Asset Price Booms and Macroeconomic Policy: a Risk-Shifting Approach*

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Abstract

This paper uses risk-shifting models to analyze some potential policy responses to asset price booms and bubbles. We argue that the presence of risk shifting can generate many of the features of such booms and so is a reasonable framework to explore these issues. Our analysis offers several insights. First, we find that determining whether there is indeed a bubble in asset markets is unimportant, since risk-shifting leads to the same inefficiencies regardless of whether it gives rise to a bubble or not. Second, while risk shifting offers a reason for intervention, we find the leading proposals for interventions against booms have ambiguous welfare implications in our model. Specifically, we show tighter monetary policy may exacerbate some inefficiencies due to risk shifting even as it mitigates others, and that leverage restrictions may fan asset prices and exacerbate excessive leverage rather than curb it.

*The views here do not represent those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

Introduction

Policymakers have long debated how to respond to asset price booms and potential bubbles, i.e., when asset prices seem to surge above the value of the dividends assets are expected to yield. One view, summarized in Bernanke and Gertler (1999) and Gilchrist and Leahy (2002), argues that policymakers should not respond in these cases, but should instead wait to see what happens to asset prices. If asset prices collapse and drag down economic activity, policymakers should step in, just as they would whenever output is depressed. An alternative view, summarized in Borio and Lowe (2002), argues that policymakers should not stand idly by during these episodes. They argue that asset booms can be anticipated to end in recessions, especially when these booms coincide with credit booms, a claim corroborated in subsequent work such as Jorda, Schularick, and Taylor (2015) and Mian, Sufi, and Verner (2017). The implication is that policymakers should raise interest rates during asset booms to dampen asset prices and mitigate the eventual crash.

The severity of the Global Financial Crisis that started in 2007 and the difficulty central banks faced in its wake to stimulate economic activity after lowering nominal short-term interest rates to zero have shifted opinion in policy circles that a proactive response to asset booms might in fact be more appropriate. But this led to a new debate over what type of intervention would be most appropriate. The two policy tools that have attracted the most attention are monetary policy and macroprudential regulation. Svensson (2017) makes the case against monetary tightening during asset price booms, arguing its costs exceed its benefits. Stein (2013) makes a case against regulatory policy on the grounds that it is likely to be circumvented in practice even if it could work in principle, arguing for monetary tightening instead.

This paper revisits the question of how policymakers should respond to asset booms through the lens of risk-shifting models, i.e., models in which the agents who ultimately finance the purchase of assets cannot easily monitor the risk associated with any individual borrower they fund. We focus on risk-shifting because it seems to play an important role in booms. First, asset booms that end badly often feature extensive lending against assets, and so assets are disproportionately purchased with borrowed funds. Second, booms are often associated with new and imperfectly understood technologies or assets like housing that are valued idiosyncractically, features that make it difficult for lenders to assess the risks on any given loan. With new technologies, lenders may find it hard to distinguishing genuinely productive applications of new technologies (dot-com, blockchain) from speculative investments that may not pan out. With housing, lenders may not be able to distinguish illiquid borrowers who value home ownership from speculators betting on house prices who would walk away from their investments if house prices fell. In both cases, lenders will have a hard time discerning how likely the agent they lend to is to default if asset prices fall.[1]

The notion of risk-shifting has spawned a large literature since it was first introduced by Jensen and

[1] While we describe situations in which limited information about what borrowers do is due to exogenous factors, Asriyan, Laeven, and Martin (2018) argue that information about assets might also deteriorate endogenously during booms if lenders have less incentive to invest in screening borrowers at those times.

Meckling (1976). Much of this literature focuses on how leverage affects investment decisions. A smaller and more recent literature has examined how these investment decisions in turn affect the prices of the assets that leveraged agents buy.[2] Allen and Gorton (1993) first showed that risk-shifting can lead to situations in which asset prices can exceed the fundamental value of the dividends assets can generate. This suggests these models may be relevant for analyzing asset booms. Subsequent work by Allen and Gale (2000), Barlevy (2014), Dow and Han (2015), and Dubecq, Mojon, and Ragot (2015) has further explored asset pricing with risk-shifting. This paper contributes to this literature in two ways. First, we explore risk-shifting in a general equilibrium setting that allows us to analyze macroeconomic policy, including both monetary policy and macroprudential regulation, in a way previous work has yet to do. Second, we explicitly account for default costs that previous work has ignored. Although we model these as the physical cost of recovering resources from borrowers, these costs can be viewed as a stand-in for how asset price collapses can lead to a contraction in output above and beyond anything that directly transpires in asset markets. The presence of such costs has important implications for policy.

Our analysis yields several new insights. The first is that even though risk-shifting can result in bubbles, such bubbles are a symptom of risk-shifting rather than the problem in and of itself. In particular, when default costs are large, assets will not exhibit bubbles and yet risk-shifting can remain a problem. Intuitively, large default costs discourage lending, which helps keep asset prices in check and equal to fundamentals. But they don't discourage lending against risky assets altogether, and whatever lending remains is distortionary. This provides a rigorous basis for the assertion by Borio and Lowe (2002) that policymakers should intervene during booms even if they cannot be sure that assets are overvalued, since whether assets are overvalued in our model is indeed irrelevant if policymakers have evidence of risk-shifting.

The other key insight is that since risk-shifting models require productive uses of credit to cross-subsidize the lending that finances speculation, policy interventions that affect both activities can have ambiguous and even surprising effects. In particular, we find that tighter monetary policy helps alleviate excessive leverage associated with risk-shifting, but that it can also exacerbate resource misallocation by shifting resources from more productive uses to speculation. This reveals a counterproductive aspect of tighter monetary policy. Nevertheless, tighter monetary policy can play a useful role if the first effect dominates, and there may be ways to mitigate some of the counterproductive aspects of tighter monetary policy if instead of tightening immediately, policymakers promise to tighten only if the boom continues into the future (and, by implication, to ease if the price of the asset collapses). We similarly find that macroprudential regulation can be counterproductive, increasing speculation rather than curbing it. This is because restricting leverage might especially curb productive investments, leaving more resources for speculation. Macroprudential regulation may well play a useful role, but we show that it can also make things worse. The essence of our findings is that while risk-shifting offers a justification for intervening during an asset boom, it also reveals the difficulty of intervening given the informational frictions that give rise to risk-shifting in the first place.

[2] Shleifer and Vishny (1997) also study asset pricing with credit and limited information, but without involving risk-shifting. Asset prices in their model deviate from fundamentals because of noise traders and limits to arbitrage. They argue lender uncertainty determines how much they lend against assets and so how much asset prices deviate from fundamentals.

Even if it were desirable to reduce speculation and lending against risky assets, policymakers will find this hard to do if they cannot identify how resources are used. Indeed, one of the reasons a promise to act in the future is more useful in our framework than direct intervention is that it affects speculators and other borrowers differently and can therefore more effectively target speculation.

The paper is organized as follows. Section 1 introduces the basic setup, focusing on a simple case where asset are riskless. We then build on this framework in Section 2 and consider the case of risky assets. There, we argue that the equilibrium of our model can capture some of the key features of asset booms. Section 3 describes the ways in which the equilibrium of our model is inefficient and allow a possible role for policy intervention. Section 4 considers monetary policy, and Section 5 considers macroprudential regulation, specifically restrictions on leverage. Section 6 concludes.

1 Credit, Production, and Assets

We now set up our framework that includes credit, production, and assets. In this section we will consider the simple case where the asset involves no risk. In the next section we build on this setup and consider the more interesting case where assets are risky. It is in the latter case where credit and asset booms emerge.

Consider an overlapping generations economy where agents live for two periods and value consumption only in their second period of life. Specifically, agents born at date t value consumption c_t and c_{t+1} at dates t and $t+1$ according to

$$u\left(c_t, c_{t+1}\right) = c_{t+1} \tag{1}$$

There is a fixed supply of identical assets normalized to one. For now, we assume these are endowed to agents at date 0, although later on we will consider the case where assets are produced. Each asset yields a constant real dividend $d > 0$ per period. In the next section we will allow for a stochastic dividend.

There is a cohort of old agents at date 0 who start out owning all the assets. A new cohort is born at each $t = 0, 1, 2, ...$ Each cohort consist of two types of agents. The first, whom we call savers, are endowed with an aggregate e units of the good when young. They cannot produce or store goods, and must either buy assets or trade intertemporally to convert their endowment into consumption when old. The second type, whom we call entrepreneurs, can convert a good at date t into $1 + y$ goods at date $t + 1$ where $y > 0$, but only up to a finite capacity of one unit of input. Each entrepreneur is endowed with $w < 1$ goods while young. Since this is below their productive capacity, there is scope for savers and entrepreneurs to trade.

In principle, w and y can vary across entrepreneurs. For most of the analysis, though, we assume $w = 0$ for all entrepreneurs, meaning they must borrow all of their inputs. We will allow w to vary across entrepreneurs in Section 5. By contrast, we do allow y to vary across entrepreneurs. Let $n\left(y\right)$ denote the density of entrepreneurs with productivity y. We assume $n(y) > 0$ for all $y \in [0, \infty)$ and

$$e < \int_0^\infty n(y)\, dy < \infty \tag{2}$$

Condition (2) implies entrepreneurs could deploy more inputs than savers are endowed with.

We assume trade between savers and entrepreneurs is subject to the following frictions:

1. Savers cannot monitor whether those they finance produce or buy assets. They also cannot observe any of the agent's wealth beyond the particular project the lender finances.

2. Trade is restricted to debt contracts, i.e., for each unit of funding agents receive at date t they must promise to pay a fixed amount $1 + R_t$ at date $t + 1$

3. If borrowers fail to pay their obligation, lenders can get a court to transfer any proceeds from the project agents invested in, but the seizure wastes Φ resources per unit invested in the project and incurs a utility cost ϕ to the borrower who defaulted.

The first friction captures the idea that asset booms are often associated with uses that lenders cannot easily evaluate. For example, mortgage lenders may not be able to distinguish liquidity constrained agents who value homeownership and speculators who would default on their loans if prices fell. The return $1 + y$ of entrepreneurs in our model would be analogous to the surplus to the former from homeownership.[3] Assuming wealth is unobservable implies borrowers face limited liability, since it means lenders can only go after the proceeds from the project the borrower invests in. The restriction to debt contracts accords with the empirical popularity of credit, which in practice is presumably due to the costs of enforcing contingent payments. Finally, although costly default is uncontroversial, the way we model these costs merits some discussion. The reason we impose a utility cost ϕ on borrowers is to ensure that they will not borrow if they expect to default with certainty. As such, we will consider the limiting case where $\phi \to 0$ to avoid explicitly accounting for ϕ. Assuming that seizure costs are proportional to the underlying investment captures the idea that auditing a borrower requires tracking what happened to all of the inputs the borrower invested. When we later allow for $w > 0$ so that borrowers may only borrow a fraction of the resources they invest, this will mean seizure is more costly for large projects even when they borrow relatively little.

To recap, each period young savers choose whether to use their endowment e to buy assets from the old who own them or to lend, preferably to entrepreneurs. When the dividends associated with the asset are deterministic, as we have assumed so far, no agent who borrows from savers will default. In that case, the inability to monitor borrowers, the restriction to debt contracts, and the cost of default are irrelevant. It is only when we assume stochastic dividends in the next section that these frictions will play a role.

[3] Of course, the surplus agents who value homeownership obtain are not a constant but depend on the price of housing. For an example of a proper risk-shifting model of housing, see Barlevy and Fisher (2018).

An equilibrium for this economy consists of paths for asset prices $\{p_t\}_{t=0}^{\infty}$ and interest rates on loans $\{R_t\}_{t=0}^{\infty}$ that ensure both asset and credit markets clear when agents act optimally. To facilitate our exposition, we will proceed as if these paths are deterministic. In Appendix A we show there are no equilibria with stochastic prices. To solve for an equilibrium, we need to derive supply and demand for assets and credit. Agents in their last period of life do not participate in credit markets and so do not enter into either supply or demand for credit. They are the source of supply in asset markets, however, selling any assets they own if the asset price $p_t > 0$. Young savers are the only agents with resources to lend. They will compare the return to lending with the return to the asset and invest only in whatever offers the highest return. Young entrepreneurs are the only ones who can produce, and so the only ones who will borrow for that purpose. Specifically, those with productivity $y > R_t$ will borrow to produce, regardless of whether they also borrow to buy assets. Finally, any young agent can borrow to buy assets. They would do so as long as the return to assets $1 + r_t \equiv \frac{d+p_{t+1}}{p_t}$ is at least as high as the interest rate on loans $1 + R_t$.

Savers use their endowment to either buy assets or make loans. Their borrowers will then either produce or buy assets. Hence, the endowment is ultimately used to either finance production or buy assets, implying

$$\int_{R_t}^{\infty} n(y)\, dy + p_t = e \tag{3}$$

Since we assume $n(y) > 0$ for all y, there is a unique market-clearing interest rate $R_t = \rho(p_t)$ for any asset price p_t. Moreover, $\rho(p_t)$ is increasing in p_t. Intuitively, a higher p_t reduces the amount of goods available for productive investment, so the interest rate on loans R_t must rise to lower demand from entrepreneurs.

Next, we argue the interest rate on loans $1+R_t$ must equal the return to buying the asset $1+r_t \equiv \frac{d+p_{t+1}}{p_t}$. For suppose $R_t < r_t$. Then agents could earn positive profits by borrowing and buying assets. Demand for borrowing would be infinite while the supply of credit can be at most e, so this cannot be an equilibrium. Next, suppose $R_t > r_t$. Then no one would buy the asset: Savers can earn a higher return from lending than buying the asset, and no agent would borrow to buy the asset knowing she would default. Since the old sell the asset whenever its price is positive, this would require $p_t \leq 0$. But if the price were nonpositive, demand for the asset would be infinite. For both the credit and asset market to clear, then, we must have

$$1 + R_t = \frac{d + p_{t+1}}{p_t} = 1 + r_t \tag{4}$$

Substituting in (3) into (4) implies

$$\begin{aligned} p_{t+1} &= (1 + \rho(p_t))\, p_t - d \\ &\equiv \psi(p_t) \end{aligned} \tag{5}$$

where $\psi'(p_t) > 0$, $\psi(0) = -d < 0$, and $\lim_{p\to\infty} \psi(p) > e$. The graph of $\psi(p)$ is illustrated in Figure 1 together with the 45$^{\mathrm{o}}$ line. The two lines intersect at the unique value p^d at which $p^d = \psi(p^d)$. For any initial condition, the law of motion $p_{t+1} = \psi(p_t)$ defines a unique path of asset prices. For any initial condition other than $p_0 = p^d$, the path will reach in finite time a value that is either negative or exceeds e, neither of which can be an equilibrium. Hence, the unique deterministic equilibrium is $p_t = p^d$ and

$R_t = \rho\left(p^d\right)$ for all t. We make note that the steady state price p^d is increasing in the dividend d. Formally, setting $p_t = p_{t+1} = p^d$ in the zero-profit condition (4), implies

$$d = \rho\left(p^d\right) p^d$$

The right hand side is increasing in p^d, so a higher d must imply a higher p^d. Graphically, a larger d will shift the curve $p_{t+1} = \psi\left(p_t\right)$ in Figure 1 down, and so the steady state p^d will rise.

In the Appendix, we confirm that there are no stochastic equilibria, so $p_t = p^d$ for all t is the unique equilibrium for this economy. We can summarize this result as follows.

Proposition 1 *The unique equilibrium when $d_t = d$ for all t features a constant asset price $p_t = p^d$ and a constant interest rate $R_t = \rho(p^d)$ for all t. Only entrepreneurs with productivity $y > R_t$ produce.*

In equilibrium, then, savers both lend to entrepreneurs and buy assets, either directly or by lending to others who pass on the proceeds from buying them back to savers. Savers are indifferent between the two activities in equilibrium, and the returns to buying assets and making loans are equal. Denote this return by $R^d = \rho\left(p^d\right)$. Consider the present value of dividends discounted at this return. This is given by

$$f_t \equiv \sum_{j=1}^{\infty}\left(\frac{1}{1+R^d}\right)^j d = d/R^d = p^d$$

The value of dividends discounted according to the return agents can earn thus coincides with the price of the asset. When $d_t = d$, the asset will not be associated with a bubble.

Remark 1: We can easily allow for multiple riskless assets. Suppose there were J assets indexed $j = 1, ..., J$, each with fixed supply of 1 but potentially different fixed dividends d_j. Let p_{jt} denote the price of the j-th asset at date t. Define $d \equiv \sum_{j=1}^{J} d_j$ as the total dividends from all J assets and $p_t \equiv \sum_{j=1}^{J} p_{jt}$ as the value of all J assets. Resources that don't finance production will be used to buy assets, so (3) continues to hold. In addition, the return on each asset $1 + r_{jt} \equiv \frac{d_j + p_{j,t+1}}{p_{jt}}$ must equal the interest rate on loans $1 + R_t$. Combining these equalities implies (4). Hence, the equilibrium conditions for p_t and R_t are unchanged. We can reinterpret p_t from our model as the total value of all assets. ■

Remark 2: With some modifications, we can also allow for a growing set of riskless assets. Suppose each period's old are endowed with a stock of new of assets normalized to 1. These assets begin to pay dividends after one period later. For aggregate dividends to remain constant, dividends on any individual asset must decay over time. Let d_{st} denote the dividend at date t on assets that arrived at date s, and assume

$$d_{st} = \begin{cases} (1-\theta)^{t-1} d & \text{if } s = 0 \\ (1-\theta)^{t-s-1} \theta d & \text{if } s = 1, 2, 3, ... \end{cases} \quad \text{for } t \geq s$$

By design, total dividends $\sum_{s=0}^{t-1} d_{st}$ in each period t sum to d. Let p_{st} denote the date-t price of the asset that arrived at date s, and set $p_t = \sum_{s=0}^{t} p_{st}$ as the total value of all assets around at date t. The market clearing condition (3) is unchanged. The return on each asset $1 + r_{st} \equiv \frac{d_{s,t+1}+p_{s,t+1}}{p_{st}}$ will equal the interest rate on loans $1 + R_t$. Aggregating over all assets available at date t yields the following alternative to (4):

$$1 + R_t = \frac{d + (p_{t+1} - p_{t+1,t+1})}{p_t}$$

The equilibrium value of all assets p_t will be constant and equal to $\frac{d}{R^d+\theta}$, where R^d denotes the equilibrium interest rate on loans. The price of any individual asset equals $p_{st} = \frac{d_{s,t+1}}{d} p_t = \frac{d_{s,t+1}}{R^d+\theta}$. ■

We conclude our discussion with a brief comment on welfare. In equilibrium, the amount savers spend to buy assets is such that the return on the asset is the same as the productivity of the marginal entrepreneur who produces in equilibrium. Is this efficient? At first, it might seem that any resources spent on the asset are wasted, since the asset will yield d regardless of how much is spent on it while lending to entrepreneurs yields additional output. However, shifting resources to production would harm the current owners of the asset, who need to sell their assets to consume. Redirecting all resources towards production is therefore not Pareto improving. In fact, the equilibrium is efficient. Intuitively, suppose current asset owners could destroy their assets so that they stop yielding any dividends going forward. Paying the old for their assets can thus be viewed as an investment to preserve the asset. Efficiency dictates the returns to all investments the young undertake should be equal at the margin, which is indeed the case in equilibrium. In the next section, we will argue that returns will not be equated at the margin when assets are risky.

2 Risky Assets, Credit Booms, and Bubbles

We now consider the case where the asset pays stochastic dividends. We return to allowing for only a single asset. Let the dividend on this asset follow a regime-switching process such that the dividend d_t starts at $D > d$ when $t = 0$ and then switches to d with a constant probability $\pi \in (0, 1)$ in each period if it has yet to switch. Once the dividend falls to d, it will remain equal to d forever.

An equilibrium still consists of paths for asset prices $\{p_t\}_{t=0}^{\infty}$ and interest rates on loans $\{R_t\}_{t=0}^{\infty}$, but it now also includes a path for the share of lending that is used to buy assets $\{\alpha_t\}_{t=0}^{\infty}$. These three paths must ensure that the share of lending used to buy assets is consistent with optimizing behavior by borrowers and lenders and that asset and credit markets clear at all dates t regardless of the value of d_t.

In what follows, it will prove convenient to distinguish for each date t whether the dividend d_t is D or d. If $d_t = D$, agents who buy the asset at date t will be unsure whether the dividends d_{t+1} they will receive at $t + 1$ will be high or low. If $d_t = d$, agents who buy the asset at date t know the dividends they will receive at $t + 1$ equal d. Let $\left(p_t^D, R_t^D, \alpha_t^D\right)$ denote equilibrium values at date t if $d_t = D$ and $\left(p_t^d, R_t^d, \alpha_t^d\right)$ denote the same equilibrium values at date t if $d_t = d$. Once dividends fall, the equilibrium will be the same as in the previous section, with $p_t^d = p^d$ and $R_t^d = R^d$ for all t. Although we did not specify the share of lending used to buy assets, recall that when $d_t = d$ those who borrow to buy assets know they will pass on all of the return from the asset to their lender, so agents are indifferent about borrowing. This implies α_t^d is indeterminate and can assume any value in $\left[0, \frac{p^d}{e}\right]$. All that remains is to solve for $\left\{\left(p_t^D, R_t^D, \alpha_t^D\right)\right\}_{t=0}^{\infty}$.

We first show that we can solve for the equilibrium price p_t^D and interest rate on loans R_t^D independently of α_t^D. It is still the case that all of the savers' endowment e must be used either to fund production or buy assets. Hence, the path of prices must continue to satisfy (3). Next, we argue that in equilibrium,

$$\left(1 + R_t^D\right) p_t^D = p_{t+1}^D + D \tag{6}$$

That is, the interest rate on loans $1 + R_t^D$ is equal to the return on the asset if $d_{t+1} = D$, which we will now show is the maximal return on the asset, i.e.,

$$p_t^D + D > p^d + d$$

For suppose $p_t^D + D \leq p^d + d$ for some t. Since $D > d$, this implies $p_t^D < p^d$. Since (3) always holds in equilibrium, the interest rate on loans R_t must equal $\rho\left(p_t\right)$. If $p_t^D < p^d$, then since $\rho'\left(\cdot\right) > 0$, we have

$$R_t^D = \rho\left(p_t^D\right) < \rho\left(p^d\right) = R^d$$

But then we would have

$$\left(1 + R_t^D\right) p_t^D < \left(1 + R^d\right) p^d = p^d + d.$$

This means that an agent who borrows to buy assets can make positive profits after paying their debt obligation at date $t+1$ if $d_{t+1} = d$. But then there would be infinite demand for borrowing to buy assets, which cannot be an equilibrium given supply of credit is finite. It follows that $p_t^D + D > p^d + d$.

To show that $\left(1 + R_t^D\right) p_t^D$ must equal $p_{t+1}^D + D$, suppose $\left(1 + R_t^D\right) p_t^D < p_{t+1}^D + D$. In this case, demand for borrowing would be infinite: Agents will earn profits if $d_{t+1} = D$ but can default if $d_{t+1} = d$. The supply of credit is finite, though, so this cannot be an equilibrium. Next, suppose $\left(1 + R_t^D\right) p_t^D > p_{t+1}^D + D$. In this case, no agent would borrow to buy the asset knowing they would default and incur the cost ϕ. Nor would any agent buy the asset, since an agent would be better off making loans than buying the asset. In particular, since no agent borrows to buy assets, the only agents who borrow are entrepreneurs with productivity $y > R_t^D$, and they will repay for sure. The return to lending R_t would then exceed the return to buying the asset. If no agent buys the asset, the price of the asset p_t^D would have to be nonpositive to ensure the old don't want to sell the asset. But this cannot be an equilibrium price, since if $p_t^D \leq 0$ there would be infinite demand for the asset. The only remaining possibility is $\left(1 + R_t^D\right) p_t^D = p_t^D + D$.

Condition (6) is identical to the condition for an asset that offers a constant dividend $d_t = D$ for all t. From the previous section, we know there is a unique path $\left\{p_t^D, R_t^D\right\}_{t=0}^{\infty}$ that satisfies both this condition and (3). The equilibrium price p_t^D is constant and equal to p^D, where p^D solves

$$\rho\left(p^D\right) p^D = D$$

Likewise, the interest rate on loans R_t^D is constant and equal to $R^D = \rho\left(p^D\right)$. The fact that the asset trades as if it delivers $d_t = D$ forever, even though the dividend can fall with a probability π that can be arbitrarily close to 1, suggests the asset is overvalued. We return to this point below.

The only part of the equilibrium we still need to solve for is α_t^D. To do this, let us first derive the expected return to buying an asset $\overline{r}_t^D$ and the expected return to lending $\overline{R}_t^D$. The expected return to buying the asset depends on values we have already solved:

$$1+\overline{r}_t^D = (1-\pi)\left(1+\frac{D}{p^D}\right)+\pi\left(\frac{d+p^d}{p^D}\right) \equiv 1+\overline{r}^D \tag{7}$$

As for the expected return to lending, recall that a fraction α_t^D of lending is used to buy assets. Since all of the proceeds from asset purchases accrue to the lender, the expected return to these loans is just the expected return to buying an asset net of default costs, $1+\overline{r}^D-\pi\Phi$. All remaining loans are used to finance production and will be repaid in full, so the return on those loans is $1+R^D$. This implies

$$\begin{aligned} 1+\bar{R}_t^D &= \left(1-\alpha_t^D\right)\left(1+R^D\right)+\alpha_t^D\left(1+\overline{r}^D-\pi\Phi\right) \\ &= \left(1-\alpha_t^D\right)\left(1+\frac{D}{p^D}\right)+\alpha_t^D\left(1+\overline{r}^D-\pi\Phi\right) \end{aligned} \tag{8}$$

If $\overline{R}_t^D > \overline{r}^D$, savers would prefer lending over buying assets. The only agents who buy assets are those who borrow to do so, in which case $\alpha_t^D = \frac{p^D}{e}$. If $\overline{R}_t^D = \overline{r}^D$, savers would be indifferent between buying assets and lending. This means α_t^D can assume any value between 0 and $\frac{p^D}{e}$. Finally, if $\overline{R}_t^D < \overline{r}^D$, savers at date t would prefer buying assets over lending. No agent would borrow to buy assets, implying $\alpha_t^D = 0$. Hence, the expected return to lending $\overline{R}_t^D$ and the share of lending used to buy assets α_t^D are jointly determined: we just described how α_t^D depends on $\overline{R}_t^D$ relative to $\overline{r}^D$, while (8) implies $\overline{R}_t^D$ depends on α_t^D.

To solve for $\overline{R}_t^D$ and α_t^D, consider first the case where $\alpha^D = \frac{p^D}{e}$. This can only be an equilibrium if $\overline{R}_t^D \geq \overline{r}^D$ when $\alpha_t^D = \frac{p^D}{e}$, i.e., only if

$$\left(1-\frac{p^D}{e}\right)\frac{D}{p^D}+\frac{p^D}{e}\left(\overline{r}^D-\pi\Phi\right) \geq \overline{r}^D$$

Rearranging this equation and substituting in for $\overline{r}^D$ implies $\alpha_t^D = \frac{p^D}{e}$ is an equilibrium only if

$$\Phi \leq \frac{e-p^D}{p^D}\left(\frac{D+p^D-d-p^d}{p^D}\right) \equiv \Phi^* \tag{9}$$

Next, consider the case where $\alpha_t^D \in \left(0, \frac{p^D}{e}\right)$. This can only be an equilibrium if $\overline{R}_t^D = r^D$ when we evaluate $\overline{R}_t^D$ at the relevant α_t^D. Since $\overline{R}_t^D$ is decreasing in α_t^D, this requires that $\overline{R}_t^D < \overline{r}^D$ when $\alpha_t^D = \frac{p^D}{e}$, or

$$\Phi > \Phi^* \tag{10}$$

In this case, the equilibrium value of α_t^D is the one that equates $\overline{R}_t^D$ and $\overline{r}^D$, which implies

$$\alpha_t^D = \frac{D+p^D-d-p^d}{D+p^D-d-p^d+\Phi p^D} \tag{11}$$

Finally, there cannot be an equilibrium in which $\alpha_t^D = 0$. This would require $\overline{R}_t^D \leq \overline{r}^D$ when $\alpha_t^D = 0$. But $\alpha_t^D = 0$ implies $\overline{R}_t^D = \frac{D}{p^D} > \overline{r}^D$. Hence, the value of α_t^D is unique and is either equal to $\frac{p^D}{e}$ or some value between 0 and $\frac{p^D}{e}$, depending on the cost of default Φ. We can summarize this result as follows:

Proposition 2 *When the dividend process follows a regime-switching process, in the limit as $\phi \to 0$, the unique equilibrium is given by*

$$(p_t, R_t) = \begin{cases} \left(p^D, R^D\right) & \text{if } d_t = D \\ \left(p^d, R^d\right) & \text{if } d_t = d \end{cases}$$

The equilibrium share of loans α_t is constant when $d_t = D$ and given by

$$\alpha_t = \alpha^D = \begin{cases} \frac{p^D}{e} & \text{if } \Phi \leq \Phi^* \\ \frac{D+p^D-d-p^d}{D+p^D-d-p^d+\Phi p^D} & \text{if } \Phi > \Phi^* \end{cases}$$

When $d_t = d$, the equilibrium α_t is indeterminate and can assume any value in $\left[0, \frac{p^d}{e}\right]$.

As long as $d_t = D$, agents will borrow to buy assets. These agents will repay their debt if dividends turn out to be high and default if they turn out to be low. In the limit as $\phi \to 0$, the profits speculators earn if $d_{t+1} = D$ also tend to 0 and all the proceeds from the asset go to the lender. Although default costs Φ do not affect asset prices or interest rates on loans, they do matter for the expected return on loans $\overline{R}^D$ and the share of lending used to buy assets α^D. Intuitively, large default costs discourage lending when too many borrowers speculate. Hence, when Φ is large, savers must buy some assets directly instead of lending to others to buy them. Although the share of lending that must buy assets α^D is uniquely determined, the identity of who borrows to buy assets is indeterminate. Given agents can hide any wealth not associated with the particular project they borrow for, any agent would be willing to borrow and buy assets, including entrepreneurs who separately borrow to produce and savers who separately lend to others.

Now that we have characterized the equilibrium when assets are risky, we argue it can capture many features of the episodes documented by Borio and Lowe (2002), Jorda, Schularick, and Taylor (2015), and Mian, Sufi, and Verner (2017). Specifically, we show that our equilibrium can be associated with asset price booms and credit booms, asset bubbles, high realized returns and cheap borrowing during the boom, and an eventual collapse in asset prices that is associated with default and low consumption.

Asset Price Booms: We begin with asset price dynamics. The equilibrium price of the asset while $d_t = D$ will be the same as in an economy in which the dividend is equal to D for all t. But as we noted earlier, the price of an asset with a fixed dividend is increasing in the value of the dividend. Hence, $p^D > p^d$. Our economy therefore starts with a high asset price that collapses when dividends fall.

Historically, though, asset price booms are characterized by both high prices and rapid price growth. The environment we considered assumes dividends are fixed within each regime. This simplifies the analysis, because it allows us to solve for a single equilibrium price per regime rather than a path of prices. But if we allow dividend to rise within each regime, we can generate a rising price path during the boom. Suppose that as long as the boom persists, dividends come from an increasing sequence $\{D_t\}_{t=0}^{\infty}$ where $D_0 \geq d$. Once the regime switches, we assume dividends remain fixed at d forever.[4] Intuitively, new technologies are often

[4] Zeira (1999) previously analyzed asset pricing when dividends grow up to a stochastic date. He emphasized that asset

not profitable immediately but might still offer a promise of an eventual stream of profits. Likewise, rents are often not high even when housing markets are hot, but there is an expectation that continuing housing demand could eventually lead to scarcity and high rents. The equilibrium interest rate on loans $1 + R_t^D$ in this case will still equal the maximum return on the asset at date t. Using the fact that $R_t^D = \rho\left(p_t^D\right)$, the equilibrium price path during the boom will satisfy a sequence of difference equations

$$p_{t+1}^D = \left(1 + \rho\left(p_t^D\right)\right) p_t^D - D_{t+1}$$

If we assume D_t is constant for $t \geq T$ for some finite T, we can use backwards induction to show there is a unique equilibrium path p_t^D that rises during the boom phase. Since this extension offers little new insight, we will continue to work with a constant D_t.

Our analysis also abstracts from how an asset boom starts. If we allowed $d_t = d$ at date 0 with the possibility that d_t rises to D thereafter, agents would have an incentive to borrow and bet on a high future dividend even when $d_t = d$. But then the asset would trade at a high price even before dividends rise, and the price wouldn't surge when d_t rises. This is analogous to the observation by Diba and Grossman (1987) that asset bubbles must be present from the inception of the asset and cannot appear suddenly. But just as Martin and Ventura (2012) overcome this result by allowing for new assets each period that can be associated with bubbles, we can similarly allow for new assets each period that can be associated with temporarily high dividends. Recall our discussion of how to allow for new riskless assets in Remark 2 above. We can equally allow for new assets whose dividends follow the regime-switching process we've assumed here.[5] Even though our model as specified only features a high initial asset price that eventually falls, it can be modified to incorporate periodic asset booms with rising prices that eventually crash.

Credit Booms: Next, we show that the asset boom is associated with a boom in lending against assets. Recall that lending by savers finances both production and asset purchases. When $d_t = D$, the amount agents borrow to buy assets is unique and given by

$$\frac{\alpha^D}{1 - \alpha^D} \int_{R^D}^{\infty} n(y)\, dy \tag{12}$$

By contrast, the amount of lending against when $d_t = d$ is indeterminate given agents are indifferent about borrowing to buy assets. However, this indeterminacy is not robust to the introduction of small transaction costs for borrowing; if it were even a little costly to borrow, agents would only buy riskless assets with their own wealth when $d_t = d$. By contrast, agents will continue to borrow both to buy risky assets and to produce when transaction costs are small. Thus, if we introduce small borrowing costs and take the limit

prices can rise with dividends and then crash once dividends stop growing even if dividends themselves do not change. By contrast, we allow dividends to fall when growth stops. Zeira also did not consider risk-shifting or credit.

[5] To ensure a riskless return on assets outside of boom periods, we may need one-off changes in the dividends of existing assets at the start of a boom to ensure the same return whether a new boom begins or not. Interestingly, even if we assume these assets pay a constant dividend for the duration of the boom, the fact that the equilibrium interest rate depends on the dividend on the high-paying asset makes the return to these assets risky. But these issues are beyond the scope of this paper.

as these costs go to zero, lending against assets will disappear when $d_t = d$. The boom in asset prices when $d_t = D$ would then be associated with a boom in borrowing to buy assets.

If we cannot distinguish lending for speculation and lending for production, a more relevant measure for credit is total borrowing. Adding the amount borrowed to produce and to buy assets at date t yields

$$\frac{1}{1-\alpha_t}\int_{R_t}^{\infty} n\left(y\right)dy$$

The term $\frac{1}{1-\alpha_t}$ will be higher when $d_t = D$ when $\alpha_t = \alpha^D \in \left(0, \frac{p^D}{e}\right)$ than when $d_t = d$ and $\alpha_t = 0$. But the interest rate on loans R_t will from R^D to R^d, so the amount agents borrow to produce $\int_{R_t}^{\infty} n\left(y\right)y$ will be lower when $d_t = D$. Total lending can therefore either rise or fall when d_t falls to d. When $\Phi \leq \Phi^*$, savers lend out all of their endowment e when $d_t = D$ but strictly less than e when $d_t = d$. Only when $\Phi \gg \Phi^*$ is it possible for total borrowing to fall with dividends. The asset boom is associated with a boom in lending to speculators, and with a boom in total borrowing if default costs are not too large.

Asset Bubbles: We now argue that during the boom assets can trade at a price above their fundamental value, i.e., the present discounted value of the dividends the asset will generate. Although in practice it is difficult to measure the fundamental value of an asset, asset booms are often suspected to be associated with bubbles given how fast asset prices rise without any reasonable changes in the expected flow of dividends to justify this increase. In the model, of course, we can exactly compute the fundamental value of an asset and determine if an asset boom is associated with a bubble.

Let us define the fundamental value of the asset in our model. For this, it will help to distinguish among the several rates or return when $d_t = D$. The first is the interest rate on loans R^D that borrowers are asked to repay. Recall this rate is the maximal possible return on the asset, i.e.,

$$1 + R^D = 1 + \frac{D}{p^D} \tag{13}$$

During the boom, lenders will not expect to always collect this interest in full, since a fraction $\alpha^D > 0$ of lending is used to finance asset purchases that may prove unprofitable. Lenders instead expect to earn a return of $\overline{R}^D$ given by

$$1 + \overline{R}^D = \left(1 - \alpha^D\pi\right)\left(1 + \frac{D}{p^D}\right) + \alpha^D\pi\left(\frac{d + p^d}{p^D} - \Phi\right) \tag{14}$$

Finally, the expected return to buying the asset in equilibrium is given by

$$1 + r^D = \frac{\left(1-\pi\right)\left(D + p^D\right) + \pi\left(d + p^d\right)}{p^D} \tag{15}$$

These three returns can be ranked, with $R^D > \overline{R}^D \geq \overline{r}^D$. To derive the last inequality, note that if the expected return to buying the asset $1 + \overline{r}^D$ exceeded $1 + \overline{R}^D$, no agent would agree to lend given they can buy the asset. But at any finite interest rate, a positive mass of entrepreneurs will borrow to produce.

We now need to take a stand on the rate at which to discount dividends when we define the fundamental value of an asset. Given the information frictions in our economy, the relevant discount rate should arguably be the rate any agent with wealth would expect to earn in the absence of any information about who they might be lending to. The expected return such agents earn in our economy is the expected return savers anticipate. Since savers always lend in our economy, this expected return is equal to $\overline{R}^D$. Using the fact that the equilibrium is stationary, we can define the fundamental value of the asset f^D recursively as

$$f^D = \frac{\pi\left(d + p^d\right) + (1-\pi)\left(D + f^D\right)}{1 + \bar{R}^D} \tag{16}$$

Equation (16) incorporates $1 + \overline{R}^D$ as the discount rate and uses the fact there $p^d = f^d$, since recall from the previous section that with constant dividends the price p^d coincides with the fundamental value of the asset d/R^d. Rearranging (16) implies

$$1 + \overline{R}^D = \frac{\pi\left(d + p^d\right) + (1-\pi)\left(D + f^D\right)}{f^D} \tag{17}$$

Comparing (17) with (15) shows that $p^D > f^D$ whenever $\overline{R}^D > \overline{r}^D$ and $p^D = f^D$ whenever $\overline{R}^D = \overline{r}^D$. Proposition 2 above implies that when $\Phi < \Phi^*$, the expected return on loans $\overline{R}^D$ exceeds the expected return on the asset $\overline{r}^D$. In this case the asset will exhibit a bubble. But when $\Phi \geq \Phi^*$, the expected return on loans $\overline{R}^D$ will equal the expected return on the asset $\overline{r}^D$. In this case the price of the asset coincides with fundamentals. As we summarize in the next proposition, whether a bubble exists depends on Φ:

Proposition 3 *Let f^D denote the value of dividends discounted at the expected return on loans $\bar{R}_t$. Then the difference between the price of the asset and its fundamental value $b^D = p^D - f^D$ is*

$$b^D = \left(\pi\left(d + p^d\right) + (1-\pi) D\right)\left[\frac{1}{\pi + \bar{r}^D} - \frac{1}{\pi + \bar{R}^D}\right] \tag{18}$$

Hence, there exists a bubble when $\Phi < \Phi^*$ but not when $\Phi \geq \Phi^*$.

Bubbles can arise because leveraged agents who have the option to default only care about the upside potential of the asset, and are willing to pay above its expected value to buy it. When Φ is large, savers will be reluctant to lend if there is too much speculation. This requires that savers buy some of the asset directly. But since savers value an asset at its fundamentals, the asset cannot be a bubble in this case.

Although bubbles are only possible when $\Phi < \Phi^*$, in the model spending on assets remains excessive regardless of Φ. To see why, recall that the productivity of the marginal entrepreneur during the boom is R^D. Since $R^D > \overline{R}^D$, the productivity of the marginal entrepreneur exceeds what society expects to earn from an asset regardless of whether $\overline{R}^D > \overline{r}^D$ or $\overline{R}^D = \overline{r}^D$. In both cases, the return to production exceeds the return to an asset. Hence, bubbles are not the problem in our model; risk shifting is. This suggests that the relevant question for policymakers thinking about intervention is not whether asset prices exceed fundamentals, but whether there is evidence of risk shifting. We return to this point in the next section.

Realized Returns and Interest Rates: We next consider rates of return during the boom. Since $R^D > R^d$, the high dividend regime will be associated with a higher realized return on investment, both for those who buy assets and for those who lend.[6] A boom will appear to be a good time for savers.

However, there are two important caveats to this result. First, even if *realized* returns are higher, *expected* returns may be lower. The expected return to lending during the boom is $\overline{R}^D$ and after the boom is R^d. The expected return $\overline{R}^D$ defined in (14) is a weighted average of $\frac{D+p^D}{p^D}$ and $\frac{d+p^d}{p^D}$ minus expected default costs. Since $D/p^D = R^D > R^d$ and $R^d = d/p^d > d/p^D$, we have

$$1 + \frac{D}{p^D} > 1 + R^d > \frac{d + p^d}{p^D}$$

If the weighted average of $1 + \frac{D}{p^D}$ and $\frac{d+p^d}{p^D}$ gives enough weight to the latter, the expected return to lending will be below $1 + R^d$ even before accounting for default costs. Asset booms can therefore be times of low expected returns even though realized returns while the boom continues are high.

Second, notwithstanding the high returns agents earn during the boom, there is a sense in which interest rates are too low during the boom. Consider an economy in which lenders could monitor borrowers. Lenders would charge borrowers different rates depending on whether they produce or buy assets. Those who borrow to buy assets would be charged an interest rate at least as high as the maximum return on the asset, $1+D/\widehat{p}^D$, where a hat denotes the asset price in the counterfactual economy with monitoring. If $\Phi > 0$, the only possible equilibrium with monitoring would involve no borrowing to buy the asset while $d_t = D$. Savers would instead have to buy the asset directly from the old. But they must also make loans to entrepreneurs. Hence, the expected return on the asset with monitoring, $(1 - \pi)\left(1 + D/\widehat{p}^D\right) + \pi\left(d + p^d\right)/\widehat{p}^D$, must be the same as the interest rate on loans to those who produce and repay in full. We can show that this implies $\widehat{p}^D < p^D$. Intuitively, savers will lend more to entrepreneurs to produce when they can monitor borrowers, meaning fewer resources will be spent to buy the asset. This implies

$$R^D = \frac{D}{p^D} < \frac{D}{\widehat{p}^D}$$

The interest rate when lenders cannot monitor borrowers will be lower than the interest rate charged to those who buy assets with perfect monitoring. Lending against risky assets thus carries too low of an interest rate. Essentially, imperfect monitoring forces entrepreneurs to cross-subsidize speculators so the latter face interest rates that don't reflect the risk of the assets they buy.

Fallout from the Crash: Finally, we turn to how asset booms end in our model. Recall that a boom ends when dividends fall. When that happens, agents who previously borrowed to buy assets will be forced to default and will impose costs Φp^D on their lenders. The collapse in asset prices thus triggers a fall in

[6] The high returns earned by those who buy assets in our model can be attributed entirely to higher dividends. In practice, the high returns during the booms seem to be driven primarily by asset price growth rather than higher dividends. As we noted above, we can introduce such growth by allowing for time-varying dividends.

the resources this cohort can consume above and beyond the decline in the dividend income they earn. By construction, the decline is proportional to the price of assets p^D during the boom. A larger boom thus implies a larger loss once the boom ends. In our model this is because recovery costs are larger the more resources were invested in assets. In practice, the economic losses after a crash are primarily due to disruptions from the choices of distressed agents. For example, intermediaries may be unable or unwilling to lend given the losses they suffered due to debt overhang. Likewise, households may be forced to deleverage after incurring losses, which can reduce output in the presence of various frictions. Both of these mechanisms suggest the decline in output after a crash is will depend on the magnitude of losses associated with the crash. Hoggarth, Reis, and Saporta (2002) and Reinhart and Rogoff (2009) estimate that the decline in output following a fall in asset prices can be significant, with a fall in GDP per capita that falls between 9 and 16%. Atkinson, Luttrell, and Rosenblum (2013) estimate the cumulative loss in output in the US in the recent crisis was even larger. This suggests Φ should be viewed as substantial in practice.

To recap, our model can capture several key features of asset booms. Since these episodes have tended to end in lower subsequent output growth, many have argued for intervention during the boom to mitigate the eventual fallout. In the remainder of the paper, we examine whether there is indeed a reason to intervene during the boom in our model and whether particular interventions are welfare-improving.

3 Inefficiency of Equilibria

In this section, we argue that there are two distinct senses in which the equilibrium of our model is inefficient when $d_t = D$. The first concerns resource misallocation: The marginal return to production during the boom exceeds the expected return on assets, so there are potential gains to redirecting some of the resources spent on assets to production. The second concerns excessive leverage: Agents who borrow to buy assets ignore the default costs Φp^D they impose on their lenders and take on too much debt. These inefficiencies suggest there may be scope for intervening to mitigate the distortions caused by leveraged speculators.

We begin with misallocation. When $d_t = D$, the productivity of the marginal entrepreneur is R^D. Even if $\Phi = 0$, the interest rate on loans R^D exceeds the expected return on loans $\overline{R}^D$, since agents who borrow to buy assets sometimes default. Since demand for borrowing to produce is always positive, it is also the case that $\overline{R}^D \geq \overline{r}^D$ to ensure savers are willing to lend. Hence, $R^D > \overline{r}^D$. Young agents could earn a higher return on their endowment if they used some of what they spend on assets to produce. But since agents who borrow to buy the asset ignore any losses that their lenders incur, their private gain to buying the asset is higher than its social return, and so the young end up spending these resources on assets. Once the boom ends, the return on the asset d/p^d will be the same as the productivity of the marginal entrepreneur R^d. The distortion only occurs during the boom.

Although we can make young agents collectively better off by shifting some of the resources used to buy assets to production, doing so would leave the old agents that sell assets with fewer resources to consume and thus worse off. Redirecting resources to production would therefore not constitute a Pareto improvement. A similar point was raised by Grossman and Yanagawa (1993). They also study an overlapping generations economy in which agents use resources to either produce or buy assets. While their model does not feature risk shifting, it does feature a production externality that implies the return to production is higher than the return on the asset. They too find that it is impossible to make all agents better off even though resources are misallocated. However, we now argue that the inability to improve upon an equilibrium with misallocation is not a general result; a Pareto improvement is possible if agents create assets rather than take the stock of assets as exogenously given. While an exogenous supply of assets may be a reasonable assumption for some assets, such as the work of a deceased artist, it does not seem relevant in the case of new technologies or housing. If the agents who sell assets must first create them, it will be possible to make all agents better off by shifting resources from risky asset creation to entrepreneurial activity.

Formally, suppose the old at date 0 are endowed with neither goods nor assets, but can convert goods into assets. Suppose further that assets can only be created at date 0. Let $c(q)$ denote the cost of producing the q-th asset, where $c'(q) > 0$. Given the price of an asset p_0 at date 0, the old will create assets up to the point q^* at which $c(q^*) = p_0$. The young will spend $p_0 q^*$ to buy assets, and so we need to replace p_t in equation (3) with $p_t c^{-1}(p_0)$, which we note is increasing in p_t for all t. It is easy to verify that our analysis goes through in this case. The equilibrium remains unique and has the same properties as before. Now, suppose we intervene and marginally reduce the quantity of assets produced at date 0. Since the old earn zero profits on the last asset produced, this intervention will leave them no worse off. As for cohorts born at dates $t \geq 0$, they will be able to redirect the p_t resources they would have spent on the last asset produced and use it for production instead. If $d_t = D$ at date t, the return from the marginal producer would be higher than the expected return on the asset. If $d_t = d$, the return from the marginal producer would be the same as the return on the asset. Since $\Pr(d_t = D) > 0$ for all t, reducing q^* makes all cohorts better off. As long as the resources the young spend on the asset at date 0 are not pure rents to the old, the misallocation during the boom will be associated with inefficiency.

As we noted above, the fact that the productivity of the marginal entrepreneur when $d_t = D$ exceeds the expected return on the asset is true even when $\Phi = 0$. When $\Phi > 0$, there is yet another source of inefficiency. Even if we hold the quantity of assets created at date 0 fixed, all agents could be made better off if lenders directly bought the assets their borrowers purchase and then reimbursed borrowers for whatever returns they would have earned. This approach avoids the default costs lenders incur when dividends fall, leaving more resources to distribute across agents. Essentially, there is no socially useful purpose for agents to borrow and buy risky assets when assets can be purchased outright. Yet agents do so in equilibrium because they privately benefit from speculating at the expense of their creditor. Since lenders cannot distinguish which agents are borrowing against risky assets, they cannot charge them a higher interest rate.

These two sources of inefficiency suggest there may be scope for intervention to either reduce the amount of assets created during a boom or to discourage borrowing against any assets that are created. But policy-makers would presumably face the same difficulties as private agents in distinguishing between speculation and more productive uses of assets, and cannot direct their actions to those who create or purchase risky assets. However, policymakers can still rely on blunt tools, e.g. using monetary policy to influence the equilibrium interest rate or restricting leverage more broadly for all borrowers.

The remainder of the paper studies the effects of these interventions. To do so, however, we need to relax some of the simplifying assumptions we have relied on so far. First, to capture the effect of monetary policy, we need to relax our assumption that the amount of resources e that each cohort can allocate to investment and asset purchases is fixed. While this assumption is convenient, the typical model of how monetary policy affects interest rates assumes that price rigidities allow economic activity to expand or contract when the monetary authority moves. In the next section, we drop the assumption that agents are endowed with a fixed amount of goods so that we can incorporate monetary policy into our model.

To capture the effects of leverage restrictions, we need to relax our assumption that entrepreneurs have no resources of their own. When borrowers lack all resources as we have assumed so far, there is no way to restrict leverage other than cutting off credit altogether. In Section 5 we return to assuming savers are endowed with a fixed amount of goods, but we assume entrepreneurs are endowed with goods as well. Entrepreneurs with different endowments turn out to prefer different degrees of leverage. Instead of a single credit market, then, we allow for multiple markets that vary by how much leverage a borrower can take on. We can then study the effect of leverage restrictions which shut down the markets with the highest leverage.

4 Monetary Policy

We begin with monetary policy. As we noted above, this requires us to abandon our assumption that savers are endowed with an exogenous amount of goods. Our approach follows Galí (2014), who also considers monetary policy in an overlapping generations economy with assets. We modify our benchmark model in two ways. First, we assume savers are endowed with labor rather than goods. Second, we introduce a monetary authority that moves after the producers who convert labor into goods set their prices but before they hire labor. This allows the real wage – and consequently output – to respond to monetary policy.

We leave the formal details of the analysis to Appendix B and only sketch the results here. Our assumptions on the incentive for savers to supply labor services as a function of the real wage ensure that in the absence of money, the equilibrium real wage that clears the labor market is constant over time and independent of d_t. Hence, absent any monetary interventions, the reduced-form representation of our economy would be the same as the model we have assumed up to now: Each cohort of savers has a constant budget e which it must allocate between entrepreneurial activity and purchasing assets.

Next, we introduce money. As in Galí (2014), we consider an equilibrium in which money doesn't circulate. Rather, the inflation rate adjusts to equate the real value of the nominal rate quoted by the monetary authority and the real return agents earn elsewhere, ensuring agents never want to hold money.

If producers can set their prices after the monetary authority moves, or if the nominal interest rate can be anticipated in advance, the monetary authority will be unable to affect the real wage or any other real variable, as producers will change their prices to match the real wage that would prevail in the absence of intervention. But if producers set their prices before the monetary authority moves and cannot perfectly anticipate what the monetary authority will do, they can only set their price as a markup over the wage they expect to prevail after the monetary authority moves. If the nominal interest rate proves to be higher (or lower) than expected, the real wage can be higher (or lower) than expected. Intuitively, an unanticipated move by the monetary authority can lead to a self-fulfilling fall in aggregate demand for goods. A lower demand for goods means producers don't need to hire as much labor, the real wage falls, and since agents earn less, demand for goods will indeed be lower. A surprise move by the monetary authority can thus reduce earnings e in the same way an income tax would. A contractionary policy at date 0 thus corresponds to reducing the earnings e_0 at date 0 below the level e that would have prevailed in the absence of intervention. We can deduce the implications of this intervention on asset and credit markets by looking that the comparative statics of changing e_0 in our model where agents are endowed with goods. The next proposition, based on results reported in Appendix B, summarizes these effects.

Proposition 4 *An unanticipated monetary intervention at date* 0 *that reduces earnings* e_0 *below the earnings* e *that would have prevailed absent any intervention leads to a lower asset price* p_0^D *and a higher real interest rate on loans* R_0^D *than would have prevailed absent any intervention.*

Given this result, we can discuss the welfare implications of a contractionary monetary intervention at date 0. Since prices are set at the beginning of each period, an intervention at date 0 will have no impact on any cohorts born at dates $t = 1, 2, 3, ...$ Agents born at date 0 can consume

$$\left[(1-\pi)\left(D+p_1^D\right)+\pi\left(d+p_1^d\right)\right]-\pi\Phi p_0^D+\int_{R_0^D}^{\infty}(1+y)\,n(y)\,dy \tag{19}$$

The first term in (19) represents the expected resources paid out by the asset. These are unaffected by anything the monetary authority does at date 0. The next term represents expected default costs. A contractionary policy at date 0 drives down the price p_0^D and lowers the expected costs of default. The final term represents the proceeds from production by entrepreneurs in this cohort. Since tighter monetary policy increases R_0^D, fewer entrepreneurs produce. A contractionary monetary policy thus mitigates excessive borrowing against assets but exacerbates the problem of too little entrepreneurial activity. The impact of the intervention on this cohort is ambiguous, although for sufficiently large Φ the first effect will dominate and this cohort will be better off. Finally, the cohort born at date 0 will be worse off given they earn p_0^D which is lower than they would have earned otherwise. However, since the effect of policy on p_0^D and R_0^D is independent of Φ, then when Φ is sufficiently large it should be possible for the cohort at date 0 to leave the old at date 0 whole and still be better off on account of the lower default costs. Hence, although contractionary monetary policy has generally ambiguous effects on welfare, it can potentially lead to a Pareto improvement. This result is reminiscent of Svensson (2017), who argues tighter monetary policy is generally costly but can lower the odds of a financial crisis. In our framework, the probability the boom ends is fixed at π, but tighter monetary policy mitigates the severity of the output decline if the boom ends.

Given that a contractionary intervention mitigates excessive borrowing during the boom but exacerbates the underprovision of production, in Appendix B we consider an alternative that avoids this tradeoff. Suppose the monetary authority does nothing at date 0 but promises to be contractionary at date 1 if the boom continues. Since producers set prices anticipating the average outcome at date 1, this means the monetary authority will be expansionary at date 1 if the asset price collapses at date 1. This is equivalent to promising a temporarily high endowment $e_1^d > e$ at date 1 if $d_1 = d$ and a temporarily low endowment $e_1^D < e$ if $d_1 = D$. Per Proposition 4, the contractionary policy at date 1 will depress p_1^D and increase R_1^D. However, as the next result states, this will reduce both p_0^D and R_0^D at date 0.

Proposition 5 *A commitment by the monetary authority at date* 0 *to set* $e_1^d > e > e_1^D$ *leads to a lower asset price* p_0^D *and a lower interest rate on loans* R_0^D *at date* 0 *than would have prevailed absent any intervention.*

A promise to tighten if a boom continues (and consequently ease if the boom ends) mitigates both excessive leverage and insufficient entrepreneurial activity at date 0, in contrast to tightening immediately. Not surprisingly, this policy can raise welfare under more general circumstances than immediate tightening. Formally, cohorts born after the intervention, so at $t = 2, 3, ...$, are unaffected, The cohort born at date 1 will be better off if $d_1 = d$; although they work more than without the intervention, the monopoly power we need to allow for price setting implies employment absent intervention is too low, and so higher employment raises welfare. Whether the cohort born at date 1 will also be better off if $d_1 = D$ is ambiguous, just as a direct intervention at date 0 was ambiguous: This cohort will fund less entrepreneurial activity given R_1^D is higher but will incur smaller default costs Φp_1^D. Even if Φ is small so default isn't very costly, as long as the probability π that dividends fall is close to 1, this cohort will be better off *ex ante*. The cohort born at date $t = 0$ will be strictly better off, since both expected default costs Φp_0^D are lower and more entrepreneurial activity is financed when R_0^D is lower. Finally, the old at date 0 will be worse off given the amount they earn from the assets they sell p_0^D will be lower. But the young at date 0 would be better off even if they had to fully compensate the old when $\Phi > 0$. Hence, this intervention can be Pareto improving when Φ is small and tightening at date 0 would not be Pareto improving.

The advantage of a commitment to tighten in the future is that it discourages speculation by punishing those who buy risky assets but not those who produce. Lenders would do the same if we allowed them to write contingent financial contracts. We ruled this out on the grounds that enforcing contingent contracting is too costly. But contingent intervention does not require enforcement and may serve as a substitute.

5 Macroprudential Regulation

We now turn to interventions that involve regulation of credit relationships. As we already anticipated, this will require us to relax our assumption that entrepreneurs are endowed with nothing. When entrepreneurs lack all resources, any arbitrarily small down-payment requirement would shut down all credit. This would eliminate speculation, but it would also end all trade between savers and entrepreneurs regardless of how productive the latter might be. To analyze leverage restrictions more realistically, we need borrowers to be able to produce and speculate even when leverage is restricted. We therefore allow entrepreneurs who are endowed with some resources. But agents with different endowments may have different preferences about the leverage they take on. This suggests allowing for multiple markets that vary by leverage.

Since our analysis no longer involves monetary policy, we return to assuming agents are endowed with goods rather than the inputs needed to produce goods. As before, each cohort consists of savers endowed with e goods who want to save them and entrepreneurs who can convert goods today into goods tomorrow. Before, we assumed all entrepreneurs were endowed with $w = 0$ but vary in productivity y. Ideally, we would generalize our analysis to any distribution $n(w, y)$ over $[0, 1) \times [0, \infty)$. For tractability, though, we are forced to restrict the distributions we consider, much as we imposed $n(w, y) = 0$ for $w \neq 0$ above. We discuss what can happen with more general distributions $n(w, y)$ at the end of this section.

First, we assume that for each $w \in [0, 1]$ the marginal density for w, or $\int_{y=0}^{\infty} n(w, y)\, dy$, is constant and equal to $2\varphi e$ where φ is some constant between 0 and 1 and e is the endowment of savers. The total endowment of all entrepreneurs is then

$$\int_0^1 w\,(2\varphi e)\, dw = \varphi e$$

The combined endowment of savers and entrepreneurs in each cohort is $(1 + \varphi)\, e$. The resources entrepreneurs need to all produce at full capacity is

$$\int_0^1 (1 - w)\,(2\varphi e)\, dw = \varphi e$$

Since $\varphi < 1$, entrepreneurs require fewer resources than savers have, in contrast to what we assumed in (2).

Second, we assume a particular lower bound y^* on the productivity of entrepreneurs, meaning $n(w, y) = 0$ for $y < y^*$. Specifically, we assume y^* exceeds the maximum possible return on the asset in equilibrium. To see that the maximum return on the asset is finite, observe that the asset price p_t at any date t is at least $(1 - \varphi)\, e$, the amount of available resources if all entrepreneurs produced at capacity, and at most $(1 + \varphi)\, e$, the amount of resources each cohort has to spend on the asset. The maximum possible return on the asset therefore occurs when the dividend $d_{t+1} = D$, the price of the asset at date t assumes its lowest value $(1 - \varphi)\, e$, and the price at $t + 1$ assumes its maximum value $(1 + \varphi)\, e$. Hence, all entrepreneurs will be more productive than the asset if

$$1 + y^* > \frac{D + (1 + \varphi)\, e - (1 - \varphi)\, e}{(1 - \varphi)\, e} = \frac{D + 2\varphi e}{(1 - \varphi)\, e} \tag{20}$$

Again, we will assume $n(w, y) > 0$ for all $y \geq y^*$ to ensure demand for credit is always positive.

These distributional assumptions imply that all entrepreneurs will want to produce at capacity and that savers can fully fund them. By contrast, up to now our assumptions implied only an endogenously determined fraction of entrepreneurs receive funding in equilibrium. Given we now have multiple markets, solving for the endogenous fraction of entrepreneurs who borrow in each market is difficult, and assuming all entrepreneurs will be funded greatly simplifies the analysis.

Since entrepreneurs have positive wealth, they can demonstrably invest it in production to reassure lenders and hopefully obtain better terms. We therefore allow for a continuum of markets indexed by $\lambda \in [0, 1)$ where λ denotes the share of a project a borrower finances herself. An agent who borrows in market λ can borrow $\frac{1-\lambda}{\lambda}$ units for each unit of her own wealth that she invests. She can thus leverage her endowment of w to finance an investment of size $\frac{w}{\lambda}$. These arrangements require that lenders observe the resources borrowers bring to the project they finance even if they cannot observe what that investment represents. We continue to assume lenders cannot observe any resources the agent has beyond what she invests in her project. Essentially, agents can borrow via shell entities and hide their other resources. We now characterize the equilibrium with multiple credit markets. We will then consider the effect of shutting down those markets where agents have to finance only a small share of their investment.

5.1 Equilibrium with Multiple Markets

With multiple credit markets, an equilibrium will now consist of a path for interest rates $\{R_t(\lambda)\}_{t=0}^{\infty}$ for each market $\lambda \in [0, 1)$, together with a path for asset prices p_t and the amount of lending used to buy assets. Rather than the share of lending used to buy assets $\alpha_t(\lambda)$ for each λ, however, we will need to track the amount borrowed in each market both to buy assets and to produce. We can use this data to deduce $\alpha_t(\lambda)$, but is also provides additional information on the total amounts agents borrow in different markets. Let $F_t^a(\lambda)$ and $F_t^p(\lambda)$ denote the cumulative amount of resources agents borrow at date t to buy assets and to produce, respectively, in markets 0 through λ. The price p_t, interest rates $R_t(\lambda)$, and the amounts borrowed $F_t^a(\lambda)$ and $F_t^p(\lambda)$ must ensure all markets clear when agents optimize.

To determine if lenders are acting optimally, we need to know what they expect for the return to lending for each $\lambda \in [0, 1)$. For active markets with lending, the expected return $\overline{R}_t(\lambda)$ must be consistent with the quantities agents borrow to produce and to buy assets in market λ, and so can be deduced from prices, interest rates, and the amount agents borrow to buy assets and to produce. But in inactive markets with no borrowing at all, the expected return is not pinned down. Hence, an equilibrium requires specifying the expected return $\overline{R}_t(\lambda)$ in all markets, including inactive ones. Optimality requires that lenders expect the same return $\overline{R}_t$ in all active markets, i.e. $\overline{R}_t(\lambda) = \overline{R}_t$ if market λ is active, and not expect to do better lending in an inactive market, i.e., $\overline{R}_t(\lambda) \leq \overline{R}_t$ if market λ is inactive. In addition, following Gale (1996) and Guerrieri, Shimer, and Wright (2010), we restrict attention to equilibria with "reasonable" beliefs that do not hinge on arguably suboptimal behavior. Specifically, consider an inactive market λ in which the interest rate $R_t(\lambda)$ is sufficiently high that anyone who borrows to buy assets will default with certainty. Even if an agent were forced to trade only in market λ, they would never choose to borrow and buy assets. By contrast, highly productive entrepreneurs would agree to borrow if forced to trade only in market λ. It seems unreasonable for lenders to worry about default in a market where no agent would borrow to buy risky assets. We therefore assume $\overline{R}_t(\lambda) = R_t(\lambda)$ in any inactive market λ in which borrowing to buy an asset results in default. We discuss what other equilibria can arise without this restriction in Appendix C.

As in Section 1, we assume the equilibrium is deterministic and confirm in Appendix C there are no stochastic equilibria. We will follow a similar approach to solving an equilibrium as before. Back when all entrepreneurs had no wealth, we characterized the (single) equilibrium interest rate on loans R_t in (6), deduced that at this rate entrepreneurs would borrow $\int_{R_t}^{\infty} n(y)\,dy$ to produce, and then used (3) to solve for p_t as total resources net this amount. Here we also begin by characterizing the schedule of interest rates $R_t(\lambda)$, use this to infer how much entrepreneurs borrow to produce, and then solve for p_t.

As before, interest rates must ensure agents cannot expect to profit from borrowing to buy an asset. If $d_t = D$, the asset will yields its maximum return when $d_{t+1} = D$, in which case the return on the asset would be $\frac{p_{t+1}^D + D}{p_t^D}$. We need to make sure that even in this case, borrowing to buy the asset isn't profitable. If agents borrow in market λ, the relevant condition is

$$(1-\pi)\left[\frac{p_{t+1}^D + D}{p_t^D} - (1-\lambda)\left(1 + R_t^D(\lambda)\right)\right] \leq \left(1 + \overline{R}_t^D\right)\lambda \tag{21}$$

That is, the expected profits to buying an asset and defaulting if returns are low cannot exceed what the agent would expect to earn lending out the resources she spends to buy assets. If (21) were violated, no agent would agree to lend given they can always borrow in market λ and buy assets. But no lending cannot be an equilibrium given demand for credit from entrepreneurs is positive for all interest rates. Rearranging (21) yields a lower bound on $1 + R_t^D(\lambda)$ which applies whether market λ is active or not:

$$1 + R_t^D(\lambda) \geq \frac{1}{1-\lambda}\left[\frac{p_{t+1}^D + D}{p_t^D} - \frac{\lambda\left(1+\overline{R}_t^D\right)}{1-\pi}\right] \tag{22}$$

In Appendix C, we show that in equilibrium the bound in (22) is a decreasing function of λ which is equal to $\frac{p_{t+1}^D + D}{p_t^D}$ when $\lambda = 0$ and tends to $-\infty$ as λ tends to 1.[7]

Suppose $1 + R_t^D(\lambda)$ were strictly higher than the bound in (22) for some λ. Then no agent would want to borrow to buy assets in market λ. If agents still borrow to produce in this market, then market λ would be active and $\overline{R}_t^D(\lambda) = \overline{R}_t^D$. If market λ were instead inactive, our restriction on beliefs requires that $\overline{R}_t^D(\lambda) = R_t^D(\lambda)$. Since $\overline{R}_t^D(\lambda) \leq \overline{R}_t^D$ in all inactive markets, then $R_t^D(\lambda) \leq \overline{R}_t^D$. At the same time, the interest rate $R_t^D(\lambda)$ in any market λ cannot be lower than $\overline{R}_t^D$. In active markets, the interest rate must be at least $\overline{R}_t^D$ to ensure an expected return of $\overline{R}_t^D$. If $R_t^D(\lambda) < \overline{R}_t^D$ in some inactive market λ, then that market would have a lower interest rate than all active markets. But entrepreneurs with $w > \lambda$ would then have an incentive to borrow all $1 - w$ units they need to produce at capacity in market λ (or some other inactive market that offered an even lower rate than market λ). That would mean that there is an inactive market in which agents would like to trade at, so it cannot be inactive. From this, we can conclude that if $1 + R_t^D(\lambda)$ exceeds the bound in (22) in some market λ, then the interest rate $R_t^D(\lambda)$ must equal $\overline{R}_t^D$.

[7] Essentially, if the expected return on loans $\overline{R}_t^D$ were lower than expected return on the asset $\overline{r}_t^D$, there would be no lending, which cannot be an equilibrium. But if $1 + \overline{R}_t^D > (1-\pi)\frac{p_{t+1}^D + D}{p_t^D} + \pi\frac{p_{t+1}^d + d}{p_t^D}$, it must also exceed $(1-\pi)\frac{p_{t+1}^D + D}{p_t^D}$.

Given the bound in (22) is decreasing in λ and its limiting values, it follows that there exists some $\lambda_t^D \in [0,1)$ such that $1+R_t(\lambda)$ is equal to the lower bound in (22) when $\lambda < \lambda_t^D$, since at low values of λ the lower bound exceeds $1+\overline{R}_t^D$ and so the interest rate cannot exceed the lower bound and be equal to $1+\overline{R}_t^D$. When $\lambda > \lambda_t^D$, the fact that $R_t^D(\lambda) \geq \overline{R}_t^D$ for all t implies the interest rate $1+R_t^D(\lambda)$ must exceed the bound in (22), and hence equal $1+\overline{R}_t^D$. The interest rate schedule $1+R_t^D(\lambda)$ is thus

$$1+R_t^D(\lambda) = \begin{cases} \frac{1}{1-\lambda}\left[\frac{p_{t+1}^D+D}{p_t^D} - \frac{\lambda\left(1+\overline{R}_t^D\right)}{1-\pi}\right] & \text{if } \lambda > \lambda_t^D \\ 1+\overline{R}_t^D & \text{if } \lambda > \lambda_t^D \end{cases} \tag{23}$$

The interest rate schedule in (23) is illustrated in Figure 2. The interest rate in market $\lambda = 0$ is equal to the maximal return on the asset $\frac{p_{t+1}^D+D}{p_t^D}$. If a borrower commits to finance a larger fraction of the project λ, she will be charged a lower interest rate that falls continuously with λ, up to a threshold λ_t^D. Once agents pay for enough of the asset on their own, they will no longer have an incentive to default. At that point, there is no reward to a borrower for offering to finance a larger share of her project.

Given the interest rate schedule (23), we next determine how much entrepreneurs borrow to produce. From (23) we know $1+R_t^D(\lambda) \leq \frac{p_{t+1}^D+D}{p_t^D}$. Since each entrepreneur's productivity $y \geq y^*$, and $1+y^* > \frac{p_{t+1}^D+D}{p_t^D}$ per (20), entrepreneurs should borrow and produce as much as possible up to capacity in any market λ.

Consider an entrepreneur with wealth w. In market $\lambda = w$, she should optimally borrow $1-w$ and earn

$$1+y-\left(1+R_t^D(w)\right)(1-w)$$

If she borrowed in some market $\lambda < w$, she should optimally borrow $1-w$ and earn

$$1+y-\left(1+R_t^D(\lambda)\right)(1-w)$$

Since $R_t^D(\lambda)$ is weakly decreasing in λ from (23), the latter can be no better than borrowing in market $\lambda = w$. Finally, if she borrowed in some market $\lambda > w$, she could borrow at a lower interest rate but would have to produce below capacity. Here, she should optimally borrow as much as possible and earn

$$\frac{w}{\lambda}\left[1+y-\left(1+R_t^D(\lambda)\right)(1-\lambda)\right]$$

Substituting in (23), it is clear there is no point to borrowing in a market $\lambda > \lambda_t^D$ given one can borrow more resources in market λ_t^D but at the same interest rate. Borrowing in market $\lambda \in (w, \lambda_t^D)$ yields profits

$$\frac{w}{\lambda}\left[1+y-\frac{p_{t+1}^D+D}{p_t^D}+\frac{\lambda\left(1+\overline{R}_t^D\right)}{1-\pi}\right]$$

This expression is decreasing in λ. Hence, an entrepreneur with wealth w will optimally borrow $1-w$ at the same interest rate as she could obtain in market $\lambda = w$, or $R_t^D(w)$. For $\lambda < \lambda^D$, market λ is the only one with interest rate $R_t^D(\lambda)$. Hence, the only entrepreneurs who borrow in market λ for $\lambda \in [0, \lambda^D)$ are those with wealth $w = \lambda$. The cumulative amount entrepreneurs borrow to produce in equilibrium is hence

$$F_t^p(\lambda) = \int_0^\lambda (1-w)(2\varphi e)\,dw = \lambda(2-\lambda)\varphi e \tag{24}$$

Entrepreneurs with $w > \lambda_t^D$ will still borrow $1 - w$ at an interest rate of $R_t^D(w)$, but they could borrow this amount in any market $\lambda \in [\lambda_t^D, w]$. Markets $\left[0, \lambda_t^D\right]$ must be active in equilibrium, but markets $(\lambda_t^D, 1)$ need not be. Since all entrepreneurs borrow to produce up to capacity, the total they borrow in all markets in order to produce, $F_t^p(1)$, is equal to φe. The amount of resources that will then be left to spend on the asset is $(1-\varphi)e$. This uniquely pins down the asset price p_t^D for any date t. By implication, it also pins down the maximal return on the asset, since $\left(p_{t+1}^D + D\right)/p_t^D = 1 + \frac{D}{(1-\varphi)e}$.

So far, we have solved for the equilibrium price p_t^D, the amount borrowed to produce $F_t^p(\lambda)$, and the interest rates $R_t^D(\lambda)$ when $d_t = D$. However, the latter is expressed in terms of the expected return to lending in any active markets $\overline{R}_t^D$ and a cutoff λ_t^D, which we have yet to solve for. To solve for the cutoff λ_t^D, observe that by continuity, the interest rate $R_t^D(\lambda)$ in the limit as $\lambda \uparrow \lambda_t^D$ will equal $\overline{R}_t^D$, and so λ_t^D can be directly inferred from $\overline{R}_t^D$ by equating the two. This condition implies

$$1 + \overline{R}_t^D = \frac{1-\pi}{1-\pi(1-\lambda_t^D)} \frac{p_{t+1}^D + D}{p_t^D} \tag{25}$$

To compute the expected return to lending $\overline{R}_t^D$, we can add up all of the earnings lenders recoup. Since entrepreneurs with wealth w borrow $1-w$ units at an interest rate of $1 + R_t^D(w)$, the amount savers collect from entrepreneurs who produce is given by

$$\int_0^1 \left(1 + R_t^D(w)\right)(1-w)(2\varphi e)\, dw$$

Savers also collect all of the proceeds from asset purchases, since any asset is purchased with a combination of funds borrowed from a saver and funds self-financed by another saver. The expected amount savers recoup from assets is therefore $\left(1 + \overline{r}_t^D\right) p_t^D$. From this, we must net out the expected costs of default. Let γ_t^D denote the fraction of assets that are pledged against debt. Expected default costs are $\gamma_t^D \Phi p_t^D$. Substituting in for the price $p_t^D = (1-\varphi)e$, the total expected collection on loans is given by

$$\left(1 + \overline{R}_t^D\right) e = \left[1 + \overline{r}^D - \pi\gamma_t^D \Phi\right](1-\varphi)e + \int_0^1 \left(1 + R_t^D(w)\right)(1-w)(2\varphi e)\, dw \tag{26}$$

To compute $\overline{R}_t^D$, we still need the fraction of assets pledged against debt γ_t^D. Here, we use the fact that $\gamma_t^D = 1$ if the return to lending $\overline{R}_t^D > \overline{r}_t^D$, since in that case no agent would want to directly buy assets without borrowing against them, but γ_t^D can be between 0 and 1 if $\overline{R}_t^D = \overline{r}_t^D$. In the latter case, γ_t^D must ensure the two returns are equal, i.e.,

$$\left[1 + \overline{r}^D - \pi\gamma_t^D \Phi\right](1-\varphi) + \int_0^1 \left(1 + R_t^D(w)\right)(1-w)(2\varphi)\, dw = 1 + \max\left\{\overline{R}_t^D, \overline{r}_t^D\right\} \tag{27}$$

We can then use (26) and (27) to jointly solve for $\overline{R}_t^D$ and γ_t^D. This allows us to fully compute the equilibrium when $d_t = D$. Knowing $\overline{R}_t^D$, and by implication λ_t^D, fully characterizes the schedule of interest rates $R_t^D(\lambda)$, and it tells us the expected return $\overline{R}_t(\lambda)$ for all λ, which from above we know must be equal to $\overline{R}_t^D$ for all markets regardless of whether they are active or not. We only need to solve for the amounts agents borrow to buy assets, $F_t^a(\lambda)$, and we know these must ensure $\overline{R}_t(\lambda) = \overline{R}_t$ for all λ. We note that the share of lending used to buy assets in each market, $\alpha_t^D(\lambda)$, must be decreasing in λ. This is because the interest rate entrepreneurs pay is decreasing in λ, and so fewer borrowers need to buy assets to keep the expected return to lending equal to $\overline{R}_t$.

This describes the equilibrium when $d_t = D$. When $d_t = d$, there will be no default. In that case, the interest rate on loans $R_t(\lambda)$ must be the same for all λ. This interest rate is necessarily equal to the return on the asset $\frac{p_{t+1}^d+d}{p_t^d}$. Since this interest rate will be below y^*, all entrepreneurs will borrow to produce at capacity. As before, this pins down the price of the asset at $p_t^d = (1-\varphi)e$. By implication, it also pins down the return on the asset, since $\frac{p_{t+1}^d+d}{p_t^d} = 1 + \frac{d}{(1-\varphi)e}$. We summarize this as follows.

Proposition 6 *In the limit as $\phi \to 0$, the unique equilibrium given our distributional assumptions on entrepreneur wealth and productivity features a constant asset price across both regimes,*

$$p_t^D = p_t^d = (1-\varphi)e$$

The equilibrium schedule of interest rates $R_t(\lambda)$ is given by (23) when $d_t = D$ and is given by

$$R_t(\lambda) = \frac{d}{(1-\varphi)e}$$

for all $\lambda \in [0,1)$ when $d_t = d$. The expected return to lending $\bar{R}_t$ while $d_t = D$ corresponds to the constant value of $\bar{R}^D$ that together with γ^D solves (26) and (27), and is given by

$$\bar{R}_t = \frac{d}{(1-\varphi)e}$$

when $d_t = d$. The total amount agents borrow to produce, $F_t^p(1)$, equals φe for all t. However, the amount agents borrow to produce in individual markets with no default is indeterminate. This includes markets $\lambda \in [\lambda^D, 1)$ when $d_t = D$ and all markets when $d_t = d$. Borrowing to produce in markets $[0,\lambda] \subset [0,\lambda^D]$ is

$$F_t^p(\lambda) = \int_0^\lambda (1-w)(2\varphi e)dw$$

when $d_t = D$. Finally, the amount borrowed to buy assets, $F_t^a(\lambda)$, must ensure the expected return to lending equals $\bar{R}_t$ for all λ given $R_t(\lambda)$ and $F_t^p(\lambda)$.

The equilibrium when entrepreneurs have positive wealth is similar in many respects to the equilibrium we already solved for when we assumed entrepreneurs have no resources. The key difference is that entrepreneurs with wealth now help to finance their investments in order to lower their interest rates. However, even in markets when agents must finance a part of their investment, there might be agents willing to borrow to buy risky assets. Such speculators would of course need to have resources to pay their share. In contrast to the case where we assumed entrepreneurs have no resources, then, which agents speculate is now determined: Although both savers and entrepreneurs have the resources needed to pay for part of the assets they borrow to buy, only savers would be willing to spend them on assets. At the same time, in markets where the share λ that agents pay is sufficiently large, i.e., when $\lambda > \lambda^D$, there will be no incentive for agents to borrow and buy risky assets. Thus, a severe enough restriction on leverage will eliminate speculation, although at the cost of forcing poorer entrepreneurs to operate well below capacity.

Before we introduce leverage restrictions to determine their full implications, let us make one remark about welfare. Our distributional assumptions imply all entrepreneurs produce at capacity in equilibrium. Unlike our analysis so far, then, there is no sense in which in equilibrium resources spent on assets could have been better deployed in production. In other words, the is no misallocation during the boom. As long as $\Phi > 0$, though, borrowing to buy assets remains socially wasteful. Macroprudential regulation that forbids high leverage may still have scope for improving welfare by curbing such borrowing.

5.2 Leverage Restrictions

To study the effects of leverage restrictions, we next introduce a floor $\underline{\lambda}$ and shut down all markets with $\lambda < \underline{\lambda}$. This is equivalent to a cap on leverage for borrowers. Agents with wealth $w < \underline{\lambda}$ would only be able to undertake projects of size $w/\underline{\lambda} < 1$, while entrepreneurs with wealth $w \geq \underline{\lambda}$ can still operate at full capacity if they so choose. For simplicity we will consider a permanent floor, although one could equally consider a floor that is only in place while $d_t = D$, when such a floor might be useful.

We have already solved for the equilibrium when $\underline{\lambda} = 0$. When we allow for $\underline{\lambda} > 0$, optimality would still imply the schedule of interest rates in (23) when $d_t = D$, although the thresholds $\overline{R}_t^D$ and λ_t^D can depend on $\underline{\lambda}$. Since the schedule of interest rates has the same structure as when $\underline{\lambda} = 0$, entrepreneurs will still want to produce at full capacity, but entrepreneurs with $w < \underline{\lambda}$ will be unable to do so. Above, we showed that profits for an entrepreneur with wealth w decrease in λ for $\lambda > w$. This implies entrepreneurs with $w < \underline{\lambda}$ will borrow in the market with smallest possible λ, or $\underline{\lambda}$. The total inputs entrepreneurs use is then

$$\begin{aligned}
\int_{w=0}^{\underline{\lambda}} 2\varphi e \left(\frac{w}{\underline{\lambda}}\right) dw + \int_{w=\underline{\lambda}}^{1} (2\varphi e)\, dw &= \frac{\varphi e}{\underline{\lambda}} w^2 \Big/_{0}^{\underline{\lambda}} + 2\varphi e \left(1 - \underline{\lambda}\right) \\
&= \underline{\lambda}\varphi e + \left(1 - \underline{\lambda}\right) 2\varphi e
\end{aligned}$$

The amount that remains to spend on the asset is $(1 + \varphi)\, e$ minus the above, which pins down its price:

$$p_t^D = \left(1 - \varphi\left(1 - \underline{\lambda}\right)\right) e \tag{28}$$

A higher $\underline{\lambda}$ will lead to a higher asset price. Intuitively, leverage restrictions force poor entrepreneurs to operate at a smaller scale. Since savers want to save a fixed amount e regardless of $\underline{\lambda}$, the decline in production will release resources to buy assets, pushing p_t^D up. Assuming $\underline{\lambda}$ is imposed permanently, the same logic implies $p_t^d = \left(1 - \varphi\left(1 - \underline{\lambda}\right)\right) e$. The expected return on the asset when $d_t = D$ is thus

$$1 + \overline{r}_t^D = \frac{(1 - \pi)\left(D + p_{t+1}^D\right) + \pi\left(d + p_{t+1}^d\right)}{p_t^D} = 1 + \frac{(1 - \pi)\, D + \pi d}{\left(1 - \varphi\left(1 - \underline{\lambda}\right)\right) e}$$

Leverage restrictions thus reduce the expected return to buying the asset. While it is difficult to characterize the effects of increasing $\underline{\lambda}$ on the entire schedule of interest rates $R_t(\lambda)$, we show in Appendix C that the expected return to lending $\overline{R}^D$ declines with $\underline{\lambda}$. The effect of increasing $\underline{\lambda}$ can be summarized as follows.

Proposition 7 *The asset price $p_t^D = (1-\varphi(1-\underline{\lambda}))e$ is increasing in $\underline{\lambda}$, while the expected returns when $d_t = D$ from the asset $\bar{r}^D = \frac{(1-\pi)D+\pi d}{(1-\varphi(1-\underline{\lambda}))e}$ and from lending $\bar{R}^D$ are decreasing in $\underline{\lambda}$ for a permanent floor $\underline{\lambda}$.*

Note the contrast between the effects of contractionary monetary policy we discussed in the previous section and the effects of restrictions on leverage above. Both policies reduce output: monetary policy reduces what is produced today, leverage restrictions reduce the amount entrepreneurs today produce for next period. However, tighter monetary policy dampens asset prices and raises the returns to saving while leverage restrictions increases asset prices and lowers the return to saving. This suggests leverage restrictions may be counterproductive, stoking asset prices rather than dampening them. This counterproductive aspect of leverage restrictions is new as far as we know. Stein (2013) argues leverage restrictions may be limited and ineffective, but his point was that borrowers can often circumvent them, not because he argued these regulations might contribute to more speculation. The logic for our result is that in risk-shifting models leverage there must be an investment activity cross-subsidizing speculation. If this other investment is particularly sensitive to leverage restrictions, restricting leverage may redirect resources toward speculation. Thus, if leverage restrictions sharply reduced demand for housing by liquidity constrained households but not the amount of funds for mortgages, leverage restrictions would end up encouraging speculation.

Proposition 7 establishes that tighter leverage restrictions drive up asset prices. But that does not necessarily mean that total borrowing against assets must rise. Even if a higher $\underline{\lambda}$ increases p^D, it could still lower the share of assets purchased with debt γ^D. Our next result shows that when $\underline{\lambda}$ is low and all assets are purchased with debt, or when $\underline{\lambda}$ is high enough that it discourages speculation, increasing $\underline{\lambda}$ will increase p^D without changing γ^D. In these cases, raising $\underline{\lambda}$ will make agents worse off. For intermediate values, increasing $\underline{\lambda}$ at some point must reduce both the share of assets purchased with debt and expected default costs. On its own this would make society better off, but increasing $\underline{\lambda}$ also reduces what poor entrepreneurs can produce. For large Φ, however, reducing default costs would be paramount.

Proposition 8 *There exist cutoffs $0 \leq \Lambda_0 \leq \Lambda_1 < 1$ such that*

1. *If $\underline{\lambda} < \Lambda_0$, increasing $\underline{\lambda}$ leaves $\gamma^D = 1$, increases expected default costs $\pi\Phi p^D$, and leaves fewer goods for cohorts to consume from date $t = 1$ on.*

2. *If $\underline{\lambda} \geq \Lambda_1$, $\gamma^D = 0$ and there is no default. Increasing $\underline{\lambda}$ then leaves fewer goods for cohorts to consume from date $t = 1$ on.*

3. *If $\Lambda_0 < \underline{\lambda} < \Lambda_1$, increasing $\underline{\lambda}$ there exist values of $\underline{\lambda}$ at which increasing $\underline{\lambda}$ lowers γ^D and expected default costs $\pi\gamma^D\Phi p^D$. In this case, increasing $\underline{\lambda}$ while $d_t = D$ can be Pareto improving for large Φ.*

In contrast to monetary policy, a threat to tighten credit conditions in the future rather than tighten them immediately will not mitigate this counterproductive aspect. Raising $\underline{\lambda}$ next period will increase p_{t+1}^D, and regardless of how it affects γ_{t+1}^D, a higher p_{t+1}^D at date $t+1$ makes speculation at date t more attractive.

However, our finding that increasing $\underline{\lambda}$ raises asset prices contemporaneously need not carry over to other distributions $n(w, y)$ of wealth and productivity across entrepreneurs. If we allowed for entrepreneurs with productivity $y < y^*$ who are on the margin between lending and producing, an increase in $\underline{\lambda}$ could induce some of these marginal entrepreneurs to switch from lending their resources to borrowing for production. If enough entrepreneurs switch from lending to borrowing in order to produce, the fall in lending and the increase in demand for production may leave fewer resources to spend on the asset and its price will fall. We confirm numerically that there exist distributions $n(w, y)$ for which increasing $\underline{\lambda}$ reduces p_t^D.

Although increasing $\underline{\lambda}$ can sometimes lower asset prices, this can only occur when $d_t = D$. When $d_t = d$, a higher $\underline{\lambda}$ increases p_t^d regardless of the distribution $n(w, y)$. To see this, note that when $d_t = d$ there is no default. $R_t^d(\lambda)$ is then equal to a constant R_t^d for all λ. This common rate R_t^d and the asset price p_t^d satisfy two equilibrium conditions similar to (3) and (4). First, since all the resources that savers and entrepreneurs are endowed with must be used to produce or buy the asset, we have

$$\int_{R^d}^{\infty} \int_0^1 \min\left\{1, \frac{w}{\underline{\lambda}}\right\} n(w, y)\, dw\, dy + p_t^d = \int_0^{\infty} \int_0^1 wn(w, y)\, dw\, dy + e \tag{29}$$

This defines R_t^d as a function $\rho_{\underline{\lambda}}\left(p_t^d\right)$ of the price p_t^d which is increasing in p_t^d for a fixed $\underline{\lambda}$ and decreasing in $\underline{\lambda}$ for a fixed p_t^d. Second, the interest rate on loans must equal the return on the asset, and so

$$\left(1 + R_t^d\right) p_t^d = d + p_{t+1}^d \tag{30}$$

Substituting in $R_t^d = \rho_{\underline{\lambda}}\left(p_t^d\right)$ implies $p_{t+1}^d = \rho_{\underline{\lambda}}\left(p_t^d\right) - d$. Figure 3 illustrates the effect of increasing $\underline{\lambda}$ graphically. Since $\rho_{\underline{\lambda}}\left(p_t^d\right)$ is decreasing in $\underline{\lambda}$ for a fixed p_t, the curve that plots p_{t+1}^d as a function of p_t^d shifts down, and the steady state p^d increases. Intuitively, the return on the asset is negatively related to its price. In the absence of risk-shifting, the the return on the asset is also equal to the interest rate. Increasing $\underline{\lambda}$ requires the interest rate on loans to fall so that credit markets clear even when leverage restrictions rein in demand for borrowing by poor entrepreneurs. Hence, the return on loans must fall and the price of the asset must rise. By contrast, with risk-shifting, the return on the asset and the interest rate on loans are different, so it will be possible for interest rates on loans to fall but the return on the asset to rise. The fact that tighter leverage restrictions only reduce asset prices when there is risk-shifting suggests a possible way for policymakers to detect it and confirm whether their intervention was warranted.

6 Conclusion

This paper examined the effects of various policy interventions against asset booms in a model of risk shifting. We show that risk-shifting can introduce a role for policy, since speculation by agents can lead to misallocation and excessive leverage. We then use the model to see what insights it offers to guide policymakers. We find that while risk-shifting can lead to asset bubbles and overvalued assets, risk-shifting leads to the same inefficiency whether asset prices exceed fundamentals or are equal to them. This suggests that given evidence of risk shifting, there is no need for policymakers to determine if there is a bubble in asset markets. Second, we find that the interventions that have dominated the debate on how to respond during these episodes have ambiguous effects and may exacerbate the distortions policy should correct. The type of policies that seem to hold the most promise are those that are inherently more targeted against speculation rather than the investment activity that cross-subsidizes speculation and allows for risk-shifting. For example, threatening to pursue a contractionary monetary policy only if the boom continues may discourage speculators but not other investment activity. As another example, restrictions on leverage are more likely to dampen asset prices when speculation is more sensitive to these restrictions than the investment activity that cross-subsidizes it would be.

We focus on risk shifting because it seems like an important aspect of many of the historical episodes of asset booms. That said, there is a large literature on asset bubbles that seeks to explain the same episodes we do but without considering risk shifting. We emphasize that the risk-shifting mechanism we consider is compatible with these alternative explanations. One strand of the literature has focused on bubbles with rational agents. This includes models of bubbles that arise because of dynamic inefficiency, e.g. Galí (2014, 2017), and binding credit market frictions, e.g. Martin and Ventura (2012), Hirano and Yanagawa (2017), Miao and Wang (forthcoming). Both of these bubbles have been modeled in overlapping generations environments closely related to the one we study here. One can therefore potentially incorporate those elements into our framework. For example, we assume entrepreneurs have a finite capacity, while models of bubbles based on borrowing constrains assume the amount entrepreneurs produce is bounded by how much they can borrow. The two aspects can certainly be studied together. A separate literature has focused on disagreement as a source of bubbles, e.g. Scheinkman and Xiong (2003), Hong, Scheinkman, and Xiong (2006), Simsek (2013), and Barberis, Greenwood, Jin, and Shleifer (2018). But this too is compatible with risk shifting. For example, as is clear from our discussion in Section 5, in our model savers lend to otherwise identical savers who then speculate on assets. If agents differ in terms of their beliefs, it will presumably be savers who are pessimistic about the assets that end up lending to savers who are optimistic about the asset and end up buying it. It stands as an open question whether risk-shifting interacts with the other forces that can generate overvaluation in interesting ways.

Our model also suggests future directions for research both in developing risk-shifting models of asset prices and in applying them. For example, we assumed lenders suffer a cost Φ when their borrowers default and lenders must recover repayment. This cost plays a significant role for welfare, since one of the key motivations behind policy is to reduce the costs of excessive borrowing that borrowers inflict on others. In practice, the main costs associated with the collapse of asset prices do not involve recovery but various distortions that arise when asset prices fall, and which both borrower and lender may fail to internalize. To get at these would require introducing financial intermediaries and models in which household balance sheets play an important role. These may have important implications for what type of interventions are best during booms, since how interventions affect outcomes once asset prices collapse will likely matter for welfare. In terms of applications, we have described the analog between our setup and the housing market. However, the cross-subsidization in the housing market involves borrowers who also buy homes but not in order to speculate. This leaves open the question of whether an intervention that shifts resources from illiquid home buyers to speculators still drive house prices up in the same way that a redistribution in our model from entrepreneurs who do not buy assets to speculators who do drives up asset prices. Another question that our analysis leaves unanswered is whether the policy implications we deduce in our model would still hold in open economy settings. For example, we argued that a threat of contractionary monetary policy if a boom continues (and easing if the boom ends) can help dampen asset prices because speculators anticipate a lower asset price if the boom continues. But if contractionary monetary policy leads to higher real rates that attract capital inflows, it is not clear whether asset prices will still fall. Extending our framework deal with these issues seems an important step before applying it in practice.

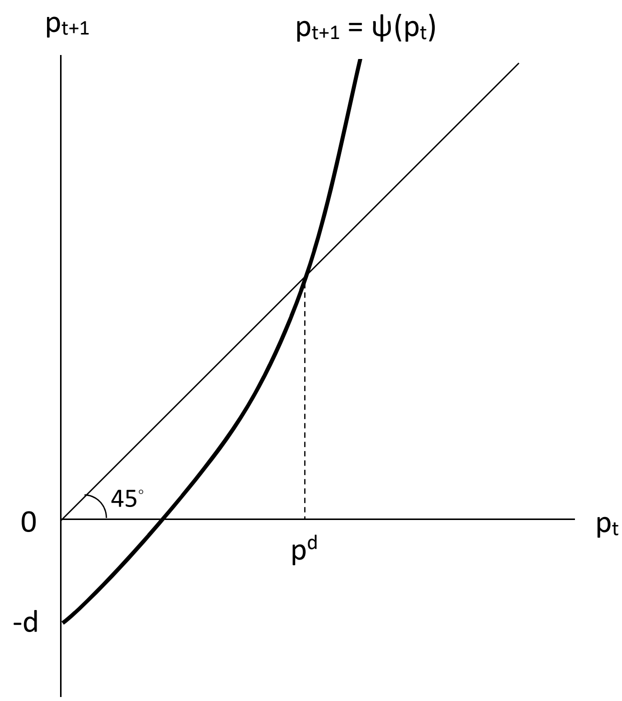

Figure 1: Determination of equilibrium price p^d with deterministic dividends

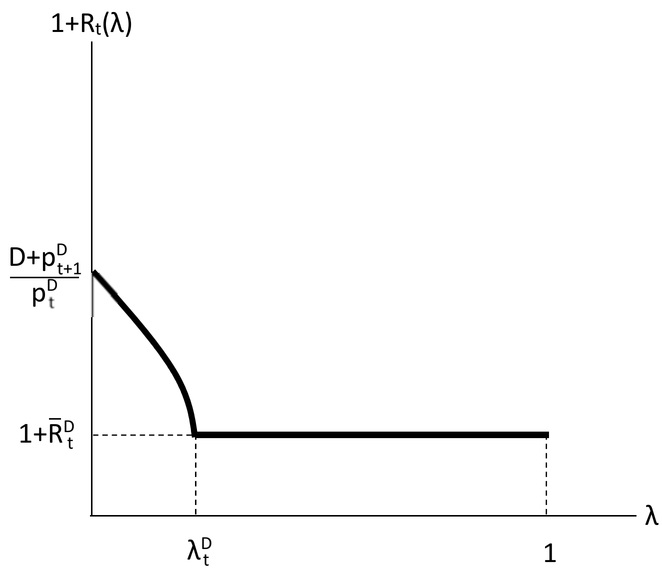

Figure 2: Interest rates as a function of share λ of investment that borrowers pay

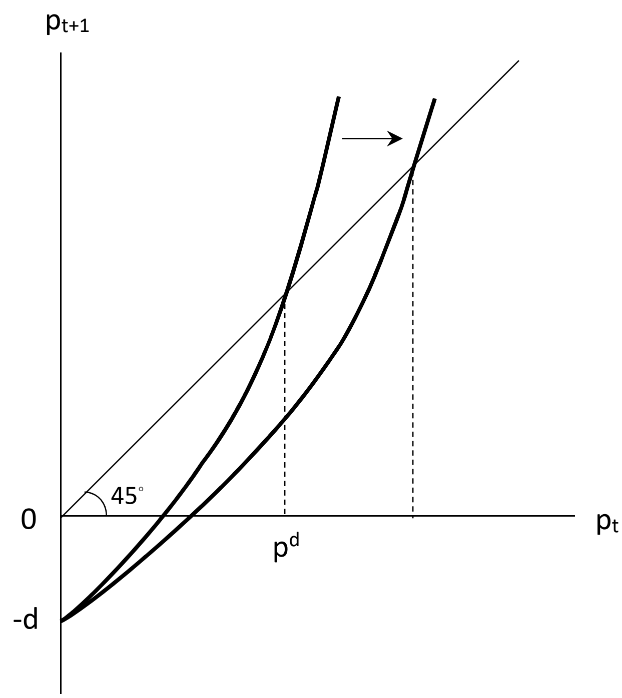

Figure 3: Effect of increasing floor $\underline{\lambda}$ with deterministic dividends

Appendix A: Proof of Proposition 1

Proof of Proposition 1: In the text, we showed there is a unique deterministic equilibrium. Here we consider equilibrium paths for $\{p_t, R_t\}_{t=0}^{\infty}$ that can be stochastic and show that the unique equilibrium in this case is the determinstic equilibrium we solve in the text.

Consider a sequence of random variables $\omega^t = \{\omega_0, ..., \omega_t\}$ and let Ω^t denote the space of all possible realizations of ω^t. Let $p_t = p(\omega^t)$ and $R_t = R(\omega^t)$ represent the equilibrium price and interest rate for each state ω^t. They must satisfy the two conditions

$$\int_{R(\omega^t)}^{\infty} n(y)\,dy + p(\omega^t) = e \tag{31}$$

$$\left[1 + R(\omega^t)\right] p(\omega^t) = d + E\left[p(\omega^{t+1})\,|\omega^t\right] \tag{32}$$

Equation (31) implies

$$R(\omega^t) = \rho\left(p(\omega^t)\right)$$

where $\rho'(\cdot) > 0$. Combing (31) and (32) yields

$$E\left[p(\omega^{t+1})\,|\omega^t\right] = \left(1 + \rho\left(p(\omega^t)\right)\right) p(\omega^t) - d \tag{33}$$

Suppose there exists some date t and some state ω^t such that $p(\omega^t) > p^d$. We recursively construct a sequence $\overline{p}_{t+s}$ where $\overline{p}_t = p(\omega^t)$ and for every $s = 1, 2, 3, ...$ we define

$$\overline{p}_{t+s+1} = \left(1 + \rho\left(\overline{p}_{t+s}\right)\right)\overline{p}_{t+s} - d$$

We know from Figure 1 that $\overline{p}_{t+s}$ shoots off to infinity, and so there exists a date T such that $p_t > e$ for $t \geq T$. We now argue by induction that if $p(\omega^t) > p^d$, then we can find a sequence of states ω^{t+s} such that $p(\omega^{t+s}) \geq \overline{p}_{t+s}$ for all s. We begin with state ω^t. By construction, $p(\omega^t) = \overline{p}_t$. Next, suppose $p(\omega^{t+s}) \geq \overline{p}_{t+s}$ for some s. From (33), we have

$$E\left[p(\omega^{t+s+1})\,|\omega^{t+s}\right] = \left(1 + \rho\left(p(\omega^{t+s})\right)\right) p(\omega^{t+s}) - d \tag{34}$$

Since $E[p(\omega^{t+1})\,|\omega^t] \leq \sup_{\omega^{t+1}} p(\omega^{t+1})$, it follows that there exists some state ω^{t+s+1} such that

$$\begin{aligned} p(\omega^{t+s+1}) &\geq \left(1 + \rho\left(p(\omega^{t+s})\right)\right) p(\omega^{t+s}) - d \\ &\geq \left(1 + \rho\left(\overline{p}_{t+s}\right)\right)\overline{p}_{t+s} - d \\ &= \overline{p}_{t+s+1} \end{aligned}$$

This confirms that there exists a sequence of states ω^{t+s} such that $p(\omega^{t+s}) \geq \overline{p}_{t+s}$ for all s. But this means $p(\omega^{t+s}) > e$ for $t + s \geq T$, which means we have a state of the world in which the price we propose cannot be an equlibrium. Hence, $p(\omega^t) \leq p^d$ for all states ω^t and all t.

Next, suppose there exists some state ω^t such that $p(\omega^t) > p^d$. By a similar type of argument, we can construct a sequence $\underline{p}_{t+s}$ where $\underline{p}_t = p(\omega^t)$ and for every $s = 1, 2, 3, ...$ we define

$$\underline{p}_{t+s+1} = \left(1 + \rho\left(\underline{p}_{t+s}\right)\right)\underline{p}_{t+s} - d$$

We know from Figure 1 that $\underline{p}_{t+s}$ turns negative by some T. By a similar logic as before, we can find a sequence of states ω^{t+s} such that $p\left(\omega^{t+s}\right) \leq \underline{p}_{t+s}$. But this means $p\left(\omega^{t+s}\right) < 0$ for $t+s \geq T$, which means we have a state of the world in which the price we propose cannot be an equlibrium. Hence, $p\left(\omega^{t}\right) = p^{d}$ for all states ω^{t} and all t.

Since $p\left(\omega^{t}\right) = p^{d}$ for all states ω^{t}, from (31) or (32) it follows that $R\left(\omega^{t}\right) = R^{d}$ for all ω^{t}. This establishes that the equilibrium in the text is unique. ■

Appendix B: Monetary Policy

This appendix introduces within-period production, a monetary authority, and nominal price rigidity into our setup. Our approach largely follows Galí (2014) in how we incorporate price rigidity and monetary policy into an overlapping generations economy with assets.

B.1 Agent Types and Endowments

A new cohort of agents is born each period. Agents still live two periods and care only about consumption when old. In our benchmark model we assumed each cohort consisted of two types – savers who were endowed with a fixed amount of goods and entrepreneurs who are endowed with no resources but who can convert goods at date t into goods at date $t+1$. We continue to assume each cohort is composed of savers and entrepreneurs. Entrepreneurs are the same as in the benchmark model. But we now assume savers are endowed with the inputs to produce goods rather than with the goods themselves. This allows for an endogenous quantity of goods that can potentially vary with the stance of monetary policy.

More precisely, we assume two types of savers, each of mass 1. Half are workers, endowed with 1 unit of labor each and required to choose how to allocate it. The other half are producers, endowed with the knowledge of how to convert labor into output but not with labor itself.[8] Producers set the price of the goods they produce and then hire the labor needed to satisfy their demand. Although producers and entrepreneurs both produce output, they differ in when and how they produce it. Producers born at date t convert labor into goods at date t. Entrepreneurs then convert the goods producers created at date t into goods at date $t+1$. Producers operate within the period; enterpreneurs operate across periods.

B.2 Production, Pricing, and Labor Supply

Workers allocate their one unit of labor between home or market production. Home production yields the same consumption good as the market, but using a concave technology $h(x)$ where $h'(0) = 1$ and $h'(1) = 0$. The reason for these particular limits will become clear below.

Workers who sell their labor on the market earn a wage W_t per unit labor. Their labor services are hired by producers, whom we index by $i \in [0, 1]$. Each producer can produce a distinct intermediate good according to a linear technology. In particular, if producer i hires n_{it} units of labor, she will produce $x_{it} = n_{it}$ units of intermediate good i. The different intermediate goods can then be combined to form final consumption goods according to a constant elasticity of substitution (CES) production function available

[8] This formulation is borrowed from Adam (2003) rather than Galí (2014). The latter assumes homogeneous types who sell labor when young and hire labor when old. We prefer the former to ensure income only accrues to agents while young.

to all agents. That is, given x_{it} of each $i \in [0,1]$, the amount of final goods X_t that can be produced is

$$X_t = \left(\int_0^1 x_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \tag{35}$$

Let P_t denote the price of the final good and P_{it} denote the price of intermediate good i. At these prices, the x_{it} that maximize the profits of a final goods producer solve

$$\max_{x_{it}} P_t \left(\int_0^1 x_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} - \int_0^1 P_{it} x_{it} di$$

The first-order condition with respect to x_{it} yields

$$x_{it} = X_t \left(\frac{P_t}{P_{it}}\right)^{\frac{1}{\sigma}} \tag{36}$$

If we set $X_t = 1$, we can compute the price of the cost of the optimal bundle of intermediate goods $x_{it} = \left(\frac{P_t}{P_{it}}\right)^{\frac{1}{\sigma}}$ needed to produce one unit of the final good:

$$\int_0^1 P_{it} x_{it} di = \int_0^1 P_{it}^{1-\frac{1}{\sigma}} P_t^{\frac{1}{\sigma}} di$$

Since any agent can produce final goods, the price P_t must equal the per unit cost of producing a good in equilibrium. Equating the two yields the familiar CES price aggregator:

$$P_t = \left(\int_0^1 P_{it}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \tag{37}$$

Each intermediate goods producer chooses their price P_{it} to maximize expected profits given demand (36) and wage W_t. To allow producers to move either before or after the monetary authority, we condition producer i's choice on their information Ω_{it} when choosing their price. Each producer will set P_{it} to solve

$$\max_{P_{it}} E\left[(P_{it} - W_t) X_t \left(\frac{P_t}{P_{it}}\right)^{1/\sigma} \middle| \Omega_{it} \right]$$

The optimal price is then

$$P_{it} = \frac{E\left[W_t X_t | \Omega_t\right]}{(1-\sigma) E\left[X_t | \Omega_t\right]} \tag{38}$$

By symmetry, all producers will charge the same price, produce the same amount, and hire the same amount of labor, i.e., $n_{it} = n_t$ for all $i \in [0,1]$. The output of consumption goods is thus

$$X_t = \left(\int_0^1 n_t^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} = n_t$$

Workers receive $(W_t/P_t)\, n_t$ of these goods and producers receive the remaining $(1 - W_t/P_t)\, n_t$. The total income of workers also includes home production $h(1-n_t)$. They maximize their income by setting

$$h'(1-n_t) = W_t/P_t \tag{39}$$

By contrast, the total resources available to youngs agents is $e_t = n_t + h(1-n_t)$, which is maximized at

$$h'(1-n_t) = 1$$

Our assumption that $h'(0) = 1$ implies total resources are maximized when $n_t = 0$ and all goods are produced in the market. The resources $e_t = n_t + h(1-n_t)$ are thus increasing in n_t for all $n_t \in [0,1]$.

B.3 Assets, Credit, and Money

Since agents want to consume when old, they will wish to save their earnings $e_t = n_t + h(1 - n_t)$. We allow them to buy assets or make loans as in our benchmark model. Without money, this specification would be equivalent to our benchmark model except that the income of savers e_t which before was exogenous and fixed is now endogenous and potentially time-varying. Equilibrium in the asset and credit markets involves the same conditions as in the benchmark model. First, regardless of the income they earn, the young will spend all of their resources either funding entrepreneurs or buying assets, and so we still have

$$\int_{R_t}^{\infty} n(y)\, dy + p_t = e_t$$

where p_t is the real price of the asset and R_t is the real interest rate on loans. The interest rate R_t must still ensure agents cannot earn profits by borrowing and buying assets. When $d_t = d$, this requires

$$\left(1 + R_t^d\right) p_t^d = d + p_{t+1}^d$$

and when $d_t = D$, this requires

$$\left(1 + R_t^D\right) p_t^D = D + p_{t+1}^D$$

We can then use R_t and p_t to solve for the expected return on loans:

$$\overline{R}_t = \begin{cases} R_t^d & \text{if } d_t = d \\ \max\left\{\overline{r}_t^D, \left(1 - \frac{p_t^D}{e_t}\right) R_t^D + \frac{p_t^D}{e_t}\left(\overline{r}_t^D - \pi\Phi\right)\right\} & \text{if } d_t = D \end{cases} \tag{40}$$

where $\overline{r}_t^D$ is the expected return to buying the asset. Below, we show that when prices are flexible or money is absent altogether, the equilibrium real wage W_t/P_t will be constant over time. Employment n_t and total earnings of all savers $e_t = n_t + h(1 - n_t)$ will then also be constant. The reduced form of our model in the absence of money thus coincides with our benchmark model.

To introduce money, we follow Galí (2014) in assuming money does not circulate in equilibrium. That is, money is the numeraire, and P_t and W_t denote the price of goods and labor relative to money. However, no agent actually holds money in equilibrium. Instead, the monetary authority announces a nominal interest rate i_t at each date t which it commits to pay at date $t+1$ to those who lend to it (with money it can always issue). It is also willing to charge i_t to those who borrow money from the monetary authority will full collateral. This is roughly in line with what central banks do in practice, paying interest on reserves and lending at the discount window against collateral. To ensure money doesn't circulate, the real return on lending to the monetary authority must equal the expected return on savings. Let $\Pi_t = P_{t+1}/P_t$ denote the gross inflation rate between dates t and $t+1$. Since agents always lend to entrepreneurs, the expected return on savings will equal $\overline{R}_t$, the expected return on loans. This implies

$$1 + i_t = \left(1 + \overline{R}_t\right) \Pi_t \tag{41}$$

When the monetary authority changes the nominal interest rate i_t, either inflation Π_t or the expected return $1 + \overline{R}_t$ or both will have to adjust to ensure agents will neither borrow nor lend to the monetary authority.

B.4 Defining an Equilibrium

Given a path of nominal interest rates $\{1+i_t\}_{t=0}^{\infty}$, an equilibrium consists of a path of prices $\{P_t, W_t, p_t, R_t\}_{t=0}^{\infty}$ and a path of employment $\{n_t\}_{t=0}^{\infty}$ such that agents behave optimally and markets clear. Collecting the relevant conditions from above yields the following five equations for these five variables:

$$
\begin{array}{ll}
\text{(1) Optimal pricing:} & P_t = \dfrac{E\left[W_t X_t|\Omega_t\right]}{(1-\sigma)\,E\left[X_t|\Omega_t\right]} \\
\text{(2) Optimal labor supply:} & h'\left(1-n_t\right) = \dfrac{W_t}{P_t} \\
\text{(3) Optimal saving:} & \int_{R_t}^{\infty} n\left(y\right) dy + p_t = e_t \\
\text{(4) Credit market clearing:} & 1+R_t = \begin{cases} \frac{D+p_{t+1}^D}{p_t^D} & \text{if } d_t = D \\ \frac{d+p_{t+1}^d}{p_t^d} & \text{if } d_t = d \end{cases} \\
\text{(5) Money market clearing:} & \Pi_t = \dfrac{1+i_t}{1+\overline{R}_t}
\end{array}
$$

where the expected return on loans $\overline{R}_t$ is given by (40).

B.5 Equilibrium with Flexible Prices

We begin with the case where producers set their prices P_{it} after observing the wage W_t. This corresponds to the case where prices are fully flexible, or alternatively where there is no money and so no sense in which nominal prices are set in advance. Producers can deduce what other producers will do and the labor workers will supply, they can perfectly anticipate total output X_t. Hence, their information set $\Omega_t = \{W_t, X_t\}$. It follows that $E\left[W_t X_t|\Omega_t\right] = W_t X_t$ and $E\left[X_t|\Omega_t\right] = X_t$. The optimal pricing rule (1) then implies

$$P_t = \frac{W_t}{1-\sigma}$$

The real wage is thus constant and equal to $1-\sigma$. Substituting this into (2) yields

$$h'\left(1-n_t\right) = 1-\sigma \tag{42}$$

Since $h\left(\cdot\right)$ is concave, n_t is equal to some constant n^* for all t. It follows that $e_t = n^* + h\left(1-n^*\right)$ is also constant for all t. We can then use (3) and (4) to solve for p_t and R_t as in the benchmark model, and then use (40) to compute $\overline{R}_t$. Finally, given $\overline{R}_t$ we can use the implied Π_t from (5) to derive $\{P_t\}_{t=1}^{\infty}$ for any initial value for P_0. The initial price level P_0 is indeterminate, in line with the Sargent and Wallace (1975) result on the price level indeterminacy of pure interest rate rules. The nominal wage $W_t = \left(1-\sigma\right) P_t$.

B.6 Equilibria with Rigid Prices

We now turn to the case where producers set the price of their intermediate good P_{it} before the monetary authority moves. That is, producers set prices, the monetary authority sets $1+i_t$, and then producers hire workers at a nominal wage W_t. This formulation implies prices are only rigid for one period.

If monetary policy is deterministic, producers can perfectly anticipate the nominal interest rate and the equilibrium nominal wage W_t, and so $\Omega_t = \{W_t, X_t\}$ and $W_t/P_t = 1-\sigma$ as before.

Next, suppose monetary policy is contingent on some random variable, i.e., $i_t = i(\xi_t)$ where $\{\xi_t\}_{t=0}^{\infty}$ is a sequence of random variables. For simplicity, consider the case where ξ_t is only random at $t=0$, i.e.,

$$\xi_0 = \begin{cases} H & \text{w/prob } \chi \\ L & \text{w/prob } 1-\chi \end{cases}$$
$$\xi_t \text{ is deterministic for } t = 1, 2, \ldots$$

From date $t=1$ on, we know from the optimal price-setting condition (1) that $W_t/P_t = 1-\sigma$. It then follows that $n_t = n^*$ and $e_t = e^* \equiv n^* + h(1-n^*)$ for all $t \geq 1$, and we can determine p_t, R_t, and $\overline{R}_t$ for $t \geq 1$ just as in the case where prices are flexible. All we need is to solve for the equilibrium at date 0.

We use a superscript $\xi \in \{H, L\}$ to denote the value of a variable as for a given realization of ξ_0. Assume wlog that $i_0^H > i_0^L$. The optimal price setting condition (1) is now

$$\frac{\chi n_0^H \frac{W_0^H}{P_0} + (1-\chi)\, n_0^L \frac{W_0^L}{P_0}}{\chi n_0^H + (1-\chi)\, n_0^L} = 1-\sigma \tag{43}$$

That is, the output-weighted average real wage over the two values of ξ is equal to $1-\sigma$. Optimal labor supply (2) then implies

$$\begin{aligned} h'\left(1-n_0^H\right) &= \min\left\{\frac{W_0^H}{P_0}, 1\right\} \\ h'\left(1-n_0^L\right) &= \min\left\{\frac{W_0^L}{P_0}, 1\right\} \end{aligned}$$

These are three equations for four unknowns, meaning the set of all equilibria can be parameterized by a single parameter. Wlog, we can choose the real wage when $\xi = H$ to be this parameter. The three equations above yield values for W_0^L/P_0, n_0^H, and n_0^L given W_0^H/P_0. From these, we can deduce earnings $e_0^\xi = n_0^\xi + h\left(1-n_0^\xi\right)$ for each $\xi \in \{H, L\}$. We can then use (3) and (4) to derive p_0^ξ and R_0^ξ by solving

$$\int_{R_0^\xi}^{\infty} n(y)\, dy + p_0^\xi = e_0^\xi \tag{44}$$

$$\left(1+R_0^\xi\right) p_0^\xi = D + p^D \tag{45}$$

and then compute the expected return on loans $\overline{R}_0^\xi$ using (40), and, via (5), the inflation rate Π_0^ξ for each $\xi \in \{H, L\}$. As before, the price level P_0 is indeterminate.The reason for multiple equilibria at date 0 is that optimal pricing only restricts the average real wage across states but not the real wage for any realization of ξ_0. The real wage can exceed $1-\sigma$ for one realization of ξ_0 if it falls below $1-\sigma$ for the other realization.

There case where monetary policy has no effect on real variables at date 0 remains an equilibrium. In this case, $W_0^H/P_0 = W_0^L/P_0 = 1-\sigma$. But price rigidity expands the set of equilibria to include ones in which real variables vary with the nominal interest rate. Since the nominal interest rate only serves as a signal to coordinate real activity rather but does not directly affect it, there are equilibria in which $W_0^H > W_0^L$ as well as equilibria in which $W_0^H < W_0^L$.[9] Empirically, higher nominal interest rates appear to be contractionary. This suggests the relevant equilibria are those in which $W_0^H/P_0 < 1-\sigma < W_0^L/P_0$, i.e., real wages are lower when the monetary authority unexpectedly raises the nominal interest rate. In this case, from (2) we know that a higher nominal interest rate will be associated with lower employment ($n_0^H < n^* < n_0^L$) and hence lower earnings ($e_0^H < e^* < e_0^L$). If we do comparative statics on (44) and (45), it follows that a higher nominal interest rate will be associated with a lower real asset price ($p_0^H < p^D < p_0^L$) and a higher real interest rate on loans ($R_0^H > R^D > R_0^L$). The real expected return to buying assets will also be higher ($\overline{r}_0^H > \overline{r}^D > \overline{r}_0^L$). However, whether the real expected return to lending $\overline{R}_0^H$ will be higher is ambiguous. (40) implies $\overline{R}_0^\xi$ is either equal to $\overline{r}_0^\xi$ or to a weighted average of R_0^ξ and $\overline{r}_0^\xi$. In the latter case, although both terms are higher when $\xi = H$ the weight on $\overline{r}_0^\xi$, which is p_0^ξ/e_0^ξ, can be higher or lower for $\xi = H$. These results are summarized in Proposition 4 in the paper.

B.7 Promises of Future Intervention

Our last point concerns the effects of a promise at date 0 to be contractionary at date 1 if the boom continues into that date. In this case, ξ_0 and ξ_t for $t \geq 2$ are deterministic, while $\xi_1 = d_1$. This specification presumes producers each period before the dividend d is revealed. Solving for equilibrium at date 1 is identical to how we solved for the equilibrium at date 0 when we assumed ξ_0 was random. Consider equilibria in which the real wage is lower if the boom continues, so

$$W_1^D/P_1 < 1-\sigma < W_1^d/P_1.$$

This implies $n_1^D < n^* < n_1^d$ and so $e_1^D < e < e_1^d$. In other words, if dividends fall and the boom ends, monetary policy must be expansionary. It follows that $p_1^D < p^D$ and $p_1^d > p^d$, and likewise that $R_1^D < R^D$ and $R_1^d > R^d$. Now, turning to date 0, conditions (3) and (4) imply

$$\begin{aligned} \int_{R_0^D}^{\infty} n\left(y\right)dy + p_0^D &= e \\ \left(1+R_0^D\right)p_0^D &= D + p_1^D \end{aligned}$$

Comparative statics of this system with respect to p_1^D reveals that $p_0^D < p^D$ and $R_0^D < R^D$. That is, while contractionary monetary policy at date 0 dampens p_0^D but raises R_0^D at date 0, a threat to enact contractionary monetary policy at date 1 if dividends remain high will dampen both p_0^D and R_0^Dat date 0. These results are summarized in Proposition 5 in the paper.

[9] One way to avoid such multiplicity is to assume dynamic monetary policy rules that are conditioned on past economic variables. This allows a central bank to takes actions that are unsustainable if a high interest rate today leads to certain outcomes, eliminating equilibria with those outcomes. See Cochrane (2011) for a discussion of these issues.

Appendix C: Macroprudential Regulation

This appendix define an equilibrium with multiple markets. We show certain aspects of it are uniquely determined. We then discuss some comparative static results.

C.1 Defining an Equilibrium

We begin with some notation. Let p_t denote the price of the asset at date t. Given asset prices, we can define the return to buying the asset at date t as

$$z_t \equiv \frac{d_{t+1} + p_{t+1}}{p_t}$$

The return z_t can be random, both since d_{t+1} is uncertain and because in principle p_{t+1} can be a random variable. Let $G_t(z)$ denote the (possibly degenerate) distribution of the return z_t. Let $r_t^{\max}$ denote the maximum possible return on the asset. As discussed in the text, we know $r_t^{\max}$ is finite, since $r_t^{\max} \leq \frac{D+2\varphi e}{(1-\varphi)e} < \infty$. We will use $\overline{r}_t$ to denote the expected return to buying the asset at date t, i.e.,

$$1 + \overline{r}_t \equiv \int_{-\infty}^{r_t^{\max}} z dG_t(z)$$

Next, we define variables for the different markets $\lambda \in [0, 1)$ agents can borrow in. Let $R_t(\lambda)$ denote the interest rate on loans in market λ, so an agent who agrees to pay a share λ of the project she undertakes will promise to pay back $1 + R_t(\lambda)$ for each unit she borrows. Since agents may default, let $\overline{R}_t(\lambda)$ denote what lenders expect to earn from lending in market λ given the possibility of default. Next, let $F_t^a(\lambda)$ and $F_t^p(\lambda)$ denote the cumulative amounts agents borrow in markets 0 through λ in order to purchase assets and to produce, respectively. The amount borrowed in market λ for purpose x is then

$$dF_t^x(\lambda) = \lim_{\Delta \to 0} \frac{F^x(\lambda + \Delta) - F^x(\lambda)}{\Delta}$$

This formulation allows for mass points. Define total borrowing in market λ as

$$dF_t(\lambda) = dF_t^a(\lambda) + dF_t^p(\lambda)$$

We will refer to a market λ as *active* at date t if $dF_t(\lambda) > 0$ and *inactive* if $dF_t(\lambda) = 0$.

Given these preliminaries, we define an equilibrium as a path $\left\{p_t, F_t^p(\lambda), F_t^a(\lambda), R_t(\lambda), \overline{R}_t(\lambda)\right\}_{t=0}^{\infty}$ that satisfies conditions (46)-(50) below to ensure that all markets clear when agents are optimizing and a particular restriction on beliefs, (51), that we motivate and discuss below.

Our first three conditions stipulate that agents act optimally. We begin with lenders. Optimality requires their agents will only invest their wealth in investments that yield the highest expected return. Let $\overline{R}_t^*$ denote the maximal expected return to lending in any market λ, i.e.,

$$\overline{R}_t^* \equiv \sup_{\lambda\in[0,1)} \overline{R}_t(\lambda)$$

Optimal lending requires that agents lend in market λ' only if it they expecft to earn $\overline{R}_t^*$ and if this rate exceeds the expected return to buying the asset, i.e.,

$$dF_t(\lambda') > 0 \text{ only if } \overline{R}_t(\lambda') = \overline{R}_t^* \text{ and } \overline{R}_t^* \geq \overline{r}_t \tag{46}$$

Next, we make sure that entrepreneurs who produce act optimally. We first argue entrepreneurs should use their endowment to produce. Recall entrepreneurs have productivity $y \geq y^*$, and from (20) we know $y^* \geq r_t^{\max} \geq \overline{r}_t$, so producing is better than buying assets. But y^* must also exceed the expected return to lending $\overline{R}_t^*$. For suppose $\overline{R}_t^*$ were higher than y^*. Since $y^* > r_t^{\max}$, then $\overline{R}_t^*$ must also exceed $r_t^{\max}$. In that case, no agent would use their endowment to buy assets, nor would any agent borrow to buy assets given the interest rate on loans in any active market must be at least $\overline{R}_t^*$. Yet assets must trade in equilibrium: Owners sell their assets whenever the asset price is positive, while demand for the asset would be infinite if its price were nonnegative. Since entrepreneurs earn the highest return from production, they should use their endowment w to produce. Entrepreneurs can further leverage their endowment to produce at a capacity above what their endowment would allow. If they borrow in market λ where $\lambda < w$, they can borrow enough to reach full capacity. The payoff to an entrepreneur with wealth w borrowing in different markets λ are described in the text. Optimality requires that there will be borrowing to produce in market λ' only if some entrepreneur finds it optimal to borrow in that market, i.e.,

$$dF_t^p(\lambda') > 0 \text{ only if } \lambda' \in \arg\max_{\lambda\in[0,1]} \left\{ \begin{array}{ll} [1 + y - (1-w)(1+R_t(\lambda))] & \text{if } \lambda \leq w \\ \frac{w}{\lambda}[1 + y - (1-\lambda)(1+R_t(\lambda))] & \text{if } \lambda > w \end{array} \right\} \text{ for some } (w,y) \tag{47}$$

Note that the choice of λ includes $\lambda = 1$, i.e. we are allowing entrepreneurs not to borrow at all.

Third, we ensure that any agents who borrow to buy assets are optimizing. A necessary condition for them to borrow in market $\lambda \in [0,1]$ and buy assets is that doing so yields a higher expected return than lending out the same resources. Define

$$x_t(\lambda) \equiv (1 + R_t(\lambda))(1-\lambda)$$

The expected profits from borrowing in market λ to buy assets is $\int_{x_t(\lambda')}^{\infty} (z_t - x_t(\lambda'))\, dG(z_t)$. For an agent to agree to borrow in market λ, these expected profits must be at least as much as $\left(1 + \overline{R}_t^*\right)\lambda$. In addition, we argue that if agents borrow in market λ to produce, the expected profits must exactly equal $\left(1 + \overline{R}_t^*\right)\lambda$. For suppose $\int_{x_t(\lambda')}^{\infty} (z_t - x_t(\lambda'))\, dG(z_t) > \left(1 + \overline{R}_t\right)\lambda'$ for some λ'. Then no one would ever lend given they can borrow in market λ' and buy assets. But no lending cannot be an equilibrium given demand for borrowing by entrepreneurs is positive at all interest rates. Hence, optimality implies

$$dF_t^a(\lambda') > 0 \text{ only if } \int_{x_t(\lambda')}^{\infty} (z_t - x_t(\lambda'))\, dG(z_t) = \left(1 + \overline{R}_t^*\right)\lambda' \tag{48}$$

Our next two conditions concern market. Since we argued above that in equilibrium assets will trade, the endowment of savers must be used to either produce or buy assets. This requires that

$$F_t^p(1) + p_t = e \tag{49}$$

Finally, we turn to equilibrium beliefs. In any market λ' where credit is traded, lenders must expect the return on lending $\overline{R}_t(\lambda)$ to conform with the actual fraction of borrowers who borrow in market λ' with the intent to produce and to buy assets, respectively. That is,

$$\overline{R}_t(\lambda) = \frac{dF_t^p(\lambda)}{dF_t(\lambda)} R_t(\lambda) + \frac{dF_t^a(\lambda)}{dF_t(\lambda)} E_t \max\left\{R_t(\lambda), \frac{d_{t+1} + p_{t+1}}{p_t} - 1\right\} \text{ if } dF_t(\lambda') > 0 \tag{50}$$

Condition (50) does not impose any restrictions on expectations in inactive markets where $dF_t(\lambda') = 0$. As we describe in the text, we impose an additional restriction on beliefs to rule out beliefs that seem unreasonable. In particular, we impose that in any market in which agents will not find it profitable to buy assets, lenders should not expect agents to default. Formally, this implies

$$\overline{R}_t(\lambda') = R_t(\lambda') \text{ whenever } \int_{x_t(\lambda')}^{1+r_t^{\max}} (z_t - x_t(\lambda'))\, dG(z_t) < \left(1 + \overline{R}_t^*\right)\lambda' \tag{51}$$

We discuss below what happens if we were to drop this restriction.

C.2 Solving for Equilibrium

As in the text, we first solve for the schedule of interest rates $R_t(\lambda)$. We then use it to characterize the amount agents borrow to produce $F_t^p(\lambda)$, and use it and (49) to solve for the price p_t.

Our first result uses (48) to characterize the schedule of interest rates.

Proposition C1: *Equilibrium interest rates will be given by*

$$1 + R_t(\lambda) = \begin{cases} \frac{\widetilde{x}_t(\lambda)}{1-\lambda} & \text{if } \lambda \in [0, \lambda_t^*) \\ 1 + \overline{R}_t^* & \text{if } \lambda \in [\lambda_t^*, 1) \end{cases} \tag{52}$$

where $\widetilde{x}_t(\lambda)$ *is the value of* x *that solves*

$$\int_{z=x}^{1+r_t^{\max}} (z - x)\, dG_t(z) = \left(1 + \overline{R}_t^*\right)\lambda \tag{53}$$

The schedule $R_t(\lambda)$ *is a decreasing and continuous function of* λ *for* $\lambda \in [0, \lambda_t^*)$.

Proof: Our proof is establishes a series of lemmas.

Lemma 1: $R_t(\lambda) \geq \overline{R}_t^*$ for all $\lambda \in [0, 1)$ whether market λ is active or inactive.

Proof of Lemma 1: From (46), we know that $\overline{R}_t(\lambda') = \overline{R}_t^*$ for all λ' for which $dF_t(\lambda') > 0$. Moreover, (50) implies that for any market λ' for which $dF_t(\lambda') > 0$, the interest rate on loans $R_t(\lambda')$ must be at least as large as the expected return on loans $\overline{R}_t(\lambda')$. Hence, $R_t(\lambda') \geq \overline{R}_t(\lambda') = \overline{R}_t^*$ for any active market λ'. This leaves inactive markets in which $dF_t(\lambda') = 0$. Suppose to the contrary that $R_t(\lambda') < \overline{R}_t^*$ for some λ' for which $dF_t(\lambda') = 0$. Entrepreneurs with wealth $w > \lambda'$ would then optimally choose to borrow in some market, call it λ'', in which the interest rate $R_t(\lambda'') \leq R_t(\lambda') < \overline{R}_t^*$: They can borrow and produce at full capacity by borrowing in market λ', so they will either borrow in that market or in some other market that offers them a still lower rate. But this implies $R_t(\lambda'') < \overline{R}_t^*$ and $dF_t(\lambda'') > 0$, which contradicts (46). It follows that $R_t(\lambda) \geq \overline{R}_t^*$ for all $\lambda \in [0,1)$ whether $dF_t(\lambda) = 0$ or not. ■

Next, we show $R_t(\lambda)$ as a function of $\lambda \in [0,1)$ is given by (52).

Lemma 2: In equilibrium, $R_t(\lambda) = \max\left\{\overline{R}_t^*, \frac{\widetilde{x}_t(\lambda)}{1-\lambda} - 1\right\}$ where $\widetilde{x}_t(\lambda)$ equals the x that solves (53).

Proof of Lemma 2: Define $x_t(\lambda) \equiv (1 + R_t(\lambda))(1-\lambda)$. Suppose there exists a $\lambda' \in [0,1)$ for which

$$\int_{z=x_t(\lambda')}^{1+r_t^{\max}} (z - x_t(\lambda'))\, dG_t(z) > \left(1 + \overline{R}_t^*\right)\lambda' \tag{54}$$

This would mean agents can earn higher expected profits borrowing in market λ' to buy assets than if they lent out any of their own resources that they would have otherwise invested. Since lending is dominated, there can be no lending in equilibrium, meaning $F_t(\lambda) = 0$ for all $\lambda \in [0,1)$. This can only be an equilibrium if $R_t(\lambda) = +\infty$ for all $\lambda \in [0,1)$, or else there would be some mass of entrepreneurs with arbitrarily high productivity who would borrow in one of the markets in which $R_t(\lambda) < \infty$. But in that case, $x_t(\lambda)$ would be infinite for any $\lambda \in [0,1)$ and it would be impossible for the expected return to buying an asset to exceed $\left(1 + \overline{R}_t^*\right)\lambda$. Hence, for all $\lambda \in [0,1)$,

$$\int_{z=x_t(\lambda)}^{1+r_t^{\max}} (z - x_t(\lambda))\, dG_t(z) \leq \left(1 + \overline{R}_t^*\right)\lambda \tag{55}$$

Consider any $\lambda' \in [0,1)$ for which this inequality is strict, i.e., $\int_{z=x_t(\lambda')}^{1+r_t^{\max}} (z - x_t(\lambda'))\, dG_t(z) < \left(1 + \overline{R}_t^*\right)\lambda'$. Since buying assets yields a lower payoff than lending, no agent will borrow in market λ' to buy assets, and so $dF_t^a(\lambda') = 0$. Agents may still borrow in market λ' to produce. If $dF_t^p(\lambda') > 0$, then (46) implies $\overline{R}_t(\lambda') = \overline{R}_t^*$ and so $R_t(\lambda') = \overline{R}_t^*$. If $dF_t^p(\lambda') = 0$, then we know from Lemma 1 that $R_t(\lambda') \geq \overline{R}_t^*$. Condition (51) implies $\overline{R}_t(\lambda') = R_t(\lambda')$, and so $\overline{R}_t(\lambda') \geq \overline{R}_t^*$. But the definition of $\overline{R}_t^*$ implies $\overline{R}_t(\lambda') \leq \overline{R}_t^*$. Hence, in this case we also have that $R_t(\lambda') = \overline{R}_t(\lambda') = \overline{R}_t^*$. Thus, for every λ, either $R_t(\lambda) = \overline{R}_t^*$ or else $R_t(\lambda) = \frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ where $\widetilde{x}_t(\lambda)$ is the x which solves (53).

We now show that $R_t(\lambda) = \max\left\{\overline{R}_t^*, \frac{\widetilde{x}_t(\lambda)}{1-\lambda} - 1\right\}$. Consider first values of λ for which $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} > 1 + \overline{R}_t^*$. Since $\int_{z=x}^{1+r_t^{\max}} (z - x)\, dG_t(z)$ is decreasing in x, this means

$$\int_{z=\left(1+\overline{R}_t^*\right)(1-\lambda)}^{1+r_t^{\max}} \left(z - \left(1 + \overline{R}_t^*\right)(1-\lambda)\right) dG_t(z) > \int_{z=\widetilde{x}(\lambda)}^{1+r_t^{\max}} (z - \widetilde{x}(\lambda))\, dG_t(z) = \left(1 + \overline{R}_t^*\right)\lambda$$

which we said cannot be an equilibrium. Hence, we cannot have $R_t(\lambda) = \overline{R}_t^*$, and we must instead have $1 + R_t(\lambda) = \frac{\widetilde{x}_t(\lambda)}{1-\lambda} > 1 + \overline{R}_t^*$. Next, consider values of λ for which $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} < 1 + \overline{R}_t^*$. In this case, if we have $1 + R_t(\lambda) = \frac{\widetilde{x}_t(\lambda)}{1-\lambda}$, then $R_t(\lambda) < \overline{R}_t^*$. But this violates Lemma 1. In this case, then, $1 + R_t(\lambda) = 1 + \overline{R}_t^* > \frac{\widetilde{x}_t(\lambda)}{1-\lambda}$. The claim follows. ■

Next, we show that $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ is a decreasing function of λ, which yields a cutoff λ_t^* above which $R_t(\lambda) = \overline{R}_t^*$.

Lemma 3: $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ is a weakly decreasing function of λ.

Proof of Lemma 3: Recall that $\widetilde{x}_t(\lambda)$ is the value of x which solves (53). Even though $G_t(z)$ can contain mass points, these are multiplied by 0, so the integral $\int_{z=x}^{1+r_t^{\max}} (z-x)\, dG_t(z)$ is continuous in x. This implies $\widetilde{x}_t(\lambda)$ is a continuous function of λ, although it may be kinked. To show it is decreasing, we need to show that all of its directional derivatives are nonpositive. Totally differentiating this equation implies

$$\frac{d\widetilde{x}_t(\lambda)}{d\lambda} = -\frac{1 + \overline{R}_t^*}{\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}$$

When $\widetilde{x}_t(\lambda)$ is a mass point of $G_t(z)$, the integral in the denominator above will be discontinuous, since $\lim_{\lambda \to \lambda^+} \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) \neq \lim_{\lambda \to \lambda^-} \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)$. However, both expressions are negative, so $\widetilde{x}_t(\lambda)$ is a decreasing function of λ.

Next, consider $\widetilde{R}_t(\lambda) \equiv \frac{\widetilde{x}_t(\lambda)}{1-\lambda} - 1$. The function $\widetilde{R}_t(\lambda)$ is also continuous in λ with a potentially discontinuous derivative. Differentiating the equation $\widetilde{x}_t(\lambda) = (1-\lambda)\left(1 + \widetilde{R}_t(\lambda)\right)$ implies

$$\frac{d\widetilde{x}_t(\lambda)}{d\lambda} = -\left(1 + \widetilde{R}_t(\lambda)\right) + (1-\lambda)\frac{d\widetilde{R}_t(\lambda)}{d\lambda}$$

Rearranging and using the expression for $\dfrac{d\widetilde{x}_t(\lambda)}{d\lambda}$ above yields

$$\begin{aligned}
\frac{d\widetilde{R}_t(\lambda)}{d\lambda} &= \frac{1}{(1-\lambda)}\left[1 + \widetilde{R}_t(\lambda) + \frac{d\widetilde{x}_t(\lambda)}{d\lambda}\right] \\
&= \frac{1}{(1-\lambda)}\left[1 + \widetilde{R}_t(\lambda) - \frac{1 + \overline{R}_t^*}{\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}\right] \\
&= \frac{1}{(1-\lambda)\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}\left[\left(1 + \widetilde{R}_t(\lambda)\right)\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) - \left(1 + \overline{R}_t^*\right)\right]
\end{aligned}$$

Once again, $\lim_{\lambda \to \lambda^+} \dfrac{d\widetilde{R}_t(\lambda)}{d\lambda} \neq \lim_{\lambda \to \lambda^-} \dfrac{d\widetilde{R}_t(\lambda)}{d\lambda}$ at values of λ for which $\widetilde{x}_t(\lambda)$ is a mass point of $G_t(z)$.

From Lemma 2, we know there are two cases to consider. The first is if $\widetilde{R}_t(\lambda) < \overline{R}_t^*$. In that case, we have

$$\left(1+\widetilde{R}_t(\lambda)\right)\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) \;<\; \left(1+\overline{R}_t^*\right)\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) \;\leq\; 1+\overline{R}_t^*$$

In that case, both $\lim_{\lambda\to\lambda^+}\frac{d\widetilde{R}_t(\lambda)}{d\lambda}<0$ and $\lim_{\lambda\to\lambda^-}\frac{d\widetilde{R}_t(\lambda)}{d\lambda}<0$.

Next, suppose $\widetilde{R}_t(\lambda)\geq\overline{R}_t^*$. From Lemma 2, we know $\widetilde{R}_t(\lambda)$ is equal to the equilibrium interest rate $R_t(\lambda)$. We now make use of the fact that the expected return from lending against an asset in market λ is given by

$$1+\overline{R}_t(\lambda)=\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}}(1+R_t(\lambda))\,dG_t(z)+\int_{-\infty}^{\widetilde{x}_t(\lambda)} z\,dG_t(z)$$

Since $\overline{R}_t^*=\sup\limits_{\lambda\in[0,1)}\overline{R}_t(\lambda)$, we have

$$\begin{aligned}1+\overline{R}_t^* &\geq \int_{x_t(\lambda)}^{1+r_t^{\max}}(1+R_t(\lambda))\,dG_t(z)+\int_{-\infty}^{x_t(\lambda)} z\,dG_t(z)\\ &\geq \int_{x_t(\lambda)}^{1+r_t^{\max}}(1+R_t(\lambda))\,dG_t(z)\end{aligned}$$

Since $\widetilde{R}_t(\lambda')=R_t(\lambda')$, it follows that, once again, both $\lim_{\lambda\to\lambda^+}\frac{d\widetilde{R}_t(\lambda)}{d\lambda}\leq 0$ and $\lim_{\lambda\to\lambda^-}\frac{d\widetilde{R}_t(\lambda)}{d\lambda}\leq 0$. This establishes the lemma. ■

From Lemmas 2 and 3, there exists at most one value λ_t^* for which $R_t(\lambda)>\overline{R}_t^*$ for all $\lambda\in[0,\lambda_t^*)$ and $R_t(\lambda^*)=\overline{R}_t$ for all $\lambda\geq\lambda_t^*$. The proposition then follows. ■

Given the schedule of interest rates $R_t(\lambda)$ in Proposition C1, we can next determine how much entrepreneurs should borrow to produce and in which markets.

Proposition C2: *In equilibrium, entrepreneurs with wealth w will borrow $1-w$ units to produce, in a market with an interest rate equal to $R_t(w)$.*

Proof: If entrepreneurs borrow in a market λ where $\lambda\leq w$, they can produce at full capacity and would only need to put down $\lambda\left(\frac{1-w}{1-\lambda}\right)$ resources to borrow $1-w$ to reach full capacity. This would earn them an expected profit of

$$1+y-(1+R_t(\lambda))(1-w)$$

This value is maximized by choosing the lowest interest rate $R_t(\lambda)$. From Proposition C1, we know $R_t(\lambda)$ is weakly decreasing in λ and is therefore maximized at $\lambda=w$.

Next, suppose they borrow in market λ where $\lambda > w$. In that case, they would be unable to produce at full capacity. Since $y \geq y^* > r_t^{\max} \geq R_t(0) \geq R_t(\lambda)$ for all $\lambda \in [0,1)$, they should produce at the maximal capcity possible, or $\frac{w}{\lambda}$. This implies profits would equal

$$\frac{w}{\lambda}(1 + y - x_t(\lambda)) \tag{56}$$

where $x_t(\lambda) = (1-\lambda)(1 + R_t(\lambda))$ is the amount a borrower is required to repay per each unit of resource she borrows. Since $R_t(\lambda) = \overline{R}_t^*$ for all $\lambda \in (\lambda_t^*, 1)$, there would be no benefit to going to market $\lambda > \lambda_t^*$: It would force the entrepreneur to produce less at the same interest rate. The only case that remains is the interval of markets $\lambda \in [w, \lambda_t^*]$. In that case, we can differentiate profits in (56) to get

$$\begin{aligned}
\frac{d}{d\lambda}\left(\frac{w}{\lambda}(1 + y - x_t(\lambda))\right) &= -\frac{w}{\lambda^2}\left[(1 + y - x_t(\lambda)) + \lambda \frac{dx_t(\lambda)}{d\lambda}\right] \\
&= -\frac{w}{\lambda^2}\left[(1 + y - x(\lambda)) - \frac{\lambda\left(1 + \overline{R}_t^*\right)}{\int_x^{1+r_t^{\max}} dG_t(z)}\right] \\
&= -\frac{w}{\lambda^2 \int_x^{1+r_t^{\max}} dG_t(z)}\left[\int_x^{1+r_t^{\max}} (1 + y - x_t(\lambda))\, dG_t(z) - \lambda\left(1 + \overline{R}_t^*\right)\right]
\end{aligned}$$

Since $y \geq y^* > \frac{D+2\varphi e}{(1-\varphi)e} > r_t^{\max}$, we have

$$\frac{d}{d\lambda}\left(\frac{w}{\lambda}(1 + y - x_t(\lambda))\right) < -\frac{w}{\lambda^2 \int_x^{1+r_t^{\max}} dG_t(z)}\left[\int_x^{1+r_t^{\max}} (z - x_t(\lambda))\, dG_t(z) - \lambda\left(1 + \overline{R}_t^*\right)\right]$$

But for $\lambda \leq \lambda_t^*$, the expression in brackets is equal to 0. Hence, borrowing in a market with $\lambda > w$ will be strictly dominated by borrowing in the market with $\lambda = w$. At the optimum, then, each entrepreneur borrow $1 - w$ at a rate of $R_t(w)$. ■

Since $R_t(\lambda)$ is weakly decreasing in, Claim 2 implies entrepreneurs with wealth $w \in [0, \lambda_t^*)$ sort perfectly as agents with wealth w choose market $\lambda = w$. All markets $\lambda \in [0, \lambda_t^*)$ are therefore active, and the cumulative borrowing to produce $F_t^p(\lambda)$ for $\lambda < \lambda_t^*$ is given by

$$F_t^p(\lambda) = \int_0^\lambda (1 - w)(2\varphi e)\, dw = \left(2\lambda - \lambda^2\right)\varphi e$$

Moreover, regardless of which markets agents with $w > \lambda_t^*$ borrow in, entrepreneurs will borrow the amount they need to produce at capacity, and so $F_t^p(1) = \varphi e$. From (49), we have $p_t = e - F_t^p(1) = (1 - \varphi)e$ for all t. This establishes that the price is nonrandom: p_t is uniquely determined at all dates. This suffices to prove Proposition 6 in the text, since now that we know the price is nonstochastic, the analysis in the text applies. In particular, we can use (50) to solve for $F_t^a(\lambda)$ for all $\lambda \in [0,1)$. The last step is to solve for $\overline{R}_t^*$, which we can do similarly as in the text.

C.3 Equilibria with Unrestricted Beliefs

Before we turn to the comparative static exercises, we briefly discuss the role of restriction (51). Suppose we drop this restriction. In that case, we can show the interest rate $R_t(0)$ in the market where agents are infinitely leveraged must equal $r_t^{\max}$ just as when we impose this constraint. Any interest rate $R_t(0) < r_t^{\max}$ cannot be an equilibrium, since in that case agents can earn positive profits from buying assets and so demand for borrowing would be infinite. Conversely, any interest rate $R_t(0) > r_t^{\max}$ would attract no borrowers who want to buy the asset, so $dF_t^a(0) = 0$. But there would be demand for credit from entrepreneurs with $w = 0$ whose productivity exceeds $R_t(0)$. From (50), this would mean $\overline{R}_t(0) = R_t(0)$, so agents would expect to earn above $r_t^{\max}$. But we already argued this couldn't be an equilibrium, since then no agent would buy assets and the market for assets wouldn't clear. Hence, market $\lambda = 0$ is active and the interest rate in this market $R_t(0)$ is uniquely determined.

However, any market $\lambda \in (0,1)$ can now be inactive. For any such λ, suppose we set $R_t(\lambda) > R_t(0)$ and $\overline{R}_t(\lambda) < \overline{R}_t(0)$. Borrowers would optimally avoid market λ given they can borrow more at a lower interest rate in market 0. Lenders would optimally avoid market λ given they can earn more lending in market 0. This implies $dF_t(\lambda) = 0$. In this case, (50) doesn't apply, so any beliefs $R_t(\lambda)$ are admissible. The analysis in the previous subsection still pins down $R_t(\lambda)$ in any active markets as a function of $\overline{R}_t^*$ Which markets are active will affect $\overline{R}_t^*$, but it will not affect the structure on equilibrium. Hence, dropping (51) will introduce additional equilibria in which some markets that would otherwise but active become inactive, sustained by a fear of default even though at the equilibrium interest rate no agent would ever find it optimal to borrow and buy assets even if the had no access to any other markets.

C.4 Comparative Statics

Finally, we consider some comparative statics of equilibria with respect to the floor $\underline{\lambda}$. These are stated in the paper as Propositions 7 and 8. Our first result concerns how the expected return on loans R^D changes with the floor $\underline{\lambda}$.

Proof of Proposition 7: In the text, we show that p^D and $\overline{r}^D$ are increasing and decreasing in $\underline{\lambda}$, respectively. Here, we show that $\overline{R}^D$ is decreasing in $\underline{\lambda}$. Recall that $\overline{R}^D$ is either equal to $\overline{r}^D$ or exceeds $\overline{r}^D$. We already know that $\overline{r}^D$ is decreasing in $\underline{\lambda}$. The result will follow if we can show that the equilibrium value of $\overline{R}^D$ when $\overline{R}^D > \overline{r}^D$ is also decreasing in $\underline{\lambda}$. Since the expected return on loans $\overline{R}^D$ is continuous in $\underline{\lambda}$, it must be continuously decreasing in $\underline{\lambda}$.

When $\overline{R}^D > \overline{r}^D$, all assets are purchased with debt, so $\gamma^D = 1$. This implies the equilibrium conditions are given by

$$\left(1-\lambda^D\right)\overline{R}^D - \left[\frac{D}{\left(1-\left(1-\underline{\lambda}\right)\varphi\right)e} - \lambda^D\left(\frac{1+\overline{R}^D}{1-\pi} - 1\right)\right] = 0 \quad (57)$$

$$1+\overline{R}^D - \left(1-\varphi\left(1-\underline{\lambda}\right)\right)\left[1+\overline{r}^D - \pi\Phi\right] - 2\varphi\int_0^1 \left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\} - w\right]\left[1+R^D\left(\max\left\{w,\underline{\lambda}\right\}\right)\right]dw = 0 \quad (58)$$

Moreover, if $\overline{R}^D > \overline{r}^D$, the floor $\underline{\lambda}$ must be below the cutoff λ^D. For suppose $\underline{\lambda} \geq \lambda^D$. Then all markets where agents might default will be shut down. But without default, the expected return on lending and the expected return on the asset must be equal to ensure both the credit market and asset market clear. Since $\underline{\lambda} < \lambda^D$, we can expand the integral term in (58) to obtain

$$\begin{aligned}\int_0^1 \left[\min\left\{\frac{w}{\underline{\lambda}}, 1\right\} - w\right]\left[1 + R^D\left(\max\left\{w, \underline{\lambda}\right\}\right)\right] &= (1 + R(\underline{\lambda}))\left(\frac{1}{\underline{\lambda}} - 1\right)\int_0^{\underline{\lambda}} w dw + \\ &\int_{\underline{\lambda}}^{\lambda^D} (1 + R(w))(1 - w)\, dw + \left(1 + \overline{R}^D\right)\int_{\lambda^D}^1 (1 - w)\, dw\end{aligned}$$

Substituting in $1 + R^D(\lambda) = \frac{1}{1-\lambda}\left[1 + \frac{D}{(1-(1-\underline{\lambda})\varphi)e} - \frac{\lambda\left(1+\overline{R}_t^D\right)}{1-\pi}\right]$ yields

$$(1 + R(\underline{\lambda}))\left(\frac{1}{\underline{\lambda}} - 1\right)\int_0^{\underline{\lambda}} w dw = \left[1 + \frac{D}{(1 - (1 - \underline{\lambda})\varphi) e} - \underline{\lambda}\left(\frac{1 + \overline{R}^D}{1 - \pi} - 1\right)\right]\frac{\underline{\lambda}}{2} \tag{59}$$

$$\int_{\underline{\lambda}}^{\lambda^D} (1 + R(w))(1 - w)\, dw = \int_{\underline{\lambda}}^{\lambda^D}\left[1 + \frac{D}{(1 - (1 - \underline{\lambda})\varphi) e} - \frac{w\left(1+\overline{R}_t^D\right)}{1-\pi}\right] dw \tag{60}$$

$$\left(1 + \overline{R}^D\right)\int_{\lambda^D}^1 (1 - w)\, dw = \frac{1}{2}\left(1 + \overline{R}^D\right)\left(1 + \left(\lambda^D\right)^2 - 2\lambda^D\right) \tag{61}$$

which we can differentiate with respect to λ^D, R^D, and $\underline{\lambda}$. More compactly, we can write (57) and (58) as

$$\begin{aligned} h_1\left(\overline{R}^D, \lambda^D\right) &= 0 \\ h_2\left(\overline{R}^D, \lambda^D\right) &= 0 \end{aligned}$$

Totally differentiating allows us to infer the comparative statics of the equilibrium $\overline{R}^D$ and λ^* with respect to any variable a as

$$\begin{bmatrix} \frac{\partial h_1}{\partial \overline{R}^D} & \frac{\partial h_1}{\partial \lambda^D} \\ \frac{\partial h_2}{\partial \overline{R}^D} & \frac{\partial h_2}{\partial \lambda^D} \end{bmatrix}\begin{bmatrix} d\overline{R}^D/da \\ d\lambda^D/da \end{bmatrix} = \begin{bmatrix} -\frac{\partial h_1}{\partial a} \\ -\frac{\partial h_2}{\partial a} \end{bmatrix}$$

Differentiating (57) and (58) using expressions (59)-(61) yields

$$\begin{aligned} &\frac{\partial h_1}{\partial \overline{R}^D} = 1 - \lambda^D + \frac{\lambda^D}{1-\pi} && \frac{\partial h_1}{\partial \lambda^D} = \frac{\pi(1+\overline{R}^D)}{1-\pi} \\ &\frac{\partial h_2}{\partial \overline{R}^D} = 1 + \varphi\left[\frac{1}{1-\pi}\left(\lambda^D\right)^2 - \left(1 - \lambda^D\right)^2\right] && \frac{\partial h_2}{\partial \lambda^D} = 0 \end{aligned}$$

When we evalute comparative statics with respect to $\underline{\lambda}$, we now have

$$\begin{aligned}\begin{bmatrix} d\overline{R}^D/d\underline{\lambda} \\ d\lambda^D/d\underline{\lambda} \end{bmatrix} &= \begin{bmatrix} \frac{\partial h_1}{\partial \overline{R}^D} & \frac{\partial h_1}{\partial \lambda^D} \\ \frac{\partial h_2}{\partial \overline{R}^D} & \frac{\partial h_2}{\partial \lambda^D} \end{bmatrix}^{-1}\begin{bmatrix} \frac{dh_1}{d\underline{\lambda}} \\ \frac{dh_2}{d\underline{\lambda}} \end{bmatrix} \\ &= \frac{\varphi}{\kappa}\begin{bmatrix} 0 & \frac{\pi}{1-\pi}\left(1 + \overline{R}^D\right) \\ 1 + \varphi\frac{\left(\lambda^D\right)^2}{1-\pi} - \varphi\left(1 - \lambda^D\right)^2 & -\left(1 - \lambda^D + \frac{\lambda^D}{1-\pi}\right)\end{bmatrix}\begin{bmatrix} -\frac{D}{(1-(1-\underline{\lambda})\varphi)^2 e} \\ -\frac{2D\left(1+\lambda^D\varphi\right)}{(1-(1-\underline{\lambda})\varphi)^2 e} - (1 + \pi\Phi) \end{bmatrix}\end{aligned}$$

where $\kappa = \frac{\pi(1+\overline{R}^D)}{1-\pi}\left(1 + \varphi\frac{\left(\lambda^D\right)^2}{1-\pi} - \varphi\left(1 - \lambda^D\right)^2\right) > 0$. It follows that

$$\frac{d\overline{R}^D}{d\underline{\lambda}} = -\varphi\left(1 + \varphi\left[\frac{1}{1-\pi}\left(\lambda^D\right)^2 - \left(1 - \lambda^D\right)^2\right]\right)^{-1}\left[\frac{2D\left(1 + \lambda^D\varphi\right)}{\left(1 - (1 - \underline{\lambda})\varphi\right)^2 e} + (1 + \pi\Phi)\right] < 0$$

Since $\overline{R}^D$ is decreasing in $\underline{\lambda}$ whether $\overline{R}^D > \overline{r}^D$ or $\overline{R}^D = \overline{r}^D$, the claim follows. ■

Next, we examine how changing $\underline{\lambda}$ affects the expected costs of default $\gamma^D \Phi p^D$. Since we already know that p^D is increasing in $\underline{\lambda}$, any changes in expected default costs occur entirely through γ^D. Our next result argues that there exists cutoffs Λ_0 and Λ_1 such that $d\gamma^D/d\underline{\lambda} = 0$ when $\underline{\lambda} < \Lambda_0$ or $\underline{\lambda} > \Lambda_1$. When $\Lambda_0 < \underline{\lambda} < \Lambda_1$, it is hard to characterize $d\gamma^D/d\underline{\lambda}$, but we can show that it must be decreasing for some $\underline{\lambda}$ in this interval.

Proof of Proposition 8: Define

$$\rho(\underline{\lambda}) = \frac{\overline{R}^D}{(1-(1-\underline{\lambda})\varphi)}$$

Then

$$\frac{d\rho(\underline{\lambda})}{d\underline{\lambda}} = \frac{d\overline{R}^D/d\underline{\lambda} - \varphi\rho(\underline{\lambda})}{1-(1-\underline{\lambda})\varphi} < 0$$

Since

$$\overline{R}^D/\overline{r}^D = [(1-\pi)D + \pi d]\,\rho(\underline{\lambda})$$

it follows that the ratio $\overline{R}^D/\overline{r}^D$ is decreasing in $\underline{\lambda}$. Hence, there exists a value $\Lambda_0 \geq 0$ such that $\overline{R}^D > \overline{r}^D$ for $\underline{\lambda} < \Lambda_0$ and $\overline{R}^D = \overline{r}^D$ for $\underline{\lambda} \geq \Lambda_0$. Since $\overline{R}^D > \overline{r}^D$ when $\underline{\lambda} < \Lambda_0$, then $\gamma^D = 1$ for $\underline{\lambda} < \Lambda_0$. It follows that expected default costs $\pi\gamma^D\Phi p^D = \pi\Phi p^D$ are increasing in $\underline{\lambda}$ in this region. A higher $\underline{\lambda}$ for $\lambda < \Lambda_0$ reduces the amount entrepreneurs produce and increases the foregone output when dividends fall. Each cohort will therefore be left with fewer goods to consume.

We next turn to the case where $\underline{\lambda} \geq \Lambda_0$. Here, we know $\overline{R}^D = \overline{r}^D$. Substituting this into (57) yields

$$\left(1-\lambda^D\right)\left(1+\overline{r}^D\right) = \left[1 + \frac{D}{(1-(1-\underline{\lambda})\varphi)\,e} - \frac{\lambda^D}{1-\pi}\left(1+\overline{r}^D\right)\right]$$

which, upon rearranging,

$$\frac{(1-\pi)(D-d)}{(1-(1-\underline{\lambda})\varphi)\,e + (1-\pi)D + \pi d} = \lambda^D$$

From this, we can conclude that $\lambda^D \geq \underline{\lambda}$ if

$$\frac{(1-\pi)(D-d)}{(1-(1-\underline{\lambda})\varphi)\,e + (1-\pi)D + \pi d} \geq \underline{\lambda}$$

or, upon rearranging, if

$$(1-\pi)(D-d) \geq \underline{\lambda}\left[(1-(1-\underline{\lambda})\varphi)\,e + (1-\pi)D + \pi d\right] \tag{62}$$

The RHS of (62) is a quadratic in $\underline{\lambda}$ with a positive coefficient on the quadratic term. The inequality is satisfied when $\underline{\lambda} = 0$ and violated when $\underline{\lambda} = 1$. This implies there exists a cutoff $\Lambda_1 \in (0,1)$ such that $\lambda^D > \underline{\lambda}$ if $\underline{\lambda} \in [0, \Lambda_1)$ and $\lambda^D < \underline{\lambda}$ if $\underline{\lambda} \in (\Lambda_1, 1)$. The fact that $\Lambda_1 \geq \Lambda_0$ comes from the fact that $\overline{R}^D = \overline{r}^D$ only when $\underline{\lambda} \geq \Lambda_0$, yet at $\underline{\lambda} = \Lambda_1$ we have

$$\overline{R}^D = R\left(\lambda^D\right) = R(\underline{\lambda}) = R(\Lambda_1)$$

But by construction, we know that $R(\underline{\lambda})$ when $\underline{\lambda} = \Lambda_1$ is equal to $\overline{r}^D$. This implies $\Lambda_1 \geq \Lambda_0$.

When $\underline{\lambda} > \Lambda_1$ we know that there is no borrowing against the asset, so $\gamma^D = 0$. Expected defalt costs are 0, and so the only effect of increasing $\underline{\lambda}$ is to reduce production. This will leave fewer goods for each cohort to consume.

Finally, we turn to the case where $\Lambda_0 < \underline{\lambda} < \Lambda_1$. We do not analyze this case in general. However, when $\lambda^D = \underline{\lambda}$, the interest rate in all active markets would equal $\overline{R}^D$, since the only active markets are those with $\lambda \geq \underline{\lambda} = \lambda^D$. Since $\underline{\lambda} \geq \Lambda_0$, we know that $\overline{R}^D = \overline{r}^D$ and so the interest rate in all active markets is $\overline{r}^D$. The equilibrium condition that determines γ^D is given by

$$\begin{aligned}
\left(1+\overline{r}^D\right) &= \left(1-(1-\underline{\lambda})\varphi\right)\left[1+\overline{r}^D-\gamma^D\pi\Phi\right]+2\varphi\int_0^1\left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\}-w\right]\left[1+R^D\left(\max\left\{w,\underline{\lambda}\right\}\right)\right]dw \\
&= \left(1-(1-\underline{\lambda})\varphi\right)\left[1+\overline{r}^D-\gamma^D\pi\Phi\right]+2\varphi\left(1+\overline{r}^D\right)\int_0^1\left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\}-w\right]dw \\
&= \left(1-(1-\underline{\lambda})\varphi\right)\left[1+\overline{r}^D-\gamma^D\pi\Phi\right]+2\varphi\left(1+\overline{r}^D\right)\left[\underline{\lambda}/2+(1-\underline{\lambda})-1/2\right] \\
&= 1+\overline{r}^D-\gamma^D\left(1-(1-\underline{\lambda})\varphi\right)\pi\Phi
\end{aligned}$$

Hence, when $\underline{\lambda} = \Lambda_1$, we have $\gamma^D = 0$. For $\underline{\lambda} < \Lambda_1$, however, $\gamma^D > 0$, since

$$\int_0^1\left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\}-w\right]\left[1+R^D\left(\max\left\{w,\underline{\lambda}\right\}\right)\right]$$

will be strictly greater than $\frac{1}{2}(1-\underline{\lambda})\left(1+\overline{r}^D\right)$.Hence, in the limit as $\underline{\lambda} \uparrow \Lambda_1$, we have $d\gamma^D/d\underline{\lambda} < 0$ expected default costs $\pi\gamma^D\Phi p^D$ must be decreasing in $\underline{\lambda}$ since it goes from a positive value to 0.

Finally, to show that could generate a Pareto improvement, observe that increasing $\underline{\lambda}$ while dividends are high will make the initial old at date 0 better off given p_0^D increases. Cohorts born after dividends have fallen will be unaffected if $\underline{\lambda}$ is only increased while dividends are high. Cohorts who are born while dividends are high expect to consume the dividends from the asset net of default costs $E\left[d_{t+1}\right] - \Phi\pi\gamma^D p_t^D$ as well as the output produced by entrepreneurs. If Φ is sufficiently large and φ is small, we can promise these agents a higher expected consumption. ■

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