Timing of the Birth:

the Role of Productivity Loss and Income Security

Helu Jiang
Washington University in St. Louis

Hsien-Ming Lien
National Chengchi University

Ping Wang
Washington University in St. Louis and NBER

Yin-Chi Wang
The Chinese University of Hong Kong

February 12, 2019

Abstract: As significant as the shift from quantity to quality in fertility decisions, a rise in the age at first birth has been commonly observed in the more developed world. This paper attempts to understand such demographic trend both theoretically and empirically. We develop a continuous-time lifecycle model, in which a married woman decides when to have her first child and how she allocates her time to human capital accumulation and market activity. We then calibrate the benchmark model using data from CPS and generalize the model to allow for heterogeneous skill levels. We find that the duration of fertility-related productivity loss and income security play a more important role than the conventional human capital channel in explaining the childbearing timing differentials between skill groups, and women are more sensitive to changes in fertility preference as opposed to leisure loss. Decomposition exercise shows that the two novel channels can explain 71.3% of the difference between skill groups. Compared with high-skilled women, low-skilled women are more vulnerable to changes in labor productivity, human capital, husband's income, fertility preference for children and leisure loss in raising children. As a result, low-skilled women push up or defer their timing of childbirth more relative to high-skilled women.

Keywords: Endogenous birth timing, education, job characteristics, fertility preferences.

JEL Classification Numbers: J13, J24, D90, D30.

Acknowledgment: We are grateful to Besty Caucutt, Terry Cheung, David de la Croix, Matthias Doepke, Sebastian Galiani, Bart Hamilton, Jim Heckman, Jin-Tan Liu, Lance Lochner, Bob Pollak, and Henry Siu for valuable suggestions, and to the Directorate-General of Budget, Accounting, and Statistics for providing the complete census data used in our empirical study. Financial support from the National Science Council (NSC 101-2410-H-004 -020 -MY2 for the first author and NSC 98-2911-H-001-001 for the second author) is gratefully acknowledged. The usual disclaimer applies.

Correspondence: Ping Wang, Department of Economics, Washington University in St. Louis, Campus Box 1208, One Brookings Drive, St. Louis, MO 63130, U.S.A.; Tel: 314-935-4236; Fax: 314-935-4156; E-mail: pingwang@wustl.edu.

1 Introduction

Demographic transition has been one of the central issues in the broad field of development economics. This is not only because of academic curiosity for understanding the causes of such a significant socioeconomic change, but also because of its strong implications for the speed of economic development and the misery of poverty traps. To study demographic transition, however, one must recognize that fertility choice includes three distinct decisions: the number of children, the quality of children, and the timing and spacing of births. A vast literature has been devoted to studying the first two of these aspects of fertility by documenting the decline in the total fertility rate over the past century and the associated rise in the quality of children. Much less discussed is the timing of fertility, which has undergone changes of the same order of magnitude as those observed in the quantity and quality dimensions. The focus of the literature on the quantity-quality trade-off is not surprising, because the quantity and quality aspects of children can be handled by standard demand and supply analysis without the need for a full dynamic model. By contrast, studying childbearing age requires a fully specified dynamic process of demographic and labor decisions over a female's entire life, which complicates the analysis greatly. In the present paper, we endeavor to examine this much less-explored dimension of fertility choice, hoping to better understand the determinants of timing and spacing of births. As such, our findings would help generate useful implications for the interplays between demographic transition and economic development.

Over the past five decades, the rise in the childless rate and the age at first birth has been commonly observed in many developed countries in Western Europe and North America as well as in fast growing countries in Asia. Such a positive trend is not only quantitatively large, but robust across regions and ethnic groups (given some noticeable disparities). For example, by the year 1990, almost half (49%) of Swedish women in the 25-29 age group were still childless. The comparable figures for the U.S., Germany and the Netherlands were 42%, 57% and 61%, respectively. Let us take a closer look at the U.S. using the Current Population Survey (CPS) data. The average and median age of first birth increased by 1.405 (2) years, from 24.584 (24) for 1940-1945 cohort to 25.989 (26) for 1950-1955 cohort. If we look at different skill groups, the age at first birth was 24.506 and 25.596 for the low-skilled group and the high-skilled group respectively, and the number of years of first spacing for the two skill groups were 2.518 and 3.249. It's clear to see that the decision on childbearing timing varies across time and between skill groups. Despite the empirical significance and important implications, a systematic analysis of the joint decisions of birth timing and other fertility and individual choices remains relatively under-investigated.

In an attempt to analyze the timing of births, we develop a continuous-time lifecycle model, extending the Ben-Porath framework along the lines proposed by Mullin and Wang (2008) by incorporating birth timing as a decision variable and allowing for heterogeneity of human capital.

To maintain tractability, we abstract from out-of-wedlock childbearing and multiple births.[1] Once a woman is married, she decides the timing of the birth, the allocation of time to work, and human capital accumulation. The model is solved in two steps. In the first stage, given a birth timing, we pin down all endogenous variables other than the birth timing. The endogenous timing of childbearing, modeled as a continuous variable, is then determined in the second stage. Other than the conventional wisdom that views better schooling (higher human capital and hence higher opportunity cost of having children) and child preference as the two main drivers determining the timing of childbirth, we embed three new channels in the model: (1) the leisure loss channel – a married woman will suffer a leisure loss for a certain period if she decides to have a child; (2) the "child penalty" channel – a woman will have a productivity loss and human capital depreciation when having a child; and (3) the income security (husband income) channel. We examine all the above channels, both theoretically and quantitatively, and study how they shape the decision on childbearing timing.

Based on this life-cycle framework, we are able to obtain the following theoretical predictions that are useful in guiding the empirical analysis. First, birth timing is delayed if human capital rises as a result of better education or improved work productivity, or if industry-and occupation-specific factors feature greater productivity loss from childbearing. Second, a reduction in income security leads to postponement in births. Third, birth timing is shortened if women have strong preferences for quality-adjusted children or less leisure loss.

We use data from CPS to construct two synthetic cohorts: 1940-1945 cohort and 1950-1955 cohort, and we allow for human capital heterogeneity by analyzing two skill groups. We calibrate the model targeting the average moments of the two cohorts. Quantitative analysis show that the duration of fertility-related productivity loss and income security (husband income) play a crucial role in understanding the differences in first spacing and the age at first childbearing between the two skill groups. These two novel channels together can explain 71.3% of the difference in the first spacing decisions between skill groups; each contributes around 35%. In particular, if we shut down the heterogeneity of the duration of fertility-related productivity loss, the gap in the first spacing is 13.13 years; if we assume the income security (husband to wife income ratio) is homogenous, the gap becomes 13.71 years; however if we only shut down the heterogeneity in human capital or productivity, the gap is 6.66 and 5.04 years respectively, the sum of which is less than the effect from productivity loss or income security alone. In addition, the counterfactual experiments show that fertility preference is more important than loss from leisure in terms of explaining different timing of childbearing between skill groups. Moreover, the low-skilled women are more vulnerable

[1] Although our findings can be generalized to including such extra features, the model becomes unnecessarily complicated without adding much additional insight.

to changes in labor productivity, leisure loss, income security, and fertility preference, implying that they will defer their childbearing timing decision to a larger extent compared to high-skilled women.

An outline of the paper follows. Section 1.1 discusses related literature. Section 2 outlines the life-cycle model. In section 3 we calibrate the model and perform counterfactual experiments. Section 4 discusses extensions, and section 5 concludes.

1.1 Related Literature

Our paper is related to the broader literature on demographic transition and economic development. Early studies along these lines focused on predicting fertility for the entire population or explaining differences in fertility across sub-populations (see Spengler and Duncan, 1956, Lee, 1987, and Becker, 1988). This analysis relied heavily on changes in the age, sex, and marital composition of the population, but rarely attempted to formally model the evolution of these inputs. The inability of these models to foresee the sharp fertility decline in the early 1930s and the subsequent rise in the 1950s instigated a call for deeper research in this area (cf. Becker, 1960; Easterlin, 1968). Since this broader literature is not as relevant, we will only highlight a few such studies.[2] In particular, fertility became an endogenous variable in more recent dynamic models. Barro and Becker (1989) and Becker, Murphy and Tamura (1990) are among the first to emphasize the interaction of the family with other macro variables related to economic development. Not only does a household's childbearing decision depend on economic conditions, but these decisions also feed back into the economy, influencing labor and capital accumulation decisions. Such interactions have been found to be empirically significant by Wang, Yip and Scotese (1994) using U.S. data. A common feature of the endogenous growth and fertility literature is its focus on the quantity-quality tradeoff in fertility decisions, leaving the decision on the timing of birth largely unexplored.

To our knowledge, there are only a handful of theoretical studies on birth timing. In their pioneering studies, Happel, Hill and Low (1984) and Cigno and Ermisch (1989) illustrate the sharp increase in the timing of first birth in the western world and provide basic microeconomic analysis along the lines of Becker.[3] Not until a decade ago have there been studies using dynamic general equilibrium approaches. In this still thin literature, Conesa (2000), Iyigun (2000) and Caucutt, Guner and Knowles (2001) construct discrete-time models whereas Mullin and Wang (2002) develop a continuous-time framework. Conesa introduces idiosyncratic uncertainty in future labor earnings and analyzes its impact on fertility decisions by regarding children as irreversible consumption durables. In the model, the evolution of human capital is treated as exogenous. Caucutt, Guner

[2] Hotz, Klerman and Willis (1997) provides a comprehensive overview of this literature.

[3] See also Yamaguchi and Ferguson (1995) from the sociological aspect.

and Knowles include marriage and the quantity and quality dimensions of children as endogenous variables. To keep their model tractable, life is divided into five periods in which in the latter three one is an adult, but only fertile for the first two of those three intervals. Thus, the timing of birth is reduced to a binary choice. In addition, the human capital of adults evolves based on time spent in the labor market (i.e., a learning-by-doing rather than an education setup), which eliminates any tradeoff between human capital accumulation and market production. In contrast to these two papers, endogenous human capital accumulation is the key element in Iyigun and Mullin and Wang. Iyigun considers a three-period overlapping-generation economy with the birth timing also modeled as a binary choice. Yet, human capital is accumulated via education and hence there is an immediate trade-off between childbearing and human capital accumulation. Mullin and Wang also model human capital accumulation to depend on education. They permit women to differ in their initial stock of human capital and examine birth timing over the distribution of heterogeneous women by calibrating to the U.S. economy. Our theoretical model complements the literature by considering occupation-specific and income security factors, as well as preference and mother's age factors, in addition to standard employment, income and education factors.

Equally thin is the empirical literature on birth timing. In their pivotal studies, Heckman and Walker (1990a,b) find that while female wages delay child birth timing in Sweden to all conceptions, husband incomes shorten it when marital status is excluded. An interesting result is that the postponement effect of female wages is the strongest through women's first births. Using Dutch data, Bloemen and Kalwij (2001) establish that more educated women, by changing their employment status, lengthen their timing of child birth. In addition to education, Merrigan and St.-Pierre (1998) also identify significant religious and regional effects on birth timing and spacing in Canada. More recently, Gutiérrez-Domènech (2008) applies Spainish data and estimates a positive effect of female employment on birth timing. Using data from developing countries, Bhalotra and van Soest (2008) document that the death of a child in India significantly reduces spacing for the next birth, whereas Tsay and Chu (2005) identify that both years of schooling and son preferences are important for birth timing in Taiwan.

One of the main obstacles in the empirical literature is to what extent the existing evidence between a mother's fertility decision and her decision of human capital can be interpreted as casual. Different from the previous studies focusing on the impact of a woman's career on her fertilty decision, Miller (2009) attempts to identify the casual effect in another direction—the imapct of delayed motherhood on a woman's career. Using the biological shocks as instruments, she found out that motherhood delay increases the wages rate by 3%, and career hours worked by 5%. Likewise, Bailey, Hershbein, and Miller (2012) study the diffusion of contraception pills and found out the pill (and the fertility reduction) can account for 30% of the convergence of the gender wage gap by

1990s.[4] In spite of the recent developments, there is still no clear evidence about the casual effect of a woman's career on her timing and spacing of birth choice.

Although the timing of births has not received much attention in the growth and development literature, the increase in the rate of unwed mothers over the last thirty years and this population's heavy dependence on government assistance has led to a vast literature on this topic and related issues amongst labor economists. The bulk of this research focuses on the effect of government transfer programs and marital prospects on the fraction of women having teenage births and the marital status of those women at the time of birth (see Hoynes, 1997 and Moffitt, 1995). More recently, this line of the literature has increased both the choices available to women and the complexity of their utility functions (see Neal, 2001 and Nechyba, 2001), but these models continue to share two common traits: (i) fertility decisions are limited to a small number of discrete decisions (e.g., teen versus adult or legitimate versus illegitimate births); and (ii) women optimize in a static environment in which there are no dynamic interactions. In contrast to this literature, our work concentrates on the effects of economic conditions on the commencement of childbearing for all women, not just those at risk of teenage or illegitimate childbearing, and accounts for the dynamic interactions between fertility decisions and other economic factors.

2 The Theoretical Framework

In this section, we extend the lifecycle model of Ben-Porath by introducing birth timing as a one of the key decision variables facing each fertile woman who has perfect foresight. We assume throughout that there is no out-of-wedlock childbearing and that the only fertility timing decision is *the age at first birth*. As such, we can restrict our attention to timing-quality trade-off by normalizing the population of each cohort of women to one (i.e., one child per woman).

2.1 The Basic Setup

Time is continuous, indexed by t. Each cohort of women is indexed by the age at which they can begin childbearing (M), which is the age at marriage under our simplifying assumption. All women will live for $T = M + F$ years, where F measures the length of family life. Her age at first birth is denoted by $M + B$. In addition to the incorporation of human capital that measures the quality dimension of fertility decisions and the associated returns, we consider three important

[4] Golden and Katz (2012) is the first one that explores the relation between the diffusion of contraception pill and a woman's marriage and career choice. They found out that the birth control pill delays a woman's age of first marriage, increases year of schooling, and raises working wages and hours.

features influencing a woman's optimzing behavior: (i) preference for children inclusive of altruism, gender bias and disutility from childrearing, (ii) income security or husband's income (ϕ), and (iii) productivity loss due to childbearing.

Denote by n an indicator function for the presence of a child. We assume that, once born, a child yields utility of U_0 throughout the remaining of the woman's lifetime but causes a utility loss of ψ and a productivity loss of δ for a duration of D years of childrearing. Let $I(t \in [M+B, M+F])$ be an indicator function whose value is one upon having a child and $I(t \in [M+B, M+B+D])$ be an indicator function whose value is one over such childrearing years. Without loss of generality, we assume throughout the paper that $B + D < F$, i.e., childrearing only results in a partial loss in productivity over the entire lifetime. We can thus measure the net utility enjoyment of having a child by,

$$NU(B) = U_0 I(t \in [M + B, M + F]) - \psi I(t \in [M + B, M + B + D])$$

For tractability, we further assume that the utility from consuming the composite good c is log-linear and that the mother's lifetime utility V is time-separable with subjective discounting at rate ρ. Aside from her childhood valuation that involves no decision-making, the mother's lifetime utility can then be specified as:

$$V = \int_{M}^{M+F} \left[\frac{c^{1-\sigma}}{1-\sigma} + U_0 I(t \in [M + B, M + F]) - \psi I(t \in [M + B, M + B + D]) \right] e^{-\rho(t-M)} dt \quad (1)$$

where σ is the inverse of the intertemporal elasticity of substitution.

For simplicity, we are abstracting from retirement decisions, assuming that all women work until the end of the her lifetime. Each woman is endowed with one unit of time throughout, which can be allocated to human capital accumulation (η) and market activity ($1 - \eta$). Since a woman suffers a productivity loss of δ during her childbearing years of duration D, her net time endowment is given by, $[1-\delta I(t \in [M+B, M+B+D])]$. Each woman with human capital h earns market wages at rate wh (so w can be referred to as the effective wage rate) and makes consumption-saving decision with a risk-free asset a paid at the market interest rate r. Assume positive assortative matching with a woman's husband income as a multiple of her own: θwh, where $\theta > 0$ measures the husband's income factor, or, more generally, the income security factor facing the woman. For simplicity, all husbands are absentees in the sense that we are abstracting from their behavioral considerations in our "kingdom of daughters." Thus, a woman of cohort M accumulates her nonhuman wealth according to:

$$\dot{a} = ra + [1 - \delta I(t \in [M + B, M + B + D])](1 - \eta)wh + \theta wh - c \quad (2)$$

That is, a woman accumulates asset with net savings, which is the sum of interest income and her own and her husband's wages net of her consumption spending.

In our economy, a woman can accumulate her human capital with her time devoted to education/learning as well as from her peers of cohort M that is in forms of noncompensated human capital spillovers ala Lucas (1988). In contrast to Lucas, such spillovers arise in the accumulation of human capital rather than market good production, and we allow for human capital heterogeneity. In the numerical section, we will have two skill groups for analysis. Denoting the aggregate human capital of cohort M as H, we can then specify a woman's human capital accumulation as follows:

$$\dot{h} = \Phi[1 - \delta I(t \in [M + B, M + B + D])]\eta h^{\gamma} H^{1-\gamma} \tag{3}$$

where $\Phi > 0$ is the maximum rate of human capital accumulation and $\gamma \in (0, 1)$ with $1-\gamma$ capturing the strength of the human capital spillover effect.

To close the model, we specify the production of the composite good at time t, which is produced with a Ricardian technology,

$$y = AL \tag{4}$$

where

$$L = \int_{M}^{M+F} \int_{i \,\in\, \text{cohort } \tau} \{[1 - \delta I(t \in [M + B, M + B + D])](1 - \eta)h\} \, di d\tau \tag{5}$$

which is aggregating over every woman of age τ, over all cohorts currently alive, and over human capital distribution. In a competitive labor market, women are hired at effective wage $w = A$.

2.2 Intertemporal Optimization

A woman of cohort t (entering the economy at period t) solves her intertemporal optimization problem in two steps. In the first and conventional step, she makes optimal consumption-saving decision, human capital investment decision. In the second stage on which our primary focus is, the woman pins down the optimal childbearing time.

The first-stage optimization problem is thus to maximize the lifetime utility specified in (1) subject to the two evolution equations (2) and (3). There are two controls (c, η) and two states (a, h). Denote the co-state variables associated with the two evolution equations as λ_a and λ_h, respectively. So the relative price of human capital investment in units of the composite good becomes $p = \lambda_h/\lambda_a$. It is convenience to denote a woman's relative human capital in cohort t as $v = h/H$. The first-order conditions can then be derived as follows:

$$c^{-\sigma} = \lambda_a \tag{6}$$

$$\Phi p = w v^{1-\gamma} \tag{7}$$

The Euler equations are given by,

$$\begin{aligned}\dot{\lambda}_a/\lambda_a &= \rho - r && (8)\\ \dot{\lambda}_h/\lambda_h &= \rho - \Phi v^{\gamma-1}\left\{[1-\delta I(t\in[M+B,M+B+D])\,(1-\eta+\gamma\eta)+\theta\right\} && (9)\end{aligned}$$

While (6) is a standard condition governing intertemporal consumption efficiency, (7) equates the marginal benefit of human capital investment with its marginal cost measured by foregone wage earnings. Equations (8) and (9) govern the evolution of the shadow prices of the composite good and the human capital stock.

Totally differentiating (6), in conjunction with (8), yields the standard Keynes-Ramsey condition governing the dynamic path of consumption:

$$\dot{c}/c = \frac{r-\rho}{\sigma} \tag{10}$$

We then follow the dual approach proposed by Bond, Wang and Yip (1996) to analyze this two-sector growth model by combining the two Euler equations to obtain:

$$\dot{p}/p = \dot{\lambda}_h/\lambda_h - \dot{\lambda}_a/\lambda_a = r - \Phi v^{\gamma-1}\left\{[1-\delta I(t\in[M+B,M+B+D])]\,(1-\eta+\gamma\eta)+\theta\right\} \tag{11}$$

This *intertemporal no-arbitrage condition* states that if holding asset yields higher return than accumulating human capital, then to have a nondegerate portfolio it must be that human capital provides a capital gain with $\dot{p}/p > 0$. Importantly, such a gain from accumulating human capital is lower if the woman suffers a productivity loss from childbearing.

2.3 Childbearing Decision

We are now prepared to solve hypothetical balanced growth equilibrium assuming infinite lifetime with $F \to \infty$ and under a fixed childbearing age B. Along a hypothetical balanced growth path, c, a and h all grow at constant rates, not necessarily common growth rate, whereas η, v and p are all constant over time. Our main task is to use this hypothetical balanced growth path to help pin down a woman's birth timing. Only for illustration purpose, the following analysis assumes the common growth rate for c, a, and h.

Under constant returns technologies, it is clear that along such a path, consumption, human capital and non-human asset wealth for each cohort must grow at the same rate $g = \frac{r-\rho}{\sigma}$. Thus, from (2) and (3), we have:

$$\frac{c}{a} = \frac{\rho+(\sigma-1)\,r}{\sigma} + [1-\delta I(t\in[M+B,M+B+D])](1-\eta)A\frac{h}{a} + \theta A\frac{h}{a} \tag{12}$$

$$\Phi[1-\delta I(t\in[M+B,M+B+D])]\eta v^{-(1-\gamma)} = \frac{r-\rho}{\sigma} \tag{13}$$

Since p is constant along this hypothetical path, intertemporal no-arbitrage (11) implies:

$$\Phi v^{\gamma-1}\left\{\left[1-\delta I(t\in[M+B,M+B+D])\right](1-\eta+\gamma\eta)+\theta\right\}=r \tag{14}$$

which can be combined with (13) to yield:

$$\frac{\eta\left(r-\Phi v^{\gamma-1}\theta\right)}{1-\eta+\gamma\eta}=\frac{r-\rho}{\sigma}$$

Rearrange the above equation:

$$v=\left[\frac{\Phi\theta\sigma\eta}{\sigma\eta r-(r-\rho)(1-\eta+\gamma\eta)}\right]^{\frac{1}{1-\gamma}} \tag{15}$$

That is, v is a function of η. Other things being equal, stronger human capital spillovers $(1-\gamma)$ imply a more severe free-rider problem, thereby discouraging human capital investment and reducing the relative human capital stock of a woman. For v to be positive, we have to impose the following condition.

Condition V $\eta>\frac{r-\rho}{\sigma r+(1-\gamma)(r-\rho)}$.

That is, the endogenous chosen fraction of time devoted to human capital accumulation cannot be too small. The expression (15) also states that when other things are held equal, we have

$$\frac{dv}{d\eta}=\frac{-1}{(1-\gamma)\eta}\left[\frac{\Phi\theta\sigma\eta}{\sigma\eta-(r-\rho)(1-\eta+\gamma\eta)}\right]^{\frac{1}{1-\gamma}}\frac{r-\rho}{\sigma\eta r-(r-\rho)(1-\eta+\gamma\eta)}<0$$

Proposition 1 Along the BGP, when other things are held constant, an overall increase in the time devoted to human capital accumulation will decrease the relative human capital of women.

The expression (15) can then be substituted into (14) to obtain time devoted to education/learning (η) and work time allocation $(1-\eta)$:

$$\eta=\frac{1}{\sigma r+(r-\rho)(1-\gamma)}\left[\frac{\theta(r-\rho)}{1-\delta I(t\in[M+B,M+B+D])}+r-\rho\right] \tag{16}$$

$$1-\eta=1-\frac{(r-\rho)\left\{\theta+\left[1-\delta I(t\in[M+B,M+B+D])\right]\right\}}{\left[\sigma r+(r-\rho)(1-\gamma)\right]\left[1-\delta I(t\in[M+B,M+B+D])\right]}$$

Proposition 2 When the human capital spillover effect is smaller $(\gamma\uparrow)$, when the assortative matching factor is higher $(\theta\uparrow)$, and when the labor productivity loss during the years with children attached is more severe $(\delta\uparrow)$, a woman will allocate more time to human capital accumulation.

The net work hours (ℓ) can be computed as follows:

$$\begin{aligned}
\ell &= \left[1-\delta I(t\in[M+B,M+B+D])\right](1-\eta) \\
&= \left[1-\delta I(t\in[M+B,M+B+D])\right]-\frac{(r-\rho)\left\{\theta+\left[1-\delta I(t\in[M+B,M+B+D])\right]\right\}}{\sigma r+(r-\rho)(1-\gamma)} \qquad (17)
\end{aligned}$$

That is, the hypothetical balanced growth equilibrium value of time devoted to education/learning (η) can be solved recursively. Once η is solved, v and ℓ are solved. From (17), we find that birth of children has an overall negative effect on work hours due to productivity loss. Moreover, from (7) the relative price of human capital investment can be solved. Given initial nonhuman wealth a_M and human capital h_M, from (12) we obtain the initial consumption at age M as (assuming that the woman does not give birth at age M):

$$c(M)=\frac{[\rho+(\sigma-1)r]}{\sigma}a_M+\left[\frac{(1+\theta)\left[\sigma r-\gamma(r-\rho)\right]}{\sigma r+(r-\rho)(1-\gamma)}\right]Ah_M \qquad (18)$$

Denote $c(M+B)$ as the consumption of the woman when she gives birth to her child. From (12), we can also derive the consumption at age $M+B$:

$$c(M+B)=\tilde{c}(M+B)\,e^{\left(\frac{r-\rho}{\sigma}\right)B}$$

where

$$\tilde{c}(M+B)=\frac{[\rho+(\sigma-1)r]}{\sigma}a_M+\left[\frac{(1+\theta-\delta)\left[\sigma r-\gamma(r-\rho)\right]}{\sigma r+(r-\rho)(1-\gamma)}\right]Ah_M$$

That is, $\tilde{c}(M+B)$ is smaller than $c(M)$ by $\left[\frac{\delta[\sigma r-\gamma(r-\rho)]}{\sigma r+(r-\rho)(1-\gamma)}\right]Ah_M$. For a woman aged between $[M+B,M+B+D]$ (i.e. for $t\in[M+B,M+B+D]$), her consumption stream along the hypothetical BGP is

$$c(t)=\tilde{c}(M+B)\,e^{\left(\frac{r-\rho}{\sigma}\right)(t-M)}$$

And for women who have not had children yet and whose children already leave the nest ($t\in[M,M+B]\cup[M+B+D,M+F]$), their consumption stream along the hypothetical BGP is

$$\begin{aligned}
c(t) &= \left\{\frac{[\rho+(\sigma-1)r]}{\sigma}a_M+\left[\frac{(1+\theta)\left[\sigma r-\gamma(r-\rho)\right]}{\sigma r+(r-\rho)(1-\gamma)}\right]\right\}e^{\left(\frac{r-\rho}{\sigma}\right)(t-M)} \\
&= c(M)e^{\left(\frac{r-\rho}{\sigma}\right)(t-M)}
\end{aligned}$$

Therefore, the lifetime utility of a woman, a function of B, is derived as (see the Appendix for derivations):

$$V(B)=C_1(B)+C_2(B)+\frac{1}{\rho}\Omega(B) \qquad (19)$$

where

$$\begin{aligned}
C_1(B) &= \frac{c(M)^{1-\sigma}}{1-\sigma}\frac{\sigma}{\rho+(\sigma-1)r}\left[1-e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]F}-\left[1-e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D}\right]e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B}\right] \\
C_2(B) &= \frac{\tilde{c}(M+B)^{1-\sigma}}{1-\sigma}\frac{\sigma}{\rho+(\sigma-1)r}\left[1-e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D}\right]e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} \\
\Omega(B) &= U_0\left(e^{-\rho B}-e^{-\rho F}\right)-\psi\left(1-e^{-\rho D}\right)e^{-\rho B}
\end{aligned}$$

Note that $C_1(B)$ is the utility coming from the lifetime without children attached, and $C_2(B)$ is the lifetime utility coming from the period when her children are attached to her. Thus, it is not surprising that $C_1(B)$ an increasing function in B and $C_2(B)$ is a decreasing function in B. Due to the productivity loss, $c(M) > \tilde{c}(M+B)$, and hence a birth postponement (higher B) will lead to a net utility gain from consumption. Whether $\Omega(B)$ is an increasing or a decreasing function in B depends on the relative magnitude of the utility from having children and the disutility when children are young. To ensure that the woman will consider to have a child ($B^* < \infty$), we have to impose the following condition.

Condition B $U_0/\psi > \left(1 - e^{-\rho D}\right)$.

Thus, under Condition B, $\Omega(B)$ is a decreasing function in B. If Condition B is not satisfied, a woman's lifetime utility will always increase in B, meaning that it is optimally for the woman not to have children. However, even when Condition B is satisfied, when the productivity loss is too severe (large δ), it is possible that the optimally chosen age of childbearing $B^* > F$, implying a case of no childbearing. On the contrary, when the net utility enjoyment from having a child is very high (large U_0), we have $B^* = 0$, implying childbearing soon after marriage.

2.4 Main Theoretical Predictions

From second stage optimization over (19), an interior child birth timing must satisfy the following first-order condition:

$$\begin{aligned} V'(B) &= \frac{c(M)^{1-\sigma} - \tilde{c}(M+B)^{1-\sigma}}{1-\sigma}\left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D}\right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} - e^{-\rho B}\left[U_0 - \psi\left(1 - e^{-\rho D}\right)\right] \qquad (20) \\ &= \Gamma_1(B) - \Gamma_2(B) \\ &= 0 \end{aligned}$$

which illustrates the trade-off in childbearing postponement between productivity gain and net utility gain. It is easy to see that the first term $\Gamma_1(B)$, the net consumption gain from postponing childbearing, is decreasing in B. $\Gamma_1(B)$ is also positively depending on productivity loss δ and negatively depending on husband's income (or income security) θ:

$$\begin{aligned} \frac{d\Gamma_1(B)}{d\delta} &= \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D}\right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} \frac{\left[\sigma r - \gamma(r-\rho)\right] A h_M}{\sigma r + (r-\rho)(1-\gamma)} > 0 \\ \frac{d\Gamma_1(B)}{d\theta} &= \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D}\right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} \frac{\left[\sigma r - \gamma(r-\rho)\right] A h_M}{\sigma r + (r-\rho)(1-\gamma)} \left[c(M)^{-\sigma} - \tilde{c}(M+B)^{-\sigma}\right] < 0 \end{aligned}$$

However, the effect of labor productivity Ah_M on $\Gamma_1(B)$ is less clear:

$$\frac{d\Gamma_1(B)}{dAh_M} = \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D}\right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} \left\{ \begin{array}{c} \underbrace{\frac{(1+\theta)\left[\sigma r - \gamma(r-\rho)\right]}{\sigma r + (r-\rho)(1-\gamma)} \left[c(M)^{-\sigma} - \tilde{c}(M+B)^{-\sigma}\right]}_{(-)} \\ + \underbrace{\frac{\delta\left[\sigma r - \gamma(r-\rho)\right]}{\sigma r + (r-\rho)(1-\gamma)} \tilde{c}(M+B)^{-\sigma}}_{(+)} \end{array} \right\}$$

If δ is big enough, $d\Gamma_1(B)/dAh_M$ is more likely to be positive.[5] Regarding $\Gamma_2(B)$, it is decreasing in B and depends positively on the utility gain of having children U_0 and negatively depends on the utility loss when children are attached to mothers.

For illustrative purpose, we shall refer to $\Gamma_1(B)$ and $\Gamma_2(B)$, respectively, as the productivity gain (PG) locus and the utility gain (UG) locus. It is noted that to have an interior solution of B, the second-order condition requires: $V''(B) < 0$, so $\Gamma_1'(B) < \Gamma_2'(B)$, implying that the UG locus is flatter than the PG locus. Figure 1 depicts the PG and UG loci over birth timing B. As shown in the Appendix, a decrease in preference for quality-adjusted children (U_0) or an increase in the utility loss during the childrearing years (ψ) shifts the UG locus to the left, whereas an increase in human capital or labor productivity (Ah_M) or productivity loss (δ), or a decrease in husband's income or income security (θ) shifts the PG locus to the right. Furthermore, an increase in the duration of childrearing (D) shifts the UG locus to the left and the PG locus to the right. Thus, any of such shift leads to a postponement in child birth.

Effects of an increase in	birth timing (B)
1. human capital or labor productivity (Ah_M)	+
2. husband's income or income security (θ)	−
3. productivity loss due to childbearing (δ)	+
4. preference for quality-adjusted children (U_0)	−
5. utility loss during childrearing years (ψ)	+
6. duration of childrearing (D)	+

From (20), we can actually solve the optimal age of childbirth B^* directly:

$$B^* = \frac{\sigma}{(\sigma-1)(r-\rho)} \ln \left\{ \frac{c(M)^{1-\sigma} - \tilde{c}(M+B)^{1-\sigma}}{(1-\sigma)\left[U_0 - \psi\left(1-e^{-\rho D}\right)\right]} \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D}\right] \right\}$$

[5] When δ increases, the first term in the big bracket will be more negative while the second term in the big bracket will be more positive. As long as the second term is more positive, $\frac{d\Gamma_1(B)}{dAh_M} > 0$.

To sum up, changes in human capital or labor productivity (A), income security (θ) and productivity loss due to childbearing (δ) represent human capital factors related to fertility decisions. The preference for quality-adjusted children (U_0) and the utility loss during childrearing years (ψ) capture the fertility and child-loving preference factors in birth timing decision-making, which may even include gender preference and preference of having young children around (i.e., son preference can be captured by a higher value of U_0; women who enjoy having young children around would have a lower ψ). Our comparative-static results indicate that birth timing is delayed if human capital/labor productivity or productivity loss rises, or if the husband's income/income security or fertility preference falls. Of particular note, labor market conditions are embedded in the human capital factor because wage rates depend positively on the (efficiency unit) human capital measure. These implications can be readily extended to a more general model with multiple births.

These theoretical predictions are useful for guiding our empirical analysis. It is straightforward for women's human capital measured by education levels to be important explanatory variables for the timing of births. While job security can be measured by public employment where job tenure is almost guaranteed, productivity loss due to childbearing can be related to occupations. For instance, financial, managerial and specialist jobs are more likely to suffer larger productivity losses. Finally, while preference factors may be partly captured by family obligations and gender bias (toward son) for married women, they naturally lead to unobserved heterogeneities.

3 Numerical Analysis

We now want to quantify our theory of birth timing by matching life-cycle model to the Current Population Survey (CPS) data. We are interested in the two cohorts of females in the United States: 1940-1945 cohort and 1950-1955 cohort, before and after the baby boom respectively. Since the focus of this paper is on childbearing timing, we restrict the sample to females over 35 years old as such age range is close to the end of fecundity cycle. Due to the sample size limitation of CPS, we perform synthetic cohort analysis, in particular, we use the samples of females age 35-40 years old in survey year 1980 , those age 36-41 years old in 1981, those age 37-42 years old in 1982, and those age 38-42 years old in 1983 to construct the first cohort, and for the second cohort, we use the samples of females age 35-40 years old in survey year 1990, those age 36-41 years old in 1992, and those age 38-43 years old in 1995. We drop the samples who got married before graduation and who had the first child before marriage since the model starts from the age that a women is married and this paper is abstracting from the out-of-wedlock childbearing. Table 1 provides the summary statistics of these two cohorts. It can ben seen that the average age at first birth increased by 1.405 years and the median age at first birth increased by 2 years. Similarly, the average first spacing rose by 1.068 years and the median rose by 1 year.

Table 1: Summary Statistics

	1940-1945 cohort		1950-1955 cohort		Difference	
	Mean	Median	Mean	Median	Mean	Median
Age at first marriage	22.044	22.000	22.411	22.000	0.368	0
Age at first birth	24.520	24.000	25.955	26.000	1.434	2.000
First spacing	2.477	2.000	3.543	3.000	1.066	1.000
Fertility	2.403	2.000	2.160	2.000	-0.243	0
Years of Schooling	14.129	14.000	14.363	14.000	0.234	0
Number of Observations	8623		6774			

3.1 Calibration

There are 14 parameters from the model. First, we pin down a number of parameters from the literature or directly from the data. Second, we calibrate the remaining parameters using model targets.

We set the inverse of the intertemporal elasticity of substitution σ, the discounting factor ρ, and interest rate r to be the standard values in the literature. For the productivity loss, Waldfogel (1998) claims that the wage loss of child penalty for a women in the United States ranges from 10% to 15%. Since our δ includes both human capital depreciation and productivity loss, we take the high end to set $\delta = 15\%$. The duration for productivity loss is taken from Phipps et al. (2001), in which they found out that the average duration of child-related interruptions followed by a return to the same job was 1.93 years and the average duration of child-related interruptions followed by a return to a different job was 5.75 years. In their sample, 42.9% went back to the same job, and hence we take a weighted average to set $D = 4.111$ years.

The initial age M is calibrated by using the average age at first marriage of two cohorts from CPS. We assume all women retire at 60 years old, and hence we get the working periods F to be 37.78 years. For husband-to-wife ratio θ, we use gender gap to calibrate, which is defined as the coefficient of the gender dummy variable regressed on log wage earnings, controlling for age, a quartic in age, industry, and states dummies. The coefficients estimated from the wage regressions range from 0.5 to 0.8, depending on the age and marital restrictions we put on the sample. We take the coefficient to be 0.5, in which words, husband-to-wife income ratio is about 1.649. For the technology A, the model predicts $A = w$; therefore similar to the estimate of gender gap, we calibrate productivity from the wage regression. First we define a group indicator variable for the two groups that are categorized on skill levels: we define low-skilled group as the females with high school degree and high-skilled group as those with at least some college experience and above. We regress log wages on the group dummy, age, a quartic in age, state dummies, class of workers, broad industry dummies, and broad occupation dummies, in which the base group is low-skilled females, and then we take exponential of the coefficient estimate of the group dummy variable from the regression to get the productivity measure.

We measure human capital following Hall and Jones (1999):

$$h = \exp\{f(s)\}$$

where h denotes human capital, s denotes years of schooling, and $f(s)$ is a piecewise linear function:

$$f(s) = \begin{cases} 0.134s \quad \text{if} \quad s < 4 \\ 0.134 * 4 + 0.101(s-4) \quad \text{if} \quad 4 \leq s \leq 8 \\ 0.134 * 4 + 0.101 * 4 + 0.068(s-8) \quad \text{if} \quad s > 8 \end{cases}$$

The rest of the four parameters Φ, γ, U_0 and ψ are calibrated using model targets. We calibrate average η as the ratio of average years of schooling and working period, which is the ratio of 14.24 and 37.78, and then we use equation 3 at the mean to calibrate the maximum human capital accumulation rate Φ using the average human capital growth rate. For the parameter that governs the human capital spillover effect γ, we use equation 14 at the mean, intertemporal no arbitrage condition. To calibrate U_0 and ψ, we target the average fertility timing and fertility timing differential between the high-skilled group and low-skilled group. In order to do that, we need group specific δ and D.

Denote the share of females returning to the same job after child-related interruptions by α, and denote the duration of interruption by π. Denote the share of high-skilled by S_H, then the remaining share for the low-skilled group is $1 - S_H$.

$$\bar{\pi} = \alpha\pi_S + (1-\alpha)\pi_D$$

Similarly we define the average duration for the two skill groups:

$$\bar{\pi}_H = \alpha_H\pi_S + (1-\alpha_H)\pi_D$$

$$\bar{\pi}_L = \alpha_L\pi_S + (1-\alpha_L)\pi_D$$

We assume $\alpha_H = 1.2\alpha$ for the high group, and we need to back out x for the low-skilled group such that $\alpha_L = x\alpha$. Since we have:

$$\bar{\pi} = S_H\bar{\pi}_H + (1-S_H)\bar{\pi}_L$$

And hence we have:

$$x = \frac{1 - 1.2S_H}{1 - S_H}$$

In this way, we calibrate $D_H = 3.783$ and $D_L = 4.725$. In a similar fashion, we assume that the human capital depreciation rate of the high-skilled is 1.3 times of the average depreciation rate, and thus we have $\delta_H = 0.195$ and $\delta_L = 0.06573$. Table 2 summarizes all the calibrated parameters.

Table 2: Calibration parameters

Parameters	Values	Description	Source/Target
σ	2.5	inverse of the intertemporal elasticity of substitution	literature
ρ	0.05	discounting factor	literature
r	12%	interest rate	literature
δ	15%	productivity loss during childrearing	Waldfogel (1998)
D	4.11	duration of childrearing	Phipps, Burton, Lethbridge (2001)
M	22.22	age at marriage	age of first marriage
F	37.78	working life span	author's calculation
θ	1.649	husband to wife log income ratio	gender gap
A_L	1.000	technology	normalization
A_H	1.240	technology	wage regression
Φ	0.0459	maximum human capital accumulation rate	average human capital growth rate
γ	0.9523	human capital spillover effect	intertemporal no arbitrage condition
U_0	0.00089	lifetime utility gain from child	fertility timing
ψ	0.00292	utility loss during childrearing	fertility timing differential

The following table 3 shows model predictions, in which the top panel illustrates the average and the difference between the two skill groups, and the bottom panel illustrates the results for two skill groups. We target the average time allocated to human capital accumulation, the average first spacing and age at first marriage as well as their differentials between skill groups. As can be seen from the table that high-skilled females are more likely to have their first child later in life, and they also allocate more time into human capital accumulation, thus having a higher relative human capital.

Table 3: Model Predictions

(a) Average and Difference between Skill Groups

		Data		Model	
Variable	Description	Average	Diff	Average	Diff
B	first spacing	2.9945	0.7313	2.9945	0.7313
$M+B$	age at first birth	25.2166	1.0905	25.2166	1.0905
η	human capital allocation	0.3770	0.0910	0.3770	0.0944
ν	relative human capital	1.0000	0.2415	1.2153	5.4016
l	time allocated to work	0.6230	-0.0910	0.6134	-0.0098

(b) High Skill and Low Skill Group

		Data		Model	
Variable	Description	High	Low	High	Low
B	first spacing	3.2490	2.5178	3.2490	2.5178
$M+B$	age at first birth	25.5962	24.5057	25.5962	24.5057
η	human capital allocation	0.4087	0.3176	0.4101	0.3157
ν	relative human capital	1.0840	0.8425	5.4304	0.0288
l	time allocated to work	0.5913	0.6824	0.6100	0.6198

3.2 Counterfactual Exercises

In order to understand the driving factors behind the divergence of childbirth timing decisions between low-skilled group and high-skilled group, we perform two types of counterfactual experiments. In the first group of experiments we shut down different sources of heterogeneity in the model, including fertility-related productivity loss, initial human capital, productivity, and husband income (income security). Second, we want to examine the effect of fertility preference and leisure loss in the preference. To sum up, we perform 6 counterfactual exercises in total:

1. Sources of heterogeneity
   (a) shut down duration of productivity loss heterogeneity by setting $D_H = D_L = D$
   (b) shut down initial human capital heterogeneity by setting initial $h_H = h_L = h$
   (c) shut down productivity heterogeneity by setting $A_H = A_L = A$

(d) shut down assortative matching heterogeneity by setting $\tilde{\theta} = \dfrac{\theta \bar{w} \bar{h}}{w_i h_i}$

2. Fertility preference and leisure loss

    (a) increase U_0 by 1%

    (b) decrease ψ by 1%

Table 4 shows the results for the first group of counterfactual experiments. Let's focus on the 4th column that indicates the implied difference between the high-skilled group and low-skilled group. As can be seen from the first two rows in all the four panels, shutting down heterogeneity in duration of productivity loss leads to a dramatic gap of the first spacing, which is 13.1330 years. However, the effect stemming from the heterogenous initial human capital or productivity (experiment 2 and 3) is much less evident; the difference of first spacing is by 6.6570 and 5.0398 years respectively. The husband income θ, which can be also interpreted as the measure of income security, also plays a critical role in explaining the first spacing differential, which can be seen from the first row in panel (d) that the difference is 13.7079 years. The same pattern holds for the age at first birth, measured by $M + B$. The conventional human capital channel serves a certain role in understanding the childbearing timing; however according to our quantitate result, the two new channels via fertility-related productivity loss and income security are much more essential.

Next table 5 illustrates the results for the second group of experiments. We only show the effect of change in U_0 and ψ on the first spacing B and the age at first birth $M + B$. The reason why we do not show the corresponding changes in η, ν, and l is that under these two experiments, the time allocated to human capital accumulation is not affected, and neither is the net working hours. Moreover, a 1% increase in U_0 has the same effect on the relative human capital as that of 1% decrease in ψ. As can be seen from the 5th and 6th columns in the table, a 1% increase in fertility preference leads to the decrease in the first spacing, 0.5420 and 0.7574 for the high skill group and low skill group respectively. Though a 1% decrease in leisure loss also implies a drop in the first spacing, the effect is not as strong as that shown in panel (a). So we argue that women are sensitive to the changes in fertility preference compared to leisure loss. Another observation from this table is that the decrease for the low-skilled group is much more pronounced than that for the high-skilled group, which implies that the low-skilled women are more vulnerable to the changes in fertility preference and leisure loss.

Table 4: Sources of heterogeneity

(a) Experiment 1: $D_H = D_L = D$

				Relative to Benchmark Model	
Variable	High Skilled	Low Skilled	Difference	High Skilled	Low Skilled
B	7.4425	-5.6905	13.1330	4.1935	-8.2083
$M+B$	29.7897	16.2974	13.4923	4.1935	-8.2083
η	0.3782	0.3748	0.0034	-0.0319	0.0591
ν	0.9612	1.0760	-0.1148	-4.4692	1.0472
l	0.6086	0.6207	-0.0122	-0.0015	0.0009
$\Delta V(B)/V(B)$	-0.0058	0.0053	-0.0111		

(b) Experiment 2: $h_H = h_L = h$

				Relative to Benchmark Model	
Variable	High Skilled	Low Skilled	Difference	High Skilled	Low Skilled
B	5.1137	-1.5433	6.6570	1.8647	-4.0611
$M+B$	27.4609	20.4446	7.0163	1.8647	-4.0611
η	0.3778	0.3751	0.0027	-0.0323	0.0594
ν	0.9743	1.0670	-0.0927	-4.4561	1.0382
l	0.6100	0.6198	-0.0098	0.0000	0.0000
$\Delta V(B)/V(B)$	-0.0996	0.1807	-0.2803		

(c) Experiment 3: $A_H = A_L = A$

				Relative to Benchmark Model	
Variable	High Skilled	Low Skilled	Difference	High Skilled	Low Skilled
B	4.6440	-0.3958	5.0398	1.3949	-2.9136
$M+B$	26.9912	21.5921	5.3991	1.3949	-2.9136
η	0.3778	0.3751	0.0027	-0.0323	0.0594
ν	0.9743	1.0670	-0.0927	-4.4561	1.0382
l	0.6100	0.6198	-0.0098	0.0000	0.0000
$\Delta V(B)/V(B)$	-0.1721	0.1336	-0.3057		

(d) Experiment 4: change in θ

				Relative to Benchmark Model	
Variable	High Skilled	Low Skilled	Difference	High Skilled	Low Skilled
B	7.2827	-6.4252	13.7079	4.0336	-8.9430
$M+B$	29.6299	15.5627	14.0672	4.0336	-8.9430
η	0.3486	0.4486	-0.1000	-0.0615	0.1329
ν	0.1804	45.5108	-45.3304	-5.2500	45.4820
l	0.6387	0.5469	0.0918	0.0286	-0.0729
$\Delta V(B)/V(B)$	-0.1775	0.2026	-0.3800		

Table 5: Fertility preference and leisure loss

(a) Experiment 5: 1% increase in U_0

				Relative to Benchmark Model	
Variable	High Skilled	Low Skilled	Difference	High Skilled	Low Skilled
B	2.7070	1.7604	0.9467	-0.5420	-0.7574
$M + B$	25.0542	23.7483	1.3059	-0.5420	-0.7574

(b) Experiment 6: 1% decrease in ψ

				Relative to Benchmark Model	
Variable	High Skilled	Low Skilled	Difference	High Skilled	Low Skilled
B	2.9409	1.9922	0.9488	-0.3081	-0.5256
$M + B$	25.2881	23.9801	1.3080	-0.3081	-0.5256

3.3 Decomposition

In this section, we perform the decomposition exercise to further understand the precise contribution of each channel to the childbirth timing decisions for two skill groups. Since the analysis on the age at first birth (M+B) and the first spacing (B) is the same, we only present results for the first spacing here, as illustrated by the following table 6.

The assortative matching channel, measured by the husband income θ, is able to explain 53.12% of the average first spacing. Initial divergence in human capital can account for 21.76%. However, the conventional human capital and productivity channel together can only contribute to around 33% of the total gap, which implies the crucial role that the income security and duration channel plays.

The result is even more interesting if we focus on the implied difference between the high-skilled group and low-skilled group. Heterogeneity in initial human capital and productivity together explain less than one third of the gap, of which the effect is much less pronounced than that from the experiment 1 and experiment 4. The heterogeneity in duration of fertility-related productivity loss along can explain around 34.82% of the difference between high-skilled group and low-skilled group. The remaining 36.44% of the gap is attributed to the income security channel. The decomposition exercise reinforces our finding that the novel productivity loss and income security channels play a much more crucial role than the conventional human capital and productivity channel in understanding the childbearing timing, especially the differences in the childrearing timing decisions between skill groups.

Table 6: Decomposition: First Spacing (B)

	Average			Difference		
	result	normalized	contribution	result	normalized	contribution
model	2.9945			0.7313		
Exp1: duration	2.8707	0.4070	13.59%	13.1330	0.2547	34.82%
Exp2: initial human capital	2.7963	0.6571	21.76%	6.6570	0.1217	16.64%
Exp3: productivity	2.8895	0.3451	11.52%	5.0398	0.0885	12.10%
Exp4: assortative matching	2.5107	1.5907	53.12%	13.7079	0.2665	36.44%
SUM			100%			100%

4 Extensions

Up so far, we have examined the influence of different sources of heterogeneity, fertility preference and leisure loss on the fertility timing differential between skill groups. In the next step, using data for the two cohorts, we will be able to compare the relative importance of the factors such as improvement in initial human capital, delay in marriage, narrower gender gap, and rising college premium. Instead of focusing on only two skill groups, we can further analyze the evolution of birth distribution. Last but not least, we can conduct robustness check using the census data for the early cohort allowing for multiple number of children and multiple spacing.

5 Conclusion

This paper has developed a life-cycle model of fertility choice that theoretically identifies key factors driving a woman's decision regarding on the timing of childbearing that is modeled as a continuous variable. On top of the conventional human capital channel and fertility preference, this paper highlights the importance of the duration of fertility-related productivity loss, income security, and leisure loss. Numerical analysis implies that in terms of explaining the gap of first spacing and age at first birth between the two skill groups, duration of productivity loss and income security have played a much more crucial role compared to education or opportunity cost. The conventional human capital together with productivity channel can account for only 28.7% of the gap, while around 34.8% of the difference between high-skilled women and low-skilled women can be explained by the duration of fertility-related productivity loss and the remaining 36.4% can be attributed to the income security (husband's income) channel. Moreover, both the low-skilled group and the high-skilled group are more sensitive to changes in child preference when determining the timing of the birth, and the low-skilled women are more vulnerable to changes in productivity and fertility preference, which explains why low-skilled women push up or defer their timing of children more relative to high-skilled women.

References

[1] Bailey, Martha J., Brad Hershbein, and Amalia R. Miller (2012) "The Opt-In Revolution? Contraception and the Gender Gap in Wages." *American Economic Journal: Applied Economics*, 4(3): 225-54.

[2] Barro, Robert J. and Gary S. Becker (1989), "Fertility Choice in a Model of Economic Growth," *Econometrica*, 57, 655-673.

[3] Becker, Gary S. (1960), "An Economic Analysis of Fertility," in NBER (ed.) *Demographic and Economic Change in Developed Countries*, Princeton, NJ: Princeton University Press.

[4] Becker, Gary S (1988), "Family Economics and Macro Behavior," *American Economic Review*, 78, 1-13.

[5] Becker, Gary S., Kevin M. Murphy and Robert Tamura (1990), "Human Capital, Fertility, and Economic Growth," *Journal of Political Economy*, 98, S12-S37.

[6] Bhalotra, Sonia and Arthur van Soest (2008), "Birth-spacing, Fertility and Neonatal Mortality in India: Dynamics, Frailty, and Fecundity," *Journal of Econometrics*, 143, 274-290.

[7] Bloemen, Hans and Adriaan S. Kalwij (2001), "Female Labor Market Transitions and the Timing of Births: A Simultaneous Analysis of the Effects of Schooling," *Labour Economics*, 8, 593-620.

[8] Caucutt, Elizabeth M., Nezih Guner, and John Knowles (2001), "Why Do Women Wait? Matching, Wage Inequality, and the Incentives for Fertility Delay," *Review of Economic Dynamics*, 5, 815-855.

[9] Cigno, Alessandro and John Ermisch (1989), "A Microeconomic Analysis of the Timing of Births," *European Economic Review*, 33, 737-760.

[10] Conesa, Juan Carlos (2000), "Educational Attainment and Timing of Fertility Decisions,"Working Paper, Universitat de Barcelona.

[11] Easterlin, Richard A. (1968), *Population, Labor Force, and Long Swings in Economic Growth*, New York: Columbia University Press.

[12] Freedman, Ronald, Ming-Cheng Chang and Te-Hsiung Sun (1994), "Taiwan's Transition from High Fertility to Below-Replacement Levels," *Studies in Family Planning*, 25, 317-331.

[13] Goldin, Claudia and Lawrence F. Katz. "The Power Of The Pill: Contraceptives And Women's Career And Marriage Decisions," *Journal of Political Economy*, 2002, 110(4), 730-770

[14] Glomm, Gerhard and B. Ravikumar (1992), "Public Versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," *Journal of Political Economy*, 100, 818-834.

[15] Greene, W.H. (2000) *Econometric Analysis*, 4th Ed. New York: Prentice Hall.

[16] Gutiérrez-Domènech, Maria (2008), "The Impact of the Labour Market on the Timing of Marriage and Births in Spain," *Journal of Population Economics*, 21, 83-110.

[17] Hall, R. E., and Jones, C. I. (1999). Why do some countries produce so much more output per worker than others?" *The quarterly journal of economics*, 114(1), 83-116.

[18] Happel, S. K., J. K. Hill and S. A. Low (1984), "An Economic Analysis of the Timing of Childbirth." *Population Studies*, 38, 299-311.

[19] Heckman, James J. and James R. Walker (1990a), "The Relationship between Wages and Income and the Timing and Spacing of Births: Evidence from Swedish Longitudinal Data," *Econometrica*, 58, 1411-1441.

[20] Heckman, James J. and James R. Walker (1990b), "The Third Birth in Sweden," *Journal of Population Economics*, 3, 235-275.

[21] Heckman, James J., Joseph V. Hotz and James R. Walker (1985), "New Evidence on the Timing and Spacing of Births," *American Economic Review, Papers and Proceedings*, 75, 179-184.

[22] Hotz, V. Joseph, Jacob A. Klerman and Robert J. Willis (1997), "The Economics of Fertility in Developed Countries," in M. R. Rosenzweig and O. Stark (eds.), *Handbook of Population and Family Economics*, Elsevier Science, Amsterdam, the Netherlands.

[23] Hoynes, Hilary (1997), "Does Welfare Play Any Role in Female Headship Decisions?" *Journal of Public Economics*, 65 89-117.

[24] Iyigun, Murat F. (2000), "Timing of Childbearing and Economic Growth," *Journal of Economic Development*, 61, 255-269.

[25] Lancaster, Tony (1990), *The Econometric Analysis of Transition Data*, Cambridge University Press, Cambridge, UK.

[26] Lee, Ronald D. (1987), "Population Dynamics of Humans and Other Animals," Presidential Address to the Population Association of America, *Demography*, 24, 443-467.

[27] Lucas, Robert E., Jr. (1988), "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22, 2-42.

[28] Miller, Amalia R, (2009) "Motherhood Delay and the Human Capital of the Next Generation," *American Economic Review Papers and Proceedings*, 99 (2), 154-58

[29] Miller, Amalia R (2011). "The Effects of Motherhood timing on Career Path," *Journal of Population Economics,* 24(3), 1071-1100.

[30] Merrigan, Philip and Yvan St.-Pierre (1998), "An Econometric and Neoclassical Analysis of the Timing and Spacing of Births in Canada from 1950 to 1990," *Journal of Population Economics*, 11, 29-51.

[31] Moffitt, Robert (1995), "The Effect of the Welfare System on Nonmarital Childbearing." In *Report to Congress on Out-of-Wedlock Childbearing*, National Center for Health Statistics, Washington DC: U.S. Government Printing Office.

[32] Mullin, Charles and Ping Wang (2002), "The Timing of Childbearing among Ability Heterogeneous Women in Dynamic General Equilibrium," NBER Working Paper No. 9231, Cambridge, MA.

[33] Neal, Derek (2001), "The Economics of Family Structure," NBER Working Paper, No. 8519, Cambridge, MA.

[34] Nechyba, Thomas J. (2001), "Social Approval, Values and AFDC: A Re-examination of the Illegitimacy Debate," *Journal of Political Economy*, 109, 637-841.

[35] Nerlove, M. (1974). "Household and Economy: Toward a New Theory of Population and Economic Growth," *Journal of Political Economy*, University of Chicago Press, 82(2), 200-218, Part II.

[36] Phipps, S., Burton, P., and Lethbridge, L. (2001). In and out of the labour market: long?term income consequences of child?related interruptions to women's paid work. " *Canadian Journal of Economics*, 34(2), 411-429.

[37] Spengler, Joseph J. and Otis D. Duncan (1956), *Demographic Analysis: Selected Readings*, Glencoe: The Free Press.

[38] Tsay, Wen-Jen and C.Y. Cyrus Chu (2005), "The Pattern of Birth Spacing During Taiwan's Demographic Transition," *Journal of Population Economics*, 18, 323-336.

[39] Waldfogel, J. (1998). Understanding the "family gap" in pay for women with children. " *Journal of economic Perspectives*, 12(1), 137-156.

[40] Wang, Ping, Chong K. Yip and Carol A. Scotese (1994), "Fertility Choice and Economic Growth: Theory and Evidence." *Review of Economics and Statistics*, 76, 255-266.

[41] Yamaguchi, Kazuo and Linda R. Ferguson (1995), "The Stopping and Spacing of Childbirths and Their Birth-History Predictors: Rational-Choice Theory and Event-History Analysis," *American Sociological Review*, 60, 272-298.

Appendix (Not intended for publication)

Derivation of lifetime utility

$$V(B) = \int_M^{M+F} \left[\frac{c^{1-\sigma}}{1-\sigma} + U_0 I(t \in [M+B, M+F]) - \psi I(t \in [M+B, M+B+D]) \right] e^{-\rho(t-M)} dt$$

$$= \int_M^{M+F} \frac{c^{1-\sigma}}{1-\sigma} e^{-\rho(t-M)} dt + U_0 \int_M^{M+F} I(t \in [M+B, M+F]) e^{-\rho(t-M)} dt$$

$$- \psi \int_M^{M+F} I(t \in [M+B, M+B+D]) e^{-\rho(t-M)} dt$$

$$= (i) + (ii) + (iii)$$

$$\begin{aligned} (i) &= \int_M^{M+F} \frac{c^{1-\sigma}}{1-\sigma} e^{-\rho(t-M)} dt \\ &= \int_M^{M+B} \frac{c^{1-\sigma}}{1-\sigma} e^{-\rho(t-M)} dt + \int_{M+B}^{M+B+D} \frac{c^{1-\sigma}}{1-\sigma} e^{-\rho(t-M)} dt + \int_{M+B+D}^{M+F} \frac{c^{1-\sigma}}{1-\sigma} e^{-\rho(t-M)} dt \\ &= (A) + (B) + (C) \end{aligned}$$

$$\begin{aligned} (A) &= \int_M^{M+B} \frac{c(M)^{1-\sigma} e^{(1-\sigma)\left(\frac{r-\rho}{\sigma}\right)(t-M) - \rho(t-M)}}{1-\sigma} dt \\ &= \frac{c(M)^{1-\sigma}}{1-\sigma} \int_M^{M+B} e^{-(t-M)\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]} dt \\ &= \frac{c(M)^{1-\sigma}}{1-\sigma} \frac{\sigma}{\rho+(\sigma-1)r} \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} \right] \end{aligned}$$

$$\begin{aligned} (B) &= \int_{M+B}^{M+B+D} \frac{\tilde{c}(M+B)^{1-\sigma} e^{(1-\sigma)\left(\frac{r-\rho}{\sigma}\right)(t-M) - \rho(t-M)}}{1-\sigma} dt \\ &= \frac{\tilde{c}(M+B)^{1-\sigma}}{1-\sigma} \int_{M+B}^{M+B+D} e^{-(t-M)\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]} dt \\ &= \frac{\tilde{c}(M+B)^{1-\sigma}}{1-\sigma} \frac{\sigma}{\rho+(\sigma-1)r} \left[e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right](B+D)} \right] \end{aligned}$$

$$\begin{aligned} (C) &= \frac{c(M)^{1-\sigma}}{1-\sigma} \int_{M+B+D}^{M+F} e^{-(t-M)\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]} dt \\ &= \frac{c(M)^{1-\sigma}}{1-\sigma} \frac{\sigma}{\rho+(\sigma-1)r} \left[e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right](B+D)} - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]F} \right] \end{aligned}$$

Hence,

$$\begin{aligned} (i) &= \frac{c(M)^{1-\sigma}}{1-\sigma} \frac{\sigma}{\rho+(\sigma-1)r} \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]F} - \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D} \right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} \right] \\ &\quad + \frac{\tilde{c}(M+B)^{1-\sigma}}{1-\sigma} \frac{\sigma}{\rho+(\sigma-1)r} \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D} \right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} \end{aligned}$$

$$\begin{aligned}
(ii) &= U_0 \int_M^{M+F} I(t \in [M+B, M+F]) e^{-\rho(t-M)} dt \\
&= U_0 \int_{M+B}^{M+F} e^{-\rho(t-M)} dt \\
&= \frac{U_0}{\rho} \left(e^{-\rho B} - e^{-\rho F} \right) \\
(iii) &= \psi \int_M^{M+F} I(t \in [M+B, M+B+D]) e^{-\rho(t-M)} dt \\
&= \psi \int_{M+B}^{M+B+D} e^{-\rho(t-M)} dt \\
&= \frac{\psi}{\rho} \left[e^{-\rho B} - e^{-\rho(B+D)} \right] \\
&= \frac{\psi}{\rho} \left[1 - e^{-\rho D} \right] e^{-\rho B}
\end{aligned}$$

Therefore, the lifetime utility, a function of B, is derived as

$$\begin{aligned}
V(B) &= \frac{c(M)^{1-\sigma}}{1-\sigma} \frac{\sigma}{\rho + (\sigma-1)r} \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]F} - \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D} \right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} \right] \\
&\quad + \frac{\tilde{c}(M+B)^{1-\sigma}}{1-\sigma} \frac{\sigma}{\rho + (\sigma-1)r} \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D} \right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} \\
&\quad + \frac{U_0}{\rho} \left(e^{-\rho B} - e^{-\rho F} \right) - \frac{\psi}{\rho} \left[1 - e^{-\rho D} \right] e^{-\rho B} \\
&= C_1(B) + C_2(B) + \frac{1}{\rho} \Omega(B)
\end{aligned}$$

where

$$C_1(B) = \frac{c(M)^{1-\sigma}}{1-\sigma} \frac{\sigma}{\rho + (\sigma-1)r} \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]F} - \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D} \right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} \right]$$

$$C_2(B) = \frac{\tilde{c}(M+B)^{1-\sigma}}{1-\sigma} \frac{\sigma}{\rho + (\sigma-1)r} \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D} \right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B}$$

and

$$\begin{aligned}
\Omega(B) &= U_0 \left(e^{-\rho B} - e^{-\rho F} \right) - \psi \left(1 - e^{-\rho D} \right) e^{-\rho B} \\
&= \left[U_0 \left(1 - e^{-\rho(F-B)} \right) - \psi \left(1 - e^{-\rho D} \right) \right] e^{-\rho B}
\end{aligned}$$

Derivation of Condition B

To derive Condition B, we differentiate $\Omega(B)$ with respect to B:

$$\begin{aligned}
\Omega'(B) &= -\rho U_0 e^{-\rho B} + \rho \psi \left(1 - e^{-\rho D} \right) e^{-\rho B} \\
&= \rho e^{-\rho B} \left[-U_0 + \psi \left(1 - e^{-\rho D} \right) \right]
\end{aligned}$$

Therefore,

$$\Omega'(B) \gtreqless 0 \quad \text{if} \quad \frac{U_0}{\psi} \lesseqgtr \left(1 - e^{-\rho D} \right)$$

To ensure that a $V(B)$ is strictly concave in B, we thus impose the condition $\frac{U_0}{\psi} > \left(1 - e^{-\rho D}\right)$.
Derivation of the first-order condition

$$
\begin{aligned}
V'(B) &= C_1'(B) + C_2'(B) + \frac{1}{\rho}\Omega'(B) \\
&= \frac{c(M)^{1-\sigma}}{1-\sigma}\left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D}\right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} - \frac{\tilde{c}(M+B)^{1-\sigma}}{1-\sigma}\left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D}\right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} \\
&\quad - e^{-\rho B}\left[U_0 - \psi\left(1 - e^{-\rho D}\right)\right] \\
&= \frac{c(M)^{1-\sigma} - \tilde{c}(M+B)^{1-\sigma}}{1-\sigma}\left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D}\right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} - e^{-\rho B}\left[U_0 - \psi\left(1 - e^{-\rho D}\right)\right] \\
&= \Gamma_1(B) - \Gamma_2(B) \\
&= 0
\end{aligned}
$$

@@@ Below is the old version.
The original formulation of L:

$$
L = \int_{M=t-F}^{t} \int_{i \,\in\, \text{cohort } M} \{[1 - \delta I(t \in [M+B, M+B+D])](1-\eta)h\} \, di dM
$$

Derivation of the comparative statics: Recall the definitions of $\Omega(B)$ and $\Lambda(B)$:

$$
\begin{aligned}
\Omega(B) &= \left[U_0\left(1 - e^{-\rho(F-B)}\right) - \psi\left(1 - e^{-\rho D}\right)\right] e^{-\rho B} \\
\Lambda(B) &= [1 - \delta I(t \in [M+B, M+B+D])] - \frac{r^{1-\gamma}(r-\rho)^{\gamma}}{\sigma^{\gamma}(1-\gamma)^{1-\gamma}\Phi}
\end{aligned}
$$

Straightforward differentiation yields:

$$
\begin{aligned}
\Omega'(B) &= -\rho e^{-\rho B}\left[U_0 - \psi\left(1 - e^{-\rho D}\right)\right] < 0 \\
\Omega''(B) &= \rho^2 e^{-\rho B}\left[U_0 - \psi\left(1 - e^{-\rho D}\right)\right] > 0
\end{aligned}
$$

which implies:

$$
\Gamma_2'(B) = -\rho e^{-\rho B}\left[U_0 - \psi\left(1 - e^{-\rho D}\right)\right] < 0
$$

Moreover, from the property of the indicator functions, we have: $\Lambda'(B) > 0$ and $\Lambda''(B) = 0$, implying:

$$
\begin{aligned}
&\frac{d}{dB}\left\{\frac{[\rho + (\sigma-1)r]}{\sigma}a_M + [\Lambda(B) + \theta]Ah_M\right\}^{-\sigma} Ah_M\Lambda'(B) \\
=\ & -\sigma\left\{\frac{[\rho + (\sigma-1)r]}{\sigma}a_M + [\Lambda(B) + \theta]Ah_M\right\}^{-\sigma-1}\left[Ah_M\Lambda'(B)\right]^2 Ah_M < 0
\end{aligned}
$$

and hence $\Gamma_1'(B) < 0$. Furthermore, we can derive:

$$
\frac{d}{d\theta}\left\{\frac{[\rho + (\sigma-1)r]}{\sigma}a_M + [\Lambda(B) + \theta]Ah_M\right\}^{-\sigma} Ah_M\Lambda'(B) < 0
$$

$$
\begin{aligned}
& \frac{d}{d\delta}\left\{\frac{[\rho+(\sigma-1)\,r]}{\sigma}a_M+[\Lambda(B)+\theta]\,Ah_M\right\}^{-\sigma}Ah_M\Lambda'(B) \\
= \quad & -\sigma\left\{\frac{[\rho+(\sigma-1)\,r]}{\sigma}a_M+[\Lambda(B)+\theta]\,Ah_M\right\}^{-\sigma-1}\left[Ah_M\Lambda'(B)\right]^2\frac{\partial\Lambda(B)}{\partial\delta}>0
\end{aligned}
$$

which yield the comparative static results in Section 3.4.

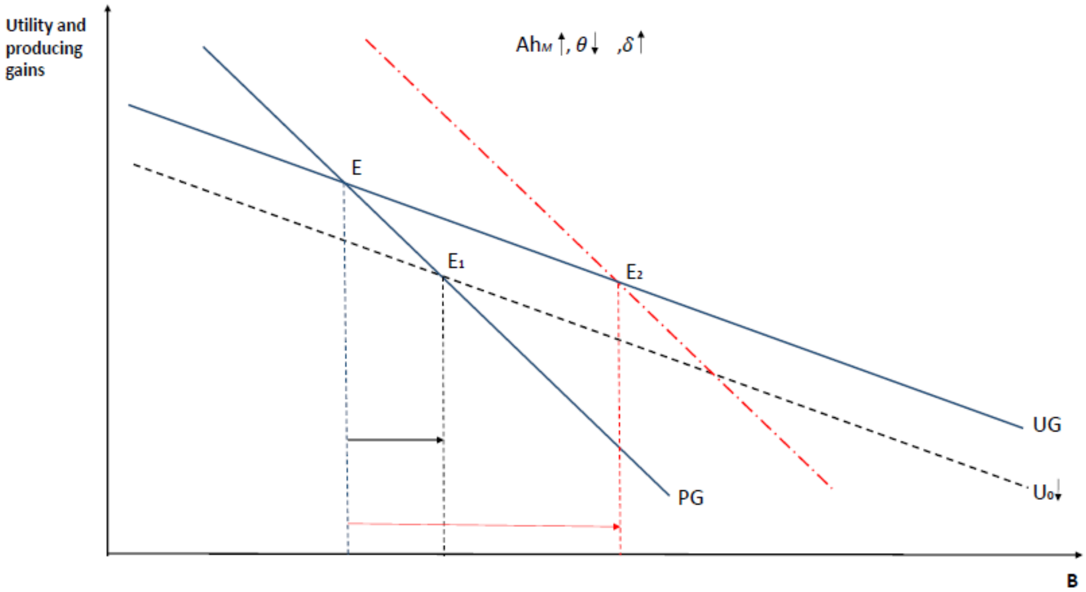

Figure 1: Comparative Statics