Asset Prices and Risk Sharing
The Valuation Effects of Capital Market Integration

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February 15, 2019

Abstract

Gains from trade in assets can always be decomposed into a consumption smoothing (volatility) and valuation (average) term. The latter follows from the changes in the present discounted value of domestic output when this is re-valued at the new equilibrium (state contingent) prices. We show that even when theory restricts the overall gains to be positive, either the smoothing or the valuation term may be negative, per effect of asymmetries in economic size and risk. We also point out that omitting the specification of the budget constraint in complete market models, a widespread practice in the quantitative literature, leads to substantial bias in welfare assessment. We provide a method to redress this problem in an accurate and efficient way.

Keywords: International Risk Sharing, Asymmetry, Risky Steady State, Welfare.

JEL Classification:.

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§The views expressed in this paper do not necessarily reflect the views of the BIS and of the US Federal Reserve System. Giancarlo Corsetti gratefully acknowledges the generous support and hospitality of the BIS when working on the first draft of this paper.

1 Introduction

Assessing the effects and the gains from market integration and financial liberalization is one of those debates in economics that has the highest policy resonance, and is most divisive in popular debates. Supporters and detractors confront each other on whether these processes improve risk sharing opportunities, rather than exposing residents in a country to higher risks (). By they same token, there is a wealth of contributions focusing on the possibility that these processes exacerbate existing distortions within a country, reducing efficiency in capital allocation and most importantly widening income and wealth disparities (). These debates clearly define key chapters in economic analysis, and a number of seminal papers have improved our understanding on empirical and theoretical grounds ().

In this paper, we call attention on yet another dimension that the analysis should not disregard. Namely, capital market integration and liberalization has first-order effects on wealth distribution not only within countries, but also across borders. As agents value assets and income streams at new equilibrium prices, they may find themselves relatively richer or poorer, and thus adjust their average consumption accordingly. The pecuniary externalities resulting from financial reform may be efficient (if the economy is frictionless), or inefficient (if there are frictions in financial, goods or factors markets). An important issue is the extent to which they are sensitive to frictions affecting international intermediation and markets.

Our main take away point is that the valuation and asset price changes resulting from financial liberalization cannot be treated as minor effects, that can be safely disregarded in models and analyses. They are an essential component of gains from trade in financial assets, and in many empirically relevant circumstances they shape and define the overall benefits.

The point of departure of our analysis is the non-controversial statement that, when two regions with segmented financial markets liberalize capital flows, households and firms residing on opposite side of the border gain access to a larger set of opportunities to diversify risk. According to a widespread view the benefits are to be assessed mainly (or exclusively) based on indicators of improved consumption smoothing—e.g. a fall in conditional consumption volatility. However, as households pursue diversification opportunities, affecting demand and supply of assets, changes in their consumption allocation translate into an equilibrium change in the way markets and agents value income streams, i.e., changes in the stochastic discount factor. Upon market integration, all markets (real and financial) indeed clear at new prices, which in turn generate adjustment in wealth and thus average consumption across borders.

To be clear: the effects of capital market integration are alway the result of both *smoothing*, reflected in the volatility of consumption, and *valuation*, driving adjustment in the level of consumption. Perhaps because many analyses in the literature rely on instances of ex-ante symmetric economies—implying that stochastic discounts are symmetric (with frictionless complete markets, this is true both ex ante and ex post)—it is a common practice to downplay, or even ignore altogether, valuation.

In this paper, we reconsider gains from trade in financial assets deriving and applying a two-way decomposition distinguishing smoothing and valuation effects, and bring the model to bear on the quantitative and empirical literature. First, we show that theory does not restrict both terms to be positive. Rather, their contribution to welfare depends systematically on structural and fundamental features of the economies, such as relative size and risk of the countries integrating their markets. Smaller countries with relatively high variance and covariance of output relative to the world tend to gain mainly in terms of smoothing—the contribution of valuation to welfare can be negative, as they may experience a fall in average consumption in equilibrium. On the contrary, countries with low variance of output tend to gain mostly in terms of valuation and average consumption—-consumption volatility may rise at the margin upon integrating markets.

Second, we show that ignoring the valuation term may lead to severe bias in assessing gains from trade: the model may predict spurious cases of welfare reversal, by which going from financial autarky to complete markets causes welfare to fall. This is indeed the case in many models in the literature, following the practice of omitting the budget constraints in solving for the complete market allocation. In the empirically relevant cases of asymmetries in size and risk across regions, this practice implies that the allocation is derived under the assumption of compensatory transfers, undoing the change in the valuation of output associated with the change in equilibrium prices. We detail a method to address this problem in quantitative model in an efficient and accurate way.

Third, we show that the relative importance of the two effects may change quite substantially across different financial arrangement. To show this, we reconsider our results generalizing the model, contrasting complete and incomplete markets, or allowing for distorted financial intermediation and/or a non-zero initial net foreign asset position. We show that, in a bond economy, relatively safe countries smooth less, but gain in wealth (experiencing an asset price booms). As their consumption level rises, they run a deficit and accumulate debt in the long run. Relative to the complete-market allocation, the balance of the gains features less smoothing, but larger valuation effects—and of course debt accumulation.

Remarkably, however, these effects are greatly moderated if cross-border intermediation is costly, e.g,, international or global banks trade assets on behalf of agents subject to a principal-agent distortion. This distortion ends up muting the asset price response to financial liberalization: independently of whether banks' profits are rebated to households, valuation effects are quite small. So are gains in terms of consumption smoothing.

Finally, changes in equilibrium prices also affect the valuation the outstanding stock of cross-border asset and liabilities—"the external wealth of nations"—, if any. We show that, as expected, valuation effects are larger if the external debt/assets are long-term and not indexed. Yet, their size is contained relative to the impact on the "domestic wealth of nation", coming from repricing physical and human capital used in the production of domestic output.

For the sake of analytical clarity and transparency, we derive these results in a standard two-country version of the Lucas (1978) endowment model, extending its set up to generalize our results to different market structures (one-bond economies, costly international intermediation) and to the present of a non-zero initial net foreign asset position. Without loss of generality, we model asymmetries in economic size, due to different population, and risk, due to different properties of the output process across borders.

The rest of the paper is organized as follows. Section 2 provides a short overview of the literature that closely relates to our paper. Section 3 introduces the baseline Lucas (1978) endowment-economy model and discusses the main channel explored in subsequent sections. The bulk of the analytical results are presented in Section 4. Section 5 introduces financial intermediaries and incomplete markets. Section 6 cycles through numerical results comparing different financial market structures and degrees of financial frictions. Section 7 summarizes our results and provides concluding remarks. An Appendix collects all the analytical derivations, proofs of propositions and the algorithm to compute the risk-sharing constant for generic DSGE models under complete markets.

2 Literature review [To Be Completed]

Given the topic, our paper is closely related (and owe) to a large number of studies.[1]. Amon these, Coeurdacier et al. (2013) analyze the welfare gains from risk sharing in models where financial integration leads to an efficient reallocation of capital. The authors show capital reallocation is particularly important in the case of market integration by riskier emerging economies with a relatively low level of capital. On the one hand, catching up tends to reallocate capital from advanced to emerging economies. On the other hand, with imperfect markets, precautionary saving due to a high level of risk dampens this effect, resulting in lower gains from integration.[2] Our paper contributes

[1] A partial list includes

[2] On a technical note, different from the approach we take in this paper, these authors use global solution methods to control for inaccuracies that perturbation could generate when non-linearities are large and integration implies significantly large changes in long-run equilibria.

to their analysis, by bringing the additional dimension of valuation vs consumption smoothing effects (contingent on financial market structures) to bear on their results.

The decomposition we pursue is discussed in van Wincoop (1994) and Lewis and Liu (2015). Both studies calculate welfare gains from risk sharing for a number of advanced economies. The study by van Wincoop (1994) emphasizes how risk asymmetries can generating different welfare gains across countries, pointing out that these gains do depend on both changes in consumption variability and changes in the international pricing of the consumption streams. He documents that countries with a low standard deviation of consumption growth and a low correlation with world consumption growth have positive price gains, but may not gain much in the form of a drop in the standard deviation of consumption. Lewis and Liu (2015) builds an empirical framework that uses data on both consumption and asset returns, and focus on the different implications of persistent and transitory risk associated with consumption stream. Gains from sharing transitory risk depend also on two effects: (1) the gain from a change in the wealth to consumption ratio and (2) the compensation to countries with a relatively better diversification potential. While the former effect derives from pooling risk, the latter depends on the share of the country in world consumption - a value that is increasing in the price of a country's consumption stream. Lewis and Liu (2015) conducts a robustness analysis allowing for differences in the mean consumption growth rates and size of countries and also a bigger number of countries. However, under risk sharing, they aggregate consumption cross countries of different size by using population shares.[3]

Relative to these important papers, our contribution provides a general theoretical analysis of how gains from trade systematically result from smoothing and valuation effects, linking welfare to asset pricing and identifying a new channel by which financial frictions impinge on the post-integration allocation.

[3]This is due to a different assumption on the risk sharing economy. Lewis and Liu (2015) considers a planners problem in which the planner cares about maximizing over utilities of individual people in each country. Specifically, in each country j there are N^j people. In our case we assume that in each country there is a representative household and thus the planner maximizes the weighted sum (where weights are the relative sizes of countries in the world economy) of utilities of the representative households in the world economy.

3 The Lucas (1978) endowment economy

To gain fundamental insight on the effects of cross-border financial integration and liberalization on the valuation of assets and on consumption smoothing, we find it instructive to frame the analysis in a standard version of the Lucas (1978) model. The world consists of two countries (country "1" and "2"), each populated by infinitely lived agents who are identical within a country while may differ across countries. To keep complexity in check, without loss of generality we restrict attention on asymmetries in (population) size and endowment.

Time t is discrete. In each period the state of the economy consists of a particular realization of the stochastic endowments processes and a particular distribution of financial assets. We use the notation x_t to indicate that the variable x depends on the state of the economy in period t.

Normalizing world population size to unity, we posit that country 1 has population n, country 2 population $(1-n)$. Per capita output in each country is modelled as an exogenous endowment flow ($D_{1,t}$ and $D_{2,t}$) following an AR(1) processes

$$D_{j,t} = (1 - \varphi_{D,j})\,\bar{D}_j + \varphi_{D,j} D_{j,t-1} + \sigma_{D,j}\varepsilon_{D,j,t};\ j = \{1, 2\}. \tag{3.1}$$

At world level, then, accounting for differences in population size, aggregate global output is

$$D_{W,t} = nD_{1,t} + (1-n)D_{2,t}. \tag{3.2}$$

and the aggregate resource constraint is

$$nC_{1,t} + (1-n)C_{2,t} = nD_{1,t} + (1-n)D_{2,t}. \tag{3.3}$$

where C denotes per capital consumption.

3.1 Consumer problem

In our analysis we consider various international financial market arrangements ranging from financial autarky through international trade in a non-contingent bond, and international trade intermediated by banks (with complete or incomplete set of assets) subject to agency problems, to complete markets.

Generally, the representative agent j ($j = \{1, 2\}$) trades in Arrow-Debreu securities A_{t+1}, at the price $\Lambda_{j,t+1|t}$, that pay one unit of consumption in period $t+1$ as well as in a non state-contingent zero-coupon bond B_t at price P_t. Hence the consumer problem is

$$\max_{C_{j,t},A_{t+1},B_t} \sum_{i=0}^{\infty} E_t \beta^i U\left(C_{j,t+i}\right) \tag{3.4}$$

where $U\left(C_{j,t+i}\right) \equiv \dfrac{C_{j,t+i}^{1-\rho} - 1}{1-\rho}$, subject to the individual budget constraint

$$C_{j,t} + E_t \Lambda_{j,t+1|t} A_{t+1} + P_t B_t = D_{j,t} + A_t + B_{t-1}. \tag{3.5}$$

The first order conditions are:

$$c_{j,t} : \; \beta U_c(c_{j,t}) - \lambda_{j,t} = 0 \tag{3.6}$$

$$A_{t+1} : \; \lambda_{j,t} - \lambda_{j,t-1} \Lambda_{j,t|t-1} = 0 \tag{3.7}$$

$$B_t : \; E_t \lambda_{j,t+1} - P_t \lambda_{j,t} = 0 \tag{3.8}$$

where $\lambda_{j,t}$ is the Lagrange multiplier on the budget constraint. Merging the the first two conditions yields the the expression for the stochastic discount factor:

$$\Lambda_{j,t+1|t} = \beta \frac{U_c(c_{j,t+1})}{U_c(c_{j,t})}. \tag{3.9}$$

while the third equation prices the non-contingent bond.

We encompass different international financial market arrangements by writing a general arbitrage equation for asset prices:

$$\mathbb{E}_t\Omega_{t+1}(\mathcal{P}_{1,t+1} - \mathcal{P}_{2,t+1}) = \gamma_t(\cdot). \tag{3.10}$$

where $\mathcal{P}_{j,t+1} = P_{j,t} = E_t\Lambda_{j,t+1|t}$ for the non-contingent bond or $\mathcal{P}_{1,t+1} = \Lambda_{j,t|t-1}$ for arrow securities, Ω_t is the discount factor of the agents who price bonds at the margin (e.g. financial intermediaries) and $\gamma_t(\cdot)$ stands for any further wedge between asset prices that could stem from constraints on financial transactions (e.g. collateral constraints). When this wedge is zero and there is no third-agent intermediating between national investors (the textbook case in which agents can borrow and lend across borders), the expression above boils down to

$$\mathbb{E}_t\mathcal{P}_{1,t+1} - \mathbb{E}_t\mathcal{P}_{2,t+1} = 0 \tag{3.11}$$

This expression subsumes both the asset-price condition under complete market (the perfect consumption-risk-sharing condition) and the the asset-price condition under one-bond assumption (i.e. the condition underlying the standard UIP condition).

The literature offers many instances in which either $\Omega_t \neq 1$ or $\gamma_t(.) \neq 0$, or both. Recently, Gabaix and Maggiori (2015) have offered insight on economies in which $\gamma_t(.)$ is proportional to net-foreign-asset (NFA) positions, while Engel (2016), and Itskhoki and Mukhin (2017) discuss the importance of this wedge in explaining a number of stylized facts concerning the exchange rate.[4]

For our purposes, the key take away point is that the market value of domestic wealth depends on the pricing kernel of traders, which in turn depends on the degree of international financial integration. Equation (3.10) imposes restrictions on the relationship between the domestic and foreign pricing kernels, and thus on the way domestic assets, and more generally wealth, are priced.

[4]Later we will also note how outstanding non-zero NFA positions can affect the gains from financial integration and their composition.

3.1.1 Decomposition of welfare gains: general formula

It is well understood that, financial market liberalization gives households access to new assets, which may be used to smooth consumption across periods and states of nature. However, in general, it also impinges on asset pricing and thus brings about changes in the present discounted value of domestic output—national wealth. As national wealth changes, households optimally adjust the level of their consumption.

To introduce our decomposition of welfare gains from financial integration, consider the following second order expansion of households' utility for country 1:

$$\widetilde{U}_{1,t} = \bar{C}_1^{-\rho}\widetilde{C}_{1,t} - \frac{1}{2}\rho\bar{C}_1^{-\rho-1}\widetilde{C}_{1,t}^2. \tag{3.12}$$

Valuation effects resulting from the repricing of domestic output at the new equilibrium prices are captured by the first term in ((3.12)). Smoothing effect translating into a change in volatility of consumption are captured by the second term in ((3.12)).

4 Welfare gains: from autarky to complete markets

In order to gain insight on the interplay of pricing and diversification in determining the net effects of financial reform, we start with the classical setting of grains from trade going from financial autarky to frictionless complete markets.

Without loss of generality, we focus on the case of one-period Arrow-Debreu securities, imposing that initially no country has outstanding claims or liabilities to other countries—essentially, before trade takes place, holding of Arrow-Debreu securities is zero. This is tantamount to posit that the two countries are initially in financial autarky.

4.1 Complete markets

In this subsection, we characterize a second-order solution for consumption in the two economies as a function of exogenous endowments.

Our solution technique for complete market allocation draws on standard theory, as discussed e.g. by Ljungqvist and Sargent (2012, Ch 8, p 274). By solving equation (3.9) backward and making use of the individual budget constraints (3.5), one can write the risk-sharing condition as

$$C_{1,t} = \kappa \left(\sigma^2_{D,1}, \sigma^2_{D,2}\right) C_{2,t} \tag{4.1}$$

where the constant is defined as $\kappa \left(\sigma^2_{D,1}, \sigma^2_{D,2}\right) \equiv \frac{C_{1,0}}{C_{2,0}}$, hence depends on time-0 initial conditions.[5] We highlight the dependence of κ on "risk" (the volatility of the underlying shocks) to point out how this term will be affected by the use of perturbation techniques in the solution of the model. For brevity, though, we will simply use κ in the rest of the paper.

This constant is an important building block in our discussion as it depends on key structural characteristics of the economies whose financial markets are integrated. Among these characteristics, relative risk plays obviously a special role—heuristically, it determines the demand and supply of insurance by different agents in the integrated markets. Likewise, we will show that the relative size of countries plays a similarly important role, since it underlies the relative demand and supply of risky assets at aggregate level.

[5] The dependency of κ on risk is not merely an assumption but the result of risk affecting economic allocations at higher orders of approximation. Likewise, the fact that κ does not depend on risk at first order follows from the fact that certainty equivalence holds using first order regular perturbation techniques.

4.1.1 The risk-sharing 'constant'

In order to solve for the risk sharing constant κ we take a second order approximation of the model around the deterministic steady state (where we assume that $\bar{C}_j = \bar{D}_j$). A second order (log) expansion of the risk sharing condition (4.1) yields:

$$\widetilde{C}_{1,t} = \widetilde{\kappa} + \widetilde{C}_{2,t} \tag{4.2}$$

Note that the first order expansion is simply $\widetilde{C}_{1,t} = \widetilde{C}_{2,t}$, as risk would not affect the level of the variables to first order.

In line with (Ljungqvist and Sargent, 2012, Ch 8), complete markets arrangement implies that the time-zero distribution of Arrow securities must be zero. In line with this, we first solve the budget constraint (equation 3.5)

$$A_{1,0} = E_0 \sum_{t=0}^{\infty} \Lambda_{1,t|0} \left(C_{1,t} - D_{1,t}\right). \tag{4.3}$$

and take a second order expansion:

$$\widetilde{A}_{1,0} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left(\bar{C}_1 - \bar{D}_1\right) \widetilde{\Lambda}_{1,t|0} + \left(\bar{C}_1 \widetilde{C}_{1,t} - \bar{D}_1 \widetilde{D}_{1,t}\right) + \right.$$
$$\left. + \widetilde{\Lambda}_{1,t|0} \left(\bar{C}_1 \widetilde{C}_{1,t} - \bar{D}_1 \widetilde{D}_{1,t}\right) + \frac{1}{2}\left(\bar{C}_1 - \bar{D}_1\right) \widetilde{\Lambda}^2_{1,t|0} + \frac{1}{2}\left(\bar{C}_1 \widetilde{C}^2_{1,t} - \bar{D}_1 \widetilde{D}^2_{1,t}\right) \right\}. \tag{4.4}$$

Appendix B derives the second-order accurate expressions for each of the terms in equation (4.4). We solve for the risk sharing constant $\widetilde{\kappa}$ by imposing the condition $\widetilde{A}_{1,0} = 0$. For ease of exposition, it is convenient to simplify the analysis by assuming that shocks are i.i.d and steady state per-head endowment is symmetric.[6] We also resort to the following normalizations: (1) total per-head endowment is 1 and (2) global risk is 1. Under these simplifying assumptions and normalization, the risk sharing constant can be represented as a straightforward function of risk aversion and relative size and

[6] We show in Appendix B that size and steady state per-head endowment affect the risk sharing constant in the same way.

risk of countries:

$$\widetilde{\kappa} = \frac{\beta}{2} \left(\rho \left(1 - 2n \right) + \left(\rho - 1 \right) \left(1 - 2\gamma \right) \right) \tag{4.5}$$

where n is relative size of country 1 and γ represents relative risk of country 1. The above expression emphasizes that per capita wealth would be perfectly symmetric in the two countries ($\widetilde{\kappa} = 0$, i.e. $\kappa = 1$) if either: (1) there were no differences in risk and size between countries, (2) countries were symmetric in size and preferences are such that $\rho = 1$, (3) countries were symmetric in risk and preferences are such that $\rho = 0$. Note that case (2) is well known in the finance literature (Cochrane, 2009): with log preferences ($\rho = 1$) and countries symmetric in size relative risk does not impinge on relative consumption.

Formally the way the relative consumption share in the world output depends on structural characteristics of countries can be summarized by the following:

Proposition 1. *Assume that steady state per-head endowment is symmetric and total risk is constant and normalized to one. Then, relative to the deterministic steady state, i) as long as $\rho > 0$ the larger the country, the smaller its consumption share in world output; and ii) with $\rho > 1$ ($\rho < 1$) the riskier the country, the smaller (larger) is its consumption share in world output. These marginal effects are invariant to size or risk.*

The proof is provided in Appendix C.

4.1.2 Asset prices under complete markets

We now study how the country specific endowment of output is revalued in the passage from financial autarky to complete markets. As noted earlier, our key message is that changes in the equilibrium asset prices are crucial in understanding welfare gains. Towards this result, we now work out the change in valuation, starting from the asset pricing equation:

$$P_{D,j,t} = E_t \Lambda_{j,t+1} \left(D_{j,t+1} + P_{D,j,t+1} \right). \tag{4.6}$$

Using results from Appendix E we show the second order approximation of this expression under complete markets for each country:

$$\widetilde{P}_{D,1,0} = \frac{\beta}{2\left(1-\beta\right)}\left[\rho^2\left(n^2\gamma + (1-n)^2(1-\gamma)\right) - \rho n(1-n) - \left(2\rho n - 1\right)\gamma\right] \tag{4.7}$$

$$\widetilde{P}_{D,2,0} = \frac{\beta}{2\left(1-\beta\right)}\left[\rho^2\left(n^2\gamma + (1-n)^2(1-\gamma)\right) - \rho n(1-n) - \left(2\rho(1-n) - 1\right)\left(1-\gamma\right)\right] \tag{4.8}$$

The first two elements are the same for both countries and are related to variance and mean of pricing kernel, respectively. The third element describes the covariance between pricing kernel and endowment of a given country.

The difference between $\widetilde{P}_{D,1,0}$ and $\widetilde{P}_{D,1,0}$ constitutes for a so called relative valuation effect and is equal to a difference in the covariances between pricing kernel and endowment of each country:

$$\widetilde{P}_{D,1,0} - \widetilde{P}_{D,2,0} = \frac{\beta}{2\left(1-\beta\right)}\left(\rho\left(1-2n\right) + \left(\rho - 1\right)\left(1-2\gamma\right)\right) = \frac{\widetilde{\kappa}}{1-\beta} \tag{4.9}$$

Equation (4.9) shows also that the relative valuation effect is tightly related to the risk sharing constant $\widetilde{\kappa}$ and thus directly affects the share of consumption in the world output.

4.1.3 Welfare under complete markets

Finally, we can represent a second order approximation to welfare for country 1 and 2 under complete markets:

$$E_0\sum_{t=0}^{\infty}\beta^t\widetilde{U}_{1,t} = -\frac{\rho\beta}{2\left(1-\beta\right)}\left((1-n)^2(1-\gamma) + n^2\gamma\right) + \frac{n(1-n)\beta}{2(1-\beta)} + \frac{(1-n)\widetilde{\kappa}}{(1-\beta)}. \tag{4.10}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{U}_{2,t} = -\frac{\rho\beta}{2\,(1-\beta)} \left((1-n)^2(1-\gamma) + n^2\gamma\right) + \frac{n(1-n)\beta}{2(1-\beta)} - \frac{n\widetilde{\kappa}}{(1-\beta)}. \quad (4.11)$$

Notice that welfare is comprised of the same components as the price gains represented by (4.7). The first component of welfare represents variance of consumption resulting from financial integration, the smoothing effect. The remaining two components approximate the level of consumption on average, summarized by the valuation effect. The level of consumption changes as the stochastic pricing kernel declines with financial integration (a decline in the pricing kernel leads also to lower asset prices on average in both countries). While the first two components are the same for both countries, the last component differs and is a result of asymmetries present between countries and is equal to the difference between asset prices. As a result, the difference in welfare between countries is solely dependent on the relative valuation effect:

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{U}_{1,t} - E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{U}_{2,t} = \frac{\widetilde{\kappa}}{(1-\beta)} = \widetilde{P}_{D,1,0} - \widetilde{P}_{D,2,0}. \quad (4.12)$$

4.2 Decomposition of welfare gains

Let us know examine the components of welfare gains resulting from complete markets relative to financial autarky.

Consumption smoothing: changes in the volatility of output

Up to second order accuracy, complete markets imply that the variance of individual consumption is identical across countries. Under the maintained assumptions of uncorrelated, i.i.d. endowment processes and our steady state normalization, the discounted sum of variance of consumption reduces to (see Appendix B):

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{C}_{1,t}^2 = E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{C}_{2,t}^2 = \frac{\beta}{1-\beta} \left(n^2 \gamma + (1-n)^2 (1-\gamma) \right). \tag{4.13}$$

In contrast, under autarky, individual consumption is exposed to national idiosyncratic shocks, hence:

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{C}_{1,aut,t}^2 = \gamma; \; E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{C}_{2,aut,t}^2 = 1-\gamma. \tag{4.14}$$

Taking the difference between the two expressions, we obtain an expression for the gains in terms of consumption smoothing:

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{C}_{1,t}^2 - E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{C}_{1,aut,t}^2 = \frac{\beta}{1-\beta} \left(2\,(1-n)\,\gamma - (1-n)^2 \right). \tag{4.15}$$

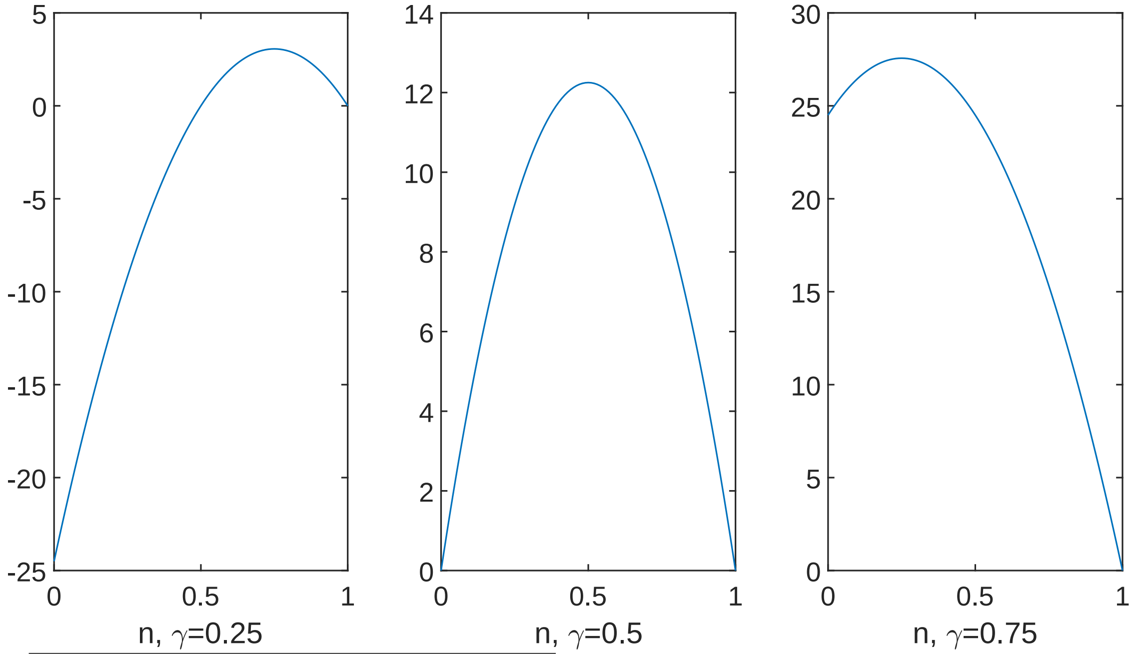
In reference to this expression, we can state the following.

Proposition 2. *In the Lucas model, assuming that endowment shocks are uncorrelated and i.i.d., the consumption smoothing gains are i) always positive and larger for the riskier country (with large variance in endowment); ii) not necessarily positive for the safer country; iii) non monotonic with respect to size, reaching a maximum at the critical value* $\gamma^* \equiv (1-n)$*; iv) larger for countries with simultaneously higher risk and smaller size.*

The proof of this proposition is provided in Appendix D. Additionally, Figure 1 shows points i)-iii) of Proposition 2.

Concerning points i) and ii) of Proposition 2, it makes sense that consumption smoothing gains can be quite different for countries with different levels of output and income volatility in financial autarky: after market integration, agents diversify their portfolios and so gain access to some average of the underlying national outputs. What is less straightforward is that consumption smoothing may worsen per effect of completing the markets: why don't agents shy away from portfolio diversification and

Figure 1: Smoothing effect, relative home country size and risk



Note: Vertical axis: Conditional discounted variance of log-consumption deviation relative to autarky. Based on equation (4.15); normalization of total risk equal to 1, $\rho = 2$ and $\beta = 0.98$.

remain close to financial autarky?

As we will see below, this is because any welfare losses in terms of reduced consumption smoothing are more than compensated by gains in terms of wealth—their consumption level rises on average.

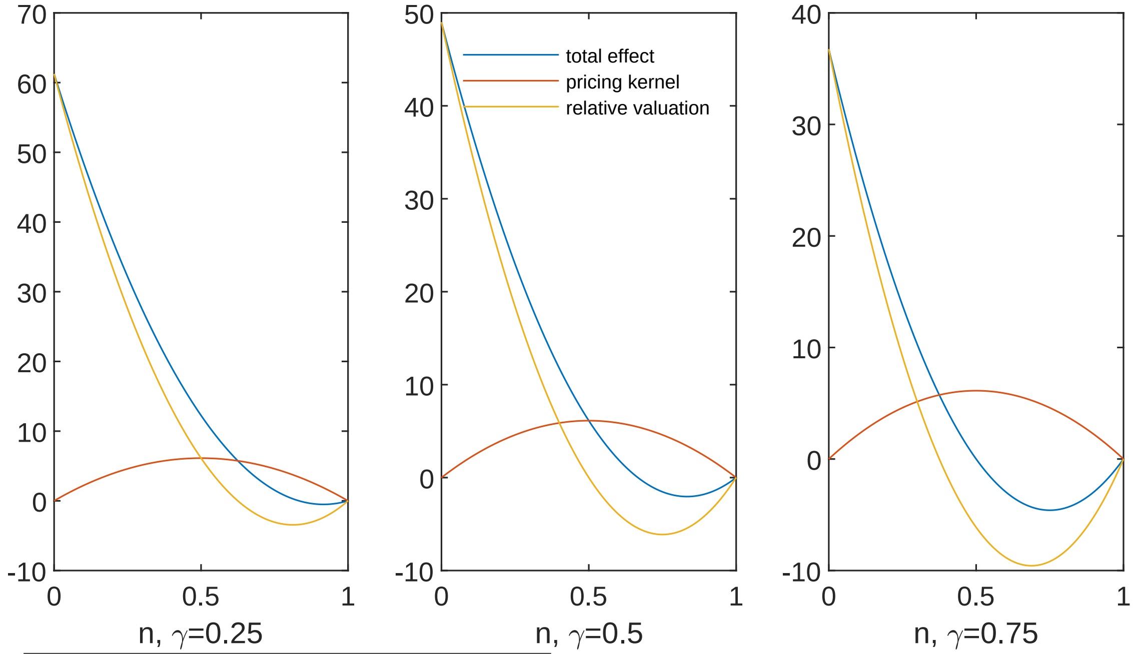
Valuation effects: changes in the present discounted value of output

The level of consumption can be approximated (see Appendix B):

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\widetilde{C}_{1,t} - \widetilde{C}_{1,aut,t} \right) = \frac{\beta(1-n)}{2(1-\beta)} \left(n + \rho(1-2n) + (\rho - 1)(1-2\gamma) \right) \tag{4.16}$$

As already mentioned above, the change in the level of consumption under complete markets comes both from a mean of the pricing kernel and the difference between covariances of each country's endowment with the pricing kernel.

Figure 2: Valuation effect, relative home country size and risk



Note: Vertical axis: Conditional discounted mean of log-consumption deviation relative to autarky. Based on equation (4.16), normalization of total risk equal to 1, $\rho = 2$ and $\beta = 0.98$.

Proposition 3. *In the Lucas model, assuming that endowment shocks are uncorrelated and i.i.d., the valuation gains: i) decrease (increase) monotonically in risk if* $\rho > 1$ *(*$\rho < 1$*), ii) are non monotonic with respect to size, reaching a minimum (maximum) in size if* $\rho > 0.5$ *(*$\rho < 0.5$*), at* $n = \frac{(2-\gamma)\,\rho-(1-\gamma)}{2\,\rho-1}$*; iii) are larger (smaller) for countries with simultaneously smaller risk and smaller size if* $\rho > 1$ *(*$\rho < 1$*).*

The proof is provided in Appendix F. Additionally, Figure 2 provides a graphical explanation of valuation effect.

Proposition 3 clarifies that provided $\rho > 1$ a *smaller* and *safer* country experiences a relatively larger increase in the present discounted value of consumption.

4.2.1 Welfare gains

We represent now the welfare gains from complete markets relative to autarky for country 1 as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\widetilde{U}_{1,t} - \widetilde{U}_{1,t}^{Aut} \right) = -\frac{1}{2} \frac{\rho\beta}{(1-\beta)} \left((1-n)^2(1-\gamma) + n^2\gamma - \gamma \right) + \\ +\frac{1}{2} n(1-n) \frac{\beta}{(1-\beta)} + (1-n) \frac{\widetilde{\kappa}}{(1-\beta)}. \tag{4.17}$$

We can simplify the above expression to:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\widetilde{U}_{1,t} - \widetilde{U}_{1,t}^{Aut} \right) = \frac{\beta(1-n)}{2(1-\beta)} \left((2\gamma - 1) + n + \rho(1-n) \right). \tag{4.18}$$

On the basis of this expression we can state the following.

Proposition 4. *In the Lucas model, moving from financial autarky to frictionless trade in a complete set of Arrow-Debreu securities generates welfare gains that are linearly increasing in domestic risk (γ), and decreasing (at an increasing rate) in domestic size (n) if $\rho > 1$ or $\rho < 1 - \frac{\gamma}{1-n}$. Furthermore, the interaction between size and risk is symmetric and negative.*

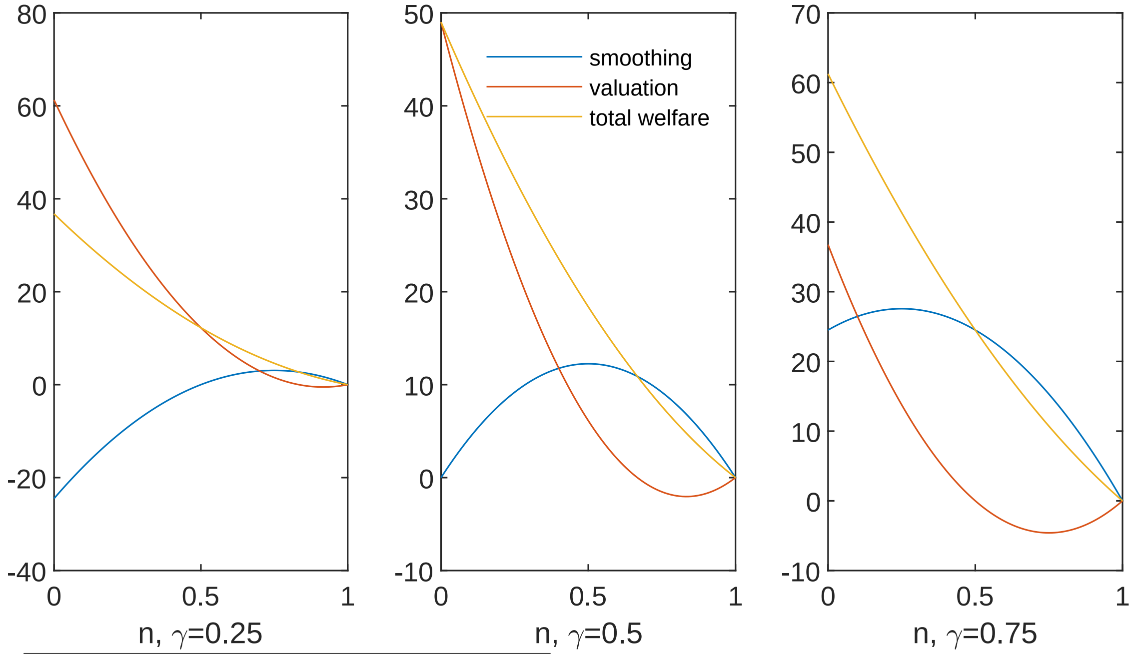
The proof is obtained by direct calculation and shown in Appendix G. Figure 3 provides a graphical explanation on how welfare gains depend on the size and risk.

From the proposition, it follows that under a parameterization that is standard in the literature ($\rho > 1$), riskier and smaller countries gain the most from risk sharing. Thus, from the welfare perspective, size and risk play a similar role.

A closer look at the role of country's size

A separate comment on size is in order. Proposition 4 shows that in the Lucas model the gains from financial integration are monotonically decreasing in size, as can be seen

Figure 3: Welfare gains decomposition, relative home country size and risk



Note: Vertical axis: Welfare units. Based on equation (4.18), normalization of total risk equal to 1, $\rho = 2$ and $\beta = 0.98$.

in Figure 3. It also shows that in terms of welfare gains, size and risk act symmetrically (though interaction is negative). Yet, for the components of the welfare gains (smoothing and valuation) are non-monotonic in size. It is thus worth inspecting the mechanism in somewhat more detail.

Size can have an effect on the gains from risk sharing only in as much it affects the way risk is valued and distributed across countries (the deterministic component cancels out by taking deviation from the symmetric deterministic steady state). Furthermore, we have seen that both the smoothing and the valuation effects change non-monotonically with respect to size.

Let's consider first the non-monotonic effect of size on smoothing. Up to first order, risk sharing implies that consumption is a weighted average of the endowments, with weights equal to country size. The variance of consumption, though, depends on the square of these weights. Thus, like the variance of a total portfolio return, the variance of consumption is non-monotonic in the weights. If country 1 is negligibly small ($n \to 0$) only the variance of the foreign country matters for domestic consumption. When n increases (but still relatively smaller than the foreign country) the diversification effect reduces domestic consumption volatility. As n increases further, the contribution of the foreign variance, and thus of diversification fades, until, with $n \to 1$, the volatility of domestic consumption depends totally on the variance of the domestic endowment. The benefit of diversification, thus, reaches a maximum at a precise "portfolio" composition (i.e. $n = (1 - \gamma)$): the larger is the variance of the foreign asset the larger must be the weight on the domestic asset, as can be seen in Figure 1.

As for the valuation effect, the conditional mean of consumption can be decomposed into two elements:

$$E_t\left(\widetilde{C}_{1,t+1}\right) = n(1-n) + \widetilde{\kappa}(1-n). \tag{4.19}$$

The first element describes the mean of pricing kernel and it achieves its maximum at $n = \frac{1}{2}$. The second element dominates the overall effect and is directly related to the risk sharind constant. As such, the second component summarizes the redistribution effect, which is the strongest when country 1 is small, $n \to 0$. In this case, upon financial integration of a small country global level state contingent prices move very little as global aggregate resources are little affected by a new country. As such, the small country gains the most. Yet, as n increases the valuation effect decreases to reach a negative value and then to increase once again to reach zero as $n \to 1$ (as state contingent prices become arbitrarily close to financial autarky).

4.3 A summary

To summarize, moving from autarky to complete markets generates relative welfare gains that are proportional to the relative gains in consumption levels. Smaller and safer countries (if $\rho > 1$) gain the most from financial market liberalization. These gains are made of changes in the volatility of consumption and changes in the value of domestic assets. Consumption smoothing gains can be *negative* if domestic risk is below a critical value (negatively related to domestic size), and are non-monotonic with respect to size. Riskier and smaller countries gain the most in terms of consumption smoothing. The valuation effect can complement or substitute for the smoothing effect in generating welfare gains. The effect of size on the valuation effect is non-monotonic, with very small countries seeing larger appreciations. The effect of risk on valuations is negative (positive) if $\rho > 1$ ($\rho < 1$).

5 The model with incomplete markets and financial intermediation

In this section we introduce international financial intermediaries (banks) drawing on Gabaix and Maggiori (2015) (GM) and Maggiori (2017). The key assumption in GM is that international financial flows are intermediated by financial institutions facing an incentive compatibility constraint similar to those discussed by Gertler and Kiyotaki (2010). Because of the the agency problem associated with intermediation, households are no longer able to insure risks efficiently—whether they can trade in a complete set of AD securities, or in a set of assets falling short of spanning risk.

In what follows, we will study the consequences of frictions due to international intermediaries in both a complete market economy and a bond economy in turn. For simplicity, following GM we assume that banks live for one period in which intermediate international capital flows.

5.1 Complete set of Arrow-Debreu securities

We start by considering the case in which households in each country can trade with the global bank in a complete set of AD securities. This bank can abscond assets—for simplicity, however, we assume it can only resell them through agents in country 1 in a competitive market.

The (competitive) bank sells/buys $E_t\Lambda_{1,t+1|t}A_{t+1}$ to/from households in country 1 and buys/sells the amount $E_t\Lambda_{2,t+1|t}A_{t+1}$ from/to households in country 2.

$$\max_{B(s_{t+1},s^t)} V_t = E_t\left[\Lambda_{1,t+1|t} - \Lambda_{2,t+1|t}\right]A_{t+1}. \tag{5.1}$$

The bank faces an incentive compatibility constraint of the form

$$\begin{aligned} V_t \geq & \Gamma\left|E_t\Lambda_{1,t+1|t}A_{t+1}\right|\left|E_t\Lambda_{1,t+1|t}A_{t+1}\right| \\ = & \Gamma\left(E_t\Lambda_{1,t+1|t}A_{t+1}\right)^2. \end{aligned} \tag{5.2}$$

Assuming that the constraint is always binding and replacing it in equation (5.1) we obtain[7]

$$\frac{\Lambda_{2,t|t-1}}{\Lambda_{1,t|t-1}} = 1 - \Gamma E_{t-1}\Lambda_{1,t|t-1}A_t. \tag{5.3}$$

Observe that, if market were frictionless markets, i.e., $\Gamma = 0$, the expression would boil down to equalization of the stochastic discount factor $\frac{\Lambda_{2,t|t-1}}{\Lambda_{1,t|t-1}} = 1$, and households would fully reap the gains of perfect risk sharing.

Rewriting equation (5.3) as

$$\left(\frac{C_{1,t}}{C_{2,t}}\right) = \left(\frac{C_{1,t-1}}{C_{2,t-1}}\right)\Theta_{t-1}. \tag{5.4}$$

where

$$\Theta_{t-1} \equiv \left(1 - \Gamma E_{t-1}\Lambda_{1,t|t-1}A_t\right)^{\frac{1}{\rho}} \tag{5.5}$$

we then take log deviations of equation (5.4) and solve forward so to obtain:

$$\widetilde{C}_{1,t} - \widetilde{C}_{2,t} = \widetilde{C}_{1,0} - \widetilde{C}_{2,0} + \widetilde{G}_t \tag{5.6}$$

[7] Note that banks choose assets state-by-state, thus the absence of the expectation operator for some of the terms in the following equations.

where

$$\widetilde{G}_t = \widetilde{\Theta}_{t-1} + \widetilde{G}_{t-1} \tag{5.7}$$

We can use the criterion that at time t=0 we have that $A_0 = 0$ and thus use the same approach discussed above to solve for $\widetilde{\kappa}$ in

$$\widetilde{C}_{1,t} - \widetilde{C}_{2,t} = \widetilde{\kappa} + \widetilde{G}_t. \tag{5.8}$$

When $\Gamma \to 0$ the model approaches the perfect and complete financial market arrangement.

5.2 Bond economy

When international financial transactions are restricted to a non-state-contingent bond, banks maximize the expected value of the bank V_t given by

$$\max_{B_t} V_t = E_t \left[\Lambda_{B,t+1|t} \left(R_{2,t} - R_{1,t}\right)\right] B_t \tag{5.9}$$

where B_t are net foreign assets of country 1 (negative of country 2 NFA), $\Lambda_{B,t+1|t}$ is the discount factor of the bank, and $R_{j,t}$ is the interest rate paid on deposits lodged at the bank (if the smallest of the two rates) or the cost of borrowing from the bank (if the largest). Note that $B_t > 0$ if $R_{2,t} - R_{1,t} > 0$, i.e. if country 2 pays a higher interest rate than country 1, then country 1 will lend to country 2. The bank rebates its ex-post profits $\left(\left(R_{2,t-1} - R_{1,t-1}\right) B_{t-1}\right)$ to both home and foreign households lump-sum, in proportion to their size.

Following Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), financial contracts are not perfectly enforceable and if the value of the bank is smaller than the value of the intermediated funds, the banker will abscond a fraction of the assets. For tractability, Gabaix and Maggiori (2015) assume that the divertible fraction is increasing in the value of the intermediated funds, i.e. divertible fraction $= \Gamma |B_t|$, where $\Gamma \geq 0$. The bank thus faces an incentive compatibility constraint of the form

$$V_t \geq \Gamma |B_t| |B_t| = \Gamma (B_t)^2 . \tag{5.10}$$

Positing that this constraint is always binding, and replacing it in the value problem of the bank, we get

$$\Gamma B_t = E_t [\beta (R_{2,t} - R_{1,t})] . \tag{5.11}$$

where for simplicity we assume that the bank is risk neutral, i.e. $\Lambda_{B,t+1|t} = \beta$. Note that the pricing condition for B_t in each country is:

$$\Lambda_{j,t+1|0} R_{j,t} = 1; \; j = \{1, 2\}. \tag{5.12}$$

so that $P_{1,t} = R_{j,t}^{-1}$. Equation (5.11) is isomorphic to the UIP condition in open-economy models, with international financial trade limited to non-contingent bonds and augmented with a premium (a common assumption in open economy models). In our version, the premium has a straightforward structural interpretation, as it derives from a limited risk-bearing ability of financial intermediaries Gabaix and Maggiori (2015).[8] When $\Gamma \to 0$ the model approaches an incomplete (though perfect) international financial market arrangement.

6 Numerical analysis

In this section we highlight key implications of our analysis by means of numerical examples. We contrast gains from trade as well as differences in consumption volatility and output valuation when moving from financial autarky to (i) complete markets, (ii) trade in a non-contingent bond and (iii,iv) trade in assets (either AD securities or a non-contingent bond) via global financial intermediaries. We do so for different

[8] See also Schmitt-Grohé and Uribe (2003) and the working paper version of Benigno (2009), i.e. Benigno (2001).

configurations of the countries' relative size, relative risk, and initial level of net foreign asset. For each configuration, we plot results in a set of 5 graphs. Two graphs show world aggregate welfare and Home country's welfare relative to the financial autarky allocation; the remaining three graphs show Home asset prices, variance of Home consumption and average level of Home consumption, also relative to financial autarky.

In all graph we show four lines, referred to as, respectively, CM (complete markets) allocation, Bond (economy), GM, for the allocation with financial intermediation á la Gabaix and Maggiori, 2015, and GMCM, for the allocation with financial intermediation and a complete set of Arrow securities. In the second graph only, showing the Home country welfare, we also include an allocation that we label "incomplete solution" for the case of complete markets.

The "incomplete solution" results from positing $\kappa = 1$ instead of solving for the risk sharing constant endogenously. We show it to point out the scope for errors in the assessments of the gains from trade when failing to include the budget constraint in the quantitative model—a practice that is surprisingly diffuse in the literature. When following this practice, the model solution de facto assumes that any endogenous asymmetry in wealth resulting from market integration is compensated by lump-sum transfers between the 'winners' and the 'losers'. These transfers do not alter the global welfare gains but affect the relative allocation, impinging on the relative welfare gains, the consumption process and therefore output pricing.

For each numerical simulation we use standard parameter values for the rate of time preferences and risk aversion, that is, $\beta = 0.98$, $\rho = 2$. The cost of intermediation is set high for the GM economy, whether with complete markets or one bond only ($\Gamma = 0.1$, as in Gabaix and Maggiori, 2015), and negligible in the bond economy, ($\Gamma = 0.001$).

In the first set of figures—Figures 4 to 7—we analyze the gains from trade varying the size of the country. Together, the figures confirm our analytical result, that smaller countries gains more from financial integration. Similarly, from the graphs, it is apparent that incomplete and imperfect financial integration produce lower gains than integration delivering perfect insurance.

What is novel in our analysis is the insight from the decomposition of the gains from trade in valuation and insurance effects. First, as shown in the upper panel of Figure 7, financial liberalization leads to large asset price changes especially for small economies. This is true whether liberalization gives households access to complete markets or to borrowing and lending only (in the bond economy). However, for a small economy, asset prices do always increase—and *by more* in the bond economy than under complete markets. This strong repricing of endowment and output when going from financial autarky to cross-border trade in bond is a result that is familiar in the textbook analysis of the current account.

To be clear on this point, in our figure we show that, at the equilibrium prices, the value of the endowment in smaller countries becomes much higher in a bond economy relative to the autarky case. This repricing is exactly what drives a key result highlighted in textbook models of the current account (see e.g. Obstfeld and Rogoff, 1996, page 75), when the rate of time preferences of a country (determining their interest rate under financial autarky) differ from the world rate. In this case, a term is appended to the baseline 'fundamental equation of the current account', reflecting the change in the present discounted value of the country's net output valued at the world interest rate rather than at the households' time preference rate. This term is sometimes labelled 'consumption tilting'—essentially, the term drives the change in average consumption reflecting equilibrium changes in wealth. A contribution from our analysis is to show that output risk generates similar effect even if the rate of time preferences is the same everywhere in the world economy.

Now, going to the second panel in the figure, it is apparent that, relative to complete markets, any relative gain in asset prices created by cross-border trade in bonds is offset by smaller gains in consumption smoothing. Trade in one non-contingent asset cannot achieve as much diversification gains. Overall, the benefits are smaller in a bond economy.

What if the liberalization is not arm-length, but rely on financial intermediaries and contracts which may be distorted because of imperfect information and/or agency problems? The figure vividly illustrates the extent to which imperfect international financial intermediation can reduce the gains from liberalization. Indeed, under the parameter values that are standard in the literature (and chosen to fit equilibrium asset prices), the costs associated with these financial frictions reduce the asset appreciation from trade, and may even cause households' welfare to decrease.[9] This is so, even if, as we assume, all profits of financial intermediation are rebated to households—wlog we posit that they are rebated in equal share across borders.

While our result calls for extensive robustness analysis and validation in larger and better specified model, we offer it as an instructive example flagging a key priority for policy design. The debate on the reform of the international financial architecture and policies under the headline of macro-pru cannot prescind from a firm grasp of the effects of wedges in asset pricing, efficiency gains and distribution of these gains from changes in the structure of asset markets.

If only as stylized instances, our set of result produce key insight for policy and empirical analysis. Liberalization of financial markets may/may not produce extensive repricing of a country's endowment of physical and human capital, in addition to valuation effects on its stock of (gross and net) foreign assets. Possible losses in consumption smoothing—corresponding to a popular view that participation in global financial markets may magnify income risk and vulnerability to global shocks—may be compensated by gains in wealth and average consumption. These gains are however heavily dependent on the way financial wedges inherent in international financial intermediation distort equilibrium pricing and allocation—even independently of issues in distribution.

Remarkably, the results obtained by varying a country size are very similar to the results obtained for symmetric economies by varying volatility of endowment (under the simplifying assumption of non cross-border correlation). We do not need to repeat

[9]One should not overlook the fact that domestic financial markets could be as or more imperfect than international financial markets. In our example we abstract from this issue and assume a frictionless domestic economy under autarky (a representative agent).

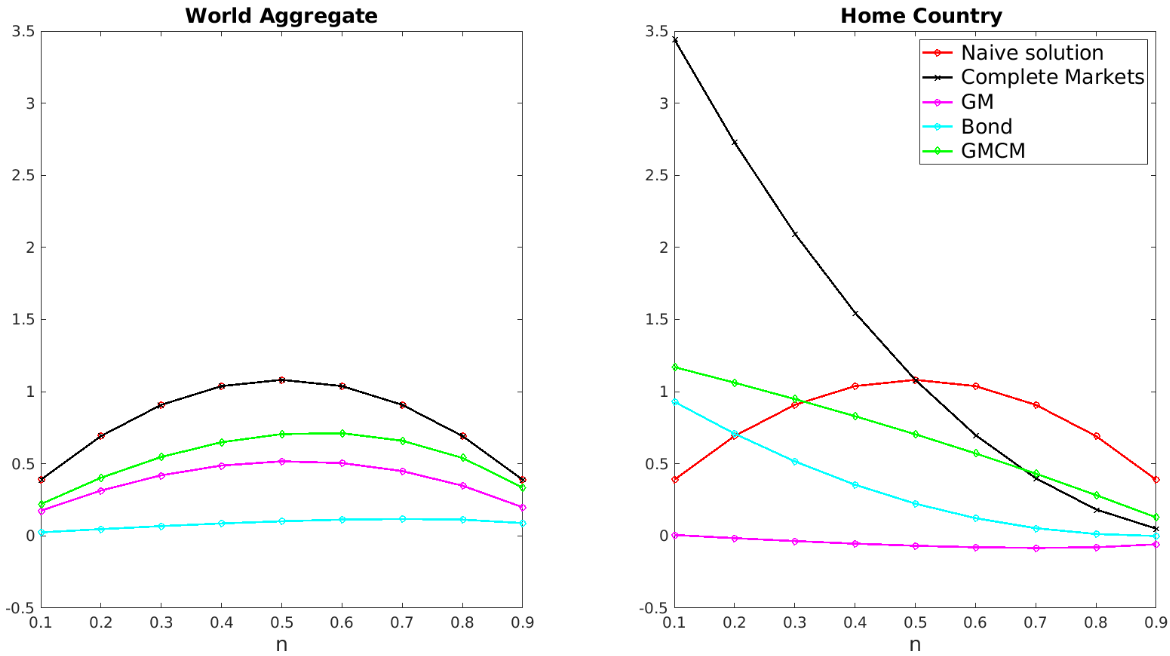
the comments made above. Rather, we call attention on the implications of a coding and modelling practice, such that the complete market economy is analyzed without including the budget constraint. The error in this practice is the presumption that the first order condition of perfect risk sharing is enough to guarantee that the allocation is efficient. This assumption overlooks that the resulting allocation is efficient up to a set of welfare weight in the planner problem. For a given initial endowment, this cannot be arbitrarily determined, but must reflect changes in the values of a country's asset—unless the researcher intends to model a programme of lump-sum transfers that keep the initial relative wealth unchanged.

Specifically, omitting the budget constraint amounts to erroneously setting the risk-sharing equal to an exogenous constant (zero), which in turn produces an incorrect assessment of the welfare gains from trade. Our figures illustrate that this error can be large and can even lead to spurious case of welfare reversals—by which a country is better off in financial autarky than under complete markets (as shown in the figure). This cannot be. The incorrect result simply highlights the content of our proposition, showing that consumption smoothing gains may be negative, under parameter configurations (i.e., features of the economy) by which most of the benefits from trade takes the form of a rise in wealth and average consumption.

In the last set of figures we show the effects of market liberalization on welfare and its constituents for different values of country 2 initial debt. Figures 8 and 9 refer to the case of variable-coupon perpetuities; Figures 10 and 11 to the case of constant-coupon perpetuities. In the first two figures, we use the label "debt" on the horizontal axis, as opposed to "tau" in the last two figures, for the case of constant coupons. Given our normalizations, the scale of debt is identical in both cases.

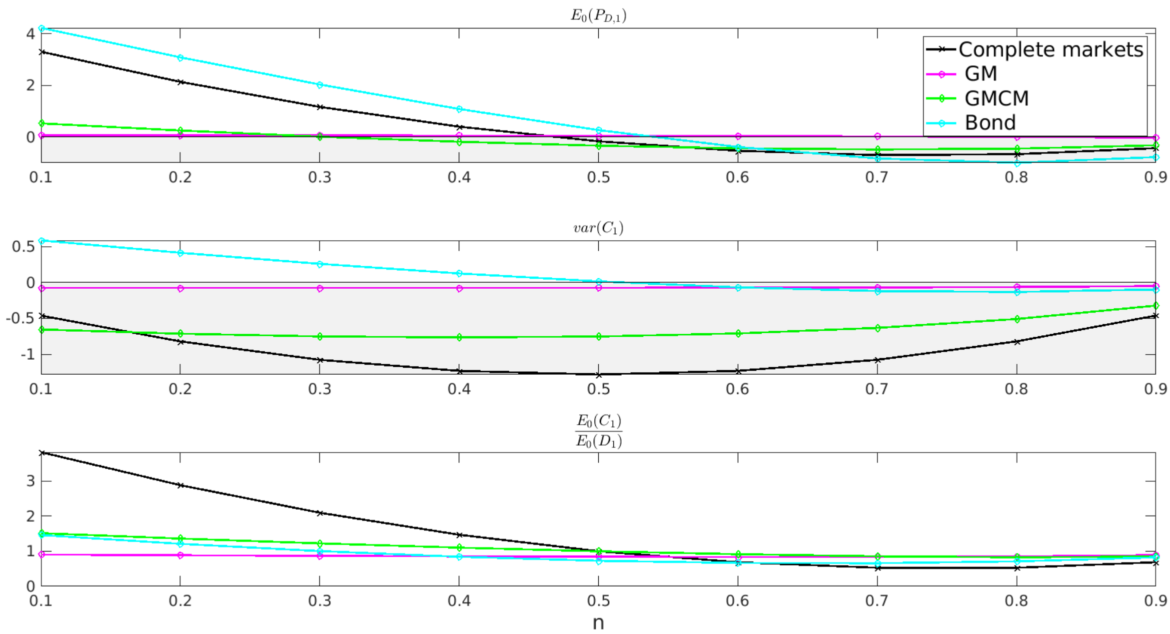
Two observations are in order. First, not surprisingly, valuation effects are larger for the constant-coupon case, as the repricing applies to the entire flow of current and anticipated future coupon. In the variable coupon case, instead, the repricing applies only to the first-period coupon (preset at time zero). Each subsequent coupon is adjusted to reflect the post-integration pricing kernel.

Figure 4: Welfare and relative home country size



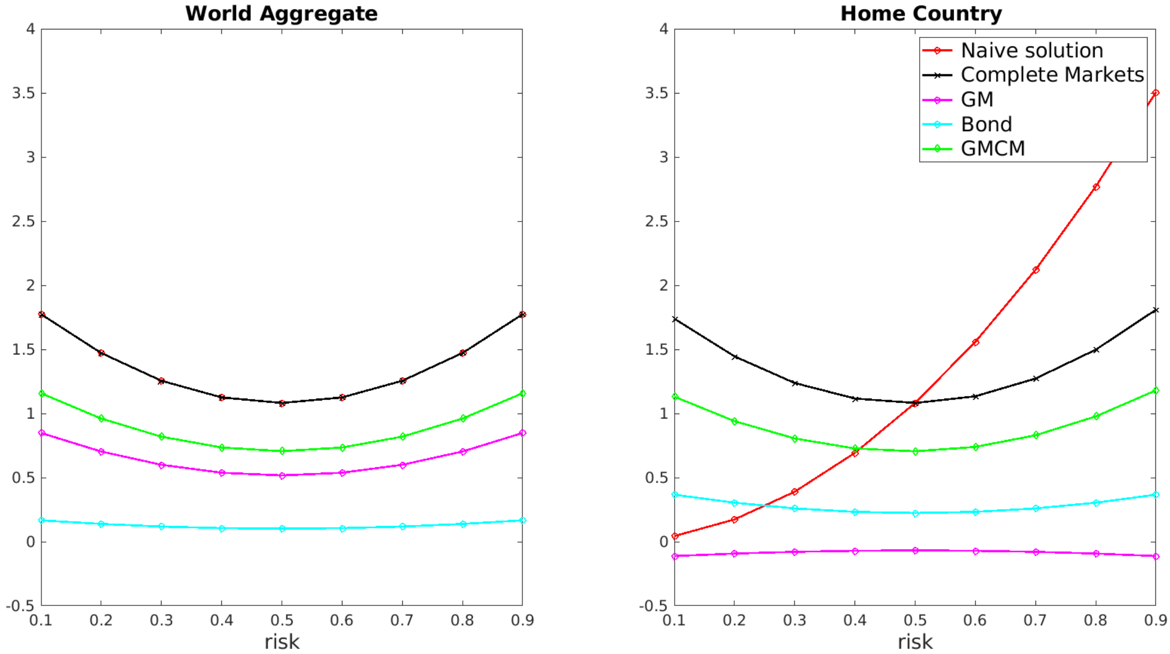
Note: Vertical axis: Welfare units relative to autarky.

Figure 5: Valuation and smoothing effects



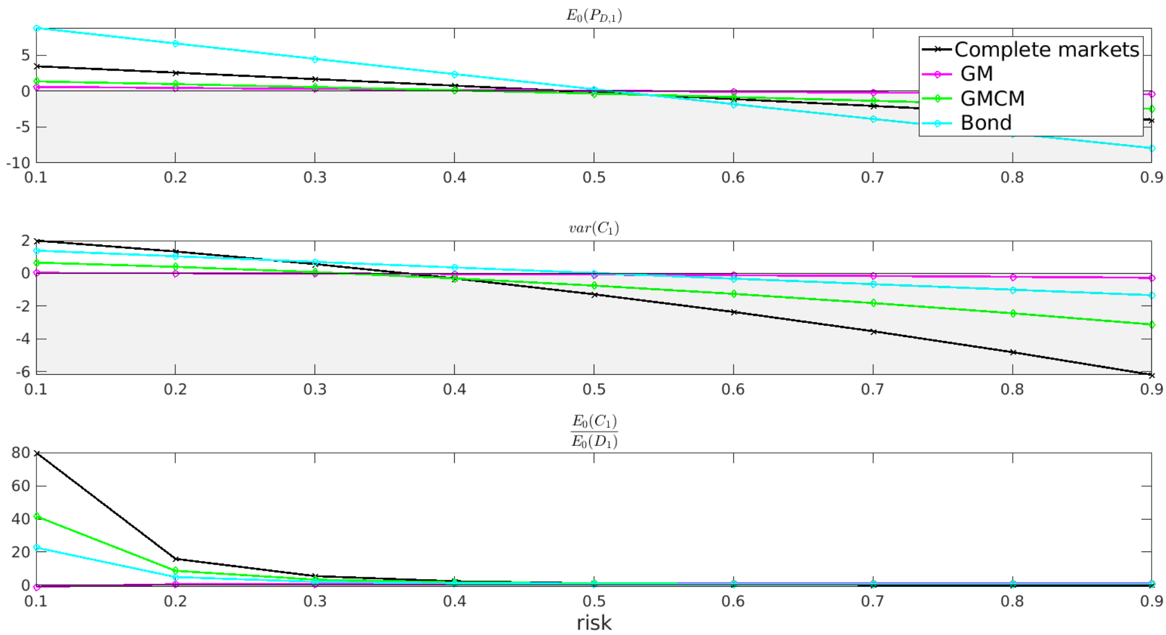
Note: In the first two panels variables are in log deviations. The bottom panel displays the ratio of conditional mean levels. In all cases values are in deviation from the autarky equilibrium.

Figure 6: Welfare and relative home country risk



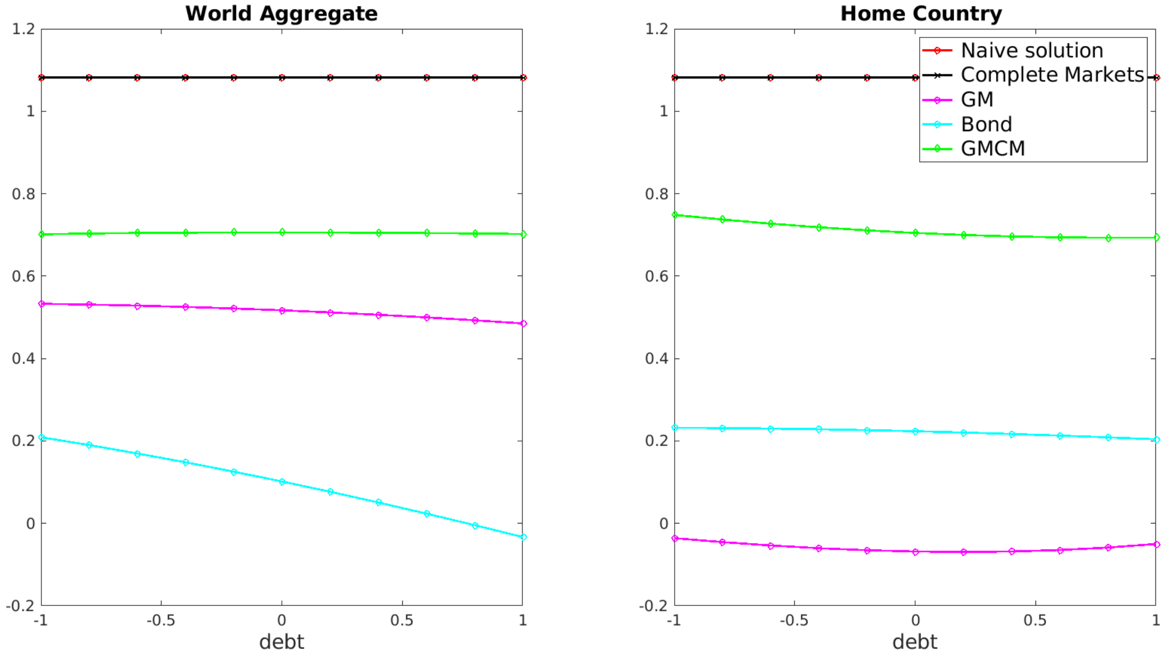
Note: Vertical axis: Welfare units relative to autarky.

Figure 7: Valuation and smoothing effect



Note: In the first two panels variables are in log deviations. The bottom panel displays the ratio of conditional mean levels. In all cases values are in deviation from the autarky equilibrium.

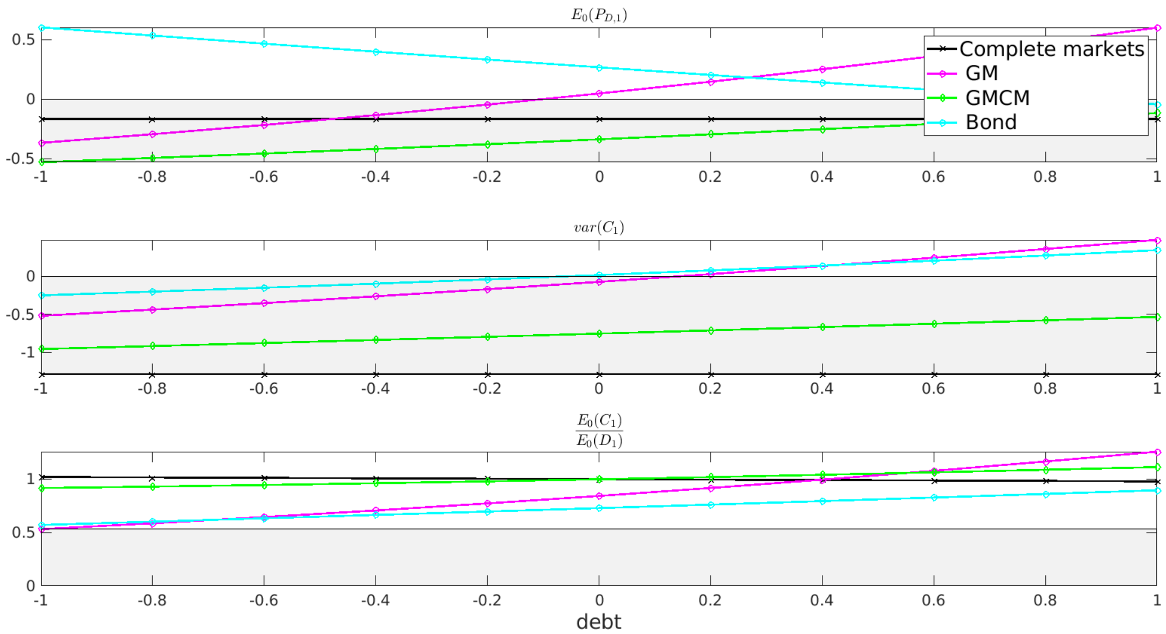
Figure 8: Welfare and country 2 variable rate debt



Note: Vertical axis: Welfare units relative to autarky.

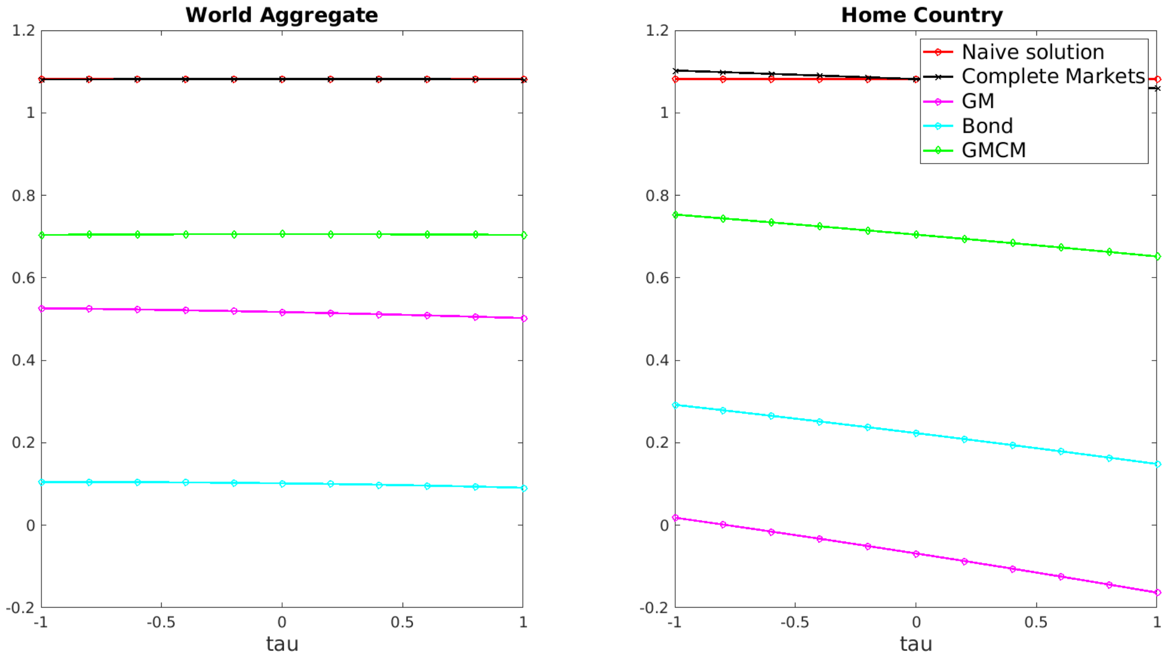
Second, for a stock of external assets/liabilities up to 100% of GDP, valuation effects and their impact on the allocation are relative contained—relative to the effects of repricing output at the new equilibrium state prices. Their combined weight is nonetheless large.

Figure 9: Valuation and country 2 variable rate debt



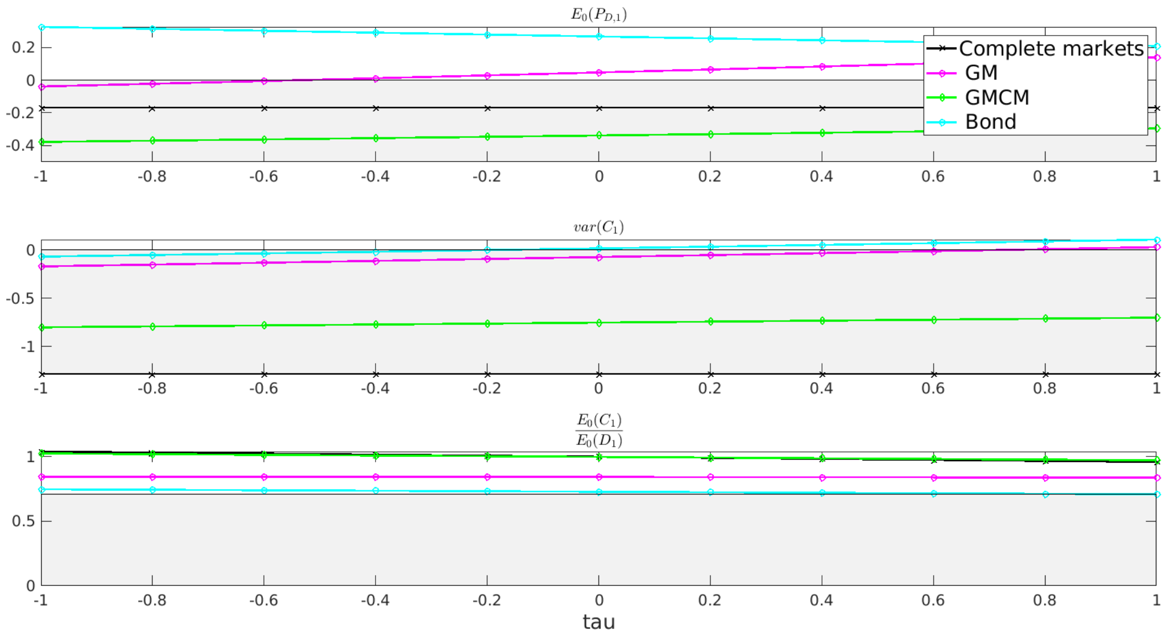
Note: In the first two panels variables are in log deviations. The bottom panel displays the ratio of conditional mean levels. In all cases values are in deviation from the autarky equilibrium.

Figure 10: Welfare and country 2 constant-coupon debt



Note: Vertical axis: Welfare units relative to autarky.

Figure 11: Valuation and country constant-coupon debt



Note: In the first two panels variables are in log deviations. The bottom panel displays the ratio of conditional mean levels. In all cases values are in deviation from the autarky equilibrium.

7 Conclusion

The success and longevity of an international financial market architecture rests crucially on its ability to generate widespread welfare improvements. Risk-sharing, among citizens and across countries, is generally seen as the main source of welfare gains, and thus as the main goal of financial integration. Oftentimes, though, risk-sharing is understood merely as mutual insurance, i.e. as a way to reduce fluctuations in consumption by diversifying the sources of income. The risk of relying exclusively on this metric is to conclude that, for example, safer countries should be wary of opening up their financial markets to trade with riskier countries.

Our paper revisits these fundamental questions. It points out the potential pitfalls of neglecting the fact that risk-sharing affects welfare through two main channels: consumption smoothing (insurance) and valuation effects. Using a simple two-country endowment model we derive an analytical and transparent representation of the constituents of the gains from risk sharing. We show that cross-country asymmetries in risk and size have sizable implications for the relative total gains from risk sharing, as well as for the distribution of the smoothing and valuation gains. Our results have important implications both for empirical research and for policy. Empirical studies on the benefits of risk sharing, and on the consequences of changes in financial market integration, can use our theoretical decomposition as a guide to data selection, modelling and inference. For example, the observation that upon joining a financial union the safer country has imported consumption volatility from the riskier ones, should not be construed as evidence of malfunctioning of risk-sharing arrangements. To reach such conclusion, evidence on the wealth-valuation effect should be collected and analyzed. Furthermore, the extent to which financial integration brings about higher social welfare, depends crucially on who can actually access financial markets, and to which extent. Financial imperfections, like asymmetric information or limited enforceability of contracts, place a limit to the ability of achieving full insurance through financial market instruments. So, even when a complete set of contingent securities would be available in principle, imperfections in financial market intermediation can drastically limit their risk-sharing benefits.

These observations have important consequences for policy. First, in designing international financial-market arrangements, policymakers should appreciate that the benefits cannot only be measured in terms of relative volatility. Second, regulatory and macroprudential policies, aimed at improving the efficiency and resilience of financial markets, should evaluate their (side-)effects not only in terms of aggregate volatility but also in terms of the valuation effects that such policies are bound to generate.

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Appendix

A Second-order accurate solution of the risk-sharing constant

In the simple Lucas' endowment model, the risk-sharing constant κ,[10] defined as

$$\kappa = \frac{C_{A,t}}{C_{B,t}} \tag{A.1}$$

[10] The term "constant" refers to time invariance. This coefficient is not invariant to risk.

depends recursively on exogenous variables, i.e.

$$\frac{1}{1+\kappa} = \frac{E_0 \sum_{t=0}^{\infty} \beta^t \left(D_{w,t}\right)^{-\rho} D_{1,t}}{E_0 \sum_{t=0}^{\infty} \beta^t \left(D_{w,t}\right)^{-\rho} D_{w,t}} \tag{A.2}$$

In general, and specifically when there is production, the right-hand-side of equation (A.2) is endogenous (e.g. the discount factor depends on consumption, and income depends on production). Therefore, to find the risk sharing constant in this more general setting we typically need to use some fixed-point algorithm.

We propose a closed-form solution for κ that is accurate up to second order. In describing our solution we follow a procedure that allows for easy numerical implementation, e.g. using Dynare.

Our solution is based on the observation that κ depends on second order terms, and in particular on the variance of the exogenous processes. We can thus represent the risk-sharing constant as

$$\log(\kappa) \equiv \bar{\kappa} E_t \varepsilon_{\kappa,t+1}^2 \tag{A.3}$$

so that

$$\bar{\kappa} E_t \varepsilon_{\kappa,t+1}^2 = \widetilde{C}_{1,t} - \widetilde{C}_{2,t} \tag{A.4}$$

where $\bar{\kappa}$ is the unknown parameter we want to solve for, and $\varepsilon_{\kappa,t}$, is a mean-zero iid auxiliary shock with variance denoted by σ_κ^2. This implies that $E_t \varepsilon_{\kappa,t+1}^2 = \sigma_\kappa^2$. So far we have thus re-scaled the original kappa by σ_κ^2.

The second-order solution of a DSGE model can be written in a second-order VAR form as (e.g. following Dynare notation)

$$y_t = A y_{t-1} + B u_t + \frac{1}{2}\left[C\left(y_{t-1} \otimes y_{t-1}\right) + D\left(u_t \otimes u_t\right) + 2F\left(y_{t-1} \otimes u_t\right)\right] + \frac{1}{2} G \vec{\Sigma}^2 \tag{A.5}$$

where $y_t \in \mathbb{R}^{n_y}$ is the vector of all the n_y variables (endogenous and exogenous excluding innovations), $u_t \in \mathbb{R}^{n_i}$ is the vector of all the n_i (iid) innovations, A, B, C, D, F, G are conformable matrices, and for any column vectors x and z, $(x \otimes z)$ is the vectorized outer product of these vectors. $\Sigma^2 \equiv E_t\left(u_{t+1}u'_{t+1}\right)$, and $\vec{\cdot}$ is the vectorization operator.[11]

The key term in equation (A.5) is the last one, which shifts the mean of variables in proportion to the exogenous risk, captured by the variance matrix Σ^2 (also referred to as the stochastic steady state in the literature).

Using regular perturbations (see e.g. Lombardo and Uhlig, 2018), none of the matrices in (A.5) depends on exogenous risk. This means that the only place where σ_κ appears is in Σ^2.

The vector y_t contains the variable measuring Arrow-Debreu securities. Assume the latter are in position i_{AD}, and that σ_κ occupies position j_{σ_κ} in the vector $\vec{\Sigma^2}$. Then we have that

$$\begin{aligned} y_t[i_{AD}] = A[i_{AD},:]y_{t-1} + B[i_{AD},:]u_t + \frac{1}{2}\left[C[i_{AD},:]\left(y_{t-1}\otimes y_{t-1}\right)\right. \\ \left. +D[i_{AD},:]\left(u_t \otimes u_t\right) + 2F[i_{AD},:]\left(y_{t-1}\otimes u_t\right)\right] + \frac{1}{2}G[i_{AD},:]\vec{\Sigma^2} \end{aligned} \tag{A.6}$$

where for a matrix X, $X[i,j]$ denotes the element in row i and column j, and where $X[i,:]$ denotes the row i of matrix X; for a vector z, $z[j_{\sigma_\kappa}]$ is the $j_{\sigma_\kappa} - th$ element in z. In particular, $\vec{\Sigma^2}$, i.e. $\vec{\Sigma^2}[j_{\sigma_\kappa}] = \sigma^2_\kappa$.

Note that if we set $\bar{\kappa} = 1$, we can solve for σ^2_κ that satisfy some restriction on $y_t[i_{AD}]$. In particular we know that under complete markets it must be that $y_0[i_{AD}] = 0$ (Ljungqvist and Sargent, 2012). One way to implement this condition is to assume that at time 0 and -1 the economy was at the stochastic steady state, i.e. all elements of

[11]To date, Dynare returns only the product $G\vec{\Sigma^2}$ in the variable "oo_.dr.ghs2". In order to implement our algorithm this product must be factorized in the two components. This can be easily done by modifying Dynare function `dyn_second_order_solver.m` at about line 173, by adding a new variable e.g. `dr.G=LHS\(-RHS);`, where LHS and RHS are variables defined in the function.

equation (A.6) are zero except the last one, i.e.[12]

$$y_0[i_{AD}] = 0 = \frac{1}{2} G[i_{AD}, :]\vec{\Sigma}^2 \tag{A.7}$$

Then we can solve for σ_κ^2 as

$$\sigma_\kappa^2 = -\frac{G[i_{AD}, j_{\sigma_\kappa}^\perp]\vec{\Sigma}^2[j_{\sigma_\kappa}^\perp]}{G[i_{AD}, j_{\sigma_\kappa}]} \tag{A.8}$$

where $j_{\sigma_\kappa}^\perp$ denotes all the elements excluding j_{σ_κ}.

Now we simply need to swap values, i.e.

$$\begin{aligned} \bar{\kappa} &\leftarrow \sigma_\kappa^2 \\ \sigma_\kappa^2 &\leftarrow 1. \end{aligned} \tag{A.9}$$

With this assignment of values κ is the second-order accurate risk-sharing constant that implements complete markets.

Our proposed algorithm, correctly implements complete markets up to second order accuracy. It should be noted also that our approach does not affect the first-order solution. This solution correctly describes growth rates of variables, since the risk-sharing constant is invariant to time (Ljungqvist and Sargent, 2012).

Our approach is reminiscent of the solution algorithm proposed by Devereux and Sutherland (2011) (DS) to solve for portfolio shares up to second order. DS introduce an auxiliary iid shock in the budget constraint of investors as a placeholder for portfolio shares. By knowing the position of this auxiliary shock DS can then use simple linear algebra to derive the shares. Although we solve a different problem, our algorithm shares with DS the idea of using auxiliary iid shocks as placeholders for parameters that would otherwise drop out of the perturbed solution.

[12] Equally easily implementable is any other condition, e.g. $Ey_0[i_{AD}] = 0$.

B Second order approximation of the model

Aggregate resource constraint:

$$n\bar{C}_1\widetilde{C}_{1,t} + (1-n)\bar{C}_2\widetilde{C}_{2,t} = \\ n\bar{D}_1\widetilde{D}_{1,t} + (1-n)\bar{D}_2\widetilde{D}_{2,t} - \frac{1}{2}\left[n\bar{C}_1\widetilde{C}^2_{1,t} + (1-n)\bar{C}_2\widetilde{C}^2_{2,t} - n\bar{D}_1\widetilde{D}^2_{1,t} - (1-n)\bar{D}_2\widetilde{D}^2_{2,t}\right] \tag{B.1}$$

We can replace (4.2) in equation (B.1) to obtain first and second order solutions of consumption in terms of exogenous processes, respectively,

$$\widetilde{C}_{1,t} = a_1\widetilde{D}_{1,t} + a_2\widetilde{D}_{2,t} \tag{B.2}$$

and

$$\widetilde{C}_{1,t} = \widetilde{\kappa}(1-n)a_2 + a_1\widetilde{D}_{1,t} + a_2\widetilde{D}_{2,t} + \frac{1}{2}a_3\left[\widetilde{D}^2_{1,t} + \widetilde{D}^2_{2,t} - 2\widetilde{D}_{1,t}\widetilde{D}_{2,t}\right] \tag{B.3}$$

where $\bar{D}_w = n\bar{D}_1 + (1-n)\bar{D}_1$, $a_1 \equiv \frac{n\bar{D}_1}{\bar{D}_w}$, $a_2 \equiv \frac{(1-n)\bar{D}_1}{\bar{D}_w}$, and $a_3 \equiv \frac{n\bar{D}_1(1-n)\bar{D}_1}{\bar{D}^2_w} = a_1a_2$.

Let's consider each term in equation (4.4) in turn, recalling that square and interaction terms must be replaced by first-order approximations, so that only the first term of equation (4.4) displays the risk-sharing constant:

$$\widetilde{C}^2_{1,t} = a_1^2\widetilde{D}^2_{1,t} + a_2^2\widetilde{D}^2_{2,t} + 2a_1a_2\widetilde{D}_{1,t}\widetilde{D}_{2,t}, \tag{B.4}$$

$$\widetilde{C}_{1,t}\widetilde{C}_{0,t} = a_1^2\widetilde{D}_{1,t}\widetilde{D}_{1,0} + a_2^2\widetilde{D}_{2,t}\widetilde{D}_{2,0} + a_1a_2\widetilde{D}_{1,t}\widetilde{D}_{2,0} + a_1a_2\widetilde{D}_{2,t}\widetilde{D}_{1,0} \tag{B.5}$$

and

$$\widetilde{C}_{1,t}\widetilde{D}_{1,t} = a_1\widetilde{D}_{1,t}^2 + a_2\widetilde{D}_{2,t}\widetilde{D}_{1,t} \tag{B.6}$$

$$\widetilde{C}_{1,0}\widetilde{D}_{1,t} = a_1\widetilde{D}_{1,t}\widetilde{D}_{1,0} + a_2\widetilde{D}_{2,0}\widetilde{D}_{1,t} \tag{B.7}$$

while the square of it is

$$\widetilde{\Lambda}_{1,t|0}^2 = \rho^2\left(\widetilde{C}_{1,t} - \widetilde{C}_{1,0}\right)^2 = \rho^2\left(\widetilde{C}_{1,t}^2 + \widetilde{C}_{1,0}^2 - 2\widetilde{C}_{1,t}\widetilde{C}_{1,0}\right) \tag{B.8}$$

The cross product of kernel and consumption is

$$\widetilde{\Lambda}_{1,t|0}\widetilde{C}_{1,t} = -\rho\left(\widetilde{C}_{1,t}^2 - \widetilde{C}_{1,t}\widetilde{C}_{0,t}\right) \tag{B.9}$$

and the cross product of kernel and dividend is

$$\widetilde{\Lambda}_{1,t|0}\widetilde{D}_{1,t} = -\rho\left(\widetilde{C}_{1,t}\widetilde{D}_{1,t} - \widetilde{D}_{1,t}\widetilde{C}_{1,0}\right) \tag{B.10}$$

Summations

For the sake of simplicity we assume that $\widetilde{D}_{1,0} = \widetilde{D}_{2,0} = 0$ and that shocks are iid, i.e. $\varphi_{D,1} = \varphi_{D,2} = 0$. Then, the present discounted sum of the terms in equation (4.4) reduce to

$$E_0\sum_{t=0}^{\infty}\beta^t\widetilde{D}_{1,t} = 0, \tag{B.11}$$

$$E_0\sum_{t=0}^{\infty}\beta^t\widetilde{D}_{1,t}^2 = \frac{\beta}{(1-\beta)}\sigma_{D,1}^2 \tag{B.12}$$

and

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{D}_{1,t} \widetilde{D}_{2,t} = 0 \tag{B.13}$$

Replacing gives a solution in terms of moments of the exogenous processes only.

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{C}_{1,t} = \frac{\widetilde{\kappa}(1-n)\bar{C}_2}{(1-\beta)\,\bar{D}_w} + \frac{1}{2} a_3 \frac{\beta}{(1-\beta)} \left(\sigma_{D,1}^2 + \sigma_{D,2}^2\right) \tag{B.14}$$

and

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{C}_{1,t}^2 = \frac{\beta}{(1-\beta)} \left(a_1^2 \sigma_{D,1}^2 + a_2^2 \sigma_{D,2}^2\right) \tag{B.15}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{C}_{1,t} \widetilde{C}_{0,t} = 0 \tag{B.16}$$

$$\widetilde{C}_{1,t} \widetilde{D}_{1,t} = a_1 \widetilde{D}_{1,t}^2 + a_2 \widetilde{D}_{1,t} \widetilde{D}_{2,t} \tag{B.17}$$

$$\widetilde{C}_{1,0} \widetilde{D}_{1,t} = 0 \tag{B.18}$$

so that

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{\Lambda}_{1,t|0} = -\frac{1}{2} \frac{\beta \rho a_3}{(1-\beta)} \left(\sigma_{D,1}^2 + \sigma_{D,2}^2\right) \tag{B.19}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{\Lambda}_{1,t|0} \widetilde{D}_{1,t} = -\frac{\beta \rho a_1}{(1-\beta)} \sigma_{D,1}^2 \tag{B.20}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{\Lambda}_{1,t|0} \widetilde{C}_{1,t} = -\frac{\beta\rho}{1-\beta} \left(a_1^2 \sigma_{D,1}^2 + a_2^2 \sigma_{D,2}^2\right) \tag{B.21}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{\Lambda}_{1,t|0}^2 = \frac{\beta\rho^2}{1-\beta} \left(a_1^2 \sigma_{D,1}^2 + a_2^2 \sigma_{D,2}^2\right) \tag{B.22}$$

Replacement

We are finally in a position to replace all terms in equation (4.4). An important point to note is that all the terms involving $\left(\bar{C}_1 - \bar{D}_1\right)$ are zero in a symmetric non-stochastic steady state, which is the point of approximation we are considering. Then we have

$$\begin{aligned} \widetilde{A}_{1,0} = \bar{D}_1 \left(\frac{\widetilde{\kappa}(1-n)\bar{C}_2}{(1-\beta)\,\bar{D}_w} + \frac{1}{2} a_3 \frac{\beta}{(1-\beta)} \left(\sigma_{D,1}^2 + \sigma_{D,2}^2\right) \right) + \\ -\bar{D}_1 \frac{\beta\rho}{1-\beta} \left(a_1^2 \sigma_{D,1}^2 + a_2^2 \sigma_{D,2}^2 - a_1 \sigma_{D,1}^2\right) + \\ +\frac{1}{2} \bar{D}_1 \frac{\beta}{(1-\beta)} \left(a_1^2 \sigma_{D,1}^2 + a_2^2 \sigma_{D,2}^2 - \sigma_{D,1}^2\right) = 0 \end{aligned} \tag{B.23}$$

which yields:

$$\widetilde{\kappa} = -\frac{\beta}{2} \frac{a_3}{a_2} \left(\sigma_{D,1}^2 + \sigma_{D,2}^2\right) + \frac{\beta}{a_2} \left(\rho - \frac{1}{2}\right) \left(a_1^2 \sigma_{D,1}^2 + a_2^2 \sigma_{D,2}^2\right) - \frac{\beta}{a_2} \left(\rho a_1 - \frac{1}{2}\right) \sigma_{D,1}^2 \tag{B.24}$$

In the main text, *wlog* we maintain the assumption that per-head steady state endowments are symmetric (per-head endowments play the same role as size of the country), and we further normalize them to 1, which implies that $\bar{D}_w = 1$ and $a_1 = n$, $a_2 = (1-n)$, and $a_3 = n(1-n)$. By the same token, to isolate the effect of relative risk from that of total risk, we normalize the latter to 1 and assume that $\sigma_{D,1}^2 = \gamma$ and $\sigma_{D,2}^2 = 1 - \gamma$ for some $\gamma \in (0,1)$.

C Proof of Proposition 1

Proposition 1 can be proved by direct calculation:

Proof. To prove the proposition, just take the partial derivatives:

$$\frac{\partial \kappa}{\partial n} = -\beta\rho < 0 \tag{C.1}$$

and

$$\frac{\partial \kappa}{\partial \gamma} = -\beta(\rho - 1) \begin{cases} \leq 0 \text{ if } \rho \geq 1 \\ > 0 \text{ otherwise} \end{cases} \tag{C.2}$$

One can verify that the Hessian of equation 4.5 is zero. □

D Proof of Proposition 2

Proposition 2 can be proved by direct inspection of the derivatives of equation (4.15). In particular

Proof.

$$\frac{\partial E_t \left(\widetilde{C}^2_{1,aut,t+1} - \widetilde{C}^2_{1,t+1} \right)}{\partial \gamma} = 2\,(1 - n) > 0 \tag{D.1}$$

which proves point i).

Point ii) is proved by noting that when a country relative riskiness is below the critical value $\frac{1}{2}\gamma^*$ with $\gamma^* = (1 - n)$, its gains are negative. Since $\gamma \in (0, 1)$ and $(1 - n) \in (0, 1)$ if $\gamma > 0.5$ (country 1 is riskier) it must be that $2\gamma > (1 - n)$, and so the riskier country always gains in terms of smoothing.

Finally, point iii) is proved by noting that

$$\frac{\partial E_t\left(\widetilde{C}^2_{1,aut,t+1}-\widetilde{C}^2_{1,t+1}\right)}{\partial(1-n)}=2\left(\gamma-(1-n)\right), \tag{D.2}$$

and that

$$\frac{\partial^2 E_t\left(\widetilde{C}^2_{1,aut,t+1}-\widetilde{C}^2_{1,t+1}\right)}{\partial(1-n)^2}=-2<0 \tag{D.3}$$

so that the gains are maximized at γ^*.

Point iv) is proved by inspection of the cross derivative, i.e.

$$\frac{\partial^2 E_t\left(\widetilde{C}^2_{1,aut,t+1}-\widetilde{C}^2_{1,t+1}\right)}{\partial\gamma\partial(1-n)}=2>0. \tag{D.4}$$

□

E Approximation of asset prices

Iterating forward the asset price equation (excluding bubbles) and taking a second order approximation yields

$$\widetilde{P}_{D,j,0}+\frac{1}{2}\widetilde{P}^2_{D,j,0}=E_0\sum_{t=1}^{\infty}\beta^t\left(\widetilde{X}_{j,t}+\frac{1}{2}\widetilde{X}^2_{j,t}\right). \tag{E.1}$$

where we used the fact that $\bar{D}_j=1$ and where

$$\widetilde{X}_{j,t}\equiv\widetilde{\Lambda}_{j,t|0}+\widetilde{D}_{j,t} \tag{E.2}$$

so that

$$\widetilde{X}^2_{j,t}=\widetilde{\Lambda}^2_{j,t|0}+\widetilde{D}^2_{j,t}+2\widetilde{\Lambda}_{j,t|0}\widetilde{D}_{j,t} \tag{E.3}$$

Importantly, this equation shows that, up to second order, assets prices have a premium component consisting of the covariance of the pricing kernel and the endowment process.

We can exploit the results in Appendix B to show that

$$E_0 \sum_{t=1}^{\infty} \beta^t \widetilde{X}_{j,t} = -\frac{1}{2} \frac{\beta \rho n(1-n)}{(1-\beta)} \tag{E.4}$$

and

$$E_0 \sum_{t=1}^{\infty} \beta^t \widetilde{X}^2_{1,t} = \frac{\beta}{(1-\beta)} \left[\rho^2 \left(n^2 \gamma + (1-n)^2 (1-\gamma) \right) - (2\rho n - 1)\, \gamma \right] \tag{E.5}$$

Up to first order we have that

$$\widetilde{P}_{D,j,0} = E_0 \sum_{t=1}^{\infty} \beta^t \widetilde{\Lambda}_{j,t|0} = 0 \tag{E.6}$$

so that $\widetilde{P}^2_{D,j,0} = 0$.

Finally we can write

$$\widetilde{P}_{D,1,0} = \frac{\beta}{2\,(1-\beta)} \left[\rho^2 \left(n^2 \gamma + (1-n)^2 (1-\gamma) \right) - (2\rho n - 1)\, \gamma - \rho n(1-n) \right] \tag{E.7}$$

Under autarky we have that

$$\widetilde{X}_{j,aut,t} = (1-\rho) \widetilde{D}_{j,t} \tag{E.8}$$

and

$$\widetilde{X}^2_{j,aut,t} = \rho^2 \left(\widetilde{D}_{j,t} - \widetilde{D}_{j,0} \right)^2 + \widetilde{D}^2_{j,t} - 2\rho \left(\widetilde{D}_{j,t} - \widetilde{D}_{j,0} \right) \widetilde{D}_{j,t} \tag{E.9}$$

and thus

$$E_0 \sum_{t=0} \beta^t \widetilde{X}_{j,aut,t} = 0 \tag{E.10}$$

$$E_0 \sum_{t=0} \beta^t \widetilde{X}^2_{1,aut,t} = (\rho - 1)^2 \frac{\beta}{1-\beta} \gamma \tag{E.11}$$

and

$$\widetilde{P}_{D,1,aut,0} = \frac{\beta}{2(1-\beta)} (\rho - 1)^2 \gamma \tag{E.12}$$

Taking the difference between prices under complete markets and under autarky yields the expression in the text.

F Proof of Proposition 3

Proposition 3 can be proved by direct calculation.

Proof. The Jacobian of equation 4.16 is

$$\frac{\partial E_0 \sum_{t=0}^{\infty} \beta^t \left(\widetilde{C}_{1,t} - \widetilde{C}_{1,aut,t} \right)}{\partial [n, \gamma]'} = \frac{\beta}{1-\beta} \begin{pmatrix} (1 - \gamma - n - \rho\,(2(1-n) - \gamma)) \\ -\,(1-n)\,(\rho - 1) \end{pmatrix} \tag{F.1}$$

and the Hessian is

$$\frac{\partial E_0 \sum_{t=0}^{\infty} \beta^t \left(\widetilde{C}_{1,t} - \widetilde{C}_{1,aut,t} \right)}{\partial [n, \gamma]' [n, \gamma]} = \frac{\beta}{1-\beta} \begin{pmatrix} (2\,\rho - 1) & (\rho - 1) \\ (\rho - 1) & 0 \end{pmatrix} \tag{F.2}$$

Therefore, there is a minimum (maximum) level effect of consumption in n if

$\rho > 0.5$ ($\rho < 0.5$), at

$$n = \frac{(2-\gamma)\ \rho - (1-\gamma)}{2\,\rho - 1}. \tag{F.3}$$

If $\rho = 0.5$ size does not affect average consumption in deviation from the autarkic equilibrium. If $\rho \geq 1$ the minimum is at $n \leq 1$. So, for typical calibrations ($\rho > 1$) average consumption (relative to autarky) is non-monotonically *decreasing* in size to reach a minimum at $n = \frac{(2-\gamma)\,\rho-(1-\gamma)}{2\,\rho-1}$ and then increase to 0 as $n \to 1$.

As for risk, if $\rho > 1$ ($\rho < 1$), average consumption decreases (increases) monotonically in risk, and the interaction between risk and size is positive (negative).

□

G Proof of Proposition 4

The proof of Proposition 4 can be obtained by direct calculation:

Proof. The Jacobian and Hessian of equation (4.17) are

$$\frac{\partial E_0 \sum_{t=0}^{\infty} \beta^t \left(\widetilde{U}_{1,t} - \widetilde{U}_{1,t}^{Aut}\right)}{\partial [n\ \gamma]'} = \frac{\beta}{1-\beta} \left[\begin{array}{cc} -\left(\gamma + (1-n)(\rho-1)\right) & (1-n) \end{array}\right] \tag{G.1}$$

and

$$\frac{\partial E_0 \sum_{t=0}^{\infty} \beta^t \left(\widetilde{U}_{1,t} - \widetilde{U}_{1,t}^{Aut}\right)}{\partial [n\ \gamma]'[n\ \gamma]} = \frac{\beta}{1-\beta} \left[\begin{array}{cc} (\rho-1) & -1 \\ -1 & 0 \end{array}\right] \tag{G.2}$$

respectively. Direct inspection of the derivatives proves proposition 4. □

H Extending the model: Valuation effects and outstanding foreign assets/liabilities

In our analysis so far, we have assumed that, at time zero, no country has any initial outstanding claim on (or liability to) the other. Holding this assumption, adding non-contingent debt securities to the complete set of Arrow Debreu securities would not alter the allocation, as they would be redundant. This is not the case if, at time zero, there are outstanding asset and debt position across borders.

In this section, we analyze the consequences of any initial outstanding positions for the welfare gains from moving from financial autarky to complete markets. In line with our analysis, we can expect changes in equilibrium prices to alter the valuation of these assets, impinging on the level of consumptions and, if only indirectly, on consumption smoothing.

We model the initial outstanding debt as perpetuities, under two assumptions regarding the coupon, either variable or fixed. With variable coupon adjusted to the equilibrium rates, the valuation effect boils down to variations in the one-period pricing kernel. With fixed coupon, the valuation effects amounts to changes in the long-run interest rate, or cumulative pricing kernel.

H.1 Variable-coupon perpetuity

To analyze the case of perpetuities with variable coupons, we assume that, some time before time 0, countries have traded in a asset T that pays in each period t the non-contingent rate R_{t-1}, set one period before. Once complete financial markets are open at time 0 (and countries start trading in one-period Arrow securities), trade in perpetuities becomes redundant and indeterminate. So, *wlog*, we posit that no new perpetuity is created at and after time 0, and that its ownership never changes.

With complete markets the price of the variable coupon perpetuity is:

$$E_t \Lambda_{1,t+1|0} R_t = 1. \tag{H.1}$$

To account for initially outstanding debt in our model, we simply need to modify a few equations. In particular, country 1 households budget constraint becomes

$$A_{1,0} = E_0 \sum_{t=0}^{\infty} \Lambda_{1,t|0} \left(C_{1,t} - D_{1,t} - \left(R_{t-1} - 1 \right) T \right), \tag{H.2}$$

or by using equation (H.1)

$$\begin{aligned} A_{1,0} = E_0 \sum_{t=0}^{\infty} \Lambda_{1,t|0} \left(C_{1,t} - D_{1,t} \right) + \\ -\Lambda_{1,0|0} \left(R_{0-1} - 1 \right) T - E_0 \sum_{t=1}^{\infty} \left(1 - \Lambda_{1,t|0} \right) T. \end{aligned} \tag{H.3}$$

. Keeping our assumption that the economy at time 0 is at the deterministic steady state (with debt), we have

$$C_{1,0} = D_{1,0} + (\frac{1-\beta}{\beta})T. \tag{H.4}$$

The second order expansion is

$$\begin{aligned} \widetilde{B}_{1,0} = E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \left(\bar{C}_1 - \bar{D}_1 \right) \widetilde{\Lambda}_{1,t|0} + \\ + \left(\bar{C}_1 \widetilde{C}_{1,t} - \bar{D}_1 \widetilde{D}_{1,t} \right) + \\ + \widetilde{\Lambda}_{1,t|0} \left(\bar{C}_1 \widetilde{C}_{1,t} - \bar{D}_1 \widetilde{D}_{1,t} \right) + \\ + \frac{1}{2} \left(\bar{C}_1 - \bar{D}_1 \right) \widetilde{\Lambda}^2_{1,t|0} + \\ + \frac{1}{2} \left(\bar{C}_1 \widetilde{C}^2_{1,t} - \bar{D}_1 \widetilde{D}^2_{1,t} \right) \Big\} + \\ + T\, E_0 \sum_{t=1}^{\infty} \beta^t \left(\widetilde{\Lambda}_{1,t|0} + \frac{1}{2} \widetilde{\Lambda}^2_{1,t|0} \right), \end{aligned} \tag{H.5}$$

and where we have used equation (H.4) and the fact that there are no deviations at time 0, or

$$\begin{aligned}\widetilde{B}_{1,0} = E_0 \sum_{t=1}^{\infty} \beta^t \Bigg\{ \bar{D}_1 \left(\frac{\bar{D}_1 + \frac{1-\beta}{\beta} T}{\bar{D}_1} \widetilde{C}_{1,t} - \widetilde{D}_{1,t} \right) + \\ + \widetilde{\Lambda}_{1,t|0} \left(\bar{C}_1 \widetilde{C}_{1,t} - \bar{D}_1 \widetilde{D}_{1,t} \right) + \\ + \frac{1}{2} \left(\bar{C}_1 \widetilde{C}^2_{1,t} - \bar{D}_1 \widetilde{D}^2_{1,t} \right) \Bigg\} + \\ + \frac{T}{\beta} E_0 \sum_{t=1}^{\infty} \beta^t \left(\widetilde{\Lambda}_{1,t|0} + \frac{1}{2} \widetilde{\Lambda}^2_{1,t|0} \right) = 0,\end{aligned} \tag{H.6}$$

or

$$\begin{aligned}\widetilde{B}_{1,0} = E_0 \sum_{t=1}^{\infty} \beta^t \Bigg\{ \left(\widetilde{C}_{1,t} - \widetilde{D}_{1,t} \right) + \frac{1-\beta}{\beta} \tau \widetilde{C}_{1,t} + \\ + \widetilde{\Lambda}_{1,t|0} \left(\widetilde{C}_{1,t} - \widetilde{D}_{1,t} \right) + \frac{1-\beta}{\beta} \tau \widetilde{\Lambda}_{1,t|0} \widetilde{C}_{1,t} + \\ + \frac{1}{2} \left(\widetilde{C}^2_{1,t} - \widetilde{D}^2_{1,t} \right) + \frac{1-\beta}{2\beta} \tau \widetilde{C}^2_{1,t} \Bigg\} + \\ + \frac{\tau}{\beta} E_0 \sum_{t=1}^{\infty} \beta^t \left(\widetilde{\Lambda}_{1,t|0} + \frac{1}{2} \widetilde{\Lambda}^2_{1,t|0} \right) = 0,\end{aligned} \tag{H.7}$$

where $\tau \equiv \frac{T}{\bar{D}_1}$, and where here $\bar{D}_1$ is per capita.

We have already computed $\widetilde{\kappa}$ when $\tau = 0$. So we simply need to evaluate the terms involving τ in equation (H.7).

Appendix B shows that

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{\Lambda}_{1,t|0} = -\frac{1}{2} \frac{\beta \rho n(1-n)}{(1-\beta)} \tag{H.8}$$

and

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{\Lambda}^2_{1,t|0} = \frac{\beta \rho^2}{1-\beta} \left(n^2 \gamma + (1-n)^2 (1-\gamma) \right) \tag{H.9}$$

for the price kernel, and

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{C}_{1,t} = \frac{\widetilde{\kappa}(1-n)\bar{C}_1}{(1-\beta)\,\bar{D}_w} + \frac{1}{2} n(1-n) \frac{\beta}{(1-\beta)} \tag{H.10}$$

and

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{C}_{1,t}^2 = \frac{\beta}{(1-\beta)} \left(n^2 \gamma + (1-n)^2 (1-\gamma) \right) \tag{H.11}$$

for consumption, and

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{\Lambda}_{1,t|0} \widetilde{C}_{1,t} = -\frac{\beta \rho}{1-\beta} \left(n^2 \gamma + (1-n)^2 (1-\gamma) \right) \tag{H.12}$$

for the cross product.

Adding these together with the weights displayed in equation (H.7) gives

$$\begin{aligned} \tau \left(\frac{\widetilde{\kappa}\bar{C}_2}{\beta \bar{D}_w} + \frac{1}{2} n(1-n) \right) + \\ + \tau \frac{1}{2} \left(n^2 \gamma + (1-n)^2 (1-\gamma) \right) + \\ - \tau \rho \left(n^2 \gamma + (1-n)^2 (1-\gamma) \right) + \\ \tau \frac{1}{2} \left(\frac{\rho^2}{1-\beta} \left(n^2 \gamma + (1-n)^2 (1-\gamma) \right) - \frac{\rho n (1-n)}{(1-\beta)} \right) \end{aligned} \tag{H.13}$$

or

$$\begin{aligned} \tau \frac{\widetilde{\kappa}\bar{C}_2}{\beta \bar{D}_w} = \\ - \tau \frac{1}{2} n(1-n) \left(1 - \frac{\rho}{(1-\beta)} \right) + \\ - \tau \frac{1}{2} \left(1 - 2\rho + \frac{\rho^2}{1-\beta} \right) \left(n^2 \gamma + (1-n)^2 (1-\gamma) \right) \end{aligned} \tag{H.14}$$

Recall that equation (H.15) gives an expression for $\widetilde{\kappa}$, when $\tau = 0$, i.e.

$$\frac{\bar{C}_2}{\beta \bar{D}_w} \widetilde{\kappa}|_{\tau=0} = -\frac{1}{2} n(1-n) + \left(\rho - \frac{1}{2} \right) \left(n^2 \gamma + (1-n)^2 (1-\gamma) \right) - \left(\rho n - \frac{1}{2} \right) \gamma. \tag{H.15}$$

Adding the two gives an expression for κ when $\tau \neq 0$:

$$\frac{1}{\beta}\left(1-n-n\frac{1-\beta}{\beta}\tau\right)(1+\tau)\widetilde{\kappa} = -\frac{1}{2}n(1-n)\left(1+\tau\left(1-\frac{\rho}{(1-\beta)}\right)\right)$$
$$+\left(\rho-\frac{1}{2}-\tau\frac{1}{2}\left(1-2\rho+\frac{\rho^2}{1-\beta}\right)\right)\left(n^2\gamma+(1-n)^2(1-\gamma)\right)-\left(\rho n-\frac{1}{2}\right)\gamma. \quad \text{(H.16)}$$

This shows that the effect of outstanding debt on relative consumption is non-linear, and that the sign of the marginal effect of debt on consumption shares ($\frac{\partial\widetilde{\kappa}}{\partial\tau}$) depends on a number of factors, like size, relative risk, risk aversion and discount factor. In the numerical analysis, in Section 6, we illustrate the effect of debt for a particular parametrization of the model.

H.2 Constant-coupon perpetuity

We now assume that the perpetuity (T) pays a constant coupon, set, *wlog*, equal to the deterministic steady-state interest rate (i.e. β^{-1}). The derivation of $\widetilde{\kappa}$, in this case is relatively simple, relative to the variable-rate case. In particular, we can directly use equation (H.5) dropping the last term, since this is specific to variable rates. Since the rest still applies, we can write the solution for the risk-sharing constant as:

$$\frac{1}{\beta}\left(1-n-n\frac{1-\beta}{\beta}\tau\right)(1+\tau)\widetilde{\kappa} = -\frac{1}{2}n(1-n)(1+\tau)$$
$$+\left(\rho-\frac{1}{2}-\tau\frac{1}{2}(1-2\rho)\right)\left(n^2\gamma+(1-n)^2(1-\gamma)\right)-\left(\rho n-\frac{1}{2}\right)\gamma. \quad \text{(H.17)}$$

We discuss the effect of constant-coupon debt only numerically in Section 6.