BUSINESS LIQUIDITY, CONSUMER LIQUIDITY, AND MONETARY POLICY*

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February 15, 2019

Abstract

Existing studies of liquidity either focus on firms or consumers. However, both hold significant cash, and firms' share increased since the '90s and fell after the 07-08 financial crisis. We propose a theory of how endogenous investment liquidity and consumption liquidity compete and interact. The consumption-investment liquidity allocation (CILA) channel amplifies the effect of monetary policy on unemployment, as it accounts for 40 percent of the effect in the calibrated model. We also show that a lower nominal interest rate directs relatively more cash to firms, whereas financial frictions induce the opposite and high unemployment, consistent with the observed patterns.

Key words: firm cash, consumer cash, liquidity allocation, unemployment

JEL classification numbers: D83, E22, E44, E52, J62, J64

*We are grateful to Liang Wang, Yu Zhu, and Randy Wright for their comments and suggestions. We would like to thank participants at the 2018 ECNU Macroeconomics Workshop for their helpful comments and discussions. Min Zhang thanks the National Natural Science Foundation of China (Grant No. 71673172) and Fundamental Research Funds for the Central Universities (Grant No. 2017ECNU-HLYT004) for the financial support. All errors are our own. Mail corresponding to: Min Zhang, East China Normal University, School of Economics, 3663 Zhongshan Road N., Shanghai, China, 200620. Email: mzhang@jjx.ecnu.edu.cn.

1 Introduction

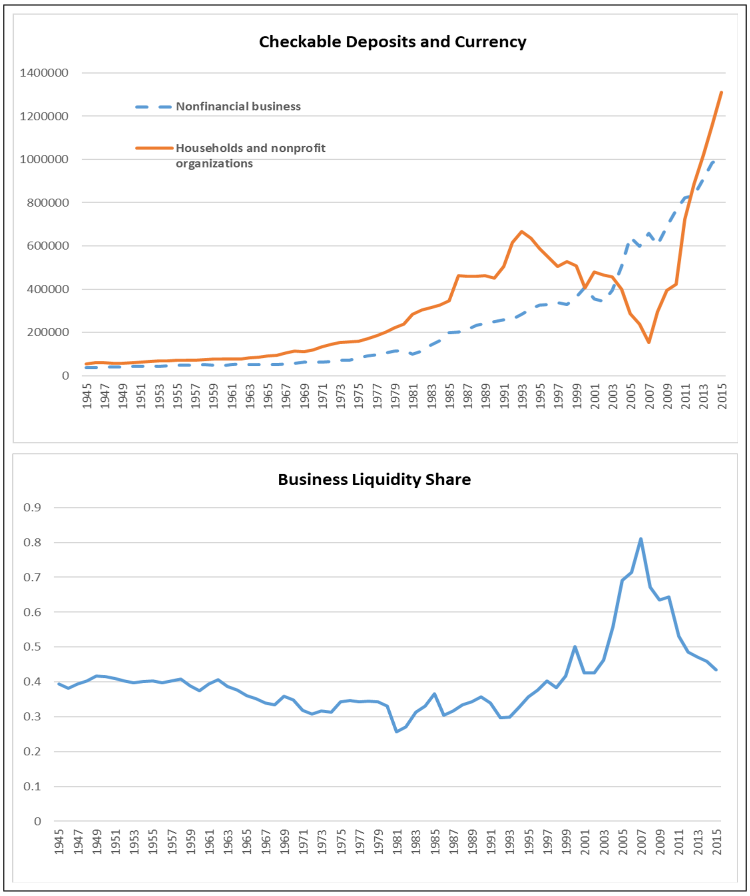
Many fundamental questions in economics concern the allocation of scarce resources. For example, how to divide income into consumption and investment. However, there is one resource that economists have long recognized its use by consumers (Baumol, 1952) and by business (Brunner and Meltzer, 1967), yet no study seriously consider how the two uses compete and interact. This resource is money.

Empirical and theoretical studies of money alike usually focus on either the money demand by consumers or that by business. The allocation is perhaps not an issue if certain dichotomy exists. However, as Figure 1 shows, both uses are significant, and their relative share changes over time. Notably, the firms' share of money holdings increased from 29.7 percent in 1992 to 81.1 percent in 2007 and declined to 43.5 percent in 2015. This paper builds on recent advances in monetary economics, as surveyed in Lagos, Rocheteau, and Wright (2017) and Rocheteau and Nosal (2017), to develop a general equilibrium model of liquidity allocation between consumers and firms. We aim to understand the policy implications of such a liquidity distribution channel and the changing patterns of liquidity allocation.

Our approach combines a theory of money demand by consumers, whereby households purchase goods from firms and a theory of money demand by firms for investment. Both consumption goods and capital goods are traded in frictional markets with limited commitment, so both transactions require money. Since unemployment is one of the central issues in discussing monetary policy, we also incorporate a labor market in the spirit of Mortensen and Pissarides (1994) and assume that job creation is associated with the investment. As far as we know, this paper is the first to incorporate three markets with searching and match frictions (capital, labor, and consumption goods). The households, as the owners of firms, need to make nontrivial choices about liquidity for consumption purposes and business purposes. The theory predicts the determinants of liquidity allocation include monetary policy (money growth rate), financial development (credit card usage), and financial frictions (efficiency of the financial market).

Modeling the two liquidity need in one model, instead of only focusing on one is useful for three reasons. First, it makes possible the study of the changing patterns of consumer-firm liquidity distribution. Second, formalizing the allocation between the two, matters as any force that shifts liquidity from one use to another will have real effects. In this paper, specifically, we study the consumption-investment liquidity allocation (CILA) channel. Third, the CILA channel allows for interesting interactions that are not present in many other distribution problems, where different uses are pure substitutes. Here, more consumer liquidity (in real terms) increases firms' profits, investment, and, thus, their need for liquidity. We call it the demand effect. Conversely, more business liquidity leads to more vacancies and trading opportunities for consumers, also raising their money demand. We call it the supply effect. These effects can only be present if we model both types of liquidity need as in our model.

Figure 1: Consumer Liquidity and Business Liquidity



NOTE: The consumer liquidity and business liquidity are both measured by the checkable deposits and currency held by households and nonprofit organizations, and nonfinancial business sector. The business liquidity share is measure as the ratio of business liquidity to the sum of consumer liquidity and business liquidity. Data source: Table L204, Checkable Deposits and Currency, Financial Accounts of the United States- Z.1, the Federal Reserve, and the series codes for consumer and business liquidities are FL153020005 and FL143020005, respectively. The link to the data is https://www.federalreserve.gov/releases/z1/.

The CILA channel propagates the effects of various economic forces. In particular, it plays a quantitatively important role in amplifying the impact of monetary policy. Calibrating the model with key features of the U.S. economy, we simulate the effects of monetary policy on unemployment. When the nominal interest rate rises from its 1994-2007 average by half to 10 percent, the steady-state unemployment increases from 5.56 percent to 7.22 percent. Monetary policy operates through two channels: a standard channel of lowering total real money balances, as in Berentsen et al. (2011), and the novel CILA channel, that is, the business liquidity share decreases from 42 percent to 29 percent. We quantify the contribution of the CILA channel by simulating the same policy change with business liquidity share fixed at 42 percent. This shows that the CILA channel alone contributes to 39.7 percent of policy effects on vacancies. The effect is also robust to a variety of model modifications. When we remove search friction in the capital market or allow credit in the goods market, the contribution of the CILA channel remains sizeable at 33.6 percent.

The model also provides insights into the changing consumer-firm liquidity distribution. It implies that the lower inflation rates after the '80s are an important reason why the share of business liquidity increased. Note that the demand effect in the CILA channel operates through the intensive margin (i.e., the revenue of each firm), whereas the supply effect works through the extensive margin (i.e., the matching probability of consumers). If the two effects offset each other, then liquidity distribution would remain unchanged. However, monetary policy has first-order effects on the real money balances, so that the demand effect dominates the supply effect, and a lower nominal interest rate raises the share of business liquidity.

What can the model say about the decrease of business liquidity share and the high unemployment after the 2008 financial crisis? To bypass the issue of measuring financial shocks, we incorporate financial frictions on the firm side and ask the following question: If we feed in financial shocks that can generate the observed change in business liquidity share (i.e., it was 71.4 percent in 2006 and 48.5 percent in 2012), how much unemployment can we explain? The main results are twofold. First, in the model, a larger financial wedge indeed lowers the share of business liquidity. Second, unemployment rises significantly from the long run steady-state value of 5.79 percent to 8.05 percent, close to its empirical counterpart of 8.08 percent in 2012.[1] A further decomposition exercise shows that the contribution of the CILA channel is 18.4 percent, and its quantitative impor-

[1] An alternative interpretation: if we feed in financial shocks that are consistent with the observed change in unemployment, then it also generate a change in liquidity distribution that is consistent with the data.

tance increases with the degree of financial friction. The simulation is also consistent with the negative correlation between business liquidity share and the relative price of capital goods observed from 2007 to 2014. We take these results as an indication of the quantitative importance of the liquidity distribution channel and the usefulness of the model in studying the aggregate liquidity distribution.

In terms of literature, the dynamic general equilibrium approach to liquidity builds on Lagos and Wright (2005). Our treatment of consumer liquidity and unemployment is similar to Berentsen et al. (2011). However, what sets this paper apart is its inclusion of both consumer liquidity and business liquidity, whereas the existing studies in this literature, which concerns frictions that make money essential, either focus on consumer liquidity (e.g., Aruoba et al., 2011) or business liquidity (e.g., Rocheteau et al., 2018). In the very different New Keynesian literature, which emphasizes nominal rigidities, some attempts feature money in the utility and firms' need for cash as working capital. In their reduced-form approach, the allocation is purely distributional: Consumer liquidity does not affect firms' incentives of production, and firms' decision does not influence households demand for money. Our approach demonstrates the rich interaction between the two uses of liquidity and the quantitative importance of the CILA channel and can be used to study long-run trends in the liquidity allocation.

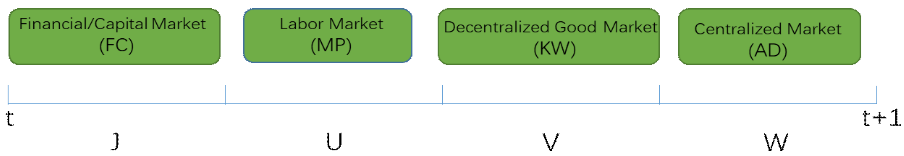
This paper also contributes to the growing literature on firm cash holdings (Bates et al., 2009). Many studies ask why some firms hold more cash than others and find that research and development, merger and acquisition, and multinational taxation, among others, are related to the increased cash holdings of firms (e.g., Begenau and Palazzo, 2017; Rempel, 2019; and Graham and Leary, 2018). Our model complements this literature by bringing consumer liquidity into the analysis. Because consumer liquidity competes and interacts with business liquidity, it is insufficient to examine firms' cash holding in isolation, even if one is only concerned with firm liquidity. Macroeconomic factors (e.g., monetary policy and financial frictions) that influence the liquidity allocation between consumers and firms will also affect the absolute cash holdings of firms. For example, the lower inflation rate also contributed to the increased business liquidity.

The paper proceeds as follows: Section 2 lays out the model featuring endogenous liquidity allocation between consumption and investment. Section 3 characterizes the equilibrium. Section 4 calibrates the model to data for the United States and quantifies the steady-state effect of a rise in inflation on unemployment. A decomposition exercise is conducted to show the role played by the CILA channel, the focus of this contribution. Section 5 extends the baseline model by considering financial development and frictions. We illustrate that the calibrated model is useful to provide insight into the changing patterns of liquidity distribution, and some implication regarding high unemployment in the post-financial-crisis era is demonstrated in the quantitative model. Section 6 concludes.

2 Model

Time is discrete, and each period is divided into four stages: first, firms obtain financing from households to buy capital goods; second, job creation (i.e., new firms with capital goods hire workers) and destruction take place; third, consumers purchase goods from firms; fourth, wage and dividend are paid, and household rebalance their asset portfolio. The second to the fourth stages feature markets in the spirit of Mortensen and Pissarides (1994), Kiyotaki and Wright (1993), and Arrow-Debreu. We call the four stages FC (financial and capital), MP, KW, and AD markets.

Figure 2: Evolution of Firms



There are three types of agents: households, firms, and capital goods suppliers. The households are also workers in the MP market and consumers in the KW market. Their measure is 1, discount factor is β, and periodic utility function is $\upsilon(q) + x$, where q and x are consumption of specialized (KW) goods and numeraire (AD) goods, $\upsilon(0) = 0$, $\upsilon'(0) > 0$, $\upsilon'(0) = \infty$, and $\upsilon''(0) < 0$.

In the FC market, there is a large measure of potential and homogeneous firms. To form a production unit, firms need to acquire κ units of capital goods and one worker. Each household chooses the number of firms to fund (equity), n, with the liquidity necessary for a production unit. Funded firms then search for capital goods suppliers, whose measure is B, determined by free entry in the previous AD market and explained below. The constant-return-to-scale (CRS) matching function in the FC market is $M^{FC}(n, B)$, and the arrival rates of firms and capital goods suppliers are $\gamma_f = M^{FC}(n, B)/n$, and $\gamma_c = M^{FC}(n, B)/B$.Due to the lack of commitment in the bilateral trade, money is needed for the transaction. Firms that fail to acquire the capital goods return the cash to households at the end of the period.

In the MP market, firms that have obtained the necessary capital goods become vacancies. Measure v of vacancies and measure u of workers form matches according to a CRS matching function $M^{MP}(v, u)$. The matching probabilities of a vacancy and a worker are $\lambda_f = M^{MP}(v, u)/v$, and $\lambda_h = M^{MP}(v, u)/u$.Previously formed matches are destroyed by nature at the rate of δ. These firms loses $(1 - \eta_f)$ fraction of their capital stock, and liquidate the rest in the AD market. The firms that fail to find workers will also liquidate their capital in the AD market. Newly unemployed workers can only find jobs in the next MP market.

In the KW market, measure s of firms with workers are active and each produce y units of numeraire goods. All households, regardless of their employment status in the MP market, enter the KW market to buy specialized goods. Firms produce q units of the specialized goods for the households they meet at the cost of $c(q)$ units of numeraire goods, with $c' > 0$ and $c'' \geq 0$. Firms require cash, as there is no way to enforce credit, and we denote the real value of the payment as d. The CRS matching function in the KW market is $M^{KW}(1, s)$, and the arrival rates of households and firms are $\alpha_h = M^{KW}(1, s)$, and $\alpha_f = M^{KW}(1, s)/s$.

In the AD market, A units of numeraire goods can be transformed into one unit of capital goods. The process can also be reversed. Firms pay wages to workers and dividend to shareholders, who also receive the value of liquidated capital goods and unspent firm cash. We assume that existing firms purchase capital goods to replenish depreciation, and the dividend is net of capital depreciation. Households consume numeraire goods, trade stocks of firms, choose the money holdings for the next period, collect debts and lend to capital goods suppliers, who borrow capital goods in the AD market, sell them in the next FC market, and repay debts in the next AD market. Since each supplier needs to prepare κ units of capital goods for sale, the households lend κ/A to a supplier at the real interest rate $r = 1/\beta - 1$.

2.1 Individual Problems

Next, we formalize the problems faced by individuals. Denote value functions of households and firms for the FC, MP, KW, and AD markets by J_{et}^{i}, U_{et}^{i}, V_{et}^{i}, and W_{et}^{i}, where e indexes employment status: $e=1$ if an agent is matched and $e=0$ otherwise; and i indexes the type of the agents: h stands for households and f stands for firms. Since the problem of capital goods suppliers are relative simple, we do not use value functions for them.

2.1.1 Households

A household enters the FC market with employment status $e \in \{0,1\}$, (real) bond b_t, money m_t, and stock of operating firms s_t. The value function in the FC market is as follows:

$$
\begin{aligned}
J_{et}^{h}\left(b_t, m_t, s_t\right) &= \max_{n_t} U_{et}^{h}\left(b_t, \hat{m}_t, s_t, n_t\right) && (1)\\
s.t.\ \hat{m}_t &= m_t - n_t \kappa p_{kt}, && (2)
\end{aligned}
$$

where n_t is the number of firms to fund, p_{kt} is the nominal price of capital goods, and $\hat{m}_t$ is the money prepared for consumption purpose. As a result, $n_t \kappa p_{kt}$ is the money balance designated for business investment purpose.

In the MP market, employment status evolves, households are passive, and their value functions are:

$$
\begin{aligned}
U_{1t}^{h}\left(b_t, \hat{m}_t, s_t, n_t\right) &= \delta V_{0t}^{h}\left(b_t, \hat{m}_t, s_{t+1}, n_t, z_t\right) + \left(1-\delta\right) V_{1t}^{h}\left(b_t, \hat{m}_t, s_{t+1}, n_t, z_t\right), && (3)\\
U_{0t}^{h}\left(b_t, \hat{m}_t, s_t, n_t\right) &= \lambda_{ht} V_{1t}^{h}\left(b_t, \hat{m}_t, s_{t+1}, n_t, z_t\right) + \left(1-\lambda_{ht}\right) V_{0t}^{h}\left(b_t, \hat{m}_t, s_{t+1}, n_t, z_t\right), && (4)
\end{aligned}
$$

where the measure of shares, s_{t+1}, and the number of destroyed matches, z_t, evolve according to the following equations:

$$
\begin{aligned}
v_t &= n_t \gamma_{ft}, && (5)\\
s_{t+1} &= v_t \lambda_{ft} + s_t\left(1-\delta\right), \text{ and} && (6)\\
z_t &= s_t \delta. && (7)
\end{aligned}
$$

Note that equations (5) says that the measure of vacancies equals the measure of funded firms that successfully meet capital suppliers.

In the KW market, the term of trade (d_t, q_t) is determined by the Nash bargaining solution, and we denote $d_t = g(q_t, \hat{m}_t\phi_t)$, where ϕ_t is the value of money in terms of the numeraire goods. Hence, the value function of a household in the KW market is

$$V_{et}^h(b_t, \hat{m}_t, s_{t+1}, n_t, z_t) = \begin{array}{l} \alpha_{ht}\left[v(q_t) + W_{et}^h(b_t, \hat{m}_t - d_t/\phi_t, s_{t+1}, n_t, z_t)\right] \\ +(1-\alpha_{ht})W_{et}^h(b_t, \hat{m}_t, s_{t+1}, n_t, z_t) \end{array}. \tag{8}$$

In the AD market, households' value function is as follows:

$$W_{et}^h(b_t, m_t, s_{t+1}, n_t, z_t) = \max_{x_t, m_{t+1}, b_{t+1}} \left\{x_t + \beta J_{et+1}^h(b_{t+1}, m_{t+1}, s_{t+1})\right\}, \tag{9}$$

$$\begin{aligned} s.t.\ x_t + b_{t+1} + \phi_t m_{t+1} = {} & ew_t + (1-e)\varsigma + \phi_t T_t \\ & + b_t(1+r) + m_t\phi_t + s_{t+1}(R_t - w_t - \delta_k\kappa/A) \\ & + n_t(1-\gamma_t)\kappa p_{kt}\phi_t + n_t\gamma_t(1-\lambda_{ft})\kappa/A + z_t\eta_f\kappa/A, \end{aligned} \tag{10}$$

where w_t, ς, T_t and R_t are real wage, unemployment payoff, (nominal) lump-sum transfers/taxes imposed by the central bank, and the expected periodic revenue of an active firm in real terms. The last four terms in (10) are from dividend (net of capital depreciation), unspent firm cash, unused capital goods, and liquidation of destroyed firms. Because utility is linear in x, households leave the AD market with degenerate distribution of money, similar to Lagos and Wright (2005).[2] The distribution of b does not matter for equilibrium.

2.1.2 Firms

We begin in the MP market. The value functions of an existing active firm and a newly formed vacancy are

$$U_{1t}^f = (1-\delta)V_{1t}^f + \delta\eta_f\kappa/A. \tag{11}$$

$$U_{0t}^f = \lambda_{ft}V_{1t}^f + (1-\lambda_{ft})(1-\delta_k)\kappa/A, \tag{12}$$

[2] Firms shares distribution is also degenerate. We can also allow households to trade the shares s_{t+1}, but that would not be necessary since they are homogeneous and we apply the law of large numbers in the evolution of the share as in equation (6).

where the last term in equation (11) represents the liquidation value of a firm hit by the seperation shock, and the last term in equation (12) says capital of unmatched vacancies depreciates at the rate δ_k and is liquidated in the AD market. Both liquidation values are given back to the shareholders, as shown in (10). Wage bargaining is introduced below.

In the KW market, the value functions of an active firm is as follows:

$$V_{1t}^{f} = \alpha_{ft} W_{1t}^{f} \left[\frac{d_t}{\phi_t}, y_t - c\left(q_t\right) \right] + \left(1 - \alpha_{ft}\right) W_{1t}^{f}\left(0, y_t\right). \tag{13}$$

where $\frac{d_t}{\phi_t}$ is the nominal payment received, and $y_t - c(q_t)$ is how much numeraire goods a firm carries after producing q_t units of specialized goods.

In the AD market, an active firm with money balance $\tilde{m}_t$ and unsold numeraire goods $\tilde{y}_t$ has the following value function

$$W_{1t}^{f}\left(\tilde{m}_t, \tilde{y}_t\right) = \tilde{m}_t \phi_t + \tilde{y}_t - w_t - \delta_k \kappa / A + \beta J_{1t+1}^{f}, \tag{14}$$

where the first four terms are paid as dividend to the shareholders, which is net of capital depreciation: $\delta_k \kappa / A$. Using equations (13) and (14) we can write the expected revenue of an active firm as

$$R_t = \alpha_{ft}\left[y_t - c\left(q_t\right) + d_t\right] + \left(1 - \alpha_{ft}\right) y_t. \tag{15}$$

Last, active firms enter the next MP market (in period $t+1$) with value function

$$J_{1t+1}^{f} = U_{1t+1}^{f}. \tag{16}$$

2.1.3 Capital Producers

First note that numeraire goods and capital goods can be transformed back and forth in the AD market. Each capital goods supplier borrows κ units of capital goods from households in the AD market, tries to sell them in the following FC market, and repays the lenders with real interest rate r. Free entry implies the following zero profit condition for every period t:

$$\gamma_{ct} \kappa p_{kt} \phi_t + \left(1 - \gamma_{ct}\right)\left(1 - \delta_k\right) \kappa / A = \left(1 + r\right) \kappa / A, \tag{17}$$

where p_{kt} is the nomial unit price of capital goods, explained in more detail below. The second term says the suppliers bear the cost of depreciation of the unsold capital goods. The equation not only makes the expected profit of capital producers zero, but also make the household indifferent about how much to lend to the capital producers.

3 Equilibrium

This section solves for the general equilibrium of the model constructed above *at the steady state*. The strategy we adopt is that we solve for each agent's problem in respective market, then derive the system of equations that are used to solve for the general equilibrium. For composition purpose, we start with the MP market, then KW, then AD, and ends up with FC.

3.1 Equilibrium Conditions for Each Market

Here we describe the equilibrium conditions of each market, including how terms of trade are determined. We begin with the MP and KW markets, then the AD market, and finally the FC market.

3.1.1 The MP Market

Real wage w_t is determined by the following generalized Nash bargaining problem with χ as the bargaining power of the firm:

$$\max_{w} \left[V_1^f - V_0^f\right]^{\chi} \left[V_1^h - V_0^h\right]^{1-\chi}. \tag{18}$$

Combining the Bellman equations (1)-(9) and (11)-(16) and solving the above bargaining problem, we have the following wage equation:

$$w = \frac{\chi\left[1-\beta\left(1-\delta\right)\right]\varsigma + \left(1-\chi\right)\left[1-\beta\left(1-\delta-\lambda_h\right)\right]\left(R+Q\kappa/A\right)}{1-\left(1-\delta\right)\beta+\left(1-\chi\right)\beta\lambda_h}, \tag{19}$$

where R is defined in Equation (15) and $Q = \beta\delta\eta_f + \beta\left(1-\delta\right)\left(1-\delta_k\right) - 1$ represents an adjusted capital depreciation rate. Note that this wage equation reduces to the one in BMW (2011) if $\kappa = 0$. Whether wages are higher here than without capital depends on η_f. If η_f is zero, then $Q < 0$, so that wage would be lower. If η_f is high enough, then $Q > 0$

and wages can be higher. The intuition is as follows: the need to replenish depreciated capital reduces the total surplus of a worker-firm match, whereas the liquidation value raises the total surplus.

3.1.2 The KW Market

In the KW market, households and firms trade bilaterally when they meet. The terms of trade for the specialized goods are determined by the solution to the following Nash bargaining problem:

$$\max_{q_t,d_t} \left[\upsilon\left(q_t\right) - d_t\right]^{1-\theta} \left[d_t - c\left(q_t\right)\right]^{\theta}, \tag{20}$$

$$s.t.\ d_t \leq \hat{m}_t \phi_t \text{ and } q_t \leq y_t, \tag{21}$$

where $\theta \in [0, 1]$ is the bargaining power of firms, and we have used the fact that households value functions are linear in d_t. The two constraints say that the two parties cannot leave with negative money balances or goods. Let $g\left(q\right)$ be the payment as a function of q, and q^* be such that $\upsilon'\left(q^*\right) = c'\left(q^*\right)$. The solution is standard: if $\hat{m}_t \phi_t > g\left(q^*\right)$, then $q = q^*$ and $d = g\left(q^*\right)$; if $\hat{m}_t \phi_t \leq g\left(q^*\right)$, then

$$d_t = \hat{m}_t \phi_t = g\left(q_t\right) \equiv \frac{\left(1-\theta\right)\upsilon'\left(q_t\right)c\left(q_t\right) + \theta\upsilon\left(q_t\right)c'\left(q_t\right)}{\left(1-\theta\right)\upsilon'\left(q_t\right) + \theta c'\left(q_t\right)}. \tag{22}$$

3.1.3 The AD market

In the AD market, it is standard to simplify the value function by eliminating the linear term, x, using the budget equation. The resulting value function is indeed linear in d_t, as discussed above. Using the envelope conditions, it is also standard to derive the following Euler equation for money demand:

$$\frac{i_t}{M^{KW}\left(1, 1-u_t\right)} = \frac{\upsilon'\left(q_{t+1}\right)}{g'\left(q_{t+1}\right)} - 1, \tag{23}$$

where $M^{KW}\left(1, 1-u\right)/1 = \alpha_{ht}$, $i_t = \left(1+\pi_{t+1}\right)/\beta - 1$ is the nominal interest rate in the AD market, and $\pi_{t+1} = \phi_t/\phi_{t+1}$ is the inflation rate. The above Euler equation says households balances the cost of carrying one unit of money, ϕ_t, with the expected marginal benefit in the KW market.[3] In the steady state, this equation is the LW curve,

[3]The expected marginal benefit is $\alpha_{ht+1}\beta\phi_{t+1}\upsilon'\left(q_{t+1}\right)/g'\left(q_{t+1}\right) + \left(1-\alpha_{ht+1}\right)\beta\phi_{t+1}$.

determining q for a given u as in Lagos and Wright (2005).

3.1.4 The FC market

Capital Price The capital price p_{kt} is determined by the generalized Nash bargaining problem with σ the bargaining power of the capital goods suppliers:

$$\max_{p_{kt}} \left[\kappa p_{kt}\phi_t - (1+r)\,\kappa/A\right]^{\sigma} \left[U_{0t}^f - \kappa p_{kt}\phi_t\right]^{1-\sigma}, \tag{24}$$

where we have used the fact that if the trade fails, then the undepreciated capital goods is not enough to fully pay off the debt, but the capital producers' payoffs are zero due to the limited liability.

Applying the usual solution gives:

$$p_{kt}\phi_t = \sigma U_{0t}^f/\kappa + (1-\sigma)(1+r)/A,$$

where

$$U_{0t}^f = \lambda_{ft}\frac{R_t - w_t - \delta_k\kappa/A + \beta\delta\eta_f\kappa/A}{1-\beta(1-\delta)} + (1-\lambda_{ft})(1-\delta_k)\,\kappa/A.$$

Simplifying gives rise to the steady state real capital price $p_{kt}\phi_t$ as

$$\begin{aligned} p_{kt}\phi_t &= \sigma\left[\lambda_{ft}\frac{(R_t - w_t)/\kappa - \delta_k/A + \beta\delta\eta_f/A}{1-\beta(1-\delta)} + (1-\lambda_{ft})(1-\delta_k)/A\right] \\ &\quad + (1-\sigma)(1+r)/A. \end{aligned} \tag{25}$$

Firm Financing By Equations (1) and (2), the first order condition with respect to n_t is

$$\partial U_{et}^h(b_t, \hat{m}_t, s_t, n_t)/\partial n_t = \kappa p_{kt}\partial U_{et}^h(b_t, \hat{m}_t, s_t, n_t)/\partial\hat{m}_t. \tag{26}$$

This first order condition states the marginal benefit (MB) of investing in one more unit of n_t by the household equals its marginal cost (MC).

On the side of MC, by Equations (**??**) and (**??**), one has $\partial U_{et}^h(b_t, \hat{m}_t, s_t, n_t)/\partial\hat{m}_t = \phi_t(1+i)$. So, the RHS of Equation (26) reduces to $\kappa p_{kt}\phi_t(1+i)$. Rewritting this term as $\kappa p_{kt}\phi_t(1+r)\frac{p_{kt+1}}{p_{kt}}$ shows that it exactly measures the number of forgone AD goods caused by investment in the business projects. Indeed, real investment $\kappa p_{kt}\phi_t$ made in period t yields $\kappa p_{kt}\phi_t (1+r)$ gains in period $t+1$, which translates into $\kappa p_{kt}\phi_t (1+r) \frac{p_{kt+1}}{p_{kt}}$ units of forgone AD good in period t after considering the price change across periods.

On the MB side, the LHS of Equation (26) is

$$\partial U_{et}^{h}(b_t, \hat{m}_t, s_t, n_t)/\partial n_t = \gamma_t \left[\lambda_{ft} V_{1t}^{f} + (1-\lambda_{ft})(1-\delta_k)\kappa/A\right] + (1-\gamma_t)\kappa p_{kt}\phi_t.$$

Intuitively, an extra new firm in FC brings the households benefits in the following three ways: first, the households might hold one more operating firm when the new firm succeeds in both capital purchase in FC and finding workers in MP. Indeed, such a gain is measured by V_{1t}^{f} given that V_{1t}^{f} is exactly the value of owning a firm. If the new firm fails in finding workers in MP, the households benefit from obtaining the $(1-\delta_k)\kappa/A$ units of AD goods returned by the inactive new firm (with capitals). Lastly, if the new firm fails in buying capitals in FC, the households receive $(1-\gamma_t)\kappa p_{kt}\phi_t$ units of AD goods returned by the new firm (without capitals).

Combining MC and MB into (26) gives rise to the Job-Creation equation (called **JC curve** hereafter):

$$\gamma_t \left[\begin{array}{c} \lambda_{ft}\frac{R_t - w_t - \delta_k\kappa/A + \beta\delta\eta_f\kappa/A}{1-\beta(1-\delta)} \\ +(1-\lambda_{ft})(1-\delta_k)\kappa/A \end{array}\right] + (1-\gamma_t)\kappa p_{kt}\phi_t = \kappa p_{kt}\phi_t(1+i), \tag{27}$$

where $\gamma_t = M^{FC}(n_t, B_t)/n_t$, $\lambda_{ft} = M^{MP}(u_t, n_t\gamma_t)/n_t\gamma_t$ given that $v_t = n_t\gamma_t$.

Note that the capital producer earns zero profit. Plugging Equation (25) into the zero profit condition (**??**) givens

$$\gamma_{ct} = \frac{1+r-(1-\delta_k)}{p_{kt}\phi_t A - (1-\delta_k)}. \tag{28}$$

Since $\gamma_{ct} = M^{FC}(n_t, B_t)/B_t = \gamma_t \frac{n_t}{B_t}$, by using the property that $M^{FC}(n_t, B_t)$ is CRS in both n_t and B_t, Equation (28) pins down the capital market tightness, n_t/B_t.

3.1.5 The trade-off in job creation

(move to proposition regarding effect of i on u) The JC curve (27) demonstrates the important trade-off caused by the liquidity allocation. To see this, on the cost side one unit rise in the investment would lead to lower money balance available for KW good purchase, which is captured by the RHS of Equation (27) as discussed above.

On the benefit side, a rise in the investment in FC is likely to increase vacancies in MP. This makes it more likely for the household to increase the holdings of operating firms s, which would bring extra profits and help afford higher consumptions in future. The *ex post* benefit is measured by the term $\frac{R_t - w_t - \delta_k \kappa / A + \beta \delta \eta_f \kappa / A}{1 - \beta(1-\delta)}$ as shown in LHS of (27), the value of an operating firm V_1^f. However, such a benefit can be realized if and only if the new firm successfully purchases capital goods in FC, matches up with workers in MP.

The job creation decision affects the matching outcomes in FC and MP, which, in return, affects the household's incentive to fund new firm. To see this, investing in more new firms (n_t goes larger) generates conjection effect in FC, so, γ_t goes down. Similarly, a rise in investment makes the households more likely to find a job in MP, while more difficult for the firms to locate the households. So, λ_{ft} goes down. The drop in both γ_t and λ_{ft} would mitigate the expected gains V_1^f from holding more operating firms, and thus, hurt the benefits from investing in new firms.

3.1.6 Law of motion of unemployment

Unemployment u_t evolves according to the following equation, where s_t is stock of operating firms (or active firms) in period t:

$$u_{t+1} = u_t \left(1 - \lambda_{ht}\right) + s_t \delta. \tag{29}$$

The steady state solution to u is

$$u \lambda_h = s \delta. \tag{30}$$

3.2 Steady State Equilibrium

Definition 1: *The steady state equilibrium is a set of functions* $(m,\ p_k \phi,\ n,\ u,\ w,\ q,\ \theta^i,\ J_e^i,\ U_e^i,\ V_e^i,\ W_p^e)$, *where* $e \in \{0,1\}$, $i \in \{h,f\}$, *which satisfy the Bellman equations (4)-(16), the definitions of market tightness in FC, MP, and KW, the wage determination equation (19), the law of motion of unemployment (30), the LW curve (**??**), the Job Creation curve (27), the capital price determination (25), and the zero profit in capital market in FC (28). The equilibrium can be characterized by the following system of equations:*

$$u\lambda_h = (1-u)\,\delta. \tag{31}$$

$$\frac{i}{M^{KW}(1,1-u)} = \frac{v'(q)}{g'(q)} - 1. \tag{32}$$

$$\gamma\left[\begin{array}{c}\lambda_f \frac{R-w-\delta_k\kappa/A+\beta\delta\eta_f\kappa/A}{1-\beta(1-\delta)} \\ +(1-\lambda_f)(1-\delta_k)\,\kappa/A\end{array}\right] + (1-\gamma)\,\kappa p_k\phi = \kappa p_k\phi\,(1+i)\,. \tag{33}$$

$$R = \alpha_f\left[g(q) - c(q)\right] + y. \tag{34}$$

$$\begin{aligned} w &= \frac{\chi\left[1-\beta(1-\delta)\right]\varsigma + (1-\chi)\left[1-\beta(1-\delta-\lambda_h)\right]\{R+Q\kappa/A\}}{1-(1-\delta)\beta + (1-\chi)\beta\lambda_h}, \\ \textit{where } Q &= \beta\delta\eta_f + \beta(1-\delta)(1-\delta_k) - 1. \end{aligned} \tag{35}$$

$$\begin{aligned} p_k\phi &= \sigma\left[\lambda_f \frac{(R-w)A/\kappa - \delta_k + \beta\delta\eta_f}{1-\beta(1-\delta)} + (1-\lambda_f)(1-\delta_k)\right]/A \\ &\quad + (1-\sigma)(1+r)/A. \end{aligned} \tag{36}$$

$$\gamma_{ct} = \frac{1+r-(1-\delta_k)}{p_{kt}\phi_t A - (1-\delta_k)}. \tag{37}$$

There are 5 unknow variables and 5 matching probabilites in the above system of equations, namely $\{u, q, p_k\phi, R, w\}$ and $\{\gamma_c, \lambda_f, \lambda_h, \alpha_f, \alpha_h\}$. For the matching probabilities in KW, one has $\alpha_f = M^{KW}(1,s)/s$ and $\alpha_h = M^{KW}(1,s)$. Given that $s = e = 1-u$ at the steady state, it follows that both α_f and α_h are functions of u. For the probabilities in MP, $\lambda_h = M^{MP}(u,v)/u = M^{MP}\left(1,\theta^{MP}\right)$, and $\lambda_f = \lambda_h/\theta^{MP}$. So, these two unknown probabilities reduce to the MP market tightness, $\theta^{MP} = v/u$. Similarly, the matching probability in FC γ_c is a function of the FC market tightness $\theta^{FC} = n/B$. So, solving γ_c is equivalent to solving θ^{FC}. As a result, solving for the steady state equilibrium is equivalent to solve 7 unknown variables $\left\{u, q, p_k\phi, R, w, \theta^{MP}, \theta^{FC}\right\}$. Once we solve for these seven unknown variables, we can solve the number of vacancies v in MP, the number of new firms n in FC, and the number of capital producers B.

4 Numerical Analysis

This section analyzes the quantitative predictions of the model laid out in Section 2. The quantitative analysis pursues two objectives. First, we show that when one takes into account the liquidity allocation adjustment made by households between good consumption and investment in business projects, then the monetary policy would generate stronger impacts on long run labor market outcomes. Second, we conduct the decomposition exercise to demonstrate the quantitative importance of this *novel* allocation channel in shaping the effects of monetary policy.

Our numerical analysis adopts the following specifications. Following Kaplan and Menzio (2016), the matching function in FC takes the form:

$$M^{FC}\left(n_t, B_t\right) = n_t B_t \left(n_t^{\zeta^{FC}} + B_t^{\zeta^{FC}}\right)^{-1/\zeta^{FC}}.$$

The matching function in MP is assumed to be Cobb-Douglas in u_t and v_t, following the labor search literature.

$$M^{MP}\left(u_t, v_t\right) = \mu^{MP}\left(u_t^{1-\zeta^{MP}} v_t^{\zeta^{MP}}\right).$$

And the matching function in KW is chosen to be the same as the one in Berentsen et al. (2011).

$$M^{KW}\left(1, s_t\right) = \frac{s_t}{1+s_t}.$$

The flow utility function and cost function in KW take the forms as:

$$u\left(q_t\right) = \frac{B\left(q_t\right)^{1-a}}{1-a}.$$

$$c\left(q_t\right) = \left(q_t\right)^{\varphi}.$$

4.1 Parameterization

Our calibration targets aim to replicate the main rates and flows in the capital market, labor market, and good market in the United States. We calibrate the model in two stages. We determine the value of nine parameters independently from the rest. We then determine the values of the remaining seven parameters to match a set of targets.

The model period is set to be one quarter. We pick values for the nine parameters in the top panel in Table 1. The discount factor (β) is set to correspond to an average annual rate of 2.8 percent. The average productivity (y) is normalized to be one. The UI benefits (ς) borrow directly from search literature. In the baseline calibration, its value is set to be 0.25 as calculated in Zhang and Faig (2012). They target the effective replacement rate measured as the product of the take-up rate (fraction of eligible unemployed workers who actually collect UI) and the observed replacement rate conditional on receiving benefits. As explained by Blank and Card (1991), the take-up rate was fairly stable around 0.7 over their period of study (1977-1987). Meanwhile, the replacement rate for those who receive benefits averaged 0.357 over the period 1972-2003. For robustness check purpose, we take two alternative values. One is the statutory UI replacement ratio, 0.4, as picked in Shimer (2005). The other is the calibrated value of non-market activities, 0.985, as reported in Hagedorn and Manovskii (2009). The calibration strategy adopted in their paper is that they pin down the value of worker's non-market activity to match the large volatility of unemployment in response to moderate productivity shocks observed in the U.S. data. The elasticity parameter in the matching function in MP $\left(\zeta^{MP}\right)$ is determined to be 0.28, the same as the estimated value in Shimer (2005). Using Hosios' rule, this suggests that the firm's bargaining power in MP (χ) is 0.28. The curvature parameter (φ) in the cost function is normalized to be one in the baseline calibration as in Berentsen et al. (2011). In the robustness check, we use some alternative values to see if the model predictions are sensitive to our choice. The exogenous eparation rate (δ) is set to be 0.033, which is the quarterly average of the monthly separation rate estimated in Shimer (2005). The fraction of capitals returned in a destroyed match is set randomly. We pick two extreme values $\left(\eta_f\right)$ in our baseline calibration, and the comparison shows how the main results react to its value. The technology parameter (A) in the capital production is pickly randomly too. In the baseline case, we set $A = 1$, and its value changes to 0.5 for robustness check purpose.

In the second stage of our calibration, we jointly pick the remaining seven parameters of the model to match the seven targets in the bottom panel in Table 1. Particularly, the scale parameter in the matching function $\left(\mu^{MP}\right)$ in MP is determined to match the long run average unemployment, 0.056. The bargaining power of the capital producers (σ) in FC is chosen to match the labor market tightness (vacancy-unemployment ratio) used in

Hall (2005), which is 0.539. In our model, each operating firm employs one worker and κ units of capitals. So, $\kappa = K/L$. With output (per worker) being normalized to one, one has $\kappa = \kappa/y = \frac{K/L}{Y/L} = \frac{K}{Y}$. Hence, the units of capital required in a job creation process (κ) is determined to matched the observed capital-output ratio 2.34 as documented in Aruoba *et al.* (2011).[4] As long as the output per worker y is normalized to one, one has $\kappa = \frac{K}{Y}$ The firm's bargaining power in KW (θ) is picked to target the mark-up ratio, 0.3, as calculated in Faig and Jerez (2005). The markup ratio in the model is measured by

$$markup = 100\left[\frac{g(q)}{c'(q)\,q} - 1\right].$$

The parameter in the matching function $\left(\zeta^{FC}\right)$ in FC is picked to match the observed liquidity allocation (business liquidity to consumption liquidity ratio). Based on the data on checkable deposits and current held by households and nonfinancial business sector, the business liquidity to consumption liquidity ratio over the period of 1980 to 2006 is 0.714.[5] The scale parameter in the utility function (B) is chosen to match the real demand for money, 0.186, as documented in Aruoba *et al.* (2011). In our model, the real demand for money is measured as

$$\frac{M}{pY} = \frac{g(q) + nkp_k\phi}{(1-u)\,R + rb}.$$

Lastly, the curvature parameter in the utility function (a) is determined to match the elasticity of money demand with respect to inflation, -0.556, the same as Berentsen et al. (2011).

[4] Alternatively, we can employ the standard Cobb-Douglas production function in MP. Assume the output per worker takes the form $y = TFP \cdot \kappa^{\alpha}$. In this case, y is normalized to one, α is set to be 0.3, $\kappa = K/L$ and is calibrated to match the observed K/Y ratio. TFP is determined to be consistent with the normalized output. Then, the assumption of Cobb-Douglas production function generates the same equilibrium outcomes as the one in the benchmark model.

[5] The data for household liquidity also contain the checkable deposits and currency held by non-profit organizations.

Table 1 Calibration targets			
Variables		**Target Descriptions**	**Target Values**
Discount factor	β	discount rate	0.973
Productivity in a formed match	y	normalization	1
UI benefits	ς	Zhang and Faig (2012), Shimer (2005)	0.25/0.4
Elasticity parameter in MP matching func.	ζ^{MP}	Shimer (2005)	0.28
Firm's bargaining power in MP	χ	Hosios' rule	0.28
Curvature parameter in $c(q)$	φ	normalization, BMW (2011)	1
Separation rate	δ	Shimer (2005)	0.033
Fra. of capital returned in destroyed match	η_f	random pick	0/1
Technology parameter in producing capital	A	random pick	1/0.5
Scale parameter in matching func. in MP	μ^{MP}	unemployment in Shimer (2005)	0.056
Capital producer's bargaining power in FC	σ	labor market tightness in Hall (2005)	0.539
Parameter in matching func. in FC	κ	K/Y in Aruoba *et al.* (2011)	2.34
Firm's bargaining power in KW	θ	mark-up ratio in Faig and Jerez (2005)	0.30
No. of capitals required in a job creation	ζ^{FC}	ratio of business liquidity to household liquidity	0.714
Scale parameter in utility func.	B	real demand for money in Aruoba *et al.* (2011)	0.186
Curvature parameter in utility func.	a	elasticity of money demand w.r.t. i in BMW (2011)	−0.556

NOTE: These are the targets that our calibration of the model aims to reproduce. Most of these targets correspond to empirical moments in the United States. Each one of the first nine targets pins down one parameter, specified in parenthesis. The rest collectively pin down the remaining parameters.

4.2 Calibration Results

The calibrated model can successfully replicate all the targets listed in Table 1. The parameter values obtained in the second stage of the calibration are reported in Table 2. The values in the first column correspond to the baseline calibration, as described above. The remaining three columns correspond to the alternative calibrations in which the values for the key parameters ς, A, and η_f are changed. These three parameters are crucial for understanding the effects of inflation on unemployment. When we increase the value of ς, the worker's outside option becomes larger, which makes him or her choosier about job opportunity, and leads to a higher unemployment. When capital production becomes less efficient (lower A), the capital price rises, which makes the investment in capital more expensive. This would discourage job creation activities, which translates into a higher unemployment. Lastly, if capital losses in a separated match become fewer, the firm's profit from operating a match becomes larger, which would promote the investment in job vacancies, and thus, help lower unemployment.

In the second column of Table 2, which is termed Benefits, ς is set at 0.4. In the third column, which is termed Technology, A is set to 0.5. In the fourth column, which is termed Depreciation, η_f is set to be 1. The intuition discussed above suggests that the predicted impacts on unemployment should increase in the cases of Benefits and Technology, and decrease in the case of Depreciation. It should be noted that although these alternative parameter values are arbitrarily set, they are useful for illustrating how the results depend on the parameter values.

Table 2 Calibration results

	Baseline	**Benefits**	**Technology**	**Depreciation**
		$\varsigma = 0.4$	$A = 0.5$	$\eta_f = 1$
μ^{MP}	0.6189	0.5694	0.6153	0.5936
σ	0.3251	0.0551	0.3282	0.0900
ζ^{FC}	4.0313	13.1113	4.0334	8.2175
θ	0.3114	0.4121	0.3184	0.4116
κ	2.3764	2.3346	1.1881	2.3685
B	0.5010	0.6334	0.5111	0.6611
a	0.3888	0.2995	0.3824	0.2846

NOTE: With these parameters the model replicates the moments in Table 1. Column 1 is the baseline calibration results. In Column 2, the UI benefits change the value from 0.25 to 0.4, the statutory replacement ratio. Column 3 reports the results where the production of capital is less efficient than the baseline case, and the efficiency parameter takes the value of 0.5. Column 4 considers a case where no capital losses occur in a match that dissolves due to an exogenous separation shock.

4.3 Effect on Unemployment

This subsection examines the effect on unemployment of monetary policy change. Particularly, we consider a monetary policy change where the average nominal interest rate R over the period of 1994 to 2007 increases by 50 percent to 10 percent. For a given real interest rate, this change suggests that inflation increases from its long run average 2.6 percent to 6 percent. We use the model calibrated in Section 3.1 to predict unemployment in response to such a policy change. As discussed in Section 3.1, the values of UI benefits, capital production efficiency and capital losses in an exogenous match dissolution affect the long run labor market outcomes. To evaluate the impacts, in addition to the baseline calibration, we consider three cases where these key parameters take different values. Moreover, the frictional financial market influences long run unemployment via prices of capital required in a job creation. Intuitively, the FC friction lowers the matching efficiency in FC. In response, capital producers raise capital prices to compensate for increasing uncertainty in selling capitals. This implies that investing in business project is more costly. Therefore, vacancies fall, and the labor market tightness deteriorates from the perspective of workers. So, unemployment increases. To see to what extent the FC friction affects the main results, we remove friction in FC by assuming that the capital producer can always meet new firms that desire capital. That is, the matching probability in FC is assumed to be one.

Table 3 reports the average unemployment rate during a money expansion (higher inflation π) predicted in our baseline calibration and the alternative cases. In the baseline calibration, the increase in inflation raises unemployment from its long run average 5.56 percent to 7.22 percent. This result remains robust when some key parameters take alternative values as shown in the remaining three columns in Table 3. Confirming the intuitions discussed above, the effect on unemployment rates increases a bit in the two cases where workers face better outside options (higher ς), and the capital production efficiency gets lower (lower A), but significantly decreases in the case where capitals experiences no losses upon an exogenous match dissolution (larger η_f). Lastly, shutting down the friction in FC reduces the rise in unemployment in all cases, in line with the intuition elaborated above.

Table 3 Impacts on unemployment

	Baseline	**Benefits**	**Technology**	**Depreciation**
		$\varsigma = 0.4$	$A = 0.5$	$\eta_f = 1$
Higher π, frictional FC	0.0722	0.0719	0.0725	0.0669
Higher π, frictionless FC	0.0675	0.0708	0.0679	0.0648

NOTE: Comparison across the baseline case and three other cases with alternative values of ς, A, and η_f shows a significant and sizable increases in unemployment in response to a rise in inflation. In Column 2, the UI benefits change the value from 0.25 to 0.4, the statutory replacement ratio. Column 3 reports the results where the production of capital is less efficient than the baseline case, and the efficiency parameter takes the value of 0.5. Column 4 considers a case where no capital losses occur in a match that dissolves exogenously. In the last line, frictionless FC suggests that there exists no uncertainty in matching capital producers and new firms that desire capitals.

4.4 Amplification Mechanism (novel)

The reasons why monetary expansion raises unemployment are twofold. First of all, the increase in inflation lowers the real profits received by firms in KW, and makes holding operating firms a less desirable asset for the households. So, the household's incentive to invest in business project is reduced, which gives rise to a lower money holdings. This is in line with the standard mechanism established in Berentsen *et al.* (2011). The simulation results square well with this working mechanism; the comparison between the first two columns in Table 4 shows that introducing a money expansion leads the total money holdings by households to drop from 0.1824 (before change) in the baseline case to 0.1215.

Table 4 Amplification mechanism

	Before change	**After change**	
	Baseline	**Frictional FC**	**Frictionless FC**
Matching prob. in FC	0.5840	0.4781	1
Capital price	1.0625	1.0806	1.0767
Money holdings	0.1824	0.1215	0.1281
Business Liquidity	0.0763 (42%)	0.0358 (29%)	0.0424 (33%)
Consumption Liquidity	0.1061 (58%)	0.0856 (71%)	0.0858 (67%)
Vacancies	0.0294	0.0138	0.0166
Unemployment	0.0556	0.0722	0.0675

NOTE: Comparison between before and after policy change shows changes in the level of money holdings and changes in its distribution over good consumption and business investment. Two cases are considered when the monetary expansion is implemented. One case is called Frictional FC where the capital market matching between the capital producers and new firms is frictional, characterized by the FC matching function. The other case is called Frictionless FC. In this case, the capital producers can always find new firms and the matching probability is therefore one.

The more interesting reason lies in the liquidity allocation adjustments, which are triggered by the money expansion triggers and prove to be sizable in our baseline simulation. To see this, Table 4 shows that with money expansions in place, the liquidity allocation moves toward good consumption, while away from business investment. Particularly, the business liquidity share decreases from 42 percent (before change) to 29 percent. This adjustment implies a lower number of new firms that get fund, which imposes an adverse pressure on the matching probability in FC. Indeed, the capital producers face a significant lower matching rate, falling sharply from 0.5840 in the baseline case to 0.4781. In response to higher uncertainty in selling capitals, the capital producers raise the capital prices by 28.96 percent to 1.0806.[6] The rise in capital price is key to understand the amplification channel stressed in our model. When capital becomes more expensive, the

[6]Note that without FC friction, the capital price should be one in the case where $A = 1$. Hence, the percentage rise of capital price is calculated by $(0.0806 - 0.0625)/0.0625 = 0.2896$.

business investment becomes more *costly.* This, together with the reduced real profits caused by the higher inflation, jointly imposes a negative effect on money holdings. So, the money holdings by households drop even *further* than the standard model without the consideration of allocation adjustment. It is straightforward to see that the additional decrease in money holdings further hurts the job creation activities. Indeed, Table 4 shows a significant reduction in number of vacancies; it falls by half, from 0.0294 to 0.0138, which leads to a large rise in unemployment.

Shutting down FC friction helps cushion the effect on unemployment. As shown in the last column in Table 4, removing friction in FC helps facilitate matching efficiency, which partially offsets the rise in capital price caused by liquidity allocation adjustments. As a result, the vacancies fall by a lower size as opposed to the baseline case. So the rise in unemployment is mitigated.

4.5 Importance of Liquidity Allocation

This subsection explores the relative contribution of the liquidity allocation to the results reported in Table 3. The strategy used to separate its contribution from the overall effect is that with money expansion present, we ask the model to allocate the same portion of liquidity over consumption and business investment as the case before the money expansion happens. So, when inflation rises, the households in this counterfactual exercise still allocate 42 percent of the money holdings to business investment, and 58 percent to consumption of goods in KW.

Table 5 Contribution of liquidity allocation

	Before change	**After change**			
	Baseline	**Frictional FC**		**Frictionless FC**	
	-	w/ adj.	w/o adj.	w/ adj.	w/o adj.
Business Liq. share	42%	29%	42%	33%	42%
Consumption Liq. share	58%	71%	58%	67%	58%
Vacancies	0.0294	0.0138	0.0200	0.0166	0.0211

NOTE: Comparison between with allocation change and without allocation change shows that the number of vacancies is higher in the case where the liquidity share is fixed. This result remains robust regardless if the FC market friction is present or not.

Table 5 reports the results. When we eliminate the adjustment of liquidity allocation, in the case with frictional FC market, the number of vacancies goes up to 0.0200, the deviation from the result of 0.0138 suggests that the relative contribution of allocation adjustment is 39.7 percent ($= \frac{0.0200-0.0138}{0.0294-0.0138}$). So, allocation channel *alone* accounts for about 40 percent of the decline in vacancies. In the case where the FC friction is shut down, we redo the same exercise, the number of vacancies goes up further to 0.0211. It is straightforward that the additional rise is caused by improved matching outcomes in FC. The calculation shows that the relative contribution of allocation adjustment is 46.8 percent ($= \frac{0.0211-0.0138}{0.0294-0.0138}$), and the relative contribution of FC friction is 7.1 percent ($= \frac{0.0211-0.0200}{0.0294-0.0138}$).

5 Financial Friction and Innovation

This section introduces financial frictions and innovations, two possible factors that might affect the way of liquidity allocation in FC by households. Particularly, we deviate from the baseline model in two ways. One is that we allow households to use credit card to purchase KW goods with some probability. The other is that we consider costs caused by financial market friction in the process of purchasing capitals by new firms.[7] That is, a fraction of funds obtained by the new firms is diverted for other use instead of the capital purchase.

The reasons for the extensions are twofolds. Theoretically, these considerations would advance our understanding of liquidity allocation channel. The results in this section help *illustrate* what are the main determinants of liquidity allocation, and what is the respective importance of each determinant?

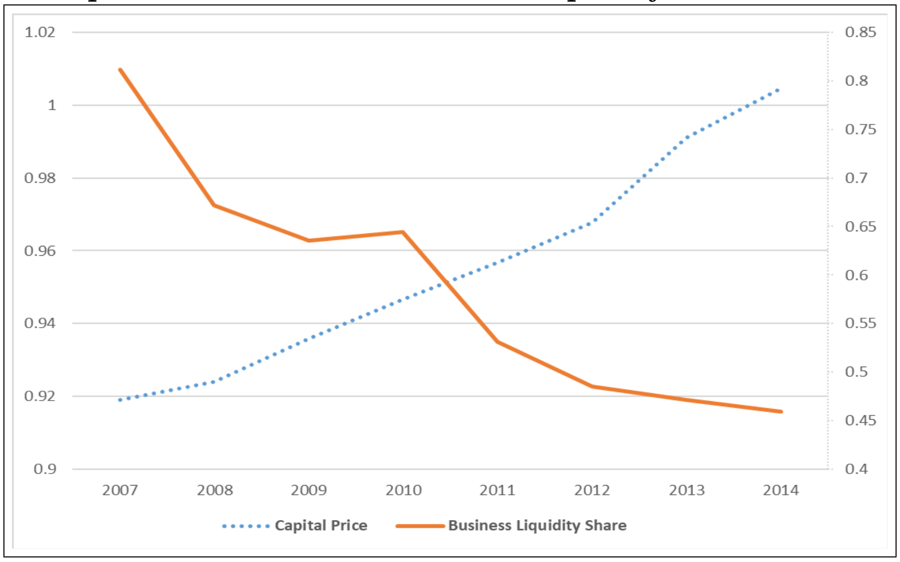
Empirically and more importantly, the wide usage of credit card and changing financial friction, indeed, shape the way how households allocate liquidity, and then, the market equilibrium outcomes. Over the period of 2007 to 2014, beyond our sample period used in baseline calibration, the business liquidity (as a ratio to overall liquidity), is *strongly negatively* correlated with the relative price of capital goods to consumption goods with the correlation being -0.906.[8] Figure 3 illustrates this strong negative relation. Moreover, the business liquidity is also negatively correlated with unemployment with the correlation being -0.343. These empirical relationships suggest that tightening liquidity constraint

[7] Financial friction could be the information asymmetry, or principle-agent problem, or other factors that affect the efficiency of the financial market in allocating resource.

[8] Price of capital goods is defined as the relative price of capital goods to consumption goods. The data is from Penn World Table website.

faced by firms, caused by the financial crisis, could be crucial in understanding the rising capital price, and high unemployment. This emprical observation motivates us to explore the importance of liquidity channel in accounting for high unemployment observed after the financial crisis.

Figure 3: Capital Price Versus Business Liquidity Share over 2007-2014



NOTE: Capital price is the relative price level of capital formation to household consumption for the United States over the period 2007-2014. Both price levels are seasonally adjusted with 2011 being normalized to one. Data source: Price Levels, Expenditure Categories and Capital, Federal Reserve Bank of St. Louis, and the series codes for the price level of capital formation and consumption are PLICPPUSA670NRUG and PLCCPPUSA670NRUG, respective.

The links are https://fred.stlouisfed.org/series/PLICPPUSA670NRUG.

https://fred.stlouisfed.org/series/PLCCPPUSA670NRUG.

5.1 Model with Financial Development and Friction

Denote the probability of using credit card in KW by μ, and the diversion cost by ϵ. The term of trade in KW is denoted by (q_t^c, d_t^c) in the credit trade, and (q_t^m, d_t^m) in the money trade. With these two changes, the value of households for type $e \in \{0,1\}$ in KW is modified as

$$\begin{aligned} V_{et}^h(b_t, \hat{m}_t, s_{t+1}, n_t, z_t) &= \mu\alpha_{ht}\left[v(q_t^c) + W_{et}^h(b_t, \hat{m}_t - d_t^c/\phi_t, s_{t+1}, n_t, z_t)\right] \\ &\quad + (1-\mu)\alpha_{ht}\left[v(q_t^m) + W_{et}^h(b_t, \hat{m}_t - d_t^m/\phi_t, s_{t+1}, n_t, z_t)\right] \\ &\quad + (1-\alpha_{ht}) W_{et}^h(b_t, \hat{m}_t, s_{t+1}, n_t, z_t), \end{aligned}$$

where $\hat{m}_t - d_t^m/\phi_t \geq 0$, while $\hat{m}_t - d_t^c/\phi_t$ can be negative (i.e., the credit card debt). We determine the terms (q_t^c, d_t^c) and (q_t^m, d_t^m) by the generalized Nash bargaining rule, the same as the baseline model. Denote the credit card limit by D. Following Randy (RES???), D is assumed to be sufficiently large. This implies two things. First, the real payment in the credit trade d_t^c satisfies $d_t^c \leq \hat{m}_t\phi_t + D$. Second, the quantity of goods in the credit trade q_t^c reaches its optimum q^*, where $v'(q^*) = c'(q^*)$. And the Nash bargain solution suggests that $d_t^c = g(q_t^*)$. The solution in the money trade remains unchanged, and one has $d_t^m = g(q_t^m) = \hat{m}_t\phi_t$.

The value of households in AD is

$$W_{et}^h(b_t, \tilde{m}_t, s_{t+1}, n_t, z_t) = \max_{m_{t+1}, x_t, b_{t+1}} \left\{ x_t + \beta J_{et+1}^h(b_{t+1}, m_{t+1}, s_{t+1}) \right\},$$

$$\begin{aligned} s.t.\ \ x_t + b_{t+1} + \phi_t m_{t+1} &= ew_t + (1-e)(\varsigma + l) + \phi_t T_t + b_t(1+r) + \phi_t \tilde{m}_t \\ &\quad + s_{t+1}(R - w - \delta_k \kappa/A) + n_t(1-\gamma_t)\kappa p_{kt}\phi_t \\ &\quad + n_t\gamma_t(1-\lambda_{ft})\kappa/A + z_t\eta_f\kappa/A. \end{aligned}$$

Note that $\tilde{m}_t$ in the budget constraint can be negative if the KW trade is completed by using credit card.

Similarly, the value of firms in KW is rewritten as

$$V_{1t}^f = \mu\alpha_{ft}W_{1t}^f(d_t^c/\phi_t, y_t - c(q_t^c)) + (1-\mu)\alpha_{ft}W_{1t}^f(d_t^m/\phi_t, y_t - c(q_t^m)) + (1-\alpha_{ft})W_{1t}^f(0, y_t).$$

Introducing the diversion cost (as a share of overall funds) ϵ affects the amount of funds required by the new firms from the households, and ultimately reshapes the households liquidity allocation. Specifically, with the presence of the diversion costs, the budget constraint for the households in FC is modified as

$$\hat{m}_t = m_t - n_t\kappa p_{kt}(1+\epsilon).$$

When $\epsilon = 0$ (no diversion cost), the new firms spend all funds on capital purchase. When $\epsilon > 0$, some funds are diverted for other purpose, which cannot be monitored by the households.

With the above changes, the system of equations at the steady state that are used to solve the model is rewritten as

$$\textbf{Law of motion of U}: u\lambda_h = (1-u)\,\delta.$$

$$\textbf{LM Curve}: \frac{i}{(1-\mu)\,\alpha_h} = \frac{v'(q^m)}{g'(q^m)} - 1.$$

$$\textbf{JC Curve}: \kappa p_k\phi\,(1+\epsilon)\,(1+i) = \gamma\left[\begin{array}{c}\lambda_f \frac{R-w-\delta_k\kappa/A+\beta\delta\eta_f\kappa/A}{1-\beta(1-\delta)} \\ +(1-\lambda_f)\,(1-\delta_k)\,\kappa/A\end{array}\right] + (1-\gamma)\,\kappa p_k\phi,$$

$$\textbf{Revenue in KW}: \; R = \mu\alpha_f\,[g(q^*) - c(q^*)] + (1-\mu)\,\alpha_f\,[g(q^m) - c(q^m)] + y.$$

$$\textbf{Wage in MP}: w = \frac{\chi\,[1-\beta(1-\delta)]\,(\varsigma+l) + (1-\chi)\,[1-\beta(1-\delta-\lambda_h)]\,(R+\kappa/AQ)}{\chi\,[1-\beta(1-\delta)] + (1-\chi)\,[1-\beta(1-\delta-\lambda_h)]},$$

where $Q = \beta\delta\eta_f - 1 + \beta(1-\delta)(1-\delta_k)$.

$$\textbf{Zero profit in FC}: \; p_k\phi A = \frac{1+r-(1-\gamma^c)(1-\delta_k)}{\gamma^c},$$

where $p_k\phi$ is determined by the Nash bargain solution in FC:

$$p_k\phi A = \sigma\left[\lambda_f \frac{(R-w)/(\kappa/A) - \delta_k + \beta\delta\eta_f}{1-\beta(1-\delta)} + (1-\lambda_f)(1-\delta_k)\right] + (1-\sigma)(1+r).$$

5.2 Quantitative Importance of Financial Development and Friction

In this subsection, we pursue to answer two questions. First, we examine to what extent introducing the financial market development *alone* would affect the main results obtained in the baseline simulation. Second, we shed light on the high unemployment observed after the 2007-2009 financial crisis. Particularly, we take the financial crisis as an exogenous shock and capture it by rising financial friction (a larger value of ϵ) in our model. With this, we aim to understand whether our model featuring liquidity allocation channel contributes to the high unemployment.

5.2.1 Implications of Financial Development

We take the wide usage of credit card as an instance of financial innovation and quantify its implication on the impact of monetary policy on the labor market outcomes. That is, we redo the quantitative analysis reported in Section 4 by introducing credit card trade in the KW market. In this case, the value of μ is set to be 0.15, the same as the choice in Aruoba *et al.* (2011). Klee (2008), by using supermarket scanner data, finds that the shoppers use credit card in 12 percent of the total transactions in the United States. Cooley and Hansen (1991), by using earlier consumer data, comes up with a measure of 16 percent. Aruoba *et al.* (2011) choose a value in between, and we follow their choice of 15 percent. With these changes, the real demand for money is modified as

$$\frac{M}{pY} = \frac{g(q^m) + nkp_k\phi}{(1-u)R + rb}.$$

Note that the total real demand for money still contains two parts: one is used to fund the business projects, $nkp_k\phi$. The other part is used to finance the cash purchase of KW goods, $g(q^m)$. It is worth pointing out that since the KW purchase happens after the decision of real demand for money, and whether the households can make payments by cash or credit card in KW is determined upon meeting with the buyers, the *ex ante* real money holdings by the households have to be the same. This explains why the term $g(q^m)$ enters the equations. In addition, the markup ratio in the model is changed into

$$markup = 100\left[\mu\frac{g(q^c)}{c'(q^c)q^c} + (1-\mu)\frac{g(q^m)}{c'(q^m)q^m} - 1\right].$$

We recalibrate the model with the usage of credit card in KW to match the targets listed in Table 1, and then impose high inflation to examine the impact of inflation on unemployment. The results are summarized in Table 6.

Table 6 Contribution of liquidity allocation

	Before change	After change			
	Baseline	With credit		Without Credit	
	-	w/ adj.	w/o adj.	w/ adj.	w/o adj.
Business Liq. share	41.7%	30.7%	41.7%	36.4%	41.7%
Consumption Liq. share	58.3%	69.3%	58.3%	63.6%	58.3%
Vacancies	0.0301	0.0146	0.0198	0.0208	0.0240

NOTE: Comparison between with allocation change and without allocation change shows that the number of vacancies is higher in the case where the liquidity share is fixed.

Table 6 shows that in the model with the usage of credit card in the KW market, the same rise in inflation as the one in Section 4.3 generates much lower number of vacancies in the model economy; vacancies fall from 0.0301 to 0.0146. And the corresponding unemployment rises from 0.0559 to 0.0731. Similar to the baseline simulation without the credit card, the allocation of liquidity changes, and the liquidity allocation moves away from business investment purpose. To see the contribution of allocation channel, we impose the liquidity allocation before the inflation change on the model economy, which suggests that the vacancies would rise from 0.0146 to 0.0198. So, the contribution of allocation channel is around 33.55 percent. The usage of the credit card helps strengthen the allocation channel. Table 6 shows that removing the credit card usage mitigates the responsiveness of the the liquidity allocation to the same inflation change. Indeed, the business liquidity share falls to 36.4 percent, significantly higher than 30.7 percent in the case with the presence of credit card. This explains why the decline in the vacancy in this case (0.0208) is less steep than the case with the credit card (0.0146).

5.2.2 Implications of Financial Friction on Post-Crisis Unemployment

In this subsection, we aim to examine if our model can provide some useful insight on the high unemployment observed after 2007-2009 financial crisis in the United States. To pursue our goal, we assume that the financial crisis makes the financial friction severer. We then use the model parameter ϵ to capture the changes in the financial market condition. When the financial market functions well, its value tends to be low. On the contrary, if the market condition gets worse, its value goes up; in this case, it becomes more costly to raise fund, which hurts job creation activities. Since financial market condition is difficult to measure, the strategy adopted is that we first randomly choose ϵ, and calibrate the model to fit the targets listed in Table 1. Then, we change the value of ϵ to mimic the declining business liquidity share observed in Figure 3. Particularly, we observe that the business liquidity share declines from 71.4 percent in 2006 to 48.5 percent in 2012. We then increase the value of ϵ such that the model-predicted business liquidity share falls by 28 percent ($= (71.4 - 48.5)/71.4$) from its long run equilibrium value of 42 percent. As shown in Table 6, such a change in business liquidity share leads the value of ϵ to rise from 0.022 to 0.072. Next, we use the model with larger ϵ to simulate unemployment.

The simulation results show that when the financial market friction increases (ϵ rises from 0.022 to 0.072), unemployment rises significantly from its long run steady state value of 0.0579 to 0.0805, very close to its empirical counterpart of 0.0808. As reported in Table 6, a larger market friction generates a lower overall liquidity as well as a change in composition; the overall liquidity falls from 0.187 (before change) to 0.142 (after change), and the business liquidity share declines pronouncedly from 42 percent to 31 percent. With these changes, the vacancies fall from 0.0307 in the baseline case to 0.0123.

To examine the role of liquidity allocation channel, we fix the business liquidity to its before-change level, which leads to a higher vacancy, 0.0157. This suggests that the contribution of the liquidity allocation channel is 18.4 $\left(=\frac{0.0157-0.0123}{0.0307-0.0123}\right)$ percent. The contribution increases with the rise in the market friction. When the friction further goes up to 0.111, the job creation activities almost shut down, falling quickly to 0.0058, and the corresponding unemployment surges to 0.1038. In this case, the business liquidity share declines even more, down to 15 percent. Restoring the liquidity share to its before-change level moves the vacancies back to 0.1491, which translates into a contribution of liquidity allocation channel to be 57.55 $\left(=\frac{0.0149-0.0058}{0.0307-0.0058}\right)$ percent.

Table 7 Contribution of liquidity allocation

	Before change	**After change**			
	Baseline	w/ adj.	w/o adj.	w/ adj.	w/o adj.
Value of ϵ	0.022	0.072		0.111	
Business Liq. share	42%	31%	42%	15%	42%
Consumption Liq. share	58%	69%	58%	85%	58%
Vacancies	0.0307	0.0123	0.0157	0.0058	0.1491

NOTE: Comparison between with allocation change and without allocation change shows that the number of vacancies is higher in the case where the liquidity share is fixed.

Of particular note is that we do not aim to to argue that our model can reproduce the high unemployment in 2012. Instead, the main purpose of this exercise is to illustrate the quantitative importance of the liquidity allocation channel; indeed, the response of liquidity allocation to deteriorated financial market *alone* allows our model to generate sizable responses of unemployment, which squares well with what we observe in the data. Moreover, the decomposition exercise illustrates the importance of allocation channel in understanding the persistent high unemployment in the post-crisis era when the friction of financial market deteriorates.

6 Conclusions

This paper has developed a theory of endogenous business liquidity and consumer liquidity. Different from much of the macro literature, we incorporate both types of liquidity. The environment has firms need cash for investment and consumers demand money for consumption. More consumer liquidity increases firms' profits, investment, and, thus, their need for liquidity. Conversely, more business liquidity also raises consumers' money demand. Therefore, the two types of liquidity compete and interact in interesting ways. A main contribution of the theory was to show that the consumption-investment liquidity allocation (CILA) channel amplifies the effect of monetary policy on unemployment, as it accounts for 40 percent of the effect in the calibrated model. Also, while many macro models are silent about the relative size of business liquidity and consumer liquidity, our theory also provides insights on the changing consumer-firm liquidity distribution. In the model, a lower nominal interest rate directs relatively more cash to firms, whereas financial frictions induce the opposite and high unemployment, consistent with the observed patterns.

Our approach of incorporating endogenous firm-consumer liquidity allocation opens new research avenues. While we focus on firm liquidity for capital investment purposes, there are other reasons why firms want to hold money, such as working capital, R&D and M&A. Future studies can explore not only how liquidity is allocated between firms and households, but also between firms' different uses of liquidity. One can also apply our firm-consumer liquidity allocation channel to the study of increased firm cash. As we have shown in this paper, macroeconomic factors (e.g., monetary policy and financial frictions) that influence the liquidity allocation between consumers and firms will also affect the absolute cash holdings of firms. Also, our model is also the first to incorporate the three frictional markets. The framework, especially how we model firm financing, can be useful for studies of the interactions of capital investment, employment, and consumption.

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