Optimal Monetary Policy in HANK Economies

Sushant Acharya[a] Edouard Challe[b] Keshav Dogra[a]

[a]New York Fed [b]CREST & Ecole Polytechnique

Question and framework

- how does imperfect insurance affect **optimal** monetary policy?
- **challenge**: social welfare function endogenous to cross-sectional distribution, which is endogenous to policy (fixed-point problem)
- **solution**: CARA-Normal HANK with closed-form expressions for
  - the distribution of agents
  - the social welfare function

Main results

- optimal policy governed by three forces

1. desirability of price stability (as in RANK models)
2. desirability of redistribution (as in Bhandari et al., 2018)
3. **impact of policy on MPC**

- decomposition of these three forces and contribution to:
  - optimal interest-rate response to aggregate (productivity) shock
  - implied path of aggregates

Related literature

- optimal monetary policy under perfect insurance
  - **RANK**: Clarida et al. (1999); Woodford (2003); Gali (2008)...
  - **TANK**: Bilbiie (2008); Debortoli and Gali (2017)...
  - **RANK & SaM**: Thomas (2008); Faia (2009); Blanchard-Gali (2010); Ravenna-Walsh (2011)...
- optimal monetary policy under imperfect insurance
  - **HANK & SaM** (zero-liquidity): Challe (2018)
  - **positive liquidity**: Nuño-Thomas (2017); Bhandari et al. (2018)
- CARA-Normal imperfect-insurance models
  - **flex-price**: Calvet (2001), Angeletos and Calvet (2005, 2006)...
  - **HANK**: Acharya & Dogra (2018)

Households

Objective and constraints

- mass 1 with survival probability θ and generational turnover

- objective:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\theta)^t \left(-\frac{1}{\gamma} e^{-\gamma c_t^i} - e^{\ell_t^i - \xi_t^i} \right), \quad \xi_t^i \sim \mathcal{N}(\bar{\xi}, \sigma^2)$$

s.t.

$$P_t c_t^i + \frac{\theta}{\underbrace{1 + i_t}_{\text{nominal rate on actuarial bonds}}} A_{t+1}^i \quad = \quad A_t^i + \underbrace{(1 - \tau_t)\tilde{w}_t}_{\text{net wage } w_t} P_t \ell_t^i + P_t T_t$$

- in real terms:

$$a_{t+1}^i = \frac{R_{t+1}}{\theta} \left(a_t^i + w_t \ell_t^i + T_t - c_t^i \right), \quad R_{t+1} = \frac{1 + i_t}{\Pi_{t+1}}$$

Households

Optimality conditions

- assume no aggregate risk (steady state or MIT shock)
- bond Euler equation

$$e^{-\gamma c_t^i} = \beta R_{t+1} \mathbb{E}_t e^{-\gamma c_{t+1}^i}$$

- optimal labor supply

$$\ell_t^i = \ln w_t - \gamma c_t^i + \xi_t^i$$

Households

Policy functions

- conjecture & verify that

$$c_t(x_t^i) = \underset{\substack{\uparrow \\ \text{output}}}{y_t} + \underset{\substack{\uparrow \\ \text{MPC}}}{\mu_t} \,.\, \underset{\substack{\uparrow \\ \text{state}}}{x_t^i}$$

  where

$$\textbf{state} \quad : \quad x_t^i = a_t^i + w_t(\xi_t^i - \bar{\xi})$$

$$\textbf{MPC} \quad : \quad \frac{1}{\mu_t} = 1 + \gamma w_t + \frac{\theta}{\mu_{t+1} R_{t+1}}$$

- and also

$$\textbf{labor sup.} \quad : \quad \ell_t(x_t^i, \xi_t^i) = \ln w_t - \gamma c_t(x_t^i) + \xi_t^i$$

$$\textbf{savings} \quad : \quad s_t(x_t^i) = (1 - (1 + \gamma w_t)\, \mu_t)\, x_t^i$$

$$\textbf{wealth} \quad : \quad a_{t+1}(x_t^i) = \frac{R_{t+1}}{\theta} (1 - (1 + \gamma w_t)\, \mu_t)\, x_t^i$$

Firms

- competitive final-goods firms and monopolistically competitive wholesale firms (with production subsidy $\tau^W = 1/\varepsilon$)

- Rotemberg pricing

- output (net of inflation cost):

$$y_t = z_t n_t - \frac{\Phi}{2}\left(\Pi_t - 1\right)^2 y_t = \frac{z_t n_t}{1 + \frac{\Phi}{2}\left(\Pi_t - 1\right)^2}$$

- NKPC:

$$\left(\Pi_t - 1\right)\Pi_t = \frac{\varepsilon}{\Phi}\left(1 - \frac{\left(1-\tau_t\right) z_t}{w_t}\right) + \left(\frac{y_{t+1} z_t w_{t+1}}{R_{t+1} z_{t+1} y_t w_t}\right)\left(\Pi_{t+1} - 1\right)\Pi_{t+1}$$

Aggregation

- let $f_t(x)$ be the cross-sectional distribution of individual states
- consumption:

$$\int c_t(x) f_t(x)\,\mathrm{d}x = y_t$$

and

$$y_t = y_{t+1} - \frac{\ln(\beta R_{t+1})}{\gamma} - \frac{\gamma\sigma^2\mu_{t+1}^2 w_{t+1}^2}{2}$$

- labor supply:

$$\int l_t(x) f_t(x)\,\mathrm{d}x = \ln w_t - \gamma y_t + \bar{\xi} = n_t$$

where

$$y_t = \frac{z_t n_t}{1 + \frac{\Phi}{2}(\Pi_t - 1)^2}$$

- nominal bonds

$$\int a_{t+1}(x) f_t(x)\,\mathrm{d}x = 0$$

Summary of aggregate dynamics

- 4 equations:

$$
\begin{aligned}
y_t &= y_{t+1} - \frac{1}{\gamma} \ln \left(\beta \frac{1+i_t}{\Pi_{t+1}} \right) - \frac{\gamma \sigma^2 \mu_{t+1}^2 w_{t+1}^2}{2} \\
y_t &= \frac{z_t \left(\ln w_t + \bar{\xi} \right)}{1 + \gamma z_t + \frac{\Phi}{2} \left(\Pi_t - 1 \right)^2} \\
\frac{1}{\mu_t} &= 1 + \gamma w_t + \frac{\theta}{\mu_{t+1} R_{t+1}} \\
\left(\Pi_t - 1 \right) \Pi_t &= \frac{\varepsilon}{\Phi} \left(1 - \frac{\left(1 - \tau_t \right) z_t}{w_t} \right) + \frac{y_{t+1} z_t w_{t+1} \left(\Pi_{t+1} - 1 \right) \Pi_{t+1}^2}{y_t z_{t+1} w_t \left(1 + i_t \right)}
\end{aligned}
$$

- 4 endogenous variables: $\{y_t, \mu_t, w_t, \Pi_t\}$
- 1 or 2 instruments: either $\{i_t, \tau_t\}$, or $\{i_t\}$ with $\tau_t = \tau$
- i_t, τ_t affect, and are affected by, **cross-sectional distribution of** x

Cross-sectional distribution(s)

- in every period $1-\theta$ of households enter/exit the economy

- state of a **newcomer** i:

$$x_t^i = w_t(\xi_t^i - \bar{\xi})$$

- state of a **survivor** i:

$$x_t^i = \frac{\mu_{t-1}}{\mu_t} x_{t-1}^i + w_t(\xi_t^i - \bar{\xi})$$

- hence the state of a hh living from time t to $t+k$ is such that:

$$x_{t+k,t}^i \sim \mathcal{N}\left(0, \frac{\sigma^2}{\mu_{t+k}^2} \sum_{j=0}^{k} \mu_{t+k}^2 w_{t+k}^2\right)$$

Cross-sectional distribution(s)

- the time-t cdf of the state is:

$$
\begin{aligned}
F_t(x) &= (1-\theta)\,\Phi\left(\frac{x}{\sigma w_t}\right) + (1-\theta)\,\theta\Phi\left(\frac{x}{\frac{\sigma}{\mu_t}\sqrt{\mu_t^2 w_t^2 + \mu_{t-1}^2 w_{t-1}^2}}\right) + \ldots \\
&\quad + (1-\theta)\,\theta\Phi\left(\frac{x}{\frac{\sigma}{\mu_t}\sqrt{\mu_t^2 w_t^2 + \mu_{t-1}^2 w_{t-1}^2 + \ldots + \mu_{t-k}^2 w_{t-k}^2}}\right) \\
&= (1-\theta)\sum_{k=0}^{\infty}\theta^k\Phi\left(\frac{x}{\frac{\sigma}{\mu_t}\sqrt{\sum_{j=0}^{k}\mu_{t-j}^2 w_{t-j}^2}}\right)
\end{aligned}
$$

- density:

$$
f_t(x) = (1-\theta)\sum_{k=0}^{\infty}\frac{\theta^k}{\frac{\sigma}{\mu_t}\sqrt{\sum_{j=0}^{k}\mu_{t-j}^2 w_{t-j}^2}}\phi\left(\frac{x}{\frac{\sigma}{\mu_t}\sqrt{\sum_{j=0}^{k}\mu_{t-j}^2 w_{t-j}^2}}\right)
$$

Cross-sectional distribution(s)

- invariant density:

$$\bar{f}(x) = \frac{1-\theta}{\theta} \sum_{k=1}^{\infty} \frac{\theta^k}{w\sigma\sqrt{k}} \phi\left(\frac{x}{w\sigma\sqrt{k}}\right)$$

- suppose the economy is in steady state until time $t-1$:

$$f_{t-1}(x) = \bar{f}(x)$$

- at time t an aggregate shock occurs; **what is** $f_t(x)$?

- use **Chapman-Kolmogorov** eq. to compute marginal density of x

Cross-sectional distribution(s)

- time-$t-1$ decision based on belief that $(\Pi_t, R_t,) = (1, R)$, hence

$$s_{t-1}(x^i_{t-1}) = (1-(1+\gamma w)\,\mu)\,x^i_{t-1} = \frac{\theta}{R}x^i_{t-1}$$

- time-t return on savings turns out to be

$$R_t = \frac{1+i_{t-1}}{\Pi_t} = \frac{R}{\Pi_t}$$

hence

$$a^i_t = \frac{R_t}{\theta}s_{t-1}(x^i_{t-1}) = \frac{x^i_{t-1}}{\Pi_t}$$

- time-t state of survivor:

$$x^i_t \quad = \quad a^i_t + w_t(\xi^i_t - \bar{\xi}) = \frac{x^i_{t-1}}{\Pi_t} + w_t(\xi^i_t - \bar{\xi})$$

- aggregate shock affects distribution of x through (Π_t, w_t)

- conditional distribution of x^i_t:

$$\mathbb{E}_t(x^i_t|x^i_{t-1}) = x^i_{t-1}/\Pi_t, \qquad \mathbb{V}_t(x^i_t|x^i_{t-1}) = \sigma^2 w^2_t$$

Cross-sectional distribution(s)

- marginal density of x:

$$f_t\left(x\right)=\underbrace{\theta\int\frac{1}{\sqrt{2\pi}\sigma w_t}e^{-\frac{1}{2}\left[\frac{x-z/\Pi_t}{\sigma w_t}\right]^2}\bar{f}\left(z\right)\mathrm{d}z}_{\equiv f_{1,t}(x)\ \text{(survivors)}}$$

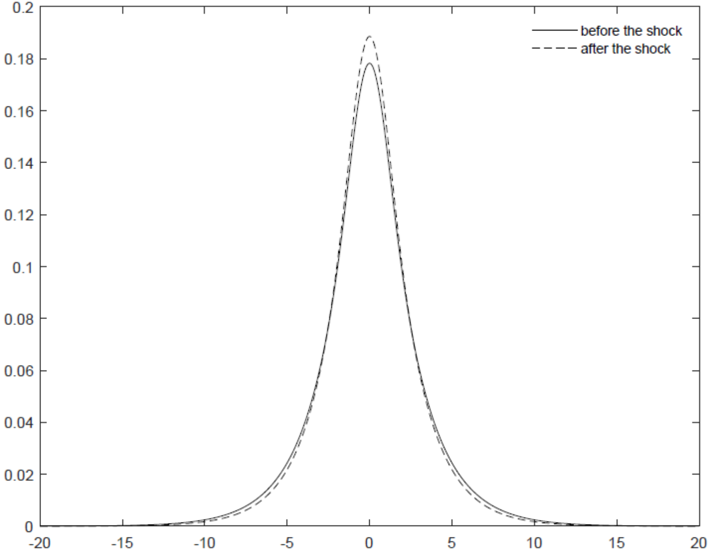
$$+\left(1-\theta\right)\underbrace{\frac{1}{\sqrt{2\pi}\sigma w_t}e^{-\frac{1}{2}\left(\frac{x}{\sigma w_t}\right)^2}}_{\equiv f_{2,t}(x)\ \text{(newcomers)}}$$

- completing the square we get:

$$f_{1,t}\left(x\right)\quad=\quad\frac{1-\theta}{\theta}\sum_{k=1}^{\infty}\theta^k\frac{e^{-\frac{1}{2}\left(\frac{x}{\sigma\sqrt{k(w/\Pi_t)^2+w_t^2}}\right)^2}}{\sqrt{2\pi}\sigma\sqrt{k\left(w/\Pi_t\right)^2+w_t^2}}$$

- inflation redistributes wealth which **reduces dispersion of state**

Impact of inflation shock



Optimal policy

- **optimal** policy $\Rightarrow$ policy affects **and is affected** by distribution

- social welfare function has 2 inputs:
  1. individual intertemporal utility conditional on state
  2. distributions of agents across states and generations

- individual intertemporal utility has closed-form expression:

$$
\begin{aligned}
V_t(x_t^i) &= \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \left(-\frac{1}{\gamma} e^{-\gamma c_{t+k}^i} - e^{\ell_{t+k}^i - \xi_{t+k}^i} \right) \\
&= -\left(\frac{1}{\gamma} + w_t \right) e^{-\gamma \left(y_t + \mu_t x_t^i \right)} + \mathbb{E}_t V_{t+1}(x_{t+1}^i) \\
&= -\frac{1}{\gamma \mu_t} e^{-\gamma \left(y_t + \mu_t x_t^i \right)}
\end{aligned}
$$

- aggregate intertemporal utilities **within** & **across** generations

Social welfare function

- total welfare of time-t survivors:

$$U_t^{t-1} = \int V_t(x) f_{1,t}(x)\,\mathrm{d}x = -\frac{1}{\gamma\mu_t} \cdot \frac{\mathrm{e}^{\frac{\gamma^2\sigma^2\mu_t^2 w_t^2}{2} - \frac{\gamma z_t(\ln w_t + \bar{\xi})}{1+\gamma z_t + \frac{\Phi}{2}(\Pi_t - 1)^2}}}{\mathrm{e}^{-\frac{\gamma^2\mu_t^2\sigma^2 w^2/\Pi_t^2}{2}} - \theta}$$

(note that inflation shows up **twice**)

- total welfare of time-t newcomers:

$$U_t^t = \int V_t(x) f_{2,t}(x)\,\mathrm{d}x = -\frac{1}{\gamma\mu_t}\mathrm{e}^{\frac{\gamma^2\sigma^2 w_t^2\mu_t^2}{2} - \frac{\gamma z_t(\ln w_t + \bar{\xi})}{1+\gamma z_t + \frac{\Phi}{2}(\Pi_t - 1)^2}}$$

- total welfare of time-$t+k$ newcomers:

$$U_t^{t+k} = \int V_{t+k}(x) f_{2,t+k}(x)\,\mathrm{d}x$$

$$= -\frac{1}{\gamma\mu_{t+k}}\mathrm{e}^{\frac{\gamma^2\sigma^2 w_{t+k}^2\mu_{t+k}^2}{2} - \frac{\gamma z_{t+k}(\ln w_{t+k} + \bar{\xi})}{1+\gamma z_{t+k} + \frac{\Phi}{2}(\Pi_{t+k} - 1)^2}}$$

Social welfare function

- SWF aggregates intertemporal utilities of all households, within and across generations:

$$
\begin{aligned}
W_t &= \lambda\theta U_t^{t-1} + (1-\theta)\sum_{k=0}^{\infty} \Xi^k U_t^{t+k} \\
&= \lambda\theta U_t^{t-1} + (1-\theta)\, W_t^n
\end{aligned}
$$

with

$$
W_t^n = U_t^t + \Xi W_{t+1}^n
$$

- complication: λU_t^{t-1} term **prevents recursive representation**
- this is due to the fact that inflation redistributes wealth at the time of the shock and only then (after that the Fisher equation kicks in)

Simple (illustrative) scenario

- compute optimal policy when $\theta = 0$
- social welfare function given by:

$$W_t = \frac{e^{\frac{\gamma^2\sigma^2 w_t^2 \mu_t^2}{2} - \frac{\gamma z_t \left(\ln w_t + \bar{\xi}\right)}{1+\gamma z_t + \frac{\Phi}{2}(\Pi_t - 1)^2}}}{\gamma \mu_t} + \Xi W_{t+1}$$

- **monetary-fiscal** policy: τ_t varies to fix imperfect-insurance externality
- **monetary policy**: central bank = residual policymaker ⇒ tradeoffs

Optimal monetary-fiscal policy

- compute optimal state-contingent tax τ_t in flex-price model

- under flexible price $\tilde{w}_t = z_t$, hence $w_t = (1 - \tau_t) z_t$

- gov't solves

$$W_t = \max_{w_t}\{-(\gamma\mu_t)^{-1} e^{\frac{\gamma^2\sigma^2 w_t^2 \mu_t^2}{2} - \gamma y_t} + \Xi W_{t+1}\}$$

s.t.

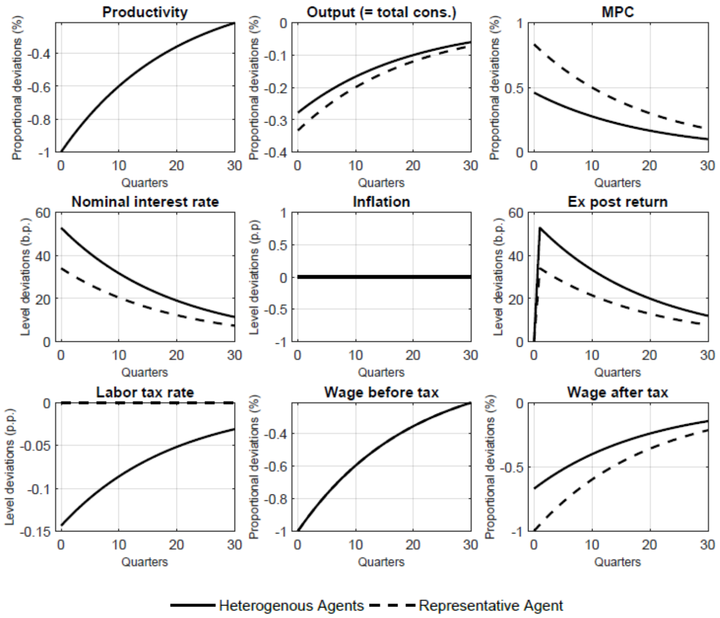
$$y_t = \frac{\ln w_t + \bar{\xi}}{z_t^{-1} + \gamma}, \quad \mu_t = \frac{1}{1 + \gamma w_t}$$

- f.o.c. w.r.t. w_t gives

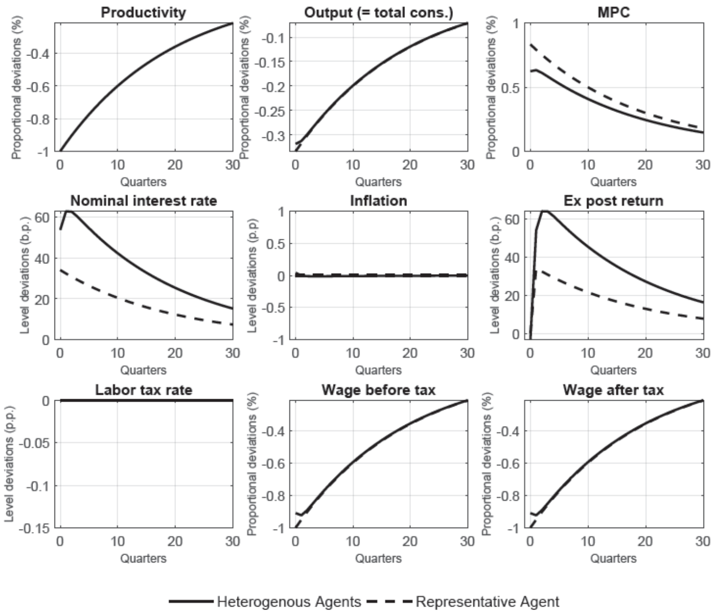
$$f(w_t; \gamma, \sigma) = \frac{(w_t^{-1} + \gamma)^2 w_t + \gamma\sigma^2}{(w_t^{-1} + \gamma)^2 - \gamma^2\sigma^2} = z_t$$

- implies **positive** and **procyclical** labor income tax rate

Optimal monetary-fiscal policy



Optimal monetary policy



Summary

- CARA-Normal HANK
- closed-form expression for cross-sectional distribution and its responsiveness to shocks (solves fixed point problem)
- key channel: **time-varying MPC** that changes the impact of monetary policy and thus optimal interest rate movements
- challenge: solving general case where social planner problem does not have a recursive representation (due to surprise redistributive effect)