^k}$, the aggregate output elasticity of sectoral capital, describes the impact of changes in sectoral capital on aggregate output. The sum of $\boldsymbol{\lambda^k}$ is simply its factor income share, $\tilde{\alpha}$.

Productivity Shock

Theorem 2 implies that under a sectoral productivity shock,

$$\frac{\mathrm{d} \ln \tilde{A}_t}{\mathrm{d} \ln A_{jt}} = v_j + \sum_i \lambda_i^k \frac{\mathrm{d} \ln x_{it}^k}{\mathrm{d} \ln A_{jt}}.$$

That is, in response to a sectoral productivity shock, there are two effects. The first is its direct effect on final demand and its indirect effect through intermediate input markets, sum-

13. The exact formula for aggregate TFP is

$$\ln(\tilde{A}_t) = \boldsymbol{v}' \boldsymbol{a_t} + \boldsymbol{\lambda^{k}}' \ln \boldsymbol{x_t^k} + \sum_i \beta_i \sum_j (1 - \eta_{ij}) + \boldsymbol{\lambda^{l}}' \ln \boldsymbol{x^l} + \sum_j v_j \sum_i \gamma_{ij} \ln \eta_{ij}.$$

14. In the neoclassical framework, capital is predetermined one period ahead, and so does the effect of capital reallocatin on aggregate TFP. The exact relation should be $\tilde{A}_t(\boldsymbol{x_{t-1}^k})$.

marized by υ_j; the second is the weighted average of capital reallocation across all producers when the initial equilibrium is inefficient.[15] Note that an aggregate productivity shock only has the first effect from technology change, but does not have the second effect, since relative MRPK remains the same. That is,

$$\frac{\mathrm{d}\ln \tilde{A}_t}{\mathrm{d}\ln \boldsymbol{A_t}} = \sum_j \upsilon_j = \upsilon_A.$$

Uncertainty Shock

However, uncertainty shocks, no matter aggregate or idiosyncratic, always affect allocative efficiency and hence aggregate TFP. And the impact of uncertainty shocks on aggregate TFP purely comes from the second effect, changes in allocative efficiency. Note that the second effect is the weighted sum of capital reallocation across all sectors. For example, under an idiosyncratic uncertainty shock,

$$\frac{\mathrm{d}\ln \tilde{A}_t}{\mathrm{d}\ln \sigma_{jt}} = \sum_i \lambda_i^k \frac{\mathrm{d}\ln x_{it}^k}{\mathrm{d}\ln \sigma_{jt}}.$$

And for a systematic uncertainty shock, we have that

$$\frac{\mathrm{d}\ln \tilde{A}_t}{\mathrm{d}\ln \boldsymbol{\sigma_t}} = \sum_j \lambda_j^k \sum_i \frac{\mathrm{d}\ln x_{it}^k}{\mathrm{d}\ln \sigma_{jt}}.$$

Unlike the effect of an aggregate productivity shock, where all sectors have same responses and maintain relative MRPK, sectors can have heterogeneous responses on endogenous wedges under a systematic uncertainty shock, and this induces a change in allocative efficiency. To build intuition, first consider the impact of an aggregate uncertainty shock. Let's continue the two sector example in Proposition 2, where $\lambda_1^k = \lambda_2^k = \lambda^k$. After being hit by a systematic uncertainty shock, we have $\Delta\tau_2^k > \Delta\tau_1^k$. Some amount of capital is reallocated from sector 2 to sector 1, $\Delta x_1^k > 0, \Delta x_2^k < 0$, and since in the initial equilibrium $x_1^k > x_2^k$, the percentage deviations would be that $|\frac{\Delta x_1^k}{x_1^k}| < |\frac{\Delta x_2^k}{x_2^k}|$. So the aggregate TFP change is that

$$\mathrm{d}\ln \tilde{A} = \lambda^k \underbrace{\mathrm{d}\ln x_1^k}_{>0} + \lambda^k \underbrace{\mathrm{d}\ln x_2^k}_{<0} < 0$$

15. This second term is related to the cross-entropy used in Baqaee and Farhi (2018).

In words, the relatively more financially constrained sectors suffer more under a systematic uncertainty shock, and this deepens allocative inefficiency, reducing aggregate TFP.

Next, consider the impact of idiosyncratic uncertainty shocks on the above two sector economy. We have

$$\frac{\mathrm{d}\ln \tilde{A}}{\mathrm{d}\ln \sigma_1} > 0, \quad \frac{\mathrm{d}\ln \tilde{A}}{\mathrm{d}\ln \sigma_2} < 0.$$

That is, an idiosyncratic uncertainty shock to the relatively less constrained producers would raise aggregate TFP, since it reduces the relative distortion between sectors by making sector 1 (lower MRPK) more financially constrained and allowing more capital being purchased by sector 2 (higher MRPK). In contrast, an idiosyncratic uncertainty shock to sector 2 would further deepen the degree of relative distortion, reducing aggregate TFP.

In general, changes in sectoral capital reallocation are weighted by their impact on aggregate output. Eq (2.17) shows that changes in aggregate TFP is maximized when $\lambda_j^k \propto \mathrm{d}\ln x_j^k$. This impact is summarized by technology parameters and input-output linkages, i.e., $\lambda_j^k = (1 - m_j)\alpha_j v_j$, while the changes of reallocation, $\mathrm{d}\ln x_j^k$, depends on the degree of financial frictions. Proposition 2 and Theorem 2 thus imply that, under uncertainty shocks, changes in allocative efficiency affects aggregate TFP the most when the relatively more financially constrained sectors are at the same time also more capital intensive, supplying more intermediate inputs to the economy (central suppliers), or having higher final demands. Changing the structure of input-output linkages may amplify or dampen the effect of allocative inefficiency on aggregate TFP, depending on the interaction between producers' location in production networks and their degree of financial frictions.

To summarize, Theorem 2 modifies Hultens's theorem in an inefficient economy and decomposes aggregate TFP change into two effects: changes in productivity and changes in allocative efficiency. While productivity shocks may have both effects, uncertainty shocks only has the second effect. An aggregate uncertainty shock always deepens allocative inefficiency, reducing aggregate TFP, while an idiosyncratic uncertainty shock may improve or worsen allocative efficiency, depending on the result of resource reallocation. Finally, whether input-output linkages amplify the effect of sectoral distortion depend on both a sector's degree of financial friction and its impact on aggregate output, and thus requires an empirical test. This is the subject of the quantitative exercises in Section 4.

3 Data, Parameter Values, and Estimation

In this section, I first present data used in the paper. I then calibrate parameter values to match data moments and discuss parameter interpretations. Finally, I estimate the autoregressive process of shocks by the maximum likelihood estimation (MLE).

3.1 Data Description

3.1.1 The Input-Output Accounts

I calibrate the model to the U.S. input-output matrix at the two-digit NAICS level, which corresponds to the direct requirement table at the sector level in the Input-Output Accounts (I-O) in the Bureau of Economic Analysis (BEA). Since there is no government in the model, I exclude the government sector (NAICS 9). I also exclude the banking sector (NAICS 521-522) from the estimation of input-output coefficients (γ_{ij}) because the model counterpart of the banking sector in the data is the financial intermediary, which does not involve production. Figure 5 is the Hinton diagram of the U.S. input-output matrix, where the area occupied by a square is proportional to the intermediate input share's value.

The row sum of the input-output matrix, which is called weighted sectoral out-degree, measures the total intermediate inputs one sector supplies to the entire economy. In the United States, the largest three common suppliers are the Manufacturing, FIRE, and Professional and Business (P&B) sectors. Figure 6 plots the weighted out-degree of each sector. The out-degree distribution exhibits a heavy tail at the high end, even at this broad level of disaggregation.[16]

The BEA uses the same accounting procedure to calculate labor cost shares in the I-O accounts as in the NIPA, and this tends to underestimate the true labor share, as pointed out in Krueger (1999) and Gomme and Rupert (2004). The issue is that proprietors' income, a sub-item in the gross operating surplus, includes components of both labor and capital income. Most studies assign two-thirds of proprietors' income as labor income from the NIPA, and the resulting value of the labor share ranges from 0.6 to 0.7. However, the amount of proprietors' income is not listed in the I-O accounts. The BEA only reports values of the noncorporate component of other gross operating surplus, and there is ambiguity to how

16. Acemoglu et al. (2012) exhibit the heavy tail phenomenon on the out-degree distribution at the disaggregation level of 523 sectors.

Figure 5: The Hinton Diagram of the U.S. Input-Output Matrix

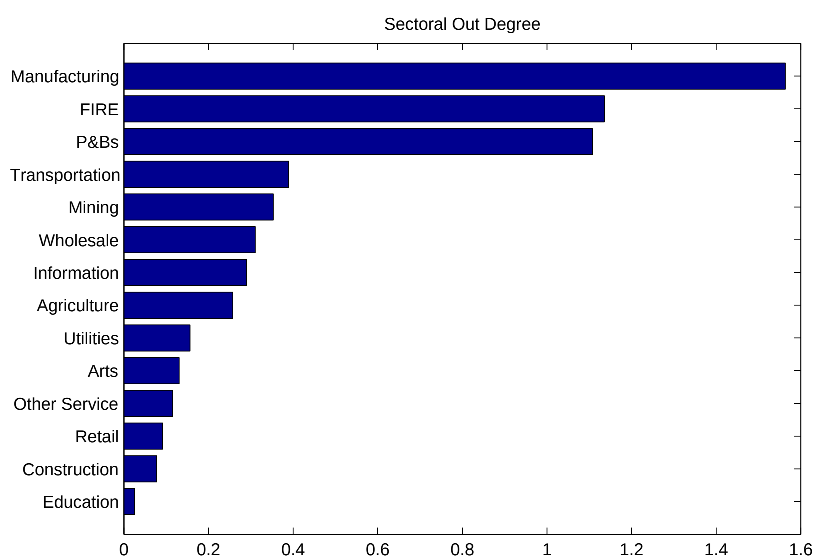

Figure 6: Sectoral Out-Degree

much should be assigned as labor income in the noncorporate component.[17] Since the BEA

17. In the direct requirement table, proprietors' income is included in the noncorporate component of other

started to publish the I-O tables annually after 1997, I use the annual direct requirement tables from 1998 to 2012. I chose the fraction of one half to match the aggregate labor share value of 0.62. This value fits the time average of the declined labor share in this period. That is, in dealing with the I-O accounts:

Labor share= Compensation of employees + 1/2 of the noncorporate part of other gross operating surplus.

Capital share = Gross operating surplus + taxes on production and imports less subsidies - 1/2 of the noncorporate part of other gross operating surplus.

To calibrate the final use share β_j, I use data reported in the annual I-O use tables. Denote final use expenditure in the data as $fe_{j,t}$, where $t = 1998, \ldots, 2012$, and $j = 1, \ldots, 14$. The final use expenditure here is the sum of personal consumption expenditure, private fixed investment, and changes in private inventories reported in the use table. I do not include final uses in exports, imports, and government expenditure here.[18] Thus, the estimated final use share for each year is:

$$\hat{\beta}_{jt} = \frac{fe_{jt}}{\sum_{j=1}^{14} fe_{jt}}.$$

And the estimated final use share is the time average over the sample period (15 years).

3.1.2 Financial Data

For financial parameters, three types of financial data are used: sectoral leverages, sectoral corporate bond spreads, and sectoral corporate bond default rates. Sectoral leverage is calculated from *Compustat*. Although I exclude the banking sector, all other firms in the FIRE sector still partially serve other types of intermediation, and they tend to bear higher leverages than firms in other production sectors. However, this high leverage in the FIRE sector does not directly correspond to its counterpart in the model, in which the leverage is an equilibrium result from default possibility, default costs, and the expected return on capital. The common measure of leverage in the literature, *balance sheet leverage* (total assets divided

gross operating surplus, which is a sub-item of gross operating surplus. The noncorporate component consists of proprietors' income without adjustments, proprietors' income with inventory valuation adjustment, rental income of persons without capital consumption adjustment, noncorporate capital consumption allowance, and noncorporate net interest.

18. The issue is that when import and export uses are taken into account, the final use of the Utilities sector is negative, since oil and gas are mostly imported. The model assumes that the value of β_j is larger than 0 because there are no imports and exports in the model.

by shareholders' equity), is not appropriate here. Since most financial firms use short-term debt to finance operations, I use the *financial leverage*, defined as [1 + financial debt/equity], to exclude short-term debt.[19] This measure of leverage better reflects the degree of external finance used for investment. I first take the median value of firms in each sector in every year, from 1985 to 2012, and then take the time average.

Sectoral bond spreads are derived from the *TRACE* database, using the period from 2003 to 2014Q1. I choose the spread between the yields of investment grade corporate bonds with around a 10-year maturity and the Treasury bill rate with a constant 10-year maturity. I select only the plain vanilla fixed coupon bounds. I first take the median value of spreads of all daily transactions in a month for each sector, and then take time average as the steady state sectoral spread value. Figure 1 depicts sectoral bond spreads of selected sectors (the two with the highest Domar weights) and the sectoral average during the sample period. To calculate the steady state value, I choose the time series average of the period of 2003Q1 to 2007Q2 and 2010Q1 to 2014Q1, disregarding the period of turmoils. In the quantitative analysis of Section 4, I use these credit spread spikes during the recession as the criterion to catch the size of uncertainty shocks.

Finally, the sectoral corporate bond default rates are reported in Moody's 2010 annual default study. They are historical default rates from 1985 to 2010, and I take the time series average. The financial data is reported in Table 1.

3.2 Calibration and Interpretation

3.2.1 Calibration Results

Table 2 lists parameters' values and their empirical targets (bold letters are vectors). For preference parameters, the coefficient of constant relative risk-aversion, ϱ, is set at 1.5, and the Frisch elasticity of labor supply, ϵ, is set at 1. The value of the Frisch elasticity I chose is larger than the estimated value from micro literature (ranging from 0.1 to 0.4), but smaller than the estimated value from macro literature (ranging from 2 to 3). The value of 1 is appropriate when the effect of unemployment is taken into account as pointed in Bigio and La'O (2017). Table 3 reports values of technology parameters.

19. This definition of financial leverage is used in accounting. Financial debt is long-term debt (DLTT) plus debt in current liabilities (DLC). Capitalized abbreviations in this section are *Compustat* mnemonics.

	Default(%)	Spread(bps)	Leverage
Agriculture	2.5	140	1.7
Mining	2.5	125	1.45
Utilities	0.2	107	2.1
Construction	2.5	187	1.8
Manufacturing	2.5	92	1.4
Wholesale	2.5	148	1.7
Retail	3	89	1.6
Transportation	2.2	126	1.9
Information	3.8	115	1.5
FIRE	0.7	140	2.1
P&Bs	2.9	134	1.4
Education	3.8	117	1.7
Arts	2.9	116	1.8
Other services	2.9	77	1.8

Table 1: Summary of financial data.

Figure 7 shows the values of calibrated influence vector, $\boldsymbol{v} = [\boldsymbol{I_N} - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta}$. We can see that the Manufacturing, FIRE, and P&B sectors are the three most influential sectors, in the sense that they are common suppliers in the input-output network, and also contribute large final use shares in the final goods production. We should expect that idiosyncratic sectoral shocks from these three sectors would have a larger impact on aggregate output than other sectors.

3.2.2 Calibration of Financial Parameters

Spreads, leverages, and default rates in Table 1 are used as targets in Eq (2.8), (2.10), and default probability ($F(\bar{\omega_j})$) in the model to pin down two vectors of parameters, $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$, and a vector of endogenous thresholds $\bar{\boldsymbol{\omega}}$ at the steady state. Entrepreneurs' survival rates $\boldsymbol{\kappa}$ are then backed out from the law of motion of net worth (Eq (A.3) in the appendix) at the steady state. Capital wedges at the steady state can then be backed out from Eq (2.10) (or Eq (A.4)) by using the sectoral corporate bond spreads. The result is reported in Table 4.

Parameter	Meaning	Target	Value
		Preference	
$\tilde{\beta}$	discount factor	3% Real Interest Rate (APR)	0.993
ϱ	coefficient of CRRA		1.5
ϵ	Frisch elasticity of labor supply		1
		Technology	
$\boldsymbol{\alpha}$	capital shares	Direct Requirement Table	Table 3
$\boldsymbol{\Gamma}$	Input-Output linkages (matrix)	from Input-Output Account	
$\boldsymbol{\beta}$	Final Use Share	Final Use in the BEA Use Table	
θ	capital adjustment cost		0.9
δ	capital depreciation rate		0.02
		Financial Parameters	
$\boldsymbol{\mu}$	bankruptcy cost	Sectoral Default rates &	Table 4
$\boldsymbol{\sigma}$	uncertainty	Corporate bond spreads &	
$\boldsymbol{\kappa}$	net worth transferring rate	Leverages	

Table 2: Parameters and Calibration Targets

Sector	Capital Share	Labor Share	Int. Inputs Share (m_j)	Final Use β
Agriculture	0.16	0.24	0.60	0.005
Mining	0.36	0.24	0.40	0.006
Utility	0.41	0.17	0.42	0.02
Construction	0.12	0.41	0.47	0.07
Manufacturing	0.13	0.21	0.66	0.19
Wholesale	0.32	0.38	0.30	0.04
Retail	0.26	0.41	0.33	0.09
Transportation	0.16	0.36	0.48	0.02
Information	0.26	0.27	0.47	0.04
FIRE	0.35	0.27	0.38	0.20
P&B	0.13	0.51	0.36	0.05
Education	0.08	0.52	0.40	0.15
Arts	0.19	0.38	0.43	0.06
Other services	0.15	0.53	0.32	0.04

Table 3: Technology Parameters

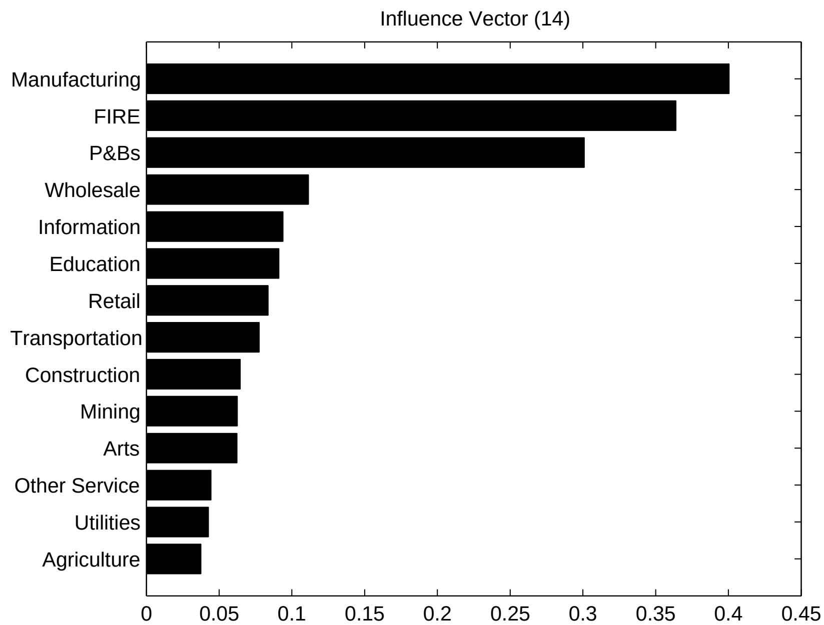

Figure 7: Influence Vector/ Domar Weights

Sector	μ	σ	κ
Agriculture	0.51	0.33	0.95
Mining	0.43	0.43	0.96
Utilities	1	0.22	0.94
Construction	0.72	0.29	0.94
Manufacturing	0.27	0.44	0.96
Wholesale	0.54	0.33	0.97
Retail	0.20	0.38	0.95
Transportation	0.52	0.27	0.95
Information	0.19	0.43	0.97
FIRE	1	0.23	0.95
Professional and Business services	0.37	0.49	0.96
Education	0.28	0.35	0.96
Arts	0.33	0.30	0.95
Other services	0.18	0.32	0.96

Table 4: Financial Parameters

3.2.3 Interpretation of Financial Parameters

The calibration of financial parameters at the disaggregated sectoral level is an important contribution of this paper. It is worth examining and explaining the calibration results in more detail here. The majority of values reported in Table 4 fall in the conventional range. The conventional values used in the previous literature are the result under a representative firm framework and thus correspond to the aggregate economy.[20] The "unconventional" values in Table 4 are the result from disaggregated sectoral data. Since the entrepreneur exit rate, κ, is a modeling technique to avoid infinite accumulation of net worth, and the calibrated values fall within the conventional range, I focus the discussion in the following on the calibration results of μ and σ.

First, the bankruptcy cost, μ, reflects all economic costs associated with the event of default in the model, and there is no appropriate and direct measure in the empirical counterpart. For example, the often used recovery rate only measures the extent to which the defaulted debt can be recovered, not including the effort and cost of lenders involved in the debt recovery.

An important result regarding μ is that the values in the FIRE and Utilities sectors are particularly high (one). This extreme value of bankruptcy cost comes from data that shows these two sectors have very low historical default rates, a medium level of spreads, and high leverages (Table 1). The high leverage and extreme low default rate imply that the two sectors are considered safe, and lenders therefore are willing to lend a lot to these sectors. But why is the price of debt high if they are safe borrowers? In the model, from Eq (A.2) and (A.4), the spread($z_{j,t+1}/R_{t+1}$) is increasing in the endogenous default rate ($F(\bar{\omega}_{j,t+1})$) and the level of bankruptcy cost (μ_j). The model shows that this is the result of these sectors having a very high level of bankruptcy cost. Some may argue that the observed low default rate in the FIRE sector may be due to government intervention or historical bailouts. But the reason behind government intervention is the enormous bankruptcy cost—and not the other way around. In addition, industries in the Utilities sector include electric power generation (fossil fuel, nuclear, and hydropwoer) and water and natural gas distribution, which also bear high bankruptcy costs. These high bankruptcy costs imply that these two sectors are particularly vulnerable to uncertainty shocks.

There is another pair of sectors worth discussing here. In Tables 3 and 1, the Wholesale and Retail sectors are similar in their technology parameters, default rate, and leverage, but

20. In Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), and Nolan and Thoenissen (2009) and Christiano, Motto, and Rostagno (2014), the calibrated values of the survival rate (κ) range from 0.94 to 0.99, the values of the uncertainty level (σ) range from 0.2 to 0.4, and the values of bankruptcy cost (μ) range from 0.2 to 0.36.

Wholesale has a higher spread than Retail. Given the financial data, the model then implies that Wholesale should have a higher level of bankruptcy cost. My explanation is that the main difference between these two sectors is their out-degree (Figure 6). All other things held equal, the sector occupying a more central position in the production network would induce a larger cost if firms in that sector fail. This larger economic cost shows up as a larger bankruptcy cost in the model.

Second, the standard deviation of idiosyncratic capital returns, σ, reflects the riskiness of investment, and there is a caveat to how we should interpret σ when we map the model into data. In the model, σ mostly affects the leverage, which is an endogenous outcome of the optimal contract. It is the balance between the credit demand (the dependence on external finance of a sector) and supply. Since entrepreneurs behave as if they are risk-neutral, the dependence on external finance channel is missing in the model. Therefore, the credit is mainly constrained by the supply side. That is, lenders are less willing to lend if the investment return is risky—the endogenous leverage is decreasing in σ. The calibration then maps a low leverage in the data to high σ in the model, and vice versa. However, in the real world, a firm may have low leverage because it does not require much external finance. It is thus important to consider the dependence on external finance of each sector when we examine the appropriateness of calibrated σ.

To do this, I follow the procedure described in Rajan and Zingales (1998) to construct a measure (RZ) of dependence on external finance for each sector.[21] Table 5 reports the equilibrium leverage in data, calibrated σ, and the RZ measure, sorted by the leverage in descending order. The RZ measure of 14 sectors ranges from 1.28 (Minging) to 0.17 (FIRE), and the median is 0.53. Here, using the median as the benchmark, sectors depend relatively more on external finance if their RZ measures are larger than 0.53. Similarly, a sector has high (low) leverage if its leverage is larger (smaller) than the median value of 1.7.

Since the model indicates that a high leverage sector is one with safe investment returns (low σ), a low RZ measure (small demand) would confirm this interpretation, and a high RZ measure suggests the model may overstate its safety. Consider the high leverage sectors, for example, the FIRE and Construction sectors have high leverages and low RZ measures, and a low value of σ_j is considered reasonable. But the values of σ_j for the Utilities, Transportation,

21. The Rajan-Zingales (RZ) measure is defined as capital expenditure minus cash flow from operations divided by capital expenditures. To calculate this, I first took the median value of firms in each sector and year in *Compustat* and then took the average from 1985 to 2012 for each sector.

	Leverage	σ	RZ
FIRE	2.10	0.23	0.17
Utilities	2.09	0.22	0.88
Transportation	1.97	0.27	0.86
Construction	1.88	0.29	0.48
Arts, Entertainment	1.84	0.30	0.89
Other services	1.76	0.32	0.48
Wholesale	1.73	0.33	0.38
Agriculture	1.70	0.33	0.58
Education and Health Care	1.70	0.35	0.53
Retail	1.58	0.38	0.68
Information	1.50	0.43	0.48
Mining	1.45	0.43	1.28
Manufacturing	1.43	0.44	0.47
P&Bs	1.37	0.49	0.40

Table 5: Equilibrium leverage, σ, and dependence on external finance (RZ).

and Arts sectors might be underestimated.

For those low leverage sectors, the risky return interpretation (high σ) is confirmed by a large value of the RZ measure, such as in the Mining sector. The Mining sector has a low leverage but a high RZ measure, reflecting higher demand on external finance but relatively smaller supply, so the risky investment interpretation—a high value of σ_j—is reasonable.[22] But for the Information, Manufacturing, and P&B sectors, one cannot rule out the possibility that they have low leverage simply because they do not need much external finance. Furthermore, the model might overestimate the values of σ_j for these sectors. As a result of this concern, I only make conclusions about sectors with reasonable matches between leverages and RZ measures regarding uncertainty shocks in the quantitative analysis. More specifically, my result in Section 4 that the FIRE sector has the largest sectoral impact is backed with a reasonable value of σ here.

22. Why are investments in the Mining sector risky? An important feature in the mining and oil industries is that their investment returns are highly related to oil price fluctuations. A plunge in the price of oil hurts mining and energy firms' profits and returns, and the default rate and spreads of their bonds often rise during these periods. Kellogg 2014 points out that firms' failures to respond to oil price volatility can lead to a significant cost (25 percent of the value of a drilling well in Texas). Since the price of oil is volatile, uncertainty in Mining investment returns is high, and this confirms the model's risky return interpretation.

3.3 Estimation of Shock Process

Since Bloom et al. (2018) has suggested that recessions are best modelled as being driven by shocks with a negative first moment and a positive second moment, to better estimate the model, I consider both productivity shocks and uncertainty shocks as the following. I assume sectoral productivity (Eq (2.2)) and sectoral uncertainty follow autoregressive (AR) process:

$$\ln(A_{j,t+1}) = \rho_j^a \ln A_{j,t} + \epsilon_{j,t+1}^a, \tag{3.1}$$

$$\ln \frac{\sigma_{j,t+1}}{\sigma_{j,ss}} = \rho_j^\sigma \ln \frac{\sigma_{j,t+1}}{\sigma_{j,ss}} + \epsilon_{j,t+1}^\sigma, \tag{3.2}$$

where $\sigma_{j,ss}$ is sector j's steady state value of the standard deviation of idiosyncratic entrepreneur's ω_j^i shocks, ρ_j^a and ρ_j^σ are autoregressive parameters, and $\epsilon_{a,t+1}$ and $\epsilon_{\sigma,t+1}$ are mutually independent and identically distributed random variables drawn from a zero mean normal distribution with the covariance matrix Ω_a and Ω_σ respectively. Instead of trying to identify sectoral and aggregate shocks, I estimate all elements of Ω_a (and Ω_σ) from the data.

I use sectoral gross output, sectoral financial leverage, aggregate investment, real interest rate, and real GDP from 1987 to 2012 to estimate autoregressive parameters and the corresponding covariance matrices by the MLE.[23] Prior to analysis, the data are transformed as follows. Sectoral gross output, aggregate investment, and GDP are transformed into real per capita term. I then take the logarithmic difference and remove the sample mean for each variable. The leverage and real interest rate are measured in level terms (%) net of their sample mean.[24] My implementation directly follows that of Ireland (2004).

23. Sectoral corporate bonds' spreads most distinguish the effect between productivity shocks and uncertainty shocks and should be included in MLE if data is available. However, the data I extracted from the *TRACE* is incomplete in time series, and cannot be used in MLE. For example, some sectors do not have bond trade records until 2004, and trade frequency decreases a lot during the period of the 2007-2008 financial crisis. As a result, the current estimation would underestimate the importance of uncertainty shocks. However, this limitation in data availability does not handicap the main purpose of this paper, which is to quantify the amplification magnitude from input-output linkages. Despite this shortcoming, the sample means can still be used to approximate the steady state values of credit spread in the model counterpart. Since data in credit spread is not useable, I chose to use sectoral financial leverage, as uncertainty shocks can generate procyclical leverage, while productivity shocks cannot.

24. As in McGrattan, Rogerson, and Wright (1997), I assign a measurement error for each observation variable and assume they are uncorrelated across variables. That is, I denote these measurement errors at time t as $\boldsymbol{u_t}$, and it follows that $\boldsymbol{u_t} = \boldsymbol{D}\boldsymbol{u_{t-1}} + \boldsymbol{\xi_t}$, where $\boldsymbol{\xi_t}$ is a vector of mean zero serially uncorrelated innovations and is normally distributed with a covariance matrix $E[\boldsymbol{\xi_t}\boldsymbol{\xi_t'}] = \boldsymbol{V}$. The matrix $\boldsymbol{D}$ and $\boldsymbol{V}$ are diagonal. The autoregressive parameters are optimized around 0.8, and standard deviations of innovations are 0.4%. I chose these values to allow for structural shocks to explain most of the variations in observation

Sector	ρ_a	ρ_σ	Ω_a (%)	Ω_σ (%)
Agriculture	0.89	0.95	0.84	8.48
Mining	0.88	0.94	1.00	7.52
Utility	0.89	0.94	0.87	7.54
Construction	0.89	0.94	1.03	7.54
Manufacturing	0.89	0.94	0.29	7.58
Wholesale	0.89	0.94	1.10	7.58
Retail	0.89	0.94	1.18	7.81
Transportation	0.89	0.94	0.87	7.61
Information	0.90	0.94	0.66	7.59
FIRE	0.89	0.94	0.82	7.58
Professional and Business services	0.90	0.94	0.57	7.64
Education	0.89	0.94	0.50	8.63
Arts	0.89	0.94	0.69	7.67
Other services	0.89	0.95	1.13	8.65

Table 6: Estimation results for shocks

I first calibrate the model and use the above calibrated parameter values to run the MLE for shock parameters. The estimation result and the diagonal elements of covariance matrices are reported in Table 6. Two remarks are in order. First, the main message in Table 6 is that the standard deviations of sectoral uncertainty shocks are roughly ten times larger than that of sectoral productivity shocks. This result is consistent with Christiano, Motto, and Rostagno (2014).

Second, with financial frictions on capital and convex capital adjustment cost, it is harder for the household to smooth consumption through production, and the model economy would be more volatile. One could use preferences with habit formation to reduce consumption volatility, but this is not the main focus of this paper. Instead, the main purpose is to explore the amplification magnitude from input-output linkages and the sectoral impact on aggregate output.

variables.

4 Quantitative Analysis

4.1 Changing the Degree of Financial Frictions

I first do a numerical demonstration to echo the implication of Proposition 2 that sectors with higher degree of default cost would have higher increase in capital wedges in response to uncertainty shocks. For expositional convenience, I use a single sector version of the model, calibrated to all the aggregate moments in Section 3. I vary the value of the default cost parameter, μ, from 0 to 1 and show its impulse response with respect to uncertainty shocks in Figure 8. $\mu = 0$ refers to the frictionless economy, and the model has no response with respect to uncertainty shocks. Without any default cost, the endogenous wedge is zero, i.e., entrepreneurs always borrow at the risk-free rate, and MRPK is equalized across sectors.

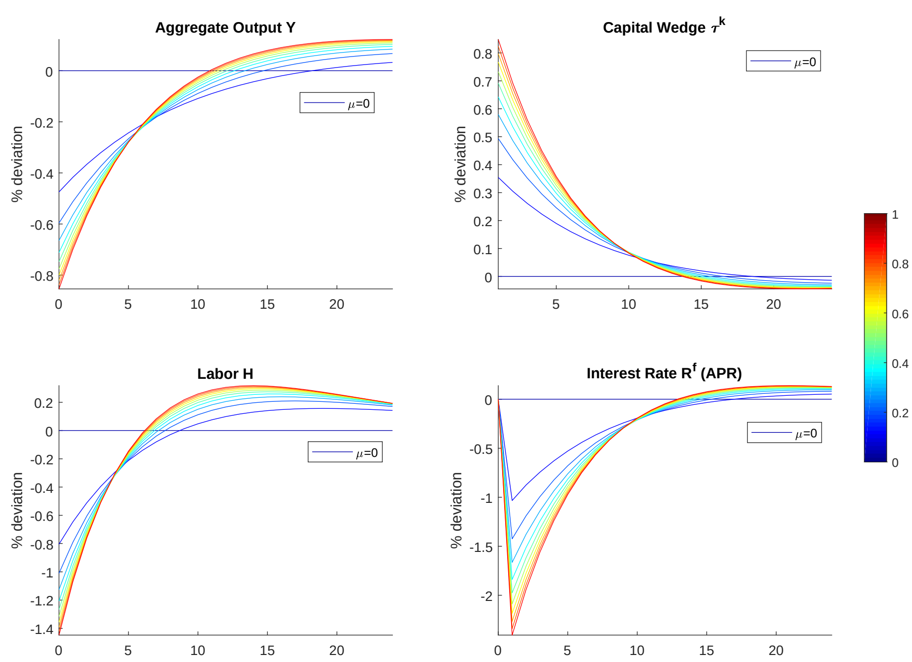

Figure 8: The impulse responses to uncertainty shocks with different μ

The general equilibrium response of Figure 8 is the following. When uncertainty level increases, default cost rises, hence the loan rate and capital wedge. Facing a higher borrowing cost, demand for credit declines. To clear the credit market, the equilibrium interest rate thus decreases, and labor supply decreases due to the intertemporal substitution effect. The decrease in equilibrium labor is the pure cause of the drop in aggregate output in this single sector economy. Figure 8 shows that all the responses are increasing in μ.

4.2 Amplification on Aggregate Shocks

To distangle and quantify the amplification magnitude from input-output linkages, I compare the main model with an equivalent multisector model without linkages, where transmission of sectoral shocks via intermediate inputs flows has been shut down. The no-linkage multisector economy is constructed as in Horvath (2000) that all sectors purchase intermediate inputs from another independent sector, which produces from its own endowment and is not subject to any stochastic variation. The production function follows Eq (2.2) but now γ_{ij} is modeled as m_j. Both economies share the same parameters values on sectoral capital and labor share, final demand, and financial frictions. So both economies are subject to the same degree of allocative inefficiency in their steady states.

In the following exercises, I first consider an aggregate uncertainty shock that drives up the average spread by about 3% from its steady state value (from 1.4% to 4.4%), as observed during the recent recession in Figure 1. This amounts to an increase of capital return volatility about two standard deviations (20%). In the second exercise, I combine a negative first-moment shock that replicates a 5.8% drop in aggregate output during the recession.

4.2.1 An Aggregate Uncertainty Shock

In all the following exercises, shocks hit the model economies at the first period. In response to an aggregate uncertainty shocks, aggregate TFP drops in both economies due to changes in allocative efficiency as in Eq (2.17), but with different magnitudes.

In the left panel of Figure 9, aggregate TFP drops in the second period after the shock because capital is predetermined one period ahead. In the right panel, capital misallocation is measured as the standard deviation of MRPK across all sectors relative to its steady state value. Both economies have similar degree of changes in capital misallocation, $\mathrm{d}\ln \boldsymbol{x}_{\boldsymbol{t}}^{\boldsymbol{k}}$, which is not surprising since both share the same parameter values on financial frictions. However, the input-output economy peaks the aggregate TFP drop by 70% compared to it in the no-linkage economy (the ratio of -0.29% to -0.17% at period 2). This large magnitude is from the difference of $\boldsymbol{\lambda}^{\boldsymbol{k}}$ between two model economies. In the input-output economy,

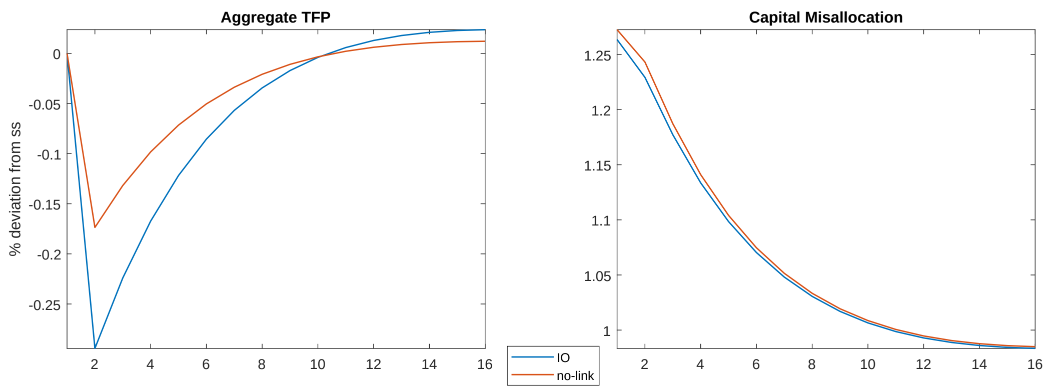

Figure 9: The impulse responses of aggregate TFP and capital misallocation (MRPK dispersion) under an aggregate uncertainty shock.

$\boldsymbol{\lambda^k}_{\text{IO}} = (1-\boldsymbol{m})\circ\boldsymbol{\alpha}\circ([I_N-\boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta})$, while in the no-linkage economy, $\boldsymbol{\lambda^k}_{\text{no-link}} = (1-\boldsymbol{m})\circ\boldsymbol{\alpha}\circ\boldsymbol{\beta}$. That is, when we shut down input-output linkages, the dynamic response of the no-linkage economy behaves as if the influence vector has been changed from $\boldsymbol{v} = [I_N - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta}$ to $\boldsymbol{\beta}$.

Denote the aggregate input-output multiplier for uncertainty shocks as υ_σ. Then $\upsilon_\sigma = \sum_j v_j \mathrm{d}\ln x^k_{jt} / \sum_i \beta_i \mathrm{d}\ln x^k_{it}$=1.7 in the calibrated model. υ_σ describes the amplification magnitude from input-output linkages on the impact of allocative efficiency (the second effect in Eq (2.17)). Thus, input-output linkages can generate significant amplification on the effect of allocative inefficiency in reducing aggregate TFP.

Next, let me discuss the differences of other general equilibrium effects between two economies in Figure 10. Aggregate output in the input-output economy drops by more than 130% to it in the no-link economy in the second period (the ratio of -1.63% to -0.71%), resulting from the larger declines in aggregate TFP and equilibrium labor in the input-output economy, while average spreads in both economies increase by 3%. Equilibrium labor declines and consumption increases due to the intertemporal substitution effect.[25] The larger decrease in the equilibrium interest rate and increase in consumption implies that the household in the input-output economy is facing a larger intertemporal substitution effect, inducing a larger shift in labor supply. In addition to this, transmission of shocks through input-output linkages also induces a relatively larger decline in labor demand. This can be seen after we control the intertemporal substitution effect. Figure 11 shows the

25. This counterfactual negative correlation between consumption and output is common in the uncertainty shock literature, e.g., see Bloom (2009) and Bloom et. al. (2018). This counterfactual relation could be muted by introducing inflation or nominal rigidities as in Christiano, Motto, and Rostagno (2014), or adding negative first-moment shocks as in Bloom et. al. (2018).

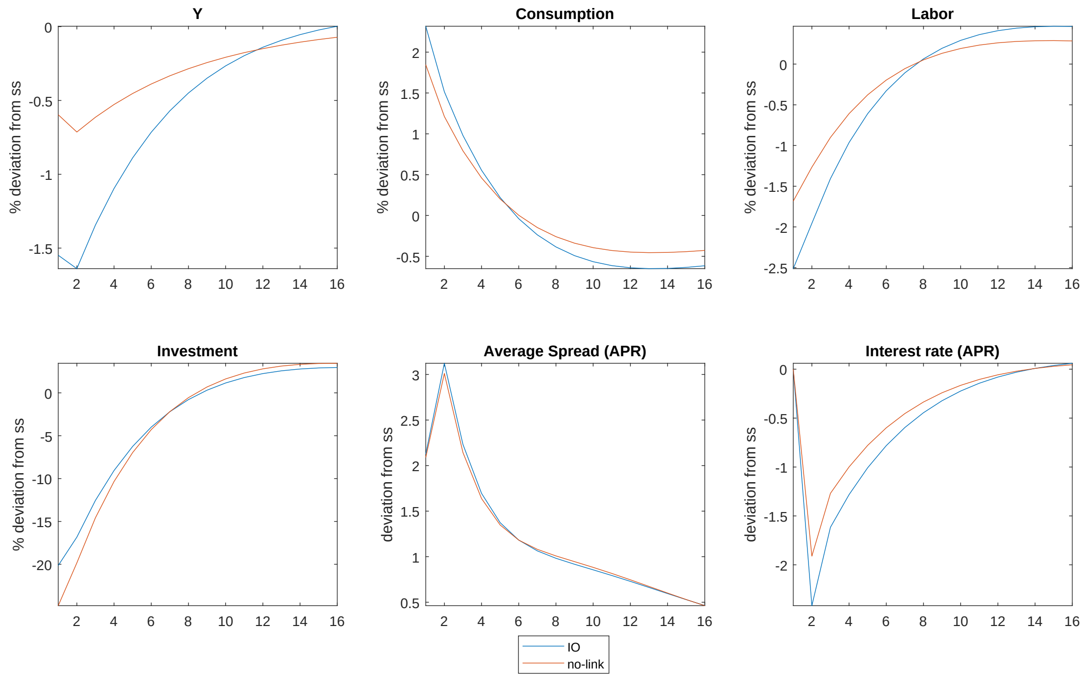

Figure 10: Impulse response functions under an aggregate uncertainty shock.

impulse response functions of the two economies, while the input-output economy is now hit by a smaller uncertainty shock such that two economies have the similar response in the interest rate. The concurrence of the impulse response of consumption indicates that now both economies have the similar degree of intertemporal substitution effect. Since the shifts in labor supply now are the same between the two economies, the smaller increase in equilibrium wage and the larger decline in equilibrium labor implies that labor demand shifts (decreases) relatively more in the input-output economy. This is because decreases of intermediate inputs from linkages also lower marginal product of labor (due to the use of Cobb-Douglas technology) and hence labor demand, while in the no-link economy, the amount of intermediate inputs is fixed. Overall, all the above effects combined give the ratio of the peaks of aggregate output drops between two economies to the value of 2.29, a significant amplification magnitude.

4.2.2 Adding a First-Moment Shock

By adding input-output linkages, uncertainty shocks that increase capital misallocation can generate a significant aggregate output drop of 1.6%. However, a pure uncertainty shock

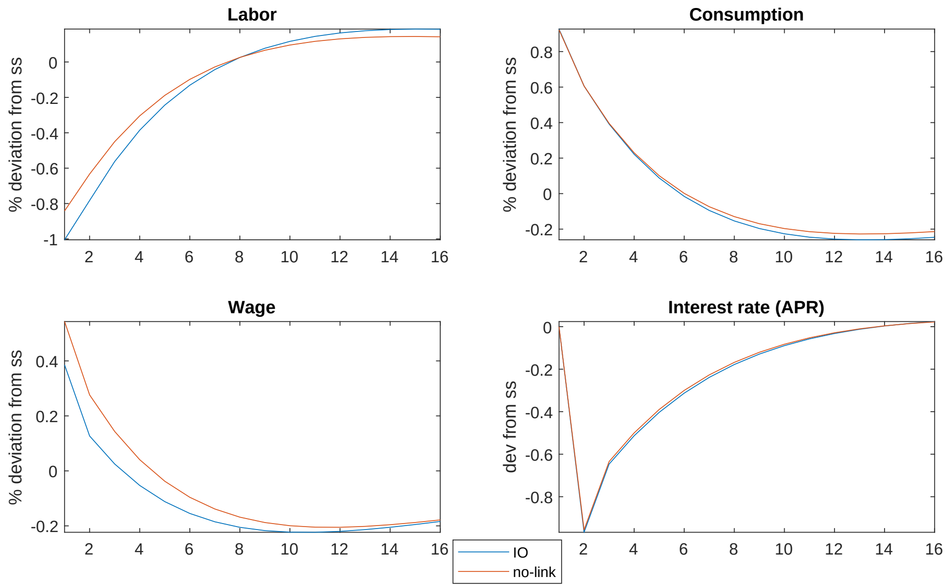

Figure 11: Impulse response functions under an aggregate uncertainty shock, controlling the intertemporal substitution effect.

has a counterfactual feature that consumption is countercyclical. Thus, I conduct a similar exercise as in Bloom et al. (2018): add a first-moment shock. To generate an empirically relevant exercise that match the aggregate output drop during the recent recession, I consider a combination of an aggregate uncertainty shock (20%) and a -2% aggregate TFP shock. By adding a TFP shock, the first effect in Eq (2.17) kicks in. The aggregate input-output multiplies of productivity shock, υ_A, now amplifies the aggregate TFP change. $\upsilon_A = \sum_j \upsilon_j / \sum_i \beta_i = \sum_j \upsilon_j = 1.8$ in the calibrated model.

In Figure 12, aggregate TFP in the no-linkage economy decreases exactly by the size of the shock (2%), while it decreases by 3.6% in the input-output economy. This is exactly the magnitude of υ_A. Aggregate output drops by 5.8% at the first period in the input-output economy, while it in the no-linkage economy is 2.6%. This is mainly resulting from the larger decreases of aggregate TFP and equilibrium labor. Overall, the ratio of aggregate output drop between two economies is 2.18.

4.3 Amplification on Sectoral Shocks

Previous results have shown significant amplification magnitudes of input-output linkages on aggregate shocks. Next, I will conduct sectoral analysis to show sectoral heterogeneity in

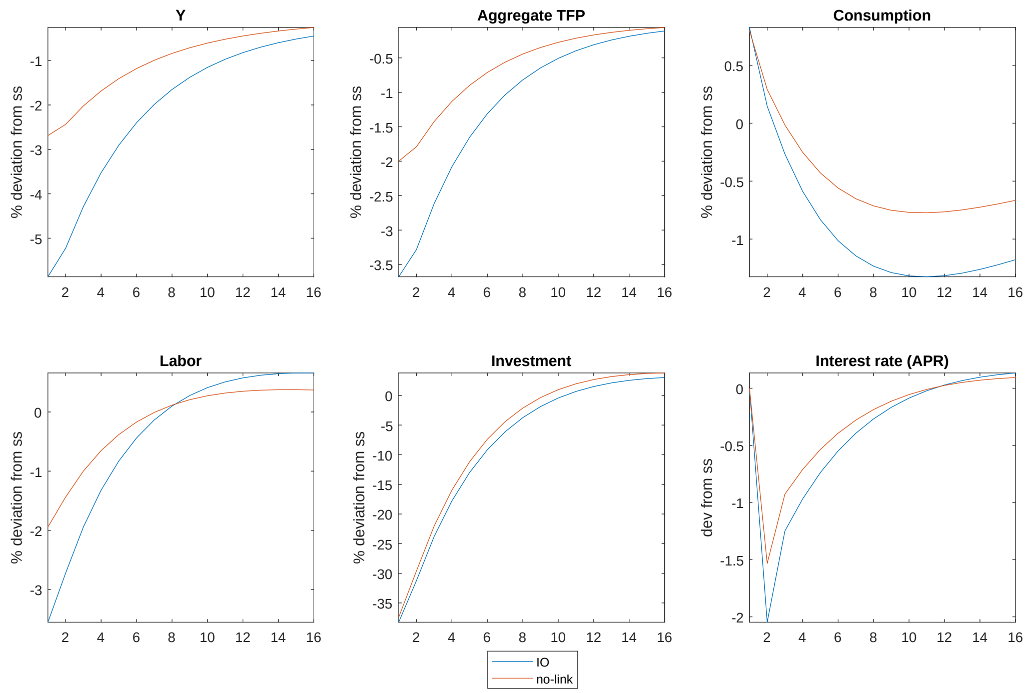

Figure 12: Impulse response functions under a combination of a -2% aggregate TFP shock and an aggregate uncertainty sock.

response to uncertainty shocks and amplification on sectoral impacts to the entire economy. I first show steady state sectoral distortions and then go to dynamic exercises.

4.3.1 Steady State Characteristics

Figure 13 shows the steady state sectoral distortions. The top panel is the model implied capital wedges for each sector, the middle panel shows the sectoral capital allocation between the model economy and another hypothetical frictionless economy, the bottom panel shows the deviation of capital allocation from its frictionless level. For example, the Construction sector is the most distorted sector; the proportion of capital allocated to the Construction sector is only 87% compared to its frictionless proportion. On the other hand, the Retail sector has the smallest capital wedge among the 14 sectors, and hence capital is over-allocated to the Retail sector, 22% larger than its frictionless proportion.

If capital is allocated hypothetically to its frictionless level, aggregate TFP will increase only by 1%.[26] This implies that overall, the degree of credit market friction is quite modest

26. The amount of TFP increase in the steady state level is consistent with the finding in Gilchrist, Sim, and Zakrajšek (2013).

in the United States. However, as discussed in the previous section, combined with input-output linkages, these small distortions can generate significant aggregate impact in the dynamic environment.

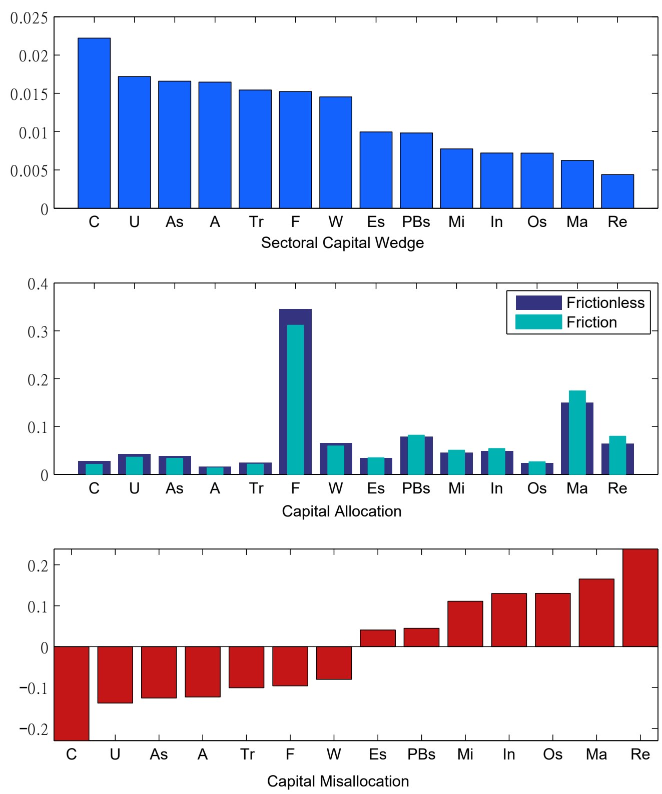

Figure 13: The Degree of Capital Misallocation in the Steady State. The sector order is: Construction, Utilities, Arts, Agriculture, Transportation, FIRE, Wholesale, Education, Professional and Business Services, Mining, Information, Other Services, Manufacturing, and Retail.

4.3.2 Amplification on Sectoral Impact

To analyze sectoral impact, I hit the model economies with a 20% uncertainty shock for each sector at a time. The left panel of Figure 14 depicts sectoral output drop (at the second period) in response to its own uncertainty shock. These sectoral heterogenous responses are contingent on the interaction of all three financial parameters: sectoral default cost, μ_j, uncertainty level, σ_j, and exist rate κ_j. Among them, default cost have a strong impact. The FIRE and Utilities sectors, which have the highest default cost (Table 4), suffer from the largest gross output declines among all sectors. These two sectors are most sensitive to an increase of capital return volatility, indicating a rise in borrowing cost. Input-output linkages do not generate much difference on sectoral output changes, since linkages mainly affect the propagation to other sectors.

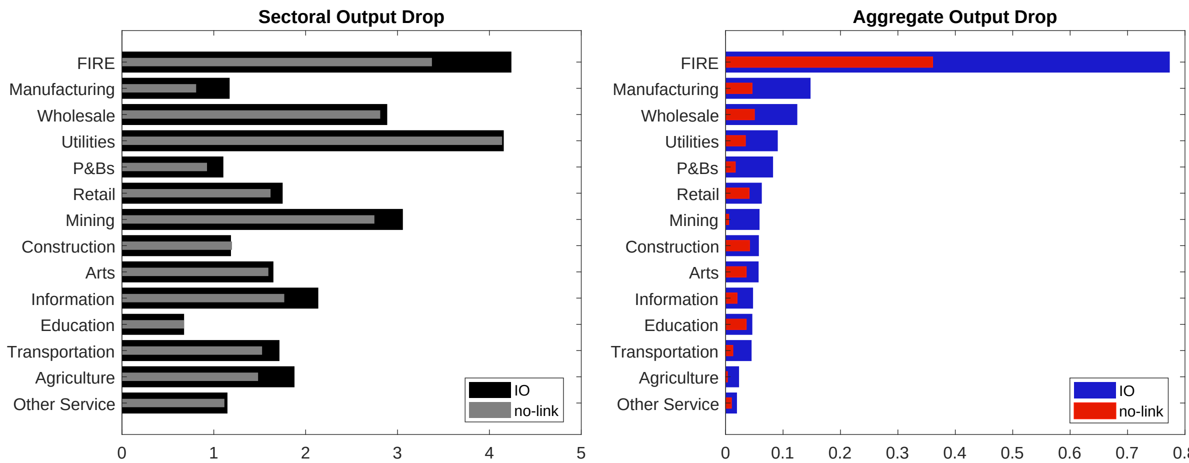

Figure 14: Sectoral and aggregate output drops under idiosyncratic sectoral shocks. The x-axis is percentage deviation in output (sectoral output on the left panel; aggregate output on the right panel).

Next, the right panel depicts the declines of aggregate output to show the impact of linkages. While Hulten's theorem tells that sectoral impact on aggregate output (under productivity shocks) is purely determined by Domar weights, this is not the case here. With uncertainty shocks, sectoral impact now depends on the combination of Domar weights and sectoral sensitivity to capital frictions.

The result that a FIRE sectoral uncertainty shock can generate the largest decline on aggregate output is because FIRE has both high Dormar weight (a key hub sector) and high sensitivity to uncertainty shocks. Among all 14 sectors, the Manufacturing sector has the largest Dormar weight, but its sectoral impact is of about only one fourth compared to the FIRE sector. On the other hand, although the Utilities sector is also highly sensitive to uncertainty shocks, its sectoral impact is still much smaller than that of the FIRE sector, since it has relatively small Domar weight. Overall, the amplification magnitude of input-output linkages on sectoral shocks ranges from 1.2 to 10 (the ratio of aggregate output drop between two model economies). For the FIRE sector, the amplification magnitude is 2.14.

Next, Figure 15 depicts the impulse response functions of other aggregate variables under a sectoral FIRE uncertainty shock that drives up the FIRE spread by about 3.8% from its

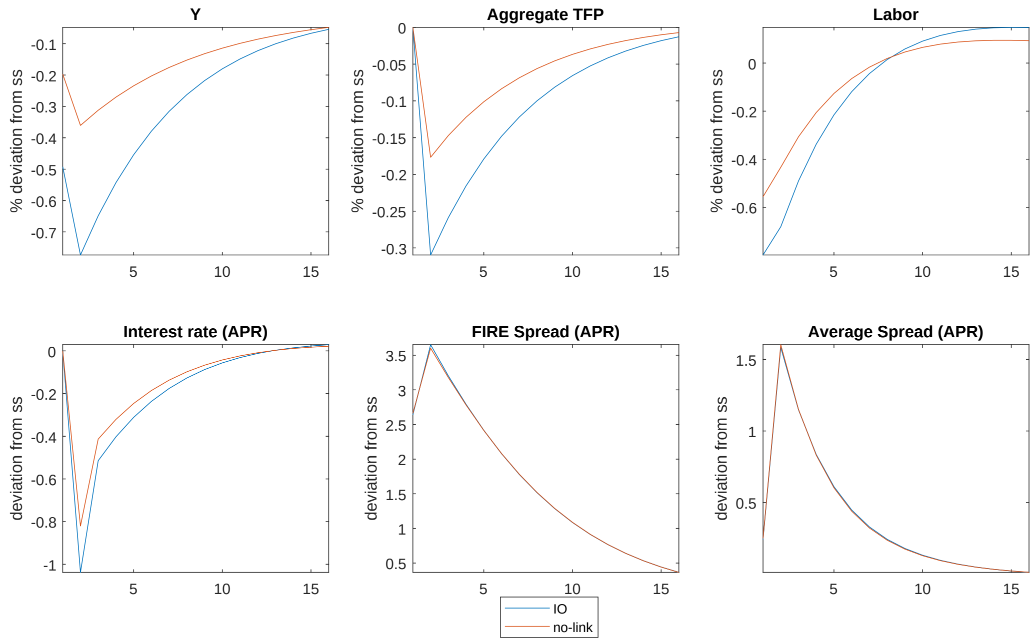

Figure 15: Impulse response functions of some aggregate variables under a sectoral FIRE uncertainty shock.

steady state value. Overall, with input-output linkages, aggregate output decreases by 0.77%, aggregate TFP drops 0.3%, equilibrium labor falls 0.8%, investment decreases by 6%, and average spread increases by 1.5%. Input-output linkages generate significant amplification on sectoral impact to aggregate TFP and aggregate output.

5 Conclusion

This paper develops a dynamic general equilibrium model with endogenous capital wedges to study how input-output linkages amplify the aggregate impact of sectoral financial distortions. I extend the result of Hulten (1978) into economies with financial distortions. The aggregate impact of a shock can be decomposed into weighted productivity changes and changes in allocative efficiency. A sectoral productivity shock has both effects, but an aggregate productivity shock only has the first effect. On the other hand, uncertainty shocks, no matter aggregate or sectoral, always induce changes in allocative efficiency. In response to an aggregate uncertainty shock, the relatively more financially constrained sectors are more vulnerable, and capital flows from them to less financially constrained sectors, reducing allocative efficiency and aggregate TFP.

In response to an aggregate uncertainty shock, input-output linkages amplify the drop in aggregate output by 2.3 times more than an equivalent multi-sector model with the same level of financial frictions but without linkages. In response to a sectoral uncertainty shock, input-output linkages amplify the drop in aggregate output between 1.2 to 10 times more than the economy without linkages. Among all sectors, the FIRE sector has the largest impact on aggregate output in response to a sectoral uncertainty shock. This is because the FIRE sector bears the largest default cost, most sensitive to changes in its financial constraint, and also has the highest Domar weight in the U.S.

Overall, I show input-output linkages can generate significant amplification magnitude on the aggregate impact of sectoral financial distortions. This has the implication that variation in capital misallocation and fluctuation in the credit markets may contribute importantly to business cycles.

References

Acemoglu, Daron, Ufuk Akcigit, and William Kerr. 2015. "Networks and the Macroeconomy: An Empirical Exploration." In *NBER Macroeconomics Annual 2015, Volume 30.* University of Chicago Press.

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2012. "The Network Origins of Aggregate Fluctuations." *Econometrica* 80 (5): 1977–2016.

Altinoglu, Levent. 2016. *The origins of aggregate fluctuations in a credit network economy.* Report. Mimeo.

Atalay, Enghin. 2017. "How important are sectoral shocks?" *American Economic Journal: Macroeconomics* 9 (4): 254–80.

Baqaee, David Rezza. 2018. "Cascading Failures in Production Networks." *Econometrica* 86 (5): 1819–1838.

Baqaee, David Rezza, and Emmanuel Farhi. 2018. "Productivity and Misallocation in General Equilibrium." Thesis.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework." In *Handbook of Macroeconomics. Volume 1C,* 1341–1393.

Bigio, Saki, and Jennifer La'O. 2017. "Financial Frictions in Production Networks." Thesis.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen Terry. 2018. "Really Uncertain Business Cycles." *Econometrica* 86 (3): 1031–1065.

Carlstrom, Charles T., and Timothy S. Fuerst. 1997. "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis." *American Economic Review* 87 (5): 893–910.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2014. "Risk Shocks." *American Economic Review* 104 (1): 27–65.

Dupor, Bill. 1999. "Aggregation and irrelevance in multi-sector models." *Journal of Monetary Economics* 43 (2): 391–409.

Durlauf, Steven N. 1993. "Nonergodic Economic Growth." *Review of Economic Studies* 60 (2): 349–366.

Foerster, Andrew T., Pierre-Daniel G. Sarte, and Mark W. Watson. 2011. "Sectoral versus Aggregate Shocks: A Structural Factor Analysis of Industrial Production." *Journal of Political Economy* 119 (1): 1–38.

Gabaix, Xavier. 2011. "The Granular Origins of Aggregate Fluctuations." *Econometrica* 79 (3): 733–772.

———. 2016. "Power Laws in Economics: An Introduction." *Journal of Economic Perspectives* 30 (1): 185–206.

Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics* 58 (1): 17–34.

Gilchrist, Simon, Jae W Sim, and Egon Zakrajšek. 2013. "Misallocation and financial market frictions: Some direct evidence from the dispersion in borrowing costs." *Review of Economic Dynamics* 16 (1): 159–176.

Gilchrist, Simon, and Egon Zakrajšek. 2012. “Credit Spreads and Business Cycle Fluctuations.” *American Economic Review* 102 (4): 1692–1720.

Gomme, Paul, and Peter Rupert. 2004. “Measuring labor’s share of income.” *FRB of Cleveland Policy Discussion Paper,* no. 7.

Gorton, Gary, and Guillermo Ordonez. 2014. “Collateral Crises.” *American Economic Review* 104 (2): 343–78.

Hayashi, Fumio. 1982. “Tobin’s Marginal q and Average q: A Neoclassical Interpretation.” *Econometrica:* 213–224.

Horvath, Michael. 1998. “Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks.” *Review of Economic Dynamics* 1 (4): 781–808.

———. 2000. “Sectoral Shocks and Aggregate Fluctuations.” *Journal of Monetary Economics* 45 (1): 69–106.

Hulten, Charles R. 1978. “Growth Accounting with Intermediate Inputs.” *Review of Economic Studies* 45 (3): 511–518.

Ireland, Peter N. 2004. “A Method for Taking Models to the Data.” *Journal of Economic Dynamics and Control* 28 (6): 1205–1226.

Jones, Charles I. 2011. “Intermediate Goods and Weak Links in the Theory of Economic Development.” *American Economic Journal: Macroeconomics* 3 (2): 1–28.

———. 2013. “Misallocation, Economic Growth, and Input-Output Economics.” In *Advances in Economics and Econometrics Tenth World Congress,* edited by Daron Acemoglu, Manuel Arellano, and Eddie Dekel, 1:419–456. Cambridge University Press.

Kellogg, Ryan. 2014. “The Effect of Uncertainty on Investment: Evidence from Texas Oil Drilling.” *American Economic Review* 104 (6): 1698–1734.

Krueger, Alan B. 1999. “Measuring Labor’s Share.” *American Economic Review* 89 (2): 45–51.

Liu, Ernest. 2018. “Industrial Policies in Production Networks.” Working paper, Princeton University.

Long, Jr., John B., and Charles I. Plosser. 1983. “Real Business Cycles.” *Journal of Political Economy* 91 (1): 39–69.

Luo, Shaowen. 2016. “Propagation of Financial Shocks in an Input-Output Economy with Trade and Financial Linkages of Firms.” Working paper, Virginia Polytechnic Institute and State University.

McGrattan, Ellen R., Richard Rogerson, and Randall Wright. 1997. “An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy.” *International Economic Review* 38 (2): 267–290.

Nolan, Charles, and Christoph Thoenissen. 2009. “Financial Shocks and the US Business Cycle.” *Journal of Monetary Economics* 56 (4): 596–604.

Rajan, Raghuram G, and Luigi Zingales. 1998. “Financial Dependence and Growth.” *American Economic Review* 88 (3): 559–586.

Shea, John S. 2002. “Complementarities and Comovements.” *Journal of Money, Credit, and Banking* 34 (2): 412–433.

Townsend, Robert M. 1979. “Optimal Contracts and Competitive Markets with Costly State Verification.” *Journal of Economic Theory* 21 (2): 265–293.

A Appendix: Technical Details on Financial Frictions

An an entrepreneur i in sector j has net worth $n^i_{j,t+1}$ and needs to borrow the amount $B^i_{j,t+1} = Q_t k^i_{j,t+1} - n^i_{j,t+1}$ from the financial intermediary with the gross loan rate, $z^i_{j,t+1}$, that the entrepreneur needs to repay. Two remarks are in order. First, specifying $B_{j,t+1}$ is equivalent to specifying the leverage, $L^i_{j,t+1} = Q_t k^i_{j,t+1}/n^i_{j,t+1}$, given net worth, $n^i_{j,t+1}$, and capital, $k^i_{j,t+1}$. Second, specifying the gross loan rate $z^i_{j,t+1}$ is equivalent to specifying the threshold, $\bar{\omega}_{j,t+1}$, such that for the lender, ex-ante,

$$\bar{\omega}_{j,t+1} R^k_{j,t+1} Q_t k^i_{j,t+1} = z^i_{j,t+1} B^i_{j,t+1},$$

for every entrepreneur. Since $E_t[\omega^i_{j,t+1}] = 1$, $R^k_{j,t+1} Q_t k^i_{j,t+1}$ is the expected payoff by investing capital $k^i_{j,t+1}$, at period t, and $\omega^i_{j,t+1} R^k_{j,t+1} Q_t k^i_{j,t+1}$ is the realized payoff at period $t+1$.

Denote $F_{jt}(\omega) = Pr[\omega^i_{jt} < \omega]$ a continuous probability distribution and $f_{jt}(\omega)$ the pdf of ω^i_{jt}. Then $F_{jt}(\bar{\omega}_{j,t})$ is the fraction of entrepreneurs in sector j who default at period t. The zero-profit condition for the financial intermediary is

$$\Big\{ \underbrace{(1-\mu_j) \int_0^{\bar{\omega}_{j,t+1}} \omega f_{j,t+1}(\omega) d\omega}_{\text{from defaulted entrepreneurs}} + \underbrace{\bar{\omega}_{j,t+1}[1 - F_{j,t+1}(\bar{\omega}_{j,t+1})]}_{\text{loan repayment}} \Big\} R^k_{j,t+1} Q_t k_{j,t+1} = R_{t+1} D_{j,t+1}, \forall j.$$

The first component of the left part is the amount of asset seized by the financial intermediary from defaulted entrepreneurs after paying the cost. The second component of the left part of is the amount of repayment the financial intermediary receives from those non-defaulted entrepreneurs. In equilibrium, a competitive financial intermediary earns zero profit in every sector. Note that $\sum_j D_{j,t+1} = D_{t+1}$. Sum the above constraint over each sector j, we get

$$\sum_{j=1}^{N} \Big\{ (1-\mu_j) \int_0^{\bar{\omega}_{j,t+1}} \omega f_{j,t+1}(\omega) d\omega + \bar{\omega}_{j,t+1}[1 - F_{j,t+1}(\bar{\omega}_{j,t+1})] \Big\} R^k_{j,t+1} Q_t k_{j,t+1} = R_{t+1} D_{t+1}.$$

To simplify the presentation, define $\Omega_{j,t+1}$ as the share of sector j's profits going to the lender at period $t+1$:

$$\Omega_{j,t+1}(\bar{\omega}_{j,t+1}, \sigma_{j,t+1}) = \int_0^{\bar{\omega}_{j,t+1}} \omega f_{j,t+1}(\omega) d\omega + \bar{\omega}_{j,t+1}[1 - F_{j,t+1}(\bar{\omega}_{j,t+1})],$$

and $\mu_j G_{j,t+1}$ the share of sector j's profits as default cost at period $t+1$:

$$\mu_j G_{j,t+1}(\bar{\omega}_{j,t+1}, \sigma_{j,t+1}) = \mu_j \int_0^{\bar{\omega}_{j,t+1}} \omega f_{j,t+1}(\omega) d\omega.$$

For the profits in sector j at period $t+1$, entrepreneurs get $(1 - \Omega_{j,t+1})$ share, and the financial intermediary gets $(\Omega_{j,t+1} - \mu_j G_{j,t+1})$ share. Thus the zero-profit condition at sector j can be written as

$$(\Omega_{j,t+1} - \mu_j G_{j,t+1}) R^k_{j,t+1} Q_t k_{j,t+1} = R_{t+1} D_{j,t+1}.$$

$\Omega_j(\bar{\omega}_{j,t+1}, \sigma_{j,t+1})$ and $G_j(\bar{\omega}_{j,t+1}, \sigma_{j,t+1})$ are functions of the endogenous threshold $\bar{\omega}_{j,t+1}$ and exogenous sectoral uncertainty $\sigma_{j,t+1}$.

The optimal contract then maximizes each entrepreneur's expected return,

$$\max_{L^i_{j,t+1}, \bar{\omega}_{j,t+1}} E_t \frac{[1 - \Omega_{j,t+1}(\bar{\omega}_{j,t+1})] R^k_{j,t+1} Q_t k^i_{j,t+1}}{R_{t+1} n^i_{j,t+1}} = E_t [1 - \Omega_{j,t+1}(\bar{\omega}_{j,t+1})] L^i_{j,t+1} \frac{R^k_{j,t+1}}{R_{t+1}},$$

by choosing the leverage and the threshold, $(L^i_{j,t+1}, \bar{\omega}_{j,t+1})$, subject to the zero-profit condition.

After solving the optimal contract for each individual, we get a linear relationship between individual capital and net worth:

$$Q_t k^i_{j,t+1} = \psi(E_t R^k_{j,t+1} / R_{t+1}) n^i_{j,t+1}.$$

Two remarks are in order here. First, after aggregating the above equation over entrepreneurs in sector j, it turns out that sectoral capital only depends on sectoral aggregate net worth, and there is no need to keep track of an individual's net worth. Second, every entrepreneur ends up with the same leverage. So sectoral leverage can be written as the ratio of the value of sectoral capital and sectoral net worth, $L_{j,t+1} = \frac{Q_t k_{j,t+1}}{N_{j,t+1}}$.

Aggregating the equilibrium condition of individual capital and net worth gives the relationship between sectoral spread and leverage as the following:

$$L_{j,t+1} = \psi(E_t R^k_{j,t+1} / R_{t+1}).$$

Rewrite the above equation gives the Eq (2.10):

$$E_t \frac{R^k_{j,t+1}}{R_{t+1}} = \psi^{-1}(L_{j,t+1}) = \varphi(\bar{\omega}_{j,t+1}, \sigma_{j,t+1}). \tag{A.1}$$

Since $L_{j,t+1}$ is the result from the optimal contract and depends on the endogenous threshold $\bar{\omega}_{j,t+1}$ and exogenous sectoral uncertainty $\sigma_{j,t+1}$.

Note that $D_{j,t+1} = Q_t k_{j,t+1} - N_{j,t+1}$, so $\frac{D_{j,t+1}}{N_{j,t+1}} = L_{j,t+1} - 1$. Dividing the zero-profit condition at sector j by $N_{j,t+1}$, we can rewrite the financial intermediary's expected zero-profit condition as the following:

$$\frac{1}{L_{j,t+1}} = 1 - E_t \frac{R^k_{j,t+1}}{R_{t+1}} (\Omega_{j,t+1} - \mu_j G_{j,t+1}). \tag{A.2}$$

At the end of period t, after the realization of ω^i_{jt}, production, and factor payments, an entrepreneur transfers all his or her asset into net worth, n^i_{t+1}, by selling capital to the capital producer. Before borrowing, $(1-\kappa_j)$ fraction of type j entrepreneurs exit and become workers, and upon doing so, they give their net worth back to the household as dividends. This dividend has two purposes. First, it makes entrepreneurs' net worth part of households' wealth. It is thus in the interest of the representative household to instruct its entrepreneurs to maximize expected net worth. Second, this setup ensures that entrepreneurs will not accumulate too much net worth and end up without the need to borrow. At the same time, a similar number of workers become type j entrepreneurs such that the relative proportion of each type is fixed. The household then gives these new entrepreneurs and defaulted entrepreneurs some startup funds as their net worth which they can use to borrow to buy new capital for the next period. For simplicity, since there is no need to keep track of individual net worth, let these startup funds for each type be $w^e_j N_{j,t}$, where w^e_j is set at 0.01 for every sector j. This small amount of wealth is used to ensure that every entrepreneur has positive net worth, for those who default and those new entrepreneurs. Therefore, the law of motion for sectoral net worth is:

$$N_{j,t+1} = \kappa_j (1 - \Omega_{jt}(\bar{\omega}_{jt}, \sigma_{jt})) R^k_{jt} Q_{t-1} k_{jt} + w^e_j N_{j,t}. \tag{A.3}$$

For each sector, the relation between the interest rate spread (the difference between $z_{j,t+1}$

and R_{t+1}) and the capital wedge (the spread between the return to capital and risk-free rate) is:

$$\frac{z_{j,t+1}}{R_{t+1}} = \bar{\omega}_{j,t+1}\frac{R^k_{j,t+1}}{R_{t+1}}\frac{L_{j,t+1}}{L_{j,t+1}-1}. \tag{A.4}$$

B Proofs

Proof of *Proposition 1*:

Influence Vector $\boldsymbol{v}$. From firms' first-order conditions of intermediate goods, we have $p_{jt} = \frac{\beta_j Y_t}{X_{jt}}, \quad M_{ijt} = \frac{\gamma_{ij} p_{it} Y_{it}}{p_{jt}}$. Multiply both sides of Eq (2.13) by p_{jt}, and plug in the above two terms:

$$\frac{\beta_j Y_t}{X_{jt}} Y_{jt} = \beta_j Y_t + \sum_{i=1}^{N} \gamma_{ij} \frac{\beta_i Y_t}{X_{it}} Y_{it}$$

Define the influence vector as $v_{jt} \equiv \frac{p_{jt} Y_{jt}}{Y_t} = \frac{\beta_j Y_{jt}}{X_{jt}}$, then the above equation becomes:

$$v_{jt} Y_t = \beta_j Y_t + \sum_{i=1}^{N} \gamma_{ij} v_{it} Y_t.$$

Divide Y_t on both sides:

$$v_{jt} = \beta_j + \sum_{i=1}^{N} \gamma_{ij} v_{it}.$$

Write in the vector form: $\boldsymbol{v_t} = \boldsymbol{\beta} + \boldsymbol{\Gamma} \boldsymbol{v_t}$. That is, the influence vector is independent of time, and $\boldsymbol{v} = [\boldsymbol{I}_N - \boldsymbol{\Gamma}]^{-1} \boldsymbol{\beta}$.

Intermediate inputs allocation matrix $\boldsymbol{\eta}$. From intermediate goods firms' first-order condition on M_{ijt}, $\gamma_{ij} p_{jt} Y_{jt} = p_{it} M_{ijt}$,

$$\gamma_{ij} p_{jt} Y_{jt} = p_{it} M_{ijt} = p_{it} \eta_{ijt} Y_{it},$$
$$\Rightarrow \frac{p_{it} Y_{it}}{p_{jt} Y_{jt}} = \frac{\gamma_{ij}}{\eta_{ijt}}.$$

Since $\frac{p_{it} Y_{it}}{p_{jt} Y_{jt}} = \frac{v_i}{v_j}$, we have $\eta_{ijt} = \frac{\gamma_{ij} v_j}{v_i}$. So η_{ji} is independent of time, and it can be written in the vector form as: $\boldsymbol{\eta} = [\frac{1}{\boldsymbol{v}} * \boldsymbol{v}'] \circ \boldsymbol{\Gamma}$.

Labor allocation variable $\boldsymbol{x^l}$. Sum intermediate goods firm's first-order condition on

l_{jt}, $(1-\alpha_j)(1-m_j)p_{jt}Y_{jt} = w_t l_{jt}$, over sectors and use the definition of $\boldsymbol{v}$:

$$\frac{(1-\alpha_j)(1-m_j)p_{jt}Y_{jt}}{l_{jt}} = w_t = \frac{\sum_{j=1}^{N}(1-\alpha_j)(1-m_j)v_jY_t}{H_t}$$

Let $l_{jt} = x_{jt}^l H_t$, then

$$\begin{aligned} x_{jt}^l =& \frac{(1-\alpha_j)(1-m_j)p_{jt}Y_{jt}}{\sum_{j=1}^{N}(1-\alpha_j)(1-m_j)v_jY_t} \\ =& \frac{(1-\alpha_j)(1-m_j)v_j}{\sum_{j=1}^{N}(1-\alpha_j)(1-m_j)v_j.} \end{aligned}$$

Proof of *Proposition 2*:

From Eq (2.7), the marginal revenue product of capital is defined as

$$\text{MRPK}_{jt} = \frac{(1-m_j)\alpha_j p_{jt}Y_{jt}}{k_{jt}}.$$

Define the capital allocation variable x_{jt}^k be that $k_{jt} = x_{jt}^k K_t$, and substitute $p_{jt}Y_{jt} = v_jY_t$ in the above equation, we get

$$\frac{k_{jt}}{k_{it}} = \frac{x_{jt}^k}{x_{it}^k} = \frac{MRPK_{it}(1-m_j)\alpha_j v_j}{MRPK_{jt}(1-m_i)\alpha_i v_i}.$$

x_{jt}^k is then solved by applying $\sum_{j=1}^{N} x_{jt}^k = 1$.

Proof of *Theorem 2*:

Take logarithm on sectoral production function in Eq (2.2). Denote $\ln Y_{jt} = y_{jt}$. Substitute k_{jt} with $x_{jt}^k K_t$, l_{jt} with $x_j^l H_t$, and M_{ijt} with $\eta_{ij}Y_{it}$:

$$\begin{aligned} y_{jt} =& \ln A_{jt} + (1-m_j)\alpha_j \ln(x_{jt}^k K_t) + (1-m_j)(1-\alpha_j)\ln(x_j^l H_t) + \sum_{i=1}^{N}\gamma_{ij}\ln(\eta_{ij}Y_{it}), \\ \Rightarrow y_{jt} =& \underbrace{\ln A_{jt} + (1-m_j)\alpha_j \ln x_{jt}^k + (1-m_j)(1-\alpha_j)\ln x_j^l + \sum_{i=1}^{N}\gamma_{ij}\ln\eta_{ij}}_{\text{denote this as } c_{yjt}} + \underbrace{(1-m_j)\alpha_j}_{\text{denote this as } \delta_{kj}} \ln K_t \\ & + \underbrace{(1-m_j)(1-\alpha_j)}_{\text{denote this as } \delta_{lj}}\ln H_t + \sum_{i=1}^{N}\gamma_{ij}\ln Y_{it}, \end{aligned}$$

$$\Rightarrow y_{jt} = \ln A_{jt} + c_{yjt} + \delta_{kj} \ln K_t + \delta_{lj} \ln H_t + \sum_{i=1}^{N} \gamma_{ij} y_{it}.$$

Denote $\ln A_{jt}$ as a_{jt}. The vector form of the above equation is

$$\begin{aligned} \boldsymbol{y_t} =& \boldsymbol{a_t} + \boldsymbol{c_{yt}} + \boldsymbol{\delta_k} \ln K_t + \boldsymbol{\delta_l} \ln H_t + \boldsymbol{\Gamma' y_t}, \\ \Rightarrow \boldsymbol{y_t} =& [\boldsymbol{I_N} - \boldsymbol{\Gamma}']^{-1}(\boldsymbol{a_t} + \boldsymbol{c_{yt}} + \boldsymbol{\delta_k} \ln K_t + \boldsymbol{\delta_l} \ln H_t). \end{aligned} \tag{B.1}$$

Replace X_{jt} with $(1-\eta_j)Y_{jt}$ in Eq (2.1). So $Y_t = \prod_{j=1}^{N}((1-\eta_j)Y_{jt})^{\beta_j}$. Take logarithm, and use Eq (B.1). The vector form of the production function in logarithm is the following:

$$\begin{aligned} \ln Y_t =& \boldsymbol{\beta'} \ln(1-\boldsymbol{\eta}) + \boldsymbol{\beta' y}, \\ \Rightarrow \ln Y_t =& \underbrace{\boldsymbol{\beta'} \ln(1-\boldsymbol{\eta}) + \boldsymbol{\beta'}[\boldsymbol{I_N} - \boldsymbol{\Gamma}']^{-1}(\boldsymbol{a_t} + \boldsymbol{c_{yt}})}_{\text{denote these terms as } \ln \tilde{A}_t} + \underbrace{\boldsymbol{\beta'}[\boldsymbol{I_N} - \boldsymbol{\Gamma}']^{-1}\boldsymbol{\delta_k}}_{\text{denote this as } \tilde{\alpha}} \ln K_t + \boldsymbol{\beta'}[\boldsymbol{I_N} - \boldsymbol{\Gamma}']^{-1}\boldsymbol{\delta_h} \ln H_t. \end{aligned} \tag{B.2}$$

Note that $\tilde{\alpha} = \boldsymbol{\beta'}[\boldsymbol{I_N} - \boldsymbol{\Gamma}']^{-1}\boldsymbol{\delta_k}$ is a constant, not a vector. And $\ln \tilde{A}_t$ is a single time-varying variable. Since the vector sum, $\boldsymbol{\delta_h} + \boldsymbol{\delta_k} = \boldsymbol{1} - \boldsymbol{m}$, we have the sum:

$$\boldsymbol{\beta'}[\boldsymbol{I_N} - \boldsymbol{\Gamma}']^{-1}\boldsymbol{\delta_k} + \boldsymbol{\beta'}[\boldsymbol{I_N} - \boldsymbol{\Gamma}']^{-1}\boldsymbol{\delta_h} = 1,$$

be a constant of 1. Denote $\lambda_j^l = (1-m_j)(1-\alpha_j)v_j$ and $\lambda_j^k = (1-m_j)\alpha_j v_j$. Thus, Eq (B.2) can be written as

$$\ln Y_t = \ln \tilde{A}_t + \tilde{\alpha} \ln K_t + (1-\tilde{\alpha}) \ln H_t,$$

$$\ln(\tilde{A}_t) = \boldsymbol{v' a_t} + \boldsymbol{\lambda^{k'}} \ln \boldsymbol{x_t^k} + \sum_i \beta_i \sum_j (1-\eta_{ij}) + \boldsymbol{\lambda^{l'}} \ln \boldsymbol{x^l} + \sum_j v_j \sum_i \gamma_{ij} \ln \eta_{ij}.$$