Production Networks and the Propagation of Commodity Price Shocks*

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Preliminary

Abstract

Commodity price fluctuations have macro economic implications not only through resource reallocation, currency value changes and monetary policy reaction, but also through production network linkages (both domestically and with the rest of the world). In this paper, we study the propagation of commodity price shocks in a multiple-sector general equilibrium model for a small open economy that exports commodity. In the small open economy, a shock to commodity prices is both aggregate and sectoral. As an aggregate shock, commodity price movements lead to changes in the value of domestic currency, impacting the macro economy due to its size and triggering monetary policy responses. As a sectoral shock, changes in commodity price impact non-commodity sectors in two aspects: impacting demand for the upstream goods and impacting the cost of production in the downstream sectors. Calibrated to the Canadian data, our model suggests that, following a positive shock to commodity prices, production and exports in the commodity sector rises, while the net impact on the rest of the economy's production is negative. The aggregate gross domestic output (GDP) increases primarily owing to growth in investment and improved trade balance. The export connection with the rest of the world in the open economy production network plays an important role in the process of adjusting to a commodity price shock, while the import connections do not matter nearly as much. We show that these propagation channels of shocks to commodity price explain a large fraction of drop in Canadian real GDP in 2015 following the sharp decline in commodity prices that started in late 2014.

Keywords: commodity price, input-output linkage, international trade.
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1 Introduction

Fluctuations of commodity prices are frequently associated with the volatility of aggregate output and prices. Their roles in transmitting inflation and in inducing macroeconomic adjustments are clearly shown through the recent boom and bust super-cycles. A broad-based boom in global commodity markets began in the early 2000s, fuelled by growing demand from emerging-market economies. Along with the rising commodity prices, adjustments took place in the economy ranging from changes in employment and output to changes in interest rates and exchange rates.

For a commodity-exporting country, rising commodity prices boosted economic activity and generated job growth in the natural resource sector. Take Canada for example, the commodity sector has been increasingly important for the Canadian economy. Commodity sector contributes 15% of aggregate output and 40% of total exports in 2014; business investment in the oil and gas extraction sector peaked at almost $80 billion. The boom in the oil sector had spill-over impacts on other sectors through resource reallocation. Between 2002 and 2013, more than a quarter of a million people moved from other provinces to the oil-producing regions. In addition, the number of workers commuting to these regions doubled during this period, rising to about 8 per cent of the regions' workforce.[1] On top of the domestic adjustment, the positive link between commodity prices and the value of currency also contributed to shifting resources from non-commodity export sectors to commodity-producing sectors and regions.

Since its historically high levels in mid-2014, the West Texas Intermediate (WTI) crude oil price declined by about 68 per cent between mid-2014 and January 2016. This has triggered a process of restructuring in the oil and gas sector, in the opposite way to what we have seen in the previous decade. The value of the Canadian dollar has fallen along with commodity prices, facilitating the adjustment to the new circumstances. Concerned about the impact of lower oil prices and the risks to inflation, the Bank of Canada lowered its policy rate in January 2015 and again in July. Commodity price fluctuations have macro implications not only through resource reallocations, currency value changes, but also through monetary policy reactions.

In this complex adjustment of the economy to commodity price super-cycles, one important channel is often overlooked: the production network. As shown in Figure 1, commodities are used primarily not only for exports (54%) but also as intermediate inputs in the domestic economy (37%). Moreover, the production in the commodity sector also uses intermediate inputs from other sectors. Table 1 shows the domestic make input-output linkages. Column 1 represents the intermediate inputs purchased by Commodity sector from all domestic sectors

[1] Patterson, Lynn: Adjusting to the Fall in Commodity Prices: One Step at a Time, March 30, 2016.

as a share of total intermediate inputs used to produce commodity. Similar to other sectors, wholesale and service sectors are the most important upstream sectors for commodity production. The linkage between commodity sector and the rest of the economy is also uneven across sectors. In this context, we would miss a big part of the picture should the discussion on the macroeconomic adjustment to commodity price movements ignore the input-output linkages and fail to capture adjustments taking place through the production network.

In this paper, we study the propagation of commodity price shocks in a multiple-sector general equilibrium model for a small open economy that exports commodity. In addition to allowing for resource reallocation, exchange rate movements and monetary policy reaction, we emphasize the role played by input-output linkages, both domestic and with the rest of the world. In the small open economy, shocks to commodity price are both aggregate and sectoral. In the sense of an aggregate shock, the super-cycle movements of commodity prices lead to the value change of domestic currency and inflation, triggering monetary policy response. As a sectoral shock, changes in commodity price impacts non-commodity sectors in two aspects. First, the rise of commodity price increases the exports and output in the commodity sector, which in turn raises the demand of this sector for the upstream goods as intermediate inputs. This tends to increase the non-commodity outputs. On the other hand, a rising commodity price increases the cost of production in the downstream non-commodity sectors since commodity is used as an intermediate input. The direction of the net effects crucially depends on the input-output linkages between commodity and other sectors, as well as the elasticity of exports with respect to exchange rate.

Calibrated to the Canadian production network, our model suggests that, following a positive shock to commodity prices, production and exports in the commodity sector rises. While the net impact on the rest of the economy's production is negative, the aggregate gross domestic output (GDP) increases primarily owing to growth in investment and improved trade balance. The export connections with the rest of the world in the open economy production network plays an important role in the process of adjusting to a commodity price shock, while the import connections do not matter nearly as much. These channels through the input-output linkages, both domestic and international, explain a large fraction of slower growth in real GDP in 2015, following the sharp decline in commodity prices in 2014 and 2015.

Our paper is closely linked to the literature that analyzes sector-specific versus aggregate sources of variations in the business cycle, see for example, Foerster et al. (2011). In explaining that sectoral variability does not average out in the index of economy-wide production variability, there are three explanations. First, there are aggregate shocks that are common across sectors, therefore becoming the dominant source of variation in aggregate economic activity. Second, there are granularity shocks (see, for example, Gabaix(2011)) impacting a handful of very large sectors in the economy. Because of their size, shocks to these sectors do not average out. Third, there are other sector-specific shocks that propagate through complementarities in production, such as input-output linkages, such that sector-specific shocks can generate substantial aggregate variability. The commodity price shock plays the role of all three in the above in explaining the variability in the aggregate. Our paper provides a structural framework to identify these transmission mechanisms and determine the quantitative importance of them. Upon a positive commodity price shock, the aggregate GDP rises despite the dampening effect on the rest of the economy, largely thanks to the size of the commodity sector as well as the lifts in investment. The upstream imports channel does very little in terms of adjusting to the commodity price shock, while the domestic upstream supply remains an important channel of adjustment. On contrast, the export connections with the rest of the world in the open economy production network play an important role in the adjustment process, though the domestic downstream impacts are small.

Our paper is also related to the literature assessing the implications of commodity price changes on the macro economy. As Kilian (2009) suggested, fluctuations in the price of oil may have very different dynamic effects on macroeconomic aggregates depending on their underlying sources of fluctuation. A number of papers have adopted structural VAR models to identify relevant shocks to the global crude oil market.[2] Though much attention has been focused on commodity-importing countries, there are a growing number of studies for countries producing and exporting commodities. In particular, Charnavoki and Dolado (2014) studies the dynamic effects of commodity shocks on the Canadian economy using dynamic factor model estimation. Our focus is different from these studies. We explicitly model the structure of the economy, laying out the propagation mechanisms. Allowing for commodity price shock to act as both an aggregate shock and a sectoral specific shock, we examine the role played by the production network, both domestic and with the rest of the world. With this structural framework, we are able to not only reproduce the main stylized features documented in the literature including a Dutch disease effect, but also identify the quantitative importance of the propagation mechanisms.

The paper proceeds as follows. Section 2 develops the multi-sector model with both domestic and foreign input-output linkages. Section 3 introduces the calibration of the model. The impacts of the 2014 plunge in oil prices and other quantitative findings are discussed in Section 4. Finally, Section 5 concludes.

2 Model

We consider a small open-economy that produces and exports commodity. The economy consists of N sectors and one representative household. Each sector produces one good,

[2] For example, Kilian and Murphy (2012), Lippi and Nobili (2012).

which can be used for final demands and as intermediate inputs. The model economy is featured with the input-output linkages, both domestic and with the rest of the world.

2.1 Sector Production The production in sector j requires a capital-labor bundle and intermediate inputs. We assume that the production technology exhibits the constant elasticity of substitution (CES) between capital-labor bundle and the intermediate input, as follows[3]

$$Q_j = A_j \left[(1-\psi_j)^{\frac{1}{\sigma_q}} \left(\left(\frac{K_j}{\alpha_{kj}} \right)^{\alpha_{kj}} \left(\frac{L_j}{\alpha_{lj}} \right)^{\alpha_{lj}} \right)^{\frac{\sigma_q - 1}{\sigma_q}} + \psi_j^{\frac{1}{\sigma_q}} M_j^{\frac{\sigma_q - 1}{\sigma_q}} \right]^{\frac{\sigma_q}{\sigma_q - 1}}.$$

We impose that $\alpha_{kj} + \alpha_{lj} = 1$. The intermediate input M_j is produced by combining domestic and imported goods, as follows

$$M_j = \left[\sum_{i=1}^{N} \left(\omega_{hij}^{\frac{1}{\sigma_m}} M_{hi,j}^{\frac{\sigma_m - 1}{\sigma_m}} + \omega_{fij}^{\frac{1}{\sigma_m}} M_{fi,j}^{\frac{\sigma_m - 1}{\sigma_m}} \right) \right]^{\frac{\sigma_m}{\sigma_m - 1}},$$

where $\sum_{i=1}^{N} (\omega_{hij} + \omega_{fij}) = 1$.

Hereafter, the import price of good j, regardless of its use, is given by $P_{fj} = SP_j^*$, where S is the nominal exchange rate, the value of domestic currency per unit of foreign currency, hence a lower value of S means an appreciation of domestic currency. P_j^* is the foreign price denominated in foreign currency.

The use of sector j output is

$$Q_j = C_{hj} + V_{hj} + \sum_{i=1}^{N} M_{hji} + X_j,$$

where C_{hj} is used as a component of aggregate consumption, V_{hj} is the contribution to the aggregate investment, and X_j is the exports by sector j. M_{hji} is the amount of good j supplied to sector i as an intermediate input. We assume that there is no trade cost, hence in equilibrium the export price (denominated in domestic currency) P_{xj} and domestic price P_{hj} must equal.

The foreign demand for sector j good is given by

$$X_j = \alpha_{xj} \left(\frac{P_{xj}}{SP^*} \right)^{-\sigma_x} Y^*.$$

Variables with an asterisk represent the prices and outputs in the rest of the world, which

[3]We omit the time subscript wherever we feel it will not create confusion.

are exogenous to the domestic economy.

2.2 Aggregate consumption and investment Aggregate consumption C_t is combined from sector-level outputs and imported goods, with the following CES technology

$$C = \left[\sum_{j=1}^{N} \left(\phi_{hj}^{\frac{1}{\sigma_c}} C_{hj}^{\frac{\sigma_c - 1}{\sigma}} + \phi_{fj}^{\frac{1}{\sigma_c}} C_{fj}^{\frac{\sigma_c - 1}{\sigma_c}} \right) \right]^{\frac{\sigma_c}{\sigma_c - 1}},$$

where $\sum_{j=1}^{N} (\phi_{hj} + \phi_{fj}) = 1$.

Capital is homogeneous across sectors, its aggregate stock evolves as

$$K_{t+1} = (1 - \delta) K_t + V_t.$$

Aggregate investment V_t is produced using domestic and imported capital goods, as follows

$$V_t = \left[\sum_{j=1}^{N} \left(\theta_{hj}^{\frac{1}{\sigma_v}} V_{hjt}^{\frac{\sigma_v - 1}{\sigma_v}} + \theta_{fj}^{\frac{1}{\sigma_v}} V_{fjt}^{\frac{\sigma_v - 1}{\sigma_v}} \right) \right]^{\frac{\sigma_v}{\sigma_v - 1}},$$

with $\sum_{j=1}^{N} (\theta_{hj} + \theta_{fj}) = 1$.

2.3 Household The representative household makes decisions on investment, domestic bond and foreign bond holdings. It maximizes utility in an infinite horizon

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t [\ln(C_t) - \xi L_t],$$

subject to

$$P_{ct} C_t + P_{vt} V_t + \frac{B_t}{R_t} + \frac{S_t B_t^*}{R_t^*} = w_t L_t + r_{kt} P_{vt} K_t + S_t B_{t-1}^* + B_{t-1} + T_t.$$

We assume a simplistic government budget constraint, $T_t = \frac{B_t}{R_t} - B_{t-1}$. That is, the lump-sum transfer T_t equals to the bond issuance.

At the aggregate level, trade surpluses are saved in the form of foreign bond. The balance of payment for the economy is

$$\sum_{i=1}^{N} P_{xi,t} X_{it} - \sum_{i}^{N} P_{fi,t} \big(V_{fi,t} + C_{fi,t} + \sum_{j}^{N} M_{fi,j,t} \big) = \frac{S_t B_t^*}{R_t^*} - S_t B_{t-1}^*.$$

2.4 Shocks In this economy, the shocks are those concerning total factor productivity A_{jt}, foreign prices P^*_{jt}, foreign interest rate R^*_t, and foreign output Y^*_{jt} for $j = 1, N$. Exchange rate is determined by the interest rate parity condition.

Nominal interest rate is determined by central bank, in a Taylor-type rule as follows

$$\ln(R_t/R) = \rho_r \ln(R_{t-1}/R) + (1-\rho_r)[\alpha_\pi \ln(\pi_t/\pi) + \alpha_z \ln(Z_t/Z)] + \epsilon_{Rt}.$$

Here, π_t is the inflation rate of consumption price P_{ct} and Z_t is the aggregate real GDP. We define Z_t as the geometric mean of consumption, investment, aggregate exports and imports[4].

2.5 The shock propagation Characterizing the optimal conditions and the equilibrium is straightforward for this model. We want to emphasize two optimal conditions that are key for the propagation mechanism, one concerning the exchange rate, the other concerning the input-output linkage. These two channels represent respectively the aggregate and sectoral nature of commodity price shocks.

2.5.1 Commodity price and exchange rate The optimal conditions with respect to domestic and foreign bonds imply uncovered interest parity condition,

$$\frac{R_t}{R^*_t} = E_t \frac{S_{t+1}}{S_t}.$$

A rise in the commodity price leads to an appreciation of domestic currency, by two channels. First, commodity exports increase, leads to possibly a larger trade balance, hence expanding the demand for foreign bonds. This tends to push downward the domestic nominal interest rate, by the uncovered interest rate parity condition. Second, the increased exports and output in the commodity sector raises the demand for capital and labor. The production in commodity sector is capital intensive, hence the demand for labor increases but by a small margin, relative to that for capital and investment. As commodity price rises, the rate of returns to capital tends to rise due to increased demand for investment, this in turn lowers consumption price. By the Taylor rule, the nominal interest rate will be lower.

Both the increased demand for foreign bonds and the lower domestic consumer price lead to a lower nominal interest rate. By the above uncovered interest rate parity condition, the expected nominal exchange rate will drop (an appreciation).

The output in non-commodity sectors tends to fall under two impacts of a positive shock

[4] We define the real GDP as $Z_t = C_t^{\omega_{ct}} V_t^{\omega_{vt}} \cdot \prod_{j=1}^N X_{jt}^{\omega_{xjt}} \cdot \prod_{j=1}^N I_{mjt}^{-\omega_{mjt}}$, where $\omega_{ct} = \frac{P_{ct}C_t}{P_{zt}Z_t}$. I_{mjt} is the total import of sector-j commodities, $I_{mjt} = C_{fjt} + V_{fjt} + \sum_{i=1}^N M_{fjit}$. We assume that ω_{ct} and other shares in the above definition are constant and equal to their respective values in steady-state value (which is endogenous).

to commodity price. The rising real returns to capital makes capital more costly for all sectors, dampening the demand for capital and output in non-commodity sectors. Moreover, for sectors with a large share of exports in gross output such as manufacturing, output can fall due to lower exports in turn due to domestic currency appreciation.

The fall in gross output does not necessarily reduce the real value added, the latter is more relevant for welfare. For sectors where the imported intermediate inputs account for a large share of total inputs, for example manufacturing, appreciated domestic currency leads to lower import prices. Whether the lower import prices increase or reduce value added depends on the value of the elasticity of substitution between individual intermediate inputs, and between capital-labor bundle and total sector intermediate input. The share of imported intermediate input i in sector j's production is given by

$$\Omega_{fijt} = \frac{P_{fit}M_{fijt}}{P_{hjt}Q_{jt}} = \omega_{fij}\psi_j P_{fit}^{1-\sigma_m}\Big[\sum_{i=1}^{N}\left(\omega_{hij}P_{hit}^{1-\sigma_m} + \omega_{fij}P_{fit}^{1-\sigma_m}\right)\Big]^{\frac{\sigma_m-\sigma_q}{1-\sigma_m}} P_{hjt}^{\sigma_q-1}A_{jt}^{\sigma_q-1}, \quad (2.1)$$

where we have substituted for P_{mjt} using $P_{mjt} = \Big[\sum_{i=1}^{N}\left(\omega_{hij}P_{hit}^{1-\sigma_m} + \omega_{fij}P_{fit}^{1-\sigma_m}\right)\Big]^{\frac{1}{1-\sigma_m}}$. When individual inputs are gross complements ($\sigma_m < 1$) and $\sigma_m > \sigma_q$, a lower import price leads to a lower share of imported goods in production. The demand for both imported and domestic inputs increases, increasing gross output. Further, if $\sigma_m > sigma_q$, a lower import price also reduces the share of domestic inputs in production. Together, the share of total intermediate inputs in sectoral production is smaller, which leads to a higher value added.

2.5.2 Propagation through production network

The negative impact of the rising commodity price on production among non-commodity sectors, as shown in the above, does not take into account the input-output linkage between commodity sector and others. Through this linkage, both the downstream and the upstream sectors of the commodity sector can be affected by commodity price. The share of domestic intermediate input i in sector j's production is given by

$$\Omega_{hijt} = \frac{P_{hit}M_{hijt}}{P_{hjt}Q_{jt}} = \omega_{hij}\psi_j P_{hit}^{1-\sigma_m}\Big[\sum_{i=1}^{N}\left(\omega_{hij}P_{hit}^{1-\sigma_m} + \omega_{fij}P_{fit}^{1-\sigma_m}\right)\Big]^{\frac{\sigma_m-\sigma_q}{1-\sigma_m}} P_{hjt}^{\sigma_q-1}A_{jt}^{\sigma_q-1}, \quad (2.2)$$

Let matrix $\Omega_{\mathbf{ht}} = [\Omega_{hijt}]_{N\times N}$, and let $F_t = [F_{jt}]_{N\times 1}$ be the vector of final uses of sector j output. We write $\mathsf{P_{ht}F_t} = [P_{hjt}F_{jt}]_{N\times 1}$ and so on. From the use equation of sector j output, $P_{hjt}Q_{jt} = P_{hjt}F_{jt} + P_{hjt}\sum_{i=1}^{N} M_{hjit}$, we obtain

$$\mathsf{P_{ht}Q_t} = \big[\mathbf{I} - \Omega_{\mathbf{ht}}\big]^{-1}\mathsf{P_{ht}F_t}. \quad (2.3)$$

Here $\mathbf{I}$ is the identity matrix. $\mathbf{H_t} = \big[\mathbf{I} - \Omega_{\mathbf{ht}}\big]^{-1}$ is the Leontief inverse matrix. As shown in Acemoglu et al. (2015) in the case of Cobb-Douglas production function in which the elements of Leontief inverse matrix are constants, the network effects materialize through the Leontief inverse matrix. When a sector-specific shock hits sector j, the demand and price of that sector change (own effect). The changed demand will impact the upstream suppliers to sector j, and the change in price of output j will impact the downstream sector that purchase inputs from sector j.

Following a positive shock in commodity price, the demand for commodity exports rises, this in turn raises the demand for non-commodity goods used as intermediate inputs to produce commodity. The output of upstream sectors therefore rises, in particular the supplying sectors that account for a large share of intermediate inputs in the commodity sector.

Importantly, the impact of a commodity price shock on the share of intermediate input in total production cost depends upon the elasticity of substitution between intermediate input and the capital-labor combination and that between imported and domestically-produced intermediate inputs. A reduction in import price leads to a lower P_{mjt}, the share of intermediate input could become higher or lower depending whether the elasticity of substitution is greater or less than 1, i.e. whether intermediate inputs and capital and labor are complements or substitutes.

3 Model Calibration

We use the Canadian data on multifactor productivity, national income and expenditure accounts, and the World Input-Output Database (WIOD), to construct inputs and outputs for eight sectors: Commodity, Construction, Utility, Manufacturing (3 sectors), Wholesale and retail, and Services. The commodity sector is constructed such that the basket of goods is the same as that used in creating Bank of Canada Commodity Price Index (BCPI), the basket includes agriculture, forestry, fishery, mining, non-metal minerals, and primary metal. The first manufacturing sectors including mainly those producing food, textile, paper, etc. The second manufacturing sector produces petroleum, chemicals and plastic products. The last manufacturing sector produces machine and equipment. Data from Statistics Canada are from 1981 to 2014. The WIOD data are from 2000 to 2014.

3.1 Production and aggregation functions

Elasticity of substitution. We estimate the elasticity of substitution in all production functions and aggregate functions. To estimate σ_q, we use the share of intermediate inputs in sector gross outputs. We use the first-differencing of the logarithm of input shares, on the right hand side are prices of intermediate inputs and the total factor productivity.[5]

[5]Unlike Foerster et al. (2011), our model allows multiple shocks, we cannot back out productivity shocks from the model. Instead, we use the total factor productivity measured by Statistics Canada, which is

Considering that the two prices are endogenous, we use the growth of prices of exports and imports as instrument variables, assuming that these latter prices are exogenous to the Canadian economy. Considering that TFP is serially correlated, we also use lagged TFP changes as instrument variable. The estimation for the period 1961-2014 leads to $\hat{\sigma}_q = 0.931$ (robust Std.Err. 0.49), the estimate is significant at 10% level. If we use the data sample since 1981, the resulting estimate is $\hat{\sigma}_q = 0.74$ (Std.Err. 0.48), but it is statistically insignificant. Overall, there is a weak evidence suggesting a declining elasticity of substitution between capital-labor bundle and the intermediate input.

To estimate σ_m, we use the domestic shares of intermediate inputs by sector, calculated using the 2016 WIOD, spanning from 2000 to 2014. For instrument variables, we use growth rates of export prices corresponding to the supply sectors and import prices corresponding to the use sectors. The resulted estimate is $\hat{\sigma}_m = 1.09$ (Std.Err. 0.20), which is at 1% significant level.

To estimate σ_c, we use domestic shares of aggregate consumption, calculated using the 2016 release of WIOD. We again use the prices of exports and imports as instrument variables. The estimate is $\hat{\sigma}_c = 0.89$ (Std.Err. 0.27), it is statistically significant at 1% level. We do a similar estimation for the elasticity of substitution in the investment aggregation function. The resulted estimate is $\hat{\sigma}_v = 1.01$ (Std.Err. 0.57), it is statistically significant at 10% level.

Weights. For the weight parameters in production and aggregation functions, we use the calibrate them to match the observed shares of individual components in production and aggregation functions in steady state. These shares in the data are calculated as averages from the multifactor productivity data for production functions, and from the WIOD for intermediate inputs, aggregate consumption and investment.

Labor shares. We calculate the share of labor in the total cost of capital-labor bundle from the multifactor productivity data, we use the average over 1981 to 2014. Labor shares from Commodity to Services are respectively, 0.39, 0.90, 0.28, 0.71, 0.48, 0.71, 0.72, and 0.62.

3.2 Price elasticity of exports

We use the estimate by Simonovska and Waugh (2014), and choose $\sigma_x = 4.6$ which corresponds to the Armington elasticity estimated by the authors.

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appropriate for our purpose. In measuring productivity, main assumptions are that the production function exhibits constant returns to scale, and perfect competition, see Baldwin et al. (2007).

3.3 Preference and depreciation rate We set $\xi = 1$, $\beta = 0.96$, and $\delta = 0.10$. Depreciation rate is based on capital stock and flow data by Statistics Canada. The implied steady state real interest rate on capital service is $r = \frac{1}{\beta} - 1 + \delta = 0.142$.

3.4 Shocks We assume that shocks of exogenous variables in logarithm all follow AR(1) processes. For commodity prices, we use the Bank of Canada Commodity Price Index (BCPI) series (in the U.S. dollars), de-trended with the HP filter. Data span from 1972 to 2018. Using the ARIMA routine in Stata, we estimate that $\widehat{\rho}_{p_{h1}} = 0.75$ (semirobust std.err. is 0.127), and the estimated standard deviation of the noise term is 0.135.

For shocks of total factor productivity, we use the measured multifactor productivity series from 1972 to 2014, removing trends with HP filter.

Estimation with original series. When using original series, without de-trending, we obtain $\widehat{\rho}_{p_{h1}} = 0.986$, and the standard deviation of the noise term is 0.125.

For total factor productivity, the results are in dropbox folder tfp*Level.txt

Foreign shocks. We estimate the shocks of foreign price and output using the price and quantitie of the gross output in the United States for the period of 1981 to 2017.[6]

The estimated serial correlation coefficient of output price is 0.796, and the standard deviation of the error term for the price process is 0.021. The serial correlation for gross output is 0.817, and the standard deviation for the error term is 0.017.

Monetary policy shocks. For Canadian monetary policy shock, we use the series created by Champagne and Sekkel (2017). At the yearly frequency, the standard deviation of this shock is 0.079.

We use the data series on shocks to Federal Funds rate as estimated in Nakamura and Steinsson (2013). Using the annual data over the period of 1995 to 2014 for the shock series, we obtain the standard deviation of the shock at 0.020.

4 Quantitative Analysis

4.1 Impact of the 2014 plunge in commodity prices We use our model to quantify the impact of sharp falls in commodity price that started in the second half of 2014. This exercise allows us to assess the conformity of the model to the data. BCPI, with the trend removed, dropped 27.5, 38.4, and 24.3 percent respectively in 2015, 2016, and 2017. Real GDP and it main components also declined. Table 3 reports the magnitude of decline in the data. Specifically, real Canadian GDP droped by 1.3% in 2016 and 2.3% in 2016. To

[6] Data are downloaded from www.bea.gov, annual industry statistics.

quantify the impact of oil price plunge in the model, we impose a one-time negative shock of the size of 27.5 percent, the same magnitude as the 2015 commodity price drop. We feed this one-time shock to the log-linearized system and plot the impulse responses. Figure 4 shows that the negative shock to BCPI led to a 1% drop in 2015 and 0.8% drop in 2016 of real GDP, suggesting that commodity price shock can explain a large fraction of fall in real GDP in 2015 and a fairly significant amount of GDP drop in 2016.[7] The GDP drop predicted by the model arises from the decline in investment and reduced net exports.[8]

According to our model, nominal exchange rate increases by close to 14% following the decline of commodity price, as is consistent with the data. The Canadian dollar relative to the U.S. dollar was above the trend by 9% in 2015 and by 13% in 2016. This depreciation in domestic currency is resulted from a lower holding of foreign bonds, which in turn because of the lower trade balance due to a sharp decline in commodity exports, the latter falls by 39% in the model.

The depreciation of Canadian dollar following a falling commodity price helps explain changes in GDP components. In the model, the reduced aggregate exports are entirely because of lower exports in commodity, while non-commodity sectors see increases in exports. Aggregate imports increase as commodity price falls, resulted from two off-setting forces. On the one hand, depreciated domestic currency encourages non-commodity exports, raising demands for imported intermediate inputs and imported investment. On the other hand, depreciated currency raises the relative price of imported goods.

Because currency depreciates, domestic producers and consumers increase demand for domestically produced goods to substitute for more expensive imports. The different elasticity of substitution between domestic and imported goods determine the domestic output prices, which turn out become higher except for one manufacturing sector. Higher domestic output prices at the end contribute to lower investment spendings.

Overall, our model is promising in capturing the differential impacts of declined commodity price among different sectors. The overall effect, i.e., the reduced real GDP, predicted by the model can explain a large fraction of slower GDP growth in the data. Finally, our model predicts that labor inputs drop in all sectors in responding to declining commodity price, which is also consistent with the data, except for the construction sector.

4.2 Baseline Impacts In this section, we examine the propagation mechanisms of commodity price shocks in the baseline model. Figure 5 to 7 illustrate the impulse responses

[7] It is worth noting that oil prices kept its plunging path in 2016, dropping another 38.4%. This would certainly contribute to the more negative growth in 2016, but it is not reflected in our exercise as we only impose a one-time 2015 shock.

[8] In our model, real consumption became higher, which is opposite to the change in the data. This could be a result of simplified setup on the consumption front. However, it is a tradeoff we take to instead focus on the production network.

of sector exports, production and other macro variables in responding to the commodity price shock. Upon a positive commodity price shock, the home country's commodity goods become relatively cheaper, thus both the production and exports in the commodity sector (sector 1) rise. Through the production network, a boosting commodity sector demands more intermediate inputs from other sectors (thus driving up other sectors' production). At the same time, a rising commodity price causes other sectors to use this imported input less intensively. On one hand, this could reduce their own production. But on the other hand, they would substitute away from imported commodity inputs to domestic intermediate inputs. Given that every sector is using inputs from other supplier industries, this is only possible if industries supplying inputs also expand their production.

On top of the production linkages, due to its capital intensive nature, the boost in the commodity sector also drives up the expected return of capital in general, leading to increased investment and reduced consumption. As a result, the price of consumption goods falls. The monetary authority lowers interest rate to fight a downward trending inflation. The cross-country interest rate differential then leads to an appreciated Canadian dollar. The rise in the value of the home currency makes other sectors' (particularly the manufacturing sectors) exports less attractive, thus dampening other sectors' exports and production.

It should be noted that all sectors not only export to the rest of the world, but also imports intermediate inputs for their own production. In particular, manufacturing sectors producing chemicals, plastics, and machinery and equipment, have both significant exports (as a share of output) and important imports (as a share of intermediate inputs). Take machinery and equipment manufacturers for example, they accounted for 37% of Canadian exports over the period of 1981-2014; while 47% of intermediate inputs used by this sector were imported over the period of 2000-2014. The appreciation of the Canadian dollar driven by the rising commodity prices would make the imported inputs relatively cheaper compared to domestic inputs, further dampening domestic production.

Form figure 5 and figure 6 we can see that, in the baseline case, the net impacts of a boosting commodity sector on other sectors are reduced production and exports. The exceptions are the construction sector and the utility sector as they hardly export. To understand how much each channel contributes to the end result, we can further carry out counterfactual experiments to disentangle the forces. It is worth noting that upon a positive commodity price shock, the aggregate GDP rises despite the dampening effect on the rest of the economy, largely thanks to the size of the commodity sector as well as the lifts in investment.

4.3 Shock Propagation and Input-Output Linkages

To disentangle the different forces behind the adjustments in this multi-sector open economy, in this section, we run four counterfactual simulations to understand the scale of impacts of the following channels.

First, we shut down the domestic upstream supply chain to the commodity sector from the domestic economy and label this case as “UpstreamH”. In other words, we assume that while all other sectors use intermediate inputs from the commodity sector for their own production; the commodity sector does not use domestic inputs for its production (they could however still import intermediate inputs). Secondly, in addition to the domestic upstream channel, we also shut down the foreign supply of intermediate inputs to the home commodity sector, and label this case as “UpstreamHF”. Thirdly, we turn to investigate the downstream impacts and shut down the downstream supply of commodity inputs to the domestic economy. This case is labelled as “DownstreamH”. Finally, on top of ceasing the domestic downstream, we shut down the exports of commodity products from the home economy. The counterfactual analysis results are presented in Figure 8 and 9.

Starting from the sectoral production charts, it is a bit hard to see, but the yellow line and red line are very close to each other in all subplots. This shows that the upstream imports channel does very little in terms of adjusting to the commodity price shock. This is probably not surprising since the commodity sector does not use many imports for its production. Among the eight sectors, imported supply from sector 5 and 6 each accounts for roughly 5% of the total intermediate inputs into the commodity sector, while the import supply from other sectors are almost negligible. That said, the domestic upstream supply remains an important channel of adjustment. Shutting down the domestic upstream supplies drives the sectoral production to be lower than the baseline case, particularly for sector 7 and 8, as the wholesale and services sectors provide significant amount of inputs to the commodity sector. Without the positive boosting effect through the upstream channel, the expected return of capital is lower compared to the baseline case. The smaller drop in consumption prices leads to less aggressive interest rate responses and less appreciation of the Canadian dollar. In terms of the overall impact on GDP, although the muted investment profile applies downward pressure, the improved export prospects appear to more than offset the downward forces, leading a slightly higher response in GDP upon the shock compared to the baseline case.

Turning to the downstream impacts of the open economy’s production network, ceasing the domestic downstream propagation channel causes sectoral production to drop a little from the baseline benchmark. In most cases, the drop is very small. On top of this, if we further shut down the export path for commodity inputs, a foreign commodity price shock leads to a flat-line response in the domestic commodity sector. It follows that this completely dampens the expectation on return to capital, reverses the positive investment reaction to negative, and significantly reduces the positive response of GDP. It is interesting to note that the export connections with the rest of the world in the open economy production network play an important role in the process of adjusting to a commodity price shock, while the import connections do not matter nearly as much.

In the current model setup, we do not build in any frictions to the adjustment process and allow all potential channels to adjust simultaneously. But it would be reasonable to assume that, upon the shock, the domestic production network impacts would happen faster while the trade part of the network takes time to adjust due to factors such as the delayed impacts of monetary policy. Under this assumption, it is not hard to understand why in 2014 when the commodity industry in Canada took a hit, the export-oriented manufacturing industries did not take over the engine of growth immediately. It is because the impacts coming from the domestic input-output linkages came before the open economy network adjustments.

5 Conclusions

To be added.

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Figure 1: The Use of Outputs, WIOD 2000-2014

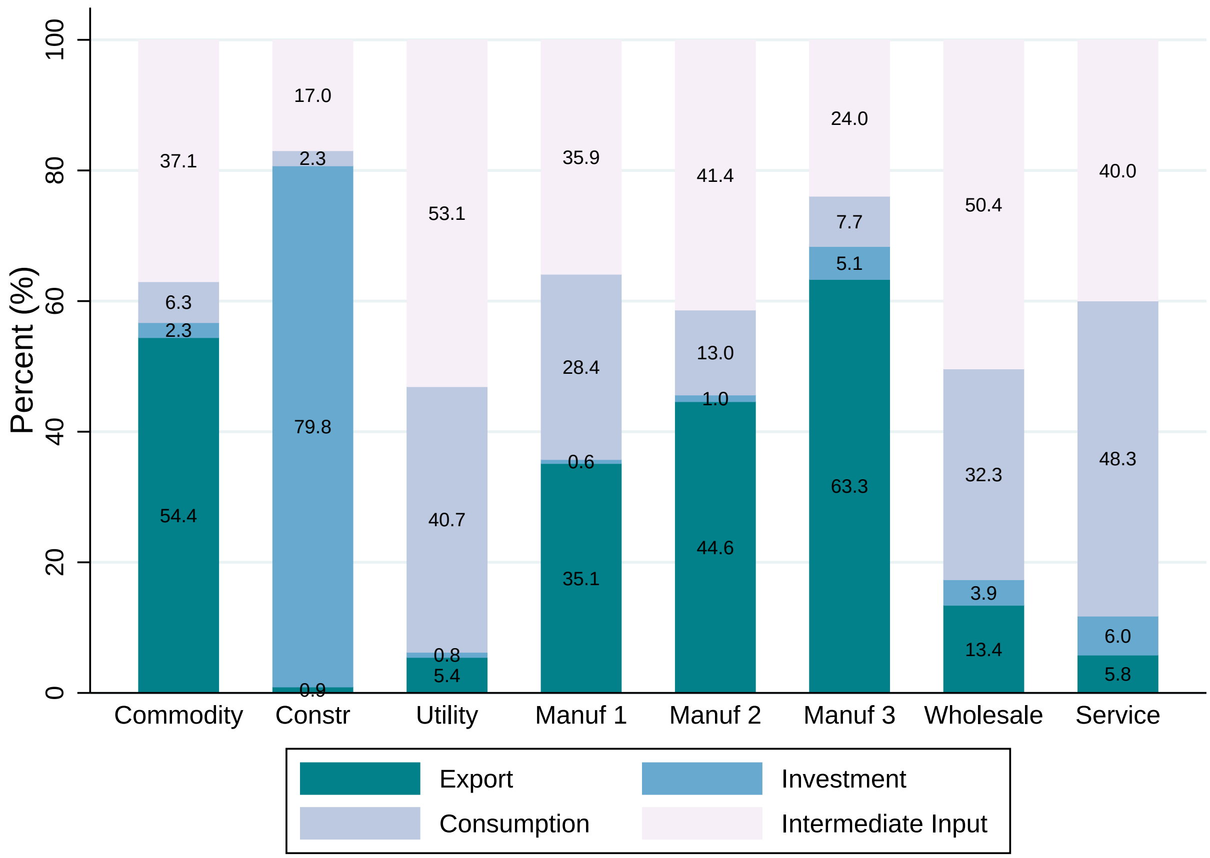

Figure 2: Sector Shares, 1981-2014

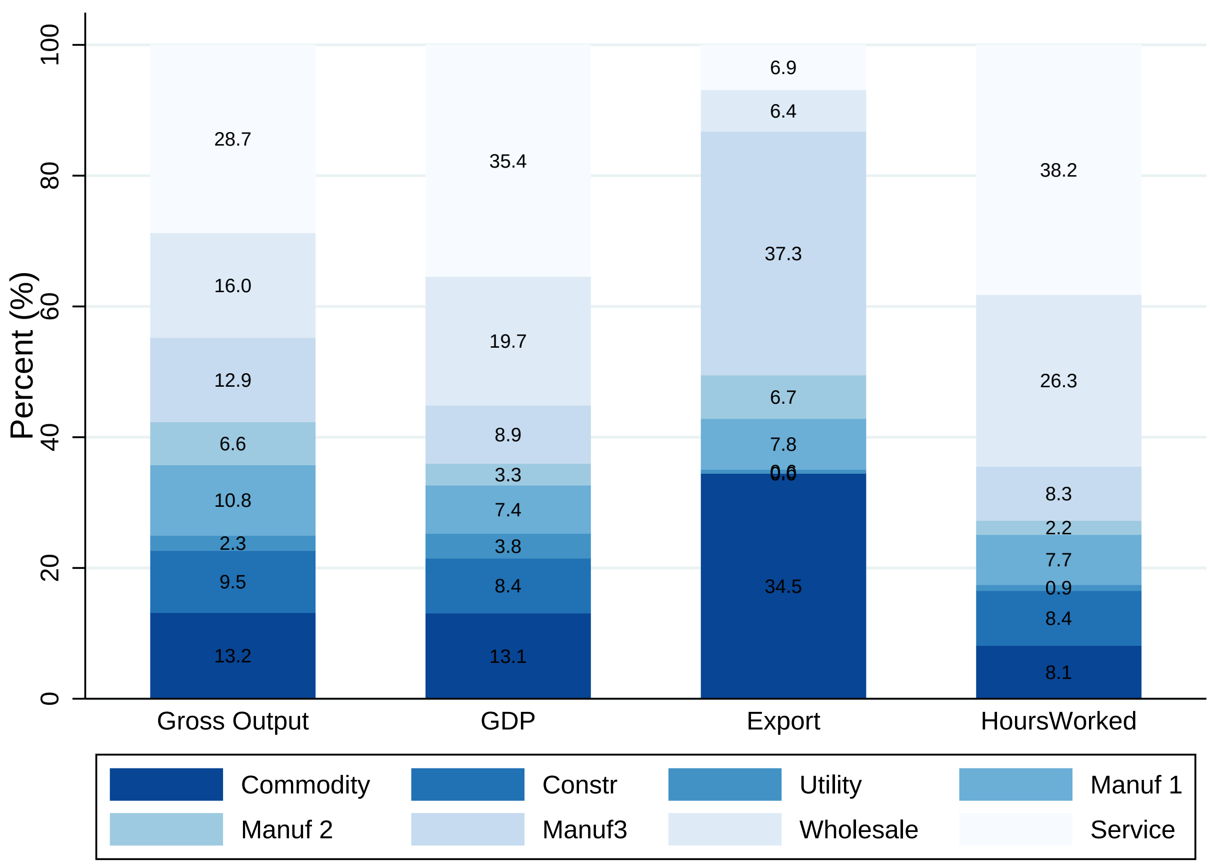

Figure 3: Contribution to Consumption and Investment, WIOD 2000-2014,

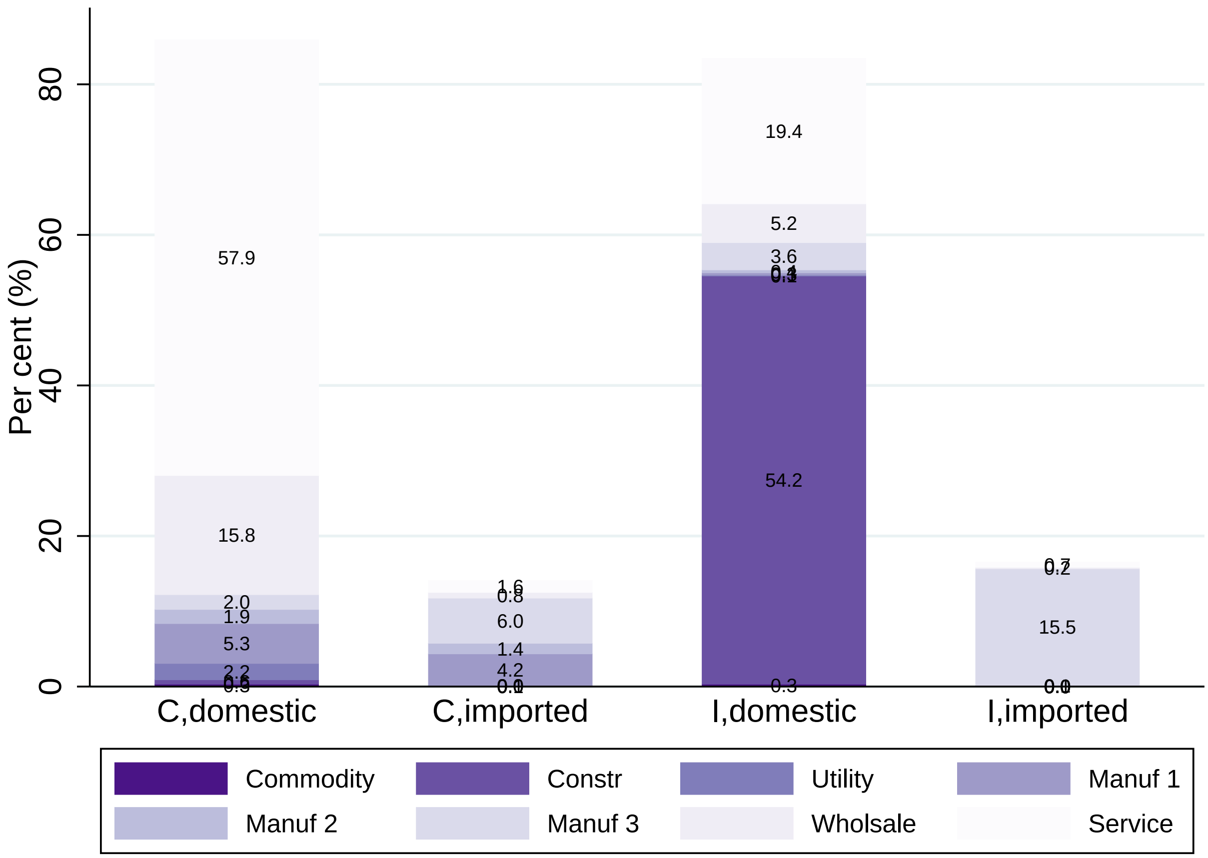

Figure 4: Impulse responses to commodity price shock in 2015

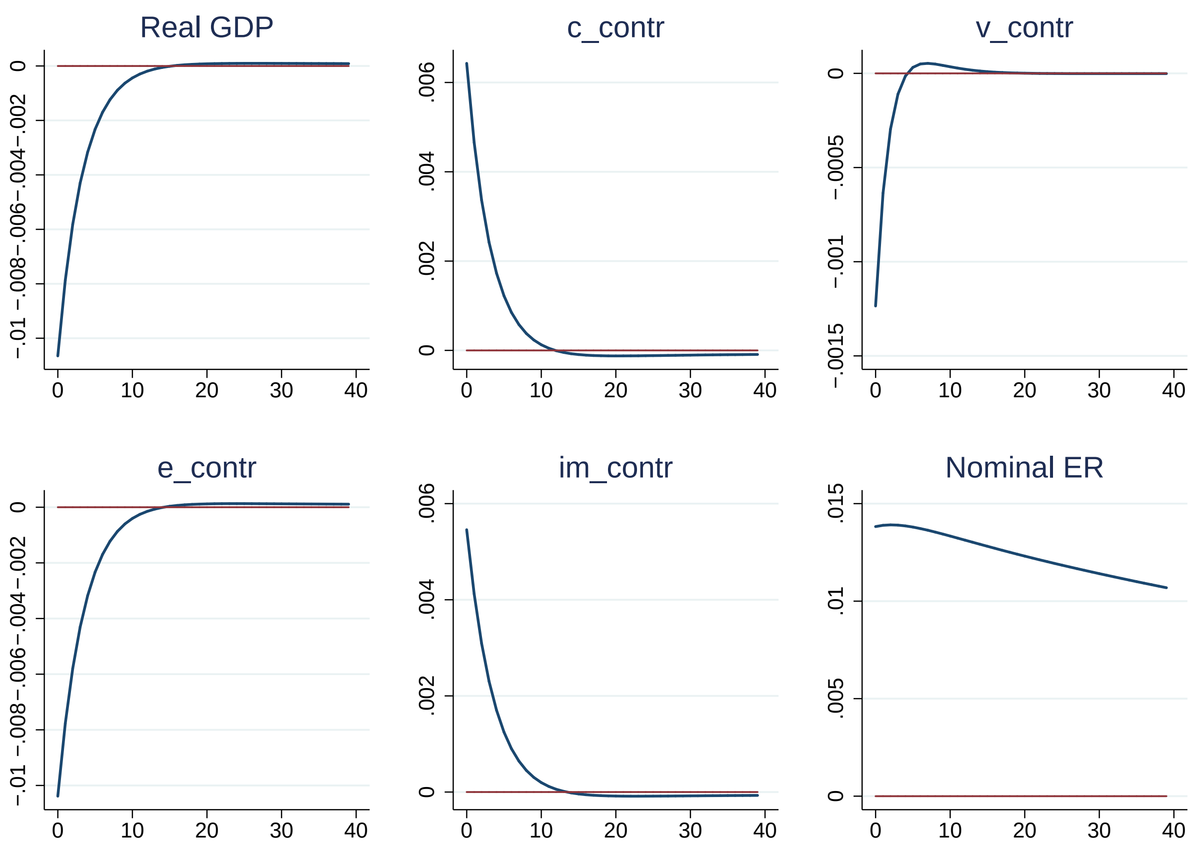

Figure 5: Baseline impacts of commodity price shocks: Exports

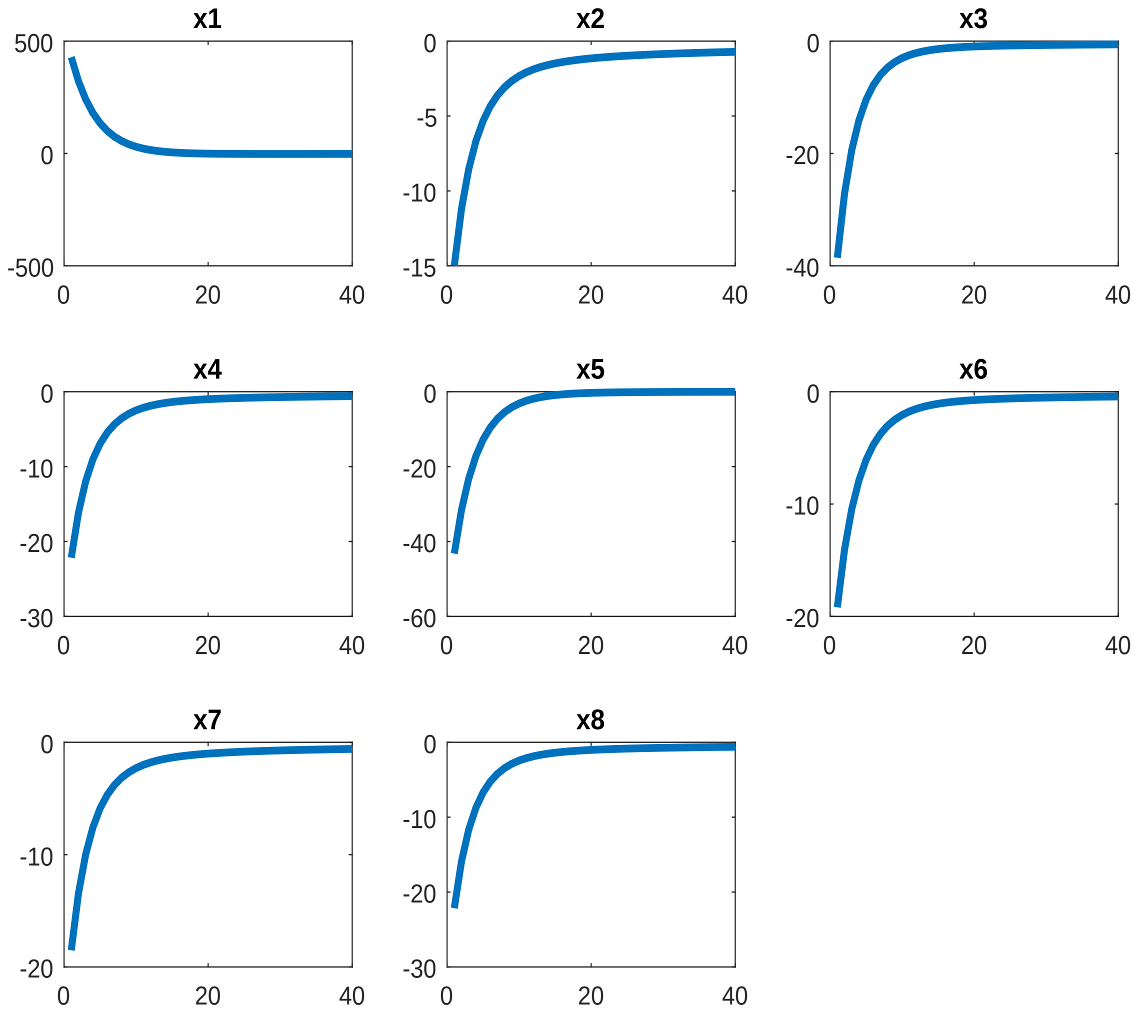

Figure 6: Baseline impacts of commodity price shocks: Sectoral production

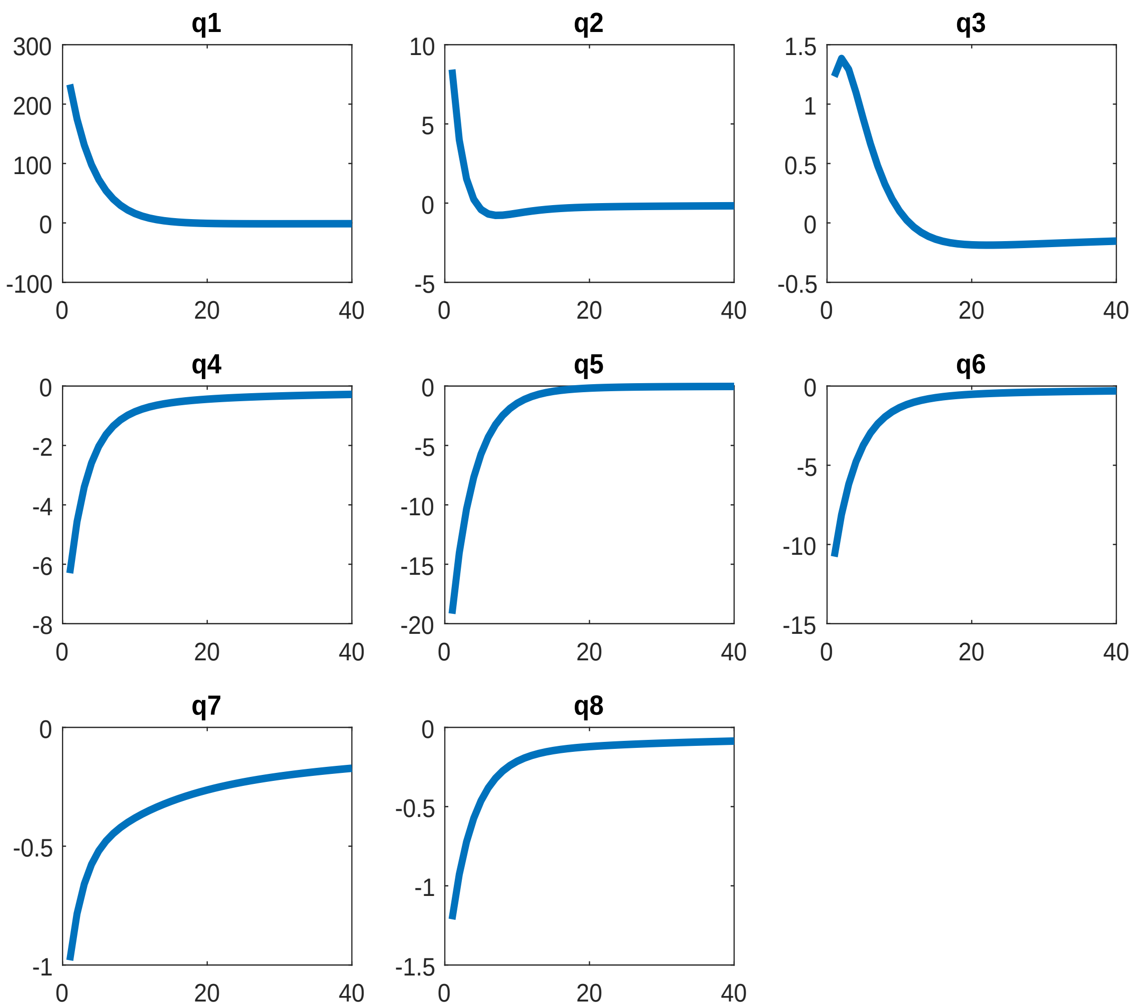

Figure 7: Baseline impacts of commodity price shocks: Macro variables

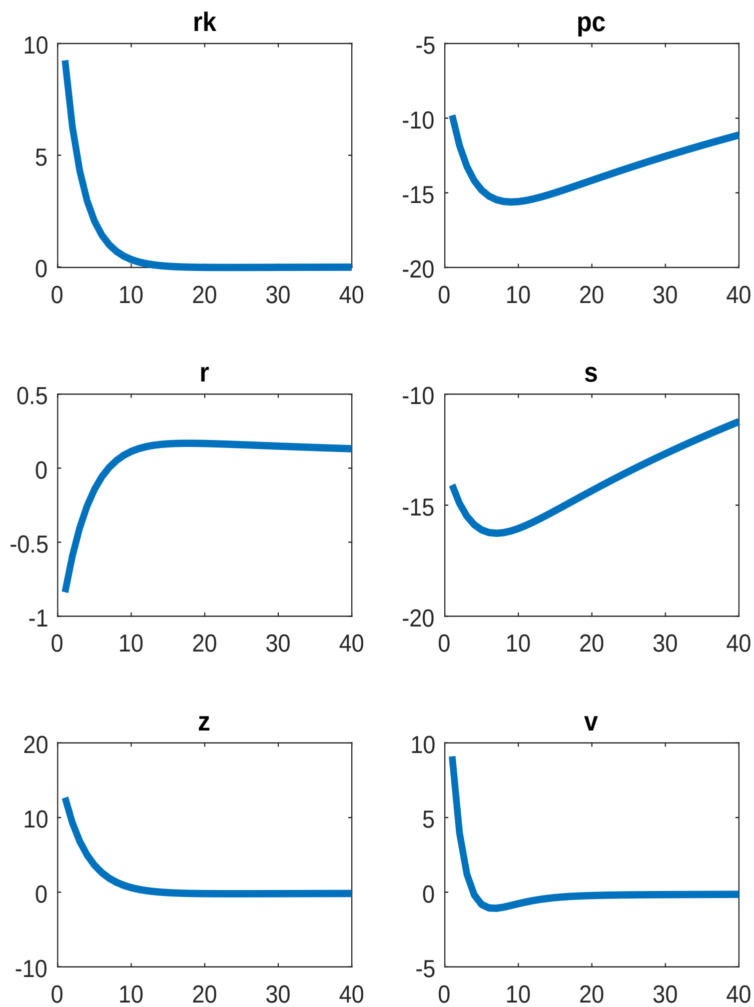

Figure 8: Counterfactual analysis: Sectoral production

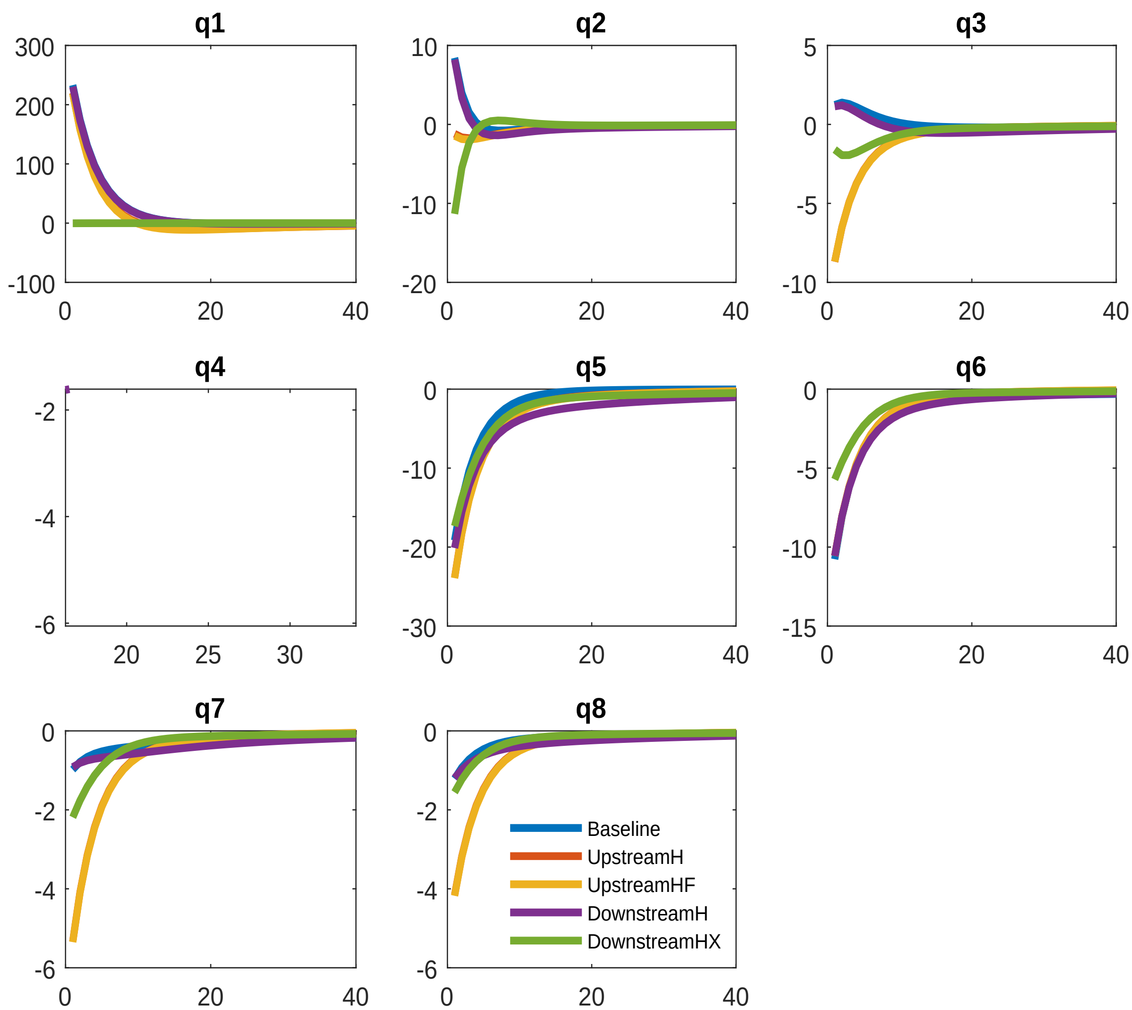

Figure 9: Counterfactual analysis: Macro variables

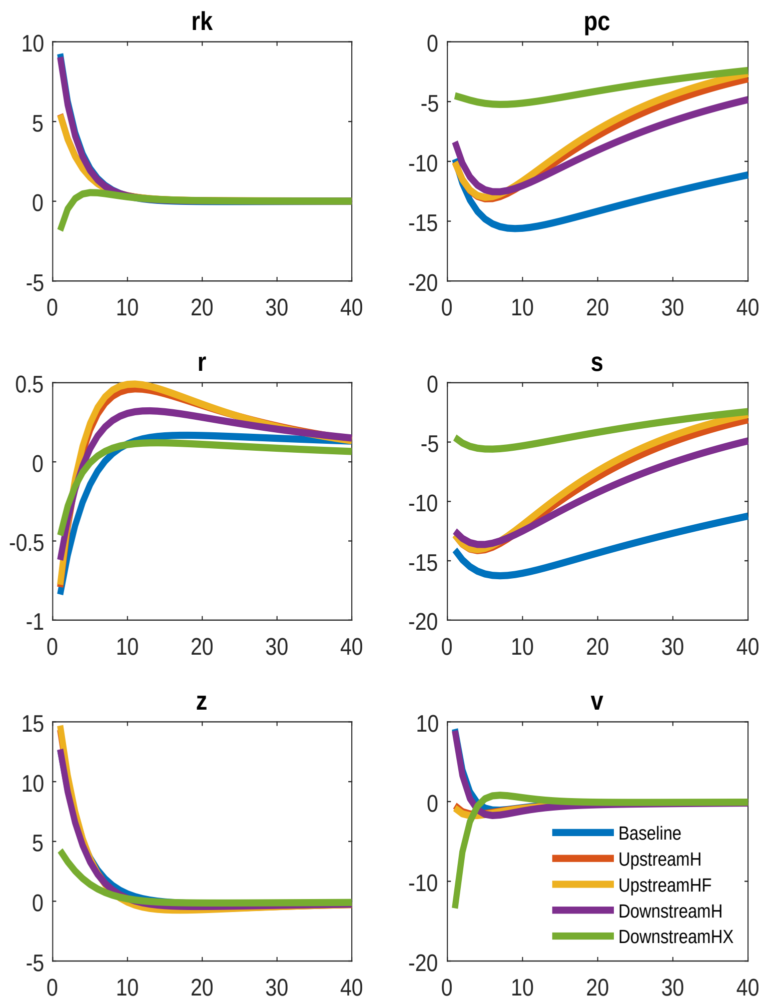

Table 1: Share of supply in use sector's total intermediate inputs(%)

Supply	Use							
	1	2	3	4	5	6	7	8
Commodity (1)	3.9	2.2	1.9	5.1	7.0	0.7	0.2	0.2
Construction (2)	2.7	1.4	15.8	0.5	0.5	0.5	3.3	5.7
Utility (3)	5.6	0.3	3.2	4.6	3.9	1.2	1.2	2.0
Manuf 1 (4)	6.7	5.8	1.2	20.9	1.7	6.3	1.1	4.4
Manuf 2 (5)	8.4	6.2	5.8	3.1	20.8	2.7	6.1	1.7
Manuf 3 (6)	3.8	9.6	4.3	2.6	1.4	9.8	4.2	4.3
Wholesale (7)	19.5	15.9	11.0	23.8	17.8	19.0	32.4	13.6
Services (8)	31.7	30.0	42.1	22.5	11.3	13.1	35.6	57.2
Total	82.3	71.3	85.3	83.2	64.5	53.2	84.1	89.1

Note: author's calculations, WIOD (2000-2014). Column j shows supply as a share of sector j total intermediate inputs.

Table 2: Share of imported supply in use sector's total intermediate inputs(%)

Imports	Use							
	1	2	3	4	5	6	7	8
Commodity (1)	2.5	1.6	1.4	0.9	5.1	1.7	0.0	0.0
Construction (2)	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.1
Utility (3)	0.1	0.0	0.1	0.1	0.2	0.0	0.0	0.0
Manuf 1 (4)	1.3	2.8	0.1	9.1	1.2	3.9	0.2	1.0
Manuf 2 (5)	5.5	4.7	2.9	4.0	25.3	3.4	3.6	1.0
Manuf 3 (6)	4.7	16.3	6.2	1.1	1.2	36.0	8.3	3.5
Wholesale (7)	0.9	0.7	0.7	0.6	1.6	0.8	1.7	1.0
Services (8)	2.6	2.5	3.2	1.0	0.9	1.0	2.0	4.3
Total	17.7	28.7	14.7	16.8	35.5	46.8	15.9	10.9

Note: author's calculations, WIOD (2000-2014). Column j shows imported supplies as a share of sector j total intermediate inputs.

Table 3: Detrended aggregates (% deviation from trends)

Year	BCPI	Real GDP	Consumption	Investment	Exports	Imports
2015	-27.5	-1.3	-1.1	-2.9	-1.5	-3.6
2016	-38.4	-2.3	-1.5	-9.8	-1.8	-6.7
2017	-24.3	-1.3	-0.9	-9.4	-2.4	-5.6

Note: author's calculations. Data Source: Statistics Canada, Bank of Canada.

A Optimal conditions

1.1 Household Let λ_t be the Lagrangian multiplier for the budget constraint, the household's optimal conditions are given by

$$
\begin{aligned}
C_t \quad &: \quad P_{ct}C_t = \frac{w_t}{\xi} = \frac{1}{\lambda_t};\\
K_{t+1} \quad &: \quad \lambda_t P_{v,t}\big[1+\Phi_1(K_{t+1},K_t)\big] = \beta \mathbf{E}_t\big\{\lambda_{t+1}P_{vt+1}\big[r_{kt+1}+1-\delta-\Phi_2(K_{t+2},K_{t+1})\big]\big\};\\
B_t \quad &: \quad \frac{1}{\beta R_t} = E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\right];\\
B_t^* \quad &: \quad \frac{1}{\beta R_t^*} = E_t\frac{S_{t+1}}{S_t}\cdot\frac{\lambda_{t+1}}{\lambda_t}.
\end{aligned}
$$

Applying the functional form, we obtain that $\Phi_1(K_{t+1},K_t) = \kappa(K_{t+1}/K_t - 1)$ and $\Phi_2(K_{t+1},K_t) = -\frac{\kappa}{2}\big(K_{t+2}^2/K_{t+1}^2 - 1\big)$. The Euler equation becomes

$$
\lambda_t P_{v,t}\big[1+\kappa(K_{t+1}/K_t-1)\big] = \beta\mathbf{E}_t\left\{\lambda_{t+1}P_{vt+1}\big[r_{kt+1}+1-\delta+\frac{\kappa}{2}\big(K_{t+2}^2/K_{t+1}^2-1\big)\big]\right\}
$$

The first two Euler equations also imply that $R_t = \mathbf{E}_t\frac{P_{vt+1}}{P_{vt}}\cdot\frac{r_{kt+1}+1-\delta+\frac{\kappa}{2}\left(K_{t+2}^2/K_{t+1}^2-1\right)}{1+\kappa(K_{t+1}/K_t-1)}$. The last two Euler equations imply that $\frac{R_t}{R_t^*} = E_t\frac{S_{t+1}}{S_t}$. The budget constraint is given by

$$
P_{ct}C_t + P_{vt}V_t + \frac{B_t}{R_t} + \frac{S_tB_t^*}{R_t^*} = w_tL_t + r_{kt}P_{vt}K_t + S_tB_{t-1}^* + B_{t-1} + T_t.
$$

The optimal aggregation consumption suggests that the optimal consumption goods are given by

$$
C_{hjt} = \phi_{hj}\left(\frac{P_{hjt}}{P_{ct}}\right)^{-\sigma_c} C_t, \quad j = 1,\cdots,N; \tag{A.1}
$$

and

$$
C_{fjt} = \phi_{fj}\left(\frac{P_{fjt}}{P_{ct}}\right)^{-\sigma_c} C_t, \quad j = 1,\cdots,N. \tag{A.2}
$$

Optimal aggregation also gives the aggregate consumption price index as follows

$$
P_{ct} = \left[\sum_{j=1}^{N}\left(\phi_{hj}P_{hjt}^{1-\sigma_c} + \phi_{fj}P_{fjt}^{1-\sigma_c}\right)\right]^{\frac{1}{1-\sigma_c}}.
$$

This price index can be obtained from the dual cost minimization problem, as the shadow price of aggregate consumption. Cost minimization also shows that profit is zero if sector j takes P_{hj} as given. Other prices indexes below are obtained in the same way.

1.2 Producers The state vector of sector j is (A_{jt}, L_{jt-1}). Producers in sector j solve the following problem

$$\max_{\{K_{jt},L_{jt},M_{jt}\}} E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \big\{ P_{hjt}Q_{jt} - r_{kt}P_{vt}K_{jt} - w_t L_{jt} - P_{mt}M_{jt} - w_t \Psi(L_{jt}, L_{jt-1}) \big\}.$$

Here, $\beta^t \lambda_t$ is the stochastic discount factor, $\lambda_t = (P_{ct}C_t)^{-1}$ is technically the Lagrange multiplier for the household's budget constraint.

Define $Y_{jt} = \left(\frac{K_{jt}}{\alpha_{kj}}\right)^{\alpha_{kj}} \left(\frac{L_{jt}}{\alpha_{lj}}\right)^{\alpha_{lj}}$, and let P_{yj} be the price of capital-labor combination. The current-price value added net of labor adjustment cost is defined as $P_{yjt}Y_j = r_t P_{vt} K_{jt} + w_t L_{jt} + w_t \Psi(L_{jt}, L_{jt-1})$. Then $P_{yjt} = (r_t P_{vt})^{\alpha_{kj}} (w_{jt})^{\alpha_{lj}}$, where w_{jt} is the marginal cost of labor input in sector j, to be defined shortly.

The producer's first-order necessary conditions are given by

$$\begin{aligned}
K_{jt} \quad &: \quad r_{kt}P_{vt}K_{jt} = \alpha_{kj}(1-\psi_j)^{\frac{1}{\sigma_q}} Y_{jt}^{\frac{\sigma_q-1}{\sigma_q}} A_{jt}^{\frac{\sigma_q-1}{\sigma_q}} P_{hj,t} Q_{jt}^{\frac{1}{\sigma_q}}. \\
M_{jt} \quad &: \quad P_{mjt}M_{jt} = \psi_j^{\frac{1}{\sigma_q}} M_{jt}^{\frac{\sigma_q-1}{\sigma_q}} A_{jt}^{\frac{\sigma_q-1}{\sigma_q}} P_{hj,t} Q_{jt}^{\frac{1}{\sigma_q}}; \\
L_{jt} \quad &: \quad P_{hjt}\frac{\partial Q_{jt}}{\partial L_{jt}} - w_t - w_t \frac{\partial \Psi_j(L_{jt}, L_{jt-1})}{\partial L_{jt}} = \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \cdot w_{t+1} \frac{\partial \Psi_j(L_{jt+1}, L_{jt})}{\partial L_{jt}}.
\end{aligned}$$

In the last condition, the derivatives are $\frac{\partial \Psi_j(L_{jt}, L_{jt-1})}{\partial L_{jt}} = \nu_j(L_{jt}/L_{jt-1} - 1)$, and $\frac{\partial \Psi_j(L_{jt+1}, L_{jt})}{\partial L_{jt}} = -\frac{\nu_j}{2}\big(\frac{L_{jt+1}^2}{L_{jt}^2} - 1\big)$. Plugging them into the optimal condition with respect to L_{jt}, we obtain

$$P_{hjt}\frac{\partial Q_{jt}}{\partial L_{jt}} - w_t - w_t \nu_j \Big(\frac{L_{jt}}{L_{jt-1}} - 1\Big) + \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \cdot w_{t+1} \frac{\nu_j}{2} \Big(\frac{L_{jt+1}^2}{L_{jt}^2} - 1\Big) = 0.$$

We define $w_{jt} = w_t + w_t \nu_j \big(\frac{L_{jt}}{L_{jt-1}} - 1\big) - \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \cdot w_{t+1} \frac{\nu_j}{2} \big(\frac{L_{jt+1}^2}{L_{jt}^2} - 1\big)$, the above optimal condition can be re-written as $w_{jt} = P_{hjt}\frac{\partial Q_{jt}}{\partial L_{jt}}$.

The optimal factor inputs satisfy

$$\begin{aligned}
Y_{jt} &= (1-\psi_j) A_{jt}^{\sigma_q - 1} \left(\frac{P_{yjt}}{P_{hjt}}\right)^{-\sigma_q} Q_{jt}; \\
K_{jt} &= \alpha_{kj}(1-\psi_j) A_{jt}^{\sigma_q - 1} \left(\frac{r_{kt}P_{vt}}{P_{yjt}}\right)^{-1} \left(\frac{P_{yjt}}{P_{hjt}}\right)^{-\sigma_q} Q_{jt}; \\
L_{jt} &= \alpha_{lj}(1-\psi_j) A_{jt}^{\sigma_q - 1} \left(\frac{w_{jt}}{P_{yjt}}\right)^{-1} \left(\frac{P_{yjt}}{P_{hjt}}\right)^{-\sigma_q} Q_{jt}; \\
M_{jt} &= \psi_j A_{jt}^{\sigma_q - 1} \left(\frac{P_{mjt}}{P_{hjt}}\right)^{-\sigma_q} Q_{jt}.
\end{aligned}$$

The share of intermediate input in total production cost (or equivalently total revenue) is given by

$$\frac{P_{mjt}M_{jt}}{P_{hjt}Q_{jt}} = \psi_j \left(\frac{P_{mjt}}{P_{hjt}}\right)^{1-\sigma_q} A_{jt}^{\sigma_q-1}.$$

The optimal capital-labor aggregate satisfies

$$\frac{P_{yjt}Y_{jt}}{P_{hjt}Q_{jt}} = (1-\psi_j)\left(\frac{P_{yjt}}{P_{hjt}}\right)^{1-\sigma_q} A_{jt}^{\sigma_q-1}. \tag{A.3}$$

The optimal conditions with respect to capital and labor imply that

$$\frac{r_{kt}P_{vt}K_{jt}}{w_{jt}L_j} = \frac{\alpha_{kj}}{\alpha_{lj}}. \tag{A.4}$$

Using the zero-profit condition for sector j gross output production, we obtain the price of gross output

$$P_{hjt} = A_{jt}^{-1}\left[(1-\psi_j)P_{yjt}^{1-\sigma_q} + \psi_j P_{mjt}^{1-\sigma_q}\right]^{\frac{1}{1-\sigma_q}}.$$

From the FONCs, we obtain the ratio of intermediate inputs over capital-labor bundle as

$$\frac{M_{jt}}{Y_{jt}} = \frac{\psi_j}{1-\psi_j}\left(\frac{P_{mjt}}{P_{yjt}}\right)^{-\sigma_q}.$$

For $\sigma_q \in (0,1)$, the above ratio decreases as P_{mj} rises. But, the cost of intermediate inputs over the cost of capital-labor bundle increases as P_{mj} rises, suggesting that the share of intermediate inputs in total production cost rises with P_{mj}.

1.3 Intermediate inputs Using the properties of the CES aggregation, for example as in the case of consumption, we can obtain the optimal intermediate input produced by sector i and used in the production in sector j, as

$$M_{hijt} = \omega_{hij}\left(\frac{P_{hit}}{P_{mjt}}\right)^{-\sigma_m} M_{jt}, \quad \text{for } i = 1, \cdots, N.$$

Similarly, the optimal imported intermediate input i for the production in sector j is

$$M_{fijt} = \omega_{fij}\left(\frac{P_{fit}}{P_{mjt}}\right)^{-\sigma_m} M_{jt}, \quad \text{for } i = 1, \cdots, N.$$

The price index of sector j intermediate input satisfies

$$P_{mjt} = \left[\sum_{i=1}^{N}\left(\omega_{hij}P_{hit}^{1-\sigma_m} + \omega_{fij}P_{fit}^{1-\sigma_m}\right)\right]^{\frac{1}{1-\sigma_m}}.$$

The share of M_{hij} in total intermediate input used in sector j production is given by

$$\frac{P_{hit}M_{hijt}}{P_{mjt}M_{jt}} = \omega_{hij}\left(\frac{P_{hit}}{P_{mjt}}\right)^{1-\sigma_m}, \quad \text{for } i = 1, \cdots, N.$$

Similarly for sector-i type of imported intermediate input, we have

$$\frac{P_{fit}M_{fijt}}{P_{mjt}M_{jt}} = \omega_{fij}\left(\frac{P_{fit}}{P_{mjt}}\right)^{1-\sigma_m}, \quad \text{for } i = 1, \cdots, N.$$

The shares of these intermediate inputs in the gross output of sector j are respectively given by

$$\begin{aligned} \frac{P_{hit}M_{hijt}}{P_{hjt}Q_{jt}} &= \omega_{hij}\psi_j P_{hit}^{1-\sigma_m} P_{mjt}^{\sigma_m-\sigma_q} P_{hjt}^{\sigma_q-1} A_{jt}^{\sigma_q-1}, \quad \text{for } i = 1, \cdots, N; \\ \frac{P_{fit}M_{fijt}}{P_{hjt}Q_{jt}} &= \omega_{fij}\psi_j P_{fit}^{1-\sigma_m} P_{mjt}^{\sigma_m-\sigma_q} P_{hjt}^{\sigma_q-1} A_{jt}^{\sigma_q-1}, \quad \text{for } i = 1, \cdots, N. \end{aligned}$$

Importantly, the impact of import prices on the share of intermediate input in total production cost depends upon the elasticity of substitution between intermediate input and the capital-labor combination and that between imported and domestically-produced intermediate inputs. A reduction in import price leads to a lower P_{mjt}, the share of intermediate input could become higher or lower depending whether the elasticity of substitution is greater or less than 1, i.e. whether intermediate inputs and capital and labor are complements or substitutes. Let $\mathbf{\Omega_h} = [\Omega_{hij}]_{N\times N}$ be an $N \times N$ matrix, where $\Omega_{hij} = \frac{P_{hit}M_{hijt}}{P_{hjt}Q_{jt}}$. Let $\mathsf{P_hQ_u}$ be a column vector, with $P_{hi}Q_{ui}$ the ith element, the total final use of sector i's output. In matrix form,

$$\mathsf{P_hQ_u} + \mathbf{\Omega_h}\mathsf{P_hQ} = \mathsf{P_hQ},$$

that is

$$\mathsf{P_hQ} = \left[\mathbf{I} - \mathbf{\Omega_h}\right]^{-1}\mathsf{P_hQ_u}. \tag{A.5}$$

The matrix $\left[\mathbf{I} - \mathbf{\Omega_h}\right]^{-1}$ is the Leontif inverse matrix.

1.4 Investment In the similar way, the optimal investment choices are given by

$$\begin{aligned} V_{hjt} &= \theta_{hj}\left(\frac{P_{hjt}}{P_{vt}}\right)^{-\sigma_v} V_t, \quad \text{for } j = 1, \cdots, N; \\ V_{fjt} &= \theta_{fj}\left(\frac{P_{fjt}}{P_{vt}}\right)^{-\sigma_v} V_t, \quad \text{for } j = 1, \cdots, N. \end{aligned}$$

The investment price index is given by

$$P_{vt} = \left[\sum_{j=1}^{N} \left(\theta_{hj} P_{hjt}^{1-\sigma_v} + \theta_{fj} P_{fjt}^{1-\sigma_v} \right) \right]^{\frac{1}{1-\sigma_v}}.$$

B Social Planner

Let $M_{hj\cdot t}$ be the output produced by sector j used as an intermediate input, similarly for $M_{fj\cdot t}$. Let $\widetilde{P}_{hjt}$ be the unit value of sector j output in terms of consumption, i.e., it is the price of sector j output relative to consumption price. We use different notations for prices of domestic good and exports, though they are equal. Let $\widetilde{P}_{fjt}$ be the ratio of import price j over consumption price. The social planner solves the following problem,

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t [\ln(C_t) - \xi L_t],$$

subject to

$$\begin{aligned}
& K_{t+1} = (1-\delta)K_t + V_t - \Phi(K_{t+1}, K_t); \\
& \sum_{j=1}^{N} \widetilde{P}_{hjt} Q_{jt} - \sum_{j=1}^{N} \widetilde{P}_{mjt} M_{jt} - \sum_{j=1}^{N} \widetilde{P}_{hjt} \Psi(L_{jt}, L_{jt-1}) = C_t + \widetilde{P}_{vt} V_t \\
& \qquad + \sum_{j=1}^{N} \widetilde{P}_{xjt} X_{jt} - \sum_{j}^{N} \widetilde{P}_{fjt} \left(C_{fjt} + V_{fjt} + M_{fj\cdot t} \right); \\
& L_t = \sum_{j=1}^{N} L_{jt}; \quad K_t = \sum_{j=1}^{N} K_{jt}; \\
& C_t = \left[\sum_{j=1}^{N} \left(\phi_{hj}^{\frac{1}{\sigma_c}} C_{hjt}^{\frac{\sigma_c-1}{\sigma}} + \phi_{fj}^{\frac{1}{\sigma_c}} C_{fjt}^{\frac{\sigma_c-1}{\sigma_c}} \right) \right]^{\frac{\sigma_c}{\sigma_c-1}}; \\
& V_t = \left[\sum_{j=1}^{N} \left(\theta_{hj}^{\frac{1}{\sigma_v}} V_{hjt}^{\frac{\sigma_v-1}{\sigma_v}} + \theta_{fj}^{\frac{1}{\sigma_v}} V_{fjt}^{\frac{\sigma_v-1}{\sigma_v}} \right) \right]^{\frac{\sigma_v}{\sigma_v-1}}; \\
& Q_{jt} = A_{jt} \left[(1-\psi_j)^{\frac{1}{\sigma_q}} Y_{jt}^{\frac{\sigma_q-1}{\sigma_q}} + \psi_j^{\frac{1}{\sigma_q}} M_{jt}^{\frac{\sigma_q-1}{\sigma_q}} \right]^{\frac{\sigma_q}{\sigma_q-1}}, \; j = 1, \cdots, N; \\
& M_{jt} = \left[\sum_{i=1}^{N} \left(\omega_{hij}^{\frac{1}{\sigma_m}} M_{hijt}^{\frac{\sigma_m-1}{\sigma_m}} + \omega_{fij}^{\frac{1}{\sigma_m}} M_{fijt}^{\frac{\sigma_m-1}{\sigma_m}} \right) \right]^{\frac{\sigma_m}{\sigma_m-1}}, \; j = 1, \cdots, N; \\
& X_{jt} = \alpha_f \left(\frac{P_{xj,t}/S_t}{P_t^*} \right)^{-\sigma_x} Y_t^*.
\end{aligned}$$

Note that the resource constraint at the sector level is redundant in social planner's problem because the social planner is concerned with only the aggregate resource constraint.

The Lagrangian function for the social planner's problem is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \ln(C_t) - \xi \sum_{j=1}^{N} L_{jt} + P_{ct} \Big[\sum_{j=1}^{N} \widetilde{P}_{hjt} Q_{jt} - \sum_{j=1}^{N} \widetilde{P}_{mjt} M_{jt} - \sum_{j=1}^{N} \widetilde{P}_{hjt} \Psi(L_{jt}, L_{jt-1}) - C_t$$
$$- \widetilde{P}_{vt} \big[\sum_{j=1}^{N} K_{jt+1} - (1-\delta) \sum_{j=1}^{N} K_{jt} + \Phi(K_{t+1}, K_t) \big] - \sum_{j=1}^{N} \widetilde{P}_{xjt} X_{jt} + \sum_{j} \widetilde{P}_{fjt} \left(C_{fjt} + V_{fjt} + M_{fj\cdot t} \right) \Big] \Big\}.$$

In the above equation, we replace Q_{jt} with the production functions. P_{ct} is the Lagrangian multipliers. Let $P_{hjt} = P_{ct} \cdot \widetilde{P}_{hjt}$ and $P_{fjt} = P_{ct} \cdot \widetilde{P}_{fjt}$. The FONCs are

$$C_t: \quad \frac{1}{C_t} = P_{ct};$$
$$L_{jt}: \quad P_{hjt} \frac{\partial Q_{jt}}{\partial L_{jt}} = \xi + P_{hjt} \frac{\partial \Psi(L_{jt}, L_{jt-1})}{\partial L_{jt}} + \beta \mathbf{E}_t P_{hjt+1} \frac{\partial \Psi(L_{jt+1}, L_{jt})}{\partial L_{jt}};$$
$$K_{jt+1}: \quad P_{vt} \big[1 + \frac{\partial \Phi(K_{t+1}, K_t)}{\partial K_{t+1}} \big] = \beta \mathbf{E}_t \Big\{ P_{vt+1} \big[(1-\delta) - \frac{\partial \Phi(K_{t+2}, K_{t+1})}{\partial K_{t+1}} \big] + P_{hjt+1} \frac{\partial Q_{jt+1}}{\partial K_{jt+1}} \Big\};$$
$$M_{jt}: \quad P_{hjt} \frac{\partial Q_{jt}}{\partial M_{jt}} = P_{mjt}.$$

The optimal values for $C_{hj}, C_{fj}, V_{hj}, V_{fj}, M_{hij}$, and M_{fij} are obtained by cost minimization for producing respectively C_t, V_t, Q_{jt}, and M_{jt}. The export is determined once the output price P_{hjt} is determined. In doing so, we notice that the marginal value of Q_{jt} in cost minimization and in the above social planer's problem must be equal. We then obtain the following

Sector output. Cost minimization is to minimize $\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t P_{ct} \Big\{ \widetilde{w}_t L_{jt} + r_{kt} \widetilde{P}_{vt} K_{jt} + \sum_{j=1}^{N} \widetilde{P}_{mjt} M_{jt} + \widetilde{w}_t \Psi(L_{jt}, L_{jt-1}) \Big\}$, subject to $Q_{jt} \leq \overline{Q}_j$. The optimal conditions are given by

$$w_{jt} = P_{hjt} \frac{\partial Q_{jt}}{\partial L_{jt}};$$
$$r_{kt} P_{vt} = P_{hjt} \frac{\partial Q_{jt}}{\partial K_{jt}};$$
$$P_{mjt} = P_{hjt} \frac{\partial Q_{jt}}{\partial M_{jt}}.$$

Here again, $w_{jt} = w_t + w_t \nu_j \big(\frac{L_{jt}}{L_{jt-1}} - 1\big) - \beta \mathbf{E}_t w_{t+1} \frac{\nu_j}{2} \big(\frac{L^2_{jt+1}}{L^2_{jt}} - 1\big)$. We see that $w_t = \xi$ in the social planner's problem.

Optimal conditions for producers are re-written as

$$K_{jt} = \alpha_{kj}(1-\psi_j) A_{jt}^{\sigma_q - 1} \left(\frac{r_{kt} P_{vt}}{P_{yjt}} \right)^{-1} \left(\frac{P_{yjt}}{P_{hjt}} \right)^{-\sigma_q} Q_{jt};$$
$$L_{jt} = \alpha_{lj}(1-\psi_j) A_{jt}^{\sigma_q - 1} \left(\frac{w_{jt}}{P_{yjt}} \right)^{-1} \left(\frac{P_{yjt}}{P_{hjt}} \right)^{-\sigma_q} Q_{jt};$$
$$M_{jt} = \psi_j A_{jt}^{\sigma_q - 1} \left(\frac{P_{mjt}}{P_{hjt}} \right)^{-\sigma_q} Q_{jt}.$$

Prices are given by

$$P_{yjt} = (r_{kt} P_{vt})^{\alpha_{kj}} (w_{jt})^{\alpha_{lj}};$$
$$P_{hjt} = A_{jt}^{-1} \left[(1-\psi_j) P_{yjt}^{1-\sigma_q} + \psi_j P_{mjt}^{1-\sigma_q} \right]^{\frac{1}{1-\sigma_q}}.$$

Consumption. Cost minimization is to minimize $\sum_{j=1}^{N} (P_{hjt} C_{hjt} + P_{fjt} C_{fjt})$, subject to $C_t \leq \overline{C}$. we notice that the marginal value of C_t in cost minimization and in the above social planer's problem must be equal. We then obtain the following

$$C_{hjt} = \phi_{hj} \left(\frac{P_{hjt}}{P_{ct}} \right)^{-\sigma_c} C_t, \quad j = 1, \cdots, N;$$
$$C_{fjt} = \phi_{fj} \left(\frac{P_{fjt}}{P_{ct}} \right)^{-\sigma_c} C_t, \quad j = 1, \cdots, N;$$
$$P_{ct} = \left[\sum_{j=1}^{N} \left(\phi_{hj} P_{hjt}^{1-\sigma_c} + \phi_{fj} P_{fjt}^{1-\sigma_c} \right) \right]^{\frac{1}{1-\sigma_c}}.$$

In this place, P_{ct} is the Lagrangian multiplier for constraint $C_t \leq \overline{C}$, and is the marginal value of consumption.

Investment. In similar fashion, we have

$$V_{hjt} = \theta_{hj} \left(\frac{P_{hjt}}{P_{vt}} \right)^{-\sigma_v} V_t, \quad \text{for } j = 1, \cdots, N;$$
$$V_{fjt} = \theta_{fj} \left(\frac{P_{fjt}}{P_{vt}} \right)^{-\sigma_v} V_t, \quad \text{for } j = 1, \cdots, N;$$
$$P_{vt} = \left[\sum_{j=1}^{N} \left(\theta_{hj} P_{hjt}^{1-\sigma_v} + \theta_{fj} P_{fjt}^{1-\sigma_v} \right) \right]^{\frac{1}{1-\sigma_v}}.$$

Intermediate inputs. For sector j, the intermediate inputs satisfies

$$M_{hijt} = \omega_{hij} \left(\frac{P_{hit}}{P_{mjt}} \right)^{-\sigma_m} M_{jt}, \quad \text{for } i = 1, \cdots, N;$$
$$M_{fijt} = \omega_{fij} \left(\frac{P_{fit}}{P_{mjt}} \right)^{-\sigma_m} M_{jt}, \quad \text{for } i = 1, \cdots, N;$$
$$P_{mjt} = \left[\sum_{i=1}^{N} \left(\omega_{hij} P_{hit}^{1-\sigma_m} + \omega_{fij} P_{fit}^{1-\sigma_m} \right) \right]^{\frac{1}{1-\sigma_m}}.$$

From the social planner's optimal condition with respect to M_{jt}, we know that $M_{jt} = \psi_j A_{jt}^{\sigma_q - 1} \left(\frac{P_{mjt}}{P_{hjt}} \right)^{-\sigma_q} Q_{jt}$. Then

$$M_{hijt} = \psi_j A_{jt}^{\sigma_q - 1} \omega_{hij} \left(\frac{P_{hit}}{P_{mjt}} \right)^{-\sigma_m} \left(\frac{P_{mjt}}{P_{hjt}} \right)^{-\sigma_q} Q_{jt}, \quad \text{for } i = 1, \cdots, N;$$
$$M_{fijt} = \psi_j A_{jt}^{\sigma_q - 1} \omega_{fij} \left(\frac{P_{fit}}{P_{mjt}} \right)^{-\sigma_m} \left(\frac{P_{mjt}}{P_{hjt}} \right)^{-\sigma_q} Q_{jt}, \quad \text{for } i = 1, \cdots, N.$$

2.1 STEADY STATE In steady state, $V = \delta K$. Also it is straightforward that $r = \frac{1}{\beta} - 1 + \delta$. Further, marginal cost of labor adjustment is zero in steady state, meaning that $w_j = w$. We substitute for M_j. The system of optimal conditions are given by the following, for all

$j = 1, \cdots, N$

$$
\begin{aligned}
V: &\quad V = \delta K; \\
C: &\quad \frac{1}{C} = P_c; \\
L_j: &\quad P_{hj} \frac{\partial Q_j}{\partial L_j} = \xi; \\
K_j: &\quad \big[\frac{1}{\beta} - 1 + \delta\big] P_v = P_{hj} \frac{\partial Q_j}{\partial K_j}; \\
C_{hj}: &\quad C_{hj} = \phi_{hj} \Big(\frac{P_{hj}}{P_c}\Big)^{-\sigma_c} C; \\
C_{fj}: &\quad C_{fj} = \phi_{fj} \Big(\frac{P_{fj}}{P_c}\Big)^{-\sigma_c} C; \\
V_{hj}: &\quad V_{hj} = \theta_{hj} \Big(\frac{P_{hj}}{P_v}\Big)^{-\sigma_v} V; \\
V_{fj}: &\quad V_{fj} = \theta_{fj} \Big(\frac{P_{fj}}{P_v}\Big)^{-\sigma_v} V; \\
M_{hij}: &\quad M_{hij} = \psi_j A_j^{\sigma_q - 1} \omega_{hij} \left(\frac{P_{hi}}{P_{mj}}\right)^{-\sigma_m} \left(\frac{P_{mj}}{P_{hj}}\right)^{-\sigma_q} Q_j,\ i = 1, \cdots, N; \\
M_{fij}: &\quad M_{fij} = \psi_j A_j^{\sigma_q - 1} \omega_{fij} \left(\frac{P_{fi}}{P_{mj}}\right)^{-\sigma_m} \left(\frac{P_{mj}}{P_{hj}}\right)^{-\sigma_q} Q_j,\ i = 1, \cdots, N; \\
P_{mj}: &\quad P_{mj} = \left[\sum_{i=1}^{N} \big(\omega_{hij} P_{hi}^{1-\sigma_m} + \omega_{fij} P_{fi}^{1-\sigma_m}\big)\right]^{\frac{1}{1-\sigma_m}}; \\
P_{yj}: &\quad P_{yj} = (r P_v)^{\alpha_{kj}} w^{\alpha_{lj}}; \\
P_{hj}: &\quad P_{hj} = A_j^{-1} \left[(1 - \psi_j) P_{yj}^{1-\sigma_q} + \psi_j P_{mj}^{1-\sigma_q}\right]^{\frac{1}{1-\sigma_q}};
\end{aligned}
$$

In addition :

$$
\begin{aligned}
P_c: \quad & P_c = \left[\sum_{j=1}^{N} \left(\phi_{hj} P_{hj}^{1-\sigma_c} + \phi_{fj} P_{fj}^{1-\sigma_c} \right) \right]^{\frac{1}{1-\sigma_c}}; \\
P_v: \quad & P_v = \left[\sum_{j=1}^{N} \left(\theta_{hj} P_{hj}^{1-\sigma_v} + \theta_{fj} P_{fj}^{1-\sigma_v} \right) \right]^{\frac{1}{1-\sigma_v}}; \\
Q_j: \quad & Q_j = C_{hj} + V_{hj} + X_j + M_{hj\cdot}; \\
X_j: \quad & X_j = \alpha_f \left(\frac{P_{hj}}{SP^*} \right)^{-\sigma_x} Y^*; \\
\text{A.R.C.}: \quad & \sum_{j=1}^{N} P_{hj} Q_j - \sum_{j=1}^{N} P_{mj} M_j = P_c C + P_v V + \sum_{j=1}^{N} P_{xj} X_j - \sum_{j}^{N} P_{fj} \left(C_{fj} + V_{fj} + M_{fj\cdot} \right); \\
K: \quad & K = \sum_{j=1}^{N} K_j; \\
L: \quad & L = \sum_{j=1}^{N} L_j.
\end{aligned}
$$

First, we solve for $\mathsf{P_h} = [P_{hj}]_{N\times 1}$. Noticing that $P_{yj} = r^{\alpha_{kj}} w^{\alpha_{lj}} P_v^{\alpha_{kj}}$, then

$$
P_{yj} = r^{\alpha_{kj}} w^{\alpha_{lj}} \left[\sum_{i=1}^{N} \left(\theta_{hi} P_{hi}^{1-\sigma_v} + \theta_{fi} P_{fi}^{1-\sigma_v} \right) \right]^{\frac{\alpha_{kj}}{1-\sigma_v}}.
$$

Let $\alpha_{\mathsf{k}} = [\alpha_{kj}]_{N\times 1}$ be a column vector, similarly for $\alpha_{\mathsf{l}}, \theta_{\mathsf{h}}, \theta_{\mathsf{f}}$, etc. We define the square diagonal matrix $\mathrm{di}(\tilde{\mathsf{r}})$ with $r^{\alpha_{kj}} w^{\alpha_{lj}}$ along the diagonal. In vector form, we have

$$
\mathsf{P_y} = \mathrm{di}(\tilde{\mathsf{r}}) \left[\theta_{\mathsf{h}}' \mathsf{P_h}^{1-\sigma_v} + \theta_{\mathsf{f}}' \mathsf{P_f}^{1-\sigma_v} \right]^{\frac{\alpha_{\mathsf{k}}}{1-\sigma_v}}.
$$

The intermediate input prices in vector form are given by

$$
\mathsf{P_m}^{1-\sigma_m} = \omega_{\mathbf{h}}' \mathsf{P_h}^{1-\sigma_m} + \omega_{\mathbf{f}}' \mathsf{P_f}^{1-\sigma_m}.
$$

Substitute for $\mathsf{P_m}$ and $\mathsf{P_y}$ in the equation for P_{hj}, we obtain

$$\begin{aligned}(A_j P_{hj})^{1-\sigma_q} &= (1-\psi_j)(r^{\alpha_{kj}} w^{\alpha_{lj}})^{1-\sigma_q}\left[\sum_{i=1}^{N}\left(\theta_{hi}P_{hi}^{1-\sigma_v}+\theta_{fi}P_{fi}^{1-\sigma_v}\right)\right]^{\frac{\alpha_{kj}(1-\sigma_q)}{1-\sigma_v}} \\ &+ \psi_j\left[\sum_{i=1}^{N}\left(\omega_{hij}P_{hi}^{1-\sigma_m}+\omega_{fij}P_{fi}^{1-\sigma_m}\right)\right]^{\frac{1-\sigma_q}{1-\sigma_m}}.\end{aligned}$$

In vector form,

$$[\mathrm{di}(\mathsf{A})\mathsf{P_h}]^{1-\sigma_q} = \mathrm{di}(\mathbf{1}-\psi)\left[[\mathrm{di}(\tilde{\mathsf{r}})]^{1-\sigma_q}\left[\theta_{\mathsf{h}}'\mathsf{P_h}^{1-\sigma_v}+\theta_{\mathsf{f}}'\mathsf{P_f}^{1-\sigma_v}\right]^{\alpha_{\mathsf{k}}\frac{1-\sigma_q}{1-\sigma_v}}\right]+\mathrm{di}(\psi)\left[\omega_{\mathbf{h}}'\mathsf{P_h}^{1-\sigma_m}+\omega_{\mathbf{f}}'\mathsf{P_f}^{1-\sigma_m}\right]^{\frac{1-\sigma_q}{1-\sigma_m}}. \tag{B.1}$$

It is noted that $(\theta_{\mathsf{h}}'\mathsf{P_h}^{1-\sigma_v}+\theta_{\mathsf{f}}'\mathsf{P_f}^{1-\sigma_v})$ is a scalar, while its exponent is a vector because α_{k} is a vector. The above vector equation has N unknowns, they cannot be solved with closed forms, we numerically solve for $\mathsf{P_h}$. Note that, in closed economy ($\theta_{fj}=\omega_{fij}=0$), there is a closed form for the steady-state prices.

Given $\mathsf{P_h}$, we obtain the Lagrangian multiplier (marginal value of consumption) P_c and investment price

$$\begin{aligned}P_c &= \left[\phi_{\mathsf{h}}'\mathsf{P_h}^{1-\sigma_c}+\phi_{\mathsf{f}}'\mathsf{P_f}^{1-\sigma_c}\right]^{\frac{1}{1-\sigma_c}}; \\ P_v &= \left[\theta_{\mathsf{h}}'\mathsf{P_h}^{1-\sigma_v}+\theta_{\mathsf{f}}'\mathsf{P_f}^{1-\sigma_v}\right]^{\frac{1}{1-\sigma_v}}.\end{aligned}$$

Other prices, $\mathsf{P_m}$ and $\mathsf{P_y}$ are also obtained from their expressions above.

Second, we solve for sector-level gross output Q_j by using the market clearing condition. Noting that $C=1/P_c$. Consumption C_{hj} is given by

$$C_{hj} = \phi_{hj}\left(\frac{P_{hj}}{P_c}\right)^{-\sigma_c}P_c^{-1}. \tag{B.2}$$

Investment V_{hj} is a share of $V=\delta K$, and $K=\sum_{i=1}^{N}K_i$. Then

$$V_{hj} = \theta_{hj}\left(\frac{P_{hj}}{P_v}\right)^{-\sigma_v}\cdot\delta\sum_{i=1}^{N}\left[\alpha_{ki}(1-\psi_i)A_i^{\sigma_q-1}\left(\frac{rP_v}{P_{yi}}\right)^{-1}\left(\frac{P_{yi}}{P_{hi}}\right)^{-\sigma_q}Q_i\right]. \tag{B.3}$$

Intermediate input $M_{hj\cdot}$ produced in sector j is given by

$$M_{hj\cdot} = \sum_{i=1}^{N}M_{hji} = \sum_{i=1}^{N}\left[\psi_i A_i^{\sigma_q-1}\omega_{hji}\left(\frac{P_{hj}}{P_{mi}}\right)^{-\sigma_m}\left(\frac{P_{mi}}{P_{hi}}\right)^{-\sigma_q}Q_i\right]. \tag{B.4}$$

Export by sector j is given by

$$X_j = \alpha_f \left(\frac{P_{hj}}{SP^*} \right)^{-\sigma_x} Y^*. \tag{B.5}$$

Substituting Equations (B.2) to (B.5) for $C_{hj}, V_{hj}, M_{hj\cdot}$ and X_j in the market clearing condition for sector j, we obtain

$$Q_j = \phi_{hj}\left(\frac{P_{hj}}{P_c}\right)^{-\sigma_c} P_c^{-1} + \theta_{hj}\left(\frac{P_{hj}}{P_v}\right)^{-\sigma_v} \cdot \delta \sum_{i=1}^{N}\left[\alpha_{ki}(1-\psi_i)A_i^{\sigma_q - 1}\left(\frac{rP_v}{P_{yi}}\right)^{-1}\left(\frac{P_{yi}}{P_{hi}}\right)^{-\sigma_q} Q_i\right]$$
$$+ \sum_{i=1}^{N}\left[\psi_i A_i^{\sigma_q-1}\omega_{hji}\left(\frac{P_{hj}}{P_{mi}}\right)^{-\sigma_m}\left(\frac{P_{mi}}{P_{hi}}\right)^{-\sigma_q} Q_i\right] + \alpha_f\left(\frac{P_{hj}}{SP^*}\right)^{-\sigma_x} Y^*. \tag{B.6}$$

Let $\mathsf{C_h}$ column vector of C_{hj} and let X be a column vector of exports. Define the following column vectors and matrices

$$\Omega_{\mathsf{vh}} = [\theta_{hj}\left(\frac{P_{hj}}{P_v}\right)^{-\sigma_v}]_{N\times 1};$$

$$\Omega_{\mathsf{k}} = [\Omega_{ki}]_{i=1,\cdots,N}, \text{ where } \Omega_{ki} = \alpha_{ki}(1-\psi_i)A_i^{\sigma_q-1}\left(\frac{rP_v}{P_{yi}}\right)^{-1}\left(\frac{P_{yi}}{P_{hi}}\right)^{-\sigma_q};$$

$$\mathbf{\Omega_{mh}} = [\Omega_{mhji}]_{j,i=1,\cdots,N}, \text{ where } \Omega_{mhji} = \psi_i A_i^{\sigma_q-1}\omega_{hji}\left(\frac{P_{hj}}{P_{mi}}\right)^{-\sigma_m}\left(\frac{P_{mi}}{P_{hi}}\right)^{-\sigma_q}.$$

Equation (B.6) in vector form is given by

$$\mathsf{Q} = \delta\Omega_{\mathsf{vh}} \cdot (\Omega_{\mathsf{k}}'\mathsf{Q}) + \mathbf{\Omega_{mh}}\mathsf{Q} + \mathsf{C_h} + \mathsf{X}. \tag{B.7}$$

Then

$$\mathsf{Q} = \left[\mathbf{I} - \delta\Omega_{\mathsf{vh}} \cdot \Omega_{\mathsf{k}}' - \mathbf{\Omega_{mh}}\right]^{-1}[\mathsf{C_h} + \mathsf{X}]. \tag{B.8}$$

We can obtain shares of all kinds, as well as relative prices, given that we have solved for $\mathsf{P_h}$ and Q.

2.2 Unknowns of the system The unknowns are

$$
\begin{aligned}
&C_{hjt}, C_{fjt},\ j = 1, \cdots, N;\\
&L_{jt},\ j = 1, \cdots, N;\\
&V_{hjt}, V_{fjt},\ j = 1, \cdots, N;\\
&M_{hijt}, M_{fijt},\ i = 1, \cdots, N;\ j = 1, \cdots, N;\\
&X_{jt},\ j = 1, \cdots, N;\\
&P_{hjt},\ j = 1, \cdots, N;
\end{aligned}
$$

The exogenous variables are variables concerning foreign countries, exchange rate S_t and/or bond rates R_t, productivity shocks A_{jt}.

C Log linearization

We log-linearize the system of equations characterizing the competitive equilibrium of our model economy.

$$
\begin{aligned}
K_{t+1}: &\quad \hat{P}_{ct} + \hat{C}_t + \hat{R}_t = E_t\hat{P}_{c,t+1} + E_t\hat{C}_{t+1};\\
V_t: &\quad \hat{K}_{t+1} = (1-\delta)\hat{K}_t + \hat{V}_t;\\
B_t: &\quad \hat{R}_t + \hat{P}_{vt} + \kappa(\hat{K}_{t+1} - \hat{K}_t) = \mathbf{E}_t\big\{\hat{P}_{vt+1} + [1-\beta(1-\delta)]\hat{r}_{kt+1} + \beta\kappa(\hat{K}_{t+2} - \hat{K}_{t+1})\big\};\\
B_t^*: &\quad \hat{R}_t - \hat{R}_t^* = E_t\hat{S}_{t+1} - \hat{S}_t;\\
w_{jt}: &\quad \hat{w}_{jt} = \hat{P}_{ct} + \hat{C}_t + \nu_j\big(\hat{L}_{jt} - \hat{L}_{jt-1}\big) - \beta\nu_j\mathbf{E}_t\big(\hat{L}_{jt+1} - \hat{L}_{jt}\big);\\
K_{jt}: &\quad \hat{r}_{kt} + \hat{P}_{vt} + \hat{K}_{jt} = (\sigma_q - 1)(\hat{A}_{jt} - (\alpha_{kj}(\hat{r}_{kt} + \hat{P}_{vt}) + \alpha_{lj}\hat{w}_{jt})) + \sigma_q\hat{P}_{hjt} + \hat{Q}_{jt};\\
L_{jt}: &\quad \hat{w}_{jt} + \hat{L}_{jt} = \hat{r}_{kt} + \hat{P}_{vt} + \hat{K}_{jt};\\
M_{hijt}: &\quad \hat{M}_{hijt} = (\sigma_q - 1)\hat{A}_{jt} - \sigma_m\hat{P}_{hit} + \sigma_q\hat{P}_{hjt} + (\sigma_m - \sigma_q)\hat{P}_{mjt} + \hat{Q}_{jt},\ i = 1, \cdots, N;\\
M_{fijt}: &\quad \hat{M}_{fijt} = \hat{M}_{hijt} + \sigma_m\hat{P}_{hit} - \sigma_m(\hat{S}_t + \hat{P}_{it}^*),\ i = 1, \cdots, N;\\
P_{hjt}: &\quad \hat{A}_{jt} + \hat{P}_{hjt} = (1-\psi_j)\left(\frac{P_{yj}}{A_jP_{hj}}\right)^{1-\sigma_q}(\alpha_{kj}(\hat{r}_{kt} + \hat{P}_{vt}) + \alpha_{lj}\hat{w}_{jt}) + \psi_j\left(\frac{P_{mj}}{A_jP_{hj}}\right)^{1-\sigma_q}\hat{P}_{mjt};\\
P_{mjt}: &\quad P_{mjt} = \sum_{i=1}^{N}\big[\omega_{hij}\big(\frac{P_{hi}}{P_{mj}}\big)^{1-\sigma_m}\hat{P}_{hit} + \omega_{fij}\big(\frac{P_{fi}}{P_{mj}}\big)^{1-\sigma_m}(\hat{S}_t + \hat{P}_{it}^*)\big];\\
X_{jt}: &\quad \hat{X}_{jt} = \hat{Y}_t^* - \sigma_x(\hat{P}_{hjt} - \hat{S}_t - \hat{P}_t^*);\\
R_t: &\quad \hat{R}_t = \rho_r\hat{R}_{t-1} + (1-\rho_r)[\alpha_\pi(\hat{P}_{ct} - \hat{P}_{c,t-1}) + \alpha_z\hat{Z}_t] + \epsilon_{Rt};\\
\text{ARC}: &\quad \hat{Q}_{jt} = \frac{C_{hj}}{Q_j}\hat{C}_{hjt} + \frac{V_{hj}}{Q_j}\hat{V}_{hjt} + \sum_{i=1}^{N}\frac{M_{hji}}{Q_j}\hat{M}_{hjit} + \frac{X_j}{Q_j}\hat{X}_{jt};\\
\text{B.C.}: &\quad \frac{P_cC}{P_zZ}(\hat{P}_{ct} + \hat{C}_t) + \frac{P_vV}{P_zZ}(\hat{P}_{vt} + \hat{V}_t) + \frac{SB^*}{R^*rpP_zZ}(\hat{S}_t + \hat{B}_t^* - \hat{R}_t^* - \hat{rp}_t)\\
&= \frac{wL}{P_zZ}((\hat{P}_{ct} + \hat{C}_t) + \hat{L}_t) + \frac{r_kP_vK}{P_zZ}(\hat{r}_{kt} + \hat{P}_{vt} + \hat{K}_t) + \frac{SB^*}{P_zZ}(\hat{S}_t + \hat{B}_{t-1}^*);\\
K_t: &\quad \sum_{j=1}^{N}(K_j/K)\hat{K}_{jt} = \hat{K}_t;
\end{aligned}
$$

$$L_t: \quad \sum_{j=1}^{N}(L_j/L)\hat{L}_{jt} = \hat{L}_t;$$

$$Z_t: \quad \hat{Z}_t = \omega_c\hat{C}_t + \omega_v\hat{V}_t + \sum_{j=1}^{N}\omega_{xj}\hat{X}_{jt} - \sum_{j=1}^{N}\omega_{mj}\hat{I}_{mjt};$$

$$\hat{C}_{hjt}: \quad \hat{C}_{hjt} = \hat{C}_t - \sigma_c(\hat{P}_{hjt} - \hat{P}_{ct});$$

$$\hat{C}_{fjt}: \quad \hat{C}_{fjt} = \hat{C}_t - \sigma_c((\hat{S}_t + \hat{P}^*_{jt}) - \hat{P}_{ct});$$

$$\hat{P}_{ct}: \quad \hat{P}_{ct} = \sum_{j=1}^{N}\left[\phi_{hj}\left(\frac{P_{hj}}{P_c}\right)^{1-\sigma_c}\hat{P}_{hjt} + \phi_{fj}\left(\frac{P_{fj}}{P_c}\right)^{1-\sigma_c}(\hat{S}_t + \hat{P}^*_{jt})\right];$$

$$\hat{V}_{hjt}: \quad \hat{V}_{hjt} = \hat{V}_t - \sigma_v(\hat{P}_{hjt} - \hat{P}_{vt});$$

$$\hat{V}_{fjt}: \quad \hat{V}_{fjt} = \hat{V}_t - \sigma_v((\hat{S}_t + \hat{P}^*_{jt}) - \hat{P}_{vt});$$

$$\hat{P}_{vt}: \quad \hat{P}_{vt} = \sum_{j=1}^{N}\left[\theta_{hj}\left(\frac{P_{hj}}{P_v}\right)^{1-\sigma_v}\hat{P}_{hjt} + \theta_{fj}\left(\frac{P_{fj}}{P_v}\right)^{1-\sigma_v}(\hat{S}_t + \hat{P}^*_{jt})\right];$$