Heterogeneous households and the portfolio rebalancing channel of monetary policy*

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Abstract

We study the heterogeneity of consumption responses across households to a common monetary policy shock. Households vary in age, wealth and ex-ante asset allocation. We combine an asset pricing framework with heterogeneous agents and incomplete markets with a life-cycle model. Within our environment, the transmission of monetary policy does not only work through the usual income and substitution motives, but also through an endogenous portfolio rebalancing effect which generates changes in equilibrium asset prices and a subsequent wealth effect on consumption. We find that, if the shock is such that prices are unchanged, the response of consumption is fully transitory, where younger households increase, and older households decrease, consumption. If equilibrium prices rise as a result of the monetary shock, wealth effects mitigate heterogeneity in current consumption responses, but introduce persistence in responses that increase significantly with age.

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I. Introduction

A household's portfolio generally consists of tradable (equity, housing, bonds) and non-tradable (labour income) assets used to make consumption plans. In making these plans, households face both aggregate risk, such as tradable asset returns, and uninsurable idiosyncratic risk like labour income shocks. Given these risks, variation in age and asset holdings, amongst other factors, can potentially generate differences in responses across households to common aggregate shocks.

In this paper, we analyse monetary policy shocks and their effect on household asset allocations and consumption plans. Household heterogeneity, in terms of age, wealth and *ex-ante* asset allocation, will notably affect the pass-through of monetary policy into consumption. In the paper we combine a life-cycle model with an asset pricing framework with heterogeneous agents and incomplete markets. This allows us to study the effect of monetary policy not only through the usual income and substitution effect, but also through an endogenous portfolio rebalancing effect and the subsequent wealth effect which results from the change in equilibrium prices of equity and housing.

We calibrate the model to Euro area countries, using microeconomic data from the Household Finance and Consumption Survey (HFCS) in order to shed light on how monetary policy shocks affect different countries. Empirical evidence shows significant variation across Euro area countries in terms of income profile, distribution of *ex-ante* asset allocations, and demographics. These factors - as we are going to show in the paper - notably interact with the pass-through of monetary policy, leading to substantial heterogeneity in the aggregate response of different countries to the same monetary policy shocks.

We first develop a life-cycle model to compute the households' optimal portfolios. The model is a sequence of 4-year trading periods. In the model, the income process combines a deterministic age profile with aggregate shocks and a combination of idiosyncratic permanent and transitory shocks. Households of different ages enter the period with a different endowment of nominal bonds, equity and housing. They observe their income realization, consume and trade in the asset market. Households can invest in bonds, equity or housing and can borrow subject to a collateral constraint with a borrowing limit proportional to the value of their housing. All assets are risky, and we model their expected return and variance based on historical moments. Assets can be bought and sold without any frictions or transaction costs.

We then derive equilibrium prices for equity and housing as in Piazzesi and Schneider (2009) and Landvoigt et al. (2015). Based on the optimal policy of households, we can compute aggregate demand and finally compute the equilibrium prices for equity and housing that equate aggregate supply and hence clear the markets in the current period. In the model, expectations of future prices are exogenous and market clearing prices establish a 'temporary equilibrium' for the current period.

Household-level endowments and the quantity of aggregate supply are derived from the data, using the Household Finance and Consumption Survey (HFCS) and the Euro Area Economic Account (EEA). The HFCS is available for 2010 and 2014 for Euro area countries. We derive household asset allocations in 2010, using directly observed figures from the HFCS and indirect exposures measured through aggregate financial data from the EEA using the methodology as in Doepke and Schneider (2006) and Adam and Zhu (2015).

Combining our model with the data, we derive, for each cohort, the asset allocation and consumption decision. We will then compare our model-implied allocation with empirical moments from the 2014 survey to assess the ability of the model to match actual household decisions. Finally, we will evaluate the impact of monetary policy shocks - in the form of a reduction in the expected bond rate - on consumption and on household portfolio allocations. The transmission will also depend on how the reduction in the rate affects the expected returns on other assets. In the paper we propose and discuss three scenarios assuming, respectively, that: i) the expected returns on other assets are held constant, ii) the expected returns on other assets vary by the same amount as the change in the bond rate and iii) the change in the expected return is calibrated to match the empirical effect monetary policy has on equity and housing prices. Our main findings are as follows.

In scenarios in which equilibrium prices are unchanged after the shocks and hence the wealth effect is absent, we observe substantial heterogeneity in household responses in current consumption. In response to a 1% shock to the expected return on bonds, young households increase current consumption by 0.3% while older agents reduce spending by up to 0.3%. For the young cohorts, the substitution effect dominates, moving consumption forward into today by becoming cheaper intertemporally. For older cohorts, however, the wealth effect dominates. For prices to remain unchanged, expected returns on all assets decline. Consequently, expected returns on current wealth fall, which is particularly painful for older agents as, unlike the younger cohorts, their future consumption is predominantly financed by current wealth. Moreover, by having a smaller life expectancy, they are less able to smooth out this temporarily negative shock to returns. As a result, consumption today declines. Across households, however, the monetary policy effect is merely transitory.

In scenarios where we let equilibrium prices rise following a reduction in the bond rate, we find that the positive wealth effect causes households across all cohorts to substantially increase current consumption. Moreover, it mitigates heterogeneity in current consumption responses. With larger initial asset holdings, price changes impact older cohorts by more, and since they consequently experience a larger wealth effect, it partly fills the gap in current responses observed under no price changes. However, we then notice significant variation in the persistence of the consumption response. Persistence in fact increase with age, and while young households only increase their consumption temporarily, older households raise their consumption for several periods. By experiencing a stronger wealth effect, smoothing desires means that the consumption response for the old is extended further out into the future. In contrast, the relevance of the budget constraint for the young causes consumption responses to be more front-loaded as current consumption is somewhat constrained by current wealth.

Scenarios with price effects also highlight the importance of monetary policy portfolio rebalancing effects that ultimately generate movements in equilibrium asset prices. In particular, after a reduction in the bond rate, a significant portion of the rise in housing demand comes from younger households who increase their leveraged investment in housing and equity substantially, thus enhancing asset price increases.

Literature review: The paper is related to different strands of literature. First, we relate to the literature of monetary policy with heterogeneous agents. Auclert (2017), Kaplan et al. (2018) and McKay et al. (2016) all assess the role of household heterogeneity in the transmission of monetary policy. In our paper we develop a life-cycle model highlighting the role of demographics, as in Wong et al. (2016). Wong et al. (2016), as well as Greenwald (2017), focus on the role of the mortgage market and housing in generating heterogeneous responses to monetary policy shocks. Cloyne et al. (2018) show the consumption response to monetary policy shocks of homeowners with mortgages. For now, we abstract from long-term debt and instead include equity as an alternative investment in the portfolio choice problem.

The paper is also related to the literature which empirically estimates household-level exposure to shocks, as in Doepke and Schneider (2006), Adam and Tzamourani (2016) and Adam and Zhu (2015). Carroll et al. (2014) estimate marginal propensities to consume for households, taking into account the distribution of wealth, also using the HFCS survey.

Using an asset pricing model á la Piazzesi and Schneider (2009), we also assess the effect of monetary policy on prices. A large number of papers estimate the effect of monetary policy on asset prices, starting from the seminal paper by Bernanke and Kuttner (2005), to Rigobon and Sack (2004) and Gurkaynak et al. (2004).

Our model, generating asset pricing appreciations following a monetary policy shock, is able to assess in a coherent framework the role of wealth effects on consumption. A recent paper by Berger et al. (2017) analyzes the effect of house prices on consumption in a heterogenous agents framework. The wealth effects on consumption have also been largely explored in previous work, such as Case et al. (2005) and Campbell and Cocco (2007).

Finally, one of the main focuses of the paper is studying the interaction between monetary policy and asset allocation. Related papers are Adrian and Shin (2010), Borio and Zhu (2012), and Hau and Lai (2016). Moreover, a strand of literature analyzes the effect of monetary policy on leverage and risk taking. Examples are Gambacorta (2009) and Altunbas et al. (2010).

II. Data

In the paper we combine microeconomic information from the Household Finance and Consumption Survey (HFCS) and aggregate data from the Euro Area Economic Account (EEA). Our goal is to measure household sector exposure to bond, equity and housing. We define bond exposure as exposure to fixed income assets and deposits net of debt. Households are exposed to bond, housing and equity not only through direct investment (which we can measure using the HFCS), but also through pension and insurance funds or mutual funds. The HFCS also reports the amount invested in investment intermediaries but there is no information on how these funds are allocated into different asset classes. In order to have an estimate of the indirect exposure we will therefore exploit the informational content of the EEA.

A. Euro Area Account

The European sector accounts provide a comprehensive and comparable overview of the European economy. They record transactions between economic agents grouped by sector. Stocks of assets and liabilities are recorded in balance sheets. The institutional sectors include Households and Non-Profit Institutions Serving Households (NPISH), Non-Financial Corporations, Financial Corporations, Non-MMF Investment Funds, Monetary Financial Institutions (MFIs), Other Financial Institutions (Financial corporations other than MFIs, insurance corporations, pension funds and non-MMFs investment fund), Insurance Corporations and Pension Funds. Transactions and financial claims between economic agents resident in the Euro area/EU28 and economic agents resident elsewhere are recorded in the Rest of the World accounts.

We define as 'mutual funds' the sector 'Non-MMF investment funds' of EEA and 'pension and insurance' the sector 'Insurance corporations and Pension Funds'. We compute mutual funds' portfolio shares in bonds, equity and housing as well as total debt. Table I details the variables we used from the EEA and how we reconcile them. We will use the EEA data on portfolio allocation to consolidate the HFCS survey with the EEA survey in order to have a better measure of the total households' exposure to the asset classes of interest. We will use the portfolio of pension/insurance funds and mutual funds to compute household indirect exposure, combining it with the information from the HFCS.

B. HFCS

The HFCS collects household-level data on a wide range of variables, with a particular focus on the composition of household savings/borrowings. It contains 20 EU member states with a sample of more than 84,000 households. The primary advantage of this survey is that data collection is harmonized across countries, permitting direct comparison of results across countries.

	EEA Variable
Equity	Unlisted shares and other equity
Equity	Listed shares
Bond	Debt securities, Short-term original maturity (up to 1 year)
Bond	Debt securities, Long-term original maturity (over 1 year or no stated maturity)
Bond	Currency and deposits, All original maturities
Debt	Loans, Short-term original maturity (up to 1 year)
Debt	Loans, Long-term original maturity (over 1 year or no stated maturity)
Debt	
Pension and insurance fund	Life insurance and annuity entitlements
Pension and insurance fund	Pension entitlements, claims of pension funds on pension managers and entitlements to non-pension benefits
Pension and insurance fund	Non-life insurance technical provisions and provisions for calls under standardized guarantees
Mutual fund	Investment fund shares/units
Mutual fund	Total financial assets/liabilities

Table I. EEA Definition of Asset and Liabilities

[1]. Thus far, there have been two waves, in 2010 and 2014, which are repeated cross-sections of similar size.

We use the HFCS data to measure the direct investments into the three asset classes, as well as the indirect investments into pension, insurance and mutual funds. Equity investment includes the value of non self-employment private business, the value of self-employment businesses and publicly traded shares. Bond investment includes the value of deposits as well as the value of bonds. Housing investment includes the value of households' main residences as well as other real estate properties. Debt includes all households' liabilities: mortgage and non-mortgage debt. Net bond investment is defined as bond investment minus debt. The technical appendix F details the variables we used to construct these series.

We will then combine this information with data from the EEA. However, before we detail how we combine the two datasets, we first discuss the main similarities and differences between the two. We therefore compute aggregated data from the HFCS and compare it to the EEA.

[1]Many countries also have panel data available, allowing for longitudinal studies

	HFCS	EEA	Ratio HFCS/EEA
Deposits	320637.92	1112410.50	0.29
Bonds	136476.83	753555.25	0.18
Listed Equity	18640.05	92458.00	0.20
Unlisted Equity	414250.22	655668.75	0.63
Loans	244862.99	689574.50	0.36

Table II. Italy: EEA vs HFCS Aggregates (EURmn)

C. Aggregate: EEA vs HFCS

The information provided by the EEA and HFCS should be similar and, for this reason, comparable. However, we notice large discrepancies once we aggregate HFCS and compare them to the EEA figures. Table II and III compare the HFCS and EEA aggregates for Italy and Germany, respectively. They show that the numbers reported by the HFCS are significantly smaller than the EEA's. Kavonius et al. (2013) already pointed out that there are major differences between the HFCS aggregated data and the EEA data, and discuss possible explanations for this gap. Our numbers are broadly in line with what they found in their aggregation exercise. While we acknowledge discrepancies are not negligible, we believe the information from the EEA is valuable in the assessment of households' indirect portfolios. We therefore proceed with the consolidation exercise, where we combine the two datasets and compute a comprehensive measure of household portfolios.

	HFCS	EEA	Ratio HFCS/EEA
Deposits	1007795.43	1738636.75	0.58
Bonds	68936.15	259370.75	0.27
Listed Equity	144956.53	178394.75	0.81
Unlisted Equity	1174832.23	234830.50	5.00
Loans	1011213.49	1518844.50	0.67

Table III. Germany: EEA vs HFCS Aggregates (EURmn)

D. Consolidation

Our focus is on portfolio allocations by age group (or cohort), and we therefore compute average direct and indirect household exposures for each cohort. While we can easily estimate the direct exposures for each age-group using the HFCS survey, for the indirect exposure we need to rely on the EEA data and for this reason we only have available aggregate numbers. That means that - for the indirect exposure - we will assume that each cohort has the same indirect portfolio allocation, which is consistent with the EEA accounts.

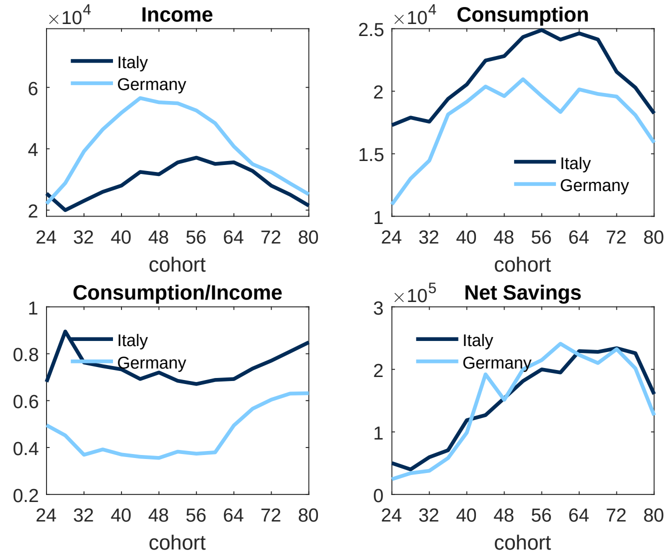

Figure 1. Consumption/Savings Decisions: Italy vs Germany (2014)

E. Household Exposure

We now summarize key statistics from the household-level data. We focus on 2014, the most recent wave, looking at the survey-weighted averages for each 4-year age cohort between 24 and 80. [2]. We compare Italy with Germany. Turning to Figure 1, the main point is that the income profile is significantly more humped for Germany. We suspect this may be a key driver for differences in savings and portfolio decisions across the two countries, particularly for younger cohorts. It also appears to suggest higher consumption/income ratios for Italy. [3]

Putting a lense on net savings to understand its composition, we move to Figure 2. Firstly, we focus exclusively on the component of savings that are investments in assets, either directly or indirectly. We exclude implicit savings from the expected value of future defined benefits pension, as they are unrelated to investment choices at the household level. Secondly, we need to apportion pension/mutual fund holdings to these instruments. In order to 'see through' the indirect holdings, we use the EEA holdings to derive their portfolio shares. These are then added to the declared direct holdings from the HFCS data, resulting in Figure 2 .

At first glance, the most glaring observation is that housing stock dwarfs all other forms of savings. This appears to be particularly apparent in Italy. Trends in savings look to be as expected: young cohorts tend to be net borrowers; borrowing increases into middle age due to

[2]Survey weights are equal to the likelihood of that household being sampled randomly from the population. These weights are particularly important given the design of the survey is to deliberately over-sample the rich, meaning sample means will be inconsistent estimates of the population mean

[3]Income figures are in gross terms and we are not confident about the consumption data. Therefore we do not take these figure at face value

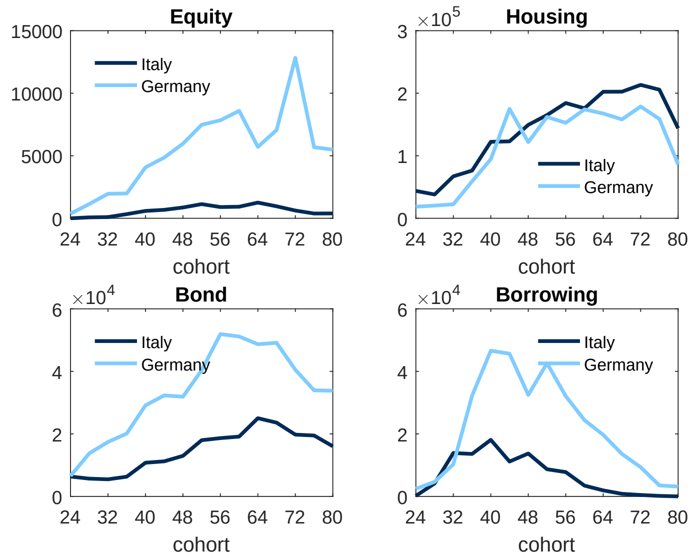

Figure 2. Portfolio Decisions: Italy vs Germany (2014)

mortgages financing household investments. As agents get older and accumulate wealth, they start borrowing less and begin to allocate savings to the three other assets.

F. *Portfolio allocation*

While we previously discussed the nominal amounts invested in different assets, we now analyze portfolio shares comparing the asset allocation over time and between countries.

Table IV and V show total asset and portfolio shares together with debt/asset ratios for Italian households at different ages. We observe a sharp reduction in the nominal amount of total assets from 2010 to 2014 for all cohorts, apart from the oldest group (68-76).

Italian household portfolio shares are heavily tilted toward housing. In fact, 80% or more of total wealth is invested in housing at every age. However, younger cohorts exhibited a reduction in their housing share from 2010 to 2014, together with an increase in their bond share.

German households invest a significantly lower share of their wealth in housing in favour of a much higher share of bond investment. In Germany we also observe a reduction in housing investment for the younger generations from 2010 to 2014, while for older generations it remained stable. The reduction in housing has been offset by an increase in bond and equity investment.

G. *Within-cohort wealth distribution*

So far we have focused on household heterogeneity by cohorts. However, within-cohort heterogeneity also plays an important role as an agent's portfolio allocation and consumption decision

	Housing	Bond	Equity	Debt	Total assets
28	0.82	0.10	0.08	0.15	80, 447.82
36	0.84	0.08	0.08	0.18	123, 477.67
44	0.81	0.08	0.11	0.09	209, 736.87
52	0.84	0.08	0.08	0.07	257, 591.48
60	0.83	0.10	0.07	0.04	289, 630.26
68	0.84	0.09	0.07	0.02	318, 587.58
76	0.89	0.09	0.02	0.01	237, 090.65

Table IV. Asset allocation: Italy 2010. Reports portfolio shares and debt as % total assets

	Housing	Bond	Equity	Debt	Total assets
28	0.78	0.14	0.08	0.10	54, 773.96
36	0.79	0.11	0.10	0.21	97, 611.77
44	0.84	0.08	0.08	0.11	162, 849.92
52	0.83	0.09	0.08	0.07	213, 915.12
60	0.81	0.10	0.09	0.05	246, 566.86
68	0.85	0.12	0.03	0.01	257, 980.28
76	0.88	0.10	0.02	0.01	253, 888.67

Table V. Asset allocation: Italy 2014. Reports portfolio shares and debt as % total assets

does not only depend on age but also on wealth. Figures 3 and 4 plot the distribution of wealth (excluding the 1% tails) for different cohorts for both the first and second wave. We notice that distributions are heavily skewed. The skewness is maximum for younger cohorts and becomes less pronounced for older households. In particular, more than 60% of young households have wealth between 0 and 40 thousands euros. For older households the share goes down to around 20%. When we compare the distribution of wealth for 2010 versus 2014, we do not notice major differences, meaning that wealth inequality has not significantly increased between the two

	Housing	Bond	Equity	Debt	Total assets
28	0.70	0.25	0.05	0.21	35, 730.18
36	0.70	0.25	0.06	0.35	81, 681.69
44	0.71	0.18	0.11	0.25	171, 513.70
52	0.65	0.20	0.14	0.19	227, 404.97
60	0.64	0.16	0.20	0.11	290, 998.77
68	0.71	0.20	0.09	0.11	220, 248.31
76	0.76	0.20	0.04	0.05	207, 216.55

Table VI. Asset allocation: Germany 2010. Reports portfolio shares and debt as % total assets

	Housing	Bond	Equity	Debt	Total assets
28	0.56	0.30	0.14	0.18	42, 337.85
36	0.60	0.30	0.10	0.36	76, 353.10
44	0.66	0.18	0.16	0.22	220, 125.32
52	0.65	0.17	0.18	0.18	239, 760.45
60	0.67	0.20	0.14	0.10	296, 868.78
68	0.70	0.21	0.09	0.09	245, 385.32
76	0.71	0.20	0.09	0.04	241, 081.34

Table VII. Asset allocation: Germany 2014. Reports portfolio shares and debt as % total assets

surveys. However, we do notice significant difference between Italy and Germany, especially for older cohorts.

In the model, the relevant variable that households are going to use to allocate their portfolio and choosing consumption is the ratio of wealth to permanent income. In the cross-section, we are not able to empirically identify a household's permanent income. However, the empirical distribution of wealth to income ratio should be close to the variable of interest: wealth to permanent income. Figures 5 and 6 show that the wealth to income ratio is also particularly skewed, but this skeweness decreases for older cohorts. We also notice that for all cohorts the highest share of households have a wealth to income ratio between 1 and 1.5. However, while for young cohorts households are concentrated in the first bin, the distribution for older people is spread out. Finally also, in the wealth-to-income ratio we find significant differences between Italy and Germany, especially for older cohorts. In fact the distribution for Germany is significantly more skewed.

H. Rest of the Economy

As we will discuss in the model section, we leave the rest of the economy (ROE) unmodelled. We use the EEA and national tables data in order to estimate the supply of assets provided by the ROE.

We estimate f^e as the new corporate equity as a percentage of total household holdings. We sum the flows from 2011 to 2014 (included) from the EEA account. Flows from the EEA account are meant to represent volumes, excluding asset appreciation. We take into account both the direct supply of assets to the household sector as well as the indirect supply through investment intermediaries and the rest of the economy. We estimate a 5% increase in equity flow for the period of interest. We assume the figure is similar for housing.

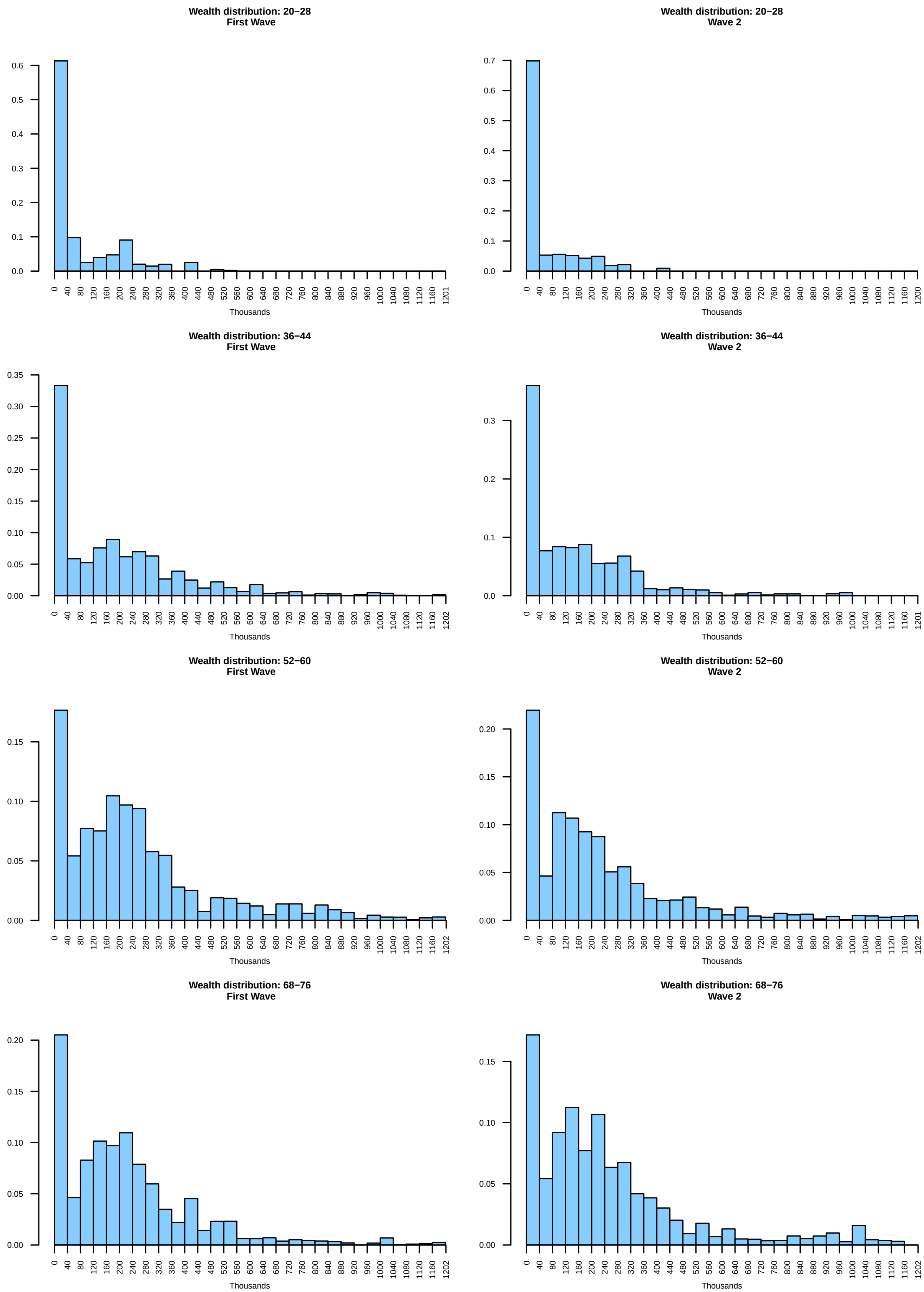

Figure 3. Italy: Wealth Distribution

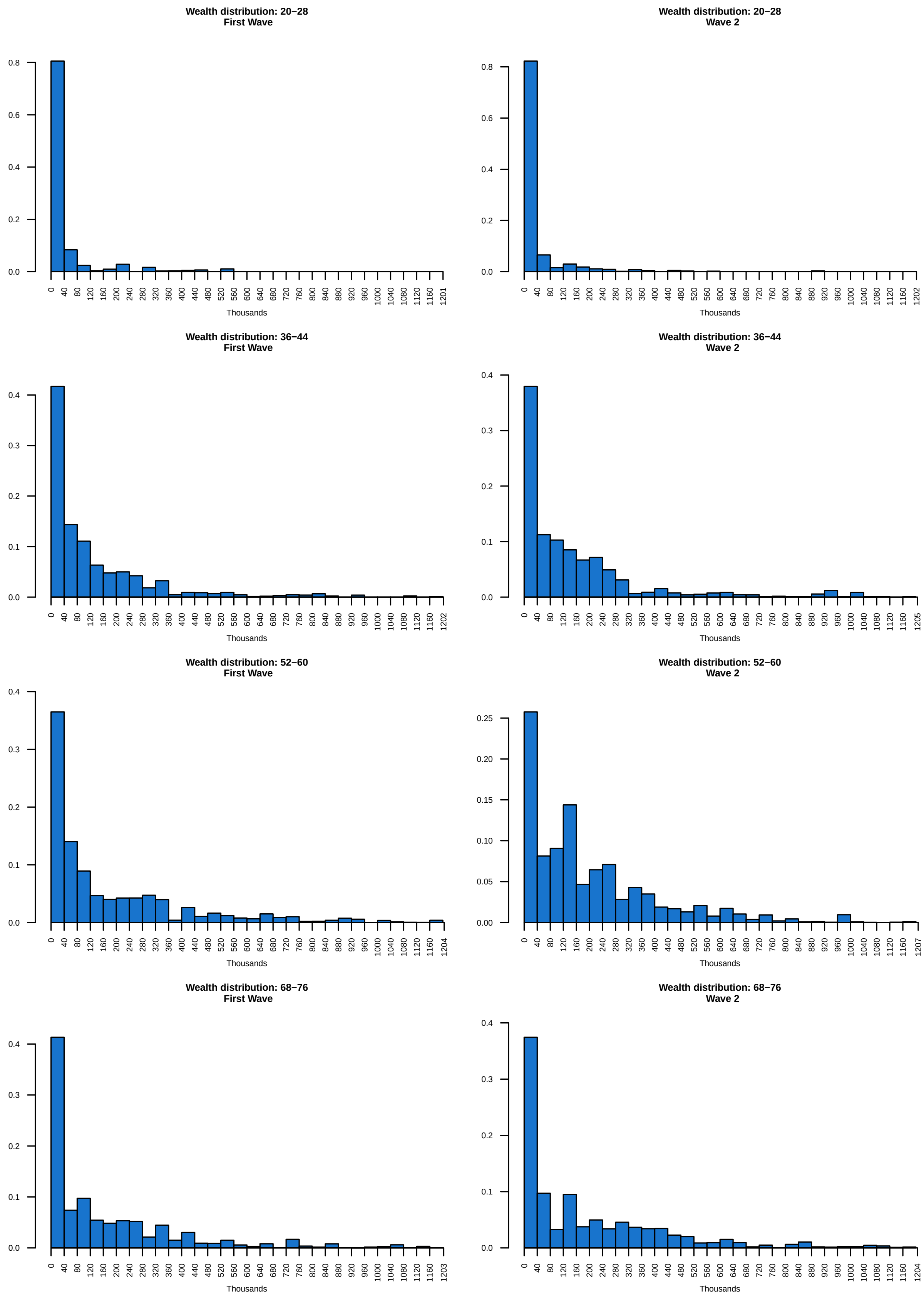

Figure 4. Germany: Wealth Distribution

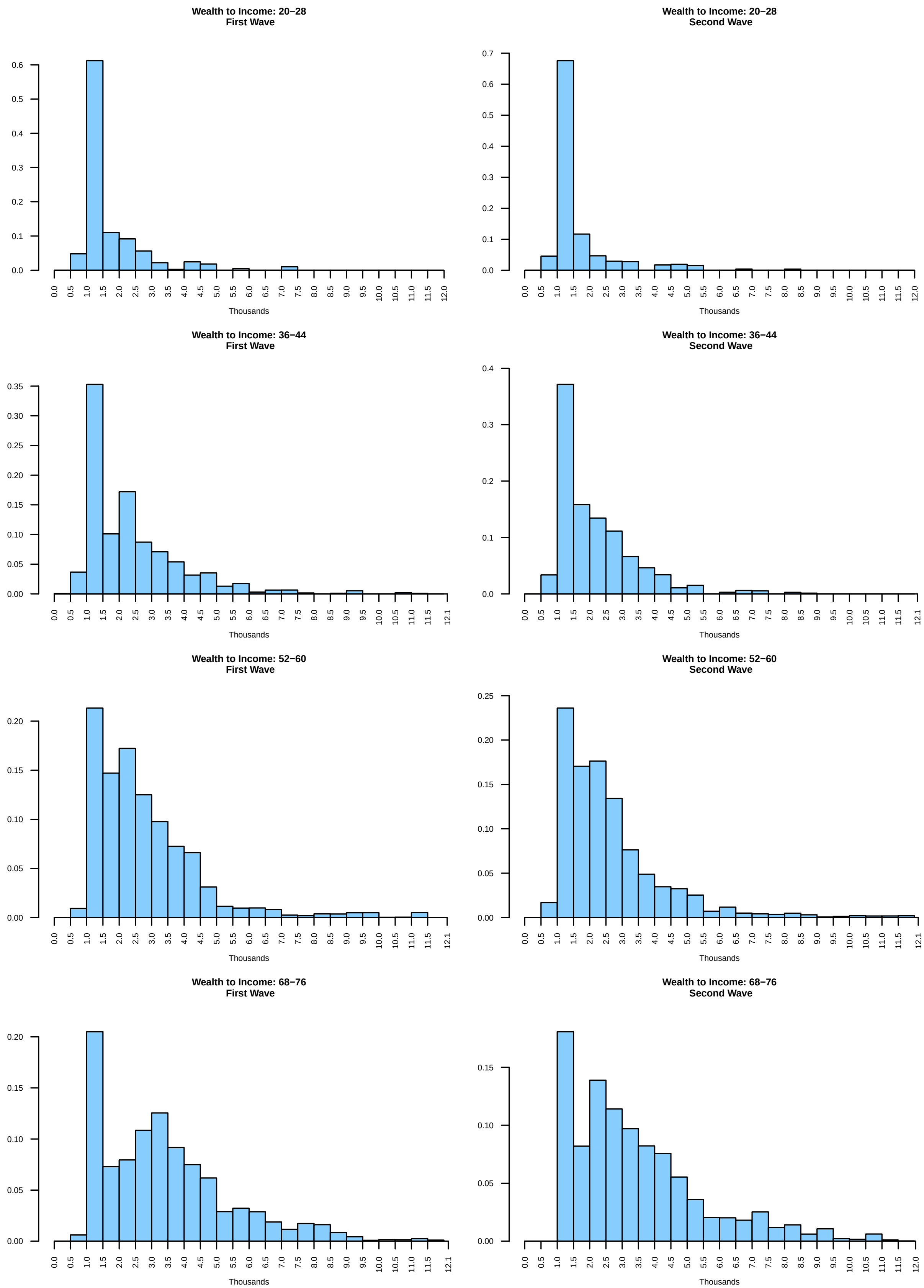

Figure 5. Italy: Distribution wealth to income

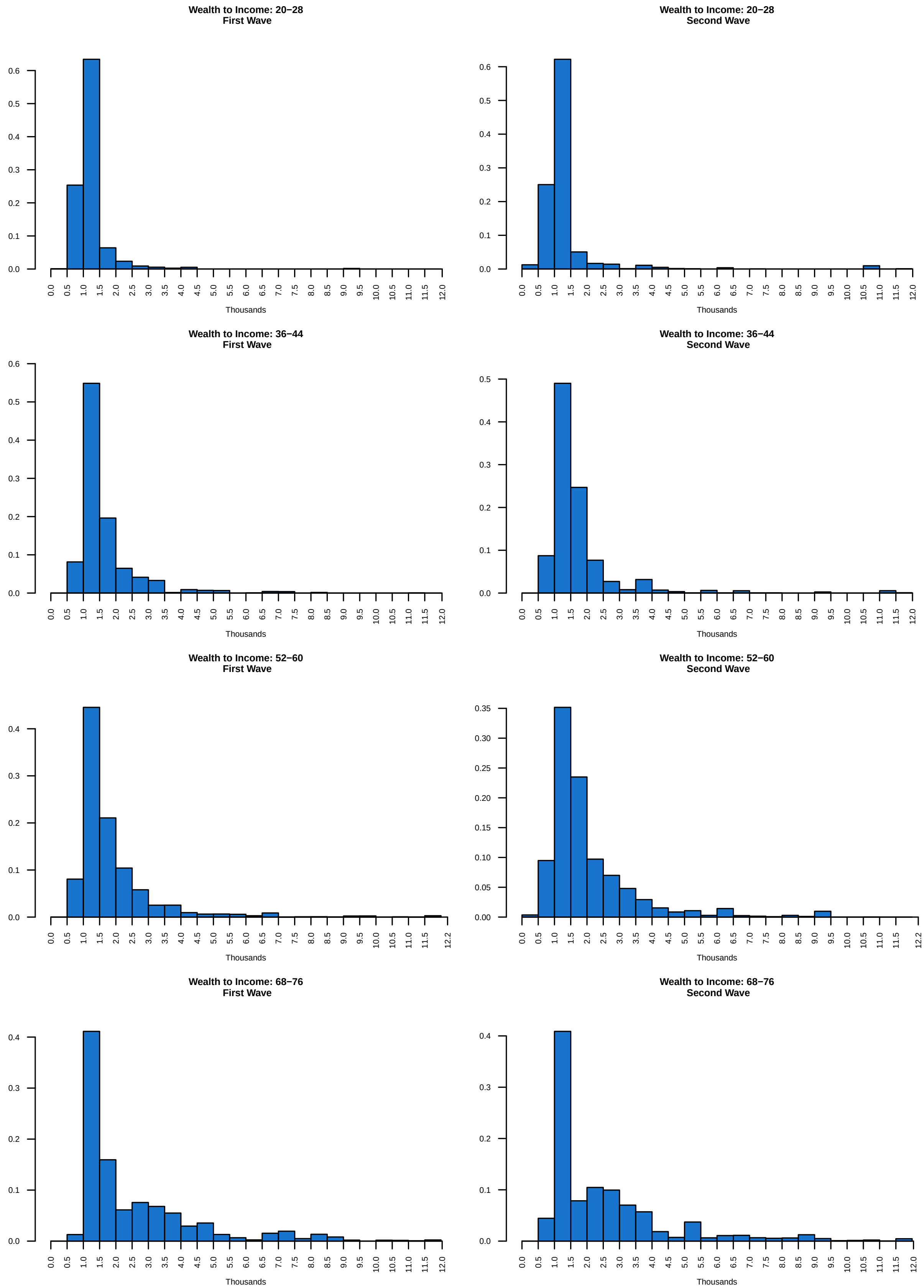

Figure 6. Germany: Distribution wealth to income

III. Model

Setup

The setup of the model is based on the framework developed by Piazzesi and Schneider (2012). The main features which make the setup suitable for our research question are:

1. **Temporary Equilibrium Concept**: Expectations of *future* asset returns are not equilibrium outcomes but instead exogenous inputs into the model. This makes finding current asset prices relatively straightforward. It also means that future returns expectations are not linked to model-implied prices. Given the model intervals are 4-year intervals, as this is the frequency with which we see household portfolio choices in the data, we argue that forming expectations from past data is reasonable vs. forming knowledge of the form of the economy that could be subject to structural change.

2. **Housing investment**: This model introduces frictions and returns characteristics that makes housing special vs equity as an asset. Understanding the distinction of housing is important, in our view, since it is the main asset held by the majority of households.

To convey this model concept more clearly, we show the model can be interpreted as having two successive layers. The first layer contains a standard portfolio choice for each household, taking into account age, initial wealth and returns expectations. The second layer introduces the temporary equilibrium concept that allows us to derive equilibrium asset prices such that total household sector demand for assets coming from the policy functions equals the value of assets supplied to the household sector by the RoE.

A. *Layer 1: Portfolio Choice Problem*

We describe the first layer of this model that focuses solely on the household sector. This generates policy functions for each household based on age and initial wealth.

Households

Households differ by age and initial wealth. Each household exhibits Epstein-Zin utility over the single homogeneous consumption good. They earn labour income, pay taxes, consume, buy assets, and bring forward assets from the previous period. When making portfolio choices, they have three assets available to invest in: nominal bonds, equity and housing. Nominal bonds are of 1-period maturity, paying real return $\frac{1}{q_t \Pi_{t+1}}$, where q_t is the price of the bond paying one nominal unit next period, and Π_t is gross inflation. Equity and housing both pay off in units of consumption. However, there are key distinctions between them. Firstly, they have different payoff profiles.[4] Secondly, only housing can be used as collateral against which the household

[4]These are estimated in the data, as explained later

borrows. Finally, housing exhibits a tax advantage in some European countries where there is exemption from capital gains tax on primary residence.

When making portfolio choices, households are subject to the following frictions: non-negativity constraints on equity and housing, and a collateral constraint where the borrowing limit is proportional to housing value (LTV constraint).

The model includes both aggregate and idiosyncratic risk. Households face aggregate growth risk from labour income, aggregate inflation risk from short-term bonds paying a fixed nominal return, and aggregate real asset returns risk on equity and housing. They also face idiosyncratic income risk that is split into a fully permanent and fully idiosyncratic component. Time-invariant variances of each of these shocks is initially taken from estimates within the literature.

Formation of Expectations

Expectations over return distributions are exogenous in our model and based on historical data. For now, we base our estimates of returns and the Variance-Covariance matrix from Piazzesi and Schneider (2012), taken from historical US returns. In the near future, we aim to use Euro Area level data. We include an idiosyncratic shock to the housing return that scales aggregate variance by 3.25, roughly in line with estimates from Piazzesi, Schneider and Tuzel (2006).

Preference Parameters

The intertemporal elasticity of substitution σ is 0.5, the coefficient of relative risk aversion γ is 5, while the discount rate β is $\exp(-0.025 * 4)$. With such a low γ, there will be a minimal desire to hold bonds because of superior Sharpe ratios. Consequently, we scale variances of stocks, equity and inflation by 3 in order to get reasonable aggregate portfolio shares.

Income Process

The income of a given agent i of age a is:

$$y^i_{a,t} = G_t A_t P^i_t U^i_t$$

where G_t grows at the common aggregate growth rate, A_t is the deterministic age profile, and (P^i_t, U^i_t) are the permanent and transitory idiosyncratic shocks, respectively. We have that $\Delta \log P^i_t, \log U^i_t$ follow i.i.d. normal distributions of mean zero. [5] Consequently, when forming expectations of $y^i_{a,t+1}$, it is sufficient to know the level $\hat{y}_{a,t} \equiv G_t A_t P^i_t$ i.e. the permanent component of income.

[5] $Var(\Delta log P^i_t) = 2\%; Var(log U^i_t) = 7\%$, taken from the Review of Economic Dynamics.

Optimization Problem

We now dig deeper into the decision problem of the household.

$$V_t(\omega_t, \hat{y}_t) = \max_{c_t,\alpha_t} \left[c_t^{1-\frac{1}{\sigma}} + \beta E_t \left[V_{t+1}(\omega_{t+1}, \hat{y}_{t+1})^{1-\gamma} \right]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\sigma}}}$$

subject to

- $c_t + \alpha_t \cdot \omega_t = \omega_t$ **- budget constraint**
- $-\alpha_t^b \omega_t \leq \phi \alpha_t^h \omega_t$ **- collateral constraint**
- $\alpha_t^h, \alpha_t^e \geq 0$

where

- $\omega_t = \alpha_{t-1}^T R_t \omega_{t-1} + y_t$

where $\alpha_t = \left(\alpha_t^b, \alpha_t^h, \alpha_t^e\right)$ are portfolio shares for bonds, housing and equity, while $R_t = \left(R_t^b, R_t^h, R_t^e\right)$ are realized returns for the respective assets. Thus, wealth is used to finance both consumption and portfolio choices, subject to the collateral constraint where the LTV ratio limit is ϕ. $\hat{y}_t$ is the permanent component of income y_t. Conditional on initial wealth, $\hat{y}_t$ is the only additional state variable since additional knowledge of overall income, y_t, is unnecessary for forming expectations since the remaining component is fully transitory.

Bequest and Inheritance

We incorporate death probabilities into the model. When agents die, they leave behind the value of their wealth to younger generations. If an agent in period t holds wealth ω_t, then the utility they receive conditional on dying in period t equals:

$$\phi_{BEQ} u(\omega_t)$$

In this case, we set $\phi_{BEQ} = 1$. This bequest is then left behind as inheritance to the cohort 32 years younger. It is modelled as a multiplicative shock on income, $U(a)$, where cohort a has income multiplied by $U(a)$ with probability $Pr_{a+32}(\text{death})$. $U(a)$ is set such that the Bequest-Inheritance market clears in period t. See Appendix E for further details. [6].

[6] To be precise, $U(a)$ depends on market-clearing asset prices that are only introduced in the second layer. However, we incorporate this within the first layer for expositional purposes

First-Order Conditions

Note that the objective function can be normalised by $\hat{y}_t$ so that we are left with just one state variable, $\frac{\omega_t}{\hat{y}_t}$ (because normalized income now contains only the fully transitory component) Thus, we re-write $V_t(\omega_t, \hat{y}_t) = \hat{V}_t\left(\frac{\omega_t}{\hat{y}_t}\right)\hat{y}_t$, and we can substitute the value functions and constraints s.t. $\frac{\omega}{\hat{y}_t}$ is the only required state variable. [7] This then generates the following FOCs:

1. $m_t^e \leq \lambda_t$ **(equity)**
2. $m_t^h \leq \lambda_t - \phi\mu_t$ **(housing)**
3. $m_t^b \leq \lambda_t - \mu_t$ **(bonds)**
4. $\lambda_t = \hat{V}_t\left(\frac{\bar{\omega}_t}{\hat{y}_t}\right)^{\frac{1}{\sigma}}\left(\frac{c_t}{\hat{y}_t}\right)^{-\frac{1}{\sigma}}$ **(consumption)**

where:

$$m_t^a = \hat{V}_t\left(\frac{\omega_t}{\hat{y}_t}\right)^{\frac{1}{\sigma}}\beta E_t\left[\left(\hat{V}_{t+1}(\frac{\omega_{t+1}}{\hat{y}_{t+1}})(\frac{\hat{y}_{t+1}}{\hat{y}_t})\right)^{1-\gamma}\right]^{\frac{\gamma-\frac{1}{\sigma}}{1-\gamma}} E_t\left[\left(\hat{V}_{t+1}(\frac{\omega_{t+1}}{\hat{y}_{t+1}})(\frac{\hat{y}_{t+1}}{\hat{y}_t})\right)^{-\gamma}\hat{V}'_{t+1}\left(\frac{\omega_{t+1}}{\hat{y}_{t+1}}\right)R_{t+1}^a\right]$$

s.t. λ_t and μ_t are the Lagrange multipliers on the budget and collateral constraint, respectively, and m_t^a represents the marginal benefit of investing in one additional unit of asset a. If positive amounts of equity investments are made, the marginal benefit coming from expected future returns must equal the marginal utility of consumption foregone. For housing, though, it also yields the benefit of relaxing the collateral constraint by providing a fraction of more collateral, while bonds also have the benefit of relaxing this constraint.

This collateral constraint is one of the key differences between equity and housing. Using housing as collateral, it permits poorer households to take on leveraged investments in housing, whose risk-reward profile may be particularly favourable towards those who are asset-poor but rich in expected future income. This investment option is likely to make the collateral constraint bind under certain circumstances, as we will show later. This leverage option is a key mechanism through which agents of different ages and asset endowments choose different portfolios.

Policy Functions

The result of this layer is a policy function $\alpha_{a,t}\left(\frac{\omega_t}{\hat{y}_t}\right)$, that varies in age, a and initial normalized wealth, $\frac{\omega_t}{\hat{y}_t}$. We now use these functions for determining market-clearing asset prices in the second layer.

[7] See appendix D for derivation

B. Layer 2: Temporary Equilibrium

Given a distribution of age and initial wealth across households, policy functions allow us to derive the value of each asset demanded by the household sector. Our next step is to introduce the temporary equilibrium concept where equilibrium asset prices are such that the amount demanded by households equals the value of assets supplied to the sector.

Aggregate asset supply to households comes from two sources. Firstly, households bring into this period assets that were acquired last period. In the portfolio choice problem, there was no concept of asset supply. Households merely allocated wealth in period $t-1$ across three available assets, and brought forward wealth into period t that was coming from realized returns on their portfolios. In this set-up, we-interpret the wealth brought forward as being the value in period t of asset quantities purchased in period $t-1$, where each asset exhibits a per-unit price. For example, the wealth accumulated from equity investments by household i, $\alpha^e_{t-1,i}R^e_t\omega_{t-1,i}$ is re-interpreted as $\theta^e_{i,t-1}p^e_t$, where $\theta^e_{i,t-1}$ is the quantity of equity brought into period t, and p^e_t is the per-unit price of equity. Consequently, we have that:

$$\alpha^T_{t-1,i}R_t\omega_{t-1,i} = \theta^e_{t-1,i}p^e_t + \theta^h_{t-1,i}p^h_t + \alpha^b_{t-1,1}R_b\omega_{t-1,i}$$

Do note that we do not re-interpret bond holdings. As they are one-period bonds, they mature in period t and so do not exhibit a price upon maturity but instead returns are all paid out in the form of principal repayment. For housing and equity, we normalize the aggregate quantity of housing and equity supply owned by the household sector to 1, so that p^e_t and p^h_t represent the market value of equity and housing, respectively, held by the household sector. We can also interpret the normalization as if there were only one, perfectly divisible, equity and housing tree and households hold a share of the trees.

Secondly, additional asset quantities are exogenously supplied to the household sector by the Rest of Economy sector. The Rest of Economy is not explicitly modelled. Instead, they merely exist to exogenously supply new assets, and to provide goods that plugs the gap in the goods market. More specifically, f^e and f^h is the new stock, in % terms, of equity and housing provided by the ROE in period t.

We then appeal to the data to measure some of these components. For housing and equity shares, we split the household sector at the cohort level, treating each cohort as a single agent in the economy. Cohort i then holds shares $(\theta^e_{i,t-1}, \theta^h_{i,t-1})$, which equals the shares of the assets held by that cohort as a percentage of aggregate household sector ownership. All values are survey-weighted. As this is for shares in period $t-1$, this will be taken from the first wave of the HFCS in 2010 and we assume they are carried by the households into the next period (which corresponds to 2014). f^e and f^h are estimated from the net new purchases of equity and housing made by the household sector between 2010 and 2014, estimated using data from the EEA. [8].

The quantities $\theta^e_{i,t-1}$, $\theta^h_{i,t-1}$ as well as the nominal bond holdings $\alpha^b_{t-1,1} R_b \omega_{t-1,i}$ are endowed to the households at time t. We can now turn to what defines an equilibrium.

C. *Equilibrium*

An **equilibrium** for time t is a vector of prices (p^h_t, p^e_t, q_t) and consumer plan collection $\{A_t(i)\}$ s.t.

1. $A_t(i)$ is part of the optimal plan $A(i)$ given T, endowments and beliefs
2. All markets clear i.e.
   - $\sum_i \varphi(i)\alpha^h_t(i, \frac{\omega_{t,i}}{\hat{y}_{t,i}}, q_t)\omega_{t,i} = p^h(1+f^h)\sum_i \varphi(i)\theta^h_{i,t-1} = p^h(1+f^h)$ **(housing)**
   - $\sum_i \varphi(i)\alpha^e_t(i, \frac{\omega_{t,i}}{\hat{y}_{t,i}}, q_t)\omega_{t,i} = p^e(1+f^e)\sum_i \varphi(i)\theta^e_{i,t-1} = p^e(1+f^e)$ **(equity)**
   - $\sum_i \varphi(i)\alpha^b_t(i, \frac{\omega_{t,i}}{\hat{y}_{t,i}}, q_t)\omega_{t,i} = D_t$ **(bonds)**

   where $\varphi(i)$ is the survey weight of cohort i, q_t is the price of a bond paying one unit next period (s.t. $E_t\left[R^b_{t+1}\right] = E_t\left[\frac{1}{q_t \Pi_{t+1}}\right]$), and D_t is the aggregate supply of nominal bonds provided by the rest of the economy.

The bond price q_t controls $E_t\left[R^b_{t+1}\right]$, which impacts the policy functions. However, given q_t, the prices of housing and equity (p^h, p^e) impact the demand side through $\omega_{t,i}$ by changing the market value of asset quantities held (whereby an appreciation (depreciation) lead to an increase (decrease) in wealth), but not the policy functions as return expectations do not change with asset prices. This means that clearing the housing and equity market is a relatively straightforward fixed point problem. See Appendix C for further details on the market clearing problem.

In our model, we have 20 cohorts, aged from 24 to 80, and we will also split cohorts into two sub-cohorts based on wealth levels, splitting at the 10th percentile of wealth.

[8] This does not include appreciation of existing assets held at the beginning of the period. Thus, we cannot back out f^e and f^h from just comparing the change in value of assets, but instead we need the value of net outright purchases of assets

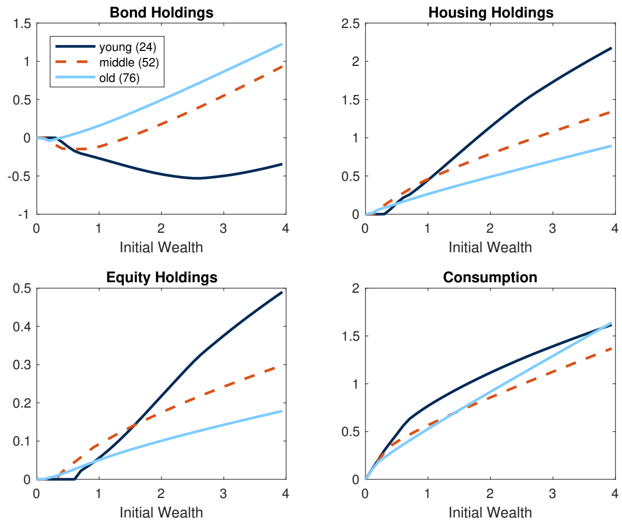

Figure 7. Policy Functions: Italy (Cohort Ages: 24,52,76)

IV. Baseline Results

A. Policy Functions

Given our model structure, we now lay out the policy functions of each agent conditional on their age and wealth. Specifically, we analyze both their portfolio choice and consumption levels. We set the expected real return on nominal bonds in period t to be the data-based expectation [9]. This results in Figure 7, which graphs policy functions for agents aged 28, 52 and 76 as a function of their state variable (normalized initial wealth).

We begin with bond holdings. For very low wealth levels, neither agent wants to save but instead prefers to consume all their wealth. Agents would ideally borrow outright, but the collateral constraint prevents this. This constraint is particularly relevant for younger agents who expect income growth in the future, hence their wealth threshold beyond which they save is higher. At very low wealth levels, this over-rides their stronger precautionary motive to save. We also notice how our model generates endogenous non-participation in the housing market for low wealth levels.

Above this wealth threshold, optimal portfolio decisions immediately become heterogeneous across ages. For the young, they use initial wealth to engage in leveraged investments. Most of young households' future wealth is coming from income, which is uncorrelated with other assets, making households willing to tolerate risky investments. For low wealth levels, household investment is concentrated in housing, but for higher wealth they diversify their portfolio with

[9]We will instead use the market-clearing expectation of nominal bond returns later when we find the equilibrium

equity. Middle-aged cohorts, apart from very low wealth levels, hold a diversified portfolio of bonds, housing and equity. Such diversification occurs at lower wealth levels because i) future income is less important simply due to lower remaining life expectancy, and ii) they anticipate a declining age profile of income. This causes them to hold a positive number of bonds at lower wealth levels. The risk tolerance for the old is even lower, for the same reasons as above, meaning a positive number of bonds are held at almost all wealth levels.

Turning to the portfolio split between equity and housing, the holdings in housing are considerably larger than in equity. Housing has a superior pre-tax mean-variance profile and also receives preferential tax treatment. At lower ages and/or wealth levels, this is accentuated by the collateral value of housing agents utilize in order to borrow. However, because housing and equity have almost uncorrelated returns, holdings of equity become non-negligible because of their usefulness as a diversification tool.

Finally, we turn to the consumption choices. Initially, at very low wealth levels, c overlaps for all cohorts as they choose to consume all of their wealth. This threshold is higher the younger the agent, hence why the kink point declines with age. However, beyond this kink, the younger cohorts' marginal savings rates are higher as they need to use this wealth to finance consumption in later life, thus flattening the current policy function for c. As this is less of a concern for the older cohort due to shorter expected remaining life, their policy function is steeper.

B. Finding Market-Clearing Prices, R

We then derive market-clearing (p^e, p^h, R) for equity and housing. [10] The two sufficient statistics in this model are 1) age and 2) initial wealth. Consequently, for each cohort, we need to evaluate model-implied wealth. For an agent i, initial wealth in period t is:

$$\bar{\omega}_t = p^h \theta^{h,i}_{t-1} + p^e \theta^{e,i}_{t-1} + \bar{b}^i_t + y^i_t \tag{1}$$

where $\theta^{h,i}_{t-1}$, $\theta^{e,i}_{t-1}$ are the empirical quantities held by agent i in housing and equity, respectively, with corresponding unit prices p^h and p^e. $\bar{b}^i_t$ is the quantity of nominal bonds brought forward from the previous period after realized returns, while y^i_t is labour income. The empirical initial equity/housing shares are normalized to sum to 1, meaning that p^h, p^e can be interpreted as the aggregate value of housing and equity in the economy. [11]

For each cohort we split into two sub-agents, one representing the top 10%, and the other representing the bottom 90%. As the wealth distribution is heavily skewed and since policy

[10] A more detailed description of the market-clearing process is in the Appendix

[11] Specifically, the right-hand-side of market-clearing conditions of housing are equity are set to one by appropriately normalising $\bar{\theta}$s

functions can be highly non-linear, the 90-10 split is particularly relevant to match the empirical moments.

Given (p^h, p^e, R), we know $\bar{\omega}_t$ for each sub-cohort, and by normalizing by the permanent component of income, we can derive model-implied portfolio and consumption choices. The goal is then to derive the combination (p^h, p^e, R) that clears all three asset markets. The key element to derive is the current real short-term rate R_t [12]. Given R_t, we derive the policy functions of all agents given initial $\left(\frac{\bar{\omega}}{\bar{y}}\right)_i$. We then find the values of $\left(p^e, p^h\right)$, which exclusively enter into agent wealth and not expected returns (since they are exogenous inputs), that clear the housing and equity market. Then, using derived $\left(p^e, p^h\right)$, we check the final bonds market and see if the model over- or under-saves vs. the data. We continually loop over R_t until we find the rate at which all markets clear.

We find that, in equilibrium, $R = 1.175$. This is essentially identical to data-based expectations for the agents from $t+1$ onwards, meaning that this is a reasonable equilibrium rate of return to clear the market. Equilibrium (p^e, p^h) imply per-capita holdings of equity and housing of $\$20,800$ and $\$100,800$, respectively.

C. *Comparing Model to the Data*

Once we have derived the market-clearing prices, we can now examine the model-implied equilibrium choices for each cohort, and compare to the data, as is performed in Figure 8 below.

The upper two quadrants plot cohort-level shares of equity and housing, with the y-axis representing the share of the aggregate supply of the asset held by each cohort. The model is able to capture cross-sectional holdings across cohorts. Consistent with the data, model-implied assets quickly accumulate into middle-age as they take advantage of the peak in the age profile of income to accumulate buffer savings for later life. This is then drawn down over the remainder of their life so as to allow themselves to smooth consumption and to de-risk by re-allocating towards bonds. Of course, their fast accumulation of housing into middle age is enhanced by being able to borrow against it.

The lower left quadrant then conveys the nominal bond holdings in the model and data. It shows that, while the shape over the life cycle is close to the one observed in the data (i.e. agents borrow to invest in risky assets when young due to high risk tolerance, but soon diversify to safer bonds as they grow older and richer), the model implies too extreme movements in bond portfolio holdings.

Finally, turning to consumption, we get a hump-shaped life-cycle profile that is consistent with the data. Intuitively, the rising profile into middle-age is due to the borrowing constraint

[12] To be specific, we fix the expected real short-term return on the bond, which is stochastic given inflation uncertainty

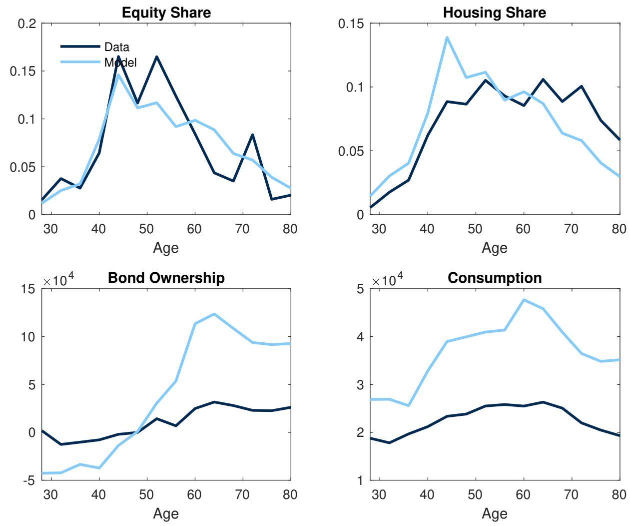

Figure 8. Optimal Choices: Model vs. Data: Italy

causing there to be an inefficiently low level of consumption when young that would otherwise allow for better smoothing into middle age. Then, consumption declines into old age because the non-negligible probability of dying disincentivizes holding any wealth. This, conditional on subsequent survival, reduces consumption.

However, the scale of consumption in our model is too high. This can be reasonably fixed by either a) moving from pre-tax to post-tax income and b) including imputed rents within consumption.

Now, it could be the case that we are getting the right cross-sectional asset ownership across cohorts because of an appropriate movement in model-implied wealth while still not getting correct portfolio shares at the cohort level. Figure 9 below suggests that this indeed is the case. The equity shares are reasonably well-matched, but housing shares, despite getting the right downward slope as cohorts age, exhibits too extreme variation across cohorts. More specifically, young agents in the model undertake excessive leveraged investments in housing, and then de-risk considerably as they age, re-allocating away from housing and towards bonds, hence why bonds also have a steep profile in the model. On the other hand, in the data, housing portfolio shares are quite sticky, suggesting that agents do not necessarily de-risk their portfolio.

Turning to wealth, we fit this reasonably well, which is a result of us finding equilibrium asset prices that imply aggregate asset values that are consistent with the data. On closer inspection, though, one can see that the model implies too high a rate of wealth accumulation when young and wealth decumulation when old. This suggests that the variation in savings rates may be a little too large in our model vs. the data, which indeed appears true when comparing the slopes in the lower left figure.

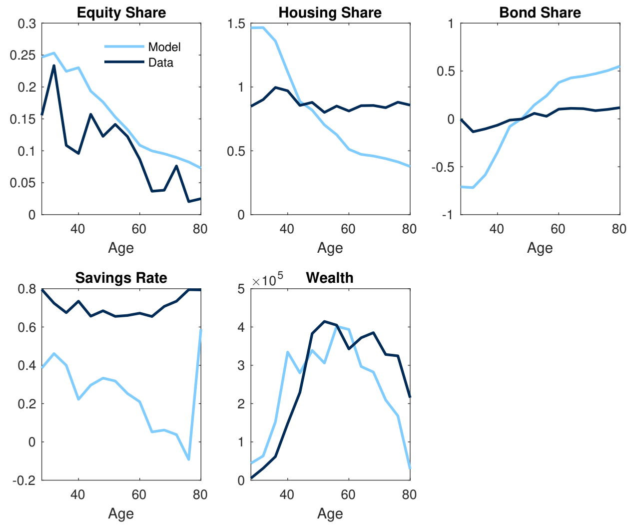

Figure 9. Portfolio Shares: Model vs. Data: Italy - Full Sample

Essentially, the discrepancy merely boils down to the model implying excessive savings in housing shares, which implies excessive leverage when young. But it may be because the collateral constraint is only relevant for agents that are not asset-rich as otherwise taking out a mortgage is not required to invest in housing. To this end, I focus on the average shares and wealth levels of those in the bottom 90% of the wealth distribution within each cohort. Results are in Figure 10 below. As you can see, the data profile of housing and bonds becomes more sloped, suggesting that leverage is more concentrated amongst the less wealthy. However, equity shares in the model are too high, meaning households don't use equity as much as they would in the model for diversification purposes.

To summarize, the model appears to capture cross-sectional variation in wealth and asset ownership across cohorts, as well as consumption profiles, quite well. However, it falls short in portfolio shares by implying excessive variation in household portfolio shares with age.

V. Monetary policy shocks and consumption elasticity

In this section we describe how monetary policy shocks impact asset allocation and consumption choices of households. Monetary policy shocks, in our framework, are modelled as an annualised 1.5% reduction in the expected real rate of return on bonds for next period. We are hence assuming that shocks are fully transitory, lasting only one period. Running comparative statics requires us to make assumptions on how this shock impacts the expected returns of other assets (housing/equity). In this section, we consider two extreme scenarios: (i) the change in the expected bond return does not affect other assets' expected returns and (ii) other assets' expected returns change one-for-one with the expected bond return. In the next section, we

Figure 10. Portfolio Shares: Model vs. Data: Italy - Bottom 90%

will also base the assumption on empirical moments.

A. Scenario 1

In the first scenario, the expected return on bonds, R_b, is not accompanied by any change in R_e or R_h. A reduction in the expected bond rate leads households to rebalance their portfolio share. In fact, leverage becomes cheaper and bond ownership is now less attractive. Figure 11 shows that, following a reduction in R_b, households increase their leverage and reduce their investment in bonds. This is particularly evident for younger households. For low levels of wealth, due to the borrowing constraint, households only invest in housing. Consequently, the increase in leveraged investment will result in an even larger housing share, as documented by Figure 13. At higher levels of wealth, bonds are instead used as an asset (and not a liability anymore). As bond investment is now less attractive, households are going to reduce their bond share in favor of equity and housing, as Figure 11 shows.

Older households exhibit higher levels of bond investing compared to young households and, notwithstanding the reduction in the bond rate, they barely change their portfolio. That is because they are particularly adverse to taking risk (given the possibility of dying over the next few periods), meaning their portfolio shares are relatively insensitive to changes in R_b.

Beyond portfolio choices, the reduction in R_b also leads to a change in the savings margin, and so consumption.

Consumption responses to monetary policy shocks notably vary across cohorts and across wealth levels, as documented by Figure 16. Inspecting the consumption response of young

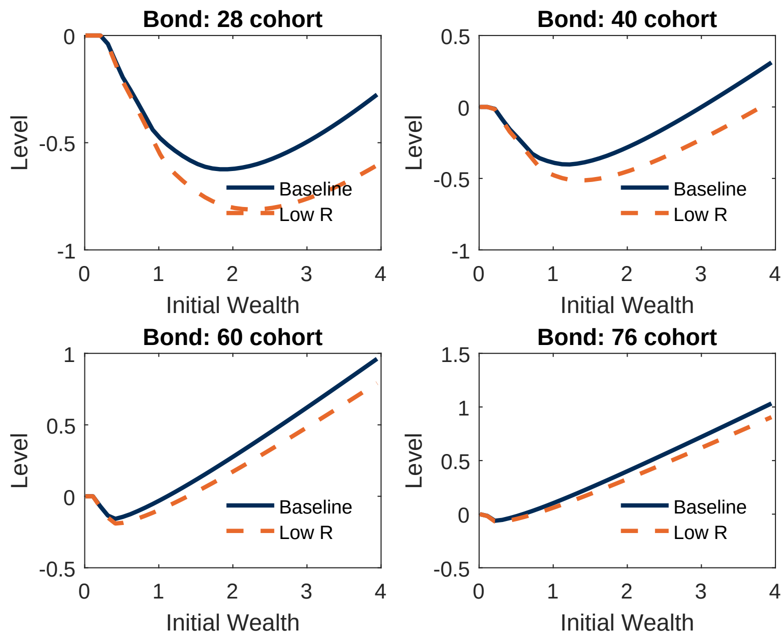

Figure 11. Policy function: Bond

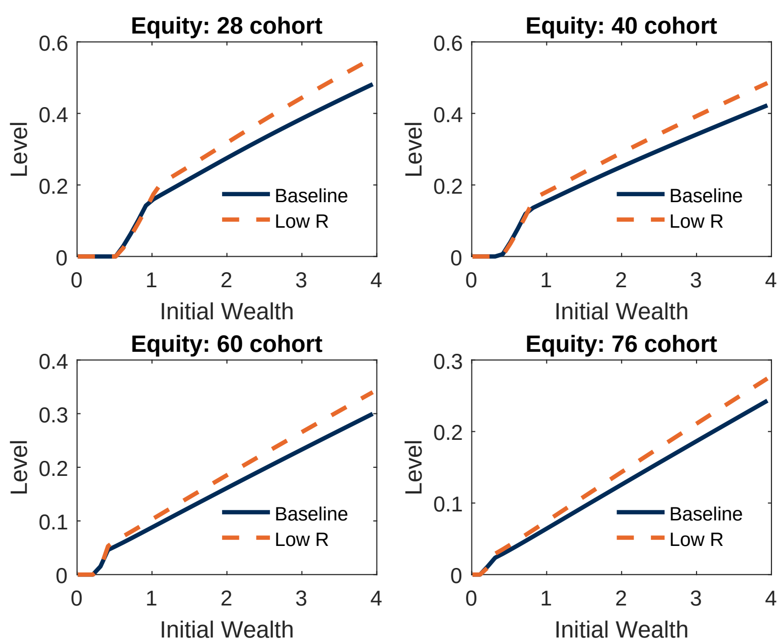

Figure 12. Policy function: Equity

households, we notice a V-shaped pattern. For low levels of wealth, households are consuming all their wealth and hence are not going to respond to the interest rate change. For slightly higher wealth levels we instead observe a fall in consumption of as much as 1%. The reduction in

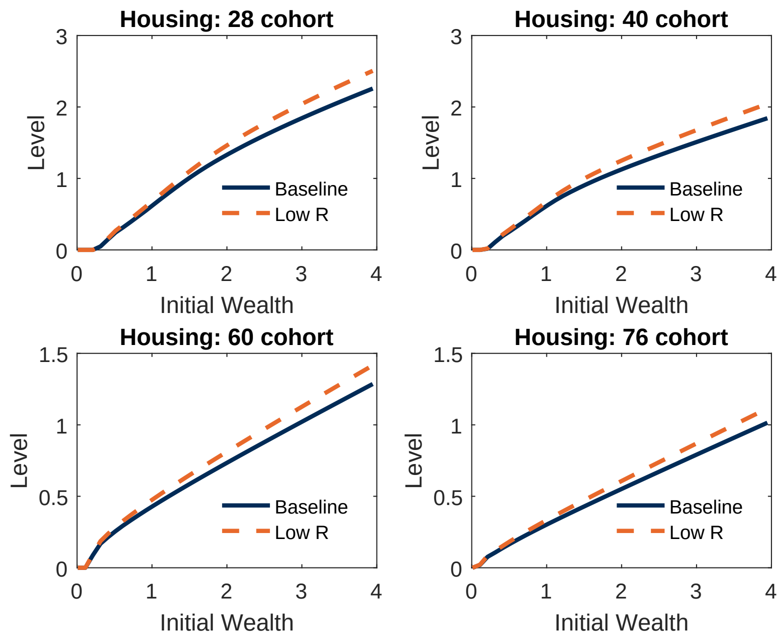

Figure 13. Policy function: Housing

consumption comes from the household's desire to increase their leveraged investment. In fact, in our model, leveraged investment requires a down-payment due to the borrowing constraint, and therefore requires households to reduce current consumption: the spread between housing and bonds is high enough to induce households to forego current consumption in order to take on more risk. For higher levels of wealth, the consumption response turns positive as the benefit from lower borrowing costs more than offsets the cost of increased leverage. The consumption response reaches its peak of around 0.8% at wealth-to-income levels of 1.5 and then slowly decreases for higher levels of wealth. That is due to the fact that households, for higher wealth levels, exhibit lower leverage or become bond net savers, meaning that the reduction in R_b generates small or negative income effects.

To illustrate the reasons behind the V-shape pattern, Figure 14 and 15 show bond and housing holdings as a percentage of household wealth before and after the shocks. We notice that for wealth-to-income levels between 0.5 and 1, households are starting to leverage up, and the reduction in the bond rate accelerates this process. The build-up of the leverage investment requires - as discussed earlier on - a down-payment. This process is creating the unusual consumption response.

Older households exhibit a similar V-shape in their consumption response. However, the level of response is different and indeed, for medium and high wealth-to-income ratios, the consumption elasticity is negative. That is explained by the negative income effect coming from the reduction in bond returns. In fact, older households are bond net savers, and since

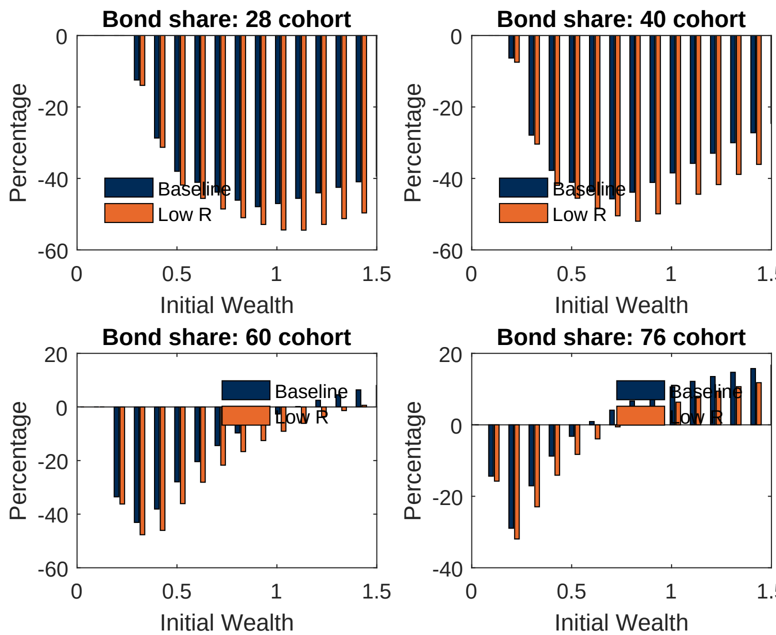

Figure 14. Bond holdings as share of wealth

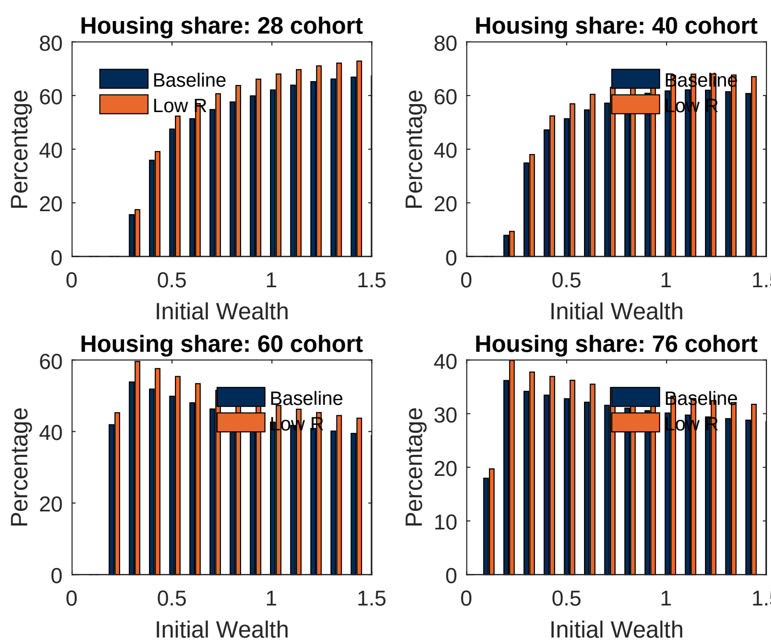

Figure 15. Housing holdings as share of wealth

they do not switch their portfolio following the rate change, their consumption is negatively affected by the reduction in the bond rate.

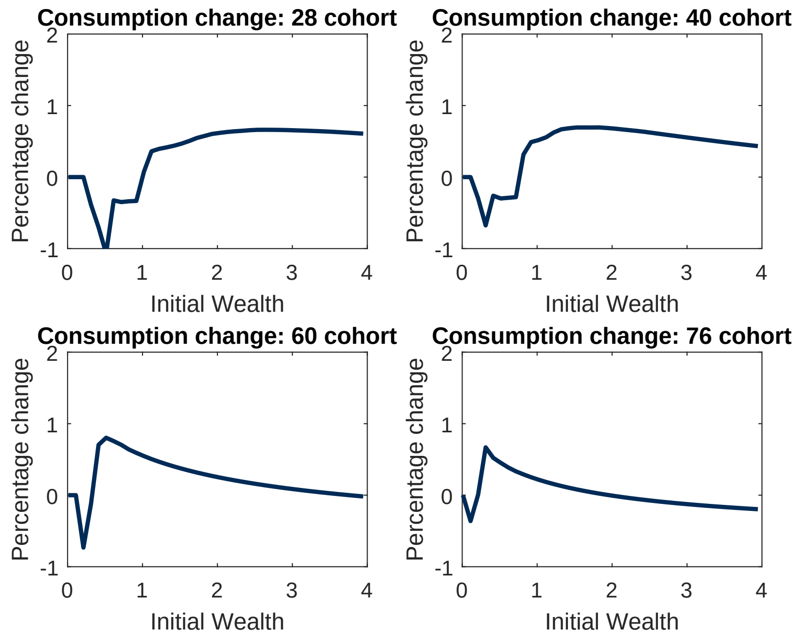

Figure 16. Consumption response

B. Scenario 2

In the second scenario we reduce the expected return on bonds, equity and housing by 1.5% (annualised). In this case, the portfolio reallocation is negligible as the difference in expected returns among assets - which led to portfolio switching in the previous scenario - is no longer relevant. However, we still notice notable changes in consumption, as documented by Figure 17. In particular, we observe how the shape of the consumption response is similar to the case in which bonds are the only available asset (see Appendix A), meaning consumption responses are in fact significantly different from the previous scenario, especially for low wealth levels. In fact, young households do not increase their leveraged investment, meaning we no longer see the required drop in consumption to finance the down-payment that was documented in the previous case.

For levels of wealth where the consumption response of young households is positive in both scenarios (wealth-to-income ratios of 1 or more), we notice that the elasticity is now much more muted. It also becomes negative for high levels of wealth, a phenomenon not observed in the previous scenario. Older households also tend to increase their consumption when their initial wealth is very low. However, the response is negative for most of the wealth levels we use in the exercise.

All in all, the consumption responses to monetary policy shocks vary significantly by the age and wealth of a household. In fact, age and wealth can not just change the scale, but also

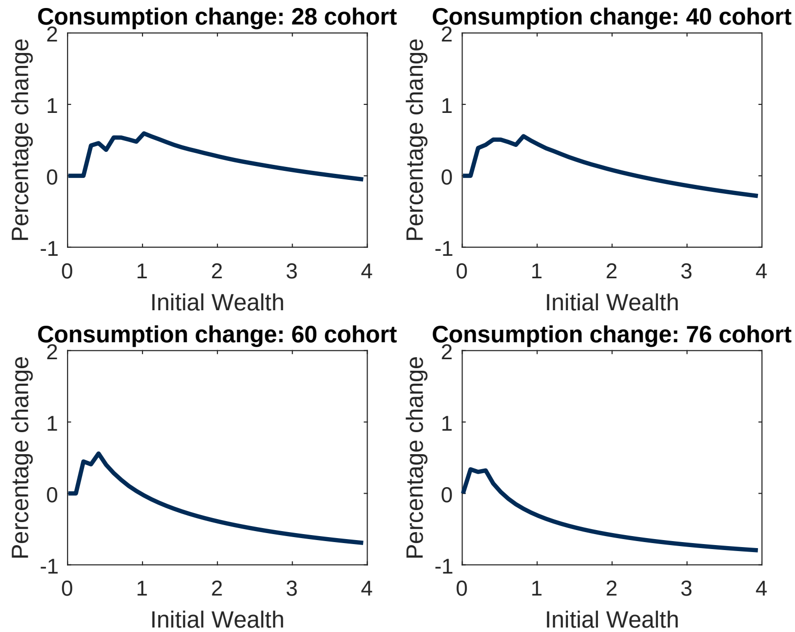

Figure 17. Consumption response

the sign of the consumption response of a given household. This suggests that the age-wealth distribution in the economy is important for the aggregate responses to monetary policy shocks.

Furthermore, we also notice substantial differences between the two scenarios we used in this section. This means that the way monetary policy affects expected returns on other assets is also a key ingredient of the transmission of monetary policy.

VI. Monetary Policy Shocks: Model-Implied Household Responses

We now know how households, for a given age and initial wealth, will react to different monetary policy shocks. What prediction does it generate for how households in our data react to the shock? To do this, we merely need to derive the model-implied initial wealth for each sub-cohort, calculated using Equation 1 above. As equilibrium prices (p^E, p^H) change after the shock, the response of the households will be a combination of 1) movements in policy functions for given wealth levels and 2) movements in initial wealth of households caused by changes in equilibrium prices.

The first step is to illustrate where each of the cohorts stand in terms of initial wealth. In the model, what matters for policy functions is not what their wealth levels are, but rather wealth levels relative to their current income [13]. Figure 18 below plots the state variable for

[13] To be more specific, it is wealth relative to permanent component of income. See Appendix B for details on estimation

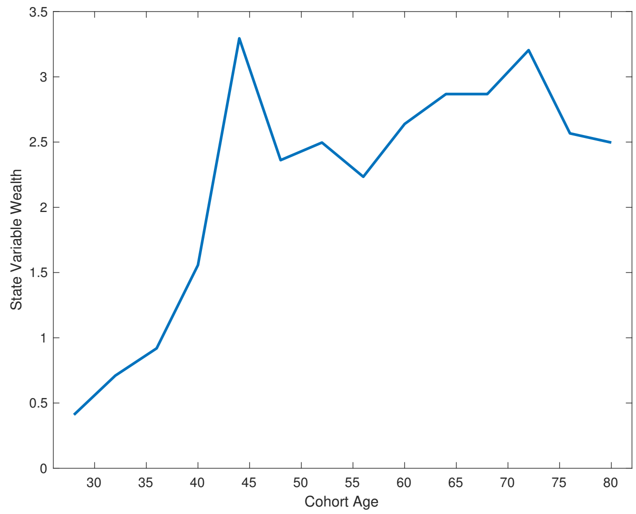

Figure 18. Initial Wealth of Cohorts

Figure 19. Changes in Portfolio/Wealth to MPS Scenario 1

each cohort. As expected, young cohorts begin with low wealth, but it quickly elevates towards 2.5-3 for older cohorts. As the policy functions are very non-linear within this region of wealth, it is suggestive that heterogeneity across cohorts matters for responses.

We will now consider three different scenarios for the MPS: two that are discussed, and a final scenario that presumes unchanged asset prices.

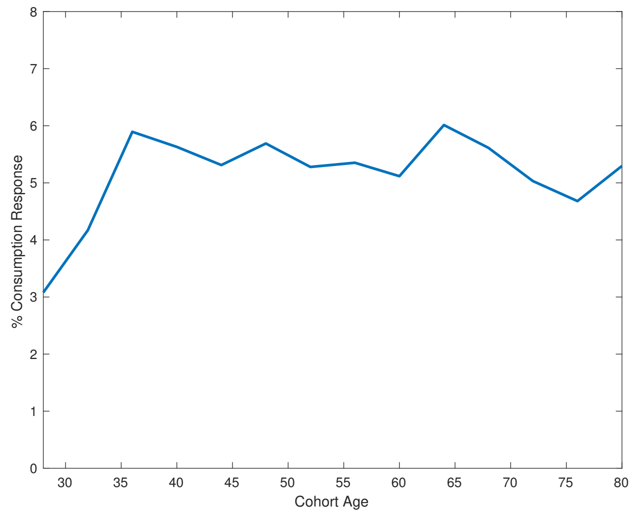

Figure 20. Consumption Responses To MPS Scenario 1

A. Scenario 1

Figure 19 conveys the results for portfolio choices and wealth. We plot each cohort's weighted average nominal outcomes of portfolio choices and initial wealth both before and after the shock. It is immediately clear that falling R^b induces a reallocation by all agents away from bonds and towards equity and housing. This is particularly strong for the young cohorts, who exploit the temporary increase in the housing risk premium to increase leveraged investments, as documented in the previous section. For the older cohorts, however, the re-allocation is less pronounced. Their higher risk aversion makes them less willing to move their savings away from safer bonds.

Notice, though, that investments in equity and housing increase by more than the divestments in bonds. This is because all cohorts experience a positive wealth effect. Why? Demand for both assets increases for all cohorts due to the desire to re-allocate. Prices must then rise (by 16% (housing) and 28% (equity)) in order for the value of supply to meet the amount demanded. As younger cohorts hold leveraged portfolios initially, their wealth increases by more in % terms, hence why their re-allocation is considerably more pronounced. This enhances the heterogeneity of responses.

We now turn to consumption responses, which are conveyed in Figure 20. For young cohorts at such low initial wealth levels, policy functions imply that consumption falls. Instead, consumption rises. Why? Young cohorts experience a positive wealth effect on their holdings of housing, which more than counter-acts their wish to substitute away from consumption today in order to exploit the superior returns on the leveraged housing investment. Indeed, this wealth effect is so strong that the cohort aged 36 raises consumption by 6%.

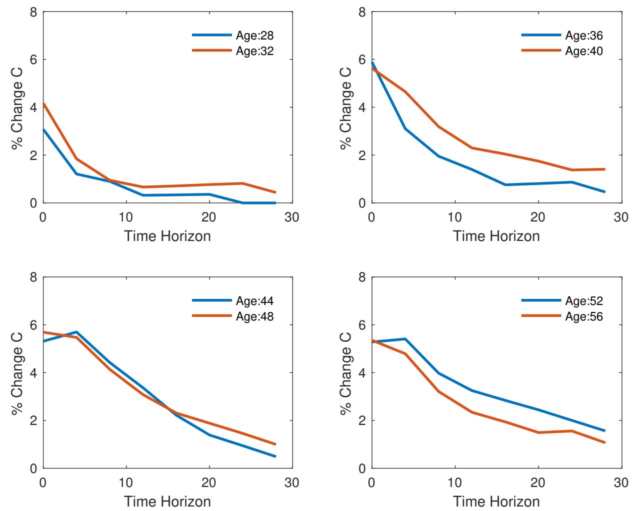

Figure 21. Consumption Path % Response to Lower R (1.175 to 1.115): Italy

As agents age, their portfolio shares in housing and equity begin to decline as they reallocate gradually towards bonds. This reduces the positive wealth effect, hence why consumption responses are smaller. This is accentuated by the fact that policy functions for older agents react less positively to a lower R^b. As they have a lower risk appetite, they don't see much of an advantage in investing in more profitable leveraged housing investments.

The heterogeneity across cohorts is also apparent when we examine the consumption plans. Figure 21 lays out the % change in the consumption plan over the horizon after the shock occurs. To do this, we track the evolution of wealth assuming that realized equal expected returns, where we use the policy functions of agents over their life cycle to understand how their portfolio choice and consumption move over time.

This shows that the heterogeneity is more stark once you look at consumption paths. Clearly, the impact is more persistent for older vs. younger cohorts. For example, agents aged 28 exhibit effects for only one period whereas those aged 48-52 experience a 2% increase in consumption 28 years out. Consequently, this fall in R^b doesn't just generate a substitution of consumption into today (although this is happening since path responses are downward-sloping), but the positive wealth effect caused by i) switching into assets with higher expected returns and ii) increases in asset prices means the impact on consumption is persistent. Persistence rises with age because wealth becomes increasingly important for financing consumption plans due to a declining future income profile. Thus, when older cohorts experience a given % change in wealth, it raises the consumption path by more.

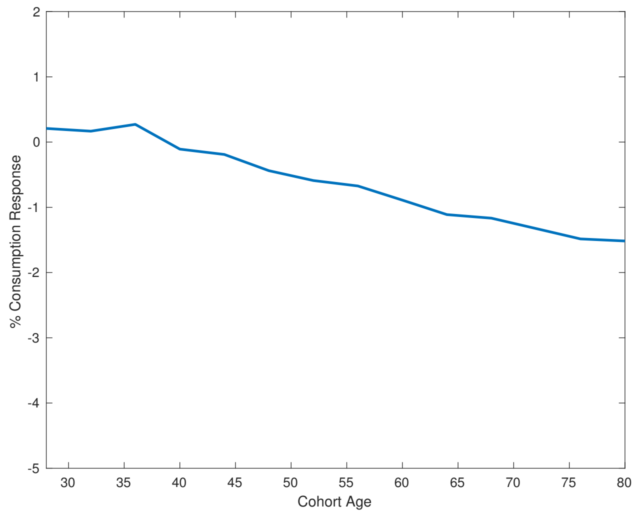

Figure 22. Consumption Response to Lower R^b, R^e, R^h (by 1.5% annually)

B. Scenario 2

An alternative formulation is to state that expected returns on equity and housing (R^e, R^h) fall by the same amount as R^b. In other words, the monetary policy shock has no impact on the risk premium.

The result of this is that there is no significant desire to re-allocate the portfolio. As a result, the demand for assets is largely unaffected, meaning equilibrium asset prices remain essentially unchanged. [14]

Consequently, movements in consumption are mostly a function of changes in the policy function. The current response of consumption is shown in Figure 22. Consistent with the discussion in the previous section, consumption responses are much more muted, even turning negative for older cohorts. For all agents, their original consumption plan has become more expensive relative to current wealth due to lower expected returns. This, however, is more relevant for older cohorts because they rely more on current wealth to finance future consumption because a) they are richer, b) they expect a declining income profile, and c) they face a higher probability of dying. The 3% drop in p^H strengthens this negative effect. As a result, consumption falls for older cohorts. [15]

More intuition can be obtained from looking at the changes in the paths (Figure 23). As you can see, the impact becomes more negative and persistent as cohorts age. The fall in p^h is more hurtful to agents who rely more on current wealth to finance future consumption. This is

[14] p^h falls marginally due to declining demand from younger cohorts as they decide to save less

[15] This is exacerbated by the more immediate probability of death. With a smaller remaining life expectancy, it is more difficult to smooth out one period of low returns

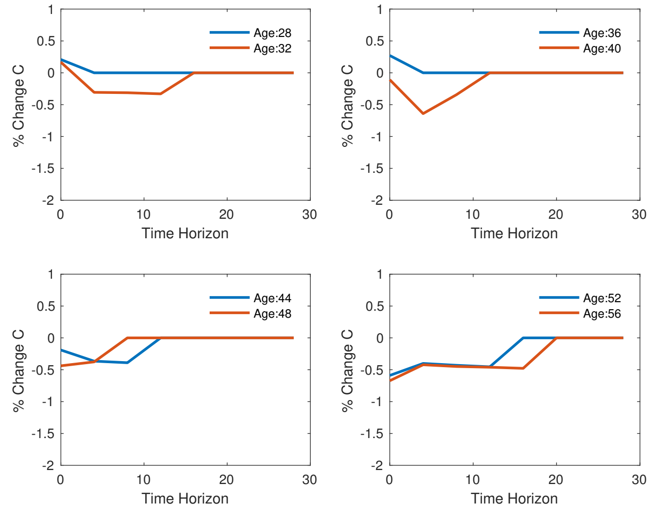

Figure 23. Consumption Response to Lower R^b, R^e, R^h (by 1.5% annually)

true for older, but not younger, cohorts. Consequently, for younger cohorts, it is predominantly a switch away from consumption the next few periods into today, whereas for the older cohorts there is also a more significant negative wealth effect that lowers consumption further out into the horizon.

C. Scenario 3: Constant Asset Prices

Thus, far, we have taken an arbitrary stand on how expected returns R^e, R^h are affected. A more desirable approach is to discipline our choice of R^e, R^h so that model outcomes match empirical outcomes. In our case, we find it most preferrable to ensure that model-implied price changes correspond to how they react in the data. This is done in the following way:

- Change R^b
- Subsequently adjust R^e and R^h to match asset price movements empirically while clearing the equity and housing markets

For now, we assume *prices remain constant*. To achieve this, we reduce expected returns (over 4 years) by the following:

$$\Delta R^b = 1.5\%; \Delta R^e = 2.1\%; \Delta R^h = 1.0\%$$

By construction, this scenario generates no wealth effects, and is exclusively driven by movements in policy functions. The reasoning is very similar to Scenario 2. The only difference is that, in Scenario 2, there was a mild drop in p^H that generated a negative wealth effect for

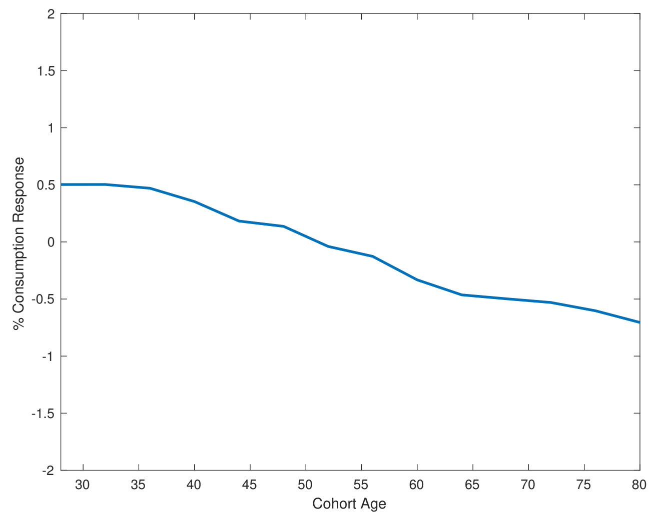

Figure 24. Consumption Response to Lower R^b, R^e, R^h s.t. (p^H, p^E) Unchanged

all agents. As a result, the consumption response shifts upwards somewhat, although the shape is unchanged.

Next, we consider how consumption paths respond to this shock. It is illustrated in Figure 24. Interestingly, one can observe that the impact on consumption is purely transitory. Consequently, price effects are crucial for generating persistence in responses.

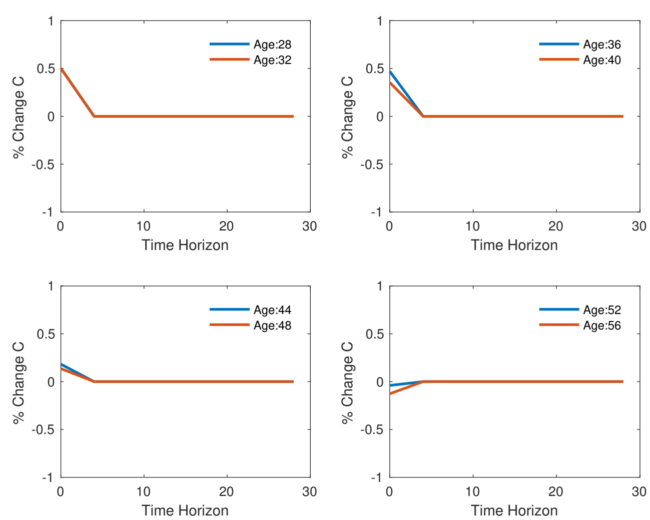

Figure 25. Consumption Path to Lower R^b, R^e, R^h s.t. (p^H, p^E) Unchanged

VII. Conclusion

In this paper we attempt to understand the degree of heterogeneity in portfolio and consumption responses across cohorts to a common monetary policy shock. We combine micro-level data from the HFCS with the aggregate Euro Area Economic Accounts in order to derive initial household endowments and aggregate asset supplies. We calibrate the model to the data using the temporary equilibrium concept that utilizes data-based expectations. Sarting with the equilibrium prices and expected returns, monetary policy shocks generate significant heterogeneity in consumption responses, taking a form that depends on the scenario we consider. If the scenario is such that price and wealth effects are absent, young households increase, while older households decrease, consumption. In this case, the monetary policy effect is fully transitory. If we analyze scenarios that involve an increase in equilibrium prices, the wealth effect mitigates the heterogeneity in current consumption responses, but introduces persistence in responses that increases significantly with age.

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A. Appendix: Baseline model and the introduction of different factors

We discuss how key ingredients of the model affect the elasticity of consumption to interest rate changes. We first describe how the model functions in its baseline form: income is deterministic and only changes with age, the only available asset is a risk free bond (where we set the variance of nominal and real returns to zero), and the only stochastic component of the model is uncertainty about death. We then sequentially add income uncertainty in the form of temporary shocks and permanent shocks. The non-availability of housing investment implies that in all three cases the household faces a zero-borrowing constraint. Figure 26 and 27 show the policy function and consumption response (in percentage change) to a one-period 1.5% change in the level of the bond rate.

In the baseline model, for low levels of wealth, consumption does not change as the borrowing constraint is binding. As soon as the borrowing constraint stops binding, the consumption response is strongest for all cohorts. The elasticity then decays as wealth increases. For younger households the overall effect is largest. This is mainly due to the income age profile and death probability. For young cohorts, the anticipation of future income growth induces them to increase consumption when the interest rate is reduced. However, for older cohorts with initial wealth more than two times their current permanent income, they decide to reduce their consumption. That is mainly due to the possibility of dying, which makes the substitution effect weaker than the income effect, as you will consume all your wealth next period. Therefore, even in these simpler model formulations, demographic factors and wealth levels shape the response of consumption to interest rate shocks. These factors will also change the direction of consumption responses.

The introduction of temporary risk reduces the level of consumption and the sensitivity to interest rate change for unconstrained agents. However, we observe that households start saving at lower levels of wealth compared to the baseline model due to a precautionary savings motive. Once we introduce temporary income shock risk, savings rise with the precautionary motive, and the income effect becomes more important. The difference between the baseline and the model with temporary shocks is in fact particularly visible for younger cohorts where the precautionary savings motive is strongest.

The introduction of permanent shocks (in addition to temporary shocks) leads to a sizable reduction in the level of consumption - as evidenced by the policy function. The sensitivity to consumption is also notably lower than in the previous two models. Moreover, for a large region of wealth levels, the response of consumption is negative. The fact that consumption now decreases in response to a fall in interest rates at high wealth levels means that permanent shock uncertainty is particularly relevant for aggregate dynamics.

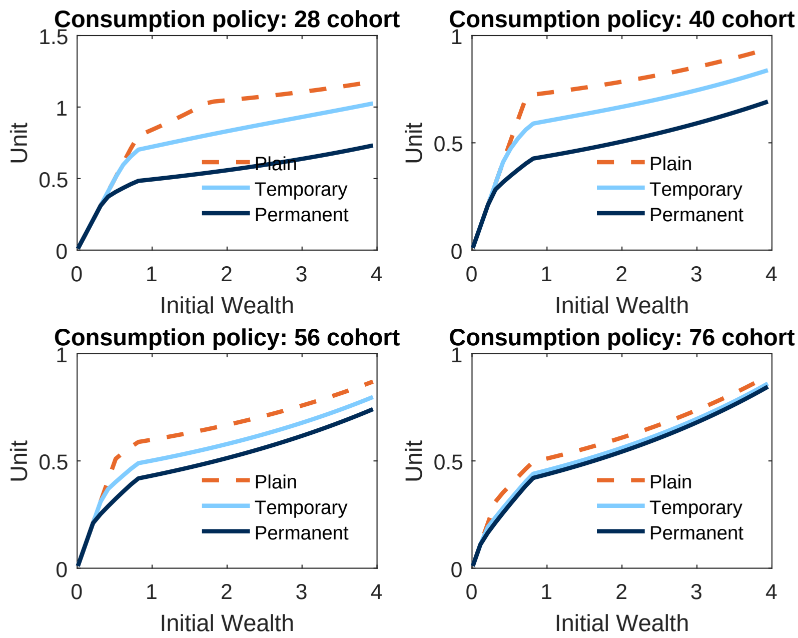

Figure 26. Policy function: Consumption

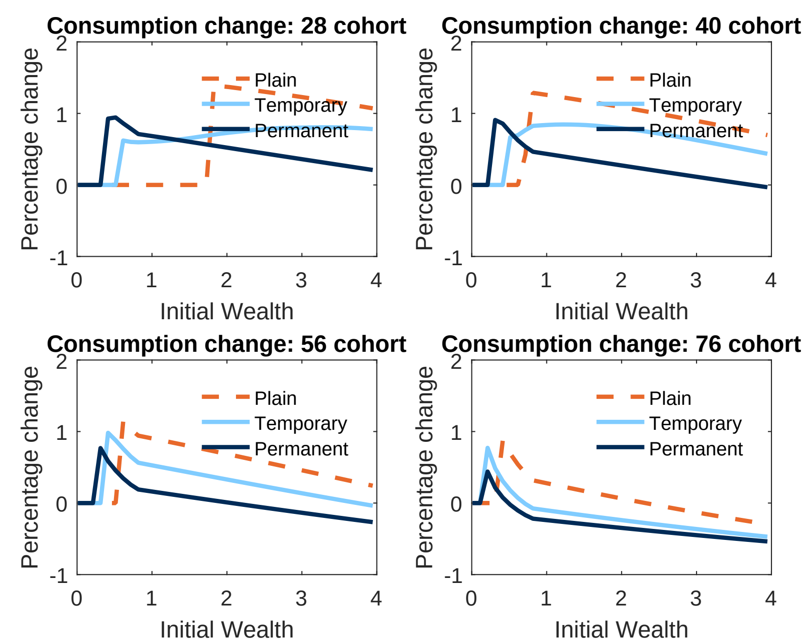

Figure 27. Consumption elasticity to interest rate changes

Introducing additional factors, such as (i) macroeconomic uncertainty in inflation and aggregate growth, (ii) inheritance and (iii) risky short-term rates has little effect on both consumption levels and consumption responses to interest rate shocks.

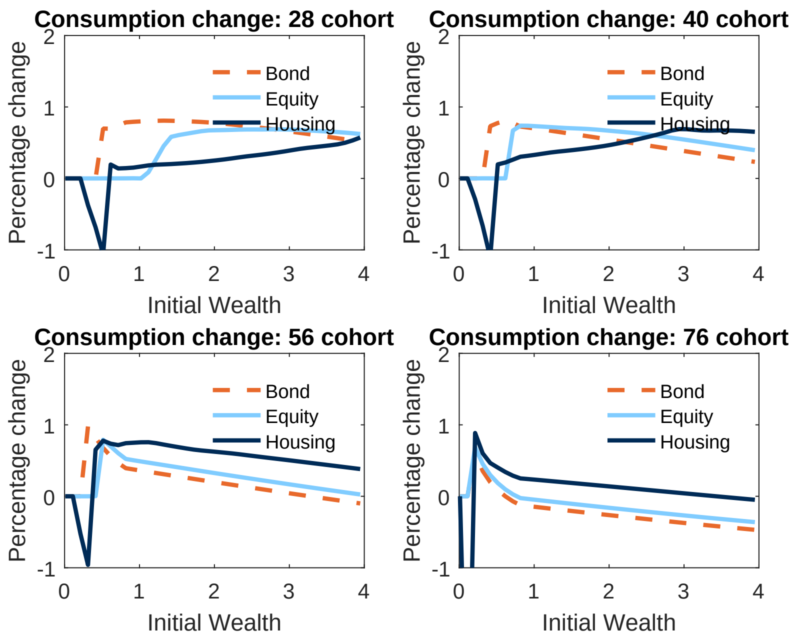

Figure 28. Consumption elasticity to interest rate changes

A. Multiple assets

We subsequently introduced additional assets: equity and housing. In this paragraph we compare consumption responses to a Scenario 1 monetary policy shock under three different models with different assets available: (i) bonds only, (ii) bonds and equity, and (iii) bonds and housing.

We start by adding equity as a possible investment choice for the households. The availability of equity induces a slight increase in the level of consumption but does not change the consumption response to rate shocks in a substantial way.

The introduction of housing instead affects the consumption response significantly. In fact, for low levels of wealth, the reduction in the interest rate induces households to increase their leveraged investment position. They are going to forego current consumption to invest in housing and increase leverage. For higher levels of wealth, the response is positive because the income effect is small as the amount invested in bonds is smaller. Households are going to increase their investment in housing, but overall they are going to decrease savings as they don't want to take on too much risk.

B. Appendix: Income Process

The process for income is:

$$y^i_{a,t} = G_t A_t P^i_t U^i_t$$

Figure 29. Consumption elasticity to interest rate changes

where

- G_t is the common aggregate growth component
- A_t^a is the deterministic age profile
- P_t^i is the permanent idiosyncratic shock
- U_t^i is the transitory idiosyncratic shock

So how do we calculate these components? To estimate A_t, we take the average of income within each age cohort, $\int_0^1 y_{a,t}^i$, and assuming a continuum of agents (as we do by using the integral), and knowing that P_t^i and U_t^i are independent and i.i.d. across agents, $\int_0^1 y_{a,t}^i = \int_0^1 G_t A_t^a P_t^i U_t^i = G_t A_t^a \int_0^1 P_t^i U_t^i = G_t A_t^a$. Then, to isolate A_t^a, we normalize $A_t^{young} = 1$ for the youngest cohort, and then we get that $A_t^a / A_t^{\text{young}} = \frac{\int_0^1 y_{a,t}}{\int_0^1 y_{yo,t}^i} = A_t^a$. We presume no cohort effects so that the deterministic age profile is time invariant. For P_t^i and U_t^i, they are drawn i.i.d. from the following distributions:

- $logU_t^i \sim N(-\sigma_u^2/2, \sigma_u^2)$ s.t. $E\left[U_t^i\right] = 1$
- $\Delta logP_t^i \sim N(-\sigma_u^2/2, \sigma_P^2)$ s.t. $E\left[\frac{P_t^i}{P_{t-1}^i}\right] = 1$

where (σ_u^2, σ_P^2) are taken from the data. Finally, $\Delta logG_t$ is drawn from the multivariate distribution with returns.

All these steps allow use to estimate the income of any agent conditional on realized shocks (U_t^i, P_t^i):

$$y_{a,t}^i = G_t A^a P_t^i U_t^i$$

as well as the permanent component of income:

$$\hat{y}_{a,t}^i = G_t A^a P_t^i$$

Moreover, we can now estimate two ratios that will prove to be very important when doing the maximisation problem when the problem is normalized to one state variable:

- $\frac{y_{a+1,t+1}^i}{\hat{y}_{a+1,t+1}^i} = U_{t+1}^i$
- $\frac{\hat{y}_{a+1,t+1}^i}{\hat{y}_{a,t}^i} = \left(\frac{A^{a+1}}{A^a}\right)\left(\frac{G_{t+1}}{G_t}\right)\left(\frac{P_{t+1}^i}{P_t^i}\right) = \left(\frac{A^{a+1}}{A^a}\right) exp(\Delta log G_{t+1}) exp(\Delta log P_{t+1}^i)$

The former refers to the transitory component of income, whereas the latter evaluates the evolution of the permanent component of income. The permanent component follows the path of the deterministic age profile of income, and also grows at the aggregate GDP rate, all of which is scaled by the change in the level of P.

C. Appendix: Market Clearing Process

In our problem where there is no need for clearing of the housing services market, we essentially have three equations and three unknowns (the fourth market is trivially cleared because of the open economy level of consumption), of the following:

1. $p^e(1+f^e) = \sum_i \alpha^e\left(\frac{\omega(i)}{\hat{y}(i)}; i, q\right)\omega_i$
2. $p^h(1+f^h) = \sum_i \alpha^h\left(\frac{\omega(i)}{\hat{y}(i)}; i, q\right)\omega_i$
3. $p^h(1+f^h) + p^e(1+f^e) + D = \sum_i S\left(\frac{\omega(i)}{\hat{y}(i)}; i, q\right)\omega(i)$

In order to solve this system of three equations and three unknowns (p^e, p^h, q), our strategy loops over q. Suppose we know q. Then we choose the values of (p^e, p^h) that clear the housing and equity market. Finding p^e and p^h is straightforward because the prices impact the wealth of agents, but not the policy functions α^e, α^h as returns expectations are invariant to current asset prices. Then, once we find (p^e, p^h), we use the final equation to try and understand what is excess savings under chosen q. If excess savings is positive, we raise q in the next loop, and vice versa, continuing until we clear the bond market.

So we now flesh out the process more explicitly here. The first thing we must do is find (p^e, p^h). So we firstly fix an initial guess of (p^e, p^h). Then we need to evaluate ultimately the state variable values for each of the cohorts. We start by calculating the initial wealth given these prices:

$$\omega_t = p^h \theta_t^h + p^e \theta_t^e + b_t + y_t$$

where $b_t = b_{t-1}(E_{t-1}^{\text{obj}}[R_t]\, E_{t-1}^{\text{obj}}[\Pi_t])$. $E^{\text{obj}}[.]$ refers to objective, not subjective expectations. [16]. This adjustment to nominal bonds is because holdings are nominal, and will earn a nominal rate of return when moving from $t-1$ to t. We presume here that they earn the expected nominal return. However, because we would prefer to attach some "real" meaning to the price of p^h, it would be appropriate to initially scale the wealth equation. This is ultimately an arbitrary choice. We decide upon:

$$\hat{y}_t^{young} \equiv \hat{y}_t^{yo} = \hat{y}_{t-1}^{yo} E_{t-1}^{\text{obj}}[G_t/G_{t-1}]\, E_{t-1}^{\text{obj}}[\Pi_t]$$

where $\hat{y}_{t-1}^{yo}$ is the average income we observe of the youngest cohort in 2010. Under our assumptions, cohort-level income grows at the nominal aggregate growth rate. This is because, given a continuum of agents at each cohort, $\int_0^1 P_t^i U_t^i = 1$, meaning what we observe as the average also averages out the shocks, so that (where $\tilde{P}_t$ is agg. price level) $\hat{y}_t^{yo} = A_t^{yo} G_t \tilde{P}_t \int_0^1 P_t^i U_t^i = A_t^{yo} G_t \tilde{P}_t = G_t \tilde{P}_t$ as $A_t^{yo} = 1$, where A refers to the age profile of income. We presume nominal aggregate growth is the expected rate. Then, we have that:

$$\frac{\omega_t}{\hat{y}_t^{yo}} = \left(\frac{p^h}{\hat{y}_t^{yo}}\right)\theta_t^h + \left(\frac{p^e}{\hat{y}_t^{yo}}\right)\theta_t^e + \frac{b_t}{\hat{y}_t^{yo}} + \frac{y_t}{\hat{y}_t^{yo}}$$

Thus, the asset prices I find are actually values relative to average income of youngest cohort in period t. We are left to decipher the values for the last two terms:

- $\frac{b_t}{\hat{y}_t^{yo}} = \frac{b_t}{\hat{y}_{t-1}^{yo}}\left(\frac{\hat{y}_{t-1}^{yo}}{\hat{y}_t^{yo}}\right) = \left(\frac{b_{t-1}}{\hat{y}_{t-1}^{yo}}\right)\left(\frac{E_{t-1}^{\text{obj}}[R_t]}{E_{t-1}^{\text{obj}}[G_t/G_{t-1}]}\right)$
- $\frac{y_t^i}{\hat{y}_t^{yo}} = A_t P_t^i U_t^i$

Both of these can be easily calculated, since $(\hat{y}_{t-1}^{yo}, b_{t-1})$ are observed, A_t is deterministic, and a series of choices on values for (P_t^i, U_t^i) for each cohort are chosen by us in order to approximate the distribution and obtain accurate cohort-level average choices. The only assumption we make here is that realized aggregate growth and real returns are the expected values.

Now, given this adjustment to provide meaning to the asset price, we now need to subsequently adjust to write wealth in terms of the relevant state variable. This becomes:

[16] **NOTE:** When we are considering ACTUAL evolution of variables, essentially wealth from $t-1$ to t here, we use actual returns expectations. Subjective expectations are used for when agents are forming expectations of future realizations when optimizing

$$\frac{\omega_t}{\hat{y}^i_t} = \frac{\omega_t}{\hat{y}^{yo}_t}\left(\frac{\hat{y}^{yo}_t}{\hat{y}^i_t}\right) = \frac{\omega_t}{\hat{y}^{yo}_t}\left(\frac{1}{A_t P^i_t}\right)$$

Note here that the state variable is increasing in U^i_t and decreasing in P^i_t In the baseline, I assume $U^i_t = P^i_t = 1$ and so there is one representative agent per cohort. This will be extended below.

Now, the estimated state variable for each cohort is then matched with the closest value on the state variable grid. We then know the amount invested in both housing and equity, which is subsequently multiplied by $A_t P^i_t$ to get it back into the units of the asset prices. We then check what is excess demand for each of the assets, where supply is fixed at $p^a(1+f^a)$. The fixed point is where excess demand is zero.

Once we find (p^e, p^h), we repeat the exact same exercise as above, but this time we are deriving optimal consumption choice. [17] Then, we compare the level of aggregate savings chosen by households with the value of available savings instruments. If there is excess supply of savings, we lower q, and vice versa, and continually adjust q until all markets clear.

Adding U^i_t, P^i_t Variation: We ideally want variation within cohorts. I will have three realizations of each, taken at CDF values of $\frac{1}{6}, \frac{3}{6}, \frac{5}{6}$, each with equally likely joint probability of $\frac{1}{9}$. This splits cohorts into 9 sub-cohorts based on these realizations. For each cohort a s.t. $a \in [1, 2, ..., 15]$, $logU^i_{a,t} \sim N(-\sigma^2_u/2, \sigma^2_u)$ and $logP^i_{a,t} \sim N(-a\sigma^2_p/2, a\sigma^2_p)$.

D. Appendix: Value Function Normalization

The original value function is:

$$V_t(\omega_t, \hat{y}_t) = \max_{c_t, \omega^a_t}\left[c_t^{1-\frac{1}{\sigma}} + \beta E_t\left[V_{t+1}(\omega_{t+1}, \hat{y}_{t+1})^{1-\gamma}\right]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\sigma}}}$$

where

- $c_t + \alpha_t \cdot \omega_t = \omega_t$
- $-\alpha^b_t \omega_t \leq \phi \alpha^h_t \omega_t$
- $\omega_t = \alpha^T_{t-1} R_t \omega_{t-1} + y_t$

Now, to show it can be normalized, we guess and verify that $V_t(\omega_t, \hat{y}_t) = \hat{V}_t\left(\frac{\omega_t}{\hat{y}_t}\right)\hat{y}_t$. The verification involves conveying that the only state variable necessary is $\frac{\omega}{\hat{y}_t}$. Plugging in the guess, we have that:

[17] Note that b must be multiplied by $\left(\frac{E^{\text{obj}}_{t-1}[R_t]}{E^{\text{obj}}_{t-1}[G_t/G_{t-1}]}\right)$

$$\hat{V}_t(\frac{\omega_t}{\hat{y}_t}) = \max_{\frac{c}{\hat{y}_t},\alpha^a} \left[(\frac{c_t}{\hat{y}_t})^{1-\frac{1}{\sigma}} + \beta E_t \left[\left(\hat{V}_{t+1}(\frac{\omega_{t+1}}{\hat{y}_{t+1}})(\frac{\hat{y}_{t+1}}{\hat{y}_t}) \right)^{1-\gamma} \right]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\sigma}}}$$

where $\frac{\omega_{t+1}}{\hat{y}_{t+1}} = \alpha_t^T R_{t+1} \left(\frac{\omega_t}{\hat{y}_t}\right)\left(\frac{\hat{y}_t}{\hat{y}_{t+1}} + \frac{y_{t+1}}{\hat{y}_{t+1}}\right)$.

Conditional on knowing $\frac{\omega_t}{\hat{y}_t}$, no additional state variable is required. To see why, note that the only two unknown fractions are:

- $\frac{\hat{y}_{t+1}}{\hat{y}_t} = \frac{A_{t+1}}{A_t}\frac{G_{t+1}}{G_t}\frac{P_{t+1}}{P_t}$
- $\frac{y_{t+1}}{\hat{y}_{t+1}} = U_{t+1}$

As the age profile A_t is deterministic, and $\frac{G_{t+1}}{G_t}, \frac{P_{t+1}}{P_t}, U_{t+1}$ follow i.i.d. processes, no information today is informative as to the realization of these random variables. Done.

E. Appendix: Inheritance

We presume that each cohort, on aggregate, receives whatever is left behind on aggregate by the cohort that is **32** years older than them. As age is top-coded at 80, we can only see what is potentially bequested by those aged 84 (as they're 80 in $t-1$), meaning we can only allow those up to age 52 to inherit. The value of inherited wealth is a function of equilibrium prices since you're inheriting housing/equity shares. We presume all assets earn the expected rate of return from $t-1$ to t (can also take realised returns from the data).

Given knowledge as to the unconditional expectation of the inheritance shock, what is its distribution? As inheritance is lumpy and doesn't follow a continuous distribution, we will state that inheritance for agent i in cohort of age a is zero with probability $(1 - Pr_{a+32}(\text{death})$ and value $U_{i,a}$ with probability $Pr_{a+32}(\text{death})$. $U_{i,a}$, however, is likely to be proportional to the level of income of agent i. This is a reduced-form characterization of inheritance being larger for richer agents, here purely from an income flow perspective. Consequently, we will model $U_{i,a}$ as being a multiplicative term on income.

So what is this value conditional on getting inheritance? Inheritances must equal bequests for matched bequest-inheritance cohorts. Each bequest from cohort aged $a+32$ is split amongst $\frac{\text{Pop}_a}{\text{Pop}_{a+32}}$ children, reflecting the different sizes of the two cohorts. The average inherited by cohort of age a in period t is equal to $A^a G(t) U(a)$ where $U(a) = U(i,a) \forall i$ receiving inheritance (permanent and transitory idiosyncratic shocks have been integrated out). [18] Letting $B(t)$

[18] It is worth noting here that I will be presuming that inheritance will grow over time, in real terms, at the real GDP growth rate, not at the rate of return on assets

be the amount bequested by a dying agent, which will be just the wealth you observe of that cohort, then bequest-inheritance clearing requires that: [19]

$$
\begin{aligned}
B(a+32,t) &= \frac{\text{Pop}_a}{\text{Pop}_{a+32}} A^a G(t) U(a) \\
\rightarrow U(a) &= \frac{\text{Pop}_{a+32}}{\text{Pop}_a} \frac{B(a+32,t)}{A^a G(t)} \\
&= \frac{\text{Pop}_{a+32}}{\text{Pop}_a} \left(\frac{p_E^{\text{guess}} \theta_{a+32}^e + p_H^{\text{guess}} \theta_{a+32}^H + b_t \frac{G_t/G_{t-1}}{\Pi_t}}{A^a} \right)
\end{aligned}
$$

where, in this case, equilibrium prices in our code are in units of the average income of the youngest cohort, equalling $G(t)$.

$U(a)$ is the size of the shock conditional on inheriting that clears the bequest-inheritance market in period t. I presume that, from $t+1$ onwards, that $U(a)$ remains the same. As there is no market-clearing from $t+1$ onwards, there are no asset prices in these periods, and so $U(a)$ can't be derived explicitly.

F. Appendix: Data Collection

Income is calculated as the sum of the following items:

- DI1200 Self-employment income
  - Sum of PG0210 Gross self employment income (profit/losses of unincorporated enterprises) for household members
- DI1200 Self-employment income
- DI1500 Income from pensions
- DI1600 Regular social transfers (except pensions)

Consumption is calculated as:

- HI0100 amount spent on food at home
- HI0200 amount spent on food outside home
- HI0210 amount spent on utilities

[19] This equation assumes that death probabilities are independent of all other shocks in the economy.

- HI0220 amount spent on consumer goods and services

Comment: we can use aggregate data statistics to scale these items to headline consumption
Equity:

- DA2104 Value of non self-employment private business
  - HD1010 value of non-selfemployment not publicly traded businesses
- DA2105 Shares, publicly traded
  - HD1510 value of publicly traded share
- DA1140 Value of self-employment businesses

We also define Unlisted Equity as:

- DA2104 Value of non self-employment private business
- DA1140 Value of self-employment businesses

We define Listed Equity:

- DA2105 Shares, publicly traded

Housing

- DA1110 Value of household's main residence. DA1110=HB0500 $*$ HB0900
  - HB0500 % of ownership of household main residence
  - HB0900 current price of household main residence
- DA1120 Value of other real estate property. DA1120=Sum of (HB270x $*$ HB280x) + HB2900
  - HB270$x other property $x: % of the property belonging to household
  - HB280$x other property $x: current value
  - HB2900 additional properties current value

Bonds:

- DA2103 Bonds
  - HD1420 market value of bond

Deposits

- DA2101 Deposits. DA2101=HD1110+HD1210
  - HD1110 value of sight accounts
  - HD1210 value of saving accounts

Debt

- DL1000 Total outstanding balance of household's liabilities. DL1000=DL1100+DL1200.
  - DL1100 Outstanding balance of mortgage debt. DL1100=DL1110+DL1120
    - DL1110 Outstanding balance of HMR mortgages. DL1110=Sum of (HB170x) + HB2100
      - HB170$x HMR mortgage $x: amount still owed
      - HB2100 money still owed on additional HMR loans
    - DL1120 Outstanding balance of mortgages on other properties. DL1120=Sum of (HB370x) + HB4100
      - HB370$x other property mortgage $x: amount still owed
      - HB4100 money still owed on additional other property loans
  - DL1200 Outstanding balance of other, non-mortgage debt. DL1200=HC0220+ HC0320 + Sum of (HC080x) + HC1100
    - HC0220 amount of outstanding credit line/overdraft balance
    - HC0320 amount of outstanding credit cards balance
    - HC080$x non-collaterised loan $x: outstanding balance of loan
    - HC1100 total amount owed for additional non-collaterised loans

Pension. We consider only non-defined benefit pension plan.
We define p_occ_pension_NDB as the sum of occupational pens

- Occupational pension plan non-defined benefit
  - Sum of PF0710 current value of all occupational pension plans that have an account.
  - Select only non-defined benefit pension (PF0800 = 2). PF0800 occupational plan has regular benefit in retirement (1 means yes, 2 means no).
- Voluntary pension.
  - Sum of PF0920 voluntary pension schemes - value of accounts
- Social security plans
  - Sum of PF0510 current value of all social security plans that have an account

Mutual fund investment:

- DA2102 Mutual funds, total. DA2102=HD1320g OR sum of(HD1320a-f)
  - HD1320x market value of mutual funds.
    * a - Funds predominantly investing in equity
    * b - Funds predominantly investing in bonds
    * c - Funds predominantly investing in money market instruments
    * d - Funds predominantly investing in real estate
    * e - Hedge funds
    * f - Other fund types (specify)
    * g - Aggregate amount of all funds together