Wealth and Demographics in the 21st Century

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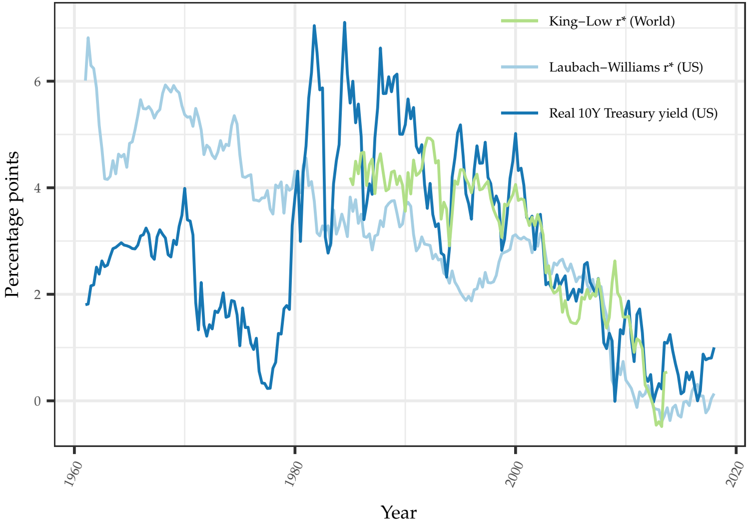
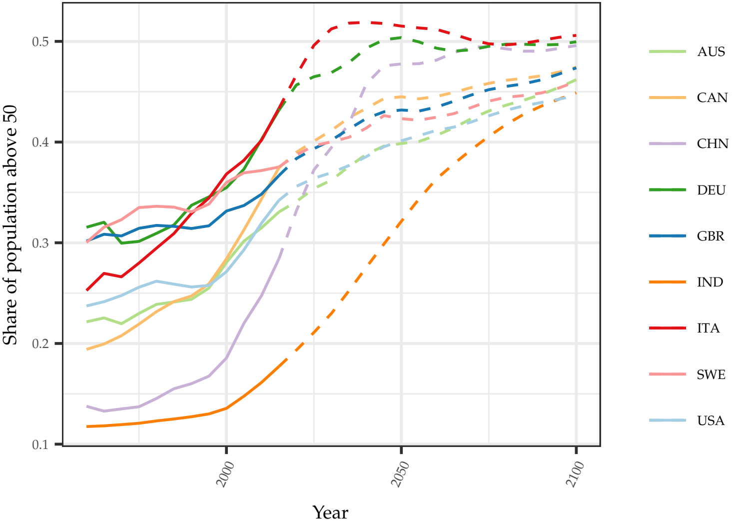
Preliminary

Safe interest rates are falling



Source: FRED (10Y Treasury yield – 5 year trailing GDP deflator), Laubach and Williams, King and Low

The World Population Is Aging



Source: UN World Population Prospects

Demographics and Interest Rates

- State of the debate
  - Aging lowers interest rates....
  - ...but how much?
- Standard strategy
  - Calibrate OLG models with realistic demography
- Different model assumptions $\rightarrow$ different interest rate responses to aging

INTEREST RATE SENSITIVITY TO AGING

- Key unknown macro quantity: interest rate sensitivity to aging (IRSA)
  - Needed to forecast real interest rate in an aging world
- Implications
  - asset prices
  - fiscal sustainability
  - conduct of monetary policy

THIS PAPER

- Develop rich OLG model with heterogeneity and flexible parametrization
- SS demand and supply framwork
  - Identify model elasticities that determine IRSA
- Calibration aims at finding good values for key elasticities
  - Model *partially* identified given key elasticities

OUTLINE

1. Intuition

2. Full model

3. Decomposition and calibration

MODEL SETUP

- OLG model in steady state. i is age, t is time.
- θ indexes demographic parameters
  - Fertility: growth rate of newborns, $N_{0t} = (1+n)^t$
  - Mortality: death rates $\phi_{i,i+1} = \frac{N_{i+1,t+1}}{N_{it}}$
  - $N_t \equiv \sum_i N_{it}$ total population, grows at rate n in s.s.
  - $\pi_i(\theta) \equiv \frac{N_{it}}{N_t}$ s.s. population share
- Labor-augmenting technology: productivity $h_i Z_t$ at age i
  - Exogenous growth $Z_t = (1+\gamma)^t$, loaded on time effects
  - Output growth rate in s.s. $1+g = (1+n)(1+\gamma)$
- Production function $Y = F(K, L)$ with constant returns to scale

$$\frac{Y}{ZN} = F\left(\frac{K}{ZN}, \sum_i \pi_i(\theta) h_i\right)$$

Asset supply: standard neoclassical model

- Capital demand (perfect competition, constant depreciation δ)

$$\frac{\partial Y}{\partial K} = r + \delta \quad \Rightarrow \quad \frac{K}{ZN} = k(r) \sum_i \pi_i(\theta) h_i$$
$$\Rightarrow \quad \frac{Y}{ZN} = y(r) \sum_i \pi_i(\theta) h_i$$

with $k(r) \equiv F_K^{-1}(r+\delta, 1)$ and $y(r) = F(k(r), 1)$
  - $\sum_i \pi_i(\theta) h_i$ is effective labor per person

- No government debt $\Rightarrow$ s.s. normalized aggregate **asset supply**

$$a^s(r, \theta) = \frac{A^s}{ZN} = \frac{K}{ZN}$$
$$= k(r) \sum_i \pi_i(\theta) h_i$$

Asset demand

- Wage per efficient unit: $\frac{\partial Y}{\partial L} = F_L\left(k\left(r\right),1\right) \equiv w\left(r\right)$
- Household problem determines savings
  - Accumulate assets from wages
  - Life-cycle motive: pretax earnings $w\left(r\right) h_i Z_t$ + pension in retirement
  - Bequest motive
  - Allows for balanced growth
- Household optimization ⇒ s.s. normalized aggregate **asset demand**

$$a^d\left(r,\theta\right) = \frac{A^d}{ZN} = \sum_i \pi_i\left(\theta\right) a_i\left(r,\theta\right)$$

Equilibrium

- Small open economy: given r, aggregate wealth to GDP ratio

$$\mathcal{W}(r,\theta) \equiv \frac{W}{Y} = \frac{A^d/ZN}{Y/ZN} = \frac{\sum_i \pi_i(\theta)\frac{a_i(r,\theta)}{y(r)}}{\sum_i \pi_i(\theta) h_i} \tag{1}$$

  - θ affects both numerator and denominator
  - From now on, normalize $\sum_i \pi_i(\theta^*) h_i = 1$ for baseline θ^*

- Closed economy: r solves

$$a^d(r,\theta) = a^s(r,\theta)$$

equivalently

$$\mathcal{W}(r,\theta) = \frac{k(r)}{y(r)} \tag{2}$$

SUFFICIENT STATISTICS FOR STEADY STATE PROJECTIONS

Consider a small change in demographics $d\theta$; $da_i \equiv \frac{\partial a_i}{\partial \theta} d\theta$, $d\pi_i \equiv \frac{\partial \pi_i}{\partial \theta} d\theta$.

- In a s.o.e., the steady state wealth-to-GDP changes by

$$d\left(\frac{W}{Y}\right) = \underbrace{\sum_i \left\{\frac{a_i}{y} - \frac{W}{Y} h_i\right\} d\pi_i}_{\Delta^s} + \underbrace{\sum_i \pi_i \frac{da_i}{y}}_{\Delta^a} \equiv \Delta \tag{3}$$

  (irrespective of γ loading on time or cohort effects.)

- In a closed economy, the steady state real interest rate changes by

$$dr = -\frac{\Delta_s + \Delta_a}{\epsilon^d - \epsilon^s} \tag{4}$$

  where ϵ^d, ϵ^s are asset demand and supply semielasticities

OUTLINE

MODEL SETUP

- OLG model in steady state
  - θ indexes demographic parameters (fertility, mortality)
  - $N_t(\theta) \equiv \sum_j N_{jt}(\theta)$ total population, $N_{j,t}$ of age j
  - $\pi_j(\theta) \equiv \frac{N_{jt}(\theta)}{N_t(\theta)}$ population share in steady state
  - Labor-augmenting technology: a household age j living at time t has exogenous productivity $h_j Z_t$

STEADY STATE FEATURES

- The steady state features
  - constant population growth rate n: $N_{0t} = (1+n)^t$, subsequently affected via Φ_j
  - constant rate of technological improvement γ: $Z_t = (1+\gamma)^t$
  - (endogenously) constant real rate r, capital rental rate $\frac{R}{P}$, wage rate per unit of efficient labor $\frac{W}{P}$, and tax rate on labor income τ

DEMOGRAPHICS

- Demographic parameters: pop. growth $n(\theta)$ and survival rates $\Phi_j(\theta)$
- Per period survival rates $\phi_{k-1,k} \rightarrow \Phi_j = \prod_{k=1}^{j} \phi_{k-1,k}$ ($\phi_{T,T+1} = 0$)
- Normalizing $t = 0$ population to 1:

$$\begin{aligned} N_{0t} &= (1+n)^t \\ N_{jt} &= \Phi_j (1+n)^{t-j} \end{aligned}$$

The total population N_t is growing at the rate of entrants

$$N_t = \sum_{j=0}^{T} N_{jt} = (1+n)^t \sum_{j=0}^{T} \frac{\Phi_j}{(1+n)^j}$$

- Write $\pi_j = \frac{N_{jt}}{N_t}$ for the population share of age j, then

$$\pi_j = \frac{\Phi_j}{(1+n)^j} \pi_0 \text{with} \quad \pi_0 = \frac{1}{\sum_{j=0}^{T} \frac{\Phi_j}{(1+n)^j}} \tag{5}$$

PRODUCTION

- Final output proced competitively by intermediate goods Y_{kt}

$$Y_t = \left(\int_0^1 Y_{kt}^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}}$$

- Y_{kt} produced with capital K_{kt} + effective labor L_{kt}

$$Y_{kt} = F\left(K_{kt}, L_{kt}\right)$$

- Monopolistic competition (to allow rents) implies price

$$P_{kt} = \frac{\epsilon}{\epsilon - 1} \frac{W_t}{F_L\left(K_{kt}, L_{kt}\right)} = \frac{\epsilon}{\epsilon - 1} \frac{R_t}{F_K\left(K_{kt}, L_{kt}\right)}$$

- Equilibrium factor prices and profits

$$\frac{R_t}{P_t} = \frac{1}{\mu} F_K\left(K_t, L_t\right) \tag{6}$$

$$\frac{W_t}{P_t} = \frac{1}{\mu} F_L\left(K_t, L_t\right) \tag{7}$$

$$\frac{\Pi_t}{P_t} = \left(1 - \frac{1}{\mu}\right) F\left(K_t, L_t\right) \tag{8}$$

Labor market

- N_{jt} workers of age j
- Worker (indexed by i)
  - idiosyncratic skill shocks h_{ijt}
  - exogenously supplies l_{ijt} hours to the market
  - Normalize such that $l_{ijt} = 1$ and $E[h_{ijt}] = h_j$
- Labor market clearing

$$L_t = \sum_j N_{jt} \mathbb{E}\left[l_{ijt} h_{ijt}\right] Z_t = N_t Z_t \sum_j \pi_j \mathbb{E}\left[h_{ijt}\right] \tag{9}$$

$$L_t = N_t Z_t \sum_j \pi_j h_t \tag{10}$$

Asset market and steady-state ratios

- A capital mutual fund owns the capital and rents it to firms.
- Every period: invests $I_t = K_{t+1} - (1-\delta) K_t$; maximizes the present discounted value of dividends

$$(1+r_{t-1}) V_t^K (K_t) = \max_{K_{t+1}} \left\{ \frac{R_t}{P_t} K_t - (K_{t+1} - (1-\delta) K_t) + V_{t+1}^K (K_{t+1}) \right\} \tag{11}$$

- Conjecture $V_t^K (K_t) = v_t^K K_t \rightarrow$ firm FOC + envelope theorem

$$v_{t+1}^K = 1 \Longrightarrow \frac{R_t}{P_t} = r_{t-1} + \delta \tag{12}$$

- Capitalized value of the capital mutual fund: $V^K (K_t) = K_t$.
- Steady state dividends: $\frac{D}{P} = (r-g) K$ with $1+g = \frac{K_{t+1}}{K_t}$

Interest rate and capital marginal product

- Combining (6) with (12)

$$F_K\left(\frac{K_t}{L_t},1\right)=\mu\left(r+\delta\right) \tag{13}$$

combining next with (7), we find

$$\frac{W}{P}=\frac{1}{\mu}F_L\left(\frac{K_t}{L_t},1\right)=\frac{1}{\mu}F_L\left(F_K^{-1}\left\{\mu\left(r+\delta\right)\right\},1\right) \tag{14}$$

finally

$$\frac{Y_t}{L_t}=F\left(\frac{K_t}{L_t},1\right)$$

- Hence, from (9), the steady state output growth rate is

$$1+g=\frac{Y_{t+1}}{Y_t}=\frac{K_{t+1}}{K_t}=\frac{L_{t+1}}{L_t}=(1+\gamma)(1+n) \tag{15}$$

and steady-state investment is

$$\frac{I}{Y}=\left(g+\delta\right)\frac{K}{Y} \tag{16}$$

PROFIT MUTUAL FUND

- A profit mutual fund passively owns the intermediate goods firms. Its value at time t is given by

$$(1+r)\,V_t^{\Pi} = \left(1-\frac{1}{\mu}\right)Y_t + V_{t+1}^{\Pi}$$

in a steady state $\frac{V_t^{\Pi}}{Y_t} = v^{\pi}$ such that

$$(1+r)\,v^{\pi} = \left(1-\frac{1}{\mu}\right) + v^{\pi}\,(1+g)$$

or

$$V_t^{\pi} = \left(1-\frac{1}{\mu}\right)\frac{1}{r-g}Y_t$$

- Intuition: markup share of GDP valued at dividend growth formula

LIFE INSURANCE COMPANY

- Life insurance company; owns both mutual funds
- Sells annuity contracts to households and has asset value

$$A_t^s = K_t + \left(1 - \frac{1}{\mu}\right) \frac{1}{r-g} Y_t \tag{17}$$

- Using (13) we have

$$K_t = F_K^{-1}\left(\mu\left(r+\delta\right)\right) L_t$$

and

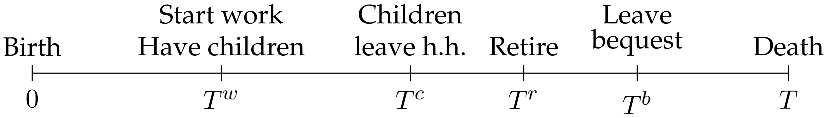
$$Y_t = F\left(F_K^{-1}\left(\mu\left(r+\delta\right)\right), 1\right) L_t$$

Then, combining with (10), we obtain "normalized normalized" asset supply $\hat{\hat{A}}^s$:

$$\frac{A_t^s}{Z_t N_t} = \left(F_K^{-1}\left(\mu\left(r+\delta\right)\right) + \left(1 - \frac{1}{\mu}\right) \frac{F\left(F_K^{-1}\left(\mu\left(r+\delta\right)\right)\right)}{r-g}\right) \left(\sum \pi_j h_j\right) \tag{18}$$

HOUSEHOLDS

- Heterogeneous households indexed by i, with ages $j = 1 \cdots T$, interpreted as single-parent families.



- Skill level given by: $h_{ijt} = \theta_i h_j \epsilon_{i,t}$
  - θ_i: household fixed effect ($\mathbb{E}[\theta_i] = 1$)
    - $\Pi^{\theta}(\theta'|\theta)$ intergenerational transmission of fixed effects.
    - $\pi^{\theta}(\theta)$: for the stationary distribution of θ.
  - h_j is the deterministic age profile of productivity,
  - $\epsilon_{i,t}$ idiosyncratic shock
    - Markov chain $\Pi^{\epsilon}(\epsilon'|\epsilon)$
    - $\mathbb{E}[\epsilon_{i,t}] = 1$ at every t.
    - $\pi^{\epsilon}(\epsilon)$: for the stationary distribution of ϵ.

VALUE FUNCTION

- Value function for household

$$V_{t,j}(\theta,\epsilon,a) = \max \psi_j u(c) + \tilde{v}_j(b) + \beta\phi_{j,j+1}\mathbb{E}\left[V_{t+1,j+1}(\theta,\epsilon',a')|\epsilon\right]$$

$$\tilde{v}_j(b) = \Upsilon Z_t^{\nu-\sigma}\mathbf{1}_{\{j=T^b\}}v(b) \tag{19}$$

$$c+b+a' = y_{jt}(\theta,\epsilon) + \frac{(1+r)\,a}{\phi_{j-1,j}} \tag{20}$$

$$y_{jt}(\theta,\epsilon) = \mathbf{1}_{\{j<T^r\}}(1-\tau)\,Z_t\frac{W}{P}\cdot\theta h_j\epsilon + \mathbf{1}_{\{j\geq T^r\}}d_t\cdot\theta$$

$$a' \geq -\bar{A}Z_t$$

  - with $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, and $v(b) = \frac{b^{1-\nu}}{1-\nu}$ with $\sigma \geq \nu$ capturing luxury bequests
    - $Z_t^{\nu-\sigma} \to$ stationary distribution of bequests
  - ψ_j age-dependent utility modifier due to child raising
  - Productivity loads on time rather than cohort effects
  - Retirement age is fixed + pensions scale with household fixed effect

BEQUESTS AND PAY-AS-YOU-GO

- Bequests given at age T^b (pooled among θ' type)

$$a_{T^w,t}(\theta') = \frac{N_{T^b,t}}{N_{T^w,t}} \sum_{\theta} \frac{\Pi^{\theta}(\theta'|\theta)\pi^{\theta}(\theta)}{\pi^{\theta}(\theta')} \mathbb{E}[b_t|\theta]$$

- The government finances PAYGO system with labor taxes

$$\tau \frac{W}{P} Z_t \sum_{\theta} \pi^{\theta}(\theta)\theta \sum_{j=T^w}^{T^r-1} N_{j,t} h_j = d_t \sum_{\theta} \pi^{\theta}(\theta)\theta \sum_{j=T^r}^{T} N_{j,t}$$

- No cross-subsidization across θ types

$$\tau \frac{W}{P} Z_t \sum_{j=T^w}^{T^r-1} N_{j,t} h_j = d_t \sum_{j=T^r}^{T} N_{j,t}$$

Aggregate consumption and assets

- Total asset demand and consumption

$$A_t^d = \sum_j N_{jt} \mathbb{E}[a_{ijt}]$$

$$C_t^d = \sum_j N_{jt} \mathbb{E}[c_{ijt}]$$

Normalization of decision rules

- Define normalized income as

$$\widehat{y_j}\left(\theta,\epsilon\right) = \mathbf{1}_{\{j<T^r\}}\left(1-\tau\right)\frac{W}{P}\cdot\theta h_j\epsilon + \mathbf{1}_{\{j\geq T^r\}}\widehat{d}\cdot\theta$$

where

$$\widehat{d} = \frac{d_t}{Z_t} = \frac{\tau\frac{W}{P}\sum_{j=T^w}^{T^r-1}\pi_j h_j}{\sum_{j=T^r}^{T}\pi_j}$$

- Normalized consumption, $\widehat{c} = \frac{c_t}{Z_t}$, bequests $\widehat{b} = \frac{b_t}{Z_t}$, etc
- Assets normalized by **last** period productivity $\widehat{a} = \frac{a_t}{Z_{t-1}}$ (so $\frac{a_t}{Z_t} = \frac{\widehat{a}}{1+\gamma}$)
  - Value function $\widehat{V} = \frac{V_t}{Z_t^{1-\sigma}}$
  - Discount factor $\widehat{\beta} = \beta\left(1+\gamma\right)^{1-\sigma}$

CONSUMER PROBLEM IN NORMALIZED FORM

- Then the problem in (20) can be rewritten as

$$\begin{aligned} \widehat{V}_j(\theta,\epsilon,\widehat{a}) \quad = \quad & \max \psi_j u(\widehat{c}) + \mathbf{Y}\mathbf{1}_{\{j=T^b\}} v\left(\widehat{b}\right) + \widehat{\beta}\phi_{j,j+1}\mathbb{E}\left[\widehat{V}_j(\theta,\epsilon',\widehat{a'})|\epsilon\right] \\ & \widehat{c} + \widehat{b} + \widehat{a'} = \widehat{y_j}\left(\theta,\epsilon\right) + \frac{(1+r)}{(1+\gamma)\,\phi_{j-1,j}}\widehat{a} \\ & \widehat{y_j}\left(\theta,\epsilon\right) = \mathbf{1}_{\{j<T^r\}}\left(1-\tau\right)\frac{W}{P}\cdot\theta h_j\epsilon + \mathbf{1}_{\{j\geq T^r\}}\widehat{d}\cdot\theta \\ & \widehat{a'} \geq -\bar{b} \end{aligned} \tag{21}$$

and

$$\widehat{a_{T^w}}(\theta') = \frac{\pi_{T^b}}{\pi_{T^w}}\sum_{\theta}\frac{\Pi^{\theta}(\theta'|\theta)\pi^{\theta}(\theta)}{\pi^{\theta}(\theta')}\mathbb{E}[\widehat{b}|\theta] \tag{22}$$

STATIONARY EQUILIBRIUM

- Stationary equilibrium: "normalized normalized" aggregate asset demand and consumption

$$\hat{\hat{A}}^d = \frac{\widehat{A_t}}{N_t} = \sum \pi_j \mathbb{E}[\widehat{a}_j] \tag{23}$$

$$\hat{\hat{C}} = \frac{\widehat{C_t}}{N_t} = \sum \pi_j \mathbb{E}[\widehat{c}_j] \tag{24}$$

- Steady state equilibrium: r that equates $\hat{\hat{A}}^s$ in (18) with $\hat{\hat{A}}^d$ in (23).

OUTLINE

1. Intuition

2. Full model

3. Decomposition and calibration

Normalized stationary equlibrium

- Calibration guided by demand/supply decomposition for small change in demographics $d\theta$.
- Start from the market clearing condition for assets in levels

$$A_t^d = A_t^s$$

- Normalize by technology and population

$$\begin{aligned} a_t^d &= \frac{A_t^d}{Z_t N_t} \\ &= \sum_j \pi_j(\theta) E[\hat{a}_{ijt}] \\ &= \frac{A_t^s}{Z_t N_t} \\ &= \left(k_t + \left(1 - \frac{1}{\mu}\right) \frac{1}{r-g} y_t\right) \sum_j \pi_j(\theta) h_j \end{aligned}$$

TOTAL DIFFERENTIATION

- Totally differentiating with respect to θ, we obtain

$$\sum_j E[\hat{a}_{ijt}]d\pi_j + \sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial \theta} d\theta + \sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial r} dr =$$

$$a_t^s \left(s_k d\log k_t + s_r d\log y_t + \sum_j \frac{d\pi_j(\theta) h_j}{\sum_k \pi_k(\theta) h_k} - \frac{1}{r-g} s_r dr \right)$$

- Rearranging terms and noticing that $\sum_k \pi_k(\theta) h_k = 1$, we have

$$\left[\sum_j \left(E[\hat{a}_{ijt}] - h_j a_t^s \right) \frac{\partial \pi_j}{\partial \theta} + \sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial \theta} \right] d\theta$$

$$= \quad - \left(\sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial r} + a_t^s \frac{\eta}{r+\delta} \left(s_k + s_r \alpha \right) + \frac{1}{r-g} s_r \right) dr$$

$$= \quad - \left(\sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial r} + a_t^s \left(\frac{\eta}{r+\delta} \left(1 - s_r(1-\alpha) \right) + \frac{s_r}{r-g} \right) \right) dr$$

Interest rate sensitivity of demographics

- Interest rate effect

$$dr = \frac{-\left[\sum_j \left(E[\hat{a}_{ijt}] - h_j a_t^s\right) \frac{\partial \pi_j}{\partial \theta} + \sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial \theta}\right] \cdot d\theta}{\left(\sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial r} + a_t^s \left(\frac{\eta}{r+\delta}\left(1 - s_r(1-\alpha)\right) + \frac{s_r}{r-g}\right)\right)}$$

- Function of
  - Observables:
    - $\left(E[\hat{a}_{ijt}] - h_j a_j^s\right)$ is **net** demand for asset for age group j
    - $\frac{\partial \pi_j}{\partial \theta} d\theta$ is the observed population share change
  - Production elasticities
    - α, δ, η
  - Rent elasticities
    - $s_r, r-g$
  - Behavioral elasticities:

$$\sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial r}$$

$$\sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial \theta} d\theta$$

CHALLENGES FOR CREDIBLE CALIBRATION

- Shocks θ: falling population growth rate + falling mortality rates
- Observables $\rightarrow$ easy
- Production elasticities
  - α,δ quantitatively unimportant in reasonable bound; η challenging
- Rent elasticities
  - Needed: separate capital from markups
- Behavioral elasticities
  - Natural experiment evidence on $\sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial r}$ (Jakobsen et al. 2018)
  - Mortality effects in $\sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial \theta} d\theta$ hopefully insensitive to parameters (tbd)
  - Bequest and consumption effects in $\sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial \theta} d\theta$ (tbd)

ROAD FORWARD

- Experiment with sensitivity of key elasticities w.r.t. parameters
  - Bequest and consumption effects in $\sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial \theta} d\theta$
  - Mortality effects in $\sum_j \pi_j \frac{\partial E[\hat{a}_{ijt}]}{\partial \theta} d\theta$
- Calibrate using weighted moment matching
  - Prioritize hitting key elasticities for counterfactual exercise in calibration
- Apply decomposition method to existing models
  - Identify locus of disagreement in literature