Entry, Exit, and Productivity Dispersion *

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PRELIMINARY

Abstract

This paper develops a dynamic stochastic general equilibrium model to analyze endogenous mechanisms of changes in the first and second moments of firm heterogeneity over the business cycle. In the model, monopolistic competition and endogenous firm entry generate procyclical marginal cost, which implies a procyclical selection mechanism (i.e., increase in a production cutoff during booms). As more firms enter during booms, competition increases in factor markets, resulting in an increase in factor prices. Such an increase in the production costs makes less productive firms shrink: an increase in the production cutoff. While this mechanism explains the countercyclical dispersion in firm-level productivity endogenously, it cannot account for the cyclical changes in the first moment (i.e., relative productivity of entering and exiting firms). We introduce initial uncertainty for entrants to generate empirically consistent movements of both first and second moments. We assume that entrants face additional uncertainty because it is more challenging to predict firm-specific productivity before they produce. Our results suggest that a part of countercyclical dispersion of productivity (at lease, 20%) can be endogenously generated in a model, without the help of the second moment shocks.

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1 Introduction

1.1 Motivation

Recent firm (or plant) level data studies have documented followings:

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- First moments
  - Entry rate is procyclical. Exit rate is less procyclical or acyclical.
  - Relative productivity and size of entering firms are countercyclical. Exiting firms' are less countercyclical or ayclical.
- Second moments
  - Dispersions of both productivity and size are countercyclical.
  - Uncertainty Shocks (Second-moment shocks).

1.2 Objective

We construct a DSGE model which endogenously explains the cyclical patterns in the first and second moments of firm heterogeneity observed in the data.

- Existing models based on Hopenhayn (1992) can match the first moments in the data but have a problem matching the second moments.
- Recent models focusing on the second moments are silent on the models' property of matching the first moments.
- Our model can derive the cyclical properties of firm heterogeneity in a straightforward way.
  - Our model's main mechanism is derived from Cyclical marginal costs + Initial uncertainty $\Rightarrow$ Selection.

1.3 What We Did

The critical features of the model are

- As in the previous literature,
  1. Endogenous entry and initial uncertainty
     - They only know the distribution of firm-specific productivity before creating business opportunity (potential firms).

    - The distribution is fixed over time. $\Rightarrow$ maybe it is the evidence of no second-moment shocks in entries.

      : The size distribution of entrants to the typical industry is stable over time. See Caves (1998) for the survey.

  2. Operating costs
    - Least productive (potential) firms do not operate (produce).

- Initial uncertainty in operating decisions
  - Firm-specific productivity: Incumbents have better information than entrants have.

    But, demand and market information: They have no difference. See Vives (1999).
  - Firms with plenty of operating experience can make better operating decisions than them of unexperienced firms because they can predict their productivity more precisely based on more information.

1.4 Related Literature

1. Empirical
   - Lee and Mukoyama (2015*a*), Woo (2016), Tan(2017):
     - Entry rate is more procyclical than exit rate.
     - The relative productivity of entering and exiting firms are countercyclical and acyclical, respectively.

2. Theoretical
   - Hopenhayn (1992), Lee and Mukoyama (2015*b*)
     - Perfectly competitive market
     - They can match the first moments of firm heterogeneity over the business cycle.
     - Countercyclical selection mechanisms cannot explain the cyclical patterns of the second moments.

- Melitz (2003): Simplified Hopenhayn (1992) open economy model[1]
  - Monopolistic competition + Endogenous entry: High entry and costs in more productive country
  - Firm heterogeneity: a firm-specific productivity level is fixed over time.
- Ghironi and Melitz (2005): DSGE version of Melitz (2003)
- Bilbiie, Ghironi and Melitz (2012): Closed economy model
  - Monopolistic competition: costs, product variety (number of firms), and entry are procyclical.
  - Essentially, there is no firm heterogeneity. All firms have the identical productivity level.

3. Firm dispersion

- A large number of recent studies has documented countercyclical firm dispersion of productivity based on various datasets and measures: Kehrig (2015), Decker, D'Erasmo and Moscoso Boedo (2016), and De Veirman and Levin (2018).
- There exist two different perspectives in the literature: exogenous approach and endogenous approach.
  - The former is to introduce uncertainty shocks, which drive or amplify business fluctuations. See Bloom (2009), Bachmann and Bayer (2013), Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry (2018) and so on.
  - The latter is to provide an endogenous mechanism in which changes in firm dispersion are generated endogenously. Kehrig (2015), Bachmann and Moscarini (2011), and Decker, D'Erasmo and Moscoso Boedo (2016) suggest production in the durable sector, consumer price changes, and intangible expenditures (market exposure) as a mechanism deriving countercyclical firm dispersion, respectively.
- In line with the latter, this paper explains this phenomenon as a result of endogenous fluctuations of productivity and entry over the business cycle.

[1] Monopolistic competition + CRS production function vs Perfect competition + DRS production function: they are homomorphic. See Hopenhayn (2014) for the details of this type models.

This paper is organized as follows. In the next section, we develop a dynamic stochastic general equilibrium model where firm heterogeneity endogenously fluctuates over the business cycles. Section 3 discusses the calibration. Section 4 presents a quantitative analysis: impulse responses and simulated moments. These results guide the model's implications for the cyclical behavior of the first and second moments of firm heterogeneity. The last section concludes.

2 Model

We extend the conventional firm dynamics model to the economy with initial uncertainty in operating decisions. It is hard to know a specific-productivity of newly created business opportunity (potential firm). The agent learns it from the experience of operation (production). Thus, young firms suffer from the lack of information about their productivity of business, which can generate misallocation.

Many conventional models with heterogeneous firms, introduced by Hopenhayn (1992), assume a perfectly competitive market and a production technology with decreasing returns. In contrast, we assume monopolistic competition and constant returns to scale production function as in Melitz (2003). Our economy is homomorphic to the economy with perfect competition and decreasing returns in production.[2]

2.1 Preference

Time is discrete and is indexed by $t = 1, 2, \cdots$. The horizon is infinite. The models specification for preferences is familiar from standard business cycle models with differentiated goods. The representative household values consumption and leisure. She has a time endowment of one unit at any point in time t. Thus, she allocate between labor L_t and leisure $1 - L_t$. There is a representative household with time separable preferences represented by the following utility function. At the initial time i, the representative household maximizes

$$\mathbb{E}_i \left[\sum_{t=i}^{\infty} \beta^{t-i} U\left(C_t, L_t\right) \right], \tag{1}$$

[2]See Hopenhayn (2014) for the details.

where $\beta \in (0,1)$ is the subjective discount factor, and $U(C_t, L_t)$ is the momentary utility function. $C_t \geq 0$ is the aggregate consumption basket defined by

$$C_t = \left\{ \int_{\omega \in \Omega} \left[c_t(\omega)\right]^{\frac{\theta-1}{\theta}} d\omega \right\}^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $\theta > 1$ is the constant elasticity of substitution across goods. In each period t, only $\Omega_t \subseteq \Omega$ is available. A price of individual good $\omega \in \Omega_t$ is denoted by $p_t(\omega) \geq 0$. The welfare based price index is

$$P_t = \left\{ \int_{\omega \in \Omega_t} \left[p_t(\omega)\right]^{1-\theta} d\omega \right\}^{\frac{1}{1-\theta}}. \tag{3}$$

Then, the demand function $c_t(\omega)$ of each good $\omega \in \Omega_t$ is the following.[3]

$$c_t(\omega) = \left[\frac{p_t(\omega)}{P_t}\right]^{-\theta} C_t \tag{4}$$

2.2 Production and Firm's Problem

Entry of potential firm requires sunk cost f_t^E in units of efficient labor. In every period t, there is a continuum of entering potential firms $N_{E,t}$. The potential firms entering at t can produce in the next period $t+1$: one period time-to-build. The exogenous exit shock occurs with constant probability $\delta \in (0,1)$ at the very end of the time period (after production and entry). A continuum of potential firms with mass N_t satisfies the following law of motion:

$$N_t = (1-\delta)\left(N_{t-1} + N_{E,t-1}\right). \tag{5}$$

The mass of potential firms represents the production capacity of economy and plays a role as the physical capital.[4] Each potential firm can produce a different variety $\omega \in \Omega$. Production requires only one factor, labor l_t. The production function is $y_t = A_t z l_t$, where y_t is the output. A_t and z are the aggregate and firm-specific productivities, respectively. We assume that a firm-specific productivity level z is fixed

[3] It is the solution to the following problem.

$$\max_{c_t(\omega), \omega \in \Omega_t} C_t \quad \text{subject to} \quad \int_{\omega \in \Omega_t} p_t(\omega)\, c_t(\omega)\, d\omega \leq P_t C_t$$

[4] Introducing physical capital does not change the main properties of our model. Thus, we assume only labor in production for simplicity.

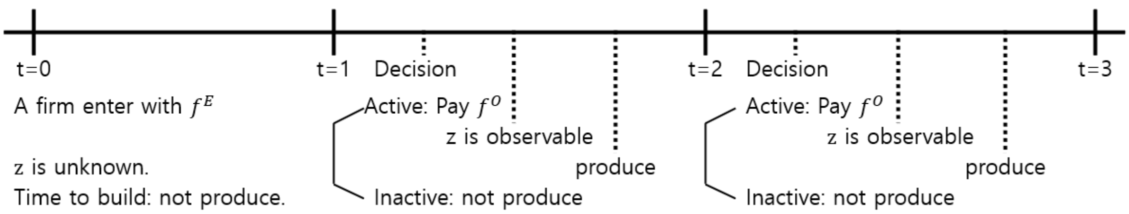

Figure 1: Time line: potential firm established in period $t = 0$

over time. Potential firms can decide to produce or not. Producing firms incur operating costs f_t^O in units of labor.

The timing is summarized in Figure 1. There is the two-type initial uncertainty for potential firms as follows. The first one is in their entry decisions as in the conventional model with endogenous entry. The firm-specific productivity z is not realized before they are established. Thus, entry of potential firm is based on its distribution. The distribution function is denoted by $G_z(\cdot)$ that is time-invariant and public information. The critical feature of our model that is different from the previous literature is the second uncertainty is related to their operating decisions. The main idea is that firms with plenty of operating experience can make better operating decisions than them of unexperienced firms because they can predict their productivity more precisely based on more information. We assume that the firm-specific productivity is unobservable when each potential firm decides to produce or not in every period. It is only observable for an operating firm after paying the operating cost. Then, it makes its hiring decision and sets the price based on observed z. In our model, operating decisions of all potential firms at birth in period t are (ex-ante) identical in period $t+1$. We assume that parameters and shocks guarantee all of them to decide to produce. Since all entrants produce and their firm-specific productivities are time-invariant, potential firms entered before period $t-1$ know the exact level of their productivities in period t. These updating information by operating yields that incumbents will have a further competitive advantage than entrants.

2.2.1 Incumbent's Problem

An incumbent is a potential firm which is already in position in a market. It knows its productivity z. The operating incumbent's employment and price choices are the solution to the following static problem:

$$\pi_t(s) = \max_{\rho_t, y_t} \quad \rho_t y_t - w_t l_t$$
$$\text{subject to} \quad y_t = \rho_t^{-\theta} C_t \text{ and } y_t = A_t z l_t,$$

where all variable are real in units of consumption basket (P_t). ρ_t is the price, w_t is the wage, and $s = z^{\theta-1}$ is the firm-specific productivity in terms of TFPR. Its distribution function is denoted by $G_s(\cdot)$. We assume that $s \in [s_{min}, s_{max})$ where $0 < s_{min} < s_{max}$ and s_{max} can be infinite. $\pi_t(s)$ is the firm's profit excluding sunk and fixed costs. Then, the markup is constant as $\mu = \theta/(\theta-1)$. The price is $\rho_t(s) = \mu(w_t/A_t) s^{\frac{1}{1-\theta}}$. The employment in production is $l_t(s) = (1-1/\theta)[\rho_t(s)]^{1-\theta} C_t/w_t$.

Operating incumbents has to hire f_t^O amount of workers in operation in addition to $l_t(s)$. Thus, their variable and fixed costs are $w_t l_t(s)$ and $w_t f_t^O$. Due to operating costs, least productive potential firms do not operate. The profit is $d_t(s) = \max\left\{0, \pi_t(s) - f_t^O w_t\right\}$. Incumbent's operating decisions can be represented by the productivity cutoff $s_{C,t}$.

$$s_{C,t} = \theta\left(\mu\frac{w_t}{A_t}\right)^{\theta-1}\frac{w_t}{C_t}f_t^O \tag{6}$$

Incumbents only produce when $s \geq s_{C,t}$. We assume $s_{C,t} > s_{min}$.

2.2.2 Entrant's Problem

Potential entrants at birth in period $t-1$ can produce in period t. They make their operating decision based on the average of their profits. There is no ex-ante heterogeneity in operating decisions. We assume that the average of profit is positive:

$$0 < \int_{s_{min}}^{s_{max}} d_t(s)\, dG_s(s) = \int_{s_{min}}^{s_{max}} \pi_t(s)\, dG_s(s) - f_t^O w_t.$$

Thus, the productivity cutoff of entrants is $s_{E,t} = s_{min}$.[5] After paying the operating cost, they hire the labor and set the price. Due to the initial uncertainty of entrants, some entrants (with $s < s_{C,t}$) have negative profits. On average, entrants are less efficient (productive) than incumbents are.

2.2.3 Average Productivity

The first moments of s with cutoffs can summarize all the information on the productivity distributions relevant for all aggregate variables as in Melitz (2003). The distribution of potential firms is fixed. The average productivity of them is $\bar{s} = \int_{s_{min}}^{s_{max}} s dG_s(s)$. Since all entrants produce, the average of operating entrants is $\bar{s}_{E,t} = \bar{s}$. However, leas productive incumbents do not produce. Thus, the average productivity of operating incumbents is $\bar{s}_{C,t} = \int_{s_{C,t}}^{s_{max}} s dG_s(s) / [1 - G_s(s_{C,t})]$. Then, the average prices of operating incumbents and entrants are $\bar{\rho}_{E,t} = \rho_t(\bar{s}_{E,t})$ and $\bar{\rho}_{C,t} = \rho_t(\bar{s}_{C,t})$, respectively. The average profits of operating incumbents and entrants are $\bar{d}_{E,t} = d_t(\bar{s}_{E,t})$ and $\bar{d}_{C,t} = d_t(\bar{s}_{C,t})$, respectively.

2.3 Free Entry Condition

Forward looking behaviors and rational expectations imply that potential firm entry is decided based on the present value of the expected future stream of profits. In period t, the value of entry is

$$\bar{\nu}_t = \mathbb{E}_t \left[\sum_{i=t+1}^{\infty} [\beta(1-\delta)]^{i-t} \left(\frac{\partial U_i}{\partial C_i} / \frac{\partial U_t}{\partial C_t} \right) \bar{d}_i \right]. \tag{7}$$

The cost of entry is f_t^E in units of efficient labor. The aggregate productivity shock affect both production and creation of new potential firms (new variety of goods).[6] Then, the free entry condition is represented by

$$\bar{\nu}_t = f_t^E \left(\frac{w_t}{A_t} \right). \tag{8}$$

We assume that shocks are small enough for this condition to hold in every period.

[5] When the average profit is negative $\int_{s_{min}}^{s_{max}} d_t(s) dG_s(s) < 0$, $s_{E,t} = s_{max}$. We set the parameters satisfying the positive profit in the stationary state. The shock is small enough not to generate the negative one over the business cycle in the simulation.

[6] See Bilbiie et al. (2012) for the details and discussions.

2.4 Household's Problem

The representative household holds two types of asset: shares x_t in mutual funds of potential firms and risk-free bonds (B_{t+1}) with real returns (r_t). The mutual funds that own all potential firms and finance entry of new potential firms. As in Ghironi and Melitz (2005), the household buys shares of mutual funds. The mutual fund pays a total profit in each period that equals the total profit of all potential firms: $N_t \bar{d}_t = (1-\delta) N_{E,t-1} \bar{d}_{E,t} + (1-\delta) N_{t-1} [1 - G_s (s_{C,t})] \bar{d}_{C,t}$ in units of consumption basket. The household buys x_{t+1} shares in the mutual fund of $N_t + N_{E,t}$ potential firms. Home entrants in period t will produce and pay dividends in the future period $t+1$. The period budget constraint (in units of consumption basket) is

$$C_t + B_{t+1} + \bar{\nu}_t (N_t + N_{E,t}) x_{t+1} = (1 + r_t) B_t + \left(\bar{\nu}_t + \bar{d}_t\right) N_t x_t + w_t L_t, \tag{9}$$

where $\bar{\nu}_t N_{E,t} x_{t+1}$ represents investments.

The household maximizes its expected intertemporal utility (Equation 1) subject to Equation (9). The intertemporal decision rules for bonds and share holdings are

$$1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 + r_{t+1}) \right] \tag{10}$$

$$\bar{\nu}_t = \beta (1-\delta) \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\bar{\nu}_{t+1} + \bar{d}_{t+1} \right) \right]. \tag{11}$$

There is no arbitrage in holding shares of mutual funds and bonds. The labor supply decision rule is given by

$$-\frac{\partial U_t}{\partial L_t} = w_t \frac{\partial U_t}{\partial C_t}. \tag{12}$$

2.5 Aggregation and Shock Process

By the definition of welfare based industry price index, the relative prices satisfy that

$$1 = \left\{ [1 - G_s (s_{C,t})] (1-\delta) N_{t-1} (\bar{\rho}_{C,t})^{1-\theta} + (1-\delta) N_{E,t} (\bar{\rho}_{E,t})^{1-\theta} \right\}^{\frac{1}{1-\theta}}. \tag{13}$$

The stock and bond markets are cleared: $B_{t+1} = 0$ and $x_{t+1} = 0$. The income equates to the spending. The aggregate accounting is

$$C_t + I_t = Y_t, \tag{14}$$

where $I_t = \bar{\nu}_t N_{E,t}$ and $Y_t = \bar{d}_t N_t + w_t L_t$ present the aggregate investment and output (gross domestic product, GDP), respectively.

The aggregate productivity shock process follows the AR(1) process:

$$\log(A_{t+1}) = \rho_A \log(A_t) + e_{A,t},, \tag{15}$$

where $e_{A,t}$ is normally distributed with zero mean and standard deviation σ_A. The stationary state value of productivity is normalized by one: $A_{ss} = 1$. We do not introduce any distribution shocks. Thus, all cyclical properties of firm heterogeneity are endogenous.

2.6 Data-consistent Numbers and Productivities of Firms

This subsection constructs the data-consistent variables representing firms dynamics and heterogeneity.

The numbers of operating (producing), entering, continuing, exiting firms are denoted by N_t^P, $N_{E,t}^P$, $N_{C,t}^P$, and $N_{X,t-1}^P$, respectively. As in Dunne, Roberts and Samuelson (1988), the total number of firms in period t is the operating (producing) firms, which includes firms enter between periods t and $t-1$. $N_{E,t}^P$ and $N_{E,t}^P$ are the number of firms enter and exit between periods t and $t-1$, respectively. We define them as follows.[7] There are $N_t = (1-\delta)(N_{t-1} + N_{E,t-1})$ potential firms at the beginning of period t. First, potential firms at birth $t-1$ always produce because of initial uncertainty in operating decisions: $(1-\delta) N_{E,t-1}$. Second, the other potential firms (at birth before $t-1$) produce only if they are more productive than $s_{C,t}$: $[1 - G_s(s_{C,t})](1-\delta) N_{t-1}$. Thus, the total number of producing firms in period t is $N_t^P = (1-\delta) N_{E,t-1} + [1 - G_s(s_{C,t})](1-\delta) N_{t-1}$. The first and second parts represent the entering

[7] Alternatively, we can define entering, continuing, exiting firms as follows. In period t, the entering firms enter between $t-1$ and t. They produce in period t but did not in period $t-1$. The continuing firms continue their business between $t-1$ and t. They produce in both periods t and $t-1$. The exiting firms exit between periods $t-1$ and t, which cover both temporary and permanent exit. The numbers of operating (producing), entering, continuing, exiting firms are $N_{E,t}^P = (1-\delta) N_{E,t-1} + \zeta_{E,t}$, $N_{C,t}^P = [1 - G_s(s_{C,t})](1-\delta) N_{t-1} + \zeta_{C,t}$, and $N_{X,t-1}^P = \delta N_{t-1}^P + G_s(s_{C,t-1})(1-\delta) N_{E,t-2}^P + \zeta_{X,t-1}$. where $\zeta_{E,t} = \max\{G_s(s_{C,t-1}) - G_s(s_{C,t}),\ 0\}(1-\delta) N_{t-1}$, $\zeta_{C,t} = -\zeta_{E,t}$, and $\zeta_{X,t-1} = \max\{G_s(s_{C,t-1}) - G_s(s_{C,t-2}),\ 0\}(1-\delta) N_{t-2}$. $\zeta_{E,t}$, $\zeta_{C,t}$, and $\zeta_{X,t}$ are negligible. We ignore them. Without $\zeta_{E,t}$, $\zeta_{C,t}$, and $\zeta_{X,t-1}$, they are the same as our benchmark definitions.

and continuing firms, respectively: $N_{E,t}^{P}=(1-\delta)N_{E,t-1}$ and $N_{C,t}^{P}=[1-G_s(s_{C,t})](1-\delta)N_{t-1}$. The exiting firms exit between periods t and $t-1$. At the end of period $t-1$, the exogenous death shock destroys δN_{t-1}^{P} mass of firms. Additionally, the massive least productive firms entering in the period $t-2$ do not produce in the period $t-1$ even though they survived from the death shock. Despite of low productivity, they produced in the period $t-2$ because they did not know their productivity. In period $t-1$, they have learned and do not produce. Their mass is $G_s(s_{C,t-1})(1-\delta)N_{E,t-2}^{P}$. Thus, the number of exiting firms between periods $t-1$ and t is $N_{X,t-1}^{P}=\delta N_{t-1}^{P}+G_s(s_{C,t-1})(1-\delta)N_{E,t-2}^{P}$. Finally, we define the entry and exit rate by $2N_{E,t}^{P}/\left(N_t^P+N_{t-1}^{P}\right)$ and $2N_{X,t-1}^{P}/\left(N_t^P+N_{t-1}^{P}\right)$, respectively.

The average productivities of entering and exiting potential firms relative to continuing potential firms are one. However, the selection mechanism yields that the average productivities of entering and exiting firms relative to continuing firms fluctuate over the business cycle. Furthermore, estimated firm-specific productivity s_t^P is normalized by the average productivity of total firms $\bar{s}_t^P$: $s_t^P=s/\bar{s}_t^P$. On average, the productivity of total, entering, continuing, and exiting firms are

$$\bar{s}_t^P=\frac{N_{E,t}^P}{N_t^P}\bar{s}_{E,t}^P+\frac{N_{C,t}^P}{N_t^P}\bar{s}_{C,t}^P \tag{16}$$

$$\bar{s}_{E,t}^P=\frac{\bar{s}_{E,t}}{\bar{s}_t^P}=\frac{\bar{s}}{\bar{s}_t^P} \tag{17}$$

$$\bar{s}_{C,t}^P=\frac{\bar{s}_{C,t}}{\bar{s}_t^P}=\frac{1}{\bar{s}_t^P}\int_{s_{C,t}}^{s_{max}} s\frac{dG_s(s)}{1-G_s(s_{C,t})} \tag{18}$$

$$\bar{s}_{X,t-1}^P=\left(\frac{\delta N_{C,t-1}^P}{N_{X,t-1}^P}\right)\bar{s}_{C,t-1}^P+\left(\frac{\delta N_{E,t-1}^P}{N_{X,t-1}^P}\right)\bar{s}_{E,t-1}^P+\left[\frac{(1-\delta)N_{E,t-2}^P}{\bar{s}_{t-1}^P N_{X,t-1}^P}\right]\int_{s_{min}}^{s_{C,t-1}^P} sdG_s(s), \tag{19}$$

respectively. Then, the relative productivity of entering and exiting firms are $\bar{s}_{E,t}^P/\bar{s}_{C,t}^P=\bar{s}_{E,t}/\bar{s}_{C,t}$ and $\bar{s}_{X,t-1}^P/\bar{s}_{C,t-1}^P=\bar{s}_{X,t-1}/\bar{s}_{C,t-1}$, respectively.

2.7 Firm Dispersion

The variance (or standard deviation) of firm specific productivity is widely used as a measure of firm dispersion. However, the variance is irrelevant because higher average productivity always increases it. Thus, the variance can over or under states. To reduce scale effect, we consider the coefficient of variation as the index of dispersion. As a measure of firm dispersion, define the coefficient of variation CV (s) is defined by CV $(\cdot)=\frac{\text{std}(\cdot)}{E[\cdot]}$ where std $(\cdot)$ is a standard deviation.[8] Thus, the dispersions of

[8]The coefficient of variation is widely used as a statistical measure of the dispersion around the mean, which is normalized. It is sometimes called unitized risk or variation coefficient. The absolute value of the coefficient of variation is known as the

potential and producing firms are given by

$$\text{CV}(s) = \sqrt{\frac{\int_{s_{min}}^{s_{max}} s^2 dG_s(s)}{\left[\int_{s_{min}}^{s_{max}} s dG_s(s)\right]^2} - 1} \quad \text{and} \quad \text{CV}(s|s \geq s_{g,t}) = \sqrt{\frac{\int_{s_{g,t}}^{s_{max}} s^2 d\tilde{G}(s|s \geq s_{g,t})}{\left[\int_{s_{g,t}}^{s_{max}} s d\tilde{G}(s|s \geq s_{g,t})\right]^2} - 1},$$

respectively. The potential firm dispersion is exogenously given. However, the observed firm dispersion is endogenously changed by the threshold. The role of selection mechanism can be represented by following proposition.

Proposition 1 *If the cut is not quite large, then the firm dispersion is decreasing in the cutoff level. Assume that* $s_{g,t} \in \{x \in (s_{min}, s_{max}) | x \leq E[s - x | s \geq x]\}$.[9] *For* $g = E, C$,

$$\frac{\partial \text{CV}(s|s \geq s_{g,t})}{\partial s_{g,t}} \leq 0 \tag{20}$$

Proof. See Appendix A. ■ Thus, the procyclical cutoff generates countercyclical dispersion of group $g = E, C$.

The dispersion of firms can be approximately measured by

$$\overline{\text{CV}}_t\left(s_{C,t}, s_{E,t}, \frac{N_{C,t}^P}{N_t^P}\right) \approx \left(\frac{N_{C,t}^P}{N_t^P}\right) \text{CV}(s|s \geq s_{C,t}) + \left(\frac{N_{E,t}^P}{N_t^P}\right) \text{CV}(s|s \geq s_{E,t}). \tag{21}$$

Continuing firms mainly determine the dispersion of total firms because the dispersion of entering firms is constant over the business cycle in our model ($s_{E,t} = s_{min}$ is constant) and the fraction of entering firms are small. That is consistent with the previous empirical finding: the dispersions of continuing firms (incumbents) and their cyclicality are similar to them of full sample (total producing firms).

3 Calibration

3.1 Functional Forms

We use the conventional functional form of CRRA preference as follows.

$$U(C_t, L_t) = \frac{C_t^{1-\gamma} - 1}{1 - \gamma} - \chi \frac{L_t^{1+1/\varrho}}{1 + 1/\varrho},$$

relative standard deviation defined by $\text{std}(s)/|E[s]| = |\text{CV}(s)|$. In our model the coefficient of variation is always defined because the productivity is nonnegative.

[9] s_{max} can be infinite.

where $\gamma > 1$ governs relative risk aversion, $\varrho > 0$ is the labor supply elasticity, and $\chi > 0$ is the disutility from work.

We assume that the firm-specific productivity denoted by s is lognormally distributed with μ_s and σ_s. To focus on endogenously changed firm distribution, we fix the parameters over time. There is no distributional shock. Then, the first and second moments of potential firms are $\mathrm{E}[s] = \exp\left(\mu_s + .5\sigma_s^2\right)$ and $\mathrm{CV}(s) = \sqrt{\exp(\sigma_s^2) - 1}$, which are time invariant. The truncated first and second moments with the productivity cutoff $s_{g,t} > 0$ are

$$\mathrm{E}_s\left[s|s \geq s_{g,t}\right] = \exp\left(\mu_s + .5\sigma_s^2\right)\Phi\left(\frac{\mu_s + \sigma_s^2 - \ln s_{g,t}}{\sigma_s}\right)/\Phi\left(\frac{\mu_s - \ln s_{g,t}}{\sigma_s}\right) \tag{22}$$

$$\mathrm{CV}\left(s|s \geq s_{g,t}\right) = \sqrt{\exp\left(\sigma_s^2\right)\frac{\Phi\left(\frac{\mu_s - \ln s_{g,t}}{\sigma_s}\right)\Phi\left(\frac{\mu_s + 2\sigma_s^2 - \ln s_{g,t}}{\sigma_s}\right)}{\left[\Phi\left(\frac{\mu_s + \sigma_s^2 - \ln s_{g,t}}{\sigma_s}\right)\right]^2} - 1}, \tag{23}$$

where $\Phi(x) = 1 - \Phi(-x)$ is the standard normal distribution function. I normalize the untruncated mean by one.

Our model tends to generate volatile number of entrants relative to the data. Thus, we introduce frictions in entry and operation to reduce its volatility. The entry and operating costs are

$$f_t^E = f^E + \eta^E\left[\exp\left(N_{E,t} - N_{E,ss}\right) - 1\right] \tag{24}$$

$$f_t^O = f^O + \eta^O\left[\exp\left(s_{C,t} - s_{C,ss}\right) - 1\right], \tag{25}$$

where $N_{E,ss}$ and $s_{C,ss}$ are the number of entrants and the productivity cutoff of incumbents in the stationary state, respectively.

3.2 Parameterization

Our parameter values are listed in Table 1. Each period represents a calendar year. Some parameters are set following the literature without solving the model. Set values of $\beta = 0.95$, $\sigma = 2$, and $\varrho = 1.5$ in preference, which are the standard choice for business cycle models. The entry cost parameter is normalized by one, which has no impact on the model's business cycle properties. For the aggregate shock process, $\rho_A = 0.685$ and $\sigma_A = 0.0163$ imply an autocorrelation of 0.91 and a standard deviation of 0.008 at the quarterly frequency.

Table 1: Model parameters

Description	Symbol	Value
Discount factor	β	0.95
Risk aversion	γ	2
Elasticity of substitution across products	θ	3.8
Labor supply elasticity	ϱ	1.5
Disutility from work	χ	11
Exogenous death shock	δ	0.045
Entry cost	f^E	1
Operating cost	f^O	0.05
Firm specific productivity: mean	μ_s	$-0.5\sigma_s$
Firm specific productivity: standard deviation	σ_s	0.32
Productivity shock: Autocorrelation	ρ_A	0.685
Productivity shock: standard deviation	σ_A	0.0163

Table 2: Calibration target

Statistics		Model: steady state	Data
Survival rate:	(age 1–5)	0.596	0.603
	(age 6–10)	0.775	0.697
	(age 11–15)	0.775	0.745
Startup rate		0.127	0.13
Entry rate		0.057	0.062
Exit rate		0.057	0.055
Relative productivity:	(entering)	0.902	0.96
	(exiting)	0.902	0.86
Relative Labor productivity:	(entering)	0.990	0.98
	(exiting)	0.990	0.92
Firm dispersion		0.267	0.26

Notes: The survival rates are non-cumulative for ages, which data, 1967 – 1977, come from Dunne, Roberts and Samuelson (1989). The startup rate is the share of new employer firms out of all employer firms, which data, 1979, come from Karahan, Pugsley and Sahin (2018). The all first and second moments firm dynamics data, 1972 – 1997, come from Lee and Mukoyama (2015*a*) and Castro, Clementi and Lee (2015) , respectively.

The remaining parameters are calibrated within the model. Table 2 lists the simulated results from the stationary distribution and their empirical counterparts. Even though its firm exit mechanism is simple, the model performs well in reproducing the survival rates from 1967 to 1977. Since the simulated moments are from the stationary distribution, the entry rate equates to the exit rate in the model. The relative productivities are identical between entering and exiting firms, too.

4 Results

This section presents the international business cycle properties of the model and its propagation mechanisms.

4.1 Impulse Responses

This section shows the dynamic path of model variables based on numerical simulations in response to transitory shocks to productivity. We report results of a version of our model without frictions in entry and operating costs – $\eta^E = \eta^O = 0$ –, which model is dented by "Benchmark".[10] To illustrate the model's propagation mechanisms, I consider two counterfactual models labeled by "No cutoff" and "No entry". No cutoff and No entry models represent the models without selection and entry margins, respectively. No cutoff model exhibits a constant cutoff over the business cycle as a result of a large value for $\eta^O = 10^{10}$. Similarly, we prevent cyclical movements of entrants by setting $\eta^E = 10^{10}$, which is no entry model.

Figure 2 describes the responses (percent deviations from steady state) of key variables to a permanent 1 percent increase in productivity. The number of years t after the shock is on the horizontal axis. The responses for all real variables are shown using the welfare-relevant price index in Benchmark, No cutoff, and No entry models.[11]

Consider the effects of the aggregate productivity shock on macroeconomic variables: output, consumption, investment, and labor. Our model's performance is qualitatively indistinguishable from the conventional model proposed by Bilbiie, Ghironi and Melitz (2012). The red lines with asterisks (Benchmark) are similar to the blue lines with diamonds (No cutoff). These results indicate that endogenous selection has the limited impacts on business cycle properties of the key macro variables.[12] No entry model's increases in investment are very small because there are constant entry and exit of potential firms. The changes in investment are only derived from the changes in the value of firms (increases in their profits). The interest rate is the returns to creating the potential firm.

The impulse responses show the role of endogenous firm entry on procyclical marginal costs. During a boom, a large entry of potential firms increases the wage in efficiency units: w_t/A_t representing the aggregate marginal costs. A large demand for goods makes firms hire more people to increase their sales, which increases factor market competition and its price (the wage). However, it is not enough

[10]The response of all key variables to the shock is qualitatively similar to that with plausible frictions.

[11]In the following section, we will introduce the real variables using the data-consistent CPI price index. The responses for all real variables using both indexes are similar. Thus, we have reported only the responses of real variables with the welfare-relevant price index because they represent economic behaviors and decisions in the model.

[12]No cutoff model is slightly different from Bilbiie, Ghironi and Melitz (2012). The original model of Bilbiie, Ghironi and Melitz (2012) has zero operating costs. But, they have essentially identical propagation mechanisms and dynamics. Further, Hopenhayn (1992) and Melitz (2003) introduce the operating fixed costs. Thus, the least productive potential firms do not operate. However, Ghironi and Melitz (2005) and Bilbiie, Ghironi and Melitz (2012) only assume the sunk entry costs.

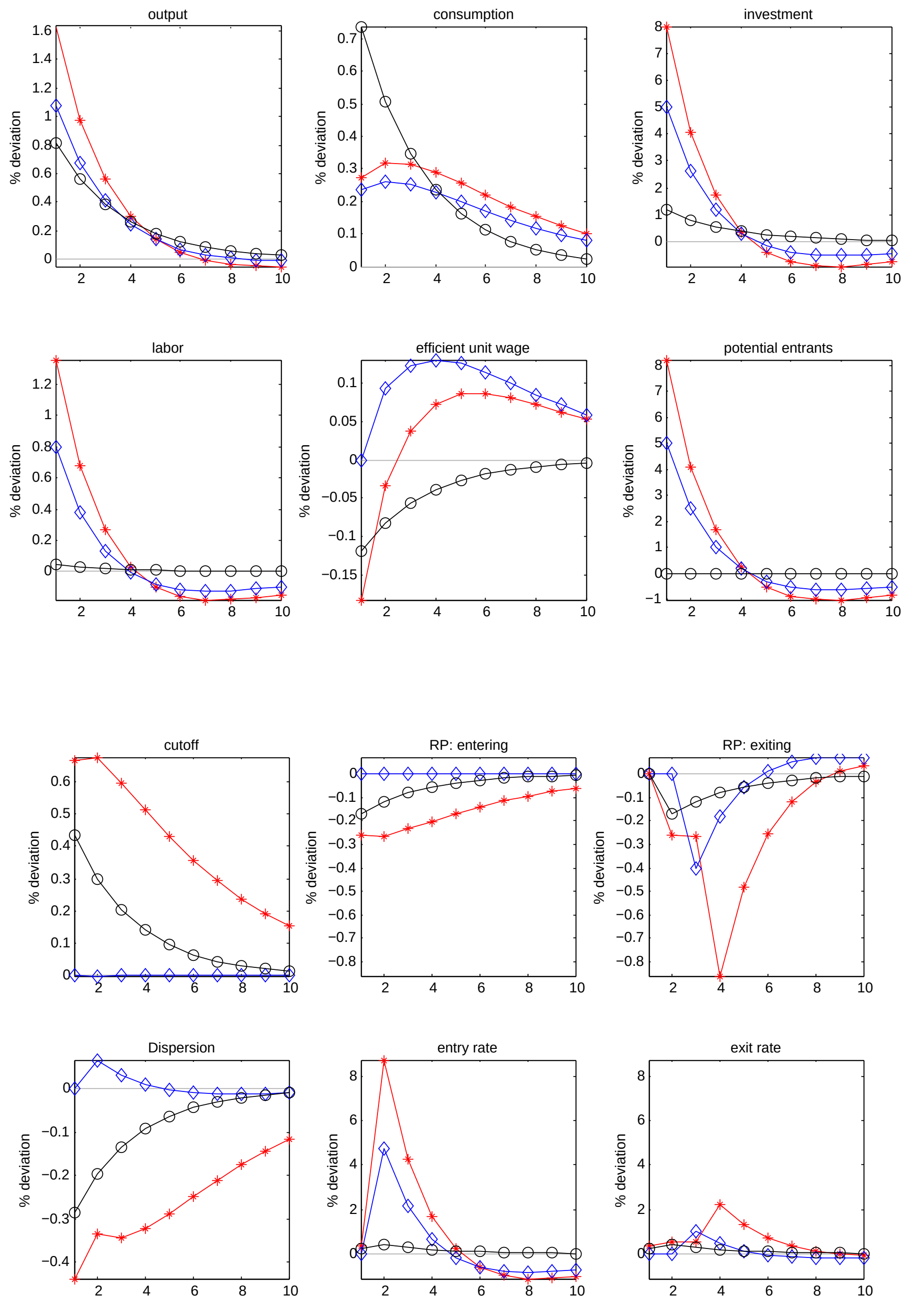

Figure 2: Impulse responses to 1% aggregate shock

Notes: The red lines with asterisks (Benchmark), blue lines with diamonds (No cutoff), and black lines with circles (No entry) are the models with $(\eta^E, \eta^O) =$, $(0, 0)$, $(0, 10^{10})$, and $(10^{10}, 0)$, respectively. The number of years after the shock is on the horizontal axis.

to generate an increase in wage in efficiency units. No entry model shows decreases in the marginal costs because the productivity increases more than the factor price. No cutoff model illustrates how firm entry yields procyclical marginal costs. Large profits (high net present value of potential firm) due to positive productivity shocks promote potential firm entry. Since entries need labor, it makes the factor market become more competitive than before. Such high competition augments the wage w_t more than the productivity A_t increases. These mechanisms need time because potential firms are a stock variable. Further, we assume the time-to-build entry technology. Thus, in the initial period, the factor price of labor in efficiency units do not increase in No cutoff model.

The procyclical wage yields procyclical selection mechanism. Least productive potential firms do not produce when they knew their firm-specific productivity and they are lower than a cutoff level. More precisely, a firm has to hire f_t^O employments when it operates. The operating fixed costs are $w_t f_t^O$. Thus, a change in the cutoff depends on the factor market competition and its price. As we have discussed, in booms the factor demand and its price are high due to intensive and extensive margins. Hence, the cutoff becomes large when a positive productivity shock is realized.

The above procyclical cutoff of incumbents generates countercyclical dispersion in Figure 2. No cutoff model's dispersion moves the opposite direction. Since dispersion of entering firms is larger than it of continuing firms, the procyclical number of entering firms imply procyclical dispersions. Further, a high cutoff level for continuing firms increases their average productivity. Both entering and exiting firms' average productivities relative to continuing firms are countercyclical.

In the impact period of the shock ($t = 1$), the number of potential entrants increases. However, the number of entering firms increases in the next period ($t = 2$) due to time-to-build. In No cutoff model, the number of producing firm does no changes in the period $t = 1$, which implies no change in entry rates. In Benchmark and No entry model, the high level of cutoff decreases the number of (producing) incumbents. Thus, the entry rate becomes positive, but it is minimal. No entry model's response of entry rates is very small. To sum, the increase in potential entrants (numerator) mainly determines the increases in entry rates with a lag. The role of producing incumbents (denominator) is limited. Now, consider exit rates. Exogenous death shocks are the main source of exit in our model. Thus, cyclical changes in exit rates are relatively smaller than them of entry rates. However, Benchmark model's response to the shock increases sluggishly over time. The mechanism behind it is that initial uncertainty in operating decisions makes survival rate of young firms lower than the others.[13] After the shock is realized, many

[13] In our model, the survival rate of firms with age 1 is smaller than it of all other firms aged 2 and above. See Table 2 for the

potential entrants enter the economy. In the next period, all of them do not know their firm-specific productivity before paying operating costs, so they always produce even though some least productive firms get negative profits (including fixed costs). In the period after next period, the least productive firms decide not to operate, which increases exit rates. Moreover, as the case of entry rates, decreases in the number of (producing) incumbents contribute to the increases in exit rates, which is quantitatively small.

4.2 Business Cycle Fluctuations: Simulated Moments

In this section, we investigate the business cycle properties of our model by computing the model generated second moments of our artificial economy for some critical macroeconomic and firm heterogeneity variables. As Ghironi and Melitz (2005) discuss, real variables in our models account variety effects: changes in the range of available products. However, our data based on the consumer price index (CPI) ignore the variety effect. Thus, we define data-consistent variables using CPI that are empirically relevant.

$$x_{R,t} = \left(N_t^P\right)^{\frac{1}{1-\theta}} x_t, \tag{26}$$

which ignores the love-of-variety effect from changes in the number of produced goods.

The all variables are HP filtered. The smoothing parameter for annual frequency is usually $\lambda = 100$ in the previous business cycle literature. However, Ravn and Uhlig (2002) suggest $\lambda = 6.25$. In the U.S. data, business cycle properties of firm dynamics variables such as numbers of entering and exiting firms, entry rate, and exit rate significantly varies with the smoothing parameter. Thus, we report both cases. We set the entry friction parameters η^E for the exercises with $\lambda = 100$ and 6.25 to be 0.85 and 0.45 to match the observed volatility of entrants, respectively. Finally, to show the role of the friction, we consider the frictionless model – $\eta^E = \eta^O = 0$ – denoted by "No friction".

4.2.1 Business Cycle Properties of Key Macroeconomic Variables

To evaluate the business cycle properties of our benchmark model, we compute the implied moments of our artificial economy for some key macroeconomic variables and compare them to those of the data in Table 3. No friction model's firm entry is more volatile than the US annual data. We have calibrated Benchmark models to match the observed standard deviations of firm entry. As in previous business cycle

detail numbers of Benchmark model.

Table 3: Business cycle properties of macro variables

	(η^E, η^O)	SD % Y_R	SD relative to GDP C_R	I_R	L	N_E^P	Correlation to GDP C_R	I_R	L	N_E^P
						Panel A: HP filtered with $\lambda = 100$				
Data		1.910	0.966	4.373	1.060	3.418	0.918	0.826	0.867	0.407
No friction	$(0, 0)$	1.808	0.281	4.475	0.738	4.439	0.940	0.995	0.994	0.353
Benchmark	$(0.85, 0)$	1.485	0.425	3.736	0.575	3.414	0.987	0.996	0.995	0.372
No cutoff	$(0.85, 10^{10})$	1.371	0.365	3.995	0.591	3.693	0.992	0.999	0.996	0.350
						Panel B: HP filtered with $\lambda = 6.25$				
Data		1.239	0.791	4.605	1.110	3.829	0.900	0.916	0.910	0.091
No friction	$(0, 0)$	1.391	0.260	4.504	0.742	4.467	0.978	0.998	0.998	0.035
Benchmark	$(0.45, 0)$	1.219	0.352	4.056	0.643	3.848	0.992	0.999	0.998	0.040
No cutoff	$(0.85, 10^{10})$	1.134	0.297	4.305	0.661	4.126	0.996	1.000	0.999	0.020

Notes: We use the log-linear approximation for the models. All variables are filtered. The sample period of data series is from 1977 to 2014. See Appendix B for the details of data.

literature, model-generated consumption, investment, and labor are too procyclical relative to the data. However, the number of entrants is less procyclical than the data. Introducing entry friction helps to solve this problem. Benchmark model perform better than No friction and No cutoff models at reproducing cyclical properties of entrants. As most dynamic stochastic general equilibrium models, our model also faces difficulties of too smooth consumption. That is because there is not enough endogenous persistence. Thus, adding entry frictions help to increase the standard deviations of consumption relative to output. Additionally, Bilbiie et al. (2012) document the model with endogenous entry reproduce too volatile investment. Our model's generated volatility of investment is similar to Bilbiie et al. (2012), but such large volatility is not problematic. Our data report low volatility of investment: 4.373 (with $\lambda = 100$), compared to the previous literature: annual 2.60 in Backus and Kehoe (1992) and quarter 2.93 in King and Rebelo (1999).[14]

We begin by comparing our model (Benchmark) to No cutoff model. As shown in the impulse responses, the cyclical changes in the incumbent's productivity cutoff amplify a positive aggregate shock. The standard deviation of output in Benchmark model is about 10% larger than it in the model with fixed cutoff. In booms, least productive incumbents exit and shrink due to high competition in the factor market, which generates productivity gains: countercyclical misallocation. Further, Benchmark model performs better than No cutoff model at reproducing consumption volatility. Why? [To be written]

[14] Backus and Kehoe (1992) use annual data with $\lambda = 100$. Their sample period of postwar is 1950 – 1983. King and Rebelo (1999) use quarterly data with $\lambda = 1600$. Their sample period is 1947:1 – 1996:4.

Table 4: Cyclical properties of firm dynamics and heterogeneity

	(η^E, η^O)	Cutoff	Relative Productivity		Entry rate	Exit rate	Dispersion
			Entering	Exiting			
			Panel A: HP filtered with $\lambda = 100$				
Data		–	–	–	0.317	-0.013	-0.398
No friction	$(0, 0)$	0.838	-0.838	0.082	0.393	0.028	-0.849
Benchmark	$(0.85, 0)$	0.904	-0.904	0.030	0.423	0.123	-0.915
No cutoff	$(0.85, 10^{10})$	0.000	0.000	0.139	0.376	-0.191	0.375
No entry	$(10^{10}, 0)$	1.000	-1.000	-0.363	0.825	0.825	-1.000
			Panel B: HP filtered with $\lambda = 6.25$				
Data		–	–	–	0.021	-0.121	-0.392
No friction	$(0, 0)$	0.908	-0.908	0.291	0.081	-0.090	-0.953
Benchmark	$(0.45, 0)$	0.937	-0.937	0.289	0.094	-0.051	-0.968
No cutoff	$(0.45, 10^{10})$	0.000	0.000	0.230	0.043	-0.218	0.042
No entry	$(10^{10}, 0)$	1.000	-1.000	-0.027	0.717	0.717	-1.000

Notes: The cyclicality is measured by the correlation to output. We use the log-linear approximation for the models. All variables are filtered. The sample period of all data series except for the firm dispersion is from 1977 to 2014. See Appendix B for the details of data. The dispersion data are constructed by using the data in Kehrig (2015) as follow. We calculate the cyclicality of firm dispersion as the sum of correlations between dispersion and output for non-durable and durable sectors: -0.293 and -0.502 with $\lambda = 100$, and -0.273 and -0.511 with $\lambda = 6.25$.

4.2.2 Business Cycle Properties of Firm Heterogeneity

Table 4 reports the correlations of first and second moments of firm heterogeneity with an output. As in the previous section, we report the results of two alternative models. Comparing Benchmark with these models provides insight on the model mechanisms: endogenous entry and selection. The first one is "No cutoff" model with $\eta^O = 10^{10}$.[15] In this case, the cutoff is fixed over the business cycle. Cyclical movements of firm dispersions are due to changes in the fraction of entrants. In the model, there are no changes in relative productivity of entering firms. Second, "No entry" model is $\eta^E = 10^{10}$. In this case, there is no change in potential entrants over the business cycle. Fluctuations of cutoff solely determine the firm dynamics and heterogeneity.

Our models generate strongly procyclical productivity cutoff for incumbents. The incumbents in booms are more productive than in recessions. However, the distribution of firm-specific productivity of entrants is constant over time. Thus, entering firms' productivity and size (relative to incumbents) are strongly countercyclical. However, them of exiting firms are weakly procyclical, because the high productivity cutoff also generates exiting firms and increases their average productivity. These results

[15] In the cases with $\lambda = 100$ and 6.25, $\eta^E = 0.85$ and 0.45, respectively.

Table 5: Countercyclical firm dispersion

	SD relative to GDP $\times$ Sign of correlation to GDP				
	Data	No friction	Benchmark	No cutoff	No entry
			Panel A: HP filtered with $\lambda = 100$		
(η^E, η^O)		$(0, 0)$	$(0.85, 0)$	$(0.85, 10^{10})$	$(10^{10}, 0)$
	-1.116	-0.223	-0.231	0.050	-0.289
			Panel B: HP filtered with $\lambda = 6.25$		
(η^E, η^O)		$(0, 0)$	$(0.45, 0)$	$(0.45, 10^{10})$	$(10^{10}, 0)$
		-0.198	-0.209	0.056	-0.289

Notes: We use the log-linear approximation for the models. All variables are filtered. According to Kehrig (2015) – filtered with $\lambda = 100$ –, the standard deviations of firm dispersion for non-durable and durable sectors are 0.043 and 0.053, respectively. The standard deviations of output for the sectors 0.026 and 0.060, respectively. We calculate the volatility of firm dispersion as $0.043 + 0.053$ divided by $0.026 + 0.060$. The standard deviations of firm dispersion for the sectors are 1.662 and 0.889, respectively. (Its average is 1.276.)

are consistent with the empirical findings of Lee and Mukoyama (2015*a*).

Our model with endogenous changes in entry and cutoff over the business cycle is consistent with the observed cyclical patterns of entry and exit rates. In the business cycle components data with the smoothing parameter $\lambda = 100$ and 6.25, the entry rates are strongly procyclical and weakly procyclical, respectively. The exit rates are acyclical and countercyclical, respectively. We conclude that the correlation of entry rate with output is larger than it of exit rate.[16] A positive shock increases the net present value of potential firms, which yields large entries of potential firms. After the next period, they start to produce. Thus, the model generates the procyclical number of entering firms and entry rate.[17] In contrast, the main mechanism of exit – exogenous death shocks – is acyclical.

The last column of Table 4 shows that procyclical movements of incumbents' productivity cutoff drive countercyclical dispersion as presented by Kehrig (2015). Even though endogenous entry yields procyclical firm dispersions because the dispersion of entrants is larger than it of incumbents, it is quantitatively small in our calibrated model. In Table 5, to quantify how much our model can explain countercyclical firm dispersions, we use the standard deviation of firm dispersion relative to output times the sign of the correlation between firm dispersion and output. In the U.S. manufacturing sector, the volatility of the manufacturing sector is 1.116 on average between non-durables and durables. Our benchmark

[16] Our interpretation of the results is similar to Lee and Mukoyama (2015*a*). **?** finds that the entry rate is procyclical and the exit rate is countercyclical.

[17] Consistent with our data and findings of **?**, the time-to-build generates that our model generated correlation of entry with lagged output is larger than it with current output.

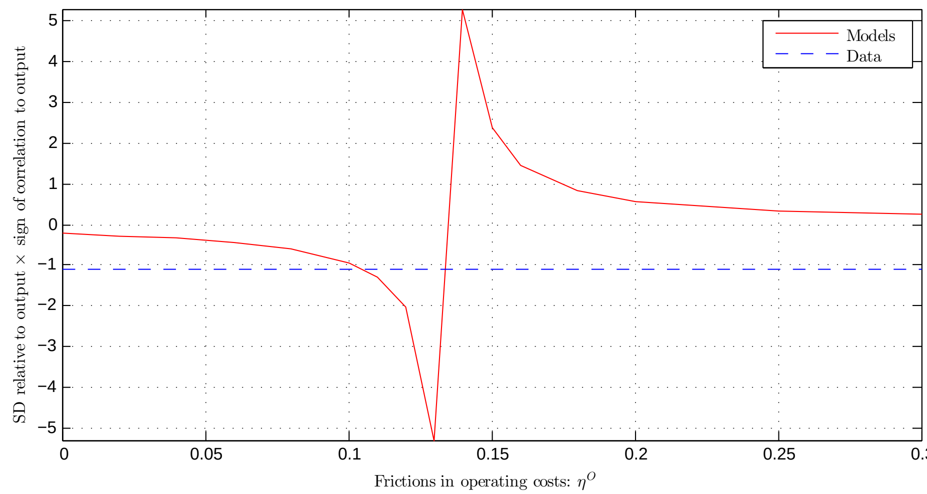

Figure 3: Firm dispersions with various frictions in operating costs

Notes: We use the log-linear approximation and HP filter with $\lambda = 100$ for the models with $\eta^E = 0.85$. The red line is the model generated volatility of firm dispersion relative to output times sign of its correlation to output. The blue dashed line is the data.

model endogenously generates 20 percent of changes in firm dispersion over the business cycle, without the help of the distributional shocks.

4.2.3 Frictions In Operating Costs

We incorporate frictions in operating costs: η^O. We explore this because our model is an extreme case where all potential entrants are ex-ante identical (in terms of their productivity and information), as well as their firm-specific productivity, is full-persistent and they observe perfectly after one experience of production. These environments in our baseline model are useful to emphasize how an aggregate shock endogenously generates business cycle fluctuations of first and second moments of firm heterogeneity. Further, they allow for the simple and easy analytical aggregations and solutions of the model. However, they are too simple and certainly unrealistic. Instead of adding idiosyncratic shocks, signals, and more mechanisms in our selection (cutoff) mechanism, we investigate the impacts of operation frictions on our main results.

Figure 3 reports the quantitative measure of model performance regarding firm dispersion. With a large value of frictions, the movements of cutoff over the business cycle become limited. As shown in No cutoff model, the firm dispersion becomes procyclical. In the range of η^O generating countercyclical firm dispersion, however, rigid operating decisions tend to increase the model's explanatory power related to firm dispersion. Why? [To be written]

5 Concluding Remarks

[To be written]

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Appendix

A Proof of Proposition 1

Proof. Assume that $G_s(\cdot)$ is twice continuously differentiable on its support, the cutoff satisfies $s_{g,t} \in (s_{min}, s_{max})$, and s_{max} can be infinite. Let $h_s(x) = G_s'(x) / [1 - G_s(x)]$ be a hazard rate at x. We obtain the derivative of firm dispersion measure with respect to the cutoff as follow. For $g = E, C$,

$$\frac{\partial \text{CV}(s|s \geq s_{g,t})}{\partial s_{g,t}} = \frac{\Xi_{g,t} h_s(s_{g,t})}{2E[s|s \geq s_{g,t}]^3 \sqrt{\text{CV}(s|s \geq s_{g,t})}}, \tag{A1}$$

where $\Xi_{g,t} = E\left[s^2 - s_{g,t}^2 | s \geq s_{g,t}\right] E[s|s \geq s_{g,t}] - 2E[s - s_{g,t}|s \geq s_{g,t}] E\left[s^2|s \geq s_{g,t}\right]$. Since $E[s^n|s \geq s_{g,t}] > 0$ for all $s_{g,t} > s_{min}$, we obtain that sign $(\partial \text{CV}(s|s \geq s_{g,t}) / \partial s_{g,t}) =$ sign $(\Xi_{g,t} h_s(s_{g,t}))$. We can rewrite $\Xi_{g,t} = -2\{0.5E[s|s \geq s_{g,t}] - s_{g,t}\} E\left[s^2|s \geq s_{g,t}\right] - s_{g,t}^2 E[s|s \geq s_{g,t}]$. Also, $\lim_{s_{g,t} \to s_{min}^+} \Xi_{g,t} = -E[s] E\left[s^2\right] < 0$. Therefore, if the cutoff $s_{g,t}$ is smaller than $0.5E[s|s \geq s_{g,t}]$, then the derivative is non-positive due to $h_s(s_{g,t}) \geq 0$. Also, we can rewrite the condition as $s_{g,t} \leq E[s - s_{g,t}|s \geq s_{g,t}]$. ■

B Data

The number of entrants data come from Business Dynamics Statistics. It is available from 1977 to 2014.

Number of Entering Firms A count of establishments born within the cell during the last 12 months

Entry Rate 100 * (entry at time t divided by the average of establishments at t and $t - 1$)

Number of Exiting Firms A count of establishments exiting from within the cell during the last 12 months.

Exit Rate 100 * (exit at time t divided by the average of establishments at t and $t - 1$)

I collect aggregate output, consumption, investment, and labor (total hours) data in Federal Reserve Economic Data (FRED) from 1977 to 2014.

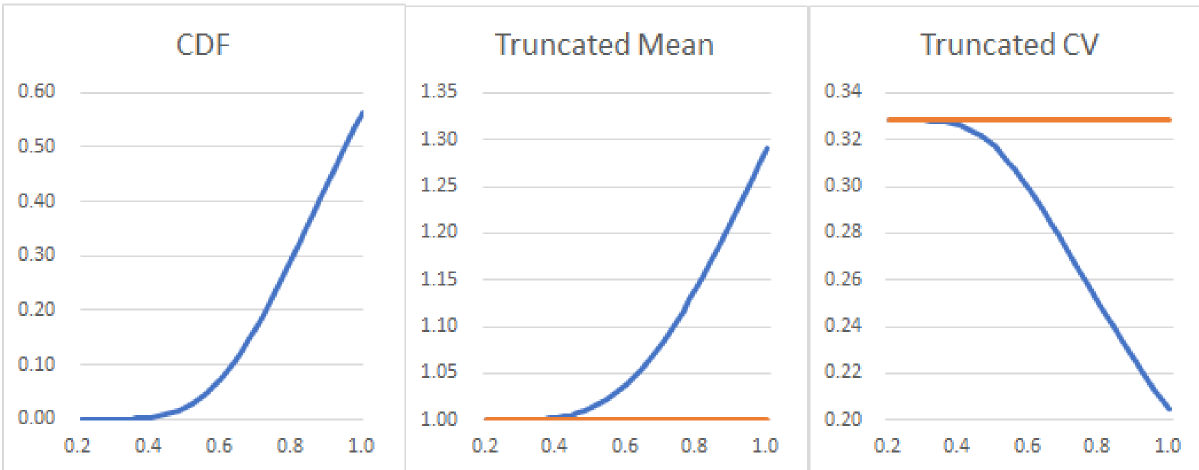
Output GDPCA: Real gross domestic product, billions of Chained 2012 dollars

Consumption PCECCA: Real personal consumption expenditures, billions of Chained 2012 dollars

Investment GPDICA: Real gross private domestic investment, billions of Chained 2012 dollars

Labor B4701C0A222NBEA: Hours worked by full-time and part-time employees, Millions of hours

C Figure



Notes:Note: Log-normal distribution with $\mu = -0.5\sigma^2$ and $\sigma = 0.32$. The yellow line is a value without truncation. CV is the coefficient of variation defined by SD/mean.