Constrained efficient borrowing with sovereign default risk*

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Preliminary and Incomplete

Abstract

We propose a tractable algorithm for solving quantitative models of sovereign default with constrained efficient borrowing (i.e., with commitment to a borrowing policy but not to a default policy). Our algorithm utilizes the government's optimality condition that, compared to the Markov condition, only requires one additional state variable that summarizes the effect of current borrowing on past consumption. Comparing the simulations of the model with and without commitment, we find that the overindebtedness chosen by the Markov government is small but accounts for most of the default risk. Higher bond prices with commitment imply that the Markov government is overindebted but underborrows. These results underscore the importance of governments' efforts to limit their future policies with fiscal rules and independent fiscal councils. Commitment does not affect significantly the procyclicality of fiscal policy, which casts doubts on the emphasis on countercyclical fiscal policy in existing fiscal rules. The government may commit to debt buybacks, showing that such policies may be part of optimal deleveraging plans. Our algorithm could be extended to study other aspects of debt management in which time inconsistency plays a role.

JEL classification: F34, F41.

Keywords: Sovereign Default, Long-term Debt, Debt Dilution, Overindebtedness, Underborrowing, Fiscal Rules, Time Inconsistency, Debt Management.

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1 Introduction

As evidenced by the widespread use and discussion of fiscal rules, the overindebtedness or deficit bias of many governments is often at the center of policy debates.[1] Overindebtedness often results from the government's lack of commitment to future borrowing (overindebtedness could also arise from externalities in a monetary union; Beetsma and Uhlig, 1999). While non-quantitative studies present stylized models in which the optimal borrowing policy with commitment is easier to characterize (Bizer and DeMarzo, 1992; Bolton and Jeanne, 2009; Kletzer, 1984), previous quantitative studies do not characterize this policy.[2] We propose a tractable algorithm for solving quantitative models with constrained efficient borrowing (this is, the optimal borrowing policy the government can commit to, considering the government cannot commit to a default policy). We use this algorithm to demonstrate the quantitative importance of the government's commitment problem and thus support ongoing discussions on the role of fiscal rules and independent fiscal councils that could help governments achieve commitment (IMF, 2017).

We study a sovereign default framework à la Eaton and Gersovitz (1981), which following Aguiar and Gopinath (2006) and Arellano (2008), is commonly used for quantitative studies of fiscal policy and sovereign debt crises. At the beginning of each period, the government first observe its endowment and i.i.d. utility of defaulting shocks. Second, when the government is not in default, it decides whether to default on its debt. Third, a government not in default can borrow. We assume the government issues long-term debt, which introduces a time consistency (debt dilution) problem that has been shown to be essential for generating plausible implications

[1]As defined by the IMF (2017), "A fiscal rule is a long-lasting constraint on fiscal policy through numerical limits on budgetary aggregates." Overindebtedness is also a relevant concern in other credit markets. For example, corporate debt contracts often include covenants intended to limit overindebtedness (Asquith et al., 2005; Smith and Warner, 1979; Rodgers, 1965; and Carey et al., 1993). Corporate debt and mortgage loans typically feature a seniority structure that may mitigate this problem. In contrast, sovereign bonds present a weaker protection against overindebtedness (due in part to the weak enforcement of sovereign debt claims).

[2]Previous quantitative studies present modifications to debt contracts that mitigate overindebtedness. Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009) study one-period bonds, Chatterjee and Eyigungor (2015) focus on seniority, and Hatchondo et al. (2016) study debt covenants that penalize the government for future borrowing. While these modifications of debt contracts mitigate overindebtedness, they have other effects. For example, they change the government's exposure to rollover risk. In contrast with the no-dilution covenants studied by Hatchondo et al. (2016), we show that the optimal borrowing policy the government wants to commit to implies debt dilution (debt dilution refers to the reduction in the value of existing debt triggered by the issuance of new debt).

for the sovereign default premium (Chatterjee and Eyigungor, 2012; Hatchondo and Martinez, 2009). Bonds are priced by competitive foreign investors.

We study the equilibrium of the model under two commitment assumptions. First, as is standard in the literature, we focus on the Markov Perfect Equilibrium, in which each period, the "Markov government" chooses how much it wants to borrow. Second, we assume that in period zero (or in the first period after a default), the "Ramsey government" can choose its income-history-contingent borrowing for every future period (until it defaults).

We propose a tractable algorithm for solving the Ramsey problem. Our algorithm utilizes the government's optimality condition and only requires one additional state variable that summarizes the effect of current borrowing on past consumption. The optimality condition for the Markov equilibrium includes a negative effect of borrowing on current consumption because borrowing lowers the price at which the government sells bonds. Thus, in this optimality condition, the derivative of the bond price appears weighted by the marginal utility of consumption and the level of issuances in the current period. We show that the optimality condition for the Ramsey problem is identical to the Markov condition, except that the derivative of the bond price appears weighted not only by the marginal utility of consumption and the level of issuances in the current period, but also by a history variable that summarizes marginal utilities and issuances in previous periods. Thus, for the same level of initial income and debt, the Ramsey's government optimality condition is the same for all income histories that imply the same weighted sum of previous periods marginal utilities and issuances (which we summarize in the new history state variable).

We impose discipline on our quantitative exercise by calibrating the baseline model without commitment to match data from Mexico, a representative economy with sovereign risk and a standard reference in the literature. The overall match between the model predictions and the data makes the model a good laboratory for the quantitative exercises we conduct in this paper. We measure the effects of commitment to future borrowing by comparing simulations of the baseline model (without commitment) with those of the model with constrained efficient borrowing.

Comparing the simulations of the standard model and the model with commitment, we find that the overindebtedness resulting from the government's lack of commitment is small. With commitment, the average debt ratio only declines from 43% to 40%. However, the government's inability to commit to a borrowing policy accounts for the majority of the default risk in the baseline simulations, as reflected in sovereign spreads. In the Ramsey economy, the mean and the volatility of the sovereign spread are only 0.2% and 0.1%, respectively. In contrast, in the Markov economy, these statistics are 2.4% and 1.0% (both matching calibration targets and, thus, representative of the data). These much higher spreads reflect much lower bond prices for the Markov government. In fact, the much lower bond prices it faces, imply that the overindebted Markov government underborrows (compared to the constrained efficient borrowing of the Ramsey government). Thus, the Ramsey government both almost eliminates sovereign risk, and consumes more than the Markov government. These results are indicative of the importance of governments' efforts to gain commitment to future borrowing (deficit) policies through fiscal rules and independent fiscal councils.

We also find that the government does not want to commit to a more countercyclical fiscal policy. The Ramsey government commits to issuing even more debt in good times and less debt in bad times. However, since the Ramsey government faces higher debt prices in bad times (as evidenced by the lower volatility of countercyclical spreads), the cyclicality of consumption does not change significantly with commitment. This result casts doubts on the common emphasis on the desirability of countercyclical fiscal policy and the consequent common use of scape clauses in fiscal rules (IMF, 2017).

We also show that the Ramsey government may want to commit to buying back debt when it is highly indebted. In contrast, the Markov government never buys back debt (consistently with the findings presented by Aguiar et al., forthcoming and Bulow and Rogoff, 1988, 1991). The Markov government does not benefit from the increase in bond prices implied by a buyback in the current period. The Ramsey government that wants to issue debt may benefit from the increase in bond prices that would result from committing to a debt buyback in the future.[3]

The rest of the article proceeds as follows. Section 2 introduces the model. Section 3 presents

[3]Bi et al. (2013) show that expectations about future fiscal consolidations are an important determinant of the success of fiscal adjustments.

the optimality conditions. Section 4 discusses the calibration. Section 5 discusses our computation algorithm. Section 6 presents the quantitative results. Section 7 concludes. The Appendix presents the derivation of optimality conditions.

2 The model

There is infinite-horizon economy with $t \in \{0, 1, 2, ...\}$ and a single tradable good. The domestic economy receives a stochastic endowment stream y of this good, where $y \in \mathcal{Y} = \{y_1, ..., y_J\}$ follows a Markov process. Let U denote the i.i.d continuation utility from defaulting. The pdf f_j and cdf F_j for U are functions of income y_j.[4] At the beginning of each period, the government observes y_j and U_j and chooses whether to default. After that, a government not in default borrows using a long-term bond. A bond issued at t pays $\{\delta, \delta(1-\delta), \delta(1-\delta)^2, ..., \}$ at $t+1, t+2, ...$ (Hatchondo and Martinez, 2009). When the government defaults, it does so on all current and future debt obligations. Bonds are priced by competitive risk-neutral lenders that discount future payments with the risk-free rate r. The government's objective is to maximize the present expected discounted value of future utility flows of the representative agent in the economy.

2.1 Markov government

We first assume that (as is standard), each period, the government chooses how much to borrow. This is, each period t, the government maximizes

$$\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j\right),$$

where $\mathbb{E}$ denotes the expectation operator, β denotes the subjective discount factor, and c_t represents consumption of private agents. Thus, the government cannot commit to future (default

[4]Assuming a stochastic continuation value of defaulting (as in, for example, Aguiar et al., forthcoming) implies that when the Ramsey government chooses future borrowing as a function of the history of income shocks, it is choosing a probability of default (as the Markov government does) and not a default decision. This allows for our approach of using the first-order condition to find the equilibrium. Assuming the distribution of U depends on income allows us to make the mean continuation value of defaulting endogenous, as it is usually done in the literature (see Section 4). Shocks to the utility cost of defaulting around this mean endogenous value can be motivated, for example, by political turnover (Hatchondo and Martinez, 2010; Hatchondo et al., 2009).

and borrowing) decisions, and one may interpret this environment as a game in which the government making decisions in period t is a player who takes as given the (default and borrowing) strategies of other players (governments) who will decide after t. We focus on Markov Perfect Equilibria. That is, we assume that in each period, the government's equilibrium default and borrowing strategies depend only on payoff-relevant state variables.

2.1.1 Recursive formulation for the Markov government

We denote with x' the next-period value of a variable x. Let $\pi_{i,j}$ denote the probability of $y' = y_j$ given $y = y_i$. Let b denote the quantity of long-term coupon obligations that mature in the current period, and q denote the bond price function. The expected utility of a government not in default with income y_i and debt b is given by

$$V_i(b) = \underset{b'}{Max} \left\{ u(c) + \beta \sum_{j=1}^{J} \pi_{i,j} \left[\; F_j\left(V_j(b')\right) V_j(b') + \int_{V_j(b')}^{\infty} U \; f_j(U) dU \; \right] \right\} \qquad (1)$$

$$s.t. \quad c = y - \delta b + q_i(b') \left[b' - (1-\delta) b \right]$$

$$q_i(b') = \sum_{j=1}^{J} \pi_{i,j} F_j\left(V_j(b')\right) \left[\delta + (1-\delta) q_j\left(\hat{b}_j(b)\right) \right],$$

where $\hat{b}$ denotes the Markov Equilibrium borrowing policy.

2.2 Ramsey government

For any t, let $y^t = \{y_0, ..., y_t\}$ denote the history of income until period t. Let $\mathcal{Y}^t$ denote the set of all possible histories until period t. In period 0, the Ramsey government chooses income-history-contingent borrowing policies for every future period. That is, at $t = 0$, the Ramsey government chooses $b_{t+1}(y^t)$, for all $y^t = \{y_0, ..., y_t\} \in \mathcal{Y}^t$ and all t, by maximizing

$$\underset{\left\{\{b_{t+1}(y^t)\}_{\forall y^t\in\mathcal{Y}^t}\right\}_{t=0}^{\infty}}{Max}\left\{u(c_0)+\sum_{t=1}^{\infty}\beta^t\sum_{\forall y^t\in\mathcal{Y}^t}Pr(y^t)\prod_{n=1}^{t-1}F_n(V_n(y^n))\left[\begin{array}{c}F_t(V_t(y^t))u(c_t(y^t))+\\ \int_{V_t(y^t)}^{\infty}U\ f_t(U)dU\end{array}\right]\right\}\quad(2)$$

$$s.t.\quad c_t(y^t)=y_t-\delta b_t(y^{t-1})+q_t(y^t)\left[b_{t+1}(y^t)-(1-\delta)b_t(y^{t-1})\right],$$

$$V_t(y^t)=u(c_t(y^t))+\beta\sum_{j=1}^{J}Pr(y_j^t)\left[\ F_{t+1}(V_{t+1}(y_j^t))V_{t+1}(y_j^t)+\int_{V_{t+1}(y_j^t)}^{\infty}Uf_{t+1}(U)dU\ \right],\quad(3)$$

$$q_t(y^t)=\sum_{j=1}^{J}Pr(y_j^t)F_{t+1}(V_{t+1}(y_j^t))\left[\delta+(1-\delta)q_{t+1}(y_j^t)\right],\quad(4)$$

where for all $y^t=\{y_0,...,y_t\}\in\mathcal{Y}^t$, $y_j^t=\{y_0,...,y_t,y_j\}$.

It should be noticed that the Ramsey problem above assumes that the borrower can commit to future borrowing but cannot commit to future default decisions, which are only optimal ex-post. Our focus on this constrained efficient borrowing policies is motivated in part by existing fiscal rules, in which governments typically attempt to commit to future borrowing and not to future default decisions (IMF, 2017). In Hatchondo et al. (2015), we argue that the focus of existing fiscal rules on borrowing is consistent with the commitment to default policies being more costly than than the commitment to borrowing limits. Our focus on constrained efficient borrowing is also consistent with the focus of the literature that study the effect of constraining borrowing with different debt instruments (Chatterjee and Eyigungor, 2012, 2015; Hatchondo and Martinez, 2009; Hatchondo et al., 2016) while assuming that default decisions continue to be decided by a Markov government. Mateos-Planas and Ríos-Rull (2015) study the case in which the government can commit to both its future borrowing and default policies.

3 Optimality conditions

This section presents the optimality conditions for both the Markov and the Ramsey governments. A detailed derivation of these conditions is presented in the Appendix.

3.1 Optimality condition for the Markov government

The next equation presents the standard optimality condition for the Markov government:

$$
\begin{aligned}
u'(c_i)q_i(b') &= \beta \sum_{j=1}^{J} \pi_{i,j} \underbrace{F_j(V_j(b'))}_{t+1 \text{ Repayment prob}} V_j'(b') - \frac{\partial q_i}{\partial b'} u'(c_i)\left[b' - (1-\delta)b\right] \\
&= \beta \sum_{j=1}^{J} \pi_{i,j} F_j(V_j(b')) u'(c_j') \left[\delta + (1-\delta) q_j'(\hat{b}_j(b))\right] - \frac{\partial q_i(b')}{\partial b'} u'(c_i)\left[b' - (1-\delta)b\right] \quad (5)
\end{aligned}
$$

The second term of the right-hand side of this equation shows that it is costly for the government to lower the bond price, which lowers the proceeds the government obtains from issuing bonds and thus consumption. Consequently, in equation (5), the derivative of the bond price is weighted by the marginal utility of consumption and the level of issuances of long-term debt $(b' - (1-\delta)b)$.

3.2 Optimality condition for the Ramsey government

For any income history $\tilde{y}^t$, the next equation presents the optimality condition for the Ramsey problem in equation (2):

$$
\begin{aligned}
&\overbrace{\frac{\partial}{\partial b_{t+1}(\tilde{y}^t)} \left[u(c_0) + \sum_{k=1}^{t-1} \beta^k Pr(\tilde{y}^k) \prod_{n=1}^{k-1} F_n(V_n(\tilde{y}^n)) \left[\begin{array}{l} F_k(V_k(\tilde{y}^k)) u(c_k(\tilde{y}^k)) + \\ \int_{V_k(\tilde{y}^k)}^{\infty} U \; f_k(U) dU \end{array} \right] \right]}^{\text{Effect of } b_{t+1}(\tilde{y}^t) \text{ on the utility in periods before } t} \\
+ \;& \beta^t Pr(\tilde{y}^t) \prod_{n=1}^{t} F_n(V_n(\tilde{y}^n)) \times \\
&\underbrace{\left[u'(c_t(\tilde{y}^t)) \left[q_t + \frac{\partial q_t}{\partial b_{t+1}(\tilde{y}^t)} \right] l_t(\tilde{y}^t) - \beta E_{\tilde{y}_t} \left[F_{t+1}(V_{t+1}) u'(c_{t+1}) \left[\delta + (1-\delta) q_{t+1} \right] \right] \right]}_{\text{Optimality condition for the Markov government}} = 0 \quad (6) \\
& s.t. \quad c_t(\tilde{y}^t) = \tilde{y}_t - \delta b_t(\tilde{y}^{t-1}) + q_t(\tilde{y}^t) \left[b_{t+1}(\tilde{y}^t) - (1-\delta) b_t(\tilde{y}^{t-1}) \right],
\end{aligned}
$$

where $l_t(\tilde{y}^t) = b_{t+1}(\tilde{y}^t) - (1-\delta)b_t(\tilde{y}^{t-1})$ denotes the level of issuances of long-term debt. Comparing equations (5) and (6) illustrates the time inconsistency problem in the standard default model with long-term debt. The last term in equation (6) shows that the Ramsey government considers the same trade-offs considered by the Markov government: borrowing allows for more consumption today at the expense of less consumption next period. However, the first term in equation (6) shows that the Ramsey government also considers the effect of period t borrowing

$(b_{t+1}(y^t))$ on the utility of every past period (which the Markov government does not do). Intuitively, the Ramsey government considers that by credibly promising to borrow more in period t and thus increasing the default probability in period $t+1$, it lowers the price of bonds issued in every period before t and thus it lowers consumption in every period before t. In contrast, as illustrated in equation (5), the Markov government is only concerned about the effect of issuances in the bond price of new issuances $b' - (1-\delta)b$, and ignores the effect on the price of bonds issued in previous periods. The following proposition presents a simpler recursive optimality condition for the Ramsey government that we can use to solve the Ramsey problem (see the Appendix for the proof).

Proposition 1 *For any history of income and borrowing such that at the beginning of a period income is y_i, debt is b, and the history variable is h, the optimality condition for the Ramsey government in equation (6) can be written as*

$$u'(c_i)q_i(b',h') = \beta \sum_j \pi_{ij} \left[F_j(V_j(b',h'))u'(c_j') \left[\delta + (1-\delta) q_j(\hat{b}_j(b',h'),h') \right] \right] - \frac{\partial q_i(b',h')}{\partial b'} h' \quad (7)$$

$$\begin{aligned} \textit{where} \quad & c_i = y_i - \delta b + q\left[b' - (1-\delta)b\right], \\ q_i(b',h') &= \sum_j \pi_{ij} F_j(V_j(b',h')) \left[\delta + (1-\delta) q_j(\hat{b}_j(b',h'),h') \right], \\ V_i(b,h) &= u(c_i) + \beta \sum_j \pi_{i,j} \left[F_j\left(V_j(b',h')\right) V_j(b',h') + \int_{V_j(b',h')} U f_j(dU) \right], \qquad (8) \\ h' &= \frac{F_i(V_i(b,h))}{f_i(V_i(b,h))\left[\delta + (1-\delta)q_i(b',h')\right] h + \beta F_i(V_i(b,h))} (1-\delta)h + u'(c_i)\left[b' - (1-\delta)b\right], \end{aligned}$$

and $\hat{b}$ denotes the Ramsey Equilibrium borrowing policy.

The history variable h summarizes the effect of borrowing in period t in the utility is previous periods (corresponding to the history y^t). Note first that $h \geq 0$. If $h = 0$, the Ramsey optimality condition in equation (7) is equal to the Markov optimality condition in equation (5). Note also that $h = 0$ when borrowing does not affect the utility of previous periods (i.e., when the first term in the the left-hand side of equation 6 is zero). For example, $h = 0$ in period 0 or after histories in which the sovereign did not borrow. In contrast, if $h > 0$, the marginal cost of issuing debt would be higher for the Ramsey government than for the Markov government (everything else equal, and assuming that $\frac{\partial q}{\partial b'} < 0$, as it is always the case in default models). In this sense, there is typically overindebtedness by Markov governments. Intuitively, h is higher for histories with higher probability, higher marginal utility, and more debt issuances in previous periods.

Comparing equations (5) and (7) also shows that with one-period debt ($\delta = 1$), the incentives of the Ramsey and Markov governments coincide. If $\delta = 1$, $h' = u'(c)\,[b' - (1-\delta)b]$ always and, therefore, h is no longer a relevant state variable and the optimality conditions for the Ramsey and Markov governments in equations (5) and (7) coincide.

4 Calibration

We present a standard calibration for a default model such that the simulations for the Markov government match data from Mexico. Mexico is a common reference for studies on sovereign risk because its business cycle displays the same properties that are observed in other economies with sovereign risk (Aguiar and Gopinath, 2007; Neumeyer and Perri, 2005). Unless specified otherwise, we use data from 1993 to 2014.

The utility function displays a constant coefficient of relative risk aversion, that is,

$$u\left(c\right) = \frac{c^{1-\gamma} - 1}{1-\gamma}, \text{ with } \gamma \neq 1.$$

The income process is a discretization of $log(y_t) = \rho log(y_{t-1}) + (1-\rho)\mu + \varepsilon_t$.

For the continuation utility of defaulting, we assume that $U \sim N(V_i^D, \sigma_U)$, where

$$V_i^D = u(y_i(1 - d_0 - d_1 y_i)) + \beta \sum_j \pi_{i,j} \left[\psi V_j(0,0) + (1-\psi) V_j^D\right], \tag{9}$$

and V denotes the expected utility of repayment in equations (1) and (8) for the Markov and Ramsey governments, respectively (and the history state variable h is irrelevant for the Markov government). This formulation endogenizes the continuation utility of defaulting, incorporating the standard assumptions on the cost of defaulting in the quantitative default literature: defaults trigger exclusion from debt markets and a loss of income. As it is standard in the literature, the duration of the exclusion from debt markets is stochastic, with ψ denoting the probability of exiting the default state. During defaults, the government looses a proportion $d_0 + d_1 y$ of its income. As in Chatterjee and Eyigungor (2012), having two parameters in the cost of defaulting gives us the flexibility to match the levels of debt and spread in the data. Note that in equation (9), the history variable h resets to 0 after a default. This assumption rules out the possibility of the Ramsey government manipulating the cost of defaulting with its post-default borrowing promises.

Table 1 presents the values given to all parameters in the model. We first pin down a subset of parameters values without using the model simulations. A period in the model refers to a quarter. The values of the risk-free interest rate and the domestic discount factor ($r = 0.01$ and $\beta = 0.98$) are standard in quantitative business cycle and sovereign default studies. The parameter values that govern the endowment process are chosen so as to mimic the behavior of logged and linearly detrended GDP in Mexico from 1980 to 2014. The estimation of the AR(1) process for the cyclical component of GDP yields $\rho = 0.0.94$ and $\sigma_\varepsilon = 0.015$. We set $\delta = 0.28$, which, with the targeted level of sovereign spreads, implies an average debt duration in the simulations of 3 years, roughly the average duration of public debt in Mexico.[5]

Our modeling of the continuation utility of defaulting assumes that on top of the most common cost of defaulting in the quantitative literature, there is an stochastic utility cost (as presented by Aguiar et al., forthcoming). Compared with standard calibrations in the literature, this necessitates the calibration of the value of an additional parameter for the volatility parameter for the utility cost, σ_U. In order to calibrate this parameter value, we choose to add as a target the volatility of the sovereign spread, which is a statistic of interest and is also strongly affected by σ_U.[6]

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[5] We use data from the central bank of Mexico for debt duration and the Macaulay definition of duration that, with the coupon structure in this paper, is given by $D = \frac{1+i_b}{\delta+i_b}$, where i_b denotes the constant per-period yield delivered by the bond.

[6] To compute the sovereign spread that is implicit in a bond price, we first compute the yield i_b, defined as the return an investor would earn if he holds the bond to maturity (forever) and no default is declared. This yield satisfies

$$q_t = \sum_{j=1}^{\infty} \delta(1-\delta)^{j-1} e^{-j i_b}.$$

The sovereign spread, r_t^s, is then computed as the difference between the yield i_b and the risk-free rate r.

r	0.01	Standard
β	0.98	Standard
ρ	0.94	Mexico GDP
σ_ε	1.5%	Mexico GDP
ψ	0.083	E(exclusion duration) = 3 years
δ	0.083	Debt duration = 3 years
σ_U	0.6	Std spread = 1%
d_0	0.19	Avg debt = 43.5%
d_1	0.73	Avg spread = 2.4%
γ	4	$\sigma(c)/\sigma(y) = 1$

Table 1: Parameter values. The standard deviation of a variable x is denoted by $\sigma(x)$.

Following Bianchi et al. (2018), we also choose to make the domestic risk aversion part of the calibration. This is a key parameter determining the government's willingness to tolerate fluctuations of consumption and thus the optimal cyclicality of fiscal policy. As Bianchi et al. (2018), we target a relative volatility of unity. While durable consumption (and total consumption) is more volatile than GDP both on average for emerging markets and for Mexico, this is not the case for non-durable consumption. The value of the risk aversion parameter that results from the calibration ($\gamma = 4$) is well within the range of values used for macro models.

Overall, we use the simulations to calibrate the value of four parameters: the two parameters of the utility cost of defaulting d_0 and d_1, the parameter σ_U determining the volatility of the income cost of defaulting, and the government's risk aversion γ. We choose these four parameters to match four targets in the data: (i) a public debt-to-income ratio of 43.5 percent (IMF Fiscal Affairs Department Historical Public Debt database), (ii) a mean level of spreads of 240 basis points, (iii) a spread volatility of 100 basis points, and (iv) a volatility of consumption relative to output equal to one. The values for the default cost d_0 and d_1 mainly determine the average debt and spread levels (Hatchondo and Martinez, 2017), σ_U mainly determines the spread volatility, and γ is determined mainly by the consumption-volatility target.

5 Computation

We solve the model using the optimality conditions in equations (5) and (7) for the Markov and Ramsey governments, respectively. Clausen and Strub (2017) show that in the default model, the objective function is continuously differentiable at the optimum and a version of the envelope theorem applies. Optimality conditions have been used before for computing Markov policies. As in previous studies (Bianchi et al., 2018; Hatchondo et al., 2010), we confirm that we can obtain the same results for the Markov government using optimality conditions or using value function iteration.

For the Ramsey government, we cannot solve the model with value function iteration, but we perform other exercises that give us confidence in our algorithm. First, we solve the model without income uncertainty ($\sigma_\varepsilon = 0$, and all other parameter values as in the benchmark calibration). In this case, there is no rollover risk to be hedged with long-term debt, and it is optimal to issue one-period bonds (Aguiar et al., forthcoming). We find that the Ramsey government with long-term debt reproduces the one-period bond allocation (which in term is superior to the allocation chosen by a Markov government). That is, without rollover risk, our algorithm for solving the problem of the Ramsey government eliminates the time inconsistency problem generated by long-term debt and thus, makes one-period and long-term debt equivalent.

Second, we focus on the four-period version of the Ramsey problem (2) with the benchmark calibration, thus including income uncertainty and rollover risk. We find that the optimal borrowing policies $\hat{b}_{t+1}(y^t)$ are equivalent to the optimal borrowing policies $\hat{b_i}(b, h)$ we find using the optimality condition in equation (7).

Third, we find that the Ramsey policy $\hat{b_i}(b, h)$ generates higher welfare than the Markov policy, than issuing one-period bonds, and than the optimal debt limit the government could commit to (Hatchondo et al., 2015). In addition, we find that equilibrium functions for the Ramsey government are well behaved. Overall, we are confident the algorithm we propose is useful for solving the Ramsey problem and thus, to study time inconsistency in default models.

	Data	Markov	Ramsey	One-period debt
Mean debt-to-income ratio (in %)	43.5	43.3	40.4	35.0
Mean spread (in %)	2.4	2.4	0.2	0.5
Std spread	1.0	1.0	0.1	0.6
$\sigma(c)/\sigma(y)$	1.0	1.1	1.2	1.3

Table 2: Data and simulations. For the moments in the simulations, we present the average of 1000 samples of 30 periods without default. The debt level in the simulations is calculated as the present value of future payment obligations discounted at the average risk-free rate, i.e., $b(\delta + r)^{-1}$. We report the annualized spread.

6 Quantitative results

We first show that the overindebtedness of the Markov government (compared to the constrained efficient debt levels chosen by the Ramsey government) is small but account for the majority of sovereign risk. In fact, due to the the much lower bond prices it faces, the overindebted Markov government underborrows. We also show that the Ramsey government does not choose a countercyclical fiscal policy, and that it may choose to commit to a debt buyback. We also show how the Ramsey allocation is superior to the allocation obtained with one-period debt.

6.1 Overindebtedness, underborrowing, and sovereign risk

Table 2 reports long-run moments in the data and in the model simulations. Note first that the simulations of the Markov government match the calibration targets.

Table 2 also shows that the overindebtedness by the Markov government is, on average, small. The average debt ratio is 43% for the Markov government and 40% for the Ramsey government.

However, this small overindebtedness accounts for a majority of the sovereign risk faced by the Markov government (calibrated to match sovereign risk in the data). Table 2 shows that compared with the overindebtedness chosen by a Markov government, the constrained efficient borrowing chosen by a Ramsey government would eliminate about 90% of both the level and the volatility of sovereign risk (as measured by the sovereign spread). The average sovereign spread would decline from 2.4% to 0.2% and the standard deviation of the spread would decline from

1.0% to 0.1%.

Furthermore, since the Ramsey government faces higher bond prices (as reflected by the lower spreads it pays), for the same debt levels, the Ramsey government borrows significantly more than the Markov government. This indicates that the Ramsey planner may not sacrifice consumption to almost eliminate sovereign risk. Figure 1 shows that in fact, on average, the Ramsey government consumes more than the Markov government, generating welfare gains.[7]

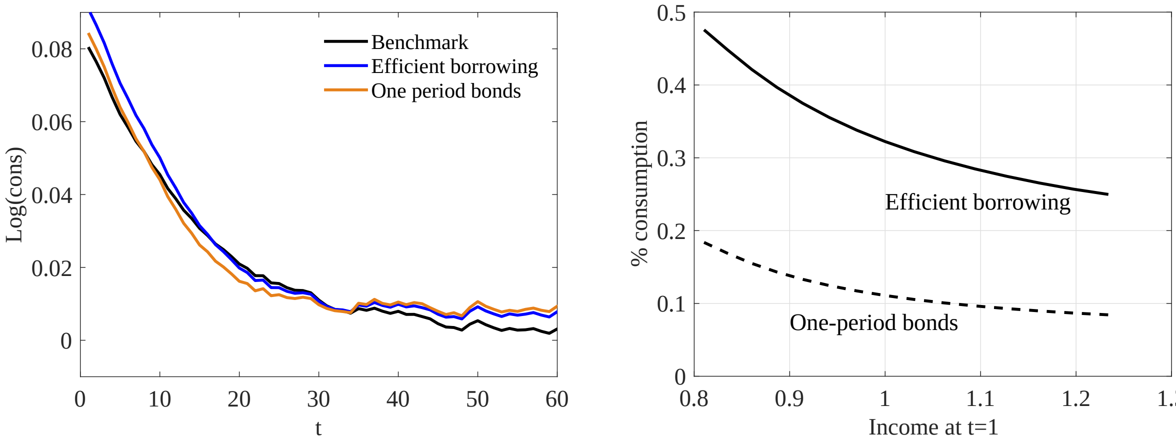

Figure 1: Consumption and welfare. The left panel presents average consumption in the simulations. The right panel presents welfare gains measured as the constant proportional change in consumption that would leave domestic consumers indifferent between staying in the benchmark economy with a Markov government and moving either to an economy with a Ramsey government (labeled as efficient borrowing) or to an economy in which the government only issues one-period debt ($\delta = 1$ and all other parameter values as in the benchmark calibration). The initial levels of debt and the history variable h are equal to zero.

Overall, compared with the constrained efficient borrowing policy that a Ramsey government would choose, the Markov government chooses a small overindebtedness that results in a more significant underborrowing and substantial and volatile sovereign risk. Our results underscore the importance of ongoing efforts by many governments in establishing credible fiscal rules and independent fiscal councils that would allow them to commit to more sensible future debt policies (as the Ramsey government does).

[7]Our model economy without production is likely to severely underestimate the welfare gains from almost eliminating sovereign risk. Bocola (2016) documents sizable effects of sovereign risk on aggregate income, which we do not model. Mendoza and Yue (2012) present a quantitative sovereign default model in which the sovereign spread affects aggregate income.

6.2 The cyclicality of fiscal policy

The left panel of Figure 2 shows that compared with Markov borrowing, constrained efficient borrowing implies issuing more debt when income is high and less debt when income is low. This is, governments would like to commit to a more procyclical debt issuance policy.

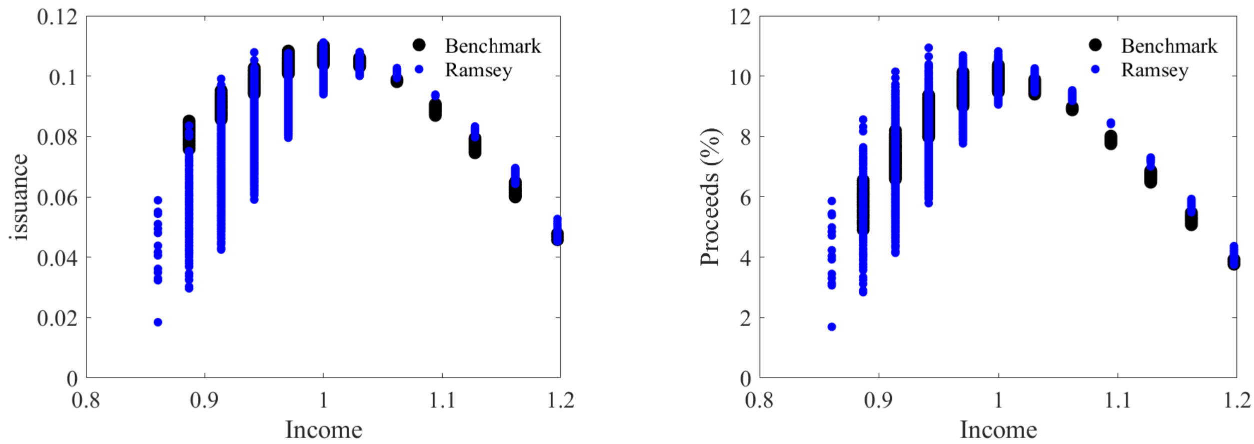

Figure 2: Debt issuances ($b' - (1-\delta)b$) and issuance proceeds ($q[b' - (1-\delta)b]$) in the simulations.

Note however, that the Ramsey government faces a much smaller decline in bond prices when income is low (as reflected in the much lower spread volatility faced by the Ramsey government in Table 2). Consequently, lower debt issuances by the Ramsey government in bad times (left panel of Figure 2) do not imply lower borrowing. In fact, as illustrated in the right panel of Figure 2, in bad times, the proceeds from debt issuances are not smaller on average for the Ramsey government. Consistently, Table 2 shows that the volatility of consumption only increases slightly with the Ramsey government.

Overall, with find that the Ramsey government does not want to commit to a fiscal policy that is less procyclical than the one chosen by the Markov government. This casts doubts on the emphasis of existing fiscal rules in promoting countercyclical fiscal policy, including with the presence of escape clauses for the limits imposed by the rules (IMF, 2017). This emphasis is motivated in part by the procyclicality of fiscal policy observed in economies with sovereign risk (Végh and Vuletin, 2011). Cuadra et al. (2010) show that in a sovereign default model the optimal fiscal policy is procyclical. We show that when the government can commit to future

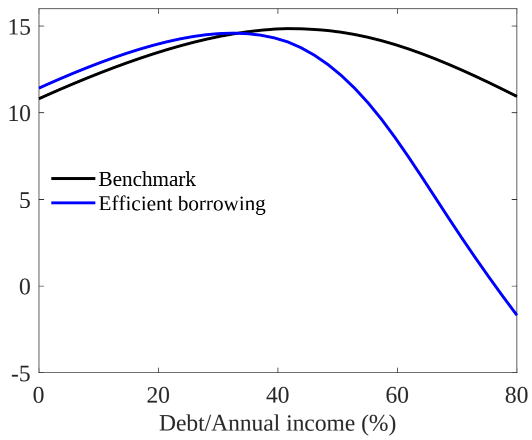

Figure 3: Optimal issuance policy ($\hat{b}(b, h, y) - (1 - \delta)b$). Initial income is the average income and the history variable h (only relevant for the Ramsey government) is equal to the initial debt level (b).

borrowing policies that eliminate most default risk, it may still want to commit to a procyclical policy.

6.3 Deleveraging and debt buybacks

Figure 3 presents the optimal issuance policy for the Ramsey and Markov governments ($\hat{b}(b, h, y) - (1 - \delta)b$, where the history variable h is only relevant for the Ramsey government). The figure shows that for low debt levels (lower than the average levels in the simulations), the Ramsey government may choose to issue even more debt than the Markov government. However, compared with the Markov government, the Ramsey government commits to a much faster deleveraging when debt levels are high. In particular, in contrast with the Markov government, the Ramsey government may choose to buy back debt.

6.4 One-period debt

Assuming one-period debt ($\delta = 1$) eliminates the time inconsistency (debt dilution) problem the Ramsey government deals with. Comparing equations (5) and (7) shows that with one-period bonds, the incentives to borrow are identical for the Markov and the Ramsey governments.

However, one-period debt does not only eliminate the time inconsistency problem, but also forces the government to use an inferior debt instrument. Therefore, comparing the economy with one-period debt and benchmark Markov economy does not allows us to measure the effect of the time inconsistency problem, as comparing the Markov and Ramsey economies does. For example, Table 2) shows that compared with the Ramsey economy, the one-period-debt economy features lower debt levels, and at the same time higher and more volatile sovereign spreads, and higher consumption volatility. Figure 1 shows that the one-period-debt economy generates lower consumption (especially in earlier periods that are more valuable for the government) and lower welfare gains than Ramsey policies with long-term debt. This is the case because one-period debt creates more rollover needs and thus exposes the government to more rollover risk.[8]

Similar issues arise with other debt instruments that mitigate the time inconsistency problem. For example, Chatterjee and Eyigungor (2015) and Hatchondo et al. (2016) show that debt covenants that introduce seniority or penalize future borrowing mitigate this problem. However, comparing economies with different debt instruments cannot perfectly measure the effects of time inconsistency, which we do by comparing the Markov and Ramsey economies.

7 Conclusions

We propose a tractable algorithm for solving quantitative models of sovereign default with with commitment to a borrowing policy (but not to a default policy). Our algorithm utilizes the government's optimality condition and, compared to the Markov equilibrium, only requires one additional state variable that summarizes the effect of current borrowing on past consumption.

Comparing the simulations of the model with and without commitment, allows us to present a quantitative assessment of the government's time inconsistency problem. We find that the overindebtedness without commitment is small but accounts for 90 percent of the default risk. However, with commitment, lower default risk implies higher bond prices and thus, the gov-

[8]The analysis of endogenous maturities is beyond the scope of this paper. Figure 1 shows that the duration of sovereign bonds in our benchmark calibration is not ex-ante optimal (in particular, one-period bonds are superior to the benchmark duration). Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016) show that with plausible calibrations of rollover risk, the default model can generate plausible endogenous debt maturities. Hatchondo et al. (2016) also show that mitigating the time inconsistency problem with debt covenants would lead the government to choose significantly longer maturities. Hatchondo and Martinez (2013) discuss the time inconsistency problem in the government's maturity choice. Our approach for computing Ramsey policies could be extended to study time inconsistency problems in the government's maturity choice and other debt management policies.

ernment without commitment is overindebted but underborrows. These results underscore the importance of governments' efforts to limit their future policies with fiscal rules and independent fiscal councils. We also find that commitment does not affect significantly the procyclicality of fiscal policy, which questions the emphasis on countercyclical fiscal policy in existing fiscal rules. In addition, we find that the constraint efficient borrowing plan the government wants to commit to includes fast deleveraging from situations of high debt, and may even include debt buybacks.

Our algorithm could be extended to study other aspects of debt management in which time inconsistency plays a role. Overindebtedness arises because Markov governments do not internalize the losses they impose to debt holders by borrowing more. Similar inefficiencies arise because Markov governments do not internalize the losses of debt holders when they issue debt to accumulate assets (Bianchi et al., 2018), when they issue debt with longer maturities (Hatchondo and Martinez, 2013), or when they issue more foreign currency debt. Extending our analysis to study and quantify these issues and thus inform the limits fiscal rules should impose on other debt management aspects is an interesting avenue for future research.

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