Asset Quality Dynamics

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Abstract

We provide a simple corporate finance environment with endogenous security design which aggregates to a standard macroeconomic model. The model is quantitatively consistent with the cyclical properties of safe corporate debt issues, in particular with the fact that those issues are less procyclical than other sources of corporate financing. It is also consistent with the countercyclicality of risk spreads on corporate debt. We then use the model to measure the effect of a protracted periods of low safe yields, one of the main features of the so-called "saving glut" the global economy is currently experiencing. A long period of low interest rates on safe debt has little impact on the level of economic activity but causes output and investment volatility to fall.

Preliminary and incomplete, comments welcome.

Keywords: Endogenous Security Markets; Corporate Debt; Savings Glut
JEL codes: E44; E30

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1 Introduction

Markets for safe US assets have received increased attention in policy and academic circles over the past decade for two prominent reasons. First, mortgage-backed securitization– the largest sources of safe US securities after Treasury markets – was at the epicenter of and is often blamed for the recent financial crisis. Second, demand for US investment-grade assets started increasing markedly in the late 1990s, fueled in part by the fast emergence of nations with high saving rates and a strong preference for safe investments, a phenomenon former Federal Reserve Board Governor Ben Bernanke termed the *Global Saving Glut*. Bernanke et al. (2011) and Gennaioli et. al (2011) among many others have argued that the saving glut has had a profound impact on financial practices in the United States and made the US financial system more prone to crises.

This paper considers a prominent producer of highly rated US securities that has received much less attention than asset-backed securitization markets: the corporate sector. One should expect increased willingness to pay for safe assets to have a significant impact on the capital structure and investment decisions by corporations. In order to quantify this effect one first needs a satisfactory model of the production of safe securities by corporations. In particular, such a model should be consistent with the cyclical properties of safe corporate issues. Our main goal in this paper is produce such a model.

To that end we begin by documenting the cyclical properties of safe corporate debt. Our main finding in this respect is that safe corporate debt is noticeably less cyclical that other corporate liabilities (risky debt and equity.) We establish a connection between this fact and the observation made recently by Covas and Den Haan (2014) that the cyclical properties of corporate liabilities vary drastically with firm size. We go on to show that a dynamic extension of Allen and Gale (1988)'s optimal security design can account for these cyclical facts.[1]

The model we build is standard on the real side but we allow the security space to respond

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[1] Another recent paper in this vein is Babus and Hachem (2019).

endogenously to changes in demand for different types of claim. Specifically, producers choose what quantity of safe claims to issue given investors' willingness to pay for such assets. In particular, ours is model where the financial structure is endogenous. As a result, in order to select a consumption and investment policy, households must form forecasts as to what the future supply of various types of securities will be for every possible sequence of aggregate shocks. Given this forecast and the resulting willingness to pay for various securities by households, the security creation decisions made by producers must be optimal. In addition to satisfying standard conditions, equilibria must therefore satisfy a consistency condition a la Allen-Gale (1988). The assumptions households make about future security markets and the resulting state prices must be consistent with the securities producers choose to issue given those prices. We propose a recursive algorithm for computing allocations and prices that solve this dynamic fixed point problem.

In our model, producers can sell financial claims to a household sector and/or to an outside market for safe securities where the willingness to pay of investors is stochastic but exogenous. We think of this latter aspect of our model as reflecting the fact that the willingness to pay for safe US assets is largely set by global markets. In the data, yields on highly rated US corporate debt are mildly countercyclical and we calibrate the exogenous process for the price of safe debt in outside markets in our model to capture that feature. This force, alone, would cause safe debt issues to be procyclical in our model. Another source of procyclicality in our model is that constraints on how much safe debt producers can issue become looser during good economic times. While this aspect of our model closely resembles the main mechanism in Jerman and Quadrini (2011), this phenomenon arises endogenously in this paper as safe borrowers require that the claims they buy pay in full even if the worst case scenario materializes.

Given these two procyclical forces, how can a model like ours account for the relative acyclicality of safe debt issues? The key difference between our model and Jerman and Quadrini's is that our model features a strongly countercyclical intensive margins: fewer producers choose to participate in safe debt markets during good times than during bad

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times. Good macroeconomic shocks increase household wealth which in turn increases their willingness to pay for the claims issued by producers, including safe claims. Good shocks thus reduce the value of accessing outside markets for producers and, therefore, fewer producers are willing to bear the costs associated with issuing securities in those markets. Quantitatively, the net effect of these offsetting channels turns out to be weakly counter-cyclical safe debt issues, consistently which what our data analysis suggests.

Having produced a model that is quantitatively consistent with the cyclical properties of safe corporate debt, we use our framework to run two counterfactual experiments. First we simulate the impact of shutting down access to outside markets for safe debts by studying an economy where all securities are priced endogenously by the US household sector. This causes the average level of capital formation to fall by 4% while average output falls by 1%. The volatility of output falls as household savings become more stable but, on the flip side, household consumption becomes significantly more volatile.

In a second experiment we lower the support of the exogenous safe yield process by a uniform 20% to mimic the recent decline in global interest rates. This phenomenon – our proxy for the aforementioned savings glut – has a small impact on the level of GDP in our model but significantly reduces the volatility of investment, output and consumption at the stochastic steady state. When safe yields go down, the return on household portfolios falls which causes domestic savings to fall as well. This, we find, almost exactly offsets the increased investment flows from outside markets. Reduced volatility in activity stems from the greater share of investment flows coming from outside markets since those flows tend to be acyclical.

2 Evidence on the cyclicality of safe corporate debt

The extent literature on the cyclicality of corporate liabilities has yielded often contradictory results. Choe et. al. (1993) and Korajczyk and Levy (2003) find that firms are more likely to issue equity during expansions while debt issues are countercyclical. Quadrini and Jerman

Figure 1: Credit rating by asset size
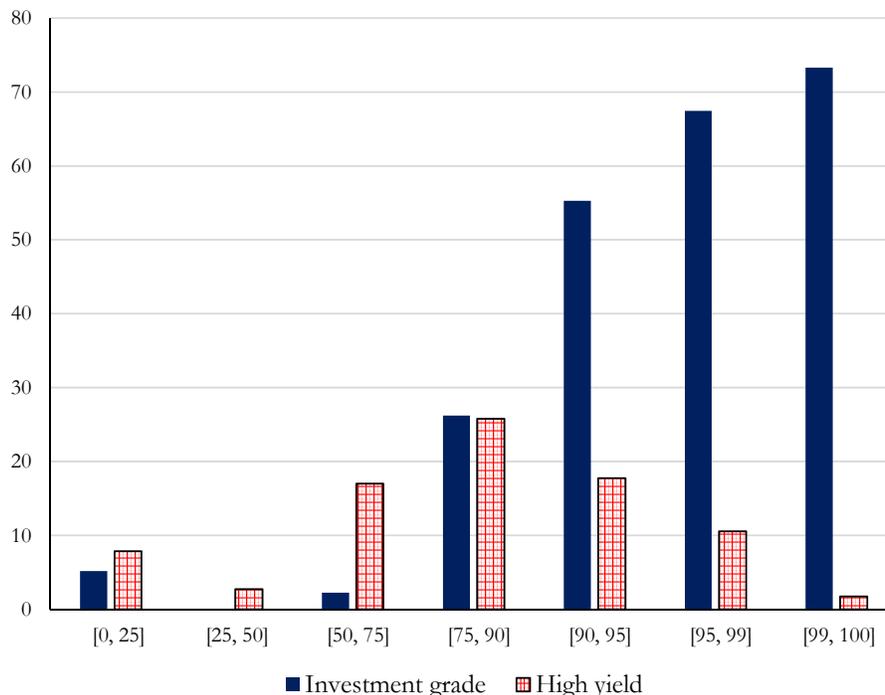
(2012) find the exact opposite.[2] Den Han and Covas (2011) suggest that those striking differences in findings could stem from the fact that different papers rely on different data sources. They also make the case that cyclical properties of security issues differ greatly across firm types. In particular, large firms appear to behave very differently from other firms. Their debt issues, in particular, are acyclical at best.

One reason why large firms behave very differently from other firms when it comes to capital raising could be that they tend to be more highly rated than other firms. Erel et. al. (2011) find that capital raising overall and debt issues in particular tend to be countercyclical for investment-grade borrowers. Kahle et. al (2010) find that large and investment-grade

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[2]Accordingly, Quadrini and Jerman (2012) propose a model where constraints on debt issues become tighter during recessions whereas Levy and Hennessy (2007) write a model where constraints on equity issues become tighter during recessions.

non-financial firms did not exhibit abnormally low net debt issuances in the aftermath the 2008 crisis, while other firms did. This section relies on Compustat data to document the main cyclical properties of safe debt issues by corporations and confirm, in particular, that safe debt issues are significantly less cyclical than other corporate liabilities. This will also establish the data benchmarks we will later use to evaluate the performance of our dynamic model of security creation.

For ease of comparability with the related literature, we follow for the most part the approach of Covas and Den Haan (2012). We focus on the 1985-2014 period and measure net debt and net liability issues exactly as Covas and Den Haan (2011), using their level approach. Compustat data for this time period comprise $366,317$ firm-year observations. Excluding foreign, utility and financial firms leaves us with $186,818$ observations while removing observations with bad or missing data and firms listed for 3 years or less further reduces our sample to $94,729$ observations.

Table 1 shows firms counts in our sample by debt rating. We rely on the issuer credit assessments performed by Standard and Poor's available in the Wharton Research Data Services version of Compustat. While many firms are not rated, our concern in this paper is with highly rated firms, firms for which rating data is available in almost all cases. Figure 1 shows the fraction of firms that are rated investment-grade (BBB or above) and of those that are rated below investment grade by asset size category. It shows a fairly strong correlation between size and credit quality. For instance, almost all the firms in the top 1% size category for which rating data is available are investment-grade rated. As we discussed above, this correlation could explain the fact emphasized by Covas and Den Haan (2011) that debt issues by the largest firms seem to behave differently from debt issues by other firms.

In the model we are about to introduce, corporations issue debt that never defaults. While it would be easy to extend this notion of safe to tolerate a small probability of default, a satisfactory data counterpart for our model's notion of debt should pay almost always. Figure 2 shows historical default rates – the frequency of failure to pay – by rating for issues rated by Moody's between 1920 and 2010. (In Moody's classification, BAA corresponds to

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Table 1: Firm counts by rating

	AAA	AA	A	BBB	<BBB	No rating
1985	13	58	125	75	163	1,680
1986	13	64	159	108	258	2,040
1987	13	59	137	117	316	2,023
1988	13	60	136	100	291	1,996
1989	14	51	140	102	280	1,980
1990	14	54	129	107	255	2,098
1991	13	55	130	114	222	2,292
1992	13	55	131	117	225	2,526
1993	13	54	134	132	270	2,697
1994	13	49	136	141	283	2,863
1995	12	48	135	155	289	3,251
1996	12	45	146	176	331	3,282
1997	12	43	149	186	336	3,175
1998	10	35	158	208	387	3,244
1999	9	32	144	218	383	3,065
2000	9	27	136	223	406	2,739
2001	8	22	124	223	425	2,522
2002	7	20	115	219	465	2,516
2003	6	15	113	220	482	2,531
2004	6	13	115	210	504	2,466
2005	6	12	113	205	509	2,466
2006	6	11	113	201	495	2,399
2007	6	11	101	203	478	2,287
2008	6	12	93	200	454	2,226
2009	6	13	86	194	461	2,190
2010	5	14	83	197	448	2,166
2011	4	13	89	209	441	2,131
2012	4	13	87	205	453	2,198
2013	4	13	88	212	472	2,232
2014	4	15	90	211	488	2,095

Notes: The sample includes all observations in the Compustat database with the exclusion of foreign, utility and financial firms and observations with missing debt data.

Table 2: Cyclicality of corporate debt by rating

| Firm rating	AAA	AAA-AA	AAA-A	$\geq$ BBB	<BBB
mean(D^S/Y)	0.81	2.93	7.66	11.8	4.16
$\rho(D^S, Y)$	-0.14	0.03	0.23	0.19	0.47
$\rho\left(\frac{D^S}{Y}, Y\right)$	-0.20	-0.09	0.06	0.00	0.38
$\rho\left(\frac{D^S}{D+E}, Y\right)$	-0.25	-0.17	-0.04	-0.15	0.35

Notes: The sample period is 1985-2014. In the left-hand column, Y is value added by the private sector as measured by the Bureau of Labor Statistics, D^s is stock of safe debt for various rating categories, $D+E$ is the book value of all liabilities. Correlation are reported for HP-filtered real series with a smoothing parameter of 100. Results obtained using the Baxter-King and Christiano-Fitzgerald filters are similar.

a BBB rating for Standard and Poor's.) Not surprisingly, ratings and realized default rates are highly correlated. AAA-rated issues show no default whatsoever during this time period while AA-rated issues only defaulted once. A-rated issues also pay in almost all cases. Based on this evidence we will use AA and above as our notion of safe in our main calibration. By way of robustness however we will also consider narrower and broader notions.

Our principal concern is the correlation of safe debt issues with economic activity. Let $\tilde{D}$ be the accumulated value of nominal debt issuances in the dataset we described while D is the accumulated value of the same issuances deflated by the Producer Price Index (PPI). Denote by $\tilde{D}^S$ and D^S the same measures but for highly rated firms only. As we discussed above, our main calibration will consider A and AA-rated firms as highly rated but we will also show results for AAA firms only, firms rated A or above and firm rated BBB or above (the standard definition of investment-grade firms.) The measure we will use for economic activity is value added by the private sector as measured by the Bureau of Labor Statistics.

According to this approach, the stock of debt that is rated, say, AAA, is the sum of the stocks of all firms that are rated AAA at the start of the year. Fluctuations in that stock, therefore, are caused by changes in the stock of debt of firms that remain AAA rated from one year to the next (the intensive margin) but also by the addition of firms that become AAA rated in a particular year, and by the subtraction of firms that lose their AAA rating

Figure 2: Frequency of default by rating, 1920-2010
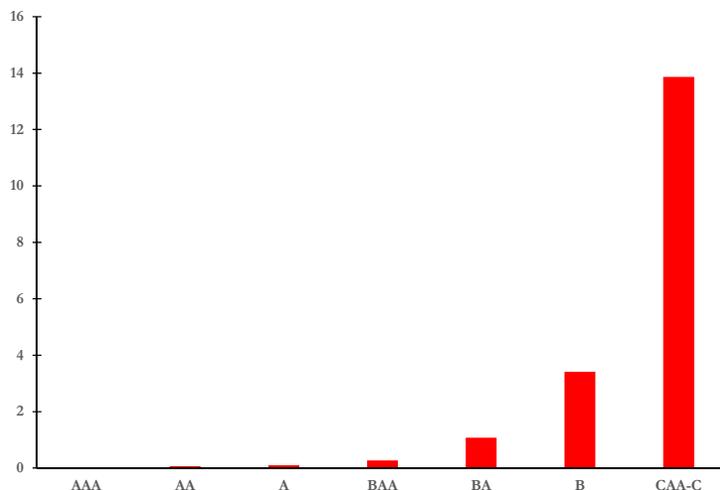
Source: Corporate Default and Recovery Rates, 1920-2010, Special Comment, Moody's Investors Service.

(the extensive margin.) In the model we describe in the next section, the same forces affect the stock of safe debt.

The first row of table 2 shows that the stock of AAA debt so issued represents around 0.8% of private sector value-added in the United States while debt that is either AAA or AA, the notion of safe debt we will use in our benchmark calibration, represents around 3% of value added. The second row shows that the debt issued by more highly rated firms is less cyclical than the debt issued by firms with a below investment-grade rating. The same holds for the ratio of debt to value added and the ratio of debt to all liabilities. The bottom line for our purposes is that the debt issued by highly-rated firms behaves very differently from overall debt issues, showing weak procyclicality at best. This is the key fact with which a satisfactory model of safe corporate debt should be consistent.

A key input in the calibration of our model is the correlation between the yield on safe debt issues and economic activity. The first row of table 3 shows that real yields on AAA-rated debt issues are mildly countercyclical. Put another way, appetite for safe assets tends

Table 3: Cyclicality of safe yields and spreads

	1985-2006	1985-2014	1947-2014
$\rho(\text{AAA yield}, Y)$	-0.0401	-0.2491	-0.3048
$\rho(\text{AAA-BAA spread}, Y)$	-0.2991	-0.3774	-0.6028

Notes: Yield and spread data are Moody's Seasoned Corporate Bond Yield downloaded from the St-Louis Fed's FRED database. We deflate the nominal yield using the CPI index and HP filter the resulting series with a smoothing parameter of 100.

to be midly procyclical. As shown in the second row and has been well-documented,[3] risk spreads tend to be countercyclical. We will use this fact to test our model.

3   The environment

Consider an economy where time is discrete and infinite and with one consumption good. The economy contains a mass one of identical, infinitely-lived households endowed with a unit of labor each period. Households order stochastic sequences of consumption according to time separable preferences with a CRRA period reward function and discount future utility at a constant rate $\beta \in (0,1)$. We will focus throughout on type symmetric equilibria. Specifically, all households start at date 0 with the same wealth $a_0 > 0$ and make the same decisions from then on.

The economy also contains a large mass of producers each characterized by a permanent skill level $z \geq 0$. The mass of producers in a given Borel set $Z \subset [0, +\infty)$ is $\mu(Z)$. Producers can each operate an establishment whose activation requires an investment of one unit of the consumption good at the start of any period. An active project operated by a producer of type $z > 0$ yields gross output

$$A_t z^{1-\alpha} n_t^\alpha + (1-\delta)$$

at the end of the period t where $\alpha \in (0,1)$, n_t is the quantity of labor employed by the

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[3]See e.g. Gourio (2012).

producer, $\delta \in (0,1)$ reflects the fraction of capital lost to depreciation, and A_t is an aggregate TFP shock common to all establishments. The aggregate shock follows a first-order Markov process whose transition function G_A is public information and takes values in $\mathcal{A} \subset I\!\!R_+$. Labor choices – unlike the decision whether or not to activate project – are made after the period TFP shock is drawn.

Producers take the price w_t of labor in a particular period as given. Conditional on having activated a project and given TFP level A_t, a producer of talent z solves

$$\Pi(A_t, w_t; z) \equiv \max_{n>0} z^{1-\alpha} A_t n^{\alpha} - nw_t$$

in net operating income. Let

$$n^*(A_t, w_t; z) \equiv \arg\max_{n>0} A_t z^{1-\alpha} n^{\alpha} - nw_t$$

denote profit-maximizing labor use given values of the aggregate shock and the wage.

Exactly as in Allen and Gale (1988) producers sell contingent claims to output at the start of each period taking the willingness to pay of investors as given. Let $q_t^H(A)$ denote households' willingness to pay for a marginal unit of consumption given a realization A of the aggregate shock at date t. Producers can also sell risk-free claims at price $q_t^W \leq 1$ in world markets provided they incur a cost $\kappa > 0$ at the start of the period.

The assumption that issuing safe debt is costly while other financing methods are free may seem to run contrary to the traditional pecking order view according to which issuing debt is cheaper than issuing equity. But our model is in fact fully consistent with that view. Most producers who are active in one period are also active in the next period. The overall payoff producers generate on risky securities exceeds the investment in those securities on average.[4] While we assume for exposition convenience that producers return all security proceeds to households each period and raise external funds anew, we could just as well assume that

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[4]Indeed, expected payoffs on risky security are investment times expected return. That expected return must exceed one under our and, for that matter, any reasonable calibration.

producers retain what they need for the next period so that most equity financing in our simulations is effectively financed by retained earnings. Retained earnings, according the the aforementioned pecking order view, are the cheapest source of funds.

One formal way to justify the safe debt issue cost is to assume that outside investors must bear a verification cost to discover the type of project used to back the securities they purchase. Paying this cost enables producers to distinguish themselves from managers with worthless projects in a manner that is visible for those outside agents. We think of κ as standing in for additional guarantees and rating requirements certain investors – institutional investors, say – require.

The key assumption we are making here is that the price of risk-free claims is set exogenously. As we discussed in the data section, real yields on safe US corporate are mildly counter-cyclical. To capture this correlation we assume that q_t^W is measurable with respect to history of past TFP shocks. That is, q_t^W is measurable with respect to $\mathcal{A}^{t-1}$. In particular and like A, it follows a Markov process.

Given q_t^H and q_t^W producers of type $z > 0$ who opt to activate their project choose a quantity of risk-free claim b^s to sell to maximize:

$$MV_t(z) \equiv \max_{b^s \geq 0} b^s q_t^W + \int_{\mathcal{A}} q_t^H(A) \left[(\Pi(A, w_t; z) + 1 - \delta) - b^s \right] dA - \left(1 + 1_{\{b^s > 0\}} \kappa \right),$$

subject to:

$$b^s \leq \Pi(\underline{A}_t, w; z), \tag{3.1}$$

where $\underline{A}_t$ is the lowest possible realization of the aggregate shock at date t i.e. the essential greatest lower bound of $G_A(\bullet | A_{t-1})$.

The constraint says that risk-free claims must be delivered with probability one i.e. even if the worst possible value of the TFP shock is drawn. We do not allow producers to collat-

eralize risk-free claims with the undepreciated part of capital. While for simplicity we make depreciation predictable and fully convertible to the consumption in the model, physical investments are obviously illiquid and risky in practice. Assuming that the undepreciated part of physical investments cannot be used as collateral for highly rated debt also turns out to imply reasonable leverage ratios as we will discuss in section 6.

Producers could sell risk-free claim to households as well as to world markets but that option will not be used in equilibrium for reasons we will discuss in section 6. Henceforth we will write $b_t^s(z)$ for the selected supply of risk free claims to world markets by producers of type $z > 0$. The price of the security sold to households by producers of type z is

$$\int_{\mathcal{A}} q_t^H(A) \left[(\Pi(A, w_t; z) + 1 - \delta) - b_t^s(z) \right] dA$$

so that the stochastic return on the same security is defined for all $A \in \mathcal{A}$ by:

$$r_t(A; z) = \frac{\Pi(A, w_t; z) + 1 - \delta - b_t^s(z)}{\int_{\mathcal{A}} q_t^H(A) \left[(\Pi(A, w_t; z) + 1 - \delta) - b_t^s(z) \right] dA}.$$

The risk-free constraint implies that $r_t(A; z) \geq 0$ for all possible TFP shocks.

The producer problem tells us the market value of a project once it is created. If z is too low, $MV_t(z) < 0$ and producers of such a type remain idle. Other producers are active and rebate their market value to the households, who own them by assumption.

Households enter a given period with wealth $a_t \geq 0$, the result of past saving decisions. It will be convenient to assume that consumption takes place at the start of the period. In order to make their consumption plans, households need to forecast the contingent path of future security menus. A security menu S is the set of securities created by producers of each active type. From the point of view of households it is enough to know the stochastic returns

$$r_t(\bullet, z) : \mathcal{A} \mapsto \mathbb{R}_+$$

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offered by producers of type $z \geq 0$ at date t given the current history $h_t \in \mathcal{A}^{t-1}$ of shocks, as well as the corresponding willingness to pay $q_t^W(h_t)$ for safe claims in global markets. Formally, households assume a mapping

$$S_t : \mathcal{A}^{t-1} \mapsto \mathcal{S}$$

that gives for every possible date and history the predicted financial structure, i.e. the set of contingent returns offered by active producers. In equilibrium, we will require this forecast to be correct.

Given this forecast and an initial wealth a_0, households choose non-negative history-contingent policies $\{c_t, a_{t+1}, b_t^d, (e_t^d(z) : z \geq 0)\}_{t=0}^{+\infty}$, where e_t^d denotes the demand of available risky securities while b_t^d are the risk-free claims purchased by households, to solve:

$$\max E \sum_{t=0}^{+\infty} \beta^t U(c_t)$$

subject to, for every date t and possible history $h_t \in \mathcal{H}_t$:

$$q_t^W b_t^d + \int e_t^d(z) d\mu(z) + c_t = a_t + \int \max \{MV_t(z), 0\} \, d\mu(z),$$

$$a_{t+1}(h_t, A) = \int e_t^d(z) r_t(A, z) d\mu(z) + b_t^d + w_t(h_t, A), \text{ for all } A \in \mathcal{A}$$

where:

$$\{q_t^W, r_t(\bullet, z) : z \geq 0\} = S_t(h_t),$$

and $w_t(h_t, A))$ is the wage rate that prevails given the history of aggregate shocks, including the current one.

As emphasized by Allen and Gale (1991), the assumption that households can only hold positive positions in available securities in all period plays a key role in this costly securitization environment. If unrestricted short-selling were allowed households could arbitrage

away any profits associated with costly cash-flow tranching and no costly tranching could take place in equilibrium. Another way to state this assumption is that short-selling, to the extent that is possible, carries at least the same transaction costs for households as security creation does for producers.

4   Equilibrium

Households enters date 0 with assets $a_0 > 0$. The aggregate state of the economy in that initial period is summarized by $\{a_0, q_0^W, A_{-1}\}$. An equilibrium, then, is history-contingent market values $\{MV_t(z)\}_{t=0}^{+\infty}$ for each producer types, wage rates $\{w_t(A)\}_{t=0}^{+\infty}$ for every $A \in \mathcal{A}$, security spaces S_t, decision plans $\{b_t^s\}_{t=0}^{+\infty}$ by producers of all types, non-negative decisions $\{c_t, a_{t+1}, b_t^d, (e_t^d(z) : z \geq 0)\}_{t=0}^{+\infty}$ for households and, finally, a pricing kernel $\{q_t^H(A) : A \in \mathcal{A}\}$ such that, at all dates t and all possible histories:

1. Given prices, decision plans solve the household and producer problems;

2. The market for labor clears:

$$\int_{\{z:MV_t(z)\geq 0\}} n^*(A, w_t(A); z) = 1 \text{ for all } A \in \mathcal{A};$$

3. The market for risk-free debt clears:

$$\int_{\{z:MV_t(z)\geq 0\}} b_t^s(z)d\mu \geq b_t^d;$$

4. The market for all risky security types $z \geq 0$ clears:

$$e_t^d(z)r_t(A, z) = \Pi(A, w_t; z) + 1 - \delta - b^s(z) \text{ for all } A \in \mathcal{A};$$

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5. The pricing kernel satisfies the Allen-Gale condition:

$$q_t^H(A_t) = \frac{\beta G_A(A_t|A_{t-1})U'\left(c_{t+1}(h_t, A_t)\right)}{U'(c_t)};$$

6. Market values satisfy, for all z,

$$MV_t(z) = b_t^s(z)q_t^W + \int_{\mathcal{A}} q_t^H(A)\left[\left(\Pi(A, w_t; z) + 1 - \delta\right) - b_t^s(z)\right]dA - \left(1 + 1_{\{b_t^s > 0\}}\kappa\right).$$

The market clearing condition for risk-free claims is an inequality because any surplus claims created by producers can be sold in world markets. Imposing it amounts to assuming that households have no direct access to risk-free investments overseas. This is immaterial for our results since, as we will argue below, all equilibria are such that $b_t^d = 0$ at all dates and histories.

5 Aggregate feasibility and GDP accounting

As usual the collection of equilibrium conditions above imply that an aggregate feasibility condition must hold each period. This section will show that this constraint has essentially the same structure as what would emanate from a traditional RBC model. Denote by K the aggregate quantity of capital used for production in a given period and let N be the total mass of employment. In equilibrium, K is the mass of establishments activated while $N = 1$ but one goal of this section is to show in full generality that GDP and conventionally-measured TFP can be given standard interpretations.

In any equilibrium, only producers whose talent is above a certain threshold $\underline{z}$ are activated, as we will formally demonstrate in the next section. It follows that:

$$K = \int_{z \geq \underline{z}} d\mu$$

which implicitly defines a threshold $\underline{z}(K)$. For labor markets to clear wages must be such that:

$$\int_{z \geq \underline{z}(K)} n^*(A, z, w) d\mu = N \tag{5.1}$$

where A and w are the current aggregate TFP shock and price of labor, respectively. In turn, standard manipulations of producer first-order conditions imply that for all $z \geq \underline{z}(K)$,

$$n^*(A, z, w) = z n^*(A, 1, w) \tag{5.2}$$

so that size is linear in z a fact we will invoke later in the calibration of the distribution μ of producer skill. After plugging this relationship into (5.1) we obtain

$$n^*(A, 1, w) = \frac{N}{KE\left[z^{\frac{1}{1-\alpha}} | z \geq \underline{z}(K)\right]}$$

Then, letting

$$F(A, K, N)$$

be aggregate output given the aggregate TFP shock, aggregate capital and aggregate labor, we have:

$$
\begin{aligned}
F(A, K, N) &= \int_{z \geq \underline{z}(K)} A z^{1-\alpha} n^*(A, z, w)^\alpha d\mu \\
&= \int_{z \geq \underline{z}(K)} A z^{1-\alpha} [z n^*(A, 1, w)]^\alpha d\mu \\
&= \int_{z \geq \underline{z}(K)} A z n^*(A, 1, w)^\alpha d\mu \\
&= A n^*(A, 1, w)^\alpha \int_{z \geq \underline{z}(K)} z d\mu \\
&= A n^*(A, 1, w)^\alpha K E\left[z | z \geq \underline{z}(K)\right] \\
&= A E\left[z | z \geq \underline{z}(K)\right]^{1-\alpha} K^{1-\alpha} N^\alpha. \tag{5.3}
\end{aligned}
$$

The final line of the derivation simply replaces n^* by what its value must be for markets to clear. The previous line uses the fact that:

$$\int_{z \geq \underline{z}(K)} z d\mu = E\left[z | z \geq \underline{z}(K)\right] K.$$

Note that F is a standard neoclassical production function with $AE\left[z | z \geq \underline{z}(K)\right]^{1-\alpha}$ playing the role of conventionally-measured TFP. In our model, measured TFP is a function both of the exogenous aggregate shock and of the average quality of active establishments.

These aggregation facts yield a convenient computational shortcut. Given the TFP realization and the quantity of capital used in production, the market-clearing wage rate can be computed as the aggregate marginal product of labor so that, simply and in all periods:

$$w(A, K) = \alpha A E\left[z | z \geq \underline{z}(K)\right]^{1-\alpha} K^{1-\alpha}.$$

And this is only the tip of the iceberg. These aggregation facts will buy us more computational shortcuts once we characterize what security markets must look like in all equilibria.

Aggregating all payments to securities now yields the following feasibility condition, given K_t, N_t, A_t and b_t^W (risk free promises to world markets):

$$\int e_t^d(z) r_t(A, z) d\mu(z) + b_t^W + N_t w_t(A) = F(A_t, K_t, N_t) + (1 - \delta) K_t.$$

In words, gross output gets spent on payments to capital and labor at the end of any given period.

To arrive at a more standard expression for GDP, first note that household assets at the start of period $t + 1$ are:

$$a_{t+1} = F(A_t, K_t, N_t) + (1 - \delta) K_t - b_t^W.$$

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At the same time, manager rents at the start of of period $t+1$ are

$$m_{t+1} = a_{t+1} + m_{t+1} - c_{t+1} - \int_{b_{t+1}^s > 0} \kappa d\mu + q_{t+1} B_{t+1} - K_{t+1}.$$

Indeed, $a_{t+1} + m_{t+1} - c_{t+1} + q_{t+1} b_{t+1}^W$ is total security purchases. Rents are the difference between those purchases and the resources spent on capital fomation and security creation. Replacing a_{t+1} by the expression above yields, after some reorganization:

$$c_{t+1} + K_{t+1} - (1-\delta) K_t + \int_{b_{t+1}^s > 0} \kappa d\mu + b_t^W - q_{t+1}^W b_{t+1}^W = F(A_t, K_t, N_t).$$

In words, GDP is spent on domestic consumption, gross fixed capital formation, security creation, and net exports.[5]

6 Security markets

This section provides equilibrium features security markets must satisfy in any given period. The security design choices of producers are fully characterized by two thresholds. As we have already mentioned, producers only engage in production hence issue securities in a given period provided their talent exceeds some history- dependent threshold $\underline{z}_t$. Whether or not they issue safe debt, likewise, is characterized by a second threshold $\bar{z}_t \geq \underline{z}_t$:

Proposition 1. *The solution to the producer security design problem at a given date t is fully described by two thresholds $0 \leq \underline{z}_t \leq \bar{z}_t$ such that:*

1. *Producers issue securities if and only if $z \geq \underline{z}_t$;*

2. *$b_t^s(z) = 0$ if $z < \bar{z}_t$;*

3. *$b_t^s(z) = (\Pi(\underline{A}_t, w; z) + 1 - \delta)$ if $z > \bar{z}_t$.*

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[5]Spending is indexed by $t+1$ rather the traditional t because of our convention that consumption takes place at the start of a given period so that period $t+1$ spending is contemporaneous with period t GDP.

Proof. Since producers maximize a linear objective over a compact, convex set the result follows almost immediately from the Extreme Value Theorem stated for instance in Ok (2007). Optimal policies for active producers must be extreme: producers either issue the most safe debt they can or none at all. For completeness and intuition's sake however, we will also provide a direct proof.

The first item of the proposition is obvious. Without loss of generality in the context of this proof but to simplify notation, assume that $\mu(z) = 1$ for all $z > \bar{z}_t$. To see the second item note that if $\int_{\mathcal{A}} q_t^H(A)dA \geq q_t^W$ then producers optimally choose to issue no risk free debt since doing so carries a fixed costs and households are willing to pay at least as much for risk-free claims as world markets are. The result holds trivially in this case.

For the rest of this proof then, assume that $\int_{\mathcal{A}} q_t^H(A)dA < q_t^W$. Then market value rises strictly with b. If a producer finds it profitable to pay the fixed cost, she finds it optimal to maximize the production of risk-free debt. The constraint must bind when some risk-free debt is issued by a producer.

That leaves us with two possibilities: produce zero risk-free debt or max-out risk-free debt. Maintaining the normalization that $\mu(z) = 1$ for all $z > \bar{z}_t$, write $MV_t^{b>0}(z)$ for the highest market value before the tranching cost κ conditional on $b = \Pi(\underline{A}_t, w)$, while $MV_t^{b=0}(z)$ is the same under the constraint that $b = 0$. We have:

$$MV_t^{b=0}(z) = \int_{\mathcal{A}} q_t^H(A) \left(\Pi(A, w_t; z) + 1 - \delta \right) dA$$

while

$$MV_t^{b>0}(z) = q_t^W \Pi(\underline{A}_t, w; z) + \int_{A > \underline{A}_t} q_t^H(A) \left(1 - \delta + \Pi(A, w_t; z) - \Pi(\underline{A}_t, w; z) \right) dA,$$

so that

$$MV_t^{b=0}(z) - MV_t^{b>0}(z) = \left(q_t^W - \int_{\mathcal{A}} q_t^H(A)dA \right) \Pi(\underline{A}_t, w; z).$$

When $\int_{\mathcal{A}} q_t^H(A)dA < q_t^W$, this difference rises with z and it follows that if $MV_t^{b=0}(z) <$

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$MV_t^{b>0}(z)+\kappa$, this remains true as z rises. This establishes the existence of a second threshold $\bar{z}_t$. This completes the proof. $\qquad\square$

These results follow from a fundamental feature of environments in the spirit of Allen and Gale (1988) such as ours: producers take state prices as given hence have a linear objective which, in turn, leads to bang-bang financial policies. One key consequence is that when producers choose to create some risk-free debt, they max out the production of such debt. In addition, only producers above a certain productivity hence size threshold issue investment-grade debt, small producers do not.

As the proof makes clear, whether a producer finds it optimal to issue risk-free debt boils down to whether the following inequality holds:

$$\left\{ q_t^W - \int_{\mathcal{A}} q_t^H(A)dA \right\} \Pi(\underline{A}_t, w; z) \geq \kappa \tag{6.1}$$

Indeed, households are willing to pay $\int_{\mathcal{A}} q_t^H(A)dA$ for a unit of risk-free payoff whereas world markets pay q_t^W for the same security. The maximum safe payoff a producer of type z can sell is $\Pi(\underline{A}_t, w; z)$. Paying participation cost κ only makes sense if the premium paid by world markets is sufficiently high. Inequality (6.1) will play a key role in the interpretation of our quantitative results.

Another key consequence of the proposition is that households do not purchase risk-free securities in equilibrium. It would make no economic sense for producers to bear the tranching cost to cater to households. This does not mean that the portfolio held by households does not have a risk-free part. It simply means that there is no need to split the risk-free part from the rest of the household portfolio.

Conversely, households hold all risky assets in the economy. It follows that, from the point of view of households and at any given history, it is as if they were in a world with one security whose stochastic payoff is

$$F(A_t, K_t, 1) + (1 - \delta)K_t - b_t^W,$$

Figure 3: Stock of debt vs yearly EBITDA among firms rated AA or above
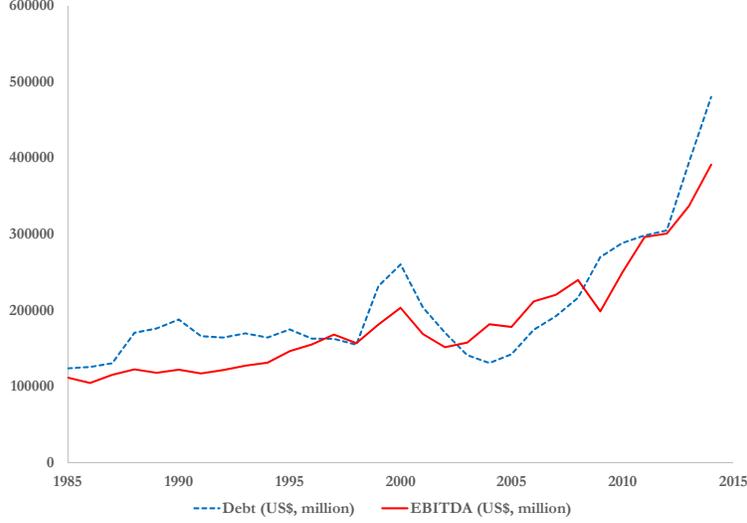
and whose price at the start of the period is

$$\int_{\mathcal{A}} q_t^H(A)\left[F(A_t, K_t, 1) + (1 - \delta)K_t - b_t^W\right]dA,$$

hence whose return, for all possible values of A_t is

$$r^H(A_t) = \frac{F(A_t, K_t, 1) + (1 - \delta)K_t - b_t^W}{\int_{\mathcal{A}} q_t^H(A)\left[F(A_t, K_t, 1) + (1 - \delta)K_t - b_t^W\right]dA}.$$

Finally, the bang-bang nature of financial policies implies that among firms that do issue safe debt, the size of the stock of debt should be near annualized net operating income. This is obviously testable using the data we described in section 2. Figure 3 compares the stock of debt to yearly EBITDA[6] The two series track each other fairly closely and this is exactly

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[6]We use OIBDP (Operating income before depreciation) as our proxy for EBITDA in Compustat. This is

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what our endogenous borrowing constraint implies.

7 A recursive approach

As usual in this sort of context, a recursive approach is the most natural way to look for an equilibrium.[7] The aggregate state of the economy at any given point in time is fully summarized by $(\bar{a}_t, A_{-1}) \in \mathbb{R}_+ \times \mathcal{A}$, i.e the wealth holdings of the household sector and the most recent TFP shock. Of course, average household wealth $\bar{a}_t$ coincides with individual wealth levels a_t but each household views those as distinct and understand, in particular, that their individual decisions do not directly affect the aggregate values.

Because $\delta > 0$ and returns to capital are decreasing in the aggregate, it is easy to see that we can confine our attention to a compact set of possible aggregate states. It will be convenient to have notation for the set of possible aggregate states at the start of a given period so define

$$\Theta = \mathbb{R}_+ \times \mathbb{R}_+ \times \mathcal{A}.$$

The purpose of this section proposes a definition of a Markov Recursive Equilibria. An equilibrium consists of the following objects:

1. $g : \Theta \times \mathcal{A} \mapsto \Theta$ is the transition function for the aggregate state given the new realization of the TFP shock;

2. $K : \Theta \mapsto \mathbb{R}_+$ is the capital stock deployed;

3. $\underline{z} \times \bar{z} : \Theta \mapsto \mathbb{R}_+^2$ are the two thresholds that define the financial structure;

4. $q^H : \Theta \times \mathcal{A} \mapsto \mathbb{R}_+$ is the willingness to pay of households for security payoffs at each possible realization of the TFP shock;

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the closest notion to our measure Π of net operating income in Compustat and only differs from EBITDA as it is typically defined in that in doesn't include non-operating income.

[7]See e.g. Krusell, Rios-Rull . . .

5. $r^H : \Theta \times \mathcal{A} \mapsto I\!R_+$ is the return on the household portfolio;

6. $e^H : \Theta \times I\!R_+ \mapsto I\!R_+$ is the household's risky investment at the start of the period;

7. $c : \Theta \times I\!R_+ \mapsto I\!R_+$ is household consumption given the aggregate state and their individual wealth level;

8. $w : \Theta \times \mathcal{A} \mapsto I\!R_+$ is the wage rate for each possible TFP shock;

9. $b^W : \Theta \mapsto I\!R_+$ is the volume of risk-free claims created;

10. $MV : \Theta \times I\!R_+ \mapsto I\!R_+$ is the market value of establishments of each possible talent level;

11. $V^H : \Theta \times I\!R_+ \mapsto I\!R$ is the household's continuation value function.

To characterize recursive equilibria, we will begin by writing out the functional equation that define V^H given other objects. Given state $\theta = (\bar{a}, A_{-1}) \in \Theta$ and the household's individual wealth $a > 0$,

$$V^H(\theta, a) = \max_{e^H \geq 0, b \geq 0} U\left(a + \int_{z \geq \underline{z}} MV(z)dz - e^H - q^W b\right) + \beta \int_{\mathcal{A}} V^H\left(g(\theta, A), a'(A)\right)dG(A|A_{-1})$$

where, for all $A \in \mathcal{A}$

$$a'(A) = b + e^H r^H(\theta, A) + w(\theta, A).$$

To understand this expression, notice that agents are atomistic hence take the evolution of the aggregate state as independent of their decisions. Here we allow for $b > 0$ even though, as we have already argued, households hold no risk-free securities in equilibrium.

Standard arguments imply that the functional equation above defines V^H uniquely in the space of bounded function as long as all laws of motion are such that all integrals are well defined, a condition we will impose throughout. Furthermore, under the premise that all underlying objects in the above functional equation are continuous, so is V^H. In fact, under that same premise, V^H is concave and differentiable in individual assets.[8] In fact, for all

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[8]See Benveniste-Scheinkman, 1979.

possible values $\theta \in \Theta$ of the aggregate state,

$$\frac{\partial V^H(\theta, a)}{\partial a} = U'(c(\theta, a)).$$

This envelope property is important because it means that the Allen-Gale approach to pricing all potential security using the marginal rate of substitutability across different periods can be motivated in our infinite horizon context exactly as it can in their two-period environment.

A fact that will greatly aid computations is that, in this environment, households hold all risky assets while world markets absorb all safe securities created. Having observed this, we can boil down the search for a RCE to the following set of conditions:

$$K(\theta) = \int_{z \geq \underline{z}} d\mu \tag{7.1}$$

$$e^H(\theta, a) = \int_{\mathcal{A}} q^H(\theta, A) \Big[F(A, K(\theta), 1) + (1 - \delta)K(\theta) - b^W(\theta) \Big] dA \tag{7.2}$$

$$\int_{z \geq \underline{z}} MV(\theta, z) dz = q^W(\theta) b^W(\theta) - K(\theta) - \int_{z \geq \bar{z}} \kappa d\mu$$

$$+ \int \int_{\mathcal{A}} q_t^H(A) \Big[(\Pi(A, w(\theta); z) + 1 - \delta) - b^s(\theta, z) \Big] dA dz \tag{7.3}$$

$$c(\theta, a) = a + \int_{z \geq \underline{z}} MV(\theta, z) dz - e^H(\theta, a) \tag{7.4}$$

$$a'(\theta, A) = e^H(\theta, A) r^H(\theta, A) + w(\theta, A) \text{ for } A \in \mathcal{A} \tag{7.5}$$

$$q^H(\theta, A) = \frac{\beta G(A|A_{-1}) U'(c^i(g(\theta, A), a'(\theta, A)))}{U'(c(\theta, a))} \text{ for } A \in \mathcal{A} \tag{7.6}$$

and the financial structure is consistent with the producers' problem given (q^H, q^W). (7.7)

The first condition defines the capital stock given the financial structure. The next three conditions express what consumption must be for households given the portfolio they must hold in equilibrium and their profit distributions. The next condition states the corresponding evolution of household assets. That also tells us what the endogenous part of the aggregate law of motion must be. Condition (7.6) is the recursive version of the Allen-Gale condition.

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The final condition is that the assumed thresholds are compatible with the willingness to pay for securities given optimal behavior by producers.

Having so reduced the set of equilibrium conditions, the search for equilibrium policy functions becomes greatly simplified. Start with a guess for household state prices for all possible values of the aggregate state. Given those, we can solve for the aggregate stock of physical capital fairly quickly. To see this, at a given aggregate state, guess K. This implies a lower bound below which producers activate their establishment. And it also gives us TFP-shock dependent wages. We can then solve for Π and, given state prices, we can solve in particular for $MV(\underline{z})$, which, in equilibrium, must be zero. We can then update K until that marginal condition holds. The next step is to find the threshold above which risk-free rate is created, given K and the pricing kernels.

These steps guarantee that labor markets clear and that producers behave optimally given pricing kernels. To have a recursive equilibrium however we need pricing kernels to be consistent with the Allen-Gale condition. For that aspect of the equilibrium search, note that knowing K and pricing kernels is enough to compute portfolios and payoffs, hence state contingent consumptions. This then gives us the implied willingness to pay by (7.6). We have an equilibrium, then, provided the values that come out of that condition are exactly the marginal willingness to pay the producers assumed in the first place.

8   The cyclical properties of safe corporate debt

This section asks whether the theory we have described is consistent with the cyclical properties of safe debt issues we documented in the data section. Before delving into the details of our quantitative strategy, it will be useful to highlight the features of our model that govern how security markets respond to TFP shocks.

8.1 Basic mechanism

The forces that govern the the cyclical properties of the stock of safe debt are fully captured by inequality 6.1. First and most obviously, the behavior of the stock of risk free debt depends on the cyclicality of the exogenous price q^W of safe claims. The data section suggests that this price process is weakly procyclical and our calibration below will reflect this.

Second, constraint (3.1) limits the quantity of safe debt producers hence the profitability of paying fixed participation cost κ. In turn, the tightness of that constraint is a function of the support of the TFP process, or more specifically of the worst-case scenario. It stands to reason – and our calibration will imply – that the support of the TFP shock should move to the left during recession giving the model a strong procyclical force for the production of safe assets. This force resembles the key mechanism in Jerman and Quadrini (2012). In their paper, the exogenous constraint on debt issues is relaxed during expansions. The same happens in our model, but for endogenous reasons.

Third, the tightness of the constraint depends not only on $\underline{A}$ but also on the price w of labor. During expansions, wages tend to go up, which acts countercyclically on debt issues in our model.

Fourth and finally, the willingness to pay for securities by the household $(\int_A q^H(A)dAi)$ is strongly procyclical. Indeed, a good TFP shocks results in higher wealth for households hence, under standard preference specifications, a greater desire to save. When household state prices rise, the profitability of participating in safe debt markets falls. This aspect of our model pushes debt issues down during expansions.

In summary, the overall cyclicality of safe assets issues in our model will depend on the relative strength of competing forces. We now turn to exploring the result of this quantitative horse race.

8.2 Parameters

In order to explore the properties of our model we first need to specify functional forms and parameters. We will think of a period as representing one year. Households' preference are in the time-separable CRRA class. We set the elasticity of substitution to 2 and the discount rate β to 0.95.

On the production side, we set the depreciation rate δ to 10% while the labor share parameter (α) is set to 65%. We will set other parameters jointly so that the model matches a set of data moments that are directly pertinent to our question.

We assume that the distribution of idiosyncratic establishment productivity is log-normal and normalize the location parameter 0. The dispersion parameter (γ) is calibrated via its implications for the size of managerial rents $\int_{\{z:MV_t(z)>0\}} b_t^s(z) MV_t(z) dz$. Intuitively, more dispersion in talent should imply greater rents and this monotonicity is borne out in all the simulations we have performed. To identify a data counterpart for these rents, we must take a stand on what the implicit producer factor represents in our model. We will think of this fixed factor as the time and skill input provided by sole proprietors and top managers.

To pin down the dispersion of talent then, we need data counterpart for $\int_{\{z:MV_t(z)\geq 0\}} MV_t(z) dz$ and for aggregate output ($Y \equiv F(A, K, N)$ where K is physical capital and N is employment) in our model. We will think of managerial rents as the <u>labor</u> income of proprietors and top managers. By top managers we mean Chief Executives and General and Operations managers (occupation codes 11-1011 and 11-1021) in the Bureau of Labor Statistics (BLS)'s National Occupational Employment and Wage Estimates survey. For proprietor's income we use the standard series available from the Bureau of Economic Analysis and make the assumption that the fraction of labor income in this measure is 65%.

As a counterpart for aggregate output Y we use value added by the private sector as measured by the Bureau of Labor Statistics minus the compensation of top managers as defined above and proprietor's income since we are treating those payments to labor as part of $\int_{\{z:MV_t(z)\geq 0\}} MV_t(z)$. For the 1998-2014 period managerial rents so measured represents

Table 4: Calibration summary

Parameter	Description	Value
β	Discount rate	0.95
σ	Utility curvature	2.00
δ	Depreciation rate	0.10
α	Labor share	0.65
γ	Project productivity dispersion	0.50
κ	Tranching cost	0.0115

around 10% of value added by the private sector hence around $\frac{0.1}{0.9} \approx 11\%$ of the appropriate counterpart for aggregate output in our model.

Next we need to parameterize the process for exogenous TFP. We assume that $\mathcal{A}$ – the support for exogenous TFP consists of three points. Output is defined as above as value added by the private sector as measured by the Bureau of Labor Statistics minus the compensation of top managers as defined above and the labor part of proprietor's income. Because below we calibrate a transition matrix based on the frequency of TFP changes, we use the entire annual series available from the BLS, which covers the 1948-2014 time period. Top manager compensation data available from the BLS do not go back that far so to approximately remove the contribution of managerial rents to value added we apply a constant factor of 0.9 to the BLS series.[9] Consistently with our specification of the labor share, we define aggregate TFP as the ratio of real value added so defined to a Cobb-Douglas index

$$\text{(Labor input)}^{.65}\text{(Capital Services)}^{.35}$$

of input use.

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[9]As long as the share of managerial rents does not fluctuate too much – an assumption which underly our entire calibration approach anyway – the specific factor one uses does not affect the TFP process. We should in principle exclude the labor provided by proprietors and top managers form the BLS series for labor services, but this would require detailed data on the hours provided by top managers and proprietors going back to 1948, data which we do not have. Here too,as long as the fraction of these categories of labor to the overall labor series does not fluctuate too much, the results for the TFP process will not change much.

We log and HP-filter the resulting series with a smoothing parameter of 100 and then classify the resulting productivity levels in three categories: bottom 25% of observations, middle 50% and top 25%. The frequency transitions between those three TFP states is summarize in the following matrix:

$$
\begin{bmatrix}
0.50 & 0.50 & 0.00 \\
0.24 & 0.50 & 0.26 \\
0.00 & 0.56 & 0.44
\end{bmatrix}.
$$

We specify G_A to equal this matrix.

One important approximation we are making here is that in our environment, TFP as we compute it using BLS data corresponds to $AE\left[z|z \geq \underline{z}(K)\right]^{1-\alpha}$ so that it mixes the exogenous part (A) of TFP and the endogenous changes in the average quality of projects. In our simulations however the endogenous part of TFP accounts for a small fraction of TFP movements so that, in practice, effective TFP is in the bottom or top 25% of values exactly when exogenous TFP is.

To specify the support of A, we calculated that HP-filtered real value added in BLS data during low (high) TFP periods is roughly 3.5% below (above) its average level in the middle TFP state. We choose a support for A to $\{0.97, 1.00, 1.03\}$ so that average output in each of the three possible aggregate states in our model matches those numbers. (The algorithm we use to generate model moments will be described in details in the next subsection.) In those simulations, the average level of aggregate output (Y) depends both on the TFP level and the endogenous response of the capital stock so that a 3% spread in exogenous TFP gets magnified to roughly a 3.5% spread in Y.

To calibrate the relationship of the willingness q^W to pay for safe securities we use Moody' seasoned AAA Corporate Bond Yield series between 1985 and 2014, averaged annually, deflated using the Consumer Price Index and HP-filtered with a smoothing parameter of 100. We normalize the resulting series so that average safe rates are 3%. The resulting support for

q^W is $\{0.972, 0.968, 0.976\}$. As we will see below, this approach turns out to imply a (mildly negative) correlation of filtered safe yields with filtered value added that comes very close to its data counterpart, even though we did not explicitly target that moment.

This leaves one parameter to calibrate, namely the tranching cost κ. Recall that the aggregate stock of safe debt in the Compustat data we employed in section 2 that is rated either AAA or AA amounts to around 3% of private value added on average between 1985 and 2014. Setting $\kappa = 0.0115$ makes the average ratio of the stock of safe debt to Y roughly 3%. In our benchmark simulations, that value of kappa implies that tranching costs represent about 0.7% of the total safe debt issues on average. Table 4 summarizes our calibration.

8.3 Algorithm

Finding an equilibrium given these parameters and functional forms boils down to solving a fixed point on the willingness to pay vector q^H. In order to locate such a fixed point, we use a version of the algorithm Telmer (1993) employed, although our search is significantly complicated by the fact that the set of securities is endogenous.

As we discussed in section 7 the state of the economy at any given point in time is fully summarized by $(\bar{a}, A_{-1}) \in I\!R_+ \times \mathcal{A}$, i.e the wealth holdings of the household sector and the most recent TFP shock. We set lower and upper bounds for asset holdings of 0.5 and 5, respectively,[10] and discretize the resulting interval using 50 equally spaced points giving us a state space with 50 distinct values overall.

Next, we need to take on two computational tasks. First, we need policy functions at the recursive equilibrium. Second, we need to estimate business cycle moment by drawing time series from the economy's stochastic steady state. For the first task, we proceed as follows:

1. Start by assuming that agents live for 2-periods ($T = 2$). In the terminal period, agents simply consume their wealth. Finding a solution to conditions $(7.1 - 7.7)$ is a version

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[10]This range is such that assets never reach those bounds in our simulations. In fact, the stochastic steady state we simulate stays with the $[3, 4]$ interval throughout our simulations.

Figure 4: Equilibrium policy functions and state prices
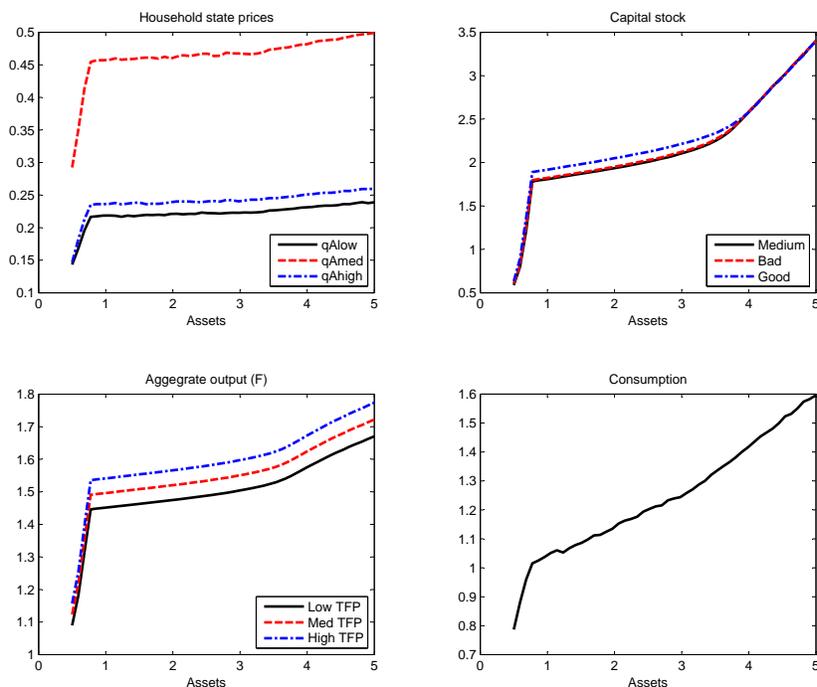
of the two-period problem studied by Allen-Gale (1988). In particular, that system has a solution. We compute the corresponding fixed point on a grid of the aggregate state space.

2. At the solution, denote by c^{T-1} the optimal consumption plan given the aggregate state. Apply the same procedure as in step 1 to compute a vector of household willingness to pay that solves $(7.1 - 7.7)$ as of date $T - 2$, so obtaining c^{T-2}.

3. Let T grow until the optimal consumption policy function become approximately invariant. This gives us the approximate equilibrium we sought.

As mentioned above, the key goal is to find a fixed point defined by equation (7.6). On the right hand side of that equation, asset values obviously fall between grid points. We use

Figure 5: Safe asset issuances
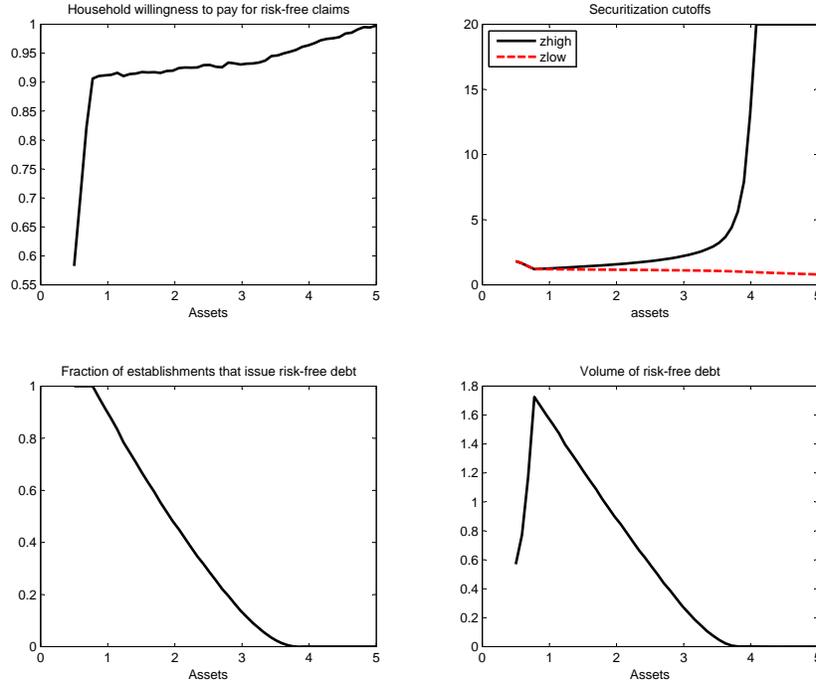
interpolation to evaluate the value of marginal rates of substitution at those interior points.

The second step consists of simulating the stochastic steady state implied by this quantitative version of our model. We follow a standard procedure and start from an arbitrary level (specifically, $a_0 = 3.5$) of household wealth and assuming that we start at the middle TFP level. We then draw a path of 500,000 TFP shocks according to G_A, trace the resulting path of assets and, using the policy functions computed above, of every other endogenous variables. We then drop the first $50,000$ periods to make sure that initial conditions have no influence on the business cycle properties we report in the next section.

Table 5: Model fit

Moment	Data	Model
Ratio of safe debt to Y	3.24%	3.18%
Ratio of management rents to Y	11.00%	11.72%
$std(log(Y))$	2.15%	2.28%
$\rho\left(\frac{1}{q^W}-1,Y\right)$	-0.25	-0.28

8.4 The Recursive Competitive Equilibrium

Figure 4 plots key endogenous variables against household assets at the recursive equilibrium we found. All policies are plotted for the case where the most recent TFP shocks is the middle state. The main patterns we emphasize in this section look similar in the other cases.

The first panel of the figure shows the willingness to pay of households for payoffs in each of the three possible aggregate states. As they become richer their ability and willingness to pay rises. This immediately implies that it becomes profitable for more producers to become active as household wealth increases and correspondingly, as shown in the second panel of the figure, more capital is deployed. As a result and for each possible realization of the aggregate shock, aggregate output Y also rises with wealth as does aggregate consumption, as shown in the two bottom panels of the figure.

Figure 5 shows the consequences of these equilibrium properties on the production of safe securities. The first panel shows the willingness to pay for safe assets by households (in other words, the sum of the three state prices shown in the first panel of figure 5) and shows, in particular, that this willingness rises with household wealth. The second panel of the figure shows the two thresholds that describe the production of securities. Below the talent level shown in the red, dotted line producers remain inactive and do not produce any securities. As mentioned in the discussion of figure 4, more producers become active as households become wealthier. The solid black line depicts the threshold above which producers choose to pay the fix cost κ and issue safe securities.

Table 6: Cyclical properties of model variables

	Y	TFP	c	a	K
Y	1.0000	0.9144	0.5074	0.4733	0.5369
TFP	0.9144	1.0000	0.1642	0.1103	0.1495
c	0.5074	0.1642	1.0000	0.9809	0.8974
a	0.4733	0.1103	0.9809	1.0000	0.9262
K	0.5369	0.1495	0.8974	0.9262	1.0000

At low household wealth levels, the gap between what households are willing to pay for safe securities and q^W is so high that every active producer chooses to pay the fix cost. As long as that remains true increases in household wealth are associated with more safe debt issues (see the fourth panel.) But past a certain household wealth level some producers choose to economize on the tranching cost. That effect is strong enough that as soon as it kicks in, the relationship between household wealth and the volume of safe debt becomes negative. As we mentioned in the calibration section, the stochastic steady state of the economy lives in the $[3, 4]$ interval and, therefore, household wealth and debt issues are in the declining portion of the figure.

The key targets that informed our calibration strategy are the average ratio of debt to gross output and the average ratio of managerial rents to gross output. The first two rows of table 5 shows that the parameters we selected do in fact roughly deliver the desired model moments in our simulations. The bottom two rows of the table show moments that we did not directly target but are important given the questions we wish to ask in this paper. First, our approach to calibrating the exogenous TFP shock delivers an output volatility that comes very close to its empirical counterpart. Second, safe yields ($\frac{1}{q^W} - 1$ is the risk-free rate in our model) display a correlation with output that also resembles its value during the 1985-2015 time period.

8.5 Cyclical properties of safe debt issues

Table 6 shows the cyclical properties of key model variables. As is standard in models whose production side are essentially that of the traditional RBC model, GDP follows TFP very closely. Aggregate consumption, wealth and capital are all positively but imperfectly correlated with GDP and TFP.

The key question we wish to ask however is whether the model can account for the relative acyclicality of safe debt issues we documented in section 2. Table 7 shows that the stock of debt whether raw, as a fraction of output or as a fraction of all liabilities displays a correlation with output that resembles the relevant evidence. We emphasize that these are moments that we did not target in any way. Furthermore, the last column of the table shows that not only does the model perform well in terms of quantities, it is also consistent with the well documented counter-cyclicality of risk premia. In the model, we measure risk premia as the expected return on the household portfolio minus the risk-free rate while in the data we use the average annual BAA-AAA spread for the 1985-2014 time period, HP filtered with a smoothing parameter of 100.

9 Safe asset markets and the business cycle

Having proposed a model that satisfactorily captures the cyclicality of safe corporate debt, we will now use it to run counterfactual experiments designed to quantify the impact of safe asset markets on the level and volatility of private sector value added. Specifically, we conduct two experiments. First we set the tranching cost so high that in the stochastic steady state no producer chooses to access safe markets. In other words, all claims are sold to the household and subject to aggregate risk in this experiment. In the second experiment, we permanently and uniformly lower the support of q^W by 20%.

Figure 9 displays the impact of shutting down debt markets by setting κ high on capital formation. The dotted red line shows the relationship between household wealth and capital

Table 7: Cyclicality of safe debt stocks and risk premia

Moment	$\rho(D,Y)$	$\rho\left(\frac{D}{Y},Y\right)$	$\rho\left(\frac{D}{\text{all }D+E},Y\right)$	$\rho(spread,Y)$
Model	-0.10	-0.13	-0.12	-0.13
Data	0.03	-0.09	-0.17	-0.25

Figure 6: Tranching and capital formation
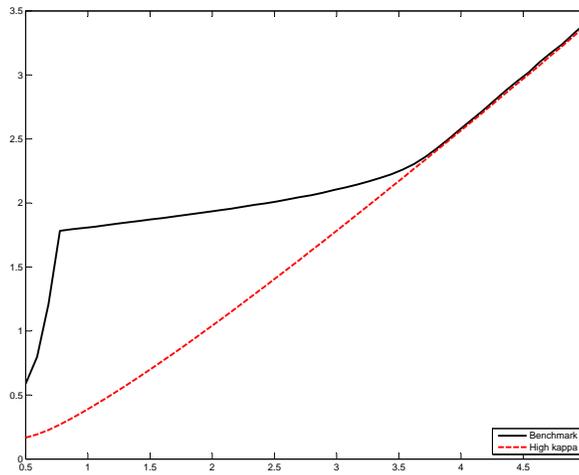
use without access to safe debt markets (assuming that the most recent TFP shock was in the middle state) while the solid black line shows the benchmark relationship. As households become poorer safe debt markets play a bigger role in the benchmark case and strongly mitigate the fall of capital use. However recall that the stochastic steady state lives in the neighborhood of $a = 3.5$ in our experiments, and part of the state space where the gap between the two lines is relatively limited. The third column of table 9 shows that aggregate capital use is about 4% lower on average in the stochastic steady state when safe markets are shut down which translates to a fall in aggregate output of about 1% on average.

Figure 10 shows the impact of a 20% downward shift in the support of safe yields on capital

	Benchmark	High kappa	Low safe yields
Mean Y	1.5894	-0.93%	+0.04%
$std(log(Y)$	0.0236	-0.42%	-0.85%
Mean c	1.3532	-0.27%	-0.27%
$std(log(c))$	0.0134	+8.21%	-2.99%
Mean Assets	3.6482	-1.34%	-1.98%
$std(log(assets))$	0.0269	-15.24%	+6.69%
Mean K	2.3426	-4.02%	+0.17%
$std(log(K))$	0.0276	+0.72%	-10.51%

formation. Capital formation is uniformly higher but the gap between the two economies is small. Not surprisingly then, the change in safe yield has little impact on average output at the stochastic steady state. While the willingness to pay in safe markets is higher households save much less due to lower returns. The increase in investment flows from safe markets is almost entirely crowded out by decreased investment by the household sector.

Output and consumption are less volatile in both counterfactual experiments than in the benchmark economy. In the high κ experiment this is caused by the fact that average household assets hence investment become significantly less volatile when corporations no longer have access to outside safe markets. In the low safe yields experiment on the other hand household savings become much less volatile. But the role of safe asset markets – a comparatively stable source of investments – increases and this effect turns out to dominate quantitatively.

10   Conclusion

In this paper we make the quantitative case that a simple dynamic extension of Allen and Gale (1988)'s optimal security design framework can account for the cyclical behavior of safe corporate debt. A satisfactory model of safe corporate debt, in turn, makes it possible to measure the impact on corporate investment and GDP of the recent increase in global appetite for safe US assets. A uniform reduction in safe yields has little impact on the level of output

as reduced investment from the household sector almost exactly offsets increased flows from world markets for safe debt. Since flows from world markets are relatively acyclical, output and investment volatility fall. Shutting down safe corporate debt markets altogether would cause a permanent 1% decrease in US output, we find, but would actually goes volatility to fall as household savings would become more stable when corporations no longer have access to outside markets for safe debt.

For tractability, we have abstracted from many features of debt markets that we intend to model in future versions of this paper. First, our model contains no role for long-term securities in part because tranching costs must be borne by producers each period. If instead we assumed that producers can enter into long-term contracts by paying a one-time fixed cost, they would want to take advantage of this option. While this will greatly increase the dimension of the state space in the dynamic programming problem we need to solve in this paper, this will produce an economy where long-term securities are issued. Second, we have limited the analysis to one specific type of tranching that amounts to extracting the part of aggregate output that is not subject to aggregate output. In an economy with more investor heterogeneity, producers would choose a more complicated and realistic capital structure.

Figure 7: Low safe yields and capital formation
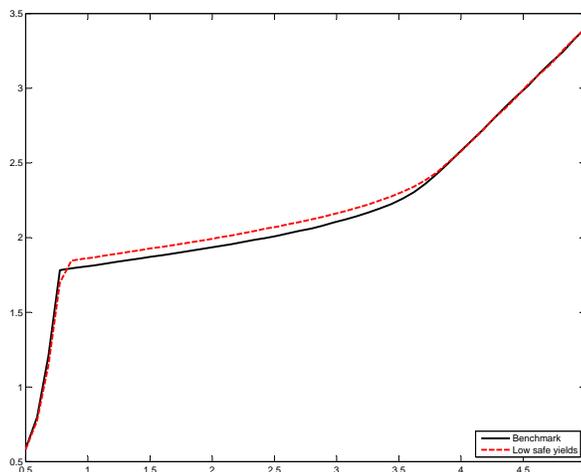
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