Central Bank Digital Currency: Welfare and Policy Implications

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Abstract

A model of multiple means of payment is constructed to analyze the effects of the introduction of central bank digital currency (CBDC). The introduction of CBDC has three beneficial effects. It mitigates crime associated with physical currency, permits the payment of interest on a key central bank liability, and economizes on scarce safe collateral. CBDC admits another instrument of monetary policy, but may require that the central bank take on private assets in its portfolio if CBDC significantly displaces privately supplied means of payment.

1 Introduction

As financial technology evolves, central banks need to continuously evaluate their role, potentially introducing new central bank assets and liabilities, and altering their approach to monetary policy decision making and implementation. In recent years, financial markets have been flooded with privately-issued cryptocurrencies, including Bitcoin and Ethereum. While such cryptocurrencies have failed to gain wide acceptance as means of payment, it is possible that the associated blockchain technologies could have applications in central banking. In addition, old-fashioned physical currency is being replaced as a means of payment in conventional transactions by credit cards, debit cards, and other electronic means of payment. Yet, the demand for central-bank-issued currency is increasing in most countries. For example, U.S. currency outstanding relative to GDP rose from 5.5% in 2007 to 8.0% in 2018. How can more currency be held when most people are using it less? As pointed out, for example by Rogoff (2016), there is strong evidence, including the fact that more than 80% of U.S. currency outstanding is in the form of $100 bills, that the strong demand for currency is generated by crime. Thus, a key liability of central banks serves to reduce the costs of criminal activity, which is a far-from-ideal state of affairs.

Central banks, including those in Sweden, Canada, and the U.K., have shown an increased interest in digital currencies (see Chapman and Wilkins 2019, Kumhoff and Noone 2018, and Bech et al. 2018), typically referred to as CBDC (central bank digital currency). Potentially, CBDC could take many forms. Ownership could be recorded and transferred on a decentralized ledger, as with cryptocurrencies, or the recordkeeping could be done in a centralized fashion, as with conventional private bank liabilities and central bank reserves. Central banks could opt for monopoly issue by the central bank of CBDC, just as for physical currency, or there could be competition among private digital currencies and CBDC, or perhaps the central bank could issue CBDC and leave the mechanics of converting CBDC into other assets to the private sector. Physical currency could be eliminated completely with the issue of CBDC, or use of physical currency could be limited by abolishing large-denomination notes.

The goal of this paper is to study the factors which could result in a welfare improvement from the issue of CBDC, and to determine how CBDC issue might change the effects of monetary policy and the optimal behavior of the central bank. In the model, the use of physical currency as a means of payment is socially costly, because currency is subject to theft. This, at least in part, captures the idea that currency promotes illegal and socially costly activity. If CBDC is issued, then it potentially drives out physical currency in the model, and therefore eliminates the illegal activity associated with physical currency. As well, CBDC is an improvement over physical currency in that the central bank can pay interest on CBDC. It was recognized, at least as early as Friedman (1969), that an alternative to a zero-nominal-interest rate policy for implementing the Friedman rule is to pay interest on currency at the appropriate rate. There are clearly practical obstacles to paying interest on physical currency, but if CBDC consists of centralized account balances, then it is straightforward to pay interest on these account balances. Finally, CBDC may play an important role in mitigating the incentive problems associated with private banking. Provided we can trust the central bank, and if the transactions costs associated with using CBDC and private bank deposits as means of payment are similar, then substituting CBDC for transactions deposits at private banks could increase welfare.

In the model, there are potentially three means of payment, physical currency, digital currency, and bank deposits. Physical currency and digital currency are issued by the central bank, while bank deposits are issued by private financial institutions. The fiscal authority can tax lump sum, and it issues one-period nominal bonds. There are also private assets in this economy. A private bank issues deposits, and holds private assets and government bonds as assets. As well, private banks are subject to limited commitment – they can abscond on their deposit liabilities, as in Williamson (2016, 2019a, 2019b) and Gertler and Kiyotaki (2011), for example. So, a bank's assets implicitly serve as collateral backing the bank's deposit liabilities. Collateral can be scarce in this economy if the government limits outstanding government debt sufficiently. Scarcity of collateral is reflected in binding collateral constraints for banks and in low real interest rates.

In a world like the current one, in which the central bank issues physical currency and there is no digital currency, theft limits the quantity of currency in circulation, and produces a social loss in the model. So long as there is theft in equilibrium, an increase in inflation is Pareto improving, if collateral constraints do not bind, and serves to increase the welfare of currency-holders and reduce the welfare of those using bank deposits in payment, if collateral constraints bind. If digital currency is no more costly for a retailer to accept as payment than physical currency, then digital currency can yield a Pareto improvement over physical currency if monetary policy is implemented appropriately. This result is due to the fact that digital currency is not subject to theft.

In a regime with digital currency and no physical currency, monetary policy works differently, in part because the central bank has two policy instruments – the nominal interest rate on short-term government debt, and the interest rate on digital currency – rather than just one. An increase in the nominal interest rate on government debt, engineered through an open market sale of government debt, will result in substitution from digital currency to bank deposits, an increase in the real interest rate, and an increase in the inflation rate. An increase in the nominal interest rate on digital currency causes substitution from bank deposits to digital currency, a decrease in the real interest rate, and an increase in the inflation rate.

There are two kinds of substitution across means of payment that can result from monetary policy. One is substitution on the supply side in that, for example, if economic agents using currency as a means of payment hold more digital currency, then the central bank must acquire more government debt, and there is less government debt available to back bank deposits. The other is on the demand side in that, for example, a lower interest rate on government debt makes bank deposits less attractive, and people could choose to substitute digital currency for bank deposits as a means of payment. Substitution on the demand side has implications for how collateral is used in the aggregate, as substitution away from bank deposits mitigates incentive problems in the aggregate. That is, if the central bank can be trusted, then digital currency could be more efficient than private bank deposits, as it uses the aggregate stock of collateral more effectively.

If the central bank is limited to holding government debt in its portfolio, this can limit the expansion of the stock of digital currency, if digital currency competes effectively with private bank deposits as a means of payment. Under such circumstances, the central bank may have to weigh the benefits and costs of taking on private assets so as to expand the reach of its liabilities.

The economics of cryptocurrencies has been studied by Abadi and Brunnermeir (2018) and Chiu and Koeppl (2018), among others. Davoodalhosseini (2018), Hendry and Zhu (2019) and Keister and Sanches (2018) analyze the role of CBDC in general equilibrium. Some papers that do a nice job of laying out the issues at stake in CBDC issue are Chapman and Wilkins (2019), Bech et al. (2018), and Kumhof and Noone (2018).

2 Model

The underlying framework follows Rocheteau and Wright (2005), with additional structure added to address the particulars of this problem. Periods are indexed by $t = 0, 1, 2, ...$, and each period there are two sub-periods, the centralized market (CM), followed by the decentralized market (DM). There is a continuum of *buyers*, with unit mass, indexed by $i \epsilon\ [0, 1]$, each of whom is infinite-lived with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[-H_t^i + \theta_t^i u(x_t^i)\right], \tag{1}$$

where $0 < \beta < 1$, H_t^i denotes labor supply in the CM, θ_t^i is an i.i.d. (across buyers and time) preference shock, realized in the CM, and x_t^i denotes consumption in the DM. Assume that $\Pr[\theta_t^i = \theta^L] = \rho$, and that $\Pr[\theta_t^i = \theta^H] = 1 - \rho$, where $\theta^H > \theta^L > 0$, and $0 < \rho < 1$.

There also exists a continuum of bankers with unit mass, each of whom has preferences

$$E_0 \sum_{t=0}^{\infty} [-H_t + X_t], \tag{2}$$

where H_t and X_t are, respectively, labor supply and consumption for the banker in the CM. In addition, there is a continuum of sellers with unit mass, each with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t], \tag{3}$$

where X_t is consumption in the CM, and h_t is labor supply in the DM. In the CM and the DM, one unit of labor supply produces one unit of the perishable consumption good. Buyers cannot produce in the DM, and sellers cannot produce in the CM.

In the CM, all agents are together in one location. At the beginning of the CM, debts acquired in the previous period are settled, then production and exchange take place, and buyers write contracts with bankers. Assets traded in the CM are physical currency, digital currency, one-period government bonds, private assets, and bank deposits.

In the DM, each buyer is matched at random with a seller. Each seller has access to technologies which permit him or her to process payments in currency, digital currency, or bank deposits. That is, the costs per transaction to accepting currency, digital currency, or bank deposits, respectively, are k^c, k^m, and k^d, in units of labor. As well, a seller can steal all of the buyer's currency at a cost k^T. We assume that $k^d > k^c$ and that $k^T > k^c$, so that the cost for a seller is higher to accepting bank deposits in exchange than accepting currency, and it is more costly to steal currency than to pay the cost to accept it in transactions, for a seller. Assume that theft of digital currency and bank deposits is not feasible. Also, suppose there is no technology that permits the trade of government debt or private assets in the DM, although banks can hold these assets and issue tradeable bank deposits as liabilities. There is limited commitment, in that no economic agent can be forced to work, so buyers cannot trade personal IOUs in the DM.

The basic assets in this economy are physical currency, digital currency, one-period nominal government debt, and private assets. There is a stock of one unit of private assets in existence, and ownership of private assets entitles the asset holder to a dividend y at the beginning of the CM. The dividend is constant for all t.

2.1 Private Banks

In the CM of period t, a bank issues deposit liabilities, each of which is a claim to one unit of consumption goods in the CM of period $t+1$. The nature of the deposit contract the bank writes with its depositors (who are buyers) depends on what a buyer can receive in exchange for a bank deposit in the DM.In a meeting between a buyer and seller in the DM, assume that the buyer makes a take-it-or-leave-it offer. Supposing the buyer possesses d bank deposits, the buyer then solves

$$\max_{d',x^d}[\theta_t^i u(x^d) - \beta d'] \tag{4}$$

subject to

$$\beta d' - x^d - k^d \geq 0, \tag{5}$$

$$d' \leq d. \tag{6}$$

That is, the buyer chooses the quantity of deposits to trade in exchange for a quantity of goods to maximize surplus (objective function (4)) , subject to constraints specifying that the seller receive nonnegative surplus (inequality (5)), and that the buyer cannot trade away more deposits than he or she possesses (inequality (6)). The solution to the above problem is then

$$x^d = \beta d' - k^d, \tag{7}$$

$$d' = \min\left(\frac{x^*(\theta_t^i) + k^d}{\beta}, d\right), \tag{8}$$

where $\theta_t^i u'(x^*(\theta_t^i)) = 1$.

We have assumed limited commitment, which applies to banks as well as to buyers, and collateral can be posted by the bank to secure its liabilities. In the CM, the bank acquires a portfolio consisting of government bonds and private assets, and the bank posts these assets as collateral against its deposit liabilities. We assume that the bank can abscond with a fraction γ of the value of its assets in the event that it defaults on its debts. As well, we will suppose that banks can hold currency as an asset, and that currency can also secure a bank's liabilities. We will assume that the bank can abscond with fraction γ^m of the value of currency it holds, should it default. Assume that $\gamma^m > \gamma$, so that it is easier to abscond with currency than with other assets. This role for currency as a potential bank asset will only come into play when the nominal interest rate on government debt is negative.

Since we will be looking for a stationary equilibrium in which quantities are constant for all t, we will drop time subscripts from the analysis. Using our solution to (4) subject to (5) and (6), we will next solve the bank's problem. Assume that k^d is sufficiently large that buyers with $\theta_t^i = \theta^L$ will never hold bank deposits in equilibrium. In equilibrium, the bank solves

$$\max_{z,d,b,a} \{ \left[-z + \theta^H u[\min\left(x^*\left(\theta^H\right), \beta d - k^d\right)\right] + \beta \min\left[0, d - \frac{x^*\left(\theta^H\right) + k^d}{\beta}\right] \} \quad (9)$$

subject to

$$z - b - \phi a + \frac{\beta R^b b}{\pi} + \beta(\phi + y)a - \beta d \geq 0, \quad (10)$$

$$(1-\gamma)\left[\frac{R^b b}{\pi} + (\phi + y)a\right] - d \geq 0. \quad (11)$$

That is, the bank chooses a banking contract (z, d) and a portfolio (b, a) to maximize the utility of the buyer (9). Here, z denotes the quantity of CM goods the buyer exchanges for d deposit claims, b is the quantity of government bonds acquired by the bank, in units of the CM consumption good, and a is the quantity of private assets acquired. As well, R^b denotes the payoff in units of money on each government bond, π is the gross inflation rate, ϕ is the price of private assets, and y is the dividend received on each unit of private assets held at the beginning of the CM.

In the bank's problem, (9) subject to (10) and (11), inequality (10) states that the net payoff to the bank from the bank contract is nonnegative, and (11) states that the bank does not default in equilibrium, i.e. the net gain from defaulting and absconding with assets is not strictly positive.

When we consider the case of negative nominal interest rates, we will need to consider the option the bank has to hold currency in its portfolio.

2.2 Government

Assume that there is no consolidated government debt outstanding at the beginning of period $t = 0$, so the period 0 consolidated government budget constraint is given by

$$\bar{c} + \bar{m} + \bar{b} = \tau_0, \quad (12)$$

where $\bar{c}$, $\bar{m}$, and $\bar{b}$ denote the quantities of physical currency, digital currency, and one-period government debt issued in period 0, all in units of the period 0 CM good. As well, τ_0 denotes the lump sum transfer to each buyer at $t = 0$. The distribution of lump sum transfers is irrelevant, given quasilinear utility. Then, in each subsequent period, again confining attention to stationary policies and stationary equilibria,

$$\bar{c} + \bar{m} + \bar{b} = \frac{\bar{c}}{\pi} + \frac{R^m \bar{m}}{\pi} + \frac{R^b \bar{b}}{\pi} + \tau \quad (13)$$

Here, R^m denotes the gross nominal interest rate on digital currency, and the lump-sum transfer τ is constant for $t = 1, 2, 3, ...$. The left-hand side of (13) is the sum of total consolidated liabilities outstanding after new liabilities are issued, while the right-hand side is the sum of total payoffs on consolidated government liabilities from the previous period, plus the transfer to buyers.

It is important for the analysis how we specify fiscal policy, as this will help determine the aggregate supply of collateral, which plays an important role in the analysis. We will assume, as in Andolfatto and Williamson (2015) and Williamson (??,??,??) that the fiscal authority sets τ_0 and τ in response to monetary policy so that the real value of the consolidated government debt is a constant, v. That is,

$$v = \bar{c} + \bar{m} + \bar{b} \tag{14}$$

Given this fiscal policy rule, the fiscal authority determines the total value of the consolidated government debt, while the central bank determines its composition.

3 Equilibrium With Physical Currency and the Potential for Theft

We will first consider a case resembling the current status quo. The central bank issues physical currency, and there is no digital currency. We will assume at this stage that buyers with $\theta_t^i = \theta^H$ use bank deposits as means of payment, and those with $\theta_t^i = \theta^L$ use currency. In what follows we will determine conditions under which this is optimal behavior.

In the DM, if a buyer arrives with c units of currency, then the seller with whom the buyer is matched steals the buyer's currency with probability $1 - \alpha$, where α is chosen optimally by the seller. That is, if $\frac{\beta c}{\pi} > k^T$, then $\alpha = 0$, if $\frac{\beta c}{\pi} < k^T$, then $\alpha = 1$, and if $\frac{\beta c}{\pi} = k^T$, then $\alpha \epsilon [0, 1]$. If the seller does not steal the buyer's currency, then the buyer makes a take-it-or-leave-it offer to the seller. Therefore, if x^c denotes the quantity of goods supplied by the seller when currency is not stolen, then

$$x^c = \frac{\beta c}{\pi} - k^c, \tag{15}$$

and so in the CM, the buyer solves

$$\max_c \left[-c + \alpha \theta^L u \left(\frac{\beta c}{\pi} - k^c \right) \right], \tag{16}$$

so the first-order condition for an optimum is

$$-1 + \frac{\alpha \beta}{\pi} \theta^L u' \left(\frac{\beta c}{\pi} - k^c \right) = 0. \tag{17}$$

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3.1 Case 1: Potential for Theft Irrelevant

In this case $\alpha = 1$, so in equilibrium

$$k^T \geq \frac{\beta c}{\pi}, \tag{18}$$

That is, theft of currency is sufficiently costly that it is optimal for a seller in the DM who meets a buyer with currency to submit to bargaining. Therefore from (15) and (17),

$$-1 + \frac{\theta^L \beta}{\pi} u'\left(x^c\right) = 0. \tag{19}$$

From the bank's problem, (9) subject to (10) and (11), asset prices satisfy

$$R^b = \frac{\pi}{\beta\left[\gamma + (1-\gamma)\theta^H u'(x^d)\right]}, \tag{20}$$

$$\phi = \frac{\beta y\left[\gamma + (1-\gamma)\theta^H u'(x^d)\right]}{1 - \beta\left[\gamma + (1-\gamma)\theta^H u'(x^d)\right]}, \tag{21}$$

where $\theta^H u'(x^d) > 1$ if the collateral constraint (11) binds, and $\theta^H u'(x^d) = 1$ otherwise. That is, when the collateral constraint does not bind, then assets are priced at their fundamental values, in that, from (20) the nominal interest rate on government debt is equal (to an approximation) to the rate of time preference plus the inflation rate (a Fisher relation), and the price of private assets, from (21), is equal to the present value of dividends.

First, consider the case in which the bank's collateral constraint does not bind. Then, from (20) and (21),

$$R^b = \frac{\pi}{\beta}, \tag{22}$$

$$\phi = \frac{\beta y}{1-\beta}, \tag{23}$$

and $x^d = x^*(\theta^H)$. In equilibrium, all asset markets clear in the CM, so

$$\rho c = \bar{c}; (1-\rho)b = \bar{b}; (1-\rho)a = 1. \tag{24}$$

Then, if we substitute in (11) using (14), we obtain

$$v + \frac{\beta y}{1-\beta} \geq \frac{(1-\rho)\left[x^*\left(\theta^H\right) + k^d\right]}{1-\gamma} + \rho\theta^L u'(x^c)(k^c + x^c). \tag{25}$$

And, from (19) and (22),

$$R^b = \theta^L u'(x^c) \tag{26}$$

Fiscal policy is given by v, the quantity of consolidated government debt, or publicly-supplied collateral (broadening our notion of collateral to include currency). We will take monetary policy as being summarized by a target for R^b, the gross nominal interest rate on government bonds. Then, given R^b, equation (26) determines x^c. Therefore, a higher nominal interest rate, from (22), implies a higher inflation rate, with the inflation rate increasing one-for-one with the nominal interest rate (approximately). A higher nominal interest rate and higher inflation reduces the real rate of return on currency, and there is less consumption in the DM in meetings in which buyers use currency as a means of payment.

Next, consider the case in which a bank's collateral constraint binds. From (15) and (19), we can express the demand for currency in terms of x^c:

$$\rho c = \rho \theta^L u'(x^c)(x^c + k^c) \tag{27}$$

Assume that

$$-x\frac{u''(x)}{u'(x)} < 1, \tag{28}$$

for all x, and that $\frac{k^c}{x^c}$ is small, over the relevant range for x^c. This then implies, from (**??**), that the demand for currency increases with consumption in the DM by buyers who use currency as a means of payment. Therefore, from (22) and (26), the demand for currency decreases when the nominal interest rate and inflation increase. Basically, (28) implies that substitution effects dominate income effects with respect to the effects of rates of return on the demand for assets.

In inequality (25), which must be satisfied for this to be an equilibrium, the left-hand side is the supply of collateral, consisting of publicly-provided collateral (v) plus private collateral – the value of private assets. On the right-hand side is the demand for collateral, consisting of collateral used by the banking system, plus the demand for currency. Note that $1-\gamma$ appears in the denominator of the first term on the right-hand side of (25), as higher γ implies that collateral held by the bank is less effective, since the bank is able to abscond with more collateral in the event of default. Alternatively γ could be interpreted as representing a regulatory constraint. With higher γ, the bank must finance more of its asset purchases with internally generated equity ("sweat equity") rather than deposit liabilities.

Note that the central bank's interest rate policy must be supported by appropriate open market operations. That is, to support a market gross nominal interest rate R^b the central bank must have enough government bonds in its portfolio that the real value of the portfolio is ρc, with x^c solving (26) and c solving (27) given x^c. This assumes that central bank profits are transferred to the fiscal authority each period.

In order to characterize the equilibrium, from (11), (14), (15), (19), (20), (21), and (24), we obtain

$$v + \frac{\beta y[\gamma + (1-\gamma)\theta^H u'(x^d)]}{1 - \beta\left[\gamma + (1-\gamma)\theta^h u'(x^d)\right]} = \frac{(1-\rho)\left(x^d + k^d\right)\left[\gamma + (1-\gamma)\theta^H u'(x^d)\right]}{1-\gamma} + \rho\theta^L u'(x^c)(x^c + k^c) \tag{29}$$

and

$$R^b = \frac{\theta^L u'(x^c)}{\theta^H[\gamma + (1-\gamma)\theta^H u'(x^d)]} \tag{30}$$

If we assume (28) and also that $\frac{k^c}{x^c}$ and $\frac{k^d}{x^d}$ are small for the range of the analysis, then (29) and (30) solve uniquely for x^c and x^d, given policy (v, R^b). As in Figure 1, higher R^b serves to increase x^d, reduce x^c, increase inflation, and increase the real interest rate. This higher interest rate policy is supported by a smaller portfolio of government debt held by the central bank. There is a permanent open market sale, which increases the quantity of available collateral, which relaxes banks' collateral constraints and expands bank deposits.

The ELB on the nominal interest rate in this case is less than zero. The ELB threshold for R^b is achieved when banks are indifferent between holding currency and government debt as assets. That is, if let $\bar{R}^b$ denote the ELB, then $\bar{R}^b$ is determined by

$$\bar{R}^b = \frac{\gamma^m + (1-\gamma^m)\theta^H u'(x^d)}{\gamma + (1-\gamma)\theta^H u'(x^d)}, \tag{31}$$

or, from (30) and (31), the ELB is achieved when

$$\theta^L u'(x^c) = \gamma^m + (1-\gamma^m)\theta^H u'(x^d). \tag{32}$$

In this case (**??**) is sufficient for $\alpha = 1$ for any nominal interest rate $R^b \geq \bar{R}^b$, but in general we only need a weaker condition.

3.2 Case 2: Potential for Theft Matters

Here, we examine the case where k^T, the cost of theft, is small enough that this affects the behavior of those using currency in transactions. In equilibrium $\alpha\epsilon(0,1)$ so

$$c = \frac{\pi}{\beta}k^T, \tag{33}$$

and

$$x^c = k^T - k^c. \tag{34}$$

Therefore, from (17) and (34),

$$-1 + \frac{\alpha\beta}{\pi}\theta^L u'\left(k^T - k^c\right) = 0. \tag{35}$$

That is, buyers who use currency in transactions reduce their currency holdings to a level such that a seller will be indifferent between stealing the buyer's currency and bargaining with the buyer to acquire the buyer's currency in exchange for goods.

First, in an equilibrium in which the bank's collateral constraint does not bind, (22) holds, so if the central bank increases the nominal interest rate, inflation increases one-for-one (approximately). But, this has no effect on x^d, nor on x^c, from (34). However, from (22) and (35),

$$\alpha = \frac{R^b}{\theta^L u'(k^T - k^c)} \tag{36}$$

so the probability of theft in the exchange of currency for goods decreases as the nominal interest rate and inflation increases. So, though a buyer who uses currency in transactions receives consumption that is invariant to monetary policy, he or she purchases more goods, in expected value terms, as the nominal interest rate and inflation rise. Note, however, that to accomplish an increase in inflation, the central bank must accommodate a higher demand for currency (from equation (33)) by holding a larger portfolio of government debt. This may seem counterintuitive, but the result arises because currency demand is constrained by the incentives of currency thieves. In order to have the threshold level of currency in the DM, after inflation has its effects, a buyer must acquire more currency, in real terms, in the CM, when inflation increases. Thus, with theft, higher inflation increases the demand for currency.

From (36), note that there is a critical value for the nominal interest rate, $\hat{R}^b$, above which theft does not take place, i.e. $\alpha = 1$ when $R^b = \hat{R}^b$, or from (36),

$$\hat{R}^b = \theta^L u'(k^T - k^c) \tag{37}$$

Second, consider the case in which the collateral constraint binds. Then, modify equations (29) and (30) using (20) and (33), and simplify to get

$$\frac{v}{\gamma + (1-\gamma)\theta^H u'(x^d)} + \frac{\beta y}{1 - \beta\left[\gamma + (1-\gamma)\theta^h u'(x^d)\right]} = \frac{(1-\rho)\left(x^d + k^d\right)}{1-\gamma} + \rho k^T R^b \tag{38}$$

and

$$\pi = R^b \beta \left[\gamma + (1-\gamma)\theta^H u'(x^d)\right] \tag{39}$$

Given policy (v, R^b), equations (38) and (39) solve recursively for x^d and π. We can show that (38) and (39) solve uniquely for x^d and π, and that an increase in R^b increases inflation, reduces the real interest rate, and reduces x^d. This happens because an increase in R^b increases inflation, which increases the demand for currency which, through the fiscal policy rule, reduces the quantity of collateral backing bank deposits, as the central bank has to purchase more government debt to increase currency supply. As well, from (36) the probability of theft falls as R^b increases.

3.3 Welfare Effects of Monetary Policy

Given (31) and (37), theft does not matter if and only if

$$R^b \geq min\left[\frac{\gamma^m + (1-\gamma^m)\theta^H u'(x^d)}{\gamma + (1-\gamma)\theta^H u'(x^d)}, \beta\theta^L u'(k^T - k^c)\right] \tag{40}$$

In general, we can write the period utility of a large-transaction buyer, from (9)-(11), as

$$U^{Hd} = -\theta^H u'(x^d)(x^d + k^d) + \theta^H u(x^d), \tag{41}$$

and from (28), the right-hand side of (41) is strictly increasing in x^d. From (15), (16), and (17), in the case in which theft does not matter, we can write the small-transaction buyer's period utility as

$$U^{Lc} = -\theta^L u'(x^c)(x^c + k^c) + \theta^L u(x^c) \tag{42}$$

Just as was the case in equation (41), the right-hand side of (42) is strictly increasing in x^c, given (28). As well, from (36), (15), (16), and (17), we can write the period utility of a small-transaction buyer when theft matters as

$$U^{Lc} = -\frac{\pi}{\beta} + \frac{\pi}{\beta} \frac{u\left(k^T - k^c\right)}{u'\left(k^T - k^c\right)}, \tag{43}$$

and the right-hand side of (43) is strictly increasing in π.

In the case in which the bank's collateral constraint does not bind, the welfare of buyers using bank deposits in exchange is unaffected by monetary policy. Changes in the nominal interest rate have no effect on the real interest rate or on consumption by buyers using bank deposits. However, from (43), welfare is increasing in inflation for those using currency in transactions, so long as there is theft in equilibrium. As inflation increases, the demand for currency in real terms increases, but since banks' collateral constraints are slack, this does not affect a bank's ability to back its deposits with collateral. Inflation is therefore welfare increasing, so long as $\alpha < 1$. So a welfare-maximizing monetary policy is to increase the nominal interest rate, and therefore inflation, until $\alpha = 1$. But, if $\alpha = 1$ when $R^b = 1$, then $R^b = 1$ is optimal. That is, from (36), an optimal monetary policy is given by

$$R^b = max[1, \theta^L u'(k^T - k^c)] \tag{44}$$

However, if banks' collateral constraints bind, then increases in the nominal interest rate, at the margin, increase the inflation rate, increase the welfare of those using currency in transactions, and reduce the welfare of those using bank deposits. However, once the nominal interest rate reaches the critical value where thieves are not active in equilibrium, further increases in the nominal interest rate reduce the welfare of those using currency in transactions, and increase the welfare of buyers using bank deposits.

4 Central Bank Digital Currency

In this section we will determine what happens if the central bank offers a digital means of payment. This digital currency will have three advantages over physical currency. First, it is not subject to theft, so that the deadweight losses associated with theft are avoided. Second, the central bank can pay interest on digital currency balances, which eliminates some welfare costs associated with inflation. Third, provided the central bank is trustworthy, digital currency permits the aggregate stock of collateral to be used more efficiently. As we will show, under the assumptions we have made, physical currency will not be held in equilibrium once digital currency is issued. We will consider two cases: (i) digital currency does not compete with private bank deposits as a means of payment; (ii) digital currency is a substitute for bank deposits.

4.1 Case 1: Digital Currency Does Not Compete with Bank Deposits

A key advantage digital currency has over physical currency, is that the central bank can pay interest on digital currency, while paying interest on physical currency is not feasible. Assume that the gross nominal interest rate on digital currency is R^m, and suppose that, in equilibrium, large-transaction buyers do not hold digital currency. Then, a small-transaction buyer who uses digital currency in transactions solves

$$\max_{m} \left[-m + \theta^L u \left(\frac{\beta R^m m}{\pi} - k^m \right) \right], \tag{45}$$

where m denotes the real quantity of digital currency acquired by the buyer in the CM, and as before the digital currency-holder makes a take-it-or-leave-it offer to the seller he or she meets in the DM. Then, the first-order condition for an optimum is

$$-1 + \frac{\beta R^m}{\pi} \theta^L u'(x^m) = 0, \tag{46}$$

where consumption in the DM, denoted x^m, is given by

$$x^m = \frac{\beta R^m m}{\pi} - k^m \tag{47}$$

Suppose first that the collateral constraints of banks do not bind. Then, (22) and (23) hold, $x^d = x^*(\theta^H)$, and the period utility of a buyer who uses bank deposits as a means of payment is

$$U^{Hd} = -\left[x^*(\theta^H) + k^d \right] + \theta^H u \left[x^*(\theta^H) \right] \tag{48}$$

Then, from (22) and (46), we get

$$\theta^L u'(x^m) = \frac{R^b}{R^m}, \tag{49}$$

which determines x^m.

If a large-transaction buyer were to use digital currency rather than bank deposits as a means of payment, then he or she solves

$$\max_{m^H} \left[-m^H + \theta^H u \left(\frac{\beta R^m m^H}{\pi} - k^m \right) \right] \tag{50}$$

The solution is characterized by

$$-1+\frac{\beta R^m}{\pi}\theta^H u'(x^{Hm})=0, \tag{51}$$

and maximized utility in this case for the large transaction buyer can be written

$$U^{Hm}=\theta^H\left[-u'(x^{Hm})(x^{Hm}+k^m)+u(x^{Hm})\right] \tag{52}$$

So, in this regime, R^m must be set so that $U^{Hm}\leq U^{Hd}$, otherwise large-transaction buyers would prefer digital currency to bank deposits.

In equilibrium, the collateral constraint must not bind. From (10), (14), and (47), this gives

$$v+\frac{\beta y}{1-\beta}\geq\rho(x^m+k^m)\frac{R^b}{R^m}+\frac{(1-\rho)(x^d+k^d)}{1-\gamma}. \tag{53}$$

Monetary policy now has two dimensions, summarized by the two gross nominal interest rates R^b and R^m, on government debt and digital currency, respectively. From (49) and (22), an increase in R^b reduces x^m and increases inflation, with no effect on x^d or the real interest rate on government debt. This increase in the nominal interest rate on government debt is supported by an open market sale of government debt by the central bank. An increase in R^m, from (49), increases x^m, and has no effect on inflation or the real interest rate. Again, the central bank must support the increase in R^m by accommodating the increase in money demand with an open market purchase of government debt. Further, note that if R^b and R^m both increase, with $\frac{R^b}{R^m}$ unchanged, then this is neutral, with the only effect being an increase in inflation. That is, the ability to pay interest on currency changes the welfare effects of inflation.

Next, consider the case in which banks' collateral constraints bind. Then, when can derive two equations, similar to (29) and (30) that solve for x^m and x^d given policy $\left(v,R^b,R^m\right)$, i.e.

$$v+\frac{\beta y[\gamma+(1-\gamma)\theta^H u'(x^d)]}{1-\beta\left[\gamma+(1-\gamma)\theta^h u'(x^d)\right]}=\frac{(1-\rho)\left(x^d+k^d\right)\left[\gamma+(1-\gamma)\theta^H u'(x^d)\right]}{1-\gamma}+\rho\theta^L u'(x^m)(x^m+k^m) \tag{54}$$

and

$$R^b=\frac{R^m\theta^L u'(x^m)}{\theta^H[\gamma+(1-\gamma)\theta^H u'(x^d)]}. \tag{55}$$

As well, from (46) we can determine the inflation rate, given x^m, i.e.

$$\pi=\beta R^m\theta^L u'(x^m) \tag{56}$$

The quantitative effects of an increase in R^b are identical to what occurs with physical currency. That is, if R^b increases, then x^d rises, x^m falls, the real interest rate on government debt rises, and the inflation rate increases. The difference with digital currency comes from the effects of a change in R^m. That is, if R^m increases, then this reduces x^d, increases x^m, and reduces the real interest rate on government debt. From (55) and (56), this results in an increase in the inflation rate. As in the case with nonbinding collateral constraints, increases in R^b and R^m that leave $\frac{R^b}{R^m}$ unchanged are neutral, but from (56) the inflation rate increases one-for-one (approximately) with the increases in nominal interest rates.

If we assume that $k^m \leq k^c$, i.e. the costs for sellers if accepting digital currency as a means of payment are not larger than for physical currency, then it is straightforward to demonstrate that, in this regime, digital currency is welfare improving. That is suppose that $k^m = k^c$. First, if there is no theft in equilibrium with physical currency, then the equilibrium allocation can be replicated with digital currency, with $R^m = 1$, i.e. a zero nominal interest rate on currency. Second, suppose an equilibrium with physical currency in which there is theft, i.e. $\alpha < 1$. Then, from (33)-(35), we can the expected period utility of small-transaction agent who holds physical currency, when digital currency is not permitted, is given by

$$U^{Lc} = \alpha\theta^L \left[-(x^c + k^c) + u(x^c)\right] \tag{57}$$

From (46) and (47), the expected utility of a digital currency user, when physical currency is not issued, is

$$U^{Lm} = \theta^L \left[-(x^m + k^c) + u(x^m)\right] \tag{58}$$

Then, if $R^m = 1$ in the digital currency regime, and given a gross nominal interest rate R^b, from (38), (39), (54), and (55), the same allocation of consumption in the DM can be supported in the digital currency regime and the physical currency regime. But, from (57) and (58), the welfare of digital currency users is higher in the digital currency regime, than for physical currency users in the alternative regime. This occurs because the physical currency users stand a chance of having their currency stolen, in which case they do not consume in the physical currency regime. Thus, in general, digital currency must dominate physical currency, simply because of the cost of theft with physical currency.

4.2 Digital Currency Competes with Bank Deposits

We have shown that, if digital currency is offered, and the cost to sellers of accepting digital currency is the same as the cost of accepting physical currency, then any equilibrium allocation with physical currency can be replicated with digital currency. But digital currency eliminates theft, so digital currency dominates physical currency. But what if digital currency competes with private bank deposits? In this subsection, we will analyze how substitution of digital currency for bank deposits matters for the effects of monetary policy, and for welfare.

Our analysis thus far has been confined to cases in which central bank digital currency is offered on terms implying its use only by small-transaction buyers. But, provided the transactions costs incurred by sellers are not too high, if the interest rate on central bank digital currency is sufficiently large, then digital currency will become sufficiently attractive that it will be held by large-transaction buyers.

In equilibrium, from (11),

$$d = (1-\gamma)\left[\frac{R^b b}{\pi} + (\phi + y)a\right], \tag{59}$$

so

$$x^d + k^d = \beta(1-\gamma)\left[\frac{R^b b}{\pi} + (\phi + y)a\right] = \frac{(1-\gamma)(b+\phi a)}{\gamma + (1-\gamma)\theta^H u'(x^d)} \tag{60}$$

Also, from (10) and (11),

$$z = b + \phi a - \beta\gamma\left[\frac{R^b b}{\pi} + (\phi + y)a\right] = \theta^H u'(x^d)\left(x^d + k^d\right) \tag{61}$$

Therefore, in equilibrium, we can express the period utility for a large-transaction buyer from using private bank deposits as

$$U^{Hd} = -\theta^H u'(x^d)(x^d + k^d) + \theta^H u(x^d), \tag{62}$$

and the period utility for a large-transaction buyer from using digital currency in transactions is

$$U^{Hm} = -\theta^H u'(x^{Hm})(x^{Hm} + k^m) + \theta^H u(x^{Hm}), \tag{63}$$

where x^{Hm} is consumption in the DM for the large-transaction buyer who uses digital currency in transactions, with this quantity satisfying the first-order condition

$$-1 + \frac{\beta R^m}{\pi}\theta^H u'(x^{Hm}) = 0. \tag{64}$$

Suppose that $k^d = k^m$, so that digital currency has no advantage over private bank deposits in making retail payments. In equilibrium, large-transaction buyers must be indifferent in this case between holding bank deposits and digital currency, so $U^{Hd} = U^{Hm}$. But, the function $\psi(x) = u'(x)(x + k^d) + u(x)$ is strictly increasing in x, given (28). Therefore, if $U^{Hd} = U^{Hm}$, then from (62) and (63), $x^d = x^{Hm}$ in equilibrium. Then, from (20) and (64)

$$R^m\theta^H u'(x^{Hm}) = R^b\left[\gamma + (1-\gamma)\theta^H u'(x^{Hm})\right], \tag{65}$$

or

$$\theta^H u'(x^{Hm}) = \frac{R^b\gamma}{R^m - R^b(1-\gamma)} \tag{66}$$

Then, from (46) and (64), we have

$$\theta^H u'(x^{Hm}) = \theta^L u'(x^m) \tag{67}$$

We can then solve (66) and (67) for x^{Hm} and x^m, and then the quantity of consumption for large-transaction buyers is $x^d = x^{Hm}$. We need to also determine the fraction of large-transaction buyers δ that uses digital currency in transactions. The equilibrium condition corresponding to (54) is

$$v + \frac{\beta y[\gamma + (1-\gamma)\theta^H u'(x^d)]}{1 - \beta\left[\gamma + (1-\gamma)\theta^h u'(x^d)\right]} = \frac{(1-\rho)(1-\delta)\left(x^d + k^d\right)\left[\gamma + (1-\gamma)\theta^H u'(x^d)\right]}{1-\gamma} + \rho\theta^L u'(x^m)(x^m + k^m) + (1-\rho)\delta\theta^H u'(x^{Hm})(x^{Hm} + k^m) \quad (68)$$

Or,

$$\overline{1-\gamma}$$

$$+\rho\theta^L u'(x^m)(x^m + k^m)$$

$$\frac{v + \frac{\beta y[\gamma + (1-\gamma)\theta^H u'(x^d)]}{1 - \beta\left[\gamma + (1-\gamma)\theta^h u'(x^d)\right]}}{1-\gamma+\rho\theta^L u'(x^m)(x^m+k^m)} = (1-\rho)\left(x^d + k^d\right)\left[\gamma + (1-\gamma)\theta^H u'(x^d) - \delta\gamma\right.$$

(70)

Then, given the solution for x^d and x^m, equation (4.2) solves for δ. An equilibrium exists if and only if this solution $\delta \epsilon (0,1)$.

How does policy matter in this equilibrium? From equation (66), an increase in R^b, given R^m, reduces x^d, x^{Hm}, and x^m, and reduces δ. That is, large-transaction buyers substitute from digital currency to private bank deposits, and this absorbs more safe collateral, because banks need the incentive not to abscond on their deposit liabilities. Then, as collateral has become effectively more scarce, the volume of exchange that can be supported is smaller, in all transactions. So, consumption falls in all transactions in the DM, and welfare of all buyers must also fall.

If R^m rises, from (66) this works qualitatively in the opposite direction to an increase in R^b. That is, x^d, x^{Hm}, and x^m all increase, and δ increases. Large-transaction buyers substitute to digital currency from private bank deposits, this frees up collateral, and therefore the aggregate effective stock of collateral can support more transactions. Consumption increases in all DM transactions, and the welfare of all buyers goes up.

4.3 Digital Currency Competes with Bank Deposits, But Private Banks Hold Only Private Collateral

We are assuming that the central bank in this model cannot hold private assets in its portfolio. Thus, it is possible that we could have an equilibrium in which private banks hold no government debt, and only hold private assets in their portfolios, with the government issuing digital currency that is used by both large-transaction and small-transaction buyers.

In this equilibrium, collateral is scarce, and the demand for public collateral, derived from the demand for digital currency, absorbs the entire supply of public collateral, determined by the fiscal authority. As in the previous subsection, ρ denotes the fraction of large-transaction buyers who hold digital currency. Then,

$$\frac{\beta y}{1-\beta\left[\gamma+(1-\gamma)\theta^h u'(x^d)\right]} = \frac{(1-\rho)(1-\delta)\left(x^d+k^d\right)}{1-\gamma} \tag{71}$$

and

$$v = \rho\theta^L u'(x^m)(x^m+k^m) + (1-\rho)\delta\theta^H u'(x^{Hm})(x^{Hm}+k^m) \tag{72}$$

That is, (71) and (72) state, respectively, that the supply of private collateral equals the demand for private collateral (from private banks), and that the supply of public collateral equals the demand for digital currency. Note, in this case, that digital currency absorbs the entire stock of consolidated government debt. The fiscal authority issues debt, which is all held by the central bank as backing for digital currency.

The nominal interest rate on government debt is irrelevant in this equilibrium, as there is no government debt outstanding. Then, since $U^{Hd} = U^{Hm}$ in equilibrium, from (62) and (63), if $k^d = k^m$, then

$$x^d = x^{Hm}, \tag{73}$$

in equilibrium. Then, (71)-(73) and (67) solve for x^d, x^m, x^{Hm}, and δ. Then, (64) determines π given R^m.

Therefore, monetary policy is just the choice of R^m in this equilibrium, which has no effect on real variables, and only increases the gross inflation rate in proportion. That is, the real interest rate is invariant to monetary policy, so increases in the nominal interest rate on digital currency serve only to increase inflation one-for-one. Monetary policy would not be neutral in this equilibrium, however, if the central bank were to conduct open market operations in private assets.

5 Conclusion

We have constructed a model of asset exchange and means of payment so as to explore the role of central bank digital currency (CBDC), and how this matters for monetary policy. Central bank digital currency potentially increases welfare for three reasons. First, in substituting for physical currency, it serves to limit criminal activity. Second, CBDC can bear interest, which introduces an additional policy instrument for the central bank, and simplifies the problem of eliminating intertemporal inefficiency typically corrected by a Friedman rule. Third, in substituting for private bank deposits as means of payment, CBDC mitigates incentive problems in private banking, provided the central bank can be trusted.

Expansion of the reach of central bank liabilities through the issue of CBDC potentially introduces a scarcity of safe collateral, if the central bank is limited to holding government debt. Potentially, this scarcity can be circumvented if the central bank acquires private assets, but this opens up other issues related to the ability of the central bank to screen private assets.

6 References

Abadi, J. and Brunnermeier, M. 2018. "Blockchain Economics," working paper, Princeton University.

Bech M. L., Faruqui, U., Ougaard, F. and Picillo, C. 2018. "Payments Are A-Changin' But Cash Still Rules." BIS Quarterly Review (March).

Chapman, J., and Wilkins, C. 2019. "Crypto 'Money': Perspective of a Couple of Canadian Central Bankers," Bank of Canada Staff Discussion Paper 2019-1.

Chiu, J. and Koeppl, T. 2018. "The Economics of Cryptocurrencies – Bitcoin and Beyond," working paper, Bank of Canada and Queen's University.

Davoodalhosseini, M. 2018. "Central Bank Digital Currency and Monetary Policy," Bank of Canada Staff Working Paper 2018-36.

Friedman, M. 1969. "The Optimum Quantity of Money," in Friedman, M. *The Optimum Quantity of Money and Other Essays*, Aldine Publishing Co.

Gertler, M., Kiyotaki, N., 2011. "Financial intermediation and credit policy in business cycle analysis," In: *Handbook of Monetary Economics,* eds. Friedman, B. and Woodford, M., Elsevier.

Hendry, S., and Zhu, Y. 2019. "A Framework for Analyzing Monetary Policy in an Economy with E-Money," Bank of Canada staff working paper 2019-01.

Keister, T., and Sanches, D. 2018. "Should Central Banks Issue Digital Currency?" working paper, Rutgers University and Philadelphia Federal Reserve Bank.

Kumhof, M. and Noone, C. 2018. "Central Bank Digital Currencies: Design Principles and Balance Sheet Implications," Staff Working Paper 725, Bank of England.

Rocheteau, G. and Wright, R. 2005. "" *Econometrica* **??**.

Rogoff, K. 2016. *The Curse of Cash*, Princeton University Press, Princeton NJ.

Williamson, S. 2016. "**??**" **??** *Journal of Economic Theory* **??**.

Williamson, S. 2019a. "Low Real Interest Rates, Collateral Misrepresentation, and Monetary Policy," forthcoming, *American Economic Journal: Macroeconomics.*

Williamson, S. 2019b. "Interest on Reserves, Interbank Lending, and Monetary Policy," forthcoming, *Journal of Monetary Economics.*