Accounting for the Slow Recovery from the Great Recession: The Role of Credit Constraints. PRELIMINARY AND INCOMPLETE

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Abstract

We study a model with heterogeneous producers that face collateral and cash-in-advance constraints. A tightening of the collateral constraint results in a credit-crunch-generated recession that reproduces several features of the financial crisis that unraveled in 2007 in the United States. As a reaction to the crisis, the US government increased substantially the net supply of its liabilities (money and bonds, which at the zero bound are perfect substitutes). A calibrated model that incorporates both the credit crunch and the policy response of the government can account for a substantial fraction of the slow recovery in investment and output, as observed since the great recession.

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1 Introduction

In this paper, we study the effect of monetary and debt policy following a negative shock to the efficiency of the financial sector. We build a model that combines the financial frictions literature, such as Kiyotaki [1998], Moll [2014], and Buera and Moll [2015], with the monetary literature, such as Lucas [1982] and Svensson [1985]. The first branch of the literature gives rise to a non trivial financial market by imposing collateral constraints on debt contracts. The second gives rise to a money market by imposing cash-in-advance constraints on purchases. We show that a calibrated version of the model can quantitatively match several salient features of the US experience since 2007. In particular, the combination of the credit crunch together with the substantial increase in government liabilities (safe assets) explains the slow recovery in productivity, investment and output that has characterized the post-crisis period.

An essential role of financial markets is to reallocate capital from wealthy individuals with no profitable investment projects to individuals with profitable projects and no wealth. The efficiency of these markets determines the equilibrium allocation of physical capital across projects and therefore equilibrium intermediation and total output. The financial frictions literature, from which we build, studies models of intermediation with these properties, the key friction being an exogenous collateral constraint on investors.[1] The equilibrium allocation critically depends on the nature of the collateral constraints: the tighter the constraints, the less efficient the allocation of capital and the lower are total factor productivity and output, so a tightening of the collateral constraint creates disintermediation and a recession. We interpret this reduction in the ability of financial markets to properly allocate capital across projects as the negative shock that hit the US economy at the end of 2007.[2]

If the shock to the collateral constraint that causes the recession is sufficiently large, the equilibrium real interest rate becomes negative and persistent as long as the shock is persistent. At the same time, investment goes down. We find these properties of the model particularly attractive because a very special feature of recent years is a substantial and persistent gap between real output and its trend, together with a substantial and persistent negative real interest rate and low investment rates.

[1]We closely follow the work of Buera and Moll [2015], who apply the model originally developed by Moll [2014] to study business cycles, to analyze the role of credit markets in economic development. See Kiyotaki [1998] for an earlier version of a related framework.

[2]As we explain in detail in Section 4.1, the behavior of the real interest rate, the variable we use to identify the shock, dates the beginning of the recession in the third quarter of 2007.

We modify this basic model by imposing a cash-in-advance constraint on households that gives rise to a demand for money balances. This allows us to study the effect of changes in the outside supply of liquid assets. This is important since the policy reaction to the recession was, among other things a large increase in government supplied liabilities, on the order of more than 40% of GDP. As it turns out, this increase in liquidity has additional real effects because Ricardian equivalence does not hold in this model: the collateral constraints imply that active entrepreneurs who are credit constrained face a real interest rate that is different from (higher than) the equilibrium real interest rate. We show that the increase in liquidity prevents the real interest rate from falling too much. We also show that this ameliorates the drop in productivity. But it also crowds out private investment, making the recovery from the recession much slower.

The reason for the drop in real interest rates is that savings must be reallocated to lower productivity entrepreneurs, but they will only be willing to do so if the real interest rate is lower. To put it differently, the "demand" for loans falls, which in turn pushes down the real interest rate. Several other properties of the recession generated by a tightening of the collateral constraint in the model are in line with the events that have unfolded since 2008, such as the persistent negative real interest rate, the sustained periods with an effective zero bound on nominal interest rates, and the substantial drops in investment, total factor productivity, and output, all of which driven by a single shock.

The model has very few parameters. A quantitative version of the model, calibrated to match the evolution of the real interest rate, performs reasonably well for most variables that have not been targeted. The one exception is labor input, which dropped substantially in the United States after 2007, explains around half of the drop in output, and is constant in the model. We therefore explore two variations to the model that improve its performance (WORK IN PROGRESS). First, instead of keeping labor constant, we assume a participation rate that exogenously declines over time, due to demographics factors, as emphasized in Aaronson et al. [2006]. Second, we allow for frictions in the setting of wages that are customary in the New Keynesian literature. We claim that the model does a good job in explaining the events that unfolded following 2008. We then use the model to discuss how productivity, investment and output ought to behave as the shock to the collateral constraints dissipates and under alternative assumptions regrading the behavior of the supply of government provided safe assets.

The paper proceeds as follows. In Section 2, we present the model and characterize the individual problems. In Section 3, we define an equilibrium and characterize its properties. In Section 4 we calibrate the full model and show how it behaves relative to the data once we take into account the injection of outside liquidity observed since 2008. We also perform several simulations under alternative assumptions regarding the future evolution of total government liabilities.

Related Literature We consider a monetary version of the model in Buera and Moll [2015] and Moll [2014];Kiyotaki [1998] is an earlier example that focuses on a two-point distribution of shocks to entrepreneurial productivity. This framework is related to a long tradition that studies the role of firms' balance sheets in business cycles and during financial crises, including Bernanke and Gertler [1989], Kiyotaki and Moore [1997], Bernanke et al. [1999], Cooley et al. [2004], Jermann and Quadrini [2012].[3]

Kiyotaki and Moore [2012] study a monetary economy in which entrepreneurs face stochastic investment opportunities and frictions to issue and resell equity on real assets. They also consider the aggregate effects of a shock to the ability to resell equity. In their environment, money is valuable provided that frictions to issue and resell equity are tight enough. They use their model to study the effect of open market operations that consist of the exchange of money for equity.

Guerrieri and Lorenzoni [2011] consider a model in which workers face idiosyncratic labor shocks where a credit crunch leads to an increase in the demand of bonds and therefore results in negative real rates. Although our model also generates a large drop in the real interest rate, the forces underlying this result are different. In our framework, the drop in the real interest rate is the consequence of a collapse in the ability of productive entrepreneurs to supply bonds (i.e., to borrow from the unproductive entrepreneurs and workers), as opposed to an increase in the demand for bonds by these agents. In our model, a credit crunch has an opposite, negative effect on investment. The closest paper is Buera and Nicolini [2017], who use the same model to explain the effect of alternative policies.

[3]See Buera and Moll [2015] for a detailed discussion of the connection between the real version of our framework and related approaches in the literature.

2 The Model

In this section, we describe the model, which closely follows the framework in Moll [2014], modified by imposing a cash-in-advance constraint on the consumer's decision problem and by assuming permanent productivity types. The analysis will be restricted to a perfect foresight economy in which, starting at the steady state, all agents learn at time zero that, starting next period, the collateral constraint will be tightened for several periods. The model is the same as the one in Buera and Nicolini.

2.1 Households

All agents have identical preferences, given by

$$\sum_{t=0}^{\infty} \beta^t \left[\nu \log c_{1t}^j + (1-\nu) \log c_{2t}^j\right], \tag{1}$$

where c_{1t}^j and c_{2t}^j are consumption of the cash good and of the credit good, for agent j at time t, and $\beta < 1$. Each agent also faces a cash-in-advance constraint,

$$c_{1t}^j \leq \frac{m_t^j}{p_t}, \tag{2}$$

where m_t^j is the beginning of period money holdings and p_t is the money price of consumption at time t.

The economy is inhabited by two classes of agents, a mass L of workers and a mass 1 of entrepreneurs, which we now describe.

Entrepreneurs Entrepreneurs are heterogeneous with respect to their productivity (which is exogenous) and their wealth (which is endogenous). We assume that the productivity of each entrepreneur, $z \in Z \subset \mathbb{R}^+$, is constant through her lifetime. We let $\Psi(z)$ be the measure of entrepreneurs of type z. Every period, each entrepreneur must choose whether to be active in the following period (to operate a firm as a manager) or to be passive and offer her wealth in the credit market. Thus, each entrepreneur has four state variables: her financial wealth (capital plus bonds), money holdings, the occupational choice (active or passive) made last period, and productivity. She must decide the labor demand if active, how much to consume of each good, and whether to be active in the following period, and, if so, how much capital to invest in her own firm. An entrepreneur's investment is constrained by her financial wealth at the end of period a and the amount of bonds she can sell $-b$, $k \leq -b + a$, where we assume that the amount of bonds that can be sold is limited by a simple collateral constraint of the form $-b^j \leq \theta k^j$, for some exogenously given $\theta \in [0, 1)$.[4]

Type-z entrepreneurs use capital and labor to produce output according to

$$y = (zk)^{\alpha} l^{1-\alpha}.$$

This constant returns technology implies that the net-of-labor-cost revenues of entrepreneurs are a linear function of capital stock, $\varrho z k$, where $\varrho = \alpha \left((1-\alpha)/w\right)^{(1-\alpha)/\alpha}$ is the return to the effective units of capital zk and w denotes the real wage. In contrast with span-of-control models, these entrepreneurs face constant marginal product of capital. Thus, as we show below, their optimal decisions exhibit a corner solution: either they are inactive or they borrow all the way up to their collateral constraint.[5]

The end-of-period investment and leverage choice of entrepreneurs with ability z and wealth a solves the following linear problem:

$$\begin{aligned} \max_{k,d} \quad & \varrho z k + (1-\delta)k + (1+r)b \\ & k \leq a - b, \\ & -b \leq \theta k, \end{aligned}$$

where r is the real interest rate. Because the problem is linear, the optimal capital and leverage choices are given by the following policy rules, with a simple threshold property,

$$k(z,a) = \begin{cases} a/(1-\theta), & z \geq \hat{z} \\ 0, & z < \hat{z} \end{cases}, \quad b(z,a) = \begin{cases} -(1/(1-\theta) - 1)a, & z \geq \hat{z} \\ a, & z < \hat{z}, \end{cases}$$

where $\hat{z}$ solves $\varrho \hat{z} = r + \delta$. Given entrepreneurs' optimal investment and leverage decisions, they face a linear return to their non monetary wealth, which is a simple

[4]If $\theta = 1$, then all capital can be pledged and individual wealth plays no role. This is equivalent to imposing no collateral constraints so the model becomes a standard representative agent Solow model with a cash-in-advance constraint.

[5]This property allows for a tight characterization of the evolution of aggregate variables.

function of their productivity

$$R(z) = \begin{cases} 1 + r, & z < \hat{z} \\ \frac{(\varrho z - r - \delta)}{(1-\theta)} + 1 + r, & z \geq \hat{z}. \end{cases} \tag{3}$$

Given these definitions, the budget constraint of entrepreneur j, with net worth a_t^j and productivity z^j, will be given by

$$c_{1t}^j + c_{2t}^j + a_{t+1}^i + \frac{m_{t+1}^j}{p_t} = R_t(z^j)a_t^j + \frac{m_t^j}{p_t} - T_t^e, \tag{4}$$

where we assume that lump-sum taxes (transfers if negative) do not depend on the productivity of entrepreneurs.[6]

These budget constraints imply that agents choose, at t, money balances m_{t+1}^i for next period, as the cash-in-advance constraints (2) make clear. Thus, we are adopting the timing convention of Svensson [1985], in which agents buy cash goods at time t with the money holdings they acquired at the end of period $t-1$.[7]

Workers There is a mass L of identical workers, endowed with a unit of time each period that they inelastically supply to the labor market. Thus, their budget constraints are given by

$$c_{1t}^W + c_{2t}^W + a_{t+1}^W + \frac{m_{t+1}^W}{p_t} = (1 + r_t)a_t^W + w_t + \frac{m_t^W}{p_t} - T_t^W, \tag{5}$$

where a_{t+1}^W and m_{t+1}^W are real financial assets and nominal money holdings chosen at time t and T_t^W are lump-sum taxes. We impose on workers a non borrowing constraint, so $a_t^W \geq 0$ for all t.[8]

[6]Note that it is efficient to transfer resources from low productivity entrepreneurs to high productivity ones. This can be achieved by productivity-specific taxes. Given the exogenous nature of the collateral constraints, we do not find those policies interesting.

[7]An advantage of this timing is that it treats all asset accumulation decisions symmetrically, using the standard timing from capital theory, where production by entrepreneurs at time t is done with capital goods accumulated at the end of period $t-1$.

[8]This is a natural constraint to impose. It is equivalent to impose on workers the same collateral constraints entrepreneurs face, since workers will never decide to hold capital in equilibrium.

2.2 Optimality Conditions

The optimal problem of agents is to maximize (1) subject to (2) and (4) for entrepreneurs or (5) for workers. To save on notation, we drop the index for individual entrepreneurs j unless strictly necessary.

We first briefly explain the zero bound equilibrium restriction on the nominal interest rate that arises from the agent's optimization problem because this is a key aspect of the model. We then discuss the other first-order conditions.

In this economy, gross savings (demand for bonds) come from inactive entrepreneurs and, potentially, from workers. Note that the return on holding financial assets for these agents is $R_t(z) = (1 + r_t)$, whereas the return on holding money—ignoring the liquidity services—is given by p_t/p_{t+1}. Thus, if there is intermediation in equilibrium, it must be the case that

$$(1 + r_t)\frac{p_t}{p_{t-1}} - 1 \geq 0 \text{ for all } t. \tag{6}$$

The first-order conditions of the household's problem imply:

$$\frac{1}{\beta}\frac{c_{2t+1}(z)}{c_{2t}(z)} = R_{t+1}(z), \quad t \geq 0, \tag{7}$$

$$\frac{\nu}{1-\nu}\frac{c_{2t+1}(z)}{c_{1t+1}(z)} = R_{t+1}(z)\frac{p_{t+1}}{p_t}, \quad t \geq 1. \tag{8}$$

Solving forward the period budget constraint (4), using the optimal conditions (7) and (8) for all periods, and assuming that the cash-in-advance constraint is binding at the beginning of period $t = 0$, we obtain the solutions for consumption of the credit good and financial assets for agents that face a strictly positive opportunity cost of money in period $t + 1$,[9]

$$\begin{aligned} c_{2t}(z) &= \frac{(1-\nu)(1-\beta)}{1-\nu(1-\beta)}\left[R_t(z)a_t - \sum_{j=0}^{\infty}\frac{T^e_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)}\right] \\ a_{t+1}(z) &= \beta\left[R_t(z)a_t - \sum_{j=0}^{\infty}\frac{T^e_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)}\right] + \sum_{j=1}^{\infty}\frac{T^e_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)}. \end{aligned} \tag{9}$$

This solution exhibits the standard property implied by log-utility: consumption

[9]Note that it could be possible that initial money holdings are so large for an active entrepreneur that the cash-in-advance constraint will not be binding in the first period. This case will not be relevant provided initial real cash balances are not too big.

is proportional to current wealth, which is equal to the current value of the assets minus the present value of taxes, the term in brackets on the right-hand side of the consumption equation. The counterpart of the proportionality of consumption is the constant savings rate once the provision for future taxes is taken into account - the second term on the right hand side of the wealth equation. These properties are key to being able to write the law of motion for macroeconomic aggregates in Section 3 below.

Note that future taxes are discounted using type-specific rates of return. These rates of return are the same as the real interest rate on government bonds for inactive entrepreneurs, but they are higher for active entrepreneurs.

We will use type-specific discounted sums as in (9) several times in what follows. To simplify the expressions, we define

$$\prod_{s=1}^{j} R_{t+s}(z) = Q_{t+j}(z) \text{ and } \prod_{s=1}^{j}(1+r_{t+s}) = q_{t+j},$$

where the first variable is agent specific and the second discounts using the real interest rate.

These equations always characterize the solution for active entrepreneurs even when nominal interest rates are zero. The reason is that for them, the opportunity cost of holding money is given by $R_t(z)p_{t+1}/p_t > (1+r_t)\, p_{t+1}/p_t \geq 1$, where the last inequality follows form (6). The solution also characterizes the optimal behavior of inactive entrepreneurs, as long as $(1+r_t)\, p_{t+1}/p_t - 1 > 0$.

The solution for inactive entrepreneurs in periods in which the nominal interest rate is zero, $(1+r_t)\, p_{t+1}/p_t - 1 = 0$, is

$$a_{t+1}(z) + \frac{m_{t+1}(z)}{p_t} - \frac{m_{t+1}^T(z)}{p_t} = \beta \left[R_t(z)a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}^e}{Q_{t+j}(z)} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}^e}{Q_{t+j}(z)},$$

where

$$\frac{m_{t+1}^T(z)}{p_t} = \frac{\nu(1-\beta)\beta}{1-\nu(1-\beta)} \left[R_t(z)a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}^e}{Q_{t+j}(z)} \right] \tag{10}$$

are the real money balances that will be used for transaction purposes in period $t+1$. Thus, $m_{t+1}/p_t - m_{t+1}^T/p_t \geq 0$ are the excess real money balances, hoarded from period

t to $t+1$.

The optimal plan for workers is slightly more involved because their income is non-homogeneous in their net worth and they will tend to face binding borrowing constraints in finite time. In particular, as long as the $(1+r_\infty)\beta < 1$, as will be the case in the equilibria we will discuss, where r_∞ is the real interest rate in the steady state, workers drive their wealth to zero in finite time and are effectively hand-to-mouth consumers in the long run. That is, for sufficiently large t,

$$c^W_{2,t} = \frac{(1-\nu)\left(w_t - T^W_t\right)}{1-\nu(1-\beta)} \text{ and } c^W_{1,t+1} = \frac{m^W_{t+1}}{p_{t+1}} = \frac{\nu(w_t - T^W_t)}{1-\nu(1-\beta)}\frac{\beta p_t}{p_{t+1}}.$$

Along a transition, workers may accumulate assets for a finite number of periods. This would typically be the case if they expect a future drop in their wages—as in the credit crunch we consider—or if they receive a temporarily large transfer, $T^W_t < 0$.

2.3 Demographics

Relative to the model in Moll [2014], we assumed that the productivity types are permanent. To guarantee that there is a non degenerated distribution of wealth shares across productivity types in the stationary equilibrium, we assume a stochastic life-cycle structure without annuities markets.[10]

Specifically, we assume that a fraction $1-\gamma$ of entrepreneurs depart for Nirvana every period and are replaced by an equal number of new entrepreneurs. The productivity z of the new entrepreneurs is drawn from the same distribution $\Psi(z)$, i.i.d. across entrepreneurs and over time. There are no annuity markets, so each new entrepreneur inherits the assets of a randomly drawn departed entrepreneur. Agents do not care about future generations, so if we let $\hat{\beta}$ be the pure discounting factor, they discount the future with the compound factor $\beta = \hat{\beta}\gamma$, which is the one we used above.

2.4 The Government

In every period, the government chooses the money supply M_{t+1}, issues one-period bonds B_{t+1}, and uses type-specific lump-sum taxes (subsidies) T^e_t and T^W_t. Government

[10] These alternative assumptions allow us to obtain simple closed-form expressions for the policy functions of entrepreneurs in the presence of lump-sum taxes, which are useful to illustrate the effect of alternative monetary and fiscal policies.

policies are constrained by a sequence of period-by-period budget constraints:

$$B_{t+1} - (1 + r_t)B_t + \frac{M_{t+1}}{p_t} - \frac{M_t}{p_t} + T_t^e + LT_t^W = 0, \quad t \geq 0. \tag{11}$$

3 Equilibrium

Given policies $\{M_t, B_t, T_t^e, T_t^W\}_{t=0}^{\infty}$ and collateral constraints $\{\theta_t\}_{t=0}^{\infty}$, an equilibrium is given by prices $\{r_t, w_t, p_t\}_{t=0}^{\infty}$ and corresponding quantities such that:

- Entrepreneurs and workers maximize their utility, taking as given prices and policies,
- The government budget constraint is satisfied, and
- Bond, labor, and money markets clear:

$$\int b_{t+1}^j dj + Lb_t^W + B_{t+1} = 0, \quad \int l_t^j dj = L, \quad \int m_t^j dj + Lm_t^W = M_t, \quad \text{for all } t.$$

To illustrate the mechanics of the model, we now provide a partial characterization of the equilibrium dynamics of the economy for the case in which the zero lower bound is never binding, $1 + r_{t+1} > p_t/p_{t+1}$ for all t, workers are hand to mouth, $a_t^W = 0$ and pay no taxes $T_t^w = 0$ for all t, and the share of cash goods is arbitrarily small, $\nu \approx 0$.

The state of the economy at any point in time is given by the capital stock K_t, the measure of wealth $\Phi_t(z)$, and the cutoff $\hat{z}_t$. We now show how the equilibrium conditions determine the new values of these three objects.

First, note that integrating the production function of all active entrepreneurs, equilibrium output is given by a Cobb-Douglas function of aggregate capital K_t, aggregate labor L, and aggregate productivity Z_t,

$$Y_t = Z_t K_t^{\alpha} L^{1-\alpha}, \tag{12}$$

where aggregate productivity is given by the wealth-weighted average of the productivity of active entrepreneurs, $z \geq \hat{z}_t$,

$$Z_t = \left(\frac{\int_{\hat{z}_t}^{\infty} z\Phi_t(dz)}{\int_{\hat{z}_t}^{\infty} \Phi_t(dz)} \right)^{\alpha}. \tag{13}$$

The higher the wealth of the high productivity entrepreneur, the larger their relative size and the higher is aggregate TFP. Note also that Z_t is an increasing function of the cutoff $\hat{z}_t$.

To obtain the evolution of aggregate capital, we integrate over the individual optimal saving decisions of entrepreneurs and use market clearing conditions. Because of the proportional optimal decision rules obtained in (9),this results in a linear function of aggregate output, the initial capital stock, and the aggregate of the (individual-specific) present value of taxes,

$$\begin{aligned} K_{t+1}+B_{t+1} &= \beta\left[\alpha Y_t+(1-\delta)K_t+(1+r_t)B_t-\int_0^\infty\sum_{j=0}^\infty\frac{T^e_{t+j}}{Q_{t+j}(z)}\Psi\left(dz\right)\right] \\ &\quad +\int_0^\infty\sum_{j=1}^\infty\frac{T^e_{t+j}}{Q_{t+j}(z)}\left(dz\right). \end{aligned} \tag{14}$$

After some algebra it is possible to show that if $Q_{t+j}(z)=(1+r_t)$ for all z, then (14) can be written as

$$K_{t+1}=\beta\left[\alpha Y_t+(1-\delta)K_t\right]$$

which implies a law of motion for capital as in Solow (1956) CHECK model: saving rates are constant. Note that this is also the solution if government debt and taxes are always zero. The reason the law of motion is different in our model is precisely because of the lack of Ricardian equivalence. The way the lack of Ricardian equivalence affects the allocation will become clear in out quantitative Section. But some intuition can be obtained by noting that the first term of the first sum on the right-hand side of (14) is time zero, whereas the first term in the second sum on the right-hand side is one. We can then use the term corresponding to $j=0$ of the sum that is inside the brackets and subtract it from the value of the debt $(1+r_t)B_t$. We can then solve forward the government budget constraint (11), using that $\nu\approx 0$, and replace the term $(1+r_t)B_t-T^e_t$ and substitute it into (14) to obtain

$$K_{t+1}=\beta\left[\alpha Y_t+(1-\delta)K_t\right]+(1-\beta)\int_0^\infty\sum_{j=1}^\infty T^e_{t+j}\left[\frac{1}{Q_{t+j}(z)}-\frac{1}{q_{t+j}}\right]\Psi(dz). \tag{15}$$

The first term gives the evolution of aggregate capital in an economy without taxes.

In this case, aggregate capital in period $t+1$ is a linear function of aggregate output and the initial level of aggregate capital. The second term captures the departure from Ricardian equivalence. For example, imagine a case in which there is positive debt and taxes are positive every period, equal to the interest payments. Since $Q_{t+j}(z) > q_{t+j}$ for all $z > \widehat{z}$, the second term is negative, reflecting the fact that public debt crowds out private investment and results in lower aggregate capital next period.

Given the capital stock at $t+1$, the evolution of the wealth measure is given by

$$\begin{aligned} \Phi_{t+1}(z) \quad = \quad & \gamma \left[\beta \left[R_t(z)\, \Phi_t(z) - \sum_{j=0}^{\infty} \frac{T_{t+j}^{e} \Psi(z)}{Q_{t+j}(z)} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}^{e} \Psi(z)}{Q_{t+j}(z)} \right] \\ & + (1-\gamma)\, \Psi(z)\, (K_{t+1} + B_{t+1}), \end{aligned} \tag{16}$$

where the first term on the right-hand side reflects the decision rules of the γ fraction of entrepreneurs that remain alive, and the second reflects the exogenous allocation of the assets of departed entrepreneurs among the new generation.

Then, given the (exogenous) value for θ_{t+1} and the wealth measure $\Phi_{t+1}(z)$, the cutoff for next period is determined by the bond market clearing condition

$$\int_0^{\hat{z}_{t+1}} \Phi_{t+1}(dz) = \frac{\theta_{t+1}}{1-\theta_{t+1}} \int_{\hat{z}_{t+1}}^{\infty} \Phi_{t+1}(dz) + B_{t+1}. \tag{17}$$

The left-hand side is total wealth of inactive entrepreneurs or, equivalently, the total supply of funds because we assumed workers to be hand to mouth. The first term on the right-hand side is total private demand for funds, which is equal to the leverage times the wealth of active entrepreneurs. The second term is government net demand for funds.

Notice that (17) implies that the amount of debt affects, through its effect on the credit market, the cutoff $\hat{z}_{t+1}$. It follows from (13), that it will also affect total factor productivity. In addition, because it changes excess demand for credit, it will also affect the real interest rate.

Finally, we describe the determination of the price level. In the previous derivations, in particular, to obtain (15) and (16), we have used that $\nu \approx 0$, and therefore, the money market clearing condition is not necessarily well defined.[11] More generally, given monetary and fiscal policy, the price level is given by the equilibrium condition

[11] To determine the price level in the cashless limit, we let $M_{t+1}, \nu \to 0$, $M_{t+1}/\nu \to \tilde{M}_{t+1} > 0$. See details in Section 3.3.3.

in the money market, for $t \geq 0$,

$$\frac{M_{t+1}}{p_t} = \frac{\nu(1-\beta)\beta}{1-\nu(1-\beta)}\Bigg[\alpha Y_t + (1-\delta)K_t + (1+r_t)B_t - \int_0^\infty \sum_{j=0}^{\infty} \frac{T^e_{t+j}}{Q_{t+j}(z)}\Psi(dz)\Bigg]. \tag{18}$$

The nominal interest rate is obtained from the inter temporal condition of inactive entrepreneurs,

$$\frac{1}{\beta}\frac{c_{2t+1}}{c_{2t}} = \frac{1+i_{t+1}}{\frac{p_{t+1}}{p_t}} = 1 + r_{t+1}, \text{ for } t \geq 0. \tag{19}$$

Note that, except for the well-known Sargent-Wallace initial price level indeterminacy result, we can think of monetary policy as sequences of money supplies, $\{M_t\}_{t=0}^{\infty}$, or sequences of nominal interest rates, $\{i_t\}_{t=0}^{\infty}$. We will think of policy as determining exogenously one of the two sequences, abstracting from the implementability problem.[12]

There are two important margins in this economy. The first is the allocation of capital across entrepreneurs, which is dictated by the collateral constraints and which determines measured TFP (see (13)). The second is the evolution of aggregate capital over time, which, in the absence of taxes, behaves as in Solow's model (see (15) and set $T^e_{t+j} = T^W_{t+j} = 0$). Clearly, fiscal policy has aggregate implications: the net supply of bonds affects (17) and taxes affect (15). However, monetary policy does not, because none of those equations depend on nominal variables. Monetary policy does have effects, because it distorts the margin between cash and credit goods, but in a fashion that resembles the effects of monetary policy in a representative agent economy. This is the case only if, as assumed above, the zero bound does not bind.

4 Calibration and Evaluation of the model: The case of constant labor and flexible wages.

In this section we first calibrate the model to the US economy and discuss how we take the model to the data. We discuss in detail the frequency we want to focus on and

[12]Because we use log utility, there is a unique solution for prices, given the sequence $\{M_t\}_{t=0}^{\infty}$.

the way we detrend the data. Second, we numerically solve the model and compare it to the data. We maintain the assumptions of exogenous constant labor supply and flexible wages, as in the pervious Section. We modify this two assumptions in the robustness section below.

4.1 Calibration

We calibrate the model such that its steady state satisfies several conditions. There are very few parameters. We first set the capital share to be one-third and the (annual) depreciation of capital to be 7%, which are standard values. We set the share of cash goods in total expenditure $\nu = 0.23$ to match the share of payments (by value) done with cash by US consumers reported by Bagnall et al. [2014].

We then set the distribution of abilities to be log-normal, $z \sim \ln\mathcal{N}(0, \sigma_z)$, and choose the standard deviation σ_z so that the log dispersion of productivity among entrepreneurs in the model matches that among manufacturing establishments in the United States, as reported by Hsieh and Klenow [2009].[13] We choose the rate at which entrepreneurs exit $1 - \gamma = 0.10$ to match the average exit rate of US establishments from the Business Dynamics Statistics (BDS). The initial parameter of the collateral constraint, $\theta = 0.69$, is chosen to match the average ratio of liabilities to non financial assets for the US non financial business sector between 1997:Q3 and 2007:Q3.[14] Given the previous parameters, we set the discount factor β equal to 0.981, so the real interest rate is 2%. Table 1 summarizes the parameter values we use.[15]

A key aspect of our calibration is related to the way in which fiscal policy is implemented. For the total size of government liabilities, we use the sum of money and bonds

[13]If there are diminishing returns to scale, all entrepreneurs will be active in equilibrium. Therefore, by matching the log dispersion of productivity among all entrepreneurs, active and inactive, the calibration is robust to the inclusion of arbitrarily small diminishing returns to scale.

[14]We measure liabilities as total liabilities in the flow of funds (FL114190005.Q+FL104190005.Q) minus the US real estate owned by foreigners (FL115114005.Q) and the foreign direct investment in the United States. (FL103192005.Q), which in the flow of funds are liabilities items for the non corporate and corporate sectors, respectively. Correspondingly, we measure non financial assets as the non financial assets in the flow of funds (FL112010005.Q+FL102010005.Q) minus the US real estate owned by foreigners (FL115114005.Q) and the foreign direct investment in the United States (FL103192005.Q).

[15]We also need to specify the relative number of workers and entrepreneurs in the economy. We assume that workers are 25% of the population, $L/(1 + L) = 1/4$. We choose a low share of workers, who in our model choose to go against their borrowing constraint in a steady state, to limit the non-Ricardian elements in the model. This number is consistent with the fraction of households with zero net liquid assets, which was 23% in the United States in 2001 [Kaplan and Violante, 2014].

Parameters	Targets
$\alpha = 1/3$, $1-(1-\delta)^4 = 0.07$	Standard values
$\nu = 0.23$	Share of payments (by value) done with cash
$z \sim \ln N(0, 3.36)$	Log dispersion of estab., US manuf.
$1-\gamma^4 = 0.10$	Avg. exit rate of US establishments
$B_0/(4Y_0)$=0.62	Total public, federal debt in the US, 2007:Q2
$\beta = 0.987$	2% real interest rate
$\theta_0 = 0.69$	Liabilities to nonfinancial assets, US nonfin. bus.

Table 1: Calibration, Initial Steady State

from 2007 until 2016 and assume the total remains constant thereafter, with taxes being collected to pay the interest on the debt.[16] Still, because of the lack of Ricardian equivalence, there is a continuum of different ways in which the taxes and transfers can be designed so as to satisfy the observed total. And each one will imply different equilibrium paths. Because of the difficulty of using data to discipline these choices, we proceed by considering two simple cases, both respecting the principle that taxes and transfers can depend on agent classes (workers and entrepreneurs). In particular, we assume that transfers are given solely to entrepreneurs, active and inactive, whereas taxes are lump-sum to all agents. In Section **??**, we consider the case with lump-sum transfers. This second case exhibits larger departures from Ricardian equivalence, as we discuss in detail below.

What remains to be calibrated is the evolution of the collateral constraint. To do so, we simulate the model, choosing the value of the collateral constraint for every period so as to reproduce the evolution of the real interest rate in the United States since the financial crisis. Specifically, we assume that starting at that steady state, all agents learn that the collateral constraint will tighten for several periods and that the Fed and the central government will substantially increase their liabilities (i.e., their supply of liquid assets), taking as given the future taxes and transfers. We chose the sequence $\{\theta_t\}_{t=0}^{\infty}$ so that the model broadly matches the evolution of the real interest rate, given the path for government liabilities.

We chose to focus on the behavior of the real interest rate in the United States following the financial crisis to calibrate the sequence $\{\theta_t\}_{t=0}^{\infty}$ for a couple of reasons. First, it is the reduction in θ_t that drives down the real interest rate. The direct

[16]Notice that the composition between money and bonds is inessential when nominal interest rates are zero, as in the period we are considering.

theoretical relationship between the unobservable shock and the real rate makes it an attractive target for the calibration.

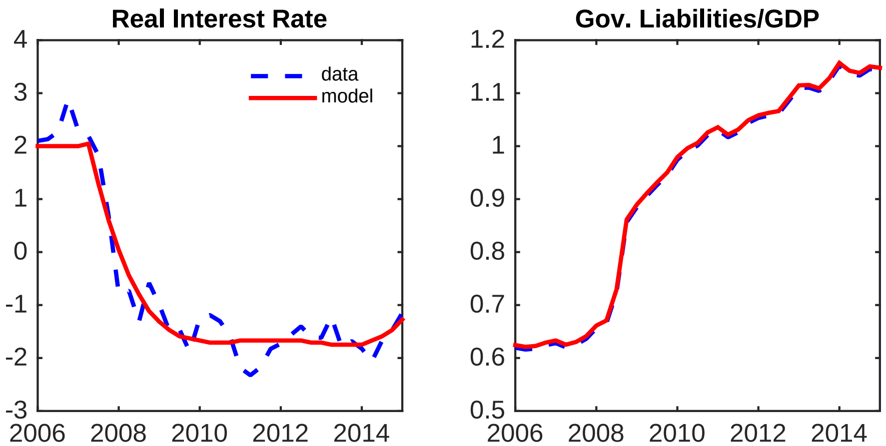

Figure 1: Real Interest Rate and Public Liquidity, Data (dashed line), and Model-Calibrated (solid line) Paths.

Second, the very long period of negative real interest rates, depicted in the left panel of Figure 1, is one of the most remarkable features of the events of the last decade and speaks to the persistence of the shock that drove the real rates below zero. The data series (dashed line) is obtained from Andrea et al. [2012], who compute it using a no-arbitrage model that jointly explains the dynamics of consumer prices as well as the nominal and real term structure of risk-free rates.[17] The solid line, labeled "model," is a smooth version of the data reported by Andrea et al. [2012]. This is our calibration target.

We use the behavior of this real rate to identify the timing, severity and persistence of the shock. As can be seen in the left panel of Figure 1, the drop in the rate is in the third quarter of 2007, so we choose this quarter as the onset of the crisis. We then chose the sequence θ_t such that the simulated series for the real interest rate matches the solid line in the left panel of Figure 1. The resulting calibrated shock implies that the collateral constraint, calibrated to be equal to 0.69 in the steady state, goes down sharply to 0.59 by the end of 2008 and continues a more gradual decline through 2014, where it gradually starts to go up (see the right panel of Figure 4). As mentioned before, the equilibrium does depend on the injection of total liquidity. Thus, in the simulation we also assume the total supply of outside liquidity to be the

[17] A very similar picture emerges if one computes ex-post real rates by subtracting observed inflation from short-term nominal interest rates, (see the Working Paper version for details).

one observed in the data starting in 2007, depicted in the second panel of Figure 1. This series is the sum of the total public federal debt and the balance sheet of the Federal Reserve Banks net of their holdings of Treasury bonds. An important part of the policy response to the crisis by the Fed was to increase the supply of liquidity providing bank reserves in exchange for mortgage-backed securities. In addition, there were large tax and transfer programs that resulted in an unprecedented increase in the level of the public debt. Furthermore, we assume a path for the money supply that is consistent with an inflation rate of 2% per year in periods in which the nominal rate is strictly positive. When the nominal rate is at the zero lower bound, the inflation rate equals the negative of the real interest rate (i.e., the observed path of inflation).

4.2 Evaluation of the Model

We now compare the simulations of the model with the US data since the third quarter of 2007 - the date identified as the beginning of the crisis by the real interest rate - until the end of our sample. In looking at variables such as output, capital, labor, or productivity, we face a difficulty that does not arise when looking at real interest rates: the trend in the data. Our model is stationary, but it can be modified to incorporate exogenous productivity growth, the same way it is done in the Solow model. To the extent that the exogenous component of productivity grows at a constant rate, allowing for this exogenous component is equivalent to removing a linear trend to the natural logarithm of the data. That is the strategy we pursue in comparing the data to the model.

In Table 2 we show the quarterly growth rate of the linear trend for the natural logarithm of output, capital, hours, and productivity using quarterly data. The linear trend has been computed by ordinary least squares regressions of the log of the corresponding variable on time. We report results for three different starting periods, 1947, 1960, and 1980. In all cases, the last period was the third quarter of 2007, the period when the crisis started according to our calibration.[18]

The results are surprisingly robust to the initial date used for output and hours, but less so for capital. However, in the case of capital, the data show a clear slowdown in growth from 1960 to 1980: the linear trend does not adjust well for the first two

[18]See the Online Appendix, Section G and H, for details on the data sources and the computation of productivity, where we explore the effect of accounting for capacity utilization.

	Initial Period		
	1947:Q3	1960:Q1	1980:Q1
Output	0.0088	0.0081	0.0080
Hours	0.0033	0.0042	0.0039
Capital	0.0092	0.0086	0.0077
Productivity	0.0027	0.0023	0.0028

Table 2: Average quarterly growth of (log) GDP, hours, capital, and TFP from the specified dates and 2007:Q3.

samples.[19] Thus, we choose the value for the sample that starts in 1980. The trend for productivity also seems relatively stable, but that hides the fact that it grew very rapidly from 1947 to 1973 (a slope of 0.0043) and then remained essentially constant until 1983. It then grew at 0.0028, the rate reported for the sample that starts in 1980.

Given this discussion, we chose the values for the trends to be the ones that result from the last period, starting in 1980, the ones reported in the last column of Table 2.

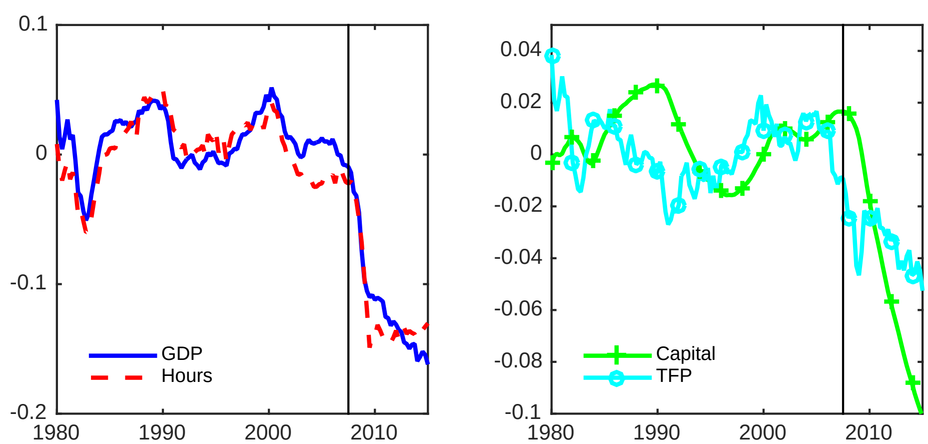

Figure 2: Detrended GDP, Hours, Capital Stock, and TFP, 1980:Q1-2015:Q1.

To show the effect of detrending in a way that makes clear the long-lasting effect of the crisis that started in 2007, in Figure 2 we depict the difference between the natural logarithm of the data and its trend (computed from 1980 to 2007) for output and hours in the left panel and for capital and productivity in the right panel. As can be seen, there are fluctuations around trend that, until 2007 never go beyond 5% in absolute value for any of the series. However, after 2007, the deviations are all negative and much larger than anything previously seen.

We now argue that, except for hours, a relevant fraction of these changes can

[19]See the Online Appendix, Section G, for details on the data sources.

be accounted for by the single shock we model and that we calibrated to match the evolution of the real interest rate.

The deviations from trend shown in Figure 2 are the ones we compare to the simulation of the calibrated model. To begin with, notice that we assumed labor supply to be constant, so the model will be unable to replicate the very large drop in hours since 2007. This result would not differ if we had leisure in the utility function. As we show in the Online Appendix, Section E.2, a shock to the collateral constraint, as we model it, does not have a significant quantitative effect on total hours.[20] If readers were hoping to learn something meaningful regarding the relationship between credit constraints and the labor participation rate, they should stop reading now.

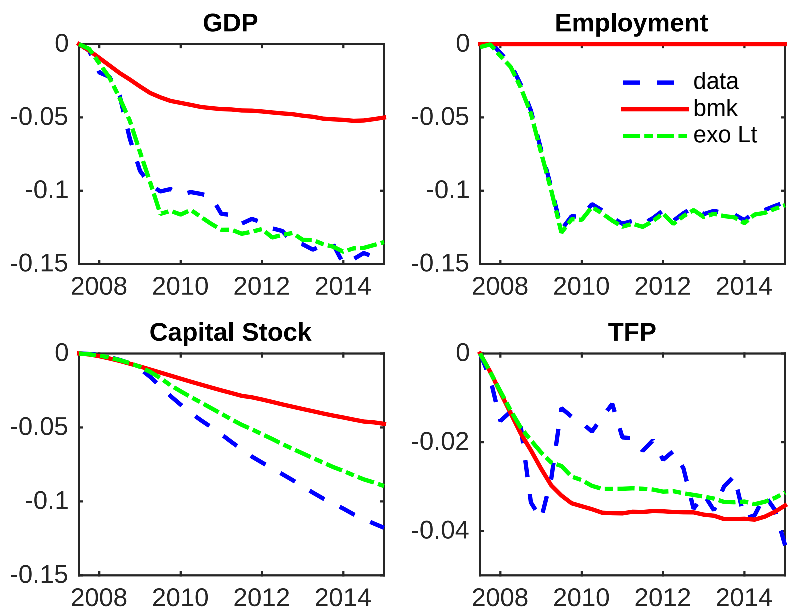

Figure 3: Great Recession in the benchmark model (solid line, bmk) and the model with exogenous hours (dash dotted line, exo Lt). The dashed line corresponds to the detrended data.

In Figure 3 we compare the detrended data for output, productivity, capital, and hours with the simulation of the model. The solid (red) line is the simulation of the model. The dashed (blue) line is the data. The two panels on the left show that the model captures the direction and persistence of the drops in capital and output - relative to trend - but misses the magnitudes: it explains only around one third of the drop in both output and capital. The lower right panel shows that the model

[20]As we show below, adding sticky wages à la Calvo cannot explain the persistent drop in labor either. The effects in the model last less than three years.

does a decent job at tracking the behavior of productivity, missing the high frequency movements. It is important to highlight that no parameter has been chosen to fit this curve - or any of the other pictures in this figure! The lower left panel shows that the model, with constant labor, misses the behavior of hours. One could conjecture, therefore, that part of the reason why the model misses the magnitudes in explaining output and capital is related to the failure in explaining labor. One way to evaluate this conjecture, given that in the model labor supply is exogenous, is to simply impose in the simulation the behavior of hours that we saw in the data. The result is depicted in Figure 3 with the dashed-dotted (green) line. Once we feed the observed value for hours into the model, it decently passes an eyeball inspection of the figure.

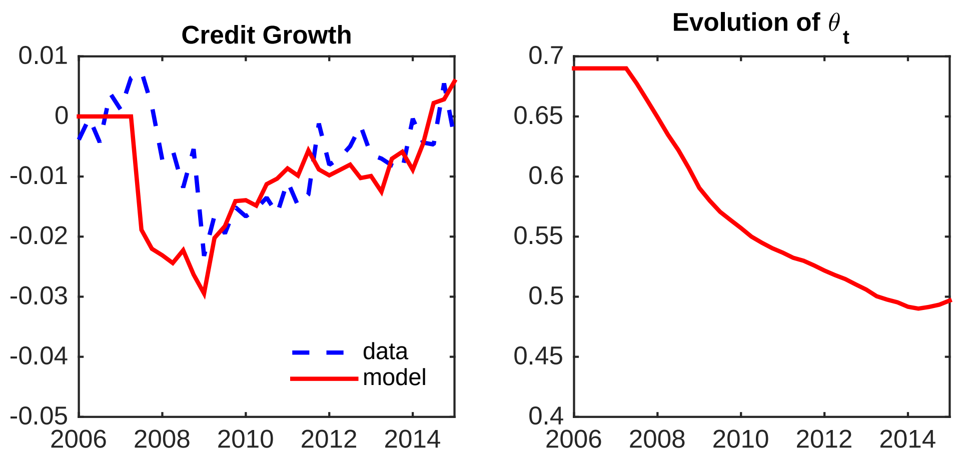

Figure 4: Credit growth and evolution of the (calibrated) collateral constraint θ. The left panel compares credit growth in the benchmark model (solid line) and the data for the nonfinancial business sector (dashed line).

As an additional evaluation of the model, we compare the behavior of credit growth in the model - equal to the growth of the product of capital times the collateral constraint - with the recent evidence in the United States. The solid line in the left panel of Figure 4 shows the path for the growth rate of credit in the calibrated model. We also show the growth rate of credit to the nonfinancial business sector, normalized by the average growth in the 1997:Q2-2007:Q2 period (dashed line). While there is substantial correlation between the data and the model, the simulation overpredicts the speed with which credit drops in the data. The right panel shows the evolution of the collateral parameters θ_t.

A natural caveat regarding the model is that debt contracts have one-period maturity, so the speed at which firms are forced to deleverage is very high. In the data, debt contracts last for many periods, and they take a few periods to get processed, approved, and executed. Thus, the comparison between the model that abstracts from all these lags and the data is trickier than what appears at first sight.[21] All in all, it can be seen that, indeed, a big change occurs in 2007:Q3, the quarter that the real interest rate identifies as the beginning of the crisis.

Notice that the growth rate of credit almost fully recovers by the end of our sample. Would this indicate that the crisis may be reverting (in the model, credit to capital starts growing when the collateral parameter starts growing) and the main variables will now begin converging to its trend, putting an end to the "secular stagnation"? This model certainly implies so, with the exception of labor input, with its corresponding impact on output and capital.

Micro evidence that is consistent with a large contraction of credit affecting business firms during the Great Recession is presented by Chodorow-Reich [2014]. Using a panel of banking relationships and employment for non financial firms, he shows that firms that had pre-crisis relationships with less healthy lenders had a lower likelihood of obtaining loans and reduced employment further following the Lehman bankruptcy. Related, Mehrotra and Sergeyev [2015] find job flow evidence consistent with the firm credit channel during the Great Recession. Consistent with our model, Kehrig [2015] presents evidence that the dispersion of productivity is countercyclical and that during the Great Recession the dispersion increase was the largest ever and pervasive across sectors.

Taken all together, these exercises provide evidence that the mechanism discussed in the model captures reasonably well many of the relevant features of the post-2007 events, with the big exception being the behavior of hours.[22]

The interpretation provided by the model, then, is that the credit crunch lasted at least eight years, with some weak indication that some reversal may be taking place: the real interest rate seems to be trending upward and credit growth, although very small, became positive at the end of 2014. The two facts are consistent with the gradual unwinding of the financial shock.

[21]We would like to highlight, though, that the maturity structure should not matter for the steady state, so we feel comfortable with using the credit to non-financial assets ratio to calibrate θ in the steady state.

[22]One could conjecture that modeling an elastic labor supply could improve the model's performance along this front. We show in the Online Appendix, Section E.2, that this is not the case.

5 Robustness WORK IN PROGRESS

References

Stephanie Aaronson, Bruce Fallick, Andrew Figura, Jonathan F Pingle, and William L Wascher. The recent decline in the labor force participation rate and its implications for potential labor supply. *Brookings Papers on Economic Activity*, 2006(1):69–154, 2006.

Andrea, , Luca Benzoni, and Olena Chyruk. Core and 'crust': Consumer prices and the term structure of interest rates. Manuscript, Research Department, Federal Reserve Bank of Chicago, 2012.

John Bagnall, David Bounie, Kim P. Huynh, Anneke Kosse, Tobias Schmidt, Scott Schuh, and Helmut Stix. Consumer Cash Usage: A Cross-Country Comparison with Payment Diary Survey Data. Working Papers 192, Oesterreichische Nationalbank (Austrian Central Bank), May 2014. URL `https://ideas.repec.org/p/onb/oenbwp/192.html`.

Ben Bernanke and Mark Gertler. Agency Costs, Net Worth, and Business Fluctuations. *American Economic Review*, 79(1):14–31, March 1989.

Ben S. Bernanke, Mark Gertler, and Simon Gilchrist. The Financial Accelerator in a Quantitative Business Cycle Framework. In J. B. Taylor and M. Woodford, editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. Elsevier, 1999.

Francisco Buera and Juan Pablo Nicolini. Liquidity traps and monetary policy: Managing a credit crunch. 2017.

Francisco J. Buera and Benjamin Moll. Aggregate Implications of a Credit Crunch: The Importance of Heterogeneity. *American Economic Journal: Macroeconomics*, 7 (3):1–42, July 2015.

Gabriel Chodorow-Reich. The Employment Effects of Credit Market Disruptions: Firm-level Evidence from the 2008-9 Financial Crisis. *Quarterly Journal of Economics*, 129(1):1–59, 2014.

Thomas Cooley, Ramon Marimon, and Vincenzo Quadrini. Aggregate Consequences of Limited Contract Enforceability. *Journal of Political Economy*, 112(4):817–847, August 2004.

Veronica Guerrieri and Guido Lorenzoni. Credit crises, precautionary savings, and the liquidity trap. Working Paper 17583, National Bureau of Economic Research, November 2011.

Chang-Tai Hsieh and Peter J. Klenow. Misallocation and Manufacturing TFP in China and India. *The Quarterly Journal of Economics*, 124(4):1403–1448, 2009.

Urban Jermann and Vincenzo Quadrini. Macroeconomic Effects of Financial Shocks. *American Economic Review*, 102(1):238–271, February 2012.

Greg Kaplan and Giovanni L. Violante. A Model of the Consumption Response to Fiscal Stimulus Payments. *Econometrica*, 82(4):1199–1239, 04 2014.

Matthias Kehrig. The Cyclical Nature of the Productivity Distribution. Manuscript, University of Texas at Austin, 2015.

Nobuhiro Kiyotaki. Credit and Business Cycles. *Japanese Economic Review*, 49(1): 18–35, March 1998.

Nobuhiro Kiyotaki and John Moore. Credit Cycles. *Journal of Political Economy*, 105 (2):211–48, April 1997.

Nobuhiro Kiyotaki and John Moore. Liquidity, Business Cycles, and Monetary Policy. Working Paper 17934, National Bureau of Economic Research, March 2012.

Robert E. Jr Lucas. Interest Rates and Currency Prices in a Two-Country World. *Journal of Monetary Economics*, 10(3):335–359, 1982.

Neil Mehrotra and Dmitriy Sergeyev. Financial shocks and job flows. Technical report, Brown University Working Paper, 2015.

Benjamin Moll. Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation? *American Economic Review*, 104(10):3186–3221, October 2014.

Lars Svensson. Money and asset prices in a cash-in-advance economy. *Journal of Political Economy*, 93(5):919–944, October 1985.