

CORPORATE DEBT MATURITY AND THE REAL ECONOMY

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Abstract

How do firms manage debt maturity in the presence of investment opportunities? I document empirically at aggregate and firm levels that corporations lengthen their average maturity of debt when output and investment rates are larger and average credit spreads are lower. To explain these findings, I construct an economic model where firms dynamically choose investment, short-term debt, and long-term debt. In equilibrium, long-term debt is more costly than short-term debt and is only used when investment opportunities present themselves in peaks of the business cycle. This framework suggests that economic stability and lower credit risk are reflected in firms that are able to hold more leverage and a higher proportion of long-term debt.

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1 Introduction

The financial crisis of the late 2000's placed debt maturity concerns at the forefront of the economic policy debate. As firms with relatively more short-term debt were exposed to rollover and liquidity crises, new questions arose as to how long-term debt could affect firm and economic stability.¹ While some academic work tackle these issues, they do not provide a link between debt maturity and investment behavior.² My work provides a comprehensive, quantitative framework that connects debt maturity choice, corporate bond yields, and the endogenous *assets* of the firm's balance sheet.

Empirically, I document several novel facts that discuss the positive link between business cycles and the long-term debt share. At the aggregate level, I use US Federal Reserve Financial Accounts data to identify the portion of total non-financial, corporate liabilities that are considered long-term. This ratio is significantly correlated with GDP and aggregate investment growth, with predictive power up to six quarters in the future. Using Compustat data at the firm level as well, I show that when firms shift their long-term debt ratio to a longer average maturity, profitability and investment rates are higher. These results are robust to controlling for a variety of macroeconomic and financial factors.

In order to understand these phenomena, I design a dynamic, heterogeneous firm, capital structure model in which corporations optimally issue equity and debt of short and long maturities. Using external financing and cash flows from production, firms finance investment into profit-generating capital. I compute and calibrate the model to target cross-sectional and aggregate data related to investment, leverage, default, and credit spreads.

The model generates a pro-cyclical long-term debt ratio through an endogenously generated, time-varying pecking order of capital market securities. The framework also implies that stable firms, which are more capitalized and have a larger portion of long-term debt, matter more for the real economy. Despite higher quantities of leverage and long-term debt, their average credit spreads are lower. Altogether, endogenous investment plays a crucial role in driving leverage and debt maturity choice, default dynamics, and credit spreads.

The conditions of Miller and Modigliani (1959) would suggest that firms are indifferent to debt of varying maturities, under a *pari passu* treatment of securities. To break this

¹It is well-documented that the over abundance of short-term liabilities on corporate balance sheets helped cause runs in commercial paper and repurchase agreement markets during the financial crisis (see eg. BNP Paribas, Bear Stearns, Lehman Brothers, General Electric). In non-crisis events as well, survey evidence suggest that CFO's take on long-term debt to "reduce risk of having to borrow in 'bad times'" (see Graham and Harvey, 2002; Servaes and Tufano, 2006).

²My focus on the endogenous asset choice of firms separates my paper from Chen et al. (2013) and He and Milbradt (2016), where cash flows are taken to be exogeneous. Ivashina and Scharfstein (2010) and Campello et al. (2011) discuss the impact of the financial crisis event on investment, but do not discuss the explicit role of long-term debt in a larger context.

indifference, the model splits the proportion of distress costs that are held by short and long-term debt. Each period, firms have access to debt issuance in a short-term, collateralized debt contract which prices in relatively less default risk. Meanwhile the longer term debt market inherits a greater burden of default risk and associated distress costs.

In a higher aggregate state of the model economy, firms optimally choose to invest more than they generate in profits. In order to do so, they need to acquire external financing. Due to a tax advantage of debt, and the fact that short-term debt is the least costly form of financing, via the collateral constraint, corporations seek to first finance their need for cash using short-term debt issuance.

For additional financing, the firm has the choice of issuing long-term debt or equity. In positive economic environments, it is desirable for corporations to take on more long-term debt as its effective cost is lower; this is due to a lower probability of default that results in lower expected distress costs. Put differently, in good states of the world, yields on long-term debt compare favorably to issuance costs in equity.

In poor aggregate states, when the marginal gains from investing are lower, the firm doesn't require as much external financing. However, if some firms do seek to externally finance more than is available through short-term markets, the marginally higher credit spreads and effective costs of longer term debt make it an unattractive option. In this case, firms would rather obtain external financing using equity than long-term debt.

As my model involves a heterogeneous firm setup, I also provide implications for the cross section. Sorting firms on distance to default, I find that firms that are closer to exiting have higher credit spreads, less capital, *less leverage*, and *less long-term debt*. The italicized statements are counter-intuitive; we would expect leverage to have a negative relationship with firm stability. These results directly imply that in the model's equilibrium cross-section, capital is a much larger driver of firm default. Furthermore, these findings suggest that economic stability is positively linked to a higher long-term debt ratio.

To underscore the importance of endogenous investment, I show that default events are largely precipitated by a joint drop in productivity levels and capital. During a sequence of negative productivity shocks in an economic recession, eventually-defaulting firms disinvest in response to a decreasing marginal product of capital. However, due to the burden of long-term debt on their balance sheet, firms eventually choose to exit. Firms do have the option to purchase back debt, but for defaulting firms, this option becomes limited as they have reduced funds from production and constraints on borrowing from short-term debt markets.

In comparison to the classical leverage and default framework of Leland and Toft (1996), there are three major differences: (i) via investment, firms shift the level of capital resulting in an endogenous dividend stream, (ii) the firm continually rebalances the book values of

short and long-term debt (a setup with “non-commitment”), (iii) when using external financing, the firm chooses between additional equity issuance and debt issuance of two different maturity types.

As documented in Chatterjee and Eyigungor (2012), models that feature long-term debt with fair pricing have difficulties with numerical convergence. Due to the discrete nature of firm default and the dependence on future policy functions, the long-term debt pricing function fluctuates greatly, which leads to non-convergence in value as well. In order to remedy this problem, I extend the computational method cited in the above paper, to allow for a simultaneous choice of both capital and debt policies. The additional choice variables complicate the problem substantially.³ I introduce zero-mean IID noise with a low amount of variance into the dividend payments. By calculating default breakpoints perfectly as a function of these noise variables and taking expectations over default and non-default regions of the noise, I am able to smooth out the discrete jumps and improve convergence of the model.

Having described the key results, I now provide a roadmap for the rest of the paper. I conclude this section by providing a literature review. In the following section, I discuss the procyclicality of the long-term debt share. The third section is dedicated to discussing the model I use to study these issues, via dynamic issuance of multiple debt maturities. Following this, I delve into the quantitative implications arising from the model, including the key mechanisms and cross-sectional implications. In the final section I conclude.

Previous Literature. This paper relates to many strands of literature regarding corporate credit spreads, capital structure, and the macroeconomy. I discuss my work in the context of each area and provide differences. The overarching theme is that I connect capital structure with multiple debt maturities, endogenous investment and output, and asset prices in a dynamic structural model.

As my model provides endogenous prices for short and long-term debt, this paper connects to the literature regarding structural models of credit. Merton (1974), Leland (1994), and Leland and Toft (1996) serve as historical benchmarks in this area. In the large majority of this work, firms are risk neutral with cash flows expressed as exogenous Gaussian diffusion processes. Using the known statistical distribution of firm value given the current state, alongside optimal choice for debt and default boundaries, we can compute closed form

³In the cited paper, consumer income is taken to be an exogenous process, while in my paper the firm’s income is chosen endogenously, as investment is a choice variable. My extension compares very similarly to the methodology used in Gordon and Guerron-Quintana (2016). The key difference is that I also have short-term debt (which I account for through a collateral constraint).

expressions for bond prices.⁴ However, as discussed in Huang and Huang (2012), the large conclusion of this literature is that structural credit models undershoot credit risk premia when matching default rates. This is partially due to the fact that model-based state prices (Arrow-Debreu prices) are not volatile or countercyclical enough. In order to correct for these problems, Bhamra et al. (2010) and Chen (2010) utilize Epstein and Zin (1989) preferences in order to increase the risk exposures of credit securities. My paper is different from these studies in that my dynamic model allows corporations to dynamically invest. Furthermore, at each point in time, firms have access to issuing multiple maturities of debt, both of which can change in book terms (“non-commitment”).

This paper connects with the vast literature of dynamic models with endogenous investment. Many of these setups include firms that have a time-varying capital structure of equity and debt. A non-exhaustive list of papers includes Gomes (2001), Whited and Wu (2006), and Hennessy and Whited (2007). Hennessy and Whited (2005) were the first to discuss debt-equity tradeoffs in a business cycle model with idiosyncratic and aggregate shocks. Livdan et al. (2009) use a structural corporate model to discuss the relationship between firm constrainedness and asset prices. Perhaps closest to my work, Kuehn and Schmid (2014) develop a partial equilibrium firm model with endogenous investment, recursive preferences-based stochastic discount factor, and long-term debt. I extend their model to have an additional, short-term debt choice that is governed through a collateral constraint. My paper also relates to work by Covas and Den Haan (2011) and Jermann and Quadrini (2012) which discuss the fact that debt issuance is procyclical. In my model, I try to match the more granular fact that *the share of long-term debt* is positively correlated with output and investment. Crouzet (2015) discusses how multiple equilibria may arise in models with multiple debt maturities and investment. The main difference between the previous paper and my own is that within the scope of my model the short-term debt is assumed to be less costly, via the collateral constraint. This is a key driver of the pro-cyclical long-term debt share in my paper. Finally, a recent paper (Alfaro et al., 2016) discusses how economic uncertainty interacts with financial frictions to cause larger shifts in short-term debt than those in long-term debt. A key difference between their work and my own is that they use collateral constraints to maintain a risk free term structure. In my model long term debt, in particular, is risky.

In terms of work regarding maturity choice, Diamond (1991) suggests that cross-sectional heterogeneity of long-term debt shares can be linked to firm level signals in the form of credit

⁴The calculation of an optimal default boundary is done through a “smooth-pasting condition” that leaves the equityholder indifferent at the boundary, between continuing to operate and dissolving. In many papers, this is also known as “strategic default.”

ratings. More recently, Greenwood et al. (2010) try to understand the time-variation of corporate debt maturity choice. They suggest that the time series patterns of corporate debt maturity are linked to investor-related substitution effects between government and corporate debt. I provide evidence and construct a model that instead utilizes firm investment as a key driver for explaining the pro-cyclical long-term debt share. He and Milbradt (2016) discuss a dynamic debt rebalancing problem with short and long-term debt. Their model features classes of equilibria, one in which firms continually shorten the overall maturity of their debt and another in which they lengthen overall maturity. There are many ways in which my model differs from theirs, but one key difference again is that I allow for an endogenous choice of assets, which provides me the opportunity to discuss the investment-related impact of debt maturity. Finally, Chen et al. (2013) discuss the impact of multiple debt maturities in a structural credit model. They are able to show that the additional use of long-term debt cuts credit spreads. Through my model, I will be able to make a similar statement regarding debt prices and debt maturity, while also speaking to the endogenous asset side of the balance sheet.

A more recent literature discusses empirical evidence regarding debt refinancing. Using syndicate loan data from the FDIC, Mian and Santos (2011) provide evidence that credit-worthy firms borrow at and extend existing loans to longer maturities when economic climates are positive in order to weather liquidity crises that might occur later. Similarly, Xu (2015) suggests that speculative grade firms issue or refinance longer maturity debt in favorable market conditions. Both of these papers support the general economic story my model displays. Other empirical work (see Ivashina and Scharfstein, 2010; Campello et al., 2011) have discussed how firms utilize alternative forms of liquidity (in particular, cash and credit lines) in order to buffer operations in the Great Recession period. As I calibrate my model to public market data, I focus my attention on tradeoffs between short and long-term debt and abstract away from alternative securities. Finally, a recent paper (Choi et al., 2016) examines the granularity or spread of debt maturity dispersion within corporations. Using data from Mergent's Fixed Income Securities Database (FISD) they suggest that younger and smaller firms have a debt maturity that is less diverse across corporate debt, while mature and larger firms display the exact opposite.

The sovereign default literature also discusses maturity tradeoffs for emerging market economies. Arellano and Ramanarayanan (2012) explain how issuing long-term debt serves as a hedge to future movements in debt prices. Broner et al. (2013) suggest that emerging economies borrow sovereign debt at a maturity that lengthens in expansions of domestic business cycles. My economic story broadly agrees with this time-varying procyclical nature of debt maturity, in the context of US firms. On the computational front, I adopt techniques

from this literature, first introduced in Chatterjee and Eyigungor (2012) and extended by Gordon and Guerron-Quintana (2016). In both of these papers, IID noise is introduced into the (effective) dividend flow payment to help smooth out the bond price calculation. This smoothing is very useful to handle the discrete jumps that come with the nature of default decisions. While I don't provide a proof for existence under the use of the IID noise (as is done in these papers), I do use this tool to help convergence greatly.

2 Pro-cyclicality of Long Term Debt Ratio

In this section, I document the fact that the share of long-term debt is positively associated with business cycles. I present evidence at both the aggregate and firm levels.

2.1 Aggregate Dynamics

Using quarterly data at the U.S. Federal Reserve Financial Accounts going back to 1952, I construct a measure of the share of long-term debt, where long-term debt includes aggregated corporate, mortgage, and municipal debt on the balance sheet of non-financial corporations.⁵ As discussed in Greenwood et al. (2010), this series contains a time-varying trend and I correct for it by extracting the cyclical component of the long-term debt share. In Figure 1, I provide a graph of this series, obtained through a Hodrick and Prescott (1997) filter. The grey bars indicate NBER recession dates and the left hand axis indicates changes in percentage points of the ratio. I find that the ratio decreases in recessions and increases in expansions of the business cycle.

This is made quantitatively clear in Figure 2. I compute cross correlation functions between cycle components of the long-term debt ratio and measures of the business cycle. All figures on the left hand side represent correlations between output growth and various cyclical measures of the long-term debt share, while those on the right hand side report correlations using investment growth. From top to bottom, the cyclical components are measured using Hodrick and Prescott, Baxter and King (1999), and Christiano and Fitzgerald (2003) filters, respectively. Across all three filters, I find that the contemporaneous correlations between economic aggregates and the long-term debt share are significantly positive. Using the HP-filtered value, for example, it is at the order of 35% while its correlation with investment growth is roughly 40%. Similar results hold for the BK-filtered debt share and the CF filter, particularly at lag zero. Altogether firms take on more long-term debt and extend the length

⁵The same measure is discussed in Chen et al. (2013). However they do not compute explicit correlations between the ratio, output, and investment growth.

of their maturity structure in business cycle expansions.

I seek to further understand the procyclical features of the long-term debt share measure, by examining predictive regressions. I project average, future output growth onto a set of controls and the HP-filtered long-term debt share, given here by $LTDR^c$:

$$\frac{1}{k} \sum_{i=1}^k \Delta y_{t+i} = \beta_0 + \beta'_X X_t + \beta_{LT} LTDR_t^c + error_{t+k}$$

The vector of controls, X_t , includes a wide array of lagged macroeconomic and financial variables known to have predictive power for business cycles, including lagged output growth, consumption growth, inflation, price-dividend ratios, credit spreads, and U.S. Treasury bond yields. The top panel of Table 1 reports the coefficients of the long-term ratio and its associated t-statistic after correcting for serial and autocorrelated errors using Newey and West (1987) adjustment. The long-term ratio predicts output at a very significant rate, up to four quarters out, even when controlling for a wide array of factors. Perhaps stemming from the higher raw correlation presented earlier, results are even stronger when I perform the same regression using investment growth as a dependent variable. In the bottom panel of the same table, I show that coefficients are larger and t-statistics are close to five in the first year. In terms of economic magnitudes, these results suggest that one percent of additional long-term debt share is associated with roughly .60% more output growth and 3% more investment growth at the annual basis. Putting both the contemporaneous and predictive facts together, the aggregate long-term debt ratio contains significant positive economic news.

2.2 Firm-Level Dynamics

Using firm-level data I seek to confirm facts shown at the aggregate level. I construct another measure of long-term debt, as that in Barclay and Smith (1995), with quarterly, Compustat data. I define the long-term debt ratio as the share of debt that is greater than one year at issuance. This measure includes any corporate bonds, mortgages and municipal debt that firms have on their balance sheet. It also includes long-term leases and wage contracts, but excludes long-term accounts payable. I do not provide full sample statistics of this measure, but in summary, the average firm holds close to 70% in the form of debt over one year. The fact that this number is so large suggests how important publically-issued long-term debt is. As financial, public, and utility firms are regulated in their capital structure behavior I remove these firms from the sample by way of their SIC codes. I also remove firm-quarter observations if there are extreme quarterly movements in market leverage or

the long-term debt ratio (in the bottom or top 1%). This is meant to remove the effects of capital structure shifts, due to merger and acquisition activity or divestitures. Due to data quality issues related to completeness, I run all tests using data following the first month of 1984.

As there is non-stationarity in many of the firm-level variables, I adjust firm quantities by their overall size and estimate the link between capital-adjusted profits (π_i/k_i) and the long-term debt share ($LTD R_i$):

$$\frac{\pi_{it}}{k_{it}} = \beta_0 + \beta'_X X_{it} + \beta_{ld1} LTD R_{it} + error_{it}$$

where X_{it} indicates a vector of firm and aggregate level controls, depending on the specification. I provide the results of this regression in Table 2. From left to right, I test multiple specifications where I successively add (i) firm-level controls, (ii) macroeconomic controls, and (iii) firm fixed effects. Following the main feature in Gilchrist et al. (2014), I include a term accounting for the historical, four-quarter volatility in profitability. I also include contemporaneous leverage, lagged investment rate, lagged market to book (“average Q”), and the long-term debt ratio. Macroeconomic controls are similar to the past subsection and include quarterly growth rates of industrial production and the consumer price index, U.S. treasury yields, and the average aggregate credit spread.

The conclusion that is consistent across all specifications is that the long-term debt ratio, at the firm level, is significantly associated with higher levels of profitability, beyond the 1% confidence level. In terms of economic magnitudes, a one standard deviation movement in the long-term debt share would be associated with a roughly 6.5% increase in profitability, from the baseline average. I can also run the same regression using contemporaneous rates of capital-adjusted investment. The results of this projection are provided in Table 3. Again, I find that the long-term debt ratio is significant in its association with investment. Economic magnitudes are similar here as a one standard deviation increase in the long-term debt share is associated with a roughly 6.8% increase in the capital-adjusted investment rate.

As in our aggregate regression results, we also check the power that firm specific long-term debt shares have in predicting profitability and investment rates. In Table 4, we display the results of both of these regressions. In the top panel, the left hand side displays average future profitability between one and four quarters out, while on the right hand side are the usual controls including the long-term debt share. It is evident that the predictive power is significant and (intuitively) declines from one quarter out to four quarters out. In economic magnitudes, a standard deviation movement in the current-long term debt share correlates with 4.1% additional future profitability (at one quarter forward) and 1.5% more

average profitability (four quarters). The bottom panel reports statistics for the investment regression, where conclusions are similar. There is significant predictive power up to four quarters.

Another interesting data experiment to run would be to check how regression results change across firm size; in particular, do particular types of firms have stronger correlations between their long-term debt shares and fundamentals. In Table 5, we do exactly this, running pooled regressions after first sorting firms into size quintiles on a monthly basis. A conclusion that is borne out of this (effective) double sort is that smaller firms have a much larger sensitivity between their respective long-term debt share and fundamental statistic. In the top panel for example, the relationship between profitability and the long-term debt share for small firms is much more significant than that for large firms, where it is in fact insignificant. The same holds in the bottom panel with respect to the investment relationship.

Through all the analysis in this section, I am not claiming that the long-term debt ratio *causes* firm and aggregate conditions to change. Rather I display these facts in order to prove its *positive correlation* with the business cycle, and in particular investment rates.

3 Economic Model

In order to better understand these empirical patterns, I introduce a dynamic, heterogeneous firm economy in which corporations maximize expected, discounted cash flows arising from endogenous investment. In order to finance their operations, they use funds from production, short-term and long-term debt issuance, and equity issuance. The optimal, simultaneous choice of investment, short and long-term debt issuance uniquely separates this framework from the literature. In the rest of this section I precisely lay out the structure of the economy and discuss the solution technique to the model.

3.1 Cash Flow Risks, Investment, and Production

The economy is populated by a large continuum of firms that are subject to both aggregate and idiosyncratic risks. The aggregate state of the economy is determined through consumption growth at t , denoted by Δc_t . It follows a first-order autoregressive process:

$$\Delta c_t = \mu_c + \rho_c \Delta c_{t-1} + \sigma_c \varepsilon_{ct} \tag{1}$$

While Δc_t governs the aggregate state of the cycle, the shocks that enter into firms' cash flows will be related to another variable, X_t , whose growth rate will be given by Δx_t :

$$\log(X_t/X_{t-1}) \equiv \Delta x_t = \mathbb{E}[\Delta c_t] + \lambda_x (\Delta c_t - \mathbb{E}[\Delta c_t]) \quad (2)$$

From the above equation, Δx_t will have the same mean as Δc_t however the volatility is larger for $\lambda_x > 1$. In terms of the model's performance, I increase the volatility of the aggregate portion of total firm productivity to increase the tie between default and the business cycle. Empirically as well, the growth rate of total factor productivity is multiple times more volatile than what is found in consumption growth data.

The firm is also exposed to idiosyncratic risks, which will create an endogenous cross-sectional distribution in quantities and prices. The idiosyncratic productivity will be given by a mean zero $x_{i,t}$:

$$x_{i,t} = \rho_x x_{i,t-1} + \sigma_x \varepsilon_{xt} \quad (3)$$

Going forward, I will denote all firm specific variables with the subscript i .

The cash flow shocks will enter into the operating profits of the firm, π_{it} . The profits are decreasing returns to scale in capital, with a factor $0 < \alpha < 1$, and taxed at a rate τ . They are given by:

$$\begin{aligned} \pi_{it} &= (1 - \tau) A_{it} k_{it}^\alpha \\ \text{s.t. } A_{it} &= \exp(\bar{x} + x_{it} + (1 - \alpha) \log(X_t)) \end{aligned} \quad (4)$$

where k_{it} measures the amount of capital the firm has on hand at the start of period t . The A_{it} term accounts for both aggregate and idiosyncratic productivity. \bar{x} is a constant, which scales the value function and does not change the core results of the model.⁶ As $\log(X_t)$ is a unit root variable that grows over time (implied by the consumption growth process), the model exhibits stochastic growth around a trend. When solving the model, we correct for the (time-varying) trend. There is more to be said about this when I discuss the solution technique to the model.

Firms make investment choices each period in response to economic conditions. Denote i_{it} as the amount of investment made at time period t . This will imply that next period capital, $k_{i,t+1}$ is known today and given by:

$$k_{i,t+1} = (1 - \delta)k_{i,t} + i_{it} \quad (5)$$

⁶The scale, \bar{x} , is selected by analytically solving a no-leverage economy and equating steady state capital to one.

Existing capital depreciates at a rate δ , which will factor into the steady state rate of investment.⁷ Adjusting capital is not costless and these costs are given by:

$$\Phi_k(k_{it}, i_{it}) = \frac{\phi_k}{2} \left(\frac{i_{it}}{k_{it}} - \frac{i_{ss}}{k_{ss}} \right)^2 k_{it} \quad (6)$$

where ϕ_k is a constant parameter and $\frac{i_{ss}}{k_{ss}}$ is the steady state investment-to-capital ratio in the model. I impose investment adjustment costs in order to slow the speed at which firms invest. In the literature investment adjustment costs are both empirically founded (Ramey and Shapiro, 2001) and help explain various features in asset pricing models. Zhang (2005), for example, suggests that a more costly downward adjustment of capital is crucial to rationalize the larger return on high book-to-market stocks.

3.2 Discount Factor

A driving force behind much of the structural asset pricing literature is an adjustment to the physical probability measure in valuing cash flows. The stochastic discount factor in many models (see Campbell and Cochrane, 1999; Bansal and Yaron, 2004) values cash flows in bad states of the world at a higher rate relative to those in good states. I also embody this intuition in my model as the firm discounts its cash flows at a countercyclical rate, using an Epstein and Zin (1989) discount factor. The use of an Epstein and Zin pricing kernel is crucial to ensure that default events are properly priced into credit spreads, while keeping risk free rates reasonably low.⁸

At time t , each firm discounts its possible cash flows at $t + 1$ using an aggregate discount factor, $M_{t+1} = M(\Delta c_t, \Delta c_{t+1})$. This pricing kernel must satisfy the following conditions:

$$\begin{aligned} M_{t+1} &= \beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{PC_{t+1} + 1}{PC_t} \right)^{\theta-1} \\ \mathbb{E}_t \left[M_{t+1} \left(\frac{C_{t+1}}{C_t} \right) \left(\frac{PC_{t+1} + 1}{PC_t} \right) \right] &= 1 \end{aligned} \quad (7)$$

where $\frac{C_{t+1}}{C_t} = e^{\Delta c_{t+1}}$ and PC_t is the level of the price consumption ratio at time t . The time discount rate is β , risk aversion is given by γ , and ψ governs the intertemporal elasticity of substitution. As is common in the literature, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. The second equation results from

⁷As there is stochastic growth in this economy, the steady state investment to capital ratio will be given by $\exp(\Delta x_{ss}) - (1 - \delta)$. If there was no growth in steady state ($\Delta x_{ss} = 0$) steady state investment to capital is δ .

⁸In cases where the Epstein and Zin friction is not present ($\gamma = \frac{1}{\psi}$), I find that the credit spread shrinks dramatically.

the first order condition of a household’s consumption-savings problem. Using this Euler condition, we can solve for M_{t+1} via numerical techniques.

It is important to note that the discount factor is not part of a larger general equilibrium problem. There are no households (or investors) that maximize over equity holdings in the continuum of firms. Connecting an aggregate household to firms with heterogeneous capital structure is more challenging and beyond the scope of this paper. I leave this for future research.

3.3 Capital Structure

Every period, the firm can issue debt of two types – short (S) and long (L). Short term debt requires repayment the following period while long-term debt only requires a fractional payment and takes the form of an annuity. Both forms of debt also have a proportional coupon, c , that provides a tax advantage for debt. Meanwhile, firms also pay a fraction κ_L of outstanding long-term book debt each period. Suppose at time t , firms issue a new amount of debt in book value terms, w_{it}^S and w_{it}^L . This will imply that the new book debt outstanding at the start of $t + 1$ are:

$$\begin{aligned} b_{i,t+1}^S &= w_{i,t}^S \\ b_{i,t+1}^L &= (1 - \kappa_L)b_{i,t}^L + w_{i,t}^L \end{aligned} \tag{8}$$

To enforce that type L is indeed longer term at issue, I will set $\kappa_L < 1$. Hence, the average duration of long-term debt will be $\frac{1}{\kappa_L}$.

Modeling debt as an annuity helps us simplify the problem as I only need to keep track of the current book value of debt as a state variable. Nonetheless, this restriction still allows me to capture the basic intuition that long-term debt provides the opportunity to pay a smaller per-period payment. As a modeling assumption, this form of debt is not new either. It is the same as the sinking fund provision used in Leland and Toft (1996), Hackbarth et al. (2006), among many others. The key difference is again, that I allow the firm to choose between two types of debt at a dynamic rate.

The firm will face a collateral constraint on its short-term debt.⁹ I impose that short-term debt to be paid off in the future, including the coupon, is no more than a fraction of

⁹It is true that collateralized short-term paper are mostly applicable for the low duration liabilities of financial firms (Kacperczyk and Schnabl, 2010), which is at odds with the non-financial data I calibrate the model to. That being said, many non-financial commercial paper contracts are associated with a standby line of credit, which helps reduce the paper’s risk properties for potential investors (Coyle, 2002). A direct implication of the model, through the alternative mechanism of the constraint, will be a reduced risk profile for short-term debt.

capital, net depreciation, next period:

$$(1 + c)b_{i,t+1}^S \leq s_0(1 - \delta)k_{i,t+1} \quad (9)$$

where $s_0 \leq 1$. Furthermore I will assume that equity holders and long-term debt holders recognize that short-term debt holders are senior claimants upon default. The reason I do so is that the combination of these assumptions will imply that short-term debt will be a risk free claim. That is to say, upon default, the firm will always have enough capital on hand to service repayment of short-term debt. Hence the price of one dollar's worth of short-term debt, p_t^S , is the risk free discounted value of $(1 + c)$:

$$p_t^S = \mathbb{E}_t [M_{t+1}(1 + c)] \quad (10)$$

While the use of a collateral constraint and seniority are simplifying assumptions, it helps us model default risk solely in the long-term debt security. This makes the model much more computationally tractable to be taken to the data. It also has roots theoretically. Diamond (1993) suggests that the seniority and collateralization of short-term debt can serve as compensation for monitoring costs of short-term creditors. This compensation will make it incentive-compatible for short-term creditors to not run on the firm, allowing the scope for future debt issuance as well.

One might ask whether the seniority and risk free nature of the model's short-term debt hold in the data. Based on the Financial Accounts data discussed in the empirical section, a very large portion of short-term debt (on average, 95%) constitute of loans. To the extent these loans are extended by banks they are almost always senior, as discussed in Welch (1997). The relatively risk free nature of bank loans can also be corroborated by examining recovery rates. In Figure 3, I provide recovery rates across debt types, as provided by Moody's recovery database for non-financial corporations. In the twenty years prior to the financial crisis, the median recovery rate for bank loans was 100%.¹⁰ Contrastingly, in the same time period, the median recovery rates for corporate bonds ranged from 67% to 2%, depending on the seniority structure of the particular debt contract. The clear differences of recovery rates suggest to us that the risk-free rate assumption for short-term bank debt is not far from reality.

¹⁰The data for recovery rates are taken from "Moody's Ultimate Recovery Database" (Emery et al. (2007)) and cover 3500 non-financial loans and bonds from 1987 – 2007. Recovery rates vary across industry, debt type, and seniority, among other categories.

3.4 Default and Debt Valuation

The equity value of a firm accounts for the discounted stream of lifetime profits. Each period, after realizing both idiosyncratic (x_i) and aggregate (X) shocks, the corporation can choose whether to (a) continue operations or (b) default and transfer residual assets to bondholders. In the model, I define a default event occurring when the value from continuing operation is too low relative to a threshold. In terms of an equation, this means that:

$$\mathbb{1}_{\{\text{Default}, it\}} = \begin{cases} 1, & \text{if } V_{it} \leq (\bar{V} X_{t-1}) \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

where V_{it} indicates the value from continuing operations and \bar{V} is a constant. Notice that this constant multiplies the business cycle shock indicating that the overall threshold value is time-varying and procyclical.

One way to interpret the default condition is that when $\bar{V} > 0$ equity holders or managers have an outside option to consider. In Eisfeldt and Papanikolaou (2013), for example, a similar outside option exists where talented labor that manage a particular type of capital, have the ability to walk away from the firm. Another, more relevant interpretation, would be a Chapter 11 reorganization as discussed in Corbae and D’Erasmus (2016). Equityholders “re-organize” such that bankruptcy proceedings determine the fraction of firm value that short and long-term debt holders receive. Following this procedure, existing equity holders retain firm value and resume operations as an unlevered firm.¹¹

When the firm goes into bankruptcy the bondholder will receive any remaining undepreciated capital and profits generated from the capital, net a repayment of the short-term debt holder. That is to say the payment given default at time $t + 1$ is:

$$X_{i,t+1}^{pd} = (1 - \xi) (\pi_{i,t+1} + (1 - \delta)k_{i,t+1} - (1 + c)b_{i,t+1}^S) \quad (12)$$

where ξ represents losses in default, which we can think to be related to legal and administrative fees paid out in bankruptcy.¹² At this point it is clear why the short-term debt holder will always be repaid in default. Because $(1 - \delta)k_{i,t+1} \geq (1 + c)b_{i,t+1}^S$ due to the collateral constraint, there will always be enough capital on hand to repay the senior claimant. This will imply that the difference between the right two terms above is always greater than zero.

Now I price the risky long-term debt. The equilibrium price, denoted by p_{it}^L , will equate

¹¹When simulating the model this is almost exactly the procedure that we conduct. The only difference however is that firm capital is reset upon default.

¹²A similar form for payment given default is used in Hennessy and Whited (2007) and Kuehn and Schmid (2014)

total lent funds to total expected proceeds next period. In period t , the firm chooses a new amount of issuance, w_{it}^L , which brings him to a book value of $b_{i,t+1}^L$. The price on the new dollar of debt will reflect the total default risk of obtaining a new level of book debt. In order to obtain a level, $b_{i,t+1}^L$ we will have:

$$p_{it}^L b_{i,t+1}^L = \mathbb{E}_t \left[M_{t+1} (1 - \mathbb{1}_{\{\text{Default}, i, t+1\}}) \times ((\kappa_L + c)b_{i,t+1}^L + (1 - \kappa_L)p_{i,t+1}^L b_{i,t+1}^L) \right] \\ + \mathbb{E}_t \left[M_{t+1} \left(\mathbb{1}_{\{\text{Default}, i, t+1\}} \right) \times X_{i,t+1}^{PD} \right] \quad (13)$$

The pricing equation can be understood in the following manner. The left hand side of the first line represents the total funds lent. On the right hand side of the first line are the payments that occur when the firm does not default. Again this includes both the effective coupon payment and the market value of remaining debt. The right hand side of the second line accounts for the payment upon default.

3.5 Equity Valuation

Shareholders seek to maximize the sum of discounted dividend payouts, taking into account the ability to potentially default in the future. Conditional on not defaulting, the firm will earn profits, choose investment, and issue short and long-term debt. The recursive formulation of each firm's problem is given by:

$$V_{it} = \max_{\{k_{i,t+1}, b_{i,t+1}^S, b_{i,t+1}^L\}} \{D_{it} - \Phi_e(D_{it}) + \mathbb{E}_t [M_{t+1} W_{i,t+1}]\} \\ D_{it} = \pi_{it} + \tau(\delta k_{it} + c b_{it}^S + c b_{it}^L) \\ - i_{it} - \Phi_k(i_{it}, k_{it}) k_{it} \\ + p_{it}^S w_{it}^S + p_{it}^L w_{it}^L - (1 + c)b_{it}^S - (\kappa_L + c)b_{it}^L - \Phi_L(w_{it}^L) \quad (14)$$

$$W_{i,t+1} = \max \{V_{i,t+1}, \bar{V} X_t\}$$

s.t. (5), (8), (9), (10), (13) hold

Note that in the top equation, current firm value is comprised of a dividend payment (D_{it}), equity issuance costs in the case that firm dividends are negative ($\Phi_e(\cdot)$), and the dynamic continuation value of the firm ($\mathbb{E}_t [M_{t+1} W_{i,t+1}]$). The additional constraints that are referred to include the laws of motion for investment and long-term debt, the collateral constraint, and the pricing equations for short and long-term debt.

The dividend to the firm will consist of after-tax profits plus a tax shield for depreciation and debt-related coupon payments. It will also include an outflow for equilibrium investment and adjustment costs on capital. The terms on the final line of D_{it} represent debt proceeds and repayment on both debt contracts, as well as issuance costs (Φ_L) in the case that the firm issues new long-term debt ($w_{it}^L > 0$). Notice that the firm pays a fractional portion $\kappa_L + c$ of long-term debt each period.

As the Bellman equation represents the value from continuing operations, the discounted future value must account for the chance of potential default in period $t + 1$. As a result, $W_{i,t+1}$ is a maximum over continuing to operate next period and choosing to take the outside option.

3.6 Discussion of Capital Structure Tradeoffs

At its core this model discusses the tradeoffs among a number of securities that can be used to finance endogenous investment. Beyond operating cash flows, the firm has the opportunity each period to take upon new short-term and long-term debt, as well as equity issuance. How does this model break the irrelevance theorem stated in Miller and Modigliani (1959)? First, as firms take on more debt (both short and long) they receive a tax shield that is proportional to the coupon payments on debt. Inherently this tax advantage creates an incentive for leverage. Beyond the tax advantage, long-term debt embodies distress costs. If the recovery parameter, $\xi > 0$, then there will be a loss in firm value upon default. This will also create a deviation from capital structure irrelevance. Finally, the model features issuance costs in both long-term debt and equity issuance.

I now discuss how the model emits a partial pecking order. Suppose the corporation would like to raise additional funds for investment beyond those garnered from current production, net of debt-related payments. The costs and benefits to issuing the three possible securities are as follows:

1. Short term debt: the benefit consists of the discounted value of the tax advantage of the future coupon payment. The costs, however, are on net zero. There is no destruction of firm value implied by the bond pricing, due to the seniority and collateral constraint.
2. Long term debt: similar to short-term debt, the benefit consists of the discounted value of the tax advantage of the future coupon payment. Conditional on being in a default region, additional long-term debt increases the likelihood of bearing distress costs.
3. Equity issuance: there is no benefit to issuing additional equity while there are flotation costs that are positive ($\Phi_e > 0$).

Among these securities, it is clear that short-term debt always provides a positive benefit. This implies that the firm takes upon as much short-term debt as it can and that the collateral constraint binds, $(1 + c)b_{i,t+1}^S = s_0(1 - \delta)k_{i,t+1}$.¹³ Beyond short-term debt, it is difficult to definitively say whether long-term debt or equity will be preferred. This will be dependent on the tradeoff between flotation costs and the time-varying net benefit of issuing corporate debt. As this will be specific to the quantitative behavior of the model, we will leave this discussion till later.

3.7 Model Solution

Due to stochastic growth over time, we solve a scaled version of the model where all time t variables are divided by the lagged level of the aggregate shock, X_{t-1} . In the discussion that follows, \hat{g} indicates the detrended value of a generic variable g . For more details on the exact system of equations that we iterate over, see Appendix A.

The model emits four states, $\{\Delta c_t, x_{it}, \hat{k}_{it}, \hat{b}_{it}^L\}$ and two controls, $\{\hat{k}_{i,t+1}, \hat{b}_{i,t+1}^L\}$. The value and bond pricing functions will be of the form:

$$\begin{aligned} \hat{V}_{it} \left(\Delta c_t, x_{it}, \hat{k}_{it}, \hat{b}_{it}^L \right) &= \max_{\{\hat{k}'_{i,t+1}, \hat{b}'_{i,t+1}^L\}} \left\{ \hat{D}_{it} - \Phi_e \left(\hat{D}_{it} \right) + e^{(\Delta x_t)} \mathbb{E}_t \left[M_{t+1} \hat{W}_{i,t+1} \right] \right\} \\ \hat{W}_{i,t+1} &= \max \left\{ \bar{V}, \hat{V}_{i,t+1} \right\} \end{aligned} \quad (15)$$

$$\begin{aligned} p_{it}^L &= \mathbb{E}_t \left[M_{t+1} \left(1 - \mathbb{1}_{\{\hat{V}_{i,t+1} \leq \bar{V}\}} \right) (\kappa_L + c + (1 - \kappa_L) p_{i,t+1}^L) \right] \\ &+ \mathbb{E}_t \left[M_{t+1} \left(\mathbb{1}_{\{\hat{V}_{i,t+1} \leq \bar{V}\}} \right) \left(\frac{\hat{X}_{i,t+1}^{PD}}{\hat{b}_{i,t+1}^L} \right) \right] \end{aligned} \quad (16)$$

A simple and intuitive algorithm to solve this system would be to start with a guess for prices and the value function. Using the guess for prices and an implied value for \hat{W} , I can compute a new value of \hat{V} that resulted from the maximization step of 15. I could then evaluate the right hand side of 16, using the previously computed \hat{V} to determine the default dummy variable and the original guess for prices, evaluated at the optimal policies.

As explained in great depth in Chatterjee and Eyigungor (2012), these sort of algorithms suffer convergence issues in models with long-term debt. The reason is the following. In order to move from one iteration of bond prices to another I need to assume a value function and policy function. If the default decision switches, for a certain set of states, from the

¹³A binding collateral constraint aids the quantitative solution of the model. I am able to eliminate one state (b_{it}^S) and control variable ($b_{i,t+1}^S$).

previous iteration to the current one, the resulting abrupt shift will create a large jump in the bond price. This jump then leads to a great shift in the next iteration of computing value and policy functions. In this pattern, I never reach joint convergence of price and value functions.

In order to remedy the convergence issues, Chatterjee and Eyigungor (2012) and Gordon and Guerron-Quintana (2016) add IID zero mean noise into the effective dividend flow. The purpose of this is two fold. First, because the IID noise enters monotonically into the dividend payout one can compute the policy functions and default decision perfectly as a function of the IID shock. Second, because the distribution of the IID noise is known perfectly, when integrating across potential default decisions in the future, as in equations 15 and 16, we can smooth out potential jumps using numerical integration techniques.¹⁴

In the same vein I add a concave function of IID noise, $g(m_{it})$, to my dividend payoff such that m_{it} is a truncated normal shock with a mean of zero and a very small variance. Furthermore g is chosen such that $\mathbb{E}(g(m_{it})) = 0$. This will imply that the value function becomes:

$$\hat{V}_{it} \left(\Delta c_t, x_{it}, \hat{k}_{it}, \hat{b}_{it}^L \right) = \max_{\{\hat{k}'_{i,t+1}, \hat{b}'_{i,t+1}\}} \left\{ \hat{D}_{it} - \Phi_e \left(\hat{D}_{it} \right) + g(m_{it}) + e^{(\Delta x_t)} \mathbb{E}_t \left[M_{t+1} \hat{W}_{i,t+1} \right] \right\}$$

Here the default decision will be a function of the noise and I compute the default rule perfectly with respect to thresholds of m . When computing the expectation of next period's continuation value, I account for the uncertainty of the noise in the expectation. To compute the expectation over m , I conduct a 15-interval numerical integration using properties of the truncated normal distribution. For a detailed description of the numerical algorithm see Appendix B.

4 Results

In this section I document the quantitative results from the model, starting with an explanation of the quarterly calibration, followed by discussions on simulated results, model mechanisms, cross-sectional behavior, and other findings.

¹⁴The implementation of IID noise in both of these papers is slightly different than in mine. In both of these papers the noise is added such that it enters into the *utility flow* of the representative household's Bellman equation. As a result optimal policies and default decisions are both affected by the noise. In my setup the noise is simply added to the (risk-neutral) dividend flow, which implies that the noise only factors into the optimal default decision. In this sense, my use of the IID noise purely helps smooth switches in the default decision. In the case of these two references, it can also help prove existence of equilibrium as policies are monotonic in the noise.

4.1 Calibration

In Table 6, I display the parameters I calibrate the baseline version of the model to. The first three rows of parameters relate to the Epstein and Zin (1989) stochastic discount factor. The values for risk aversion ($\gamma = 2$) and the intertemporal elasticity of substitution ($\psi = 2$) would suggest the firm has a preference for a resolution of early uncertainty ($\gamma > \frac{1}{\psi}$). As well known in the Long Run Risks literature, such preferences would suggest that shocks to current aggregate states will heavily influence future utility, which will then feed into the firm’s discount factor. This creates a large “distortion” in state prices to help accurately capture default and credit spread patterns.¹⁵

The next three lines of the table relate to the production parameters of the model. I use a curvature parameter ($\alpha = .65$) that is close in value to the estimates of Hennessy and Whited (2007). The depreciation parameter is standard in the literature ($\delta = .025$). The capital adjustment parameter ($\phi_k = 1$) is chosen to help curb investment rate volatility in the cross section. An additional criterion I use to set this parameter is that default rates are decreasing in ϕ_k . As firms are more exposed to adjustment costs they are more cautious in adjusting their capital stock. These cautious adjustments make firms less susceptible to a large drop in equity value, in the event that an adverse productivity shock hits.

The model is calibrated to feature one quarter short-term debt and five year long-term debt. Setting $\kappa_L = .05$ suggest that it will take 20 quarters, on average, to pay off long-term debt. The coupon rate ($c = .01$) is arbitrarily set and does not have a substantive effect on the results. As the collateral constraint will bind, the average ratio of short-term debt to total assets is $s_0(1 - \delta)$. I set $s_0 = .08$ to capture the mean ratio in Compustat data. The next three parameters relate to issuance costs for both long-term debt and equity. The fixed cost parameter for long-term debt, $\Phi_{L,a} = .006$ is set to target the frequency of long-term debt issuances in the cross section. Similarly the flotation cost parameter, $\Phi_{e,a} = .06$ is used to target the frequency of equity issuance. The parameter used for the proportional cost of equity, $\Phi_{e,b} = .05$, comes from Hennessy and Whited (2007).

The bottom set of numbers refer to productivity parameters. The productivity constant, $\bar{x} = -2.50$, is chosen to scale the economy such that detrended capital is roughly equal to one in a non-leverage economy. The autocorrelation and volatility of idiosyncratic factor productivity are taken from Kuehn and Schmid (2014). All parameters for consumption growth are set to match the mean, volatility, and autocorrelation of quarterly, real per-capita consumption growth from NIPA tables. Finally, I set $\lambda_x = 3.5$ to scale up aggregate volatility in the firm TFP. More generally, it aids to induce a default that is more countercyclical. The

¹⁵While β is not high enough to match the *level* of the risk free rate, all qualitative features of the model hold in this environment.

last parameter in the table, $\bar{V} = 1.425$ is set as an outside value to the firm. It is set to help match default rates as seen in the data.¹⁶

4.2 Model Fit

I solve the model for the previously described set of values and simulate the model. The simulation consists of a panel with 3000 firms over 500 quarters, including a burn-in period of 500 quarters. In the model results I describe, I remove defaulted firms each period. In Table 7, I provide cross-sectional statistics related to profitability, investment, debt, and default. The data comes from Compustat, onwards from 1984, and the numbers in parentheses represent time series bootstrapped standard errors. $\mathbb{E}_t(\cdot)$ and $\sigma_t(\cdot)$ refer to the cross sectional mean and volatility, respectively.

The model performs reasonably well with respect to investment and book leverage. There is a direct link in the model between book leverage and the long-term debt ratio. The portion of book leverage that is due to short-term debt is a fixed ratio – $s_0(1 - \delta)$. Any additional book leverage beyond this is through long-term debt. The model does particularly well with respect to default (.970% in the model vs. 1.08% annually in the data). This results in a credit spread of 1.84% annually which is close to the empirical target. The key statistic that the model does poorly on is profitability. The likely reason why this occurs is the fact that I do not have fixed costs in production as used in Gomes (2001). As the profitability is too high this is probably causing the need for a positive outside option ($\bar{V} > 0$) to induce default.

In Table 8 I display the aggregate statistics of the model. The first three rows provide the moments of consumption growth (mean, volatility, autocorrelation) which are set exogenously, in line with quarterly data. Aggregate investment growth and output growth move positively with consumption growth and are also close to data. Leverage in the model is also procyclical as firms issue more book debt in economic booms. Finally as desired, the model is able to match the stylized fact that the long-term debt ratio is strongly pro-cyclical. In the model, the aggregated long-term debt ratio has a correlation of .396 with consumption growth and .664 with output growth. In the next subsection, we will thoroughly discuss the mechanism that leads to this dynamic. As in the data, default rates vary negatively with the aggregate state of the economy. When firms become unproductive and have lower stocks of capital, this brings them closer to the default boundary. The smaller distance to default

¹⁶When I set $\bar{V} = 0$ (“optimal default”), an average firm never defaults. In this region the tradeoffs between short and long-term debt are not as clear. Absent of a collateral constraint which explicitly limits issuance of short-term debt, the firm is indifferent between both debt choices. Both debt contracts are risk free as well.

then generates large credit spreads.

4.3 Model Mechanism

The patterns that the model generates are best summarized by Figure 5. Here I display a set of aggregated series related to output, investment, book leverage, and the long-term debt ratio. The picture confirms the numerical evidence presented in the last sub-section. In particular, additions to book leverage, via long-term debt, are strongly connected to changes in investment. Moreover, firms take on more leverage to finance investment.¹⁷

This mechanism is particularly explained by Figure 6. Both panels represent the time series average of simulated data, across aggregate states – hence there are five bars. In the top figure, I describe what I call the funding deficit, which I define to be:

$$\sum_i \left(\hat{D}_{it} - p_{it}^S \hat{w}_{it}^S - p_{it}^L \hat{w}_{it}^L \right)$$

This deficit represents all dividends, net of short-term and long-term debt proceeds, or, how much the firm seeks to raise out of debt markets. This number is negative so I take its absolute value and index it to the median state. In terms of interpretation, it is clear that firms need to raise less debt in the first state (about 45% less) relative to the median state. In the fifth state, firms require much more, to the order of 70%. In particular the need for additional debt in the last state is driven by a higher marginal productivity of capital.

In the second panel I describe where this funding is obtained. In particular the figure displays the time-series average of:

$$\sum_i (p_{it}^L \hat{w}_{it}^L) / \sum_i \left(\hat{D}_{it} - p_{it}^S \hat{w}_{it}^S - p_{it}^L \hat{w}_{it}^L \right)$$

across states. This figure suggests that firms tend to fund more of their investment needs in good times using long-term debt proceeds. The reasoning for this is two-fold. First, in times when the marginal product of investment is high, firms are limited in funding through short-term debt markets, due to the collateral constraint.

The second reason why this occurs relates to the dynamic pecking order *between long-term debt and equity*. In order to obtain additional funding, firms can (i) issue additional long-term debt or (ii) issue equity, and the choice between the two is a choice between bearing expected distress costs and paying flotation costs, respectively. Certainly, issuing long-term

¹⁷In this figure, there might seem to be a timing discrepancy between aggregate output and investment versus leverage and the long-term debt ratio. This discrepancy exists because the former group represents a set of choice variable decided at time t while leverage related variables are chosen at time $t - 1$.

debt might provide additional default risk. However, as the firm is further away from default due to the state of the economy, the expected losses from distress are reduced. As a result, the firm ends up issuing more long-term debt.

Nonetheless, this preference changes with respect to the business cycle. The bottom panel also suggests that when aggregate conditions sour, the firm would rather buy back long-term debt using a combination of short-term debt and equity issuance. While I don't present the result here, for firms who do seek to issue equity, their issuance increases dramatically in low consumption growth states.

Moreover, the model endogenously generates a time-varying pecking order. Regardless of the state of the world, the firm first issues short-term debt. Any additional external financing will depend on consumption growth. In high consumption growth states, long-term debt will be preferred to equity. In lower growth states, firms will, if need be, prefer equity issuance.

4.4 Cross-Sectional Behavior and Rollover Risk

As the model features a set of heterogeneous firms, we can analyze characteristics across corporations. In Table 9, I sort simulated firms into quintiles each period by their detrended market value \hat{V}_{it} . For each statistic I build a panel time series of average statistics across quintiles. The characteristics of the basic sort are given in the first line, where firms in quintile 1 are 15% smaller, in terms of market capitalization, than the median firm on average. Large firms are 20% larger.

Furthermore, I find that detrended capital varies monotonically with respect to market value. Firms in quintile 5 are 27% larger than quintile 3, while those in quintile 1 are 22% smaller. The most surprising result from this table is that while stable firms have more capital, they also *have more leverage and long-term debt*. This is surprising in that we would expect increased leverage to further decay firm value and increase credit spreads. The equilibrium cross-section in this model implies that capital is the largest driver for firm stability. In terms of key firm policy variables, profitability and investment increase as a function of market value. Low market cap firms are 3% less profitable and invest 3% less than high market cap firms. Economic stability is clearly reflected in credit spreads. Firms in quintile 1 average a cost of capital that is fifteen times larger than those in the top quintile.

In Figure 7, I plot firm behavior in the eight quarters that precede default, computed by taking the average of series generated across corporate default episodes in simulation. The bottom axis provides the number of quarters in relation to default. Firms that experience default undergo a series of shocks that degrades the level of total productivity (\hat{A}_{it}) by 40%. This is displayed in the first panel. As a result of the shocks, and the lower marginal

productivity of capital, firms then dis-invest as shown in the second panel.

As productivity and capital both decrease firm value, making debt payments relatively more costly, corporations would like to buy back some of the long term debt on its balance sheet. However, this becomes difficult due to two reasons: (i) reduced capital and productivity result in less profits, which provides a shortage of internal funds to buy back debt and (ii) the collateral constraint, which is tied to the firm’s decreasing capital stock, greatly limits the amount of short-term debt that can be used to liquidate long-term. As a result in panel 3, we see that the leverage ratio actually increases. It is clear that the combination of these endogenous state variables changing leads to the value function behavior. Furthermore, it is striking to see how much credit spreads react over the course of the default episode. Over eight quarters the credit spreads rise from $\sim 0\%$ to $\sim 220\%$, when the firm is on the precipice of default. Moreover, this discussion confirms to us that the endogenous investment channel matters greatly for the model. As firms lower their capital due to a lower marginal product of capital, this has a substantial effect on firm value and the re-issuance costs of debt.

4.5 Further Considerations

While the model captures many salient features of investment behavior, corporate financing decisions, and asset prices there are a few places where it does not do as well. In particular, the model does not deliver the predictability results presented in the empirical section, between the long-term debt ratio and investment growth. The reason why this does not occur, in my estimation, is that investment growth rates in the model are not auto-correlated enough to begin with. In the data, the first order autocorrelation of investment growth is roughly .20 and slowly decreases to 0 over the course of three lags. In the model however, the first order autocorrelation is -.24 and is volatile. Moreover the *level* of aggregate investment is too strongly tied to the aggregate shock in the model, which has negative empirical consequences for the model’s investment growth. One way to tackle these issues is to add adjustment costs to the growth rate of investment (see Christiano et al., 2005). This will directly induce an autocorrelation in investment growth. Additionally, the use of a “time-to-build” assumption (see Boldrin et al., 2001) will also generate the desired characteristics for investment growth.¹⁸

In reality firms have access to additional securities which allow them to potentially avoid default – in particular, credit lines. In the financial crisis it is well documented that firms drew down their credit lines in order to fund operations. For example, Campello et al. (2011) suggests via survey data that small, financially constrained firms drew close to 60% of their

¹⁸The literature on financial frictions (see Carlstrom and Fuerst, 1997; Bernanke et al., 1999) suggests that financial intermediary related constraints on investment can create autocorrelation in investment growth.

available lines of credit in 2009. In my model however, as firms effectively get priced out of public debt markets (due to low capital, high leverage, and rising long-term credit spreads), they are forced to default. One way to augment the model to address the “lack” of debt market choices is to add a cash asset, that allows firms to save for the rainy day. These savings would allow corporations to keep additional reserves on hand in the face of future productivity drops.¹⁹ I leave this for future research.

5 Conclusion

Corporate debt maturity is a time-varying phenomenon that is linked to business cycles. In this paper I study the extent of these linkages and show how they can arise in an economic model. I empirically document that firms extend their debt maturity in peaks of the business cycle, when aggregate output and investment are high, and corporate credit spreads are low. These results also extend to the corporation level, where investment rates and profitability are linked to firm-specific long-term debt ratios.

To understand why the ratio time varies and is linked to the business cycle requires a theoretically motivated explanation. I provide one by developing a dynamic heterogeneous firm economy where corporations trade off debt maturity choice in the face of investment opportunities. Long term debt inherits relatively more distress costs than short-term debt which creates an initial preference for lower duration liabilities. However, limits on short-term debt make the potential distress costs worth it, in order to take advantage of the pro-cyclical investment opportunities. The combination of investment and the collateral structure lead to the pro-cyclical long-term debt ratio. Moreover the model sheds light on the macro-economy as it implies that firms with higher amounts of long-term debt are more systemically important to aggregate fluctuations.

¹⁹Mechanically, this would involve allowing $b_{i,t+1}^S$ to drift to negative regions. This would be equivalent to implementing firm-level retained earnings, as in Livdan et al. (2009).

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Appendices

A Detrended Model Equations

As there is stochastic growth in the model, we normalize each variable, var_t , such that:

$$v\hat{a}r_t = \frac{var_t}{X_{t-1}}$$

The complete list of equations that govern the model consist of:

Exogenous Processes (outside of model solution):

$$\begin{aligned} \log(C_t/C_{t-1}) \equiv \Delta c_t &= \mu_c + \rho_c \Delta c_{t-1} + \sigma_c \epsilon_{ct} \\ \log(X_t/X_{t-1}) \equiv \Delta x_t &= \mathbb{E}(\Delta c_t) + \lambda_x (\Delta c_t - \mathbb{E}(\Delta c_t)) \\ x_{it} &= \rho_x x_{i,t-1} + \sigma_x \epsilon_{xt} \end{aligned}$$

$$\begin{aligned} M_{t+1} &= \beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{PC_{t+1} + 1}{PC_t} \right)^{\theta-1} \\ 1 &= \mathbb{E}_t \left[M_{t+1} \left(\frac{C_{t+1}}{C_t} \right) \left(\frac{PC_{t+1} + 1}{PC_t} \right) \right] \end{aligned}$$

The above lines refer to equations (1), (2), (3), and (7) from the text.

Investment and Leverage Constraints:

$$\begin{aligned} (1+c)\hat{b}_{i,t+1}^S &= s(1-\delta)\hat{k}_{i,t+1} \\ e^{(\Delta x_t)}\hat{k}_{i,t+1} &= (1-\delta)\hat{k}_{it} + \hat{i}_{it} \\ e^{(\Delta x_t)}\hat{b}_{i,t+1}^L &= (1-\kappa)\hat{b}_{it}^L + \hat{\omega}_{it}^L \\ e^{(\Delta x_t)}\hat{b}_{i,t+1}^S &= \hat{\omega}_{it}^S \end{aligned}$$

The above lines refer to equations (5), (8), and (9) from the text. The constraint for short-term debt binds due to the strict preference for short-term debt.

Firm Value:

$$\begin{aligned} \hat{V}_{it} \left(\Delta c_t, x_{it}, \hat{k}_{it}, \hat{b}_{it}^L \right) &= \max_{\{\hat{k}'_{i,t+1}, \hat{b}'_{i,t+1}\}} \left\{ \hat{D}_{it} - \Phi_e \left(\hat{D}_{it} \right) + e^{(\Delta x_t)} \mathbb{E}_t \left[M_{t+1} \hat{W}_{i,t+1} \right] \right\} \\ \hat{W}_{i,t+1} &= \max \left\{ \bar{V}, \hat{V}_{i,t+1} \right\} \\ \hat{D}_{it} &= (1-\tau)e^{(\bar{x}+x_{it}+(1-\alpha)\Delta x_t)}\hat{k}_{it}^\alpha + \tau \left(\delta\hat{k}_{it} + c\hat{b}_{it}^S + c\hat{b}_{it}^L \right) - \hat{i}_{it} - \Phi_k \left(\hat{i}_{it}, \hat{k}_{it} \right) \hat{k}_{it} \\ &\quad + p_{it}^S \hat{\omega}_{it}^S + p_{it}^L \hat{\omega}_{it}^L - (1+c)\hat{b}_{it}^S - (\kappa+c)\hat{b}_{it}^L - \Phi_L \left(\hat{\omega}_{it}^L \right) \end{aligned}$$

The above lines refer to equation (14) from the text.

Debt Pricing:

$$p_{it}^S = \mathbb{E}_t [M_{t+1} (1 + c)]$$

$$\begin{aligned} p_{it}^L \hat{b}_{i,t+1}^L &= \mathbb{E}_t \left[M_{t+1} \left(1 - \mathbb{1}_{\{\hat{V}_{t+1} \leq \bar{V}\}} \right) \left((\kappa_L + c) \hat{b}_{i,t+1}^L + (1 - \kappa_L) p_{i,t+1}^L \hat{b}_{i,t+1}^L \right) \right] \\ &+ \mathbb{E}_t \left[M_{t+1} \left(\mathbb{1}_{\{\hat{V}_{t+1} \leq \bar{V}\}} \right) \left(\hat{X}_{i,t+1}^L \right) \right] \end{aligned}$$

$$\hat{X}_{i,t+1}^L = (1 - \xi) \left(\hat{\pi}_{i,t+1} + (1 - \delta) \hat{k}_{i,t+1} - (1 + c) \hat{b}_{i,t+1}^S \right)$$

The above lines refer to equations (10), (12), and (13) from the text.

B Numerical Solution

In this section, I outline the numerical solution that I use. As the model contains long-term debt, I use techniques from Chatterjee and Eyigungor (2012) to help convergence. The main difference is that in my model I have an additional choice variable (capital) which does not allow for the use of a monotonicity assumption. To get around this problem, I use a similar methodology as used in Gordon and Guerron-Quintana (2016), while also handling the additional short-term debt.²⁰

The two key parts of the model are given by:

Equity Value:

$$\begin{aligned}\hat{V}(\Delta c_t, x_{it}, \hat{k}_{it}, \hat{b}_{it}^L, m_{it}) &= \max_{\{\hat{k}_{i,t+1}, \hat{b}_{i,t+1}^L\}} \left\{ \hat{D}_{it} - \Phi_e(\hat{D}_{it}) + g(m_{it}) + e^{(\Delta x_t)} Z(\Delta c_t, x_{it}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^L) \right\} \\ Z(\Delta c_t, x_{it}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^L) &= \mathbb{E}_t \left[M_{t+1} \times \max \{ \bar{V}, \hat{V}_{i,t+1} \} \right]\end{aligned}$$

Pricing of Long Term Debt:

$$\begin{aligned}p^L(\Delta c_t, x_{it}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^L) &= \mathbb{E}_t \left[M_{t+1} \left(1 - \mathbb{1}_{\{\hat{V}_{i,t+1} \leq \bar{V}\}} \right) (\kappa_L + c + (1 - \kappa_L) p_{i,t+1}^L) \right] \\ &+ \mathbb{E}_t \left[M_{t+1} \left(\mathbb{1}_{\{\hat{V}_{i,t+1} \leq \bar{V}\}} \right) \left(\frac{\hat{X}_{i,t+1}^{PD}}{\hat{b}_{i,t+1}^L} \right) \right]\end{aligned}$$

where $g(\cdot)$ indicates a concave function of the noise, m_{it} . The noise is I.I.D. with a truncated $\mathcal{N}(0, \sigma_m)$, over support $[-\bar{m}, \bar{m}]$. The algorithm broadly operates as follows:

0. Set aggregate grids and solve for the stochastic discount factor, $M(\Delta c_t, \Delta c_{t+1})$, using iterative techniques on the price to consumption ratio in equation 7. Start with guesses for the expected continuation value, $Z^0(\cdot)$ and bond pricing, $p^{L0}(\cdot)$. **Neither** is a function of the noise, m .
1. Input $\{Z^0, p^{L0}\}$ into the right hand side of the equity value function and solve for one iteration of firm value. As the optimal policy is not dependent on m , we will receive two policy functions: $\{\hat{k}^{L1}(\cdot), \hat{b}^{L1}(\cdot)\}$. The policy functions are specified over the entire state space.
2. For every state vector, $S_{it} = \{\Delta c_t, x_{it}, \hat{k}_{it}, \hat{b}_{it}^L\}$, compute the default decision (D) over the support of m . Because \hat{V} is monotonic in m , we check which of three possible cases hold:

$$(a) \left\{ \hat{V}(S_{it}, \bar{m}) \leq \bar{V} \right\}$$

In this case the firm will always default over the support. The optimal decision will

²⁰There is one major difference in the way I compute my model relative to the above references. While I allow for the noise to impact the default decision, I do not allow it to affect policy functions, as it is only additive and separable in the Bellman equation. Because of this, the policy functions will not receive the smoothing benefits of the noise. The key reason I do not allow for the interaction between policies and the IID noise is because I would not like to change the concavity on the dividend flow. If I did change the concavity, this would alter the basic structure of the problem.

be given by:

$$D(S_{it}, m_{it}) = 1 \quad \forall m_{it}$$

$$(b) \left\{ \hat{V}(S_{it}, -\bar{m}) \leq \bar{V} \quad \text{and} \quad \hat{V}(S_{it}, \bar{m}) > \bar{V} \right\}$$

In this case, we know that the firm switches its default decision over the support of m . Because of the concave nature of g , we know there exists an \tilde{m} in the support of m such that:

$$\hat{V}(S_{it}, \tilde{m}) = \bar{V}$$

With \tilde{m} defined in such a manner the default decision will be given by:

$$D(S_{it}, m_{it}) = \begin{cases} 1, & \text{if } m_{it} \leq \tilde{m} \\ 0, & \text{otherwise} \end{cases}$$

$$(c) \left\{ \hat{V}(S_{it}, -\bar{m}) > \bar{V} \right\}$$

In this case the firm will never default over the support. The optimal decision will be given by:

$$D(S_{it}, m_{it}) = 0 \quad \forall m_{it}$$

Note that in order to compute values across different m , we use the optimal policies computed in step 1 of the procedure. Another implication of this procedure is that, if applicable, $\tilde{m} = \tilde{m}(S_{it})$.

3. To this point we have computed optimal policies which matter in the case firms do not default, as well as indicator variables for default. In some cases the default decision may vary over m , so we have also computed default breakpoints. Using this information we can compute our next iteration of expected continuation value and bond price, $\{Z^1, p^{L1}\}$.

- (a) Computing Expected Continuation Value. The discounted, continuation value will be given by:

$$\mathbb{E}_t \left[M_{t+1} \times \max \left\{ \bar{V}, \hat{V}_{i,t+1} \right\} \right] = \mathbb{E}_t \left[M_{t+1} \times \underbrace{\mathbb{E} \left[\max \left\{ \bar{V}, \hat{V}_{i,t+1} \right\} \mid \Delta c_{t+1}, x_{i,t+1} \right]}_{\text{reflects expectation solely over } m} \right]$$

Using the law of iterated expectations as shown above, we can focus on first computing the expectations of the continuation value over $m_{i,t+1}$ and then taking an expectation over uncertainty regarding the fundamental states. For the purposes of my work, the second expectation is taken using a standard discrete-state approach. Hence, I will focus on explaining the bracketed expectation below.

$$\mathbb{E} \left[\max \left\{ \bar{V}, \hat{V}_{i,t+1} \right\} \mid \Delta c_{t+1}, x_{i,t+1} \right] = \mathbb{E} \left[D(S_{i,t+1}, m_{i,t+1}) \bar{V} + (1 - D(S_{i,t+1}, m_{i,t+1})) \hat{V}_{i,t+1}(S_{i,t+1}, m_{i,t+1}) \right]$$

where $S_{i,t+1}$ incorporates the relevant future productivity states. To compute the right hand side expectation, for each given state $(S_{i,t+1})$, there will be two cases:

- I. There is **no switch** in default over m . In the case where the firm always defaults, the expectation will be given by \bar{V} . In the case where the firm always continues to operate, the expectation will be given by:

$$\int_{-\bar{m}}^{\bar{m}} \hat{V}(S_{i,t+1}, m_{i,t+1}) dm = \sum_{j=1}^{14} \left[\hat{V} \left(S_{i,t+1}, \frac{m_j + m_{j+1}}{2} \right) \times Pr(m_j < m_{i,t+1} < m_{j+1}) \right]$$

where Pr denotes the probability. Notice above that we approximate the integral using a numerical integral with 14 intervals. All m_j refer to elements from an equally spaced vector over the support of m , $[m_1, m_2, \dots, m_{14}, m_{15}]$.

- II. There is **a switch** in default over m , which occurs at \tilde{m} . We now write the expectation from above as:

$$\begin{aligned} \int_{-\bar{m}}^{\bar{m}} \max \left\{ \bar{V}, \hat{V}(S_{i,t+1}, m_{i,t+1}) \right\} dm &= \int_{-\bar{m}}^{\tilde{m}} \bar{V} dm + \int_{\tilde{m}}^{\bar{m}} \hat{V}_{i,t+1}(S_{i,t+1}, m_{i,t+1}) dm \\ &= \bar{V} \times Pr(m_{i,t+1} < \tilde{m}) \\ &\quad + \int_{\tilde{m}}^{\bar{m}} \hat{V}_{i,t+1}(S_{i,t+1}, m_{i,t+1}) dm \end{aligned}$$

Without loss of generality, suppose that \tilde{m} falls between m_{k-1} and m_k . Then the last integral will be computed as:

$$\begin{aligned} \int_{\tilde{m}}^{\bar{m}} \hat{V}_{i,t+1}(S_{i,t+1}, m_{i,t+1}) dm &= \hat{V}_{i,t+1} \left(S_{i,t+1}, \frac{\tilde{m} + m_k}{2} \right) \times Pr(\tilde{m} < m_{i,t+1} < m_k) \\ &\quad + \sum_{j=k}^{14} \left[\hat{V} \left(S_{i,t+1}, \frac{m_j + m_{j+1}}{2} \right) \times Pr(m_j < m_{i,t+1} < m_{j+1}) \right] \end{aligned}$$

After computing this at a given state, we are left with $\tilde{Z}^1(S_{i,t+1})$. Using this object we can proceed and compute our desired object using standard discrete methods:

$$Z^1(\Delta c_t, x_{it}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^L) = \mathbb{E}_t \left[M_{t+1} \tilde{Z}^1(S_{i,t+1}) \right]$$

- (b) Computing the Bond Price. Similar to the computation of the expected continuation value the bond price will be a function of a default portion and a non-default portion.

$$\begin{aligned} p^L(\Delta c_t, x_{it}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^L) &= \mathbb{E}_t \left[M_{t+1} (1 - D_{i,t+1}) (\kappa_L + c + (1 - \kappa_L) p_{i,t+1}^L) \right] \\ &\quad + \mathbb{E}_t \left[M_{t+1} D_{i,t+1} \left(\frac{\hat{X}_{i,t+1}^{PD}}{\hat{b}_{i,t+1}^L} \right) \right] \end{aligned}$$

where $D_{i,t+1}$ is the potential default decision at time $t+1$. It is important to realize that the future bond price, $p_{i,t+1}^L$ is a function of a number of future policies:

$$p_{i,t+1}^L = p^L(\Delta c_{t+1}, x_{i,t+1}, \hat{k}_{i,t+2}, \hat{b}_{i,t+2}^L) = p^L(\dots, \hat{k}'(\Delta c_{t+1}, x_{i,t+1}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^L), \hat{b}^{L'}(\dots))$$

where $\hat{b}^{L'}$ is a function of the same variables as \hat{k}' . In the course of the algorithm we use the policy functions computed in step 1 to express the future price as a function of

$S_{i,t+1}$. All of this being said we are left with a familiar problem:

$$\begin{aligned}
p^L(\cdot) &= \mathbb{E}_t [M_{t+1} \times ((1 - D(S_{i,t+1}, m_{i,t+1}))f_1(S_{i,t+1}) + D(S_{i,t+1}, m_{i,t+1})f_2(S_{i,t+1}))] \\
f_1(S_{i,t+1}) &= \kappa_L + c + (1 - \kappa_L)p^L(S_{i,t+1}) \\
f_2(S_{i,t+1}) &= \frac{\hat{X}_{i,t+1}^{PD}}{\hat{b}_{i,t+1}^L} = xb(S_{i,t+1})
\end{aligned}$$

Because the inner piece that multiples M_{t+1} is also dependent on the realization of the noise, we again use law of iterated expectations to focus on computing the inner expectation over m , given by:

$$\tilde{p}^{L1}(S_{i,t+1}) = \mathbb{E} [((1 - D(S_{i,t+1}, m_{i,t+1}))f_1(S_{i,t+1}) + D(S_{i,t+1}, m_{i,t+1})f_2(S_{i,t+1}))]$$

The computation of this expectation will be conceptually identical to the technique used in part (a). As a result I won't go into further detail here. The final derived bond price will be:

$$p^{L1}(\Delta c_t, x_{it}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^L) = \mathbb{E}_t [M_{t+1} \tilde{p}^{L1}(S_{i,t+1})]$$

4. Having computed expected continuation value and bond prices we check convergence:

$$\varepsilon = \max \{ \|Z^0 - Z^1\|, \|p^{L0} - p^{L1}\| \}$$

If ε is small enough we are done. Otherwise, choose a new starting guess based on the recent outcomes:

$$\begin{aligned}
Z^{NEW} &= \xi_z Z^0 + (1 - \xi_z) Z^1 \\
p^{NEW} &= \xi_p p^{L0} + (1 - \xi_p) p^{L1}
\end{aligned}$$

Let $Z^0 = Z^{NEW}$, $p^{L0} = p^{NEW}$ and go back to step 1. In practice, I set $\xi_z = 0$ and choose $\xi_p = .95$.

Tables and Figures

Table 1: **Aggregate Predictive Regressions**

	$k = 1Q$	2	3	4	5	6	7	8
$\frac{1}{k} \sum_{i=1}^k \Delta y_{t+i} = \beta_0 + \beta'_X X_t + \beta_{LT} LTDR_t^c + error_{t+k}:$								
β_{LT}	0.165***	0.151***	0.124***	0.102**	0.078	0.059	0.044	0.036
$t(\beta_{LT})$	4.343	3.773	2.783	2.166	1.612	1.298	1.106	1.062
Adjusted R^2	0.312	0.424	0.423	0.441	0.405	0.391	0.381	0.356
$\frac{1}{k} \sum_{i=1}^k \Delta i_{t+i} = \beta_0 + \beta'_X X_t + \beta_{LT} LTDR_t^c + error_{t+k}:$								
β_{LT}	0.939***	0.921***	0.797***	0.637***	0.511***	0.363*	0.245	0.187
$t(\beta_{LT})$	4.486	4.974	4.400	3.442	2.675	1.958	1.565	1.538
Adjusted R^2	0.286	0.414	0.397	0.425	0.416	0.408	0.389	0.382

This table examines the relationship between average future, real per-capita output growth and investment growth, and the HP-filtered cyclical component of the aggregate long-term debt share ($LTDR_t^c$). Each panel displays the results of regressing an economic aggregate on a vector of controls, X_t , and the cyclic component. The first line of each panel measures the sensitivity, β_{LT} , with respect to the $LTDR$ measure; the second line measures the t-statistic when accounting for Newey-West standard errors; the final row represents adjusted R^2 measures. The column heading provides the horizon of the average future dependent variable, in terms of k quarters. Controls are at time t and include: real per-capita GDP growth, real per-capita consumption growth, CPI inflation, log PD ratios, the difference between Moody's BAA and AAA interest rates, the yield on the 3M US Treasury bill, the difference between the 10Y US Treasury bond and 3M Treasury bill yields, and the growth rate of aggregate debt. All economic growth variables are in log terms, quarter over quarter, while financial prices are in level terms, from 1954 onwards. The number of stars correspond to double-sided significance at the 90%*, 95%***, and 99%*** confidence levels.

Table 2: **Firm-Level Profitability and Long Term Debt**

Explanatory Variable	(1)	(2)	(3)
$\frac{\pi_{it}}{K_{it}} = \beta_0 + \beta'_X X_{it} + \beta_{LT} LTDR_{it} + error_{it}$			
$\sigma(\Pi_i/K_i)$	-3.65***	-3.66***	-3.12***
t-statistic	-18.3	-18.3	-15.0
B_i/K_i	-.031***	-.030***	-.034***
t-statistic	-29.7	-29.5	-31.2
$I_{i,-1}/K_{i,-1}$.068***	.062***	.045***
t-statistic	11.0	9.91	7.11
$Q_{i,-1}$.003**	.003***	.003***
t-statistic	12.1	11.5	13.3
$LTDR_i / 100$.475***	.497***	.290***
t-statistic	7.13	7.50	4.22
No. of Firms	10,872	10,872	10,872
No. of Observations	217,967	217,967	217,967
Adjusted R^2	.028	.032	.014
Firm Level Controls	Yes	Yes	Yes
Macro Controls	No	Yes	Yes
Firm Fixed Effects	No	No	Yes

This table regresses firm profitability onto firm and macro level variables. Robust standard errors are clustered at the firm level. Firm-level controls include historical one year volatility on profitability, current book leverage, lagged investment rate, lagged ratio of market value to book value of assets, and the long-term debt share. Macro controls include the quarterly growth rates of industrial production and the consumer price index, the level of the 3M treasury bill, the difference between the 10Y treasury bond and 3M bill, and the difference between Moody's measures of BAA and AAA corporate bond yields. All data is from 1984 onwards. The number of stars correspond to double-sided significance at the 90%*, 95%** , and 99%*** confidence levels.

Table 3: **Firm-Level Investment and Long Term Debt**

Explanatory Variable	(1)	(2)	(3)
$\frac{i_{it}}{k_{it}} = \beta_0 + \beta'_X X_{it} + \beta_{LT} LTDR_{it} + error_{it}$			
$\sigma(\Pi_i/K_i)$	-.080*	-.112***	.028
t-statistic	-1.89	-2.62	.550
B_i/K_i	-.007***	-.007***	-.013***
t-statistic	-21.3	-23.2	-26.7
$I_{i,-1}/K_{i,-1}$.393***	.384***	.267***
t-statistic	60.4	59.0	38.5
$Q_{i,-1}$.001***	.001***	.001***
t-statistic	17.9	16.8	19.7
$LTDR_i / 100$.334***	.362***	.161***
t-statistic	16.9	18.4	6.50
No. of Firms	10,598	10,598	10,598
No. of Observations	204,211	204,211	204,211
Adjusted R^2	.312	.314	.254
Firm Level Controls	Yes	Yes	Yes
Macro Controls	No	Yes	Yes
Firm Fixed Effects	No	No	Yes

This table regresses firm profitability onto firm and macro level variables. Robust standard errors are clustered at the firm level. Firm-level controls include historical one year volatility on profitability, current book leverage, lagged investment rate, lagged ratio of market value to book value of assets, and the long-term debt share. Macro controls include the quarterly growth rates of industrial production and the consumer price index, the level of the 3M treasury bill, the difference between the 10Y treasury bond and 3M bill, and the difference between Moody's measures of BAA and AAA corporate bond yields. All data is from 1984 onwards. The number of stars correspond to double-sided significance at the 90%*, 95%** , and 99%*** confidence levels.

Table 4: **Predicting Firm-Level Variables using Long-Term Debt**

Explanatory Variable	k = 1	2	3	4
$\frac{1}{k+1} \sum_{j=0}^k \frac{\pi_{i,t+j}}{k_{i,t+j}} = \beta_0 + \beta'_X X_{it} + \beta_{LT} LTDR_{it} + error_{it}$				
<i>LTDR</i> _{<i>i</i>} / 100	.317***	.244***	.179***	.113*
t-statistic	4.83	3.71	2.73	1.71
No. of Firms	10,377	9,880	9,427	8,946
No. of Observations	207,166	196,785	186,852	177,613
Adjusted <i>R</i> ²	.012	.005	.003	.001
Firm Level Controls	Yes	Yes	Yes	Yes
Macro Controls	Yes	Yes	Yes	Yes
Firm Fixed Effects	No	No	No	No

$\frac{1}{k+1} \sum_{j=0}^k \frac{i_{i,t+j}}{k_{i,t+j}} = \beta_0 + \beta'_X X_{it} + \beta_{LT} LTDR_{it} + error_{it}$				
<i>LTDR</i> _{<i>i</i>} / 100	.298***	.242***	.191***	.164***
t-statistic	13.4	9.81	7.24	5.89
No. of Firms	9,943	9,143	8,620	8,066
No. of Observations	185,836	169,889	157,363	146,032
Adjusted <i>R</i> ²	.337	.311	.295	.269
Firm Level Controls	Yes	Yes	Yes	Yes
Macro Controls	Yes	Yes	Yes	Yes
Firm Fixed Effects	No	No	No	No

This table regresses average, future profitability and investment onto firm and macro level variables. Robust standard errors are clustered at the firm level. Firm-level controls include historical one year volatility on profitability, current leverage, lagged investment rate, lagged ratio of market value to book value of assets, and the long-term debt share. Macro controls include the quarterly growth rates of industrial production and the consumer price index, the level of the 3M treasury bill, the difference between the 10Y treasury bond and 3M bill, and the difference between Moody's measures of BAA and AAA corporate bond yields. All data is from 1984 onwards. The number of stars correspond to double-sided significance at the 90%*, 95%**, and 99%*** confidence levels.

Table 5: **Firm-Level Variables and Long Term Debt, by Book Size**

Explanatory Variable	Small	2	3	4	Large
$\frac{\pi_{it}}{k_{it}} = \beta_0 + \beta'_X X_{it} + \beta_{LT} LTDR_{it} + error_{it}:$					
<i>LTDR_i</i> / 100	.587***	.247***	.419***	.112	.034
t-statistic	3.98	2.17	4.15	1.12	.270
Average Log Size	2.57	4.11	5.22	6.37	8.32
No. of Firms	4,158	4,469	3,960	3,013	1,756
No. of Observations	46,030	45,697	44,819	43,723	37,697
Adjusted <i>R</i> ²	.105	.009	.064	.151	.265
Firm Level Controls	Yes	Yes	Yes	Yes	Yes
Macro Controls	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	No	No	No	No	No

$$\frac{i_{it}}{k_{it}} = \beta_0 + \beta'_X X_{it} + \beta_{LT} LTDR_{it} + error_{it}:$$

<i>LTDR_i</i> / 100	.342***	.397***	.382***	.322***	.101**
t-statistic	9.05	10.5	9.28	7.34	2.00
Average Log Size	2.57	4.11	5.22	6.37	8.32
No. of Firms	3,998	4,308	3,782	2,882	1,693
No. of Observations	43,026	42,897	42,138	41,133	35,016
Adjusted <i>R</i> ²	.186	.298	.354	.399	.399
Firm Level Controls	Yes	Yes	Yes	Yes	Yes
Macro Controls	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	No	No	No	No	No

This table regresses profitability and investment onto firm and macro level variables, running separate pooled regressions for firms of different size quintile. Five size quintiles are computed period by period, based on the book value of assets. Robust standard errors are clustered at the firm level. Firm-level controls include historical one year volatility on profitability, current leverage, lagged investment rate, lagged ratio of market value to book value of assets, and the long-term debt share. Macro controls include the quarterly growth rates of industrial production and the consumer price index, the level of the 3M treasury bill, the difference between the 10Y treasury bond and 3M bill, and the difference between Moody's measures of BAA and AAA corporate bond yields. All data is from 1984 onwards. The number of stars correspond to double-sided significance at the 90%*, 95%** , and 99%*** confidence levels.

Table 6: **Calibration Parameters for Baseline Model**

Parameter	Value	Note
β	.98	Time Discount
γ	7.5	Risk Aversion
ψ	2	Intertemporal Elasticity of Substitution
δ	.025	Depreciation of Capital
α	.65	Production Exponent on Capital
ϕ_k	1	Coefficient on Capital Adjustment Costs
κ_L	.05	Governs Average Maturity of Debt
c	.01	Coupon Rate
s_0	.08	Selected to hit b^S/k ratio in the data
$\Phi_{L,a}$.006	Fixed Issuance Cost of Long Term Debt
$\Phi_{e,a}$.06	Fixed Issuance Cost of Equity
$\Phi_{e,b}$.05	Proportional Issuance Cost of Equity
\bar{x}	-2.50	Productivity constant
ρ_z	.85	Firm Productivity Autocorrelation
σ_z	.15	Firm Productivity Conditional Volatility
μ_c, ρ_c, σ_c	–	Set to match mean, volatility, and autocorrelation of Δc_t
λ_x	3.50	
\bar{V}	1.425	Chosen to hit default rate

This table provides the calibrated parameters for the baseline version of the model. The calibration is at a quarterly basis. For a discussion of the specific parameters see the main text.

Table 7: Model Versus Data: Firm-Level Statistics

Variable	Description	Model	Data (2.5, 97.5%)
$\mathbb{E} \left(\mathbb{E}_t \left(\frac{\pi_{it}}{k_{it}} \right) \right)$	Cross-Sec Mean of Profitability	.066	.022 (.020, .024)
$\mathbb{E} \left(\sigma_t \left(\frac{\pi_{it}}{k_{it}} \right) \right)$	Cross-Sec Stdev of Profitability	.017	.050 (.047, .053)
$\mathbb{E} \left(\mathbb{E}_t \left(\frac{i_{it}}{k_{it}} \right) \right)$	Mean of Investment Rate	.029	.040 (.035, .045)
$\mathbb{E} \left(\sigma_t \left(\frac{i_{it}}{k_{it}} \right) \right)$	Stdev of Investment Rate	.045	.057 (.050, .064)
$\mathbb{E} \left(\mathbb{E}_t \left(\frac{b_{it}^S + b_{it}^L}{k_{it}} \right) \right)$	Mean of Book Leverage	.193	.249 (.238, .263)
$\mathbb{E} \left(\sigma_t \left(\frac{b_{it}^S + b_{it}^L}{k_{it}} \right) \right)$	Stdev of Book Leverage	.079	.183 (.180, .185)
$\mathbb{E} \left(\mathbb{E}_t \left(\frac{b_{it}^L}{b_{it}^S + b_{it}^L} \right) \right)$	Mean of Long Debt Ratio	.520	.694 (.669, .719)
$\mathbb{E} \left(\sigma_t \left(\frac{b_{it}^L}{b_{it}^S + b_{it}^L} \right) \right)$	Stdev of Long Debt Ratio	.225	.323 (.314, .331)
$400 \times \mathbb{E} \left(\mathbb{E}_t \left(\frac{\kappa_L + c}{p_{it}^L} - \frac{\kappa_L + c}{p_{it}^{*L}} \right) \right)$	Mean of Credit Spread	1.84	1.25 (.909, 1.65)
$400 \times \mathbb{E} \left(\sigma_t \left(\frac{\kappa_L + c}{p_{it}^L} - \frac{\kappa_L + c}{p_{it}^{*L}} \right) \right)$	Stdev of Credit Spread	12.30	–
$400 \times \mathbb{E} \left(\mathbb{E}_t \left(\mathbb{1}_{\text{Default},it} \right) \right)$	Mean of Default Rate	.968	1.08 (.422, 1.68)

This table provides firm-level statistics generated from a panel simulation of 3000 firms across 500 quarters. For variable x_{it} , $\mathbb{E}(\mathbb{E}_t(x_{it}))$ refers to the time series mean of the cross-sectional mean. Meanwhile, $\mathbb{E}(\sigma_t(x_{it}))$ refers to the time series mean of the cross-sectional standard deviation of x_{it} . All model computations remove defaulted firms. Quarterly data for profitability, investment rates, book leverage, and the long-term debt ratio are taken from Compustat, 1984 – 2015. The credit spread is defined as the difference between the Moody’s BAA and AAA corporate bond yields. The annual default rate series is also Moody’s. The numbers in parentheses represent bootstrapped time series errors at the 2.5% and 97.5% bounds.

Table 8: Model Versus Data: Aggregate Statistics

Variable	Description	Model	Data (2.5, 97.5%)
$\mathbb{E}(\Delta c_t)$	Mean Cons. Growth	.446	.493 (.397, .590)
$\sigma(\Delta c_t)$	Stdev Cons. Growth	.454	.466 (.413, .510)
$\rho(\Delta c_t, \Delta c_{t-1})$	AR(1) Cons. Growth	.499	.450 (.345, .536)
$\rho(\Delta c_t, \Delta y_t)$	Corr(Cons. Growth, Output Growth)	.741	.804 (.750, .863)
$\rho(\Delta c_t, \Delta i_t)$	Corr(Cons. Growth, Investment Growth)	.444	.596 (.460, .743)
$\rho\left(\Delta c_t, \frac{\sum_i (b_{it}^S + b_{it}^L)}{\sum_i k_{it}}\right)$	Corr(Cons. Growth, Leverage)	.400	-
$\rho\left(\Delta c_t, \frac{\sum_i (b_{it}^L)}{\sum_i (b_{it}^S + b_{it}^L)}\right)$	Corr(Cons. Growth, Agg. LTDR)	.396	.262 (.151, .356)
$\rho\left(\Delta y_t, \frac{\sum_i (b_{it}^L)}{\sum_i (b_{it}^S + b_{it}^L)}\right)$	Corr(Output Growth, Agg. LTDR)	.664	.323 (.264, .388)
$\rho(\text{AggDefault}_t, \Delta c_t)$	Corr(Agg. Default Rate, Cons. Growth)	-.365	-.223 (-.315, -.053)
$\rho(\text{MeanCreditSpread}_t, \Delta c_t)$	Corr(Agg. Credit Spread, Cons. Growth)	-.325	-.505 (-.659, -.345)

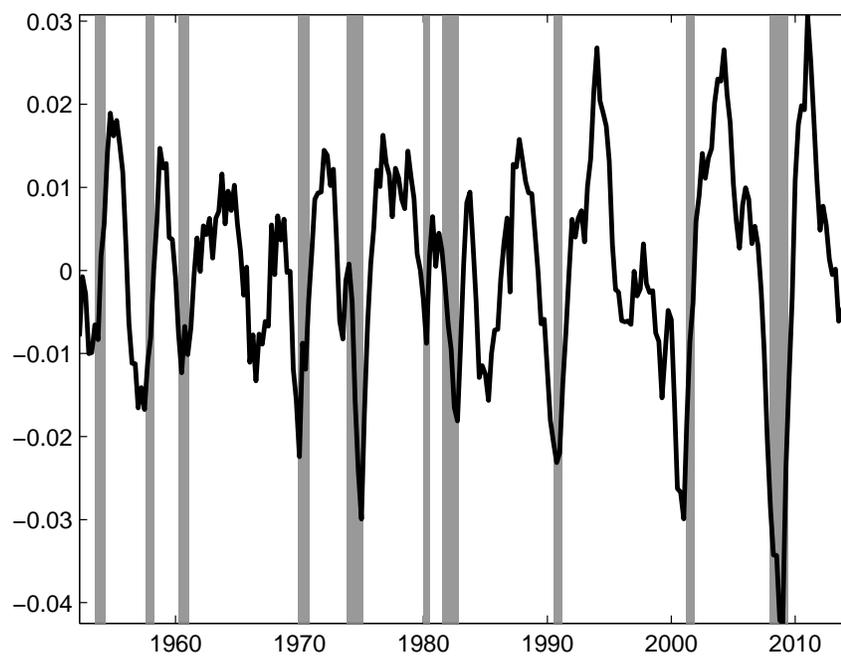
This table provides aggregate statistics generated from the aggregation of a panel simulation of 3000 firms across 500 quarters. All model computations remove defaulted firms. Data for consumption and output series are taken from NIPA accounts, from 1954 onwards. The data counterpart of the long-term debt series comes from the HP filtered version of the Flow of Funds measure. The credit spread is defined as the difference between the Moody's BAA and AAA corporate bond yields. The annual default rate series is also Moody's. The numbers in parentheses represent bootstrapped time series errors at the 2.5% and 97.5% bounds.

Table 9: **Sorting on Distance to Default**

Mean of Variable	Quintile 1	2	3	4	5
Detrended Value (\hat{V}_{it})	2.33	2.51	2.65	2.81	3.10
<i>Index</i>	86.51	94.60	100.0	106.0	119.4
Detrended Capital (\hat{k}_{it})	1.07	1.24	1.35	1.45	1.67
<i>Index</i>	78.25	92.14	100.0	108.0	126.6
Profitability ($\frac{\pi_{it}}{k_{it}}$)	.052	.058	.065	.071	.082
Investment Rate ($\frac{i_{it}}{k_{it}}$)	.012	.026	.031	.036	.042
Book Leverage ($\frac{b_{it}^S + b_{it}^L}{k_{it}}$)	.156	.186	.205	.213	.204
Book Long Term Debt Ratio ($\frac{b_{it}^L}{b_{it}^S + b_{it}^L}$)	.417	.521	.553	.557	.530
Long Term Credit Spread (% , Annual)	6.56	.901	.686	.583	.468

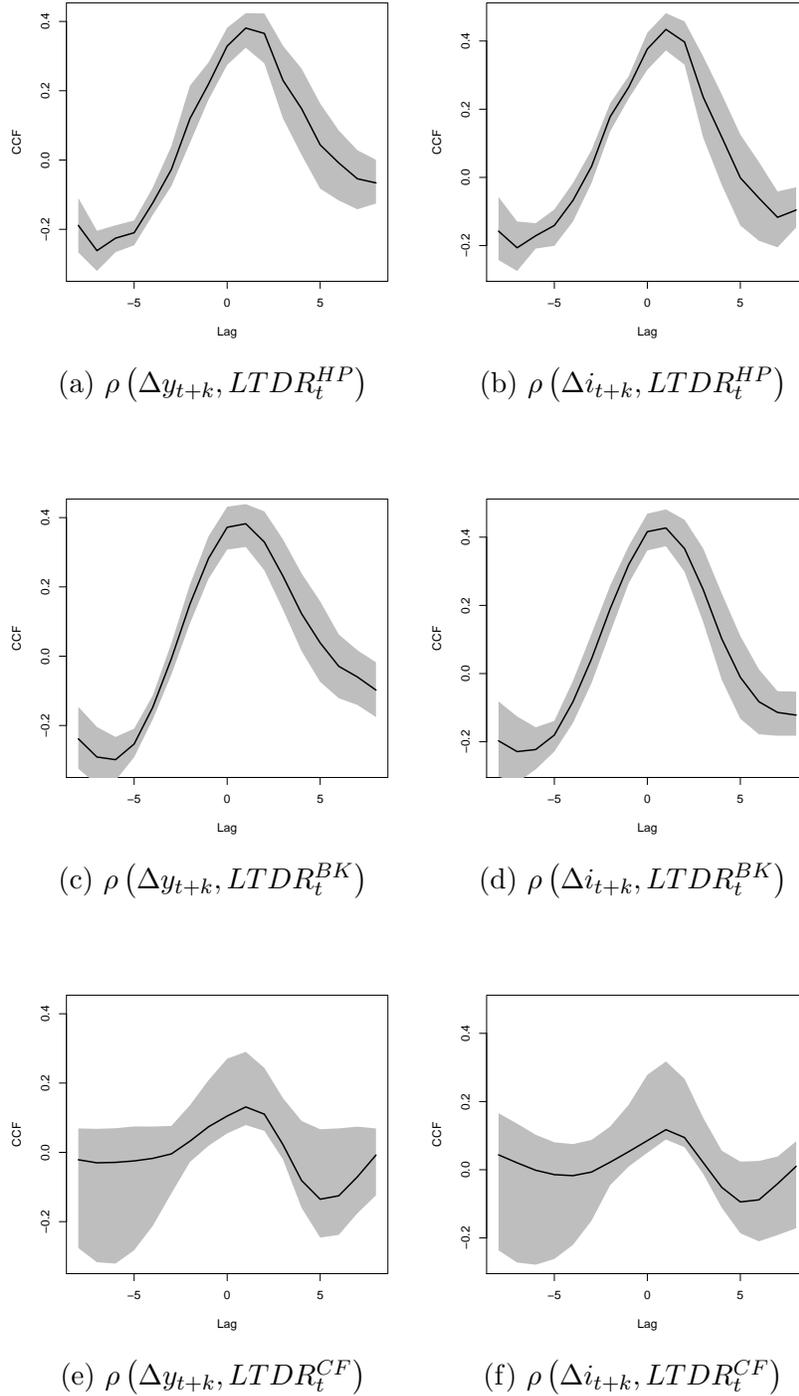
This table provides cross-sectional model statistics generated from a panel simulation of 3000 firms across 500 quarters. Each period I sort non-defaulted firms by their detrended value (\hat{V}_{it}) into five quintiles. After generating five time series for each variable I compute the average which is reported above. Under detrended value and capital I also include values that are indexed to the middle quintile.

Figure 1: Cyclical Component of the Long Term Debt Share



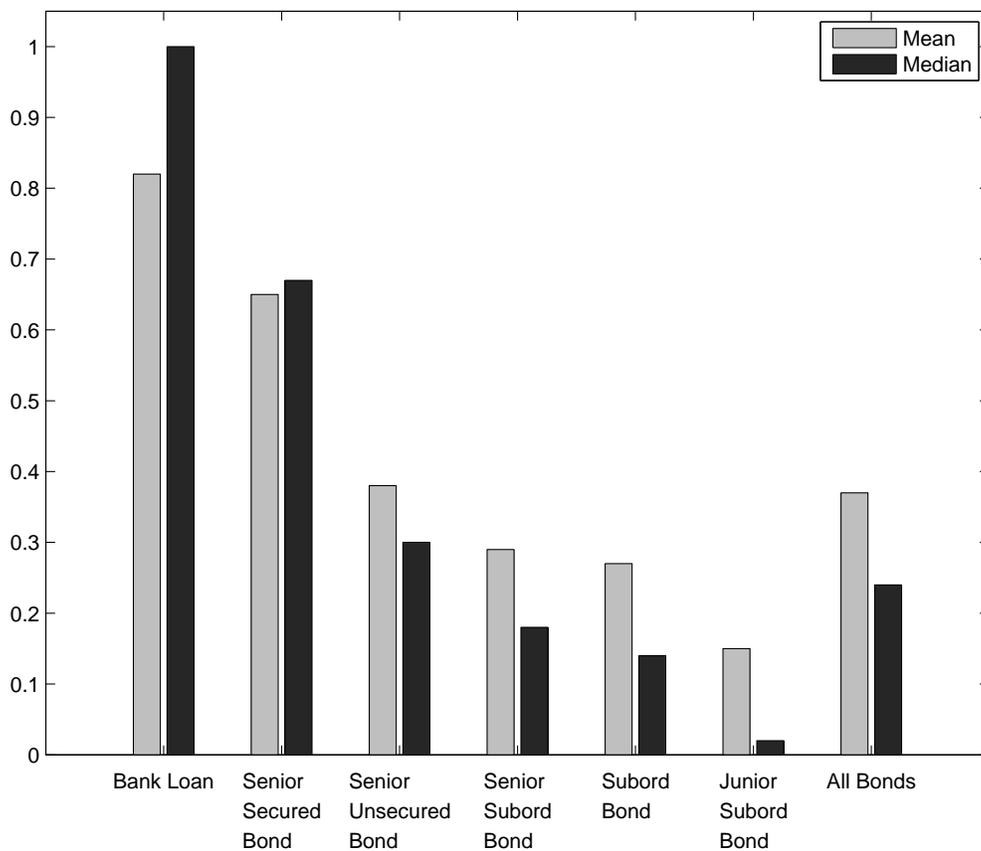
This figure displays the time series of the cyclical component of the long-term debt share, using a Hodrick and Prescott (1997) filter. Grey bars indicate NBER-defined recession dates. Data related to the long-term debt share comes from the Federal Reserve Flow of Funds. The frequency is quarterly from 1952Q2 through 2014Q2.

Figure 2: Cross Correlation of Economic Aggregates and LTDR Cycles



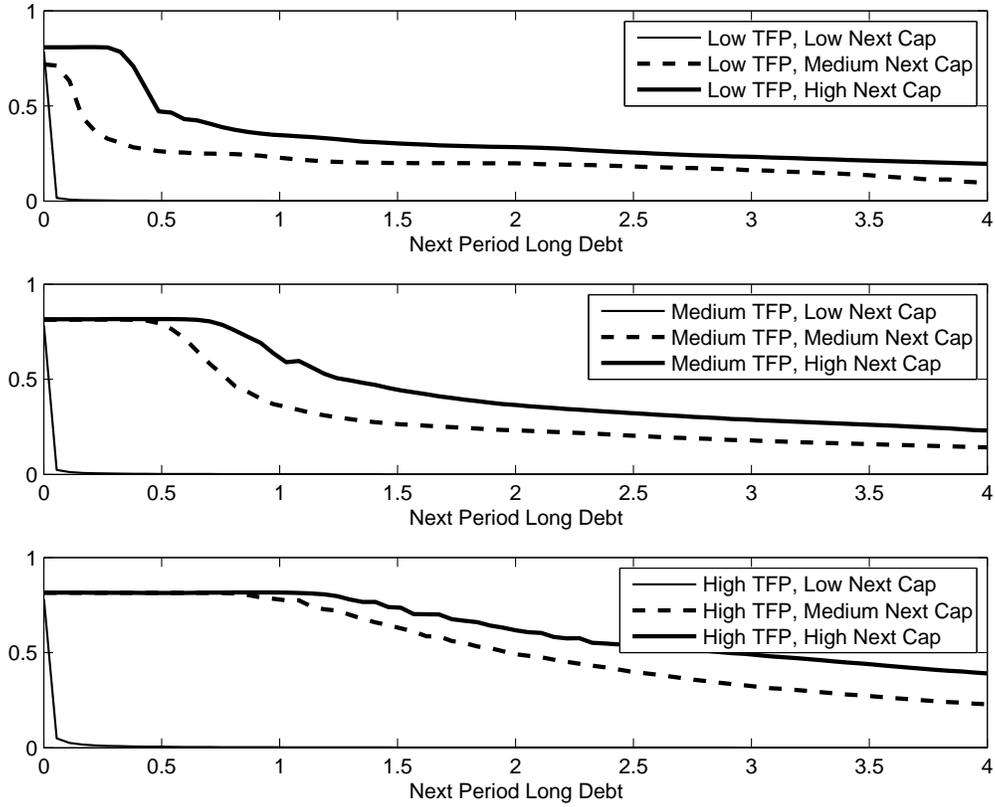
This figure displays cross correlation functions between cyclical components of the long-term debt share and economic aggregates. Figures in the left column correlate aggregate output growth and the cyclical component of the long-term debt share while those in the right column test investment growth. From top to bottom, we examine filtered values of the long-term debt share using: (1) Hodrick and Prescott ($LTDR_t^{HP}$), (2) Baxter and King band-pass ($LTDR_t^{BP}$) and (3) Christiano and Fitzgerald band-pass ($LTDR_t^{CF}$) filters. The x-axis provides the number of forward lags for each economic aggregate while the y-axis provides the cross correlation. All data is quarterly from 1952Q2 through 2014Q2. Bootstrapped 95% confidence intervals are computed at each lag and given by the gray bands.

Figure 3: Recovery Rates Across Debt Types



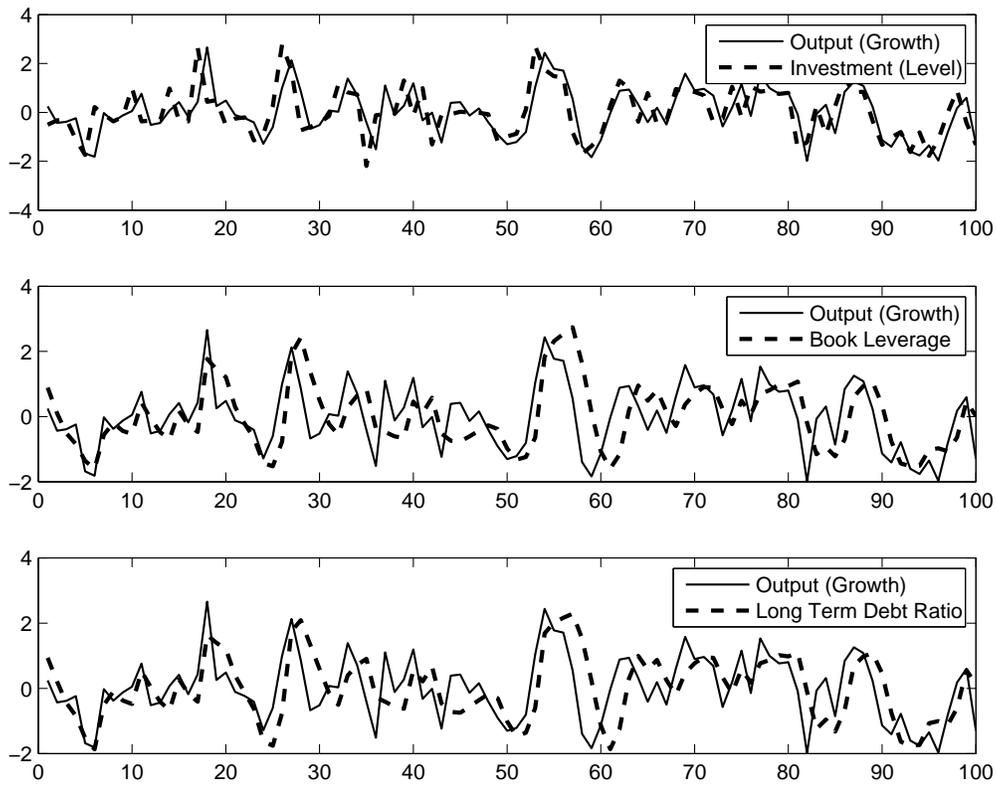
This figure displays the mean and median recovery rates across different loan and bond seniority types. All data is from “Moody’s Ultimate Recovery Database” and spans approximately 3500 loans and bonds over 720 US non-financial corporate default events. All data refers to the twenty years preceding the financing crisis (1987 – 2007). From left to right, bars represent statistics related to: bank loans, senior secured corporate bonds, senior unsecured, senior subordinated, subordinated, junior subordinated, and all corporate bonds.

Figure 4: Model-Implied Bond Prices



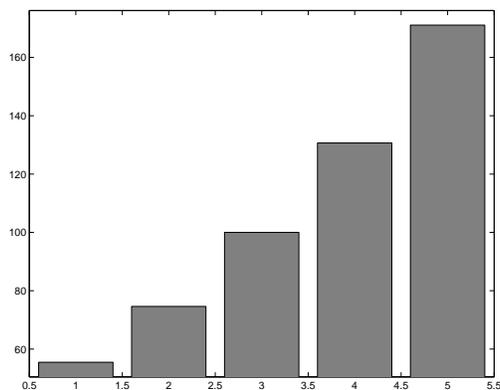
This figure displays the equilibrium value for $p_{it}^L(\hat{A}_{it}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^L)$ across states, where \hat{A}_{it} indicates the joint aggregate and idiosyncratic productivity states. The top panel focuses on the low joint TFP state (low aggregate and low idiosyncratic). The second panel focuses on the medium aggregate and idiosyncratic, while the bottom panel relates to high productivity states. In each panel the different lines (from bottom to top) represent different choices for capital next period ($\hat{k}_{i,t+1}$). The bottom axis represents the choice of next period long-term debt ($\hat{b}_{i,t+1}^L$).

Figure 5: Aggregate Behavior of the Model

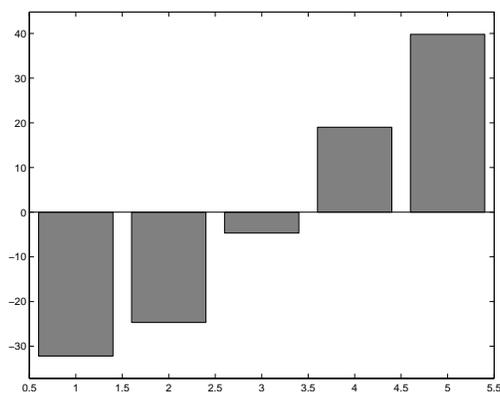


This figure displays the aggregate behavior of the model in a sample simulation of 100 quarters. In all three graphs the solid black line represents output growth. From top to bottom, the dashed line represents investment, book leverage, and the long-term debt ratio. All values are standardized and provided as z-scores.

Figure 6: **Funding Investment through Long Term Debt**



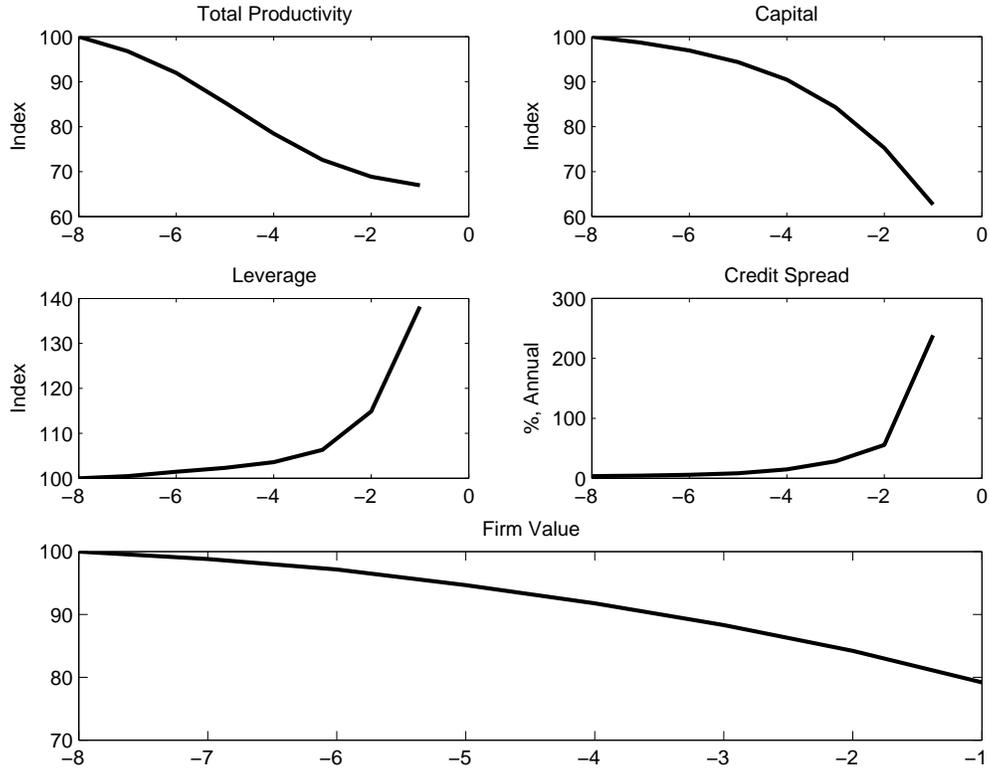
(a) Funding Deficit (Index)



(b) Long Term Debt Funding (%)

Figure (a) displays the time series average of the simulated aggregate funding deficits across five aggregate states. The left most bar relates to the lowest consumption growth state while the right most to highest. All of the state values are indexed to the middle state. To build the funding deficit we simulate the model to compute the gap between total dividend and the amount raised through short and long-term debt ($\hat{D}_{it} - p_{it}^S \hat{w}_{it}^S - p_{it}^L \hat{w}_{it}^L$). To receive the aggregate number, we sum across non-defaulted firms. Figure (b) displays the time series average of the percentage of the funding deficit that comes from the long-term debt proceeds ($p_{it}^L \hat{w}_{it}^L$). For cases where this percentage is negative, this suggests that long-term debt was purchased back.

Figure 7: Firm Behavior Preceding Default



This figure displays the average behavior of the firm eight quarters in advance of default. The panels represent, from left to right, total productivity (\hat{A}_{it}), de-trended capital (\hat{k}_{it}), leverage ($\frac{\hat{b}_{it}^S + \hat{b}_{it}^L}{\hat{k}_{it}}$), the credit spread ($\frac{\kappa_L + c}{p_{it}^L} - \frac{\kappa_L + c}{p_{it}^{L*}}$), and detrended firm-value (\hat{V}_{it}). The bottom axis provides the number of quarters in relation to default which occurs at time 0. Productivity, capital, leverage, and firm value are all indexed to the initial value, 8 quarters in advance of default, while the credit spread is expressed in annual percentage terms.