Flying or Trapped?

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Abstract

We develop a unified theory under which not only a poverty trap but a middle income trap may also exist. In an otherwise standard growth model, we consider endogenous technology choice in human/knowledge capital accumulation, which enables us to establish a rich array of equilibrium development paradigms, including poverty trap, middle income trap and flying geese growth. We then generalize the baseline structure and establish conditions for different development paradigms to arise. By calibrating the general model to fit the data from several representative economies with different income and growth patterns, we identify various prolonged flying geese episodes and middle income traps. Our results suggest that improving human capital accumulation efficacy is most important, mitigating barriers to human capital accumulation is overall more rewarding than advancing total factor productivity.

Keywords: Flying Geese Development, Middle Income Trap, Technology Choice for Human Capital Upgrading.

JEL Classification Numbers: O4, E2, D2

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1 Introduction

During the post WWII era, one has witnessed a widening world income disparity with the per capita real income ratio of the richest to poorest 10% rising from below 20 in 1960 to above 50 since the turn of the new millennium. This further promotes the study of poverty trap to better understand why the poorest countries failed to advance. Not until recently, more development economists has realized the possibility of middle income trap where many previously fast growing middle income countries suffered sluggish growth before receiving the elite membership to join the club of high income societies. This is not only a concern to the trapped middle income country but a worldwide issue because many such emerging or newly industrialized countries are primary forces advancing global growth.

Empirical evidence for the existence of middle income trap has been provided by Eichengreen, Park and Shin (2013) among others. In their work, middle income trap is identified as a substantive fall (2% or more) in per capita real income growth of a previously fast growing (3.5% or higher) middel income country (with per capita real income exceeding US$10,000 in 2005 constant PPP prices) for a considerable duration (7 year before and after the structural break). Under these criteria in conjunction with Chow tests, they find many such traps in Asian, European and Latin American countries in different years depending on their stages of development, including: (i) in the 1970s, Greece (1972), Finland (1974), Japan (1974), Venezuela (1974), Ireland (1978), (ii) in the 1980s, Singapore (1980), Mexico (1981), Puerto Rico (1988), Cyprus (1989), Korea (1989), and, (iii) in the 1990s, Portugal (1990), Hong Kong (1993), Taiwan (1995). Upon checking a number of possible drivers, they find that more educated at secondary and higher education levels is a robust factor leading to lower likelihood for a country to fall into the middle income trap.

Two natural questions arise. First, can one establish a unified theory under which not only a poverty trap but a middle income trap may also exist? Second, can human capital, or, more generally knowledge capital, play a key role affecting the presence of the middle income trap to support the aforementioned empirical finding? In this paper, we will address both questions within an optimal growth framework with endogenous technology choice. More specifically, we consider a representative agent to maximize her lifetime utility subject to periodic budget constraints that allocate income to consumption and investments in physical and human or knowledge capital. While investment in physical capital directly contribute to its stock one for one, investment in human capital depends crucially on the knowledge accumulation technology. The knowledge accumulation technology exhibits an important feature. While better technology is more productive, it is associated with higher scale barrier. As a result of this trade-off, only those with higher existing stock of human capital may adopt better technology.

In this otherwise simple growth model, we are able to establish necessary and sufficient conditions under which a poverty trap and at least one middle income trap co-exist. We find that a middle income trap is more likely to arise when the productivity of the prevailing technology is not too large, the scale barrier of the prevailing technology is sufficiently high and the productivity jump to the better technology is sufficiently big. But what happens if an economy exhibits no such middle income trap? We establish the conditions under which the economy features a flying geese development paradigm à la Akamatsu (1962), with technology upgrading in human/knowledge accumulation over time. We also show that with negligible productivity upgrading an economy can be permanently trap in poverty and that with negligible increase in technology scale barrier an economy can grow rapidly into an advanced society.

We then generalize our baseline model to permit general technical progress and employment variations over time. By applying our modified conditions for flying geese paradigm and middle income trap to a representative set of 15 countries using data from Penn World Tables (PWT9.0), we find that even in the four developed countries and four fast growing Asian Tigers where they have experienced flying geese at prolonged periods, there still exist occasions of traps. In the four emerging growing economies, such traps become more ofter, but still less frequent than flying geese. In the three development laggards featuring chopped and slower growth paths, they are trapped as much as flying. Although with different sample of countries, we are able to compare our findings with those in the literature by focusing on common samples. We find that several, but not all, traps identified in our paper are in line with those in previous studies. Overall, we find that stagnated human capital technology is the primary source for a country to fall into a middle income trap. While barriers to human capital accumulation also play a role, total factor productivity (TFP) slowdowns turn out to be the least important.

The main message delivered by this paper is that human capital upgrading can play a key role in economic development, determining whether a country may be flying or trapped. Our quantitative analysis yields an important policy implication: public police toward improving human capital accumulation efficacy or mitigating barriers to human capital accumulation is likely more rewarding than that toward advancing the TFP.

Literature

There is a sizable literature showing showing the existence of a poverty trap. The argument is based on multiple steady states: co-existence of degenerate low equilibrium (trap) and a high equilibrium (see a comprehensive survey by Azariadis and Stachurski 2005). By limiting our attention to human capital related studies, poverty trap may arise due to human capital threshold externality (Azariadis and Drazen 1990; Redding 1996), moral hazard problem associated with human capital investment (Tssidon 1992), or barrier to investment in children's human capital (Galor and Weil 2000). Yet their frameworks cannot be used to generate middle income trap.

There is a small body of research on the flying geese paradigm with microfounded theory. In particular, the flying geese paradigm may arise as a result of product upgrading (Matsuyama 2002), industry upgrading (Wang and Xie 2004; Ju, Lin and Wang 2015) or staged assimilation of global technologies (Wang, Wong and Yip 2018). None of these papers fully characterizes the local and global dynamics of the model.

In Wang, Wong and Yip (2018), the possibility of middle income trap via assimilation of global technologies is established. Specifically, technology assimilation in a country is more effective if its factor endowment gap from the technology source country is small. When a country accumulates the disadvantage capital to reduce such a gap in the process of assimilation, the mismatch is mitigated and output grows faster. But a country may over-accumulate eventually, causing more serious mismatch and growth slowdown and yielding a middle income trap. By contrast, not only we propose a very different mechanism, but we also fully establish the explicit conditions for a middle income trap and fully characterize the dynamics to ensure stability property of such a trap.

2 Model

Consider a discrete-time model with time indexed by t. The economy consists of identical infinitely lived agents. There is a single general good that can be used for consumption or investment purposes. In addition to labor, there are two capital inputs: physical and human capital. The key feature of the model is that the human capital updating technology is endogenously determined by discrete choice.

2.1 The environment

A representative agent at period t produces general goods Y_t by applying a Cobb-Douglas production technology $Y_t = AH_{t-1}^{\alpha}K_{t-1}^{\beta}, L_t^{\gamma}$ ($\alpha, \beta, \gamma \in (0,1)$, $\alpha+\beta+\gamma=1$), the inputs of which are physical capital K_{t-1}, labor L_t, and human capital H_{t-1}– which should be taken more generally to include knowledge capital and know-how. Note that the production of general goods takes one gestation period in which the human capital, H_{t-1}, and the physical capital, K_{t-1}, are prepared one period before the production occurs (time-to-build). The general goods are consumed or used for investments by the representative agent. The aggregate budget constraint is given by

$$AH_{t-1}^{\alpha}K_{t-1}^{\beta}L_t^{\gamma} = C_t + I_t^h + I_t^k, \tag{1}$$

where C_t is aggregate consumption, and I_t^h and I_t^k are investments for production of human capital and physical capital, respectively. The per worker budget constraint is obtained from Eq. (1) as follows:

$$y_t := Ah_{t-1}^{\alpha}k_{t-1}^{\beta} = c_t + i_t^h + i_t^k, \tag{2}$$

where $h_{t-1} := H_{t-1}/L_t$, $k_{t-1} := K_{t-1}/L_t$, $c_t := C_t/L_t$, $i_t^h := I_t^h/L_t$, and $i_t^k := I_t^k/L_t$ are per worker variables. Although the growth in labor force is introduced in the quantitative analysis in section 5, the population of labor force is normalized to $L_t = 1$ for all $t \geq 0$ in the theoretical analysis up to section 4.

2.2 Optimization Problem

Let us first specify a representative agent's optimization problem by maximizing her lifetime utility subject to three constraints in addition to the typical nonnegativity constraints on {}:

$$\max \sum_{\tau=t}^{\infty} \delta^{\tau-t} \ln c_{\tau}$$

subject to

$$y_{\tau} \quad : \quad = Ah_{\tau-1}^{\alpha}k_{\tau-1}^{\beta} = c_{\tau} + i_{\tau}^h + i_{\tau}^k \tag{3}$$

$$k_{\tau} \quad = \quad g_0(i_{\tau}^k) \tag{4}$$

$$h_{\tau} \quad = \quad \max_{m=1,2,\ldots,M}\{g_m(i_{\tau}^h; \bar{h}_{\tau-1}, \bar{y}_{\tau})\}, \tag{5}$$

for $\tau \geq t$, where $\delta \in (0,1)$ is the subjective discount factor. In Eqs. (3)-(5), both physical and human capital depreciates entirely in one period. The representative agent is endowed with two types of investment projects: one is to produce physical capital and the other is to produce human capital. In the first investment project, a one-for-one simple linear technology produces capital from general goods. That is, in Eq. (4), the production function for physical capital, $g(i_\tau^k)$, is given by

$$g_0(i_\tau^k) = i_\tau^k, \tag{6}$$

which implies that physical capital is produced from the general goods with a one-for-one technology.

In Eq. (5), $g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)$ is a production function for human capital when the representative agent applies the mth technology. Human capital is produced from the general goods where the human capital formation is subject to technology choice: the representative agent chooses the best technology among a number of M technologies for the human capital formation. It may be thought of as after the highest technology M, the economy will be on a perpetual balanced growth path. That is, along such a balanced growth path, human capital production is simply linear with $g_{m>M}(i_\tau^h) = \theta_{\max} i_\tau^h$, $\theta_{\max} > 0$, whereas the production exhibits constant returns in the two capitals with $\alpha + \beta = 1$. This this highest stage of development corresponds to the Rostovian state of Mass Consumption. Because this equilibrium is trivial, we, for the sake of brevity, will characterize it explicitly.

Throughout the analysis, we shall begin by analyzing the case in which there are three technologies, subsequently followed by generalization with M technologies. In each period, the agent chooses the best technology for human (or knowledge) capital formation and solves her maximization problem, exogenously given the externalities from the (average) past human capital, $\bar{h}_{\tau-1}$, and the (average) current-period output, $\bar{y}_\tau$, in the economy. We impose Assumption 1 below for regulating the functional form of $g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)$.

Assumption 1. *(i)* $\partial g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)/\partial i_\tau^h > 0$ *(ii)* $\partial^2 g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)/\partial i_\tau^h \partial \bar{h}_{\tau-1} > 0$, *(iii)* $\partial^3 g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)/\partial i_\tau^h \partial \bar{h}_{\tau-1}^2 < 0$, *and (iv)* $\partial^2 g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)/\partial \bar{h}_{\tau-1} \partial \bar{y}_\tau < 0$.

Assumption 1-(i) guarantees the positive marginal product of human capital investment, i_τ^h. Assumption 1-(ii) implies that the past knowledge accumulation, which is condensed into the past human capital formation $\bar{h}_{\tau-1}$, has a positive external effect on the marginal product promoting the human capital formation. However, from Assumption 1-(iii), the effect of the positive externality diminishes as the past knowledge more accumulates. Assumption 1-(iv) mitigates the scale effect of human capital externality. Since as the economy develops, $\bar{h}_{\tau-1}$ and $\bar{y}_{\tau}$ continue to rise, so this assumption limits the scale of knowledge spillovers.

Under Assumption 1, the representative agent chooses the best technology depending upon the human capital accumulation. If the productivity with respect to a certain technology becomes small, another suitable technology for human capital formation may be chosen as noted from the max operator in the right-hand side of Eq. (5).

To make the following analysis concrete, we specify $g_m(i^h_{\tau}; \bar{h}_{\tau-1}, \bar{y}_{\tau})$ as

$$g_m(i^h_{\tau}; \bar{h}_{\tau-1}, \bar{y}_{\tau}) = \frac{B_m(\bar{h}_{t-1})}{\bar{y}_t} i^h_{\tau}, \tag{7}$$

where $B_m(\bar{h}_{t-1}) := \theta_m(\bar{h}_{t-1} - \eta_m)^{\sigma}$ for $\bar{h}_{t-1} \geq \eta_m$, with $\alpha < \sigma \in (0,1)$, $\theta_m \in [1, \infty)$, and $\eta_m \in [0, \infty)$.[1] Thus, human or knowledge capital accumulation depends on the society's existing stock in the spirit of the knowledge spillovers in Romer (1986) and human capital spillovers in Lucas (1988). However, the externality of $\bar{h}_{t-1}$ is effective only when it exceeds a certain border, η_m. Thus, one may view the presence of η_m as a result of scale barrier in knowledge accumulation. The scale barrier in human capital considered here also captures the argument in Buera and Kaboski (2012) where higher skill is required for production of goods with greater complexity. Our scale barrier setup generates similar implication to the appreciate technology model in Caselli and Coleman (2006) where skilled labor abundant rich countries tend to choose technologies more efficient to skilled workers. As imposed a parameter condition for $B_m(\bar{h}_{t-1})$ in Assumption 2 below, there is a trade-off between the productivity, θ_m, and the scale barrier, η_m.

[1] The assumption $\alpha < \sigma$ guarantees the net effect of $\bar{h}_{t-1}$ is always positive.

Assumption 2. *(i)* $1 = \theta_1 < \theta_2 < \cdots < \theta_M$ *and (ii)* $0 = \eta_1 < \eta_2 < \cdots < \eta_M$.

Assumption 2 implies that human capital accumulation needs to exceed the higher border for an economy to utilize the higher productivity technology.

The Lagrangian for the utility maximization problem is set up as follows:

$$L_t := \sum_{\tau=t}^{\infty} \delta^{\tau-t} \ln c_\tau + \sum_{\tau=t}^{\infty} \delta^{\tau-t} \lambda_\tau \left[A h_{\tau-1}^{\alpha} k_{\tau-1}^{\beta} - i_\tau^h - i_\tau^k - c_\tau \right]$$
$$+ \sum_{\tau=t}^{\infty} \delta^{\tau-t} p_\tau^h \left[b(\bar{h}_{\tau-1}, \bar{y}_\tau) i_\tau^h - h_\tau \right] + \sum_{\tau=t}^{\infty} \delta^{\tau-t} p_\tau^k \left[i_\tau^k - k_\tau \right],$$

where $b(\bar{h}_{\tau-1}, \bar{y}_\tau) := \max_m\{B_m(\bar{h}_{\tau-1})\}/\bar{y}_\tau$, and λ_τ, p_τ^h, and p_τ^k are the shadow prices of general goods, human capital, and physical capital, respectively. The first-order conditions are given by

$$\lambda_t = \frac{1}{c_t}, \tag{8}$$
$$\lambda_t = \left(\frac{\delta \alpha b(\bar{h}_{t-1}, \bar{y}_t) y_{t+1}}{h_t} \right) \lambda_{t+1}, \tag{9}$$
$$\lambda_t = \left(\frac{\delta \beta y_{t+1}}{k_t} \right) \lambda_{t+1}, \tag{10}$$
$$\lambda_t = p_t^k = b(\bar{h}_{t-1}, \bar{y}_t) p_t^h. \tag{11}$$

The necessary and sufficient conditions for the optimality of this maximization problem consist of Eqs. (8)-(11) as well as the transversality conditions, $\lim_{t\to\infty} \delta^t p_t^k k_t = \lim_{t\to\infty} \delta^t p_t^h h_t = 0$.

3 Equilibrium

We are now prepared to define and establish the dynamic competitive equilibrium and to characterize the dynamics. A key strategy is reduce the dynamical system to the two states – physical and human capital – and the values of these two capitals. This strategy greatly simplifies the analysis because the two capital values are proportional and their limits are governed by the transversality conditions.

From Eqs. (9) and (10), it follows that $h_t = (\alpha b(\bar{h}_{t-1}, \bar{y}_t)/\beta)k_t$. From this equation and Eqs. (4)-(7), we obtain $i_t^k + i_t^h = k_t(1-\gamma)/\beta$. These two equations allow us to rewrite the flow budget constraint (3) as

$$c_t + \frac{1-\gamma}{\beta}k_t = A\left(\frac{\alpha b(\bar{h}_{t-2}, \bar{y}_{t-1})}{\beta}\right)^{\alpha} k_{t-1}^{1-\gamma}, \tag{12}$$

for all $t \geq 1$. Additionally, substituting $h_t = (\alpha b(\bar{h}_{t-1}, \bar{y}_t)/\beta)k_t$ into Eq. (9) yields

$$\lambda_{t-1} = \lambda_t \delta \alpha A b(\bar{h}_{t-2}, \bar{y}_{t-1})^{\alpha}\left(\frac{\alpha}{\beta}\right)^{\alpha-1} k_{t-1}^{-\gamma}. \tag{13}$$

Multiplying λ_t to both sides of Eq. (12) leads to

$$\lambda_t c_t + \frac{1-\gamma}{\beta}\lambda_t k_t = \lambda_t A\left(\frac{\alpha b(\bar{h}_{t-2}, \bar{y}_{t-1})}{\beta}\right)^{\alpha} k_{t-1}^{1-\gamma}. \tag{14}$$

Applying Eqs. (8) and (13) to the left-hand and right-hand sides of Eq. (14) respectively, we obtain

$$q_t^k = \frac{1}{\delta(1-\gamma)}q_{t-1}^k - \frac{\beta}{1-\gamma}, \tag{15}$$

where $q_t^k := p_t^k k_t = \lambda_t k_t$, which is the value of physical capital in period t. It follows from Eq. (11) and $k_t = (\beta/(\alpha b(\bar{h}_{\tau-1}, \bar{y}_\tau)))h_t$ that $q_t^k = p_t^k k_t = (\beta/\alpha)p_t^h h_t$. Substituting $q_t^k = p_t^k k_t = (\beta/\alpha)p_t^h h_t$ into Eq. (15) yields

$$q_t^h = \frac{1}{\delta(1-\gamma)}q_{t-1}^h - \frac{\alpha}{1-\gamma}, \tag{16}$$

where $q_t^h := p_t^h h_t$, which is the value of human capital in period t. Eqs. (15) and (16) govern the dynamics of the values of the two capitals recursively, independent of other states or controls or technology choice.

From Eqs. (10) and (11), it follows that $p_t^k y_t = q_{t-1}^k/(\delta\beta)$. The use of this equation, Eqs. (5), (7) and (15) with $i_t^h = (\alpha/\beta)k_t$ yields

$$h_t = \alpha \max_m \{B_m(h_{t-1})\} \left[\frac{1}{1-\gamma} - \frac{\beta\delta}{1-\gamma} \left(\frac{1}{q_{t-1}^k} \right) \right], \tag{17}$$

where we have used the fact that $\bar{h}_{t-1} = h_{t-1}$ in equilibrium. Additionally, from $h_t = (\alpha b(\bar{h}_{\tau-1}, \bar{y}_\tau)/\beta)k_t$ and Eq. (17), we obtain

$$k_t = \beta A h_{t-1}^\alpha k_{t-1}^\beta \left[\frac{1}{1-\gamma} - \frac{\beta\delta}{1-\gamma} \left(\frac{1}{q_{t-1}^k} \right) \right]. \tag{18}$$

We can now define a dynamic competitve equilibrium as a sequence, $\{q_t^h, q_t^k, h_t, k_t\}$, for $t \geq 0$ that satisfies Eqs. (15)-(18) and the transversality condition, given $h_0 \geq 0$ and $k_0 \geq 0$. In so far, we have, however, not yet analyzed the technology choice, $\max_m\{B_m(h_{t-1})\}$, in Eq. (17), to which we now turn.

3.1 Technology choice

We consider technology choice $\max_m\{B_m(h_{t-1})\}$ in Eq. (17). Given the trade-off under Assumption 2, the key is to determine the cutoff human capital scales between each pair of technologies.

Define such cutoffs as w_1 and w_2 so that $B_1(w_1) = B_2(w_1)$ and $B_2(w_2) = B_3(w_2)$. Note that w_1 is the cutoff of h between the first and the second technologies whereas w_1 is the cutoff between the second and the third technologies. It is straightforward to show that w_1 and w_2 satisfy

$$w_1^\sigma = \theta_2(w_1 - \eta_2)^\sigma \tag{19}$$

and

$$\theta_2(w_2 - \eta_2)^\sigma = \theta_3(w_2 - \eta_3)^\sigma, \tag{20}$$

respectively, from which w_1 and w_2 are uniquely determined as $w_1 = \theta_2^{\frac{1}{\sigma}}\eta_2/(\theta_2^{\frac{1}{\sigma}} - 1)$ and $w_2 = (\theta_3^{\frac{1}{\sigma}}\eta_3 - \theta_2^{\frac{1}{\sigma}}\eta_2)/(\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}})$.

Lemma 1. *Under Assumption 2, the following hold.*

- *If* $\eta_2 \leq h_{t-1} < w_1$ *(resp.* $h_{t-1} > w_1$*), it holds that* $B_1(h_{t-1}) > B_2(h_{t-1})$ *(resp.* $B_1(h_{t-1}) < B_2(h_{t-1})$*).*
- *If* $\eta_3 \leq h_{t-1} < w_2$ *(resp.* $h_{t-1} > w_2$*), it holds that* $B_2(h_{t-1}) > B_3(h_{t-1})$ *(resp.* $B_2(h_{t-1}) < B_3(h_{t-1})$*).*

Proof. See the Appendix.

Proposition 1. *Suppose that Assumption 2 holds. Then, in* $\max_m\{B_m(h_{t-1})\}$*, the representative agent optimally chooses the first technology* ($m = 1$) *if* $0 \leq h_{t-1} < w_1$*, the second technology* ($m = 2$) *if* $w_1 < h_{t-1} < w_2$*, and the third technology* ($m = 3$) *if* $w_2 < h_{t-1}$*.*

Proof. See the Appendix.

Note that if $h_{t-1} = w_1$, the choice between the first and second technologies is indifferent. In this case, it is assumed that the second technology is chosen over the first technology. Likewise, if $h_{t-1} = w_2$, the choice between the second and third technologies is indifferent and it is assumed that the third technology is chosen over the second technology in this case.

It is clear that Proposition 1 can be readily generalized to M technologies: the representative agent optimally chooses the first technology if $0 \leq h_{t-1} < w_1$, the mth technology if $w_{j-m} \leq h_{t-1} < w_m$ ($m = 2, ...M-1$), and the Mth technology if $w_{M-1} \leq h_{t-1}$.

3.2 Steady states

The steady states can be solved in a recursive manner by solving the capital values first. Eqs. (15)-(18) form the dynamical system in equilibrium, which we recite in the following:

$$
\begin{cases}
q_t^h = \frac{1}{\delta(1-\gamma)} q_{t-1}^h - \frac{\alpha}{1-\gamma} \\
q_t^k = \frac{1}{\delta(1-\gamma)} q_{t-1}^k - \frac{\beta}{1-\gamma} \\
h_t = \alpha \max_m \{B_m(h_{t-1})\} \left[\frac{1}{1-\gamma} - \frac{\beta\delta}{1-\gamma} \left(\frac{1}{q_{t-1}^k} \right) \right] =: J_1(q_{t-1}^k, h_{t-1}, k_{t-1}) \\
k_t = \beta A h_{t-1}^\alpha k_{t-1}^\beta \left[\frac{1}{1-\gamma} - \frac{\beta\delta}{1-\gamma} \left(\frac{1}{q_{t-1}^k} \right) \right] =: J_2(q_{t-1}^k, h_{t-1}, k_{t-1}).
\end{cases}
\tag{21}
$$

As a result of the recursive property mentioned above, the values of human and physical capital in the steady state, q^{h*} and q^{k*}, are independent of human and physical capital and the technology choice

$$
q^{h*} = \frac{\alpha\delta}{1-\delta(1-\gamma)} \text{ and } q^{k*} = \frac{\beta\delta}{1-\delta(1-\gamma)}. \tag{22}
$$

In contrast, physical capital and human capital in the steady state depend crucially on the technology choice. Suppose that the jth technology in human capital production is optimally chosen, i.e., $\max_m\{B_m(h_{t-1})\} = B_j(h_{t-1})$ ($j = 1, 2$ or 3). Since $B_j(h_{t-1})$ is concave, we can "potentially" obtain at most two steady states, say, $h_{j,1}^*$ and $h_{j,2}^*$, from the technology. From (21), it follows that

$$
h_{j,s}^* = \alpha\delta\theta_j (h_{j,s}^* - \eta_s)^\sigma \tag{23}
$$

and

$$
k_{j,s}^* = \beta\delta A (h_{j,s}^*)^\alpha (k_{j,s}^*)^\beta, \tag{24}
$$

for $s = 1, 2$. Assuming that $h_{j,1}^* < h_{j,2}^*$, we call $(q^{h*}, q^{k*}, h_{j,1}^*, k_{j,1}^*)$ and $(q^{h*}, q^{k*}, h_{j,2}^*, k_{j,2}^*)$ the low steady state and the high steady state, respectively.

From Eq. (23), the potential two steady states of human capital, $h_{j,1}^*$ and $h_{j,2}^*$, satisfy the following equation:

$$
\Pi_j(h) := h^{\frac{1}{\sigma}} - (\delta\alpha\theta_j)^{\frac{1}{\sigma}} h + (\delta\alpha\theta_j)^{\frac{1}{\sigma}} \eta_j = 0. \tag{25}
$$

Because $\Pi_j(h)$ is convex and $\hat{h}_j := \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_j)^{\frac{1}{1-\sigma}}$ gives a minimum of $\Pi_j(h)$, there exist two distinct real number solutions of Eq. (24) if and only if $\Pi_j(\hat{h}_j) < 0$. Formally, we have Lemma 2 below.

Lemma 2. *There exist two distinct real number solutions of Eq. (25) if and only if*

$$\eta_j - \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_j)^{\frac{1}{1-\sigma}}(1-\sigma) < 0. \tag{26}$$

Proof. The claim of Lemma 2 follows from the fact that $\Pi_j(h)$ is convex, $\Pi_j(0) > 0$, and $\Pi_j(\hat{h}) < 0 \iff$ Eq. (26). □

In what follows, we derive conditions that each technology actually has two steady states (including the trivial one) in the dynamical system.

3.2.1 First technology: $j = 1$

Note that under Assumption 2, Eq. (25) with $j = 1$ has two solutions, $h^*_{1,1} = 0$ and

$$h^*_{1,2} := (\delta\alpha)^{\frac{1}{1-\sigma}}, \tag{27}$$

From Eq. (24), we have

$$k^*_{1,2} := (\delta\beta A)^{\frac{1}{1-\beta}}(h^*_{1,2})^{\frac{\alpha}{1-\beta}}. \tag{28}$$

Proposition 2. *Under Assumption 2, suppose that the following parameter condition holds:*

$$(\delta\alpha)^{\frac{1}{1-\sigma}} < \eta_2. \tag{29}$$

Then, a non-trivial steady state, $(q^{h*}, q^{k*}, h^*_{1,2}, k^*_{1,2})$, associated with the first technology exists in the dynamical system.

Proof. It suffices to show that $h^*_{1,2} < w_1$. From Eq. (28), it holds that $h^*_{1,2} < \eta_2 < w_1$, and Eq. (29) gives $k^*_{1,2}$. Then, the desired conclusion holds. □

3.2.2 Second technology: $j = 2$

We next turn to the case with $j = 2$ under which there are still two steady states prevailed.

Proposition 3. *Under Assumption 2, suppose that the following parameter conditions hold:*

$$(\delta\alpha)^{\frac{1}{1-\sigma}} < \eta_2 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} \tag{30}$$

$$(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3. \tag{31}$$

Then, two steady states, $(q^{h*}, q^{k*}, h^*_{2,1}, k^*_{2,1})$ *and* $(q^{h*}, q^{k*}, h^*_{2,2}, k^*_{2,2})$*, associated with the second technology exist in the dynamical system.*

Proof. See the Appendix.

3.2.3 Third technology: $j = 3$

When the highest technology is chosen ($j = 3$), it once again features two steady states.

Proposition 4. *Under Assumption 2, suppose that the following parameter condition holds:*

$$(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_3)^{\frac{1}{1-\sigma}}. \tag{32}$$

Then, two steady states, $(q^{h*}, q^{k*}, h^*_{3,1}, k^*_{3,1})$ *and* $(q^{h*}, q^{k*}, h^*_{3,2}, k^*_{3,2})$*, associated with the third technology exist in the dynamical system.*

Proof. See the Appendix.

3.2.4 Summary

Propositions 2-4 imply that six steady states (including a trivial one) exist in the dynamical system if the following inequalities hold:

$$(\delta\alpha)^{\frac{1}{1-\sigma}} < \eta_2 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} \tag{33}$$

$$(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_3)^{\frac{1}{1-\sigma}}, \tag{34}$$

in which each technology yields two steady states.

It is straightforward to extend the analysis to the case in which the number of technologies is M with $1 = \theta_1 < \cdots < \theta_M$ and $0 = \eta_1 < \cdots < \eta_M$. In this case, $2M$ steady states appear if the following inequalities hold:

$$\begin{aligned}
&(\delta\alpha)^{\frac{1}{1-\sigma}} < \eta_2 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} \\
&(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_3)^{\frac{1}{1-\sigma}} \\
&\qquad\qquad\vdots \\
&(\delta\alpha\theta_{M-1})^{\frac{1}{1-\sigma}} < \eta_M < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_M)^{\frac{1}{1-\sigma}}.
\end{aligned}$$

3.3 Local dynamics

Suppose that the jth technology has two steady states: a lower one, $(q^{h*}, q^{k*}, h^*_{j,1}, k^*_{j,1})$, and a higher one, $(q^{h*}, q^{k*}, h^*_{j,2}, k^*_{j,2})$, where $h^*_{j,1} < h^*_{j,2}$. Linearization of the dynamical system (21) with the jth technology around one of the steady states, $(q^{h*}, q^{k*}, h^*_{j,s}, k^*_{j,s})$ ($s = 1$ or 2), implies

$$\begin{pmatrix} q^h_t - q^{h*} \\ q^k_t - q^{k*} \\ h_t - h^*_{j,s} \\ k_t - k^*_{j,s} \end{pmatrix} = \begin{pmatrix} \frac{1}{\delta(1-\gamma)} & 0 & 0 & 0 \\ 0 & \frac{1}{\delta(1-\gamma)} & 0 & 0 \\ 0 & J_{1,\lambda}(q^{k*}, h^*_{j,s}, k^*_{j,s}) & \alpha\delta B'_j(h^*_{j,s}) & 0 \\ 0 & J_{2,\lambda}(q^{k*}, h^*_{j,s}, k^*_{j,s}) & J_{2,k}(q^{k*}, h^*_{j,s}, k^*_{j,s}) & \beta \end{pmatrix} \begin{pmatrix} q^h_{t-1} - q^{h*} \\ q^k_{t-1} - q^{k*} \\ h_{t-1} - h^*_{j,s} \\ k_{t-1} - k^*_{j,s} \end{pmatrix}, \tag{35}$$

where $J_{n,q^k}(q^k, h, k) := \partial J_n(q^k, h, k)/\partial\lambda$ and $J_{n,k}(\lambda, h, k) := \partial J_n(\lambda, h, k)/\partial k$ for $n = 1, 2$. The eigenvalues of this dynamical system are given by $\rho_1 := 1/(\delta(1-\gamma))$, $\rho_2 := 1/(\delta(1-\gamma))$, $\rho_3 := \alpha\delta B_j'(h_{j,s}^*)$, and $\rho_4 = \beta$.

In the dynamical system (21), whereas h_t and k_t are state variables that cannot jump being predetermined variables, q_t^h and q_t^k can jump, which are determined by expectations. Due to the resursive property, the dynamical system can be fully characterized. In particular, since ρ_1, $\rho_2 > 1$ and $0 < \rho_4 < 1$, the property of the local dynamics depends entirely on $\alpha\delta B_j'(h_{j,s}^*)$.

Lemma 3. *Under Assumption 2, suppose that inequalities (33) and (34) are satisfied. Then, for $j = 1, 2$ and 3, it holds that $\alpha\delta B_j'(h_{j,1}^*) > 1$ and $0 < \alpha\delta B_j'(h_{j,2}^*) < 1$.*

Proof. See the Appendix.

The two state variables, h_t and k_t, cannot jump, whereas q_t^h and q_t^k can jump. It is seen from Lemma 3 that in the linearized dynamical system around the lower steady state, $(q^{h*}, q^{k*}, h_{j,1}^*, k_{j,1}^*)$, the three eigenvalues are greater than one and the absolute value of the one eigenvalue is less than one. Therefore, no equilibrium sequence, $\{q_t^h, q_t^k, h_t, k_t\}$, exists that converges to the lower steady state. Also from Lemma 3, one can see that in the linearized dynamical system around the higher steady state, $(q_t^h, q_t^k, h_{j,2}^*, k_{j,2}^*)$, the two eigenvalues are greater than one and the absolute values of the two eigenvalues are less than one. Therefore, a unique equilibrium sequence, $\{q_t^h, q_t^k, h_t, k_t\}$, exists around the higher steady state that converges to this steady state. We summarize these results in Proposition 5 below.

Proposition 5. *Under Assumption 2, suppose that inequalities (33) and (34) are satisfied. Consider the jth technology. Then, the following hold:*

- *There exists no equilibrium sequence, $\{q_t^h, q_t^k, h_t, k_t\}$, around the lower steady state, $(q^{h*}, q^{k*}, h_{j,1}^*, k_{j,1}^*)$, that converges to this steady state.*

- *There exists a unique equilibrium sequence, $\{q_t^h, q_t^k, h_t, k_t\}$, around the higher steady state, $(q^{h*}, q^{k*}, h_{j,2}^*, k_{j,2}^*)$, that converges to this steady state.*

Proof. The discussion just before Proposition 5 proves the claims. □

4 Global analysis

In the previous section, we analyzed the local dynamic property. In the analysis, it is still unclear whether an equilibrium sequence exists around the higher steady state. Moreover, even if an equilibrium sequence exists around the higher steady state, the analysis of local dynamics does not clarify where it goes. In this section, we address these questions.

4.1 Phase diagrams

In Eq. (21), the difference equations with respect to q_t^h and q_t^k, i.e., Eqs. (15) and (16), are independent of h_t and k_t, and they are solvable analytically. Solving Eqs. (15) and (16) forward, we obtain

$$q_t^k = [\delta(1-\gamma)]^u q_{t+u}^k + \beta\delta + \beta\delta[\delta(1-\gamma)] + \cdots + \beta\delta[\delta(1-\gamma)]^{u-1}. \tag{36}$$

and

$$q_t^h = [\delta(1-\gamma)]^u q_{t+u}^h + \alpha\delta + \alpha\delta[\delta(1-\gamma)] + \cdots + \alpha\delta[\delta(1-\gamma)]^{u-1}. \tag{37}$$

It follows from the transversality condition, $\lim_{u\to\infty} \delta^u q_{t+u}^k = \lim_{u\to\infty} \delta^u q_{t+u}^h = 0$, that $\lim_{u\to\infty}[\delta(1-\gamma)]^u q_{t+u}^k = \lim_{u\to\infty}[\delta(1-\gamma)]^u q_{t+u}^h = 0$. Therefore, Eqs. (36) and (37) yield $q_t^k = \beta\delta/(1-\delta(1-\gamma))$ and $q_t^h = \alpha\delta/(1-\delta(1-\gamma))$, respectively, which implies that the equilibrium sequences of $\{q_t^k, q_t^h\}$ are uniquely determined, being equal to $\{q_t^k, q_t^h\} = \{q^{k*}, q^{h*}\}$.

Then, from Proposition 1, Eqs. (17) and (18) become

$$h_t = \begin{cases} \alpha\delta h_{t-1}^{\sigma} & \text{if } 0 \leq h_{t-1} < w_1 \\ \alpha\delta\theta_2(h_{t-1} - \eta_2)^{\sigma} & \text{if } w_1 \leq h_{t-1} < w_2 \\ \alpha\delta\theta_3(h_{t-1} - \eta_3)^{\sigma} & \text{if } w_2 \leq h_{t-1} \end{cases} \tag{38}$$

and

$$k_t = \beta\delta A h_{t-1}^{\alpha} k_{t-1}^{\beta}. \tag{39}$$

Figure 1 provides the phase diagram of Eq. (38) when inequalities (32) and (33) hold under Assumption 2 and Figure 2 provides the conditional phase diagram of Eq. (39) with h_{t-1} given.

4.2 Take-off and flying geese

If the productivity of human capital formation for each technology, $B_m(\bar{h}_{t-1})/\bar{y}_t$, is high, the economy does not fall in the low- or middle-income traps. The parameters related to the productivity are θ_m and η_m. Given $\bar{h}_{t-1}$, if θ_m is large or η_m is small, the productivity of human capital formation becomes high. Proposition 6 below provides a condition that the economy does not fall in the low- or middle-income traps and converges to the high steady state of the third technology even if the initial human capital is very low.

Proposition 6. *Suppose that Assumption 2 holds. Then, there exist only two steady states in the dynamical system, which are a trivial one and the high steady state of the third technology if and only if the following parameter condition holds:*

$$(\delta\alpha)^{\frac{1}{1-\sigma}} > \max\{\Phi_1, \Phi_2\}, \tag{40}$$

where

$$\Phi_i := \frac{\left(\theta_{i+1}^{\frac{1}{\sigma}}\eta_{i+1} - \theta_i^{\frac{1}{\sigma}}\eta_i\right)^{\frac{1}{1-\sigma}}}{\theta_i^{\frac{1}{1-\sigma}}\theta_{i+1}^{\frac{1}{1-\sigma}}\left(\theta_{i+1}^{\frac{1}{\sigma}} - \theta_i^{\frac{1}{\sigma}}\right)(\eta_{i+1} - \eta_i)^{\frac{\sigma}{1-\sigma}}}.$$

Proof. See the Appendix.

Figure 3 provides the phase diagram of Eq. (38) when inequality (40) holds under Assumption 2.

The outcome of proposition 6 can be extended to the case in which the number of technologies is M. Suppose that $1 = \theta_1 < \cdots < \theta_M$ and $0 = \eta_1 < \cdots < \eta_M$. In this case, there exist two steady states: one is a trivial one and the other is the high steady state of the Mth technology if and only if $(\delta\alpha)^{\frac{1}{1-\sigma}} > \max\{\Phi_1, \ldots, \Phi_{M-1}\}$. When this inequality holds, the economy develops from a low human capital state to a high human capital state proceeding along the M technologies.

By examining this inequality, one can see that if either changes in θ_i or changes in η_i becomes negligible (but not both), then Φ_i tends to be larger and the flying geese paradigm is less likely to arise. The two polar cases are actually different. When the barrier to human capital technology upgrading is essentially unchanged, the development pattern exhibits immediate upgrading to the highest technology M as soon as the level of human capital exceeds the barrier. On the contrary, when the productivity gain from upgrading is nil, the economy is trapped in the lowest technology.

4.3 Middle income trap

What if a certain Φ_j at an intermediate technology j breaks the inequality $(\delta\alpha)^{\frac{1}{1-\sigma}} > \max\{\Phi_1, \ldots, \Phi_{M-1}\}$ stated in the previous subsection? More precisely, if there is a Φ_j such that $\Phi_j > (\delta\alpha)^{\frac{1}{1-\sigma}} > \max\{\Phi_1, \ldots, \Phi_{j-1}, \Phi_{j+1}, \ldots, \Phi_{M-1}\}$, then the economy converges to the high steady state of the jth technology if the economy starts with low human capital. This phenomenon is depicted in Figure 4.

While the trivial equilibrium is frequently referred to as the low income trap, the equilibrium at the jth technology can be called a middle income trap. In general, there can be more than one technology at which a middle income country may be trapped. To establish this main result, define the set of all globally available technogies as $T := \{1, \cdots, M\}$ and the set of "interior" technologies as $I := \{2, \cdots, M-1\}$. is convenienctly summarized below.

Proposition 7. *Suppose that Assumption 2 holds. Then, there exist nontrivial steady states in the dynamical system, featuring middle income trap at various technologies in* $J \subseteq I$ *if and only if the following parameter condition holds:*

$$\min_{j \in J}\{\Phi_j\} > (\delta\alpha)^{\frac{1}{1-\sigma}} > \max_{m \in T \setminus J}\{\Phi_m\}. \tag{41}$$

We next turn to characterizing under what circumstances a middle income trap is more likely to arise. By the definition of Φ_j, we know that the necessary condition for trap at the jth technology is:

$$\frac{\left(\theta_{j+1}^{\frac{1}{\sigma}}\eta_{j+1} - \theta_j^{\frac{1}{\sigma}}\eta_j\right)^{\frac{1}{1-\sigma}}}{\theta_j^{\frac{1}{1-\sigma}}\theta_{j+1}^{\frac{1}{1-\sigma}}\left(\theta_{j+1}^{\frac{1}{\sigma}} - \theta_j^{\frac{1}{\sigma}}\right)(\eta_{j+1} - \eta_j)^{\frac{\sigma}{1-\sigma}}} > (\delta\alpha)^{\frac{1}{1-\sigma}} \tag{42}$$

Let $g_{\theta_j} = \frac{\theta_{j+1}}{\theta_j}$ and $g_{\eta_j} = \frac{\eta_{j+1}}{\eta_j}$ capture, respectively, the productivity gap and the barrier gap between the $j{+}1$th and the jth technologies. Then the above inequality reduces to

$$\frac{(\eta_j)^{1-\sigma}\left[\left(g_{\theta_j}\right)^{\frac{1}{\sigma}} g_{\eta_j} - 1\right]}{\theta_j \left(g_{\theta_j}\right)^{\frac{1}{\sigma}} \left(g_{\eta_j} - 1\right)^{\sigma}} > \delta\alpha$$

It is straightforward to show that the left-hand side of this inequality is strictly decreasing in θ_j and strictly increasing in η_j and g_{θ_j}.

Proposition 8. *Suppose that Assumption 2 holds. Then, a middle income trap of the* j*th technology for* $1 < j < M$ *is likely to arise if the productivity of the* j*th technology is not too large, the scale barrier of the* j*th technology is sufficiently high and the productivity gap between the* $j{+}1$*th and the* j*th technologies is sufficiently big.*

It is noted that how changes in the gap from η_j to η_{j+1} depends crucially on the sign of $\left[(1-\sigma)\left(g_{\eta_j}\left(g_{\theta_j}\right)^{\frac{1}{\sigma}} - 1\right) - \left(\left(g_{\theta_j}\right)^{\frac{1}{\sigma}} - 1\right)\right]$: for sufficiently small σ, a larger scale barrier gap between the $j{+}1$th and the jth technologies would raise the likelihood of a middle-income trap.

While we have established necessary and sufficient conditions for broadly defined middle income trap to arise, one may inquire whether our model may support a more restricted middle income trap, such as to satisfy the conditions outlined by Eichengreen, Park and Shin (2013). We will elaborate on this issue using Figure 5. First of all, our technology choice allows for a sustained flying geese paradigam that may even feature jumps from, for example, the ith technology to the jth technology ($j > i+1$). Continual technology upgrading in conjunction with some technology leap frogging (jumps) would ensure that the country under consideration experiences fast growth, thus satisfying the growth condition (say, at least 3.5% annually). However, when the conditions stated (41) hold at the jth technology, a country is stucked therein. When barriers to the next generations of technologies are high and the corresponding productivity gaps are big, there could be several generations of technologies not worth adopting (such as $B_{j''}$, $j < j'', j'$). As a result, the country may stay put at the jth technology for years before pulling out (to the j+1th technology), thus causing relatively low growth (say, at least 2% lower than the pre-trap era). Of course, given appropriate values of A_j and the country's factor endowments (k, h) prevailing, it is not difficult to satisfy the middle income condition (say, at least US\$10,000 in 2005 contant PPP prices). That is, our model may support more narrowly defined middle income trap.

Finally, we note that it is possible for a country to experience multiple traps at different points of time and that technology downgrading may arise when TFP falls and physical capital decumulates. We shall illustrate all such possibilities in the quantitative analysis to which we now turn.

5 Generalization and Applications

Before conducting quantitative analysis, we would like to note that adequate generalization is needed in order to apply the barebone theoretical model to the real world. In this section, we will discuss how to generalize the setup, followed by modifying the conditions for flying geese and middle income trap paradigms and then quantitative analysis.

5.1 Generalization

There are two strategies to apply the theoretical model to the real world by means of calibration. One is to use a measure of human/knowledge capital, say, years of schooling based human capital measure, and then compute the TFP whose mean is A. An alternative is to take the TFP index series directly from PWT9.0 (rtfpna, with $2011 = 1$). As long as we are not making cross-country compaison, using such an index is innocuous, so do we choose in our quantitative analysis. Specifically, let the time-varying TFP be $\tilde{A}_t$, so per worker output is $y_t = \tilde{A}_t k_{t-1}^{\beta}$. Comparing this to the per worker production function, $y_t = A h_{t-1}^{\alpha} k_{t-1}^{\beta}$, we learn that, in our model, $\tilde{A}_t = A h_{t-1}^{\alpha}$. Assuming that A is constant throughout the periods and that $\hat{A}_t := \tilde{A}_t / A$ matches the TFP index of $\tilde{A}_t$ in the data, we then obtain $\hat{A}_t = h_{t-1}^{\alpha}$, or equivalently, $h_{t-1} = \hat{A}_t^{1/\alpha}$.

Whereas it has been assumed that the population of labor force is equal to one in the theoretical analysis, the growth in labor force is observed in the actual data. Introducing it into the model, we can modify Eq. (38) as follows:

$$h_t = \begin{cases} \frac{\alpha\delta}{n_{t+1}} h_{t-1}^{\sigma} & \text{if } 0 \leq h_{t-1} < w_1 \\ \frac{\alpha\delta\theta_2}{n_{t+1}} (h_{t-1} - \eta_2)^{\sigma} & \text{if } w_1 \leq h_{t-1} < w_2 \\ \frac{\alpha\delta\theta_3}{n_{t+1}} (h_{t-1} - \eta_3)^{\sigma} & \text{if } w_2 \leq h_{t-1}, \end{cases} \tag{43}$$

where $n_{t+1} := L_t / L_{t+1}$. The derivation of Eq. (43) is demonstrated in the Appendix. Substituting $h_{t-1} = \hat{A}_t^{1/\alpha}$ into Eq. (43), we have a dynamic equation of $\hat{A}_t$ alone when the jth technology is chosen as in the following:

$$\hat{A}_{t+1}^{\frac{1}{\alpha}} = \frac{\alpha\delta\theta_j}{n_{t+1}} \left(\hat{A}_t^{\frac{1}{\alpha}} - \eta_j \right)^{\sigma}. \tag{44}$$

Moreover, denoting the capital-output ratio (under the time-to-build setup) as $\kappa_t :=$

$\frac{k_{t-1}}{y_t}$ and plugging this and Eq. (39) into $y_t = A_t h_{t-1}^{\alpha} k_{t-1}^{\beta}$ implies $k_t = \beta\delta A_t h_{t-1}^{\alpha} k_{t-1}^{\beta}$

$$\frac{k_t}{k_{t-1}} = \frac{\beta\delta}{\kappa_t}.$$

Thus, the speed at which capital growth slows down toward a potential middle income trap is entirely governed by the (time-varying) capital-output ratio.

Updating Eq. (44) by one period yields

$$\hat{A}_{t+2}^{\frac{1}{\alpha}} = \frac{\alpha\delta\theta_j}{n_{t+2}} \left(\hat{A}_{t+1}^{\frac{1}{\alpha}} - \eta_j\right)^{\sigma}. \tag{45}$$

Assuming the jth technology being prevailed from period t to $t+2$, we then take a straightforward manipulation to Eqs. (44) and (45) to derive

$$\eta_j = \frac{n_{t+2}^{1/\sigma} \hat{A}_{t+2}^{\frac{1}{\alpha\sigma}} \hat{A}_t^{\frac{1}{\alpha}} - n_{t+1}^{1/\sigma} \hat{A}_{t+1}^{\frac{1}{\alpha\sigma}} \hat{A}_{t+1}^{\frac{1}{\alpha}}}{n_{t+2}^{1/\sigma} \hat{A}_{t+2}^{\frac{1}{\alpha\sigma}} - n_{t+1}^{1/\sigma} \hat{A}_{t+1}^{\frac{1}{\alpha\sigma}}}, \tag{46}$$

which can then be plugged into Eq. (43) to obtain

$$\theta_j = \frac{n_{t+1} \hat{A}_{t+1}^{\frac{1}{\alpha}}}{\alpha\delta \left(\hat{A}_t^{\frac{1}{\alpha}} - \eta_j\right)^{\sigma}}. \tag{47}$$

5.2 Modified conditions for flying geese and middle income trap

If Assumption 2 is actually satisfied and there is no population growth of workers, the necessary condition for trap at the jth technology is given by Eq. (42). However, the calibrated pairs of θ and η do not necessarily satisfy Assumption 2. There are various patterns of the calibrated θ and η that lead a country to traps. Technically speaking, it is more straightforward to derive a condition under which a country experiences flying-geese growth (the flying geese condition) than to derive the trap condition. If the flying geese condition does not hold in switching technologies, it is highly likely that a country falls in a trap. In this case, we diagnose whether the country actually falls in a trap by taking into account historical episodes and other macroeconomic variables such as output growth, TFP growth, etc.

Suppose that a country switches technologies from technology j to technology $j+1$ at period $T+1$. In this case, the transitional dynamics of $\hat{A}_t^{1/\alpha}$ from T to $T+2$ is given by

$$\begin{cases} \hat{A}_{T+1}^{1/\alpha} = \frac{\alpha\delta\theta_j}{n_{T+1}}(\hat{A}_T^{1/\alpha} - \eta_j)^\sigma \\ \hat{A}_{T+2}^{1/\alpha} = \frac{\alpha\delta\theta_{j+1}}{n_{T+2}}(\hat{A}_{T+1}^{1/\alpha} - \eta_{j+1})^\sigma. \end{cases} \tag{48}$$

To investigate the flying-geese condition, we define functions such that $f_j(x) := (\alpha\delta\theta_j/n_{T+1})(x-\eta_j)^\sigma$ and $f_{j+1}(x) := (\alpha\delta\theta_{j+1}/n_{T+2})(x-\eta_{j+1})^\sigma$, which are the right-hand sides of (48). We also define z that denotes a potential intersection of the transition equations in (48) such that $f_j(z) = f_{j+1}(z)$ or equivalently

$$z := \frac{\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}}\eta_{j+1} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}}\eta_j}{\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}}}. \tag{49}$$

Furthermore, we define z_j and z_{j+1} such that $f_j'(z_j) = f_{j+1}'(z_{j+1}) = 1$ or equivalently

$$z_j := \eta_j + \left(\frac{\alpha\delta\sigma\theta_j}{n_{T+1}}\right)^{\frac{1}{1-\sigma}} \tag{50}$$

and

$$z_{j+1} := \eta_{j+1} + \left(\frac{\alpha\delta\sigma\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{1-\sigma}}. \tag{51}$$

The flying-geese condition is categorized into three patterns as illustrated in Figure 6. In the first case, whereas it holds that $\eta_{j+1} > \eta_j$ as in Assumption 2, it may not necessarily hold that $\theta_{j+1} > \theta_j$. In this case, the minimum requirement for the TFP to utilize the $j+1$th technology is grater than that of the jth technology, and the productivity parameter, θ_{j+1}, should be sufficiently large that the flying geese occurs. In the second (the third case), it holds (may hold) that $\eta_{j+1} < \eta_j$, which contradicts Assumption 2.

In such a case, although the minimum requirement for the TFP to utilize the $j+1$th technology is smaller than that of the jth technology, the productivity parameter, θ_{j+1}, should be sufficiently large that the flying geese occurs as in the first case.

First case

It follows from Figure 6 that the parameter condition for the first case is given by $\eta_j < \eta_{j+1} < z$ and $f_j(z) > z$. From Eq. (49), these inequalities can be transformed into

$$\begin{cases} \eta_j < \eta_{j+1} \\ \frac{\theta_{j+1}}{n_{T+2}} > \frac{\theta_j}{n_{T+1}} \\ \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} \eta_{j+1} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}} \eta_j < \alpha\delta \left(\frac{\theta_j}{n_{T+1}}\right)\left(\frac{\theta_{j+1}}{n_{T+2}}\right)(\eta_{j+1}-\eta_j)^{\sigma}\left[\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}. \end{cases} \tag{52}$$

Second case

As illustrated in Figure 6, the second flying-geese condition is given by $\eta_{j+1} < \eta_j < z$, $f_j(z) < z$, and $f_{j+1}(z_{j+1}) > z_{j+1}$. Eqs. (49)-(51) rewrite these inequalities as follows:

$$\begin{cases} \eta_{j+1} < \eta_j \\ \frac{\theta_{j+1}}{n_{T+2}} < \frac{\theta_j}{n_{T+1}} \\ \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}} \eta_j - \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} \eta_{j+1} > \alpha\delta \left(\frac{\theta_j}{n_{T+1}}\right)\left(\frac{\theta_{j+1}}{n_{T+2}}\right)(\eta_j-\eta_{j+1})^{\sigma}\left[\left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \\ \eta_{j+1} < \sigma^{\frac{\sigma}{1-\sigma}}(1-\sigma)\left(\frac{\alpha\delta\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{1-\sigma}}. \end{cases} \tag{53}$$

Third case

We obtain the necessary condition for the third case such that (i) $\eta_{j+1} < \eta_j$ and $z < \eta_j$, which is consistent with $f_{j+1}(x)$'s solid line in Figure 6, or (ii) $\eta_{j+1} > \eta_j$ and $z > \eta_j$, which is consistent with $f_{j+1}(x)$'s dotted line, ignoring the case in which $\eta_{j+1} = \eta_j$. The conditions (i) and (ii) are unified as $(\eta_{j+1} - \eta_j)(z - \eta_{j+1}) > 0$. In addition to this condition, $z < z_j$, and $z_{j+1} < f(z_{j+1})$ are necessary for the third case. Eqs. (49)-(51)

rewrite these inequalities as follows:

$$\begin{cases} \frac{\theta_{j+1}}{n_{T+2}} > \frac{\theta_j}{n_{T+1}} \\ \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} (\eta_{j+1} - \eta_j) < \left(\frac{\alpha\delta\sigma\theta_j}{n_{T+1}}\right)^{\frac{1}{1-\sigma}} \left[\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}}\right] \\ \eta_{j+1} < \sigma^{\frac{\sigma}{1-\sigma}}(1-\sigma)\left(\frac{\alpha\delta\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{1-\sigma}}. \end{cases} \tag{54}$$

5.2.1 Summary

Thus, it is said that a country experiences the flying geese if and only if inequalities (52), (53), or (54) hold. If inequalities (52), (53), and (54) all fail to hold and if both human capital technology fails to advance (θ not rising) and human capital growth is stalled (human capital not growing or falling), it is said that a country falls in a middle income trap. Again, the terminology of middle income is used more generally for any level of development before reaching a perpetually growing balanced growth path (that is, for all $m < M$).

5.3 Quantitative analysis

It is standard in growth models to set $\delta = 0.95$ and $\alpha = \beta = \gamma = 1/3$ (Mankiw, Romer and Weil 1992). The value of σ is, however, unable to pin down by the model even if one measures human/knowledge capital directly because of technology updating over time. Our strategy is therefore to set $\sigma = 0.5$ to begin with and then conduct robustness check with $\sigma = 0.4, 0.6$. For a given value of σ, we can apply the TFP series from PWT9.0 to calibrate η_j and θ_j using Eqs. (46) and (47), respectively. We note that, in computing η_j, one must set a minimum at 0. Moreover, when the computed value of η_j such that $\hat{A}_t^{\frac{1}{\alpha}} - \eta_j \leq 0$, we set $\theta_j = \theta_{j-1}$, i.e., no upgrading.

Two remarks are in order. First, although in calibrating the values of $\{\theta_j, \eta_j\}$, we have incorporated the information from the TFP and the formation of human capital, we have not accounted from any sources of slowdown due to capital or trade barriers. As a result, we, on the one hand, may miss some capital or trade-led traps and, on the other, may find that in some of our human capital-based traps, output may not grow slowly. Second, in practice, outputs flucturate, partly driven by short-run shocks which are unrelated to the consideration of middle income traps. Thus, some smoothing strategies must be adopted. To do so, we first take three-year moving average of each of time series to remove uninteresting short-run movements. We then compute the values of $\{\theta_j, \eta_j\}$ in five-year intervals to further eliminate variations from short-run matters.

We now turn to country selection, based on four critera. First, we consider four groups based on development patterns: (i) adanced countries, (ii) fast growing economies, (iii) emerging economies with decent development speed and (iv) development laggards with frequent growth slowdowns despite earlier development. Second, countries chosen are with long time series starting no later than early 1960s (starting year varies a bit based on data availability in PWT9.0). Third, all countries are qualified as middle-low, middle-high or high income countries by mid-1980s (the middle year of our sample). Finally, we try to balance between continents, namely, Asia, Europe, North America and South America. The representative 15 countries from the four groups are as follows (see the Appendix for a summary of key average growth rates of each country):

(i) adanced countries: the U.S., the U.K., Germany, and Japan, with relatively stable growth ranging from 1.8% per year (the U.S.) to 4.4% (Japan);

(ii) fast growing economies: Hong Kong, S. Korea, Singapore, Taiwan, the so-called Asian Tigers, all growing at more than double of the speed of the U.S., ranging from 4% (Hong Kong) to 5.4% (Singapore);

(iii) emerging growing economies: Brazil, China, Greece, and Malaysia, all growing faster than 3.2% but slower than fast growing economies;

(iv) development laggards: Argentina, Mexico, and Philippines, featuring chopped growth paths with coefficient of variation all exceeding one and at slower speed than emerging growing economies.

As also observed by Wang, Wong and Yip (2018), most countries have decent physical capital growth but sluggish year-of-schooling-based human capital growth. Moreover, while advanced and fast growing countries have experienced decent TFP growth no lower than 1% annually, the TFP growth in the three development laggards have been mediocre at rates below 0.3%.

We now turn to the calibrated values of $\{\theta_j, \eta_j\}$ in each of the five-year intervals (for brevity, called episodes) for all of the 15 economies. To understand how countries perform in human capital technology, we find it best to show graphically the rates of change in human capital technology efficacy θ and the rates of change in human capital accumulation barriers η (see Figures 7-10, for each of the four groups of countries, respectively). We observe that, in advanced and fast growing countries, the rates of change in human capital technology efficacy are mostly positive over the episodes, whereas large increases in the barriers to human capital accumulation are only occasional. In development laggards, however, we can see that they have more frequently experience downgrading in human capital technology efficacy.

Our primary task is to identify potential traps and prolong periods of flying geese in the first three categories. For the cases of traps, we will also pin down the drivers of traps, possibly due to upgrading technology slowdown, technology barriers or TFP slowdown. This enables us to compare our findings with those in Eichengreen, Park and Shin (2013) where traps are identified by empirical structural breaks and with those in Wang, Wong and Yip (2018) where traps are caused by factor endowment reversal under mismatch in technology assimilation. Of course, due to different samples of countries, such comparison would be restricted to common samples only.

Table 1 summarizes our quantitative results based on $\sigma = 0.5$, where $\nearrow$ denotes flying, $\searrow$ middle income trap, and $\leftrightarrow$ inconclusive for not flying but not necessarily trapped.

It is clearly seen that most of the advanced countries and fast growing economies

Table 1. Flying or trapped

Patterns	late 50s	early 60s	late 60s	early 70s	late 70s	early 80s	late 80s	early 90s	late 90s	early 00s	late 00s
(i) Advaned Countries											
Germany	↗	↗	↗	↔	↗	↗	↔	↔	↗	↗	↘
Japan	↗	↗	↗	↔	↗	↘	↔	↗	↔	↗	↗
U.K.	↔	↘	↔	↔	↔	↗	↘	↔	↗	↘	↔
U.S.	↗	↗	↘	↘	↔	↗	↘	↗	↘	↗	↗
(ii) Fast Growing Economies											
Hong Kong			↔	↘	↗	↗	↔	↗	↘	↔	↘
S. Korea		↘	↗	↗	↗	↔	↗	↘	↔	↗	↗
Singapore			↗	↔	↗	↔	↗	↔	↗	↗	↘
Taiwan	↗	↗	↗	↘	↘	↗	↔	↗	↔	↗	↗
(iii) Emerging Growing Economies											
Brazil	↗	↘	↗	↗	↗	↘	↔	↔	↗	↘	↗
China		↘	↘	↘	↗	↘	↗	↔	↔	↔	↗
Greece	↗	↗	↗	↔	↗	↔	↔	↔	↗	↘	↔
Malaysia		↔	↗	↘	↔	↗	↘	↗	↘	↔	↔
(iv) Development Laggards											
Argentina	↘	↘	↔	↘	↗	↘	↘	↗	↔	↗	↘
Mexico	↘	↗	↘	↔	↘	↘	↘	↔	↘	↘	↔
Philippines	↔	↔	↔	↘	↔	↘	↘	↘	↔	↘	↘

Notes. The quantitative results are based on $\sigma = 0.5$. ↗ denotes flying, ↘ middle income trap, and ↔ for not flying but not necessarily trapped.

have experienced prolonged periods of flying geese paradigm with lower frequencies in traps. Of course, there are also some inconclusive cases which show not flying but still possibly trapped. In these two groups of countries, Eichengreen, Park and Shin (2013) identify several middle income traps in the cases of Hong Kong in 1993, Japan in 1974, Korea in 1989, Singapore in 1980, Taiwan in 1995, and U.K. in 1988. Wang, Wong and Yip (2018) find traps in the cases of Hong Kong in 1984 and Taiwan in 1999. Different from their works, we do not restrict the sample to fall into the World Bank ranges of middle income group. By comparison, we learn that:

- Hong Kong: flying in early 1980s and early 1990s (inconsistent with Eichengreen et al. and Wang et al.), but trapped in early 1970s during the first oil crisis, in late 1990s around the Asian financial crises, as well as in late 2000s during the Great Recession (which is beyond the sample period used by Eichengreen et al.);
- Japan: possibly trapped in early 1970s (consistent with Eichengreen et al.), but most certainly trapped in early 1980s during the second oil crisis;

- Korea: trapped in early 1990s (not far from that identified by Eichengreen et al.);
- Singapore: possibly trapped in early 1980s (consistent with Eichengreen et al.), but most certainly trapped in late 2000s (again, beyond the sample period used by Eichengreen et al.);
- Taiwan: possibly trapped in the second half of 1990s (roughly consistent with Eichengreen et al. and Wang et al.);
- U.K.: trapped in early 1960s and late 1980s (the latter consistent with Eichengreen et al.).

Turning to the third group where Greece is the only sample in common, both Eichengreen et al. and Wang et al. identify a single trap in 1972, while such a trap is possibly in our case. Relative to the first two groups, more traps are identified in our paper. For example, we earlier traps in China over the period from early 1960s to mid-1970s when its human capital upgrading was stalled by the Great Leap Forward policy and the Cultural Revolution.

Finally, moving to the last group with Mexico as the only common sample, a trap in 1981 is identified by Eichengreen et al., which is consistent with one of the many traps found in our paper. Similar to Mexico, we also find several traps in Argentina and Philippines. In all three countries, flying geese paradigm has been rare and mostly short lived with a duration of only five years – in the case of Philippines, we find no obvious flying geese episode over half a century.

Our robustness check for the cases of $\sigma = 0.4$ or 0.6 is reported in the Appendix. As it can been seen, our main findings are quite robust. In some countries, there may be some switches, usually between flying and inconclusive or between trap and inconclusive, which are largely acceptable. Only in six occasions, out of a total of 155 episodes, changing between flying and trapped, namely, UK early 00s, US early 60s, Brazil early 70s, Greece early 00s, Argentina late 70s, and Mexico late 50s. While we would view these six cases as inconclusive, our general findings are unchanged.

One may then inquire what the . In particular, we focus on (i) large technology downgrading with θ falling by more than 5%, (ii) large increase in barriers with η rising by more than 5%, and (iii) TFP slowdowns with negative TFP growth. We summarize our findings in Table 2.

Overall, the results suggest that large drops in human capital technology efficacy are overwhelmingly the primary force for a country to fall into a middle income trap. Large increases in barriers to human capital accumulation are also important, playing a bigger role in advanced and growing countries. Only in development laggards, TFP slowdowns are more important than increased severity in human capital accumulation barriers. By looking into more details, we gain further insights below:

Table 2. Underlying drivers to a middle income trap

Drivers	large technology downgrading	large increase in barriers	TFP slowdowns
Advaned	Germany: late 00s Japan: early 80s U.K.: early 60s/late 80s/early 00 U.S.: late 60s/late 80s/late 90s	Germany: late 00s Japan: early 80s U.S.: late 60s	
Fast Growing	Hong Kong: early 70s/late 90s/late 00s S. Korea: early 60s Singapore: late 00s Taiwan: late 70s	Hong Kong: early 70s/late 90s late 00s S. Korea: early 90s Singapore: late 00s Taiwan: early 70s/late 70s	Hong Kong: late 90s S. Korea: early 60s Singapore: late 00s
Emerging	Brazil: early 60s/early 80s/early 00s China: early 60s/late 60s/early 70s early 80s Greece: early 2000s Malaysia: early 70s/late 80s/late 90s	Brazil: early 60s, early 00s China: late 60s, early 80s Malaysia: early 70s, late 90s	Brazil: early 80s/early 00s China: early 60s Malaysia: late 90s
Laggards	Argentina: early 80s/late 80s/late 00s Mexico: late 60s/late 70s/early 80s late 80s/late 90s/early 00s Philippines: early 70s/early 80s/late 80s early 90s/early 00s/ late 00s	Argentina: late 50s/early 70s late 00s Mexico: late 90s Philippines: early 00s	Argentina: early 70s/early 80s late 80s Mexico: early 80s/late 80s early 00s Philippines: early 80s/early 90s

Notes. We focus on (i) large technology downgrading with θ falling by more than 5%, (ii) large increase in barriers with η rising by more than 5%, and (iii) TFP slowdowns with negative TFP growth.

- In advanced countries, slowdown in TFP plays no role in the occasional traps identified.

- In fast growing economies, increased severity in human capital accumulation barriers is as important as large drops in human capital technology efficacy for their explaining their occasional traps.

- In three of the four emerging growing economies (except Greece), all three factors play a nonnegligible role; particularly in the case of Brazil, their roles for explaining different episodes of the traps are all comparable.

- In all three development laggards, all three factors also play a nonnegligible. While their roles are comparable in the case of Argentina, increased severity in human capital accumulation barriers plays a much lesser role in the cases of Mexico and Philippines.

6 Concluding Remarks

We have constructed a simple growth model with endogenous technology choice in human/knowledge capital accumulation in which we are able to establish a rich array of equilibrium development paradigms, including poverty trap, middle income trap and flying geese growth. We have identified the productivity of the prevailing technology, the productivity jump of technology upgrading and the hike in technology scale barrier as the key drivers for different paradigms to arise in equilibrium. Different combination of these factors in conjunction of the TFP help understand different development patterns facing different countries.

Along these lines, an interesting avenue of future research is to introduce limitation to knowledge formation. This would allow for growing over cycles rooted on human capital. As such, it would complement the R&D based theory of growth and cycles pioneered by Matsuyama (1999) and Jovanovic (2009). Another interesting line of extension is to incoporate trade into the current framework to see how the interations between trade and globale talent flows may result in different development pattens, particularly for those economies with high trade dependence. Finally, it is also interesting to consider multiple dimensions of technology choice with physical and human capital upgrading and barriers. In so doing, one may separate different sources of traps into physical capital and human capital based. Of course, to accomplish any of these would require further simplification of the basic structure and more restrictive assumptions. Yet they may be potentially rewarding lines for future research.

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Appendix

In the Appendix, we provide proofs of various Lemmas and Propositions.

Proof of Lemma 1

To prove the first claim, define $\Psi_1(h) := B_1(h) - B_2(h)$ for $h \geq \eta_2$. Then, we have

$$\Psi_1(h) = h^\sigma - \theta_2(h - \eta_2)^\sigma \tag{A.1}$$

and

$$\Psi'(h) = \sigma[h^{\sigma-1} - \theta_2(h - \eta_2)^{\sigma-1}]. \tag{A.2}$$

Since $\Psi_1'(h) < \sigma[(h-\eta_2)^{\sigma-1} - \theta_2(h-\eta_2)^{\sigma-1}] < 0$ under Assumption 2, $\Psi_1(h)$ is a decreasing function for $h \geq \eta_2$. Additionally, it follows that $\Psi_1(\eta_2) > 0$ and $\lim_{h\to\infty} \Psi_1(h) = \lim_{h\to\infty} h^\sigma(1 - \theta_2(1 - \eta_2/h)^\sigma) = -\infty$. Therefore, the first claim of Lemma 1 holds. To prove the second claim, define $\Psi_2(h) := B_2(h) - B_3(h)$ for $h \geq \eta_3$. Then, we have

$$\Psi_2(h) = \theta_2(h - \eta_2)^\sigma - \theta_3(h - \eta_3)^\sigma \tag{A.3}$$

and

$$\Psi_2'(h) = \theta_2\sigma(h - \eta_2)^{\sigma-1} - \theta_3\sigma(h - \eta_3)^{\sigma-1}. \tag{A.4}$$

Since $\Psi_2'(h) < \sigma[\theta_2(h-\eta_3)^{\sigma-1} - \theta_3(h-\eta_3)^{\sigma-1}] < 0$ under Assumption 2, $\Psi_2(h)$ is a decreasing function for $h \geq \eta_3$. Additionally, it follows that $\Psi_2(\eta_3) > 0$ and $\lim_{h\to\infty} \Psi_2(h) = \lim_{h\to\infty} \theta_2(h - \theta_2)^\sigma[1 - (\theta_3/\theta_2)[(h - \eta_3)/(h - \eta_2)]^\sigma] = -\infty$.Therefore, the second claim of Lemma 1 holds. □

Proof of Proposition 2

From Lemma 1, it follows that $B_3(h_{t-1}) > B_2(h_{t-1})$ if $h_{t-1} > w_2$. Thus, if $h_{t-1} > w_2$, the third technology is preferred to the second technology. From Lemma 1, it follows that $B_2(h_{t-1}) > B_3(h_{t-1})$ if $\eta_3 \leq h_{t-1} < w_2$. Moreover, if $\eta_2 \leq h_{t-1} < \eta_3$, the third technology is not applicable. Thus, if $\eta_2 \leq h_{t-1} < w_2$, the second technology is preferred to the third technology. Likewise, from Lemma 1, it follows that $B_2(h_{t-1}) > B_1(h_{t-1})$ if $h_{t-1} > w_1$. Thus, if $h_{t-1} > w_1$, the second technology is preferred to the first technology. From Lemma 1, it follows that $B_1(h_{t-1}) > B_2(h_{t-1})$ if $\eta_2 \leq h_{t-1} < w_1$. Moreover, if $0 \leq h_{t-1} < \eta_2$, the second technology is not applicable. Thus, if $0 \leq h_{t-1} < w_1$, the first technology is preferred to the second technology. Then, we have a desired conclusion. □

Proof of Proposition 3

From the convexity of $\Pi_2(h)$ and since $k^*_{2,s}$ is obtained from Eq. (24), it suffices to show the following three claims: *Claim 1*: Eq. (25) with $j = 2$ has two distinct real number solutions, *Claim 2*: $w_1 < \hat{h}_2 < w_2$, and *Claim 3*: $\Pi_2(w_1) > 0$ and $\Pi_2(w_2) > 0$.

Claim 1

The second inequality of (30) is equivalent to inequality (26) with $j = 2$. From Lemma 2, Claim 1 is proven.

Claim 2

Under Assumption 2, it follows from inequalities (30) that $\theta_2 > 1/\sigma^\sigma$, or equivalently,

$$\frac{\theta_2^{\frac{1}{\sigma}}}{\theta_2^{\frac{1}{\sigma}} - 1} < \frac{1}{1-\sigma}. \tag{B.1}$$

From inequality (B.1) and the second inequality of (30), we obtain

$$w_1 = \frac{\theta_2^{\frac{1}{\sigma}} \eta_2}{\theta_2^{\frac{1}{\sigma}} - 1} < \sigma^{\frac{\sigma}{1-\sigma}} (\delta \alpha \theta_2)^{\frac{1}{1-\sigma}} = \hat{h}_2. \tag{B.2}$$

Under Assumption 2, inequality (31) yields

$$\hat{h}_2 = \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < (\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3 < \frac{\theta_3^{\frac{1}{\sigma}}\eta_3 - \theta_2^{\frac{1}{\sigma}}\eta_2}{\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}}} = w_2. \tag{B.3}$$

Claim 2 is proven by inequalities (B.2) and (B.3).

Claim 3

From Eq. (19), it holds that $w_1 = \theta_2^{\frac{1}{\sigma}}(w_1 - \eta_2)$. Therefore, it follows that

$$\Pi_2(w_1) = w_1(w_1^{\frac{1-\sigma}{\sigma}} - (\delta\alpha)^{\frac{1}{\sigma}}). \tag{B.4}$$

Since $\eta_2 < w_1$, from Eq. (B.4) and the first inequality of (30), we obtain

$$\Pi_2(w_1) = w_1(w_1^{\frac{1-\sigma}{\sigma}} - (\delta\alpha)^{\frac{1}{\sigma}}) > w_1(\eta_2^{\frac{1-\sigma}{\sigma}} - (\delta\alpha)^{\frac{1}{\sigma}}) > 0. \tag{B.5}$$

From Claim 1, it follows that

$$\Pi_2(\hat{h}_2) < 0. \tag{B.6}$$

From inequality (31), we have $\hat{h}_2 < \eta_3$ and

$$\Pi_2(\eta_3) := \eta_3(\eta_3^{\frac{1-\sigma}{\sigma}} - (\delta\alpha\theta_2)^{\frac{1}{\sigma}}) + (\delta\alpha\theta_2)^{\frac{1}{\sigma}}\eta_2 > 0. \tag{B.7}$$

From inequalities (B.6), (B.7), and $\eta_3 < w_2$ with the convexity of $\Pi_2(h)$, it holds that

$$\Pi_2(w_2) > 0. \tag{B.8}$$

Claim 3 is proven by inequalities (B.5) and (B.8). $\square$

Proof of Proposition 4

From the convexity of $\Pi_3(h)$ snd since $k^*_{3,s}$ is obtained from Eq. (24), it suffices show the following three claims: *Claim 1*: Eq. (25) with $j = 3$ has two distinct real number solutions, *Claim 2*: $w_2 < \hat{h}_3$, and *Claim 3*: $\Pi_3(w_2) > 0$.

Claim 1

The second inequality of (32) is equivalent to inequality (26) with $j = 3$. From Lemma 2, Claim 1 is proven.

Claim 2

Under Assumption 2, it follows from inequalities (32) that $\theta_2^{\frac{1}{\sigma}}/\theta_3^{\frac{1}{\sigma}} < \sigma$, or equivalently,

$$\frac{\theta_3^{\frac{1}{\sigma}}}{\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}}} < \frac{1}{1-\sigma}. \tag{C.1}$$

From inequality (C.1) and the second inequality of (32), we obtain

$$w_2 = \frac{\theta_3^{\frac{1}{\sigma}}\eta_3 - \theta_2^{\frac{1}{\sigma}}\eta_2}{\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}}} < \frac{\theta_3^{\frac{1}{\sigma}}\eta_3}{\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}}} < \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_3)^{\frac{1}{1-\sigma}} = \hat{h}_3, \tag{C.2}$$

which is Claim 2.

Claim 3

From Eq. (20), it holds that $\theta_2^{\frac{1}{\sigma}}(w_2 - \eta_2) = \theta_3^{\frac{1}{\sigma}}(w_2 - \eta_3)$. Therefore, it follows that

$$\Pi_3(w_2) = w_2(w_2^{\frac{1-\sigma}{\sigma}} - (\delta\alpha\theta_2)^{\frac{1}{\sigma}}) + \eta_2(\delta\alpha\theta_2)^{\frac{1}{\sigma}}. \tag{C.3}$$

Since $\eta_3 < w_2$, from Eq. (C.3) and the first inequality of (32), we obtain

$$\Pi_3(w_2) = w_2(w_2^{\frac{1-\sigma}{\sigma}} - (\delta\alpha\theta_2)^{\frac{1}{\sigma}}) + \eta_2(\delta\alpha\theta_2)^{\frac{1}{\sigma}} > w_2(\eta_3^{\frac{1-\sigma}{\sigma}} - (\delta\alpha\theta_2)^{\frac{1}{\sigma}}) + \eta_2(\delta\alpha\theta_2)^{\frac{1}{\sigma}} > 0, \tag{C.5}$$

which is Claim 3. □

Proof of Lemma 3

Eq. (23) rewrites $\alpha\delta B_j'(h_{j,s}^*)$ as

$$\alpha\delta B_j'(h_{j,s}^*) = \frac{\sigma(\alpha\delta\theta_j)^{\frac{1}{\sigma}}}{(h_{j,s}^*)^{\frac{1-\sigma}{\sigma}}}. \tag{D.1}$$

Since $h_{j,1}^* < \hat{h}_j = \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_j)^{\frac{1}{1-\sigma}} < h_{j,2}^*$, the use of Eq. (D.1) yields

$$\alpha\delta B_j'(h_{j,1}^*) > \frac{\sigma(\alpha\delta\theta_j)^{\frac{1}{\sigma}}}{(\hat{h}_j)^{\frac{1-\sigma}{\sigma}}} = 1 \tag{D.2}$$

and

$$0 < \alpha\delta B_j'(h_{j,2}^*) < \frac{\sigma(\alpha\delta\theta_j)^{\frac{1}{\sigma}}}{(\hat{h}_j)^{\frac{1-\sigma}{\sigma}}} = 1 \tag{D.3}$$

Inequalities (D.2) and (D.3) are desired conclusions. □

Proof of Proposition 6

From Eq. (23) and the definitions of w_1 and w_2, it suffices to show that $w_i < \alpha\delta\theta_{i+1}(w_i - \eta_{i+1})^\sigma$ for $i = 1, 2$. $w_i < \alpha\delta\theta_{i+1}(w_i - \eta_{i+1})^\sigma$ is equivalent to $(\alpha\delta)^{\frac{1}{1-\sigma}} > \Phi_i$. From the last inequality, we obtain the desired conclusion. □

Derivation of Eq. (43)

When the growth in labor force is introduced into the model, Eqs. (4) and (5) are changed into

$$k_\tau = g_0(i_\tau^k)/n_{\tau+1} \tag{F.1}$$

and

$$h_\tau = \max_{m=1,2,\ldots,M}\{g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)\}/n_{\tau+1}, \tag{F.2}$$

where $n_{\tau+1} = L_{\tau+1}/L_\tau$ whereas Eq. (3) remains the same. Then, the first-order conditions are obtained as follows:

$$\lambda_t = \frac{1}{c_t}, \tag{F.3}$$

$$\lambda_t = \left(\frac{\delta\alpha b(\bar{h}_{t-1}, \bar{y}_t) y_{t+1}}{n_{t+1} h_t} \right) \lambda_{t+1}, \tag{F.4}$$

$$\lambda_t = \left(\frac{\delta\beta y_{t+1}}{n_{t+1} k_t} \right) \lambda_{t+1}, \tag{F.5}$$

$$n_{t+1}\lambda_t = p_t^k = b(\bar{h}_{t-1}, \bar{y}_t) p_t^h. \tag{F.6}$$

The same manipulations as those in section 3 yield exactly the same equations with respect to $q_t^k = p_t^k k_t$ and $q_t^h = p_t^h h_t$ as Eqs. (15) and (16), respectively. Additionally, with respect to h_t and k_t, it follows that

$$h_t = \frac{\alpha \max_m \{B_m(h_{t-1})\}}{n_{t+1}} \left[\frac{1}{1-\gamma} - \frac{\beta\delta}{1-\gamma} \left(\frac{1}{q_{t-1}^k} \right) \right], \tag{F.7}$$

and

$$k_t = \frac{\beta A h_{t-1}^{\alpha} k_{t-1}^{\beta}}{n_{t+1}} \left[\frac{1}{1-\gamma} - \frac{\beta\delta}{1-\gamma} \left(\frac{1}{q_{t-1}^k} \right) \right]. \tag{F.8}$$

Again, from the same manipulations as those in section 4, we obtain

$$h_t = \begin{cases} \frac{\alpha\delta}{n_{t+1}} h_{t-1}^{\sigma} & \text{if } 0 \leq h_{t-1} < w_1 \\ \frac{\alpha\delta\theta_2}{n_{t+1}} (h_{t-1} - \eta_2)^{\sigma} & \text{if } w_1 \leq h_{t-1} < w_2 \\ \frac{\alpha\delta\theta_3}{n_{t+1}} (h_{t-1} - \eta_3)^{\sigma} & \text{if } w_2 \leq h_{t-1}, \end{cases} \tag{F.9}$$

which is equivalent to Eq. (43).

6.1 Data summary

The average growth rates of output (y), physical capital (k), human capital (h) and TFP ($\tilde{A}$) of each country are summarized below.

Table A.1. Average Growth Rates

Average Growth (%)	y	k	h	$\tilde{A}$
(i) Advaned Countries				
Germany	3.5	3.5	0.2	2.0
Japan	4.4	8.2	-0.2	1.1
U.K.	2.1	2.0	0.2	1.0
U.S.	1.8	1.8	-0.7	1.0
(ii) Fast Growing Economics				
Hong Kong	4.0	4.1	-1.0	1.6
S. Korea	5.1	6.2	-1.2	1.6
Singapore	5.4	3.7	-2.1	1.2
Taiwan	5.1	5.6	-0.9	2.2
(iii) Emerging Growing Economies				
Brazil	3.2	1.4	-1.6	1.1
China	3.7	5.9	-0.9	0.9
Greece	3.4	3.3	0.4	0.8
Malaysia	3.6	3.5	-1.6	1.0
(iv) Development Laggards				
Argentina	3.2	1.5	-0.9	0.2
Mexico	1.4	1.4	-2.1	0.3
Philippines	2.2	1.7	-1.6	0.2

6.2 Robustness check

To check the robustness of our findings, we conduct quantitative analysis under $\sigma = 0.4, 0.6$. We find the flying geese episodes and middle income traps identified in Table 1 are mostly robust. In the cases of Japan, Hong Kong and Philippines, all results are unchanged. In other cases, we list all the 20 changes out of 155 episodes as follows:

- Germany: early 90s, change from $\leftrightarrow$ to $\nearrow$ for $\sigma = 0.4, 0.6$
- UK: early 00s $\searrow$ to $\nearrow$ for $\sigma = 0.4$; early 90s $\leftrightarrow$ to $\nearrow$ for $\sigma = 0.6$
- US: early 60s $\nearrow$ to $\searrow$ and early 80s $\nearrow$ to $\leftrightarrow$ for $\sigma = 0.4$
- S. Korea: late 60s/late 70s/late 80s all from $\nearrow$ to $\leftrightarrow$ for $\sigma = 0.4$
- Singapore: early 80s $\leftrightarrow$ to $\nearrow$ for $\sigma = 0.6$
- Taiwan: late 80s $\leftrightarrow$ to $\nearrow$ for $\sigma = 0.4$

- Brazil: early 70s $\nearrow$ to $\searrow$ for $\sigma = 0.6$
- China: late 00s $\nearrow$ to $\leftrightarrow$ for $\sigma = 0.4$
- Greece: early 00s $\searrow$ to $\nearrow$ for $\sigma = 0.4$; early 60s $\nearrow$ to $\leftrightarrow$ for $\sigma = 0.6$
- Malaysia: early 60s $\leftrightarrow$ to $\nearrow$ for $\sigma = 0.4$ and $\leftrightarrow$ to $\searrow$ for $\sigma = 0.6$
- Argentina: early 70s $\searrow$ to $\leftrightarrow$ for $\sigma = 0.4$; early 60s $\leftrightarrow$ to $\nearrow$ and late 70s $\nearrow$ to $\searrow$ for $\sigma = 0.6$
- Mexico: late 50s $\searrow$ to $\nearrow$ for $\sigma = 0.4$; late 00s $\leftrightarrow$ to $\nearrow$ for $\sigma = 0.6$

Notably, 14 of them are switches between inclusive and flying/trapped, which are acceptable. There are only six occasions changing between flying and trapped: UK early 00s, US early 60s, Brazil early 70s, Greece early 00s, Argentina late 70s, and Mexico late 50s. We view these six cases as inconclusive.

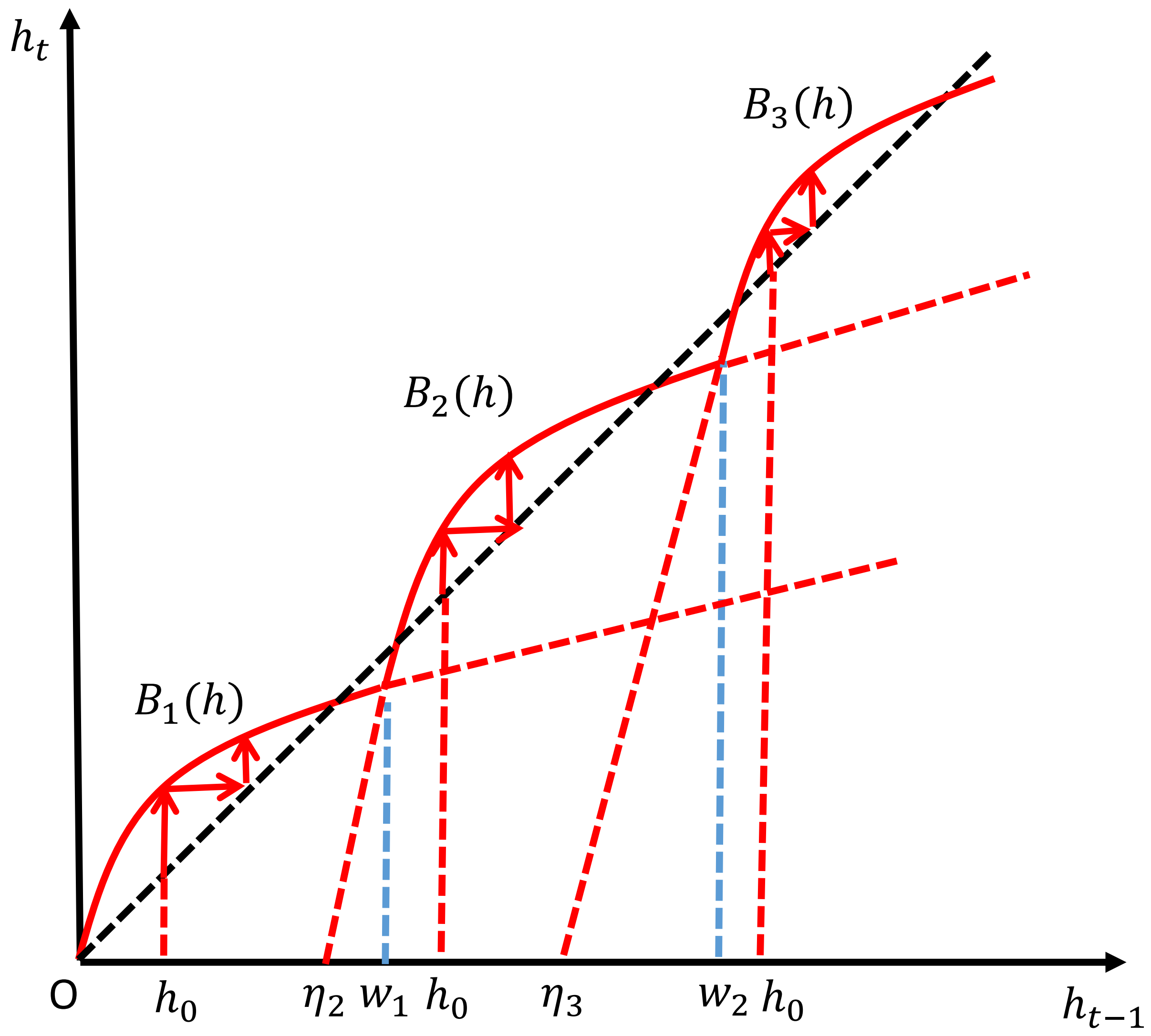


Figure 1. Phase diagram of h_t

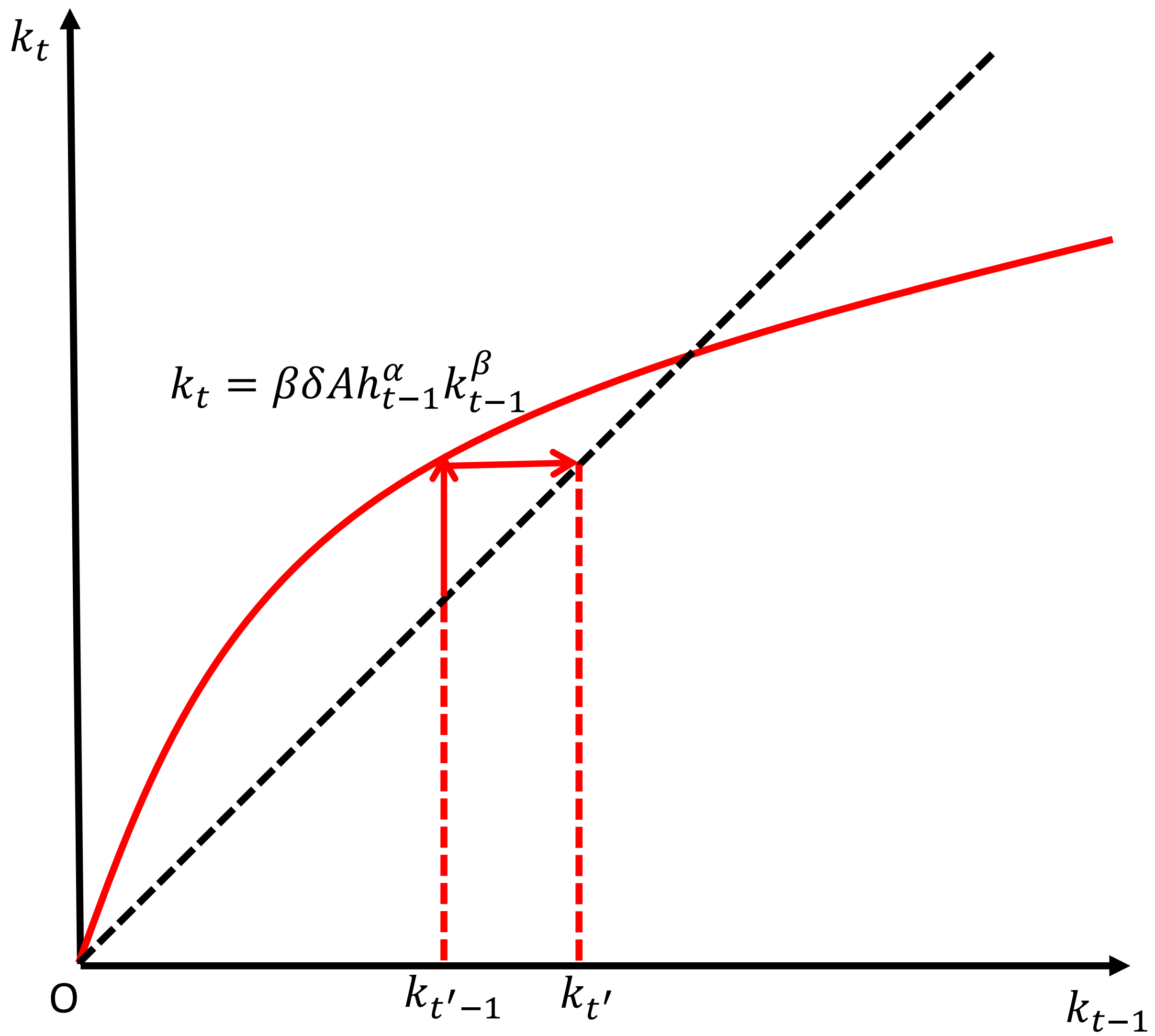


Figure 2. Conditional phase diagram of k_t given h_{t-1}

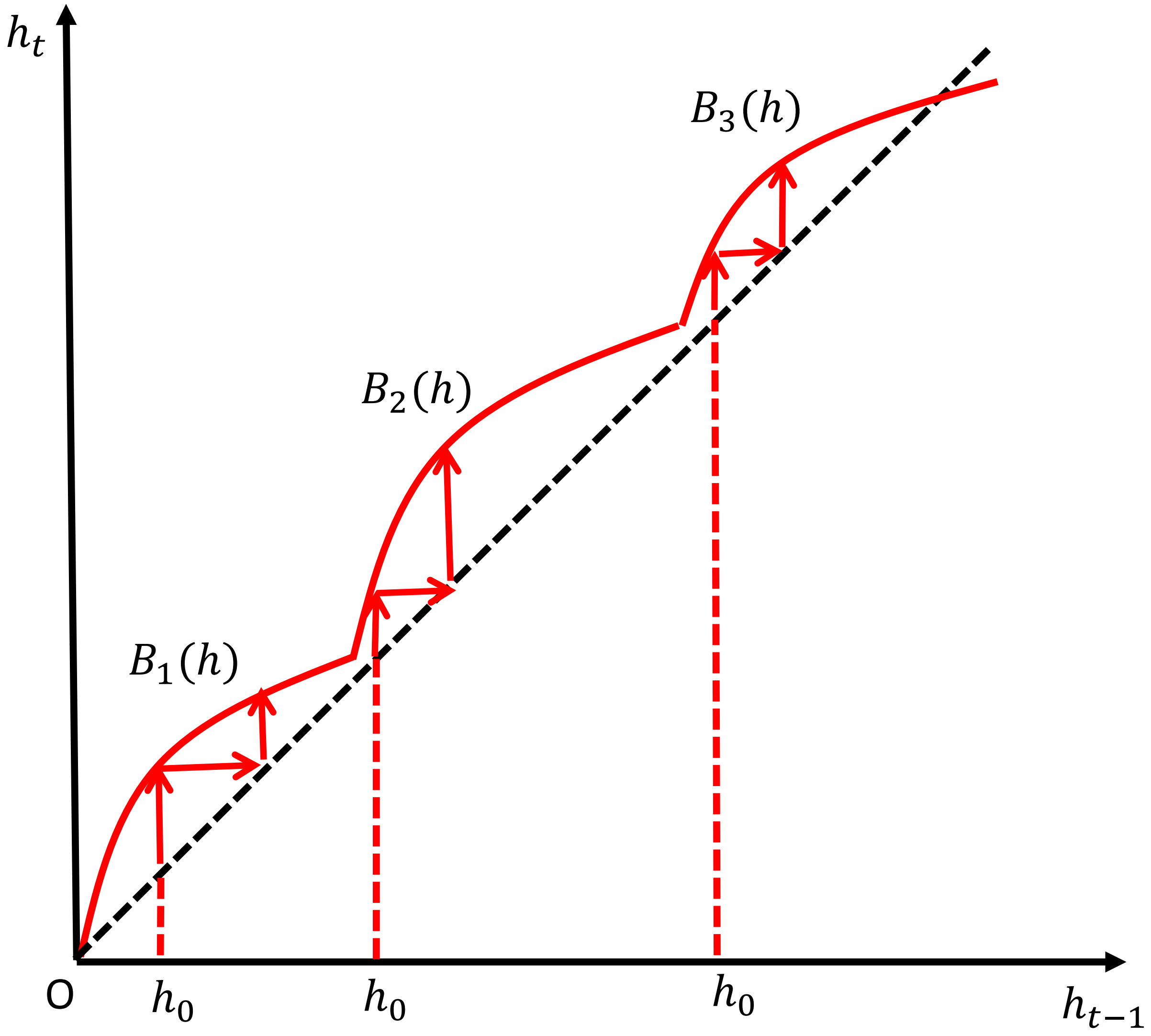


Figure 3. Flying geese pattern

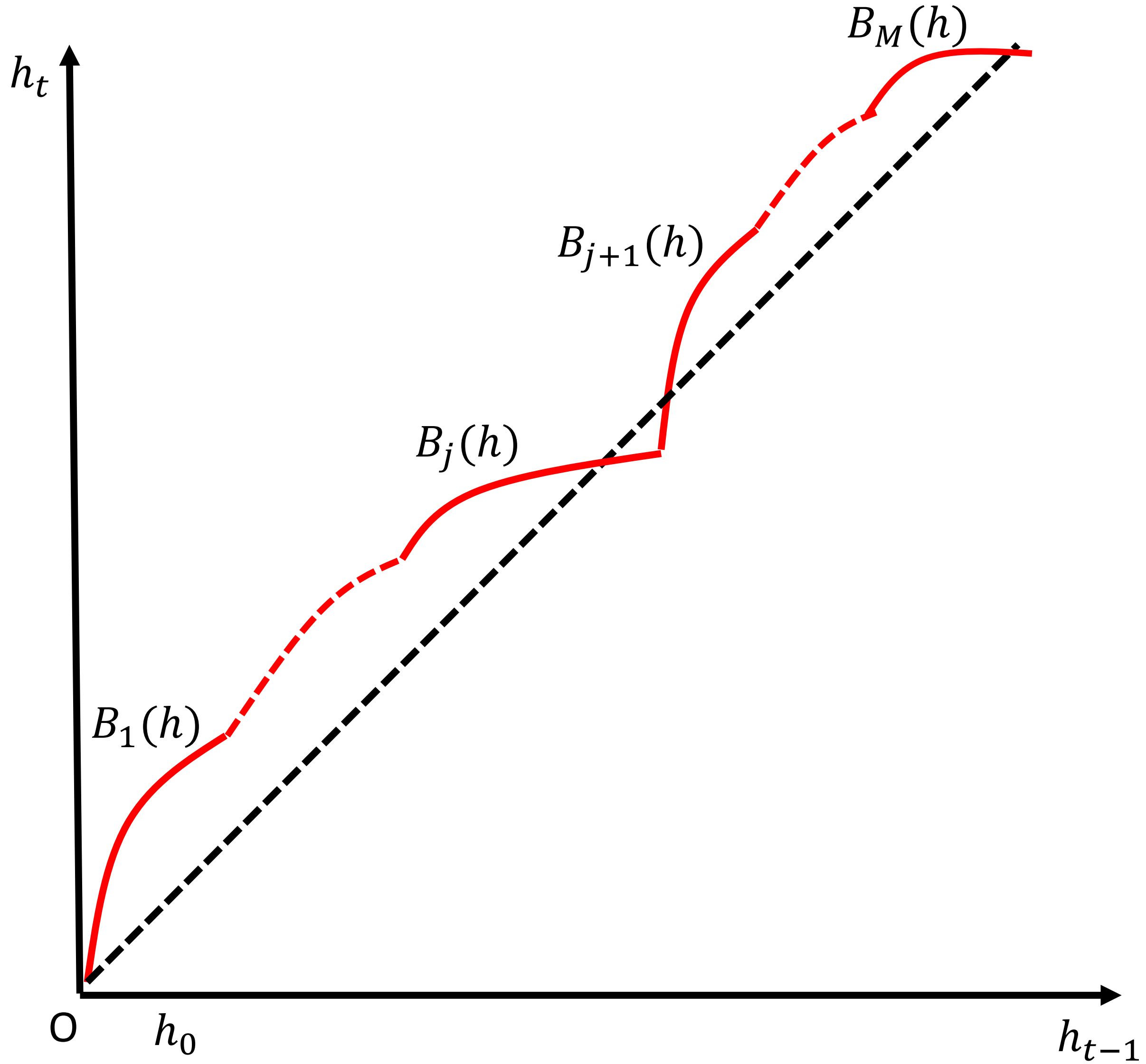


Figure 4. Trap in the jth technology

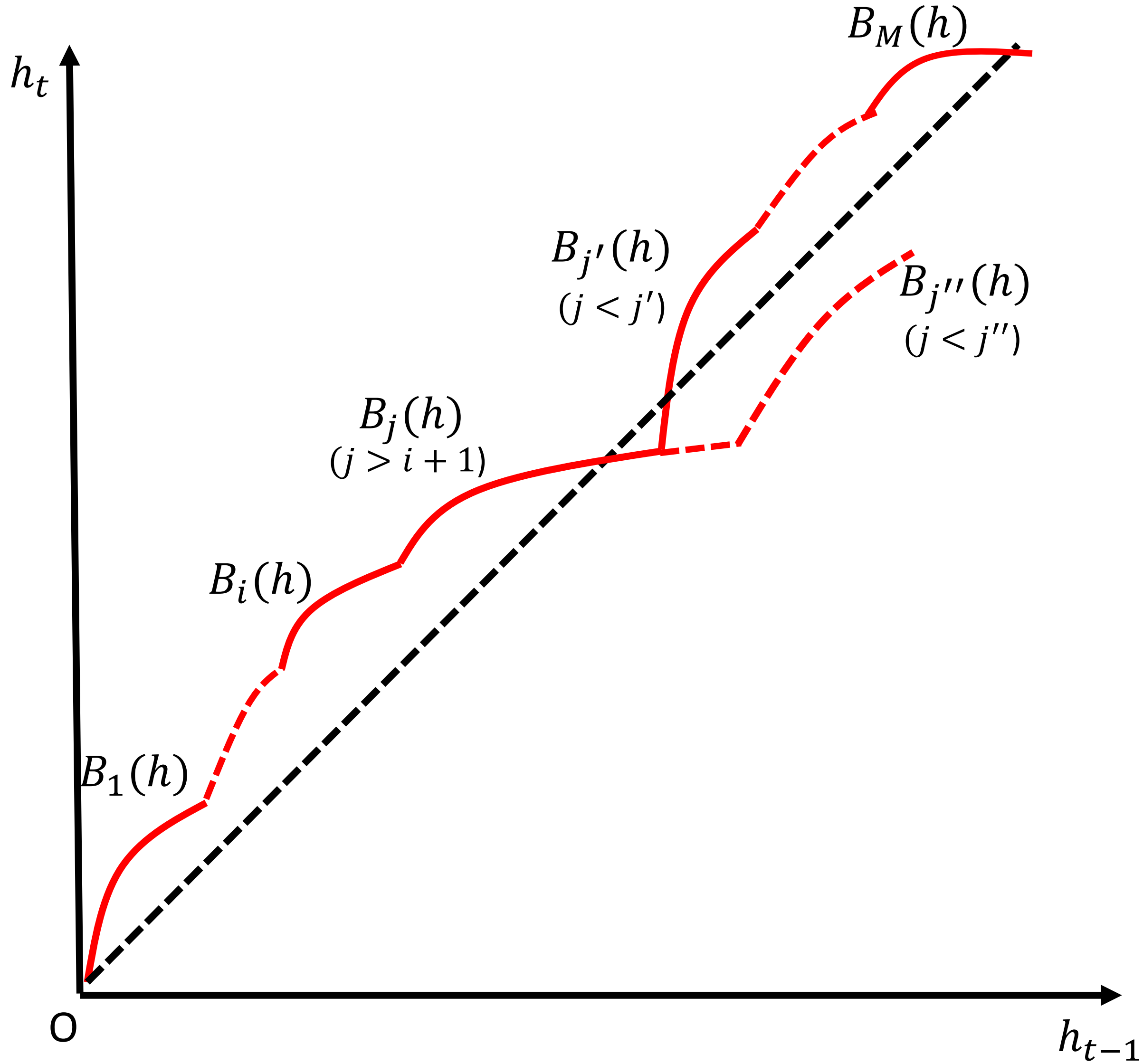


Figure 5. Restricted middle income trap

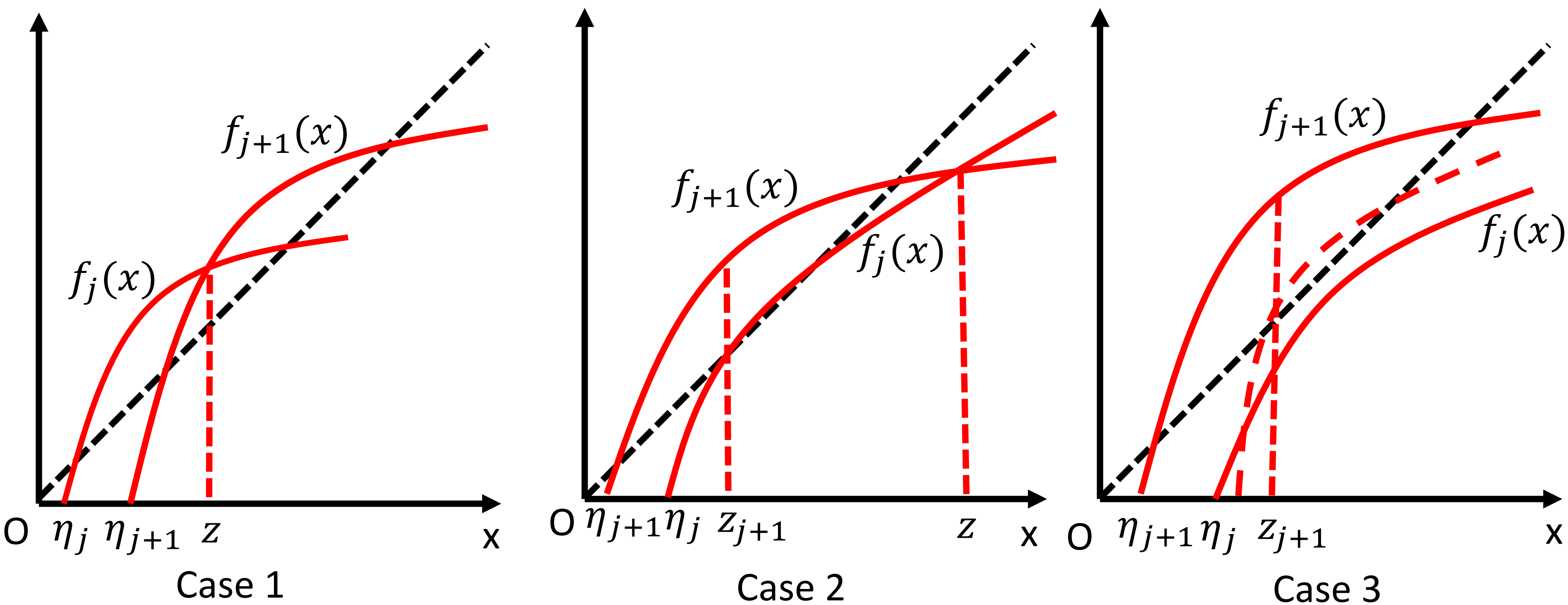


Figure 6. Flying geese condition

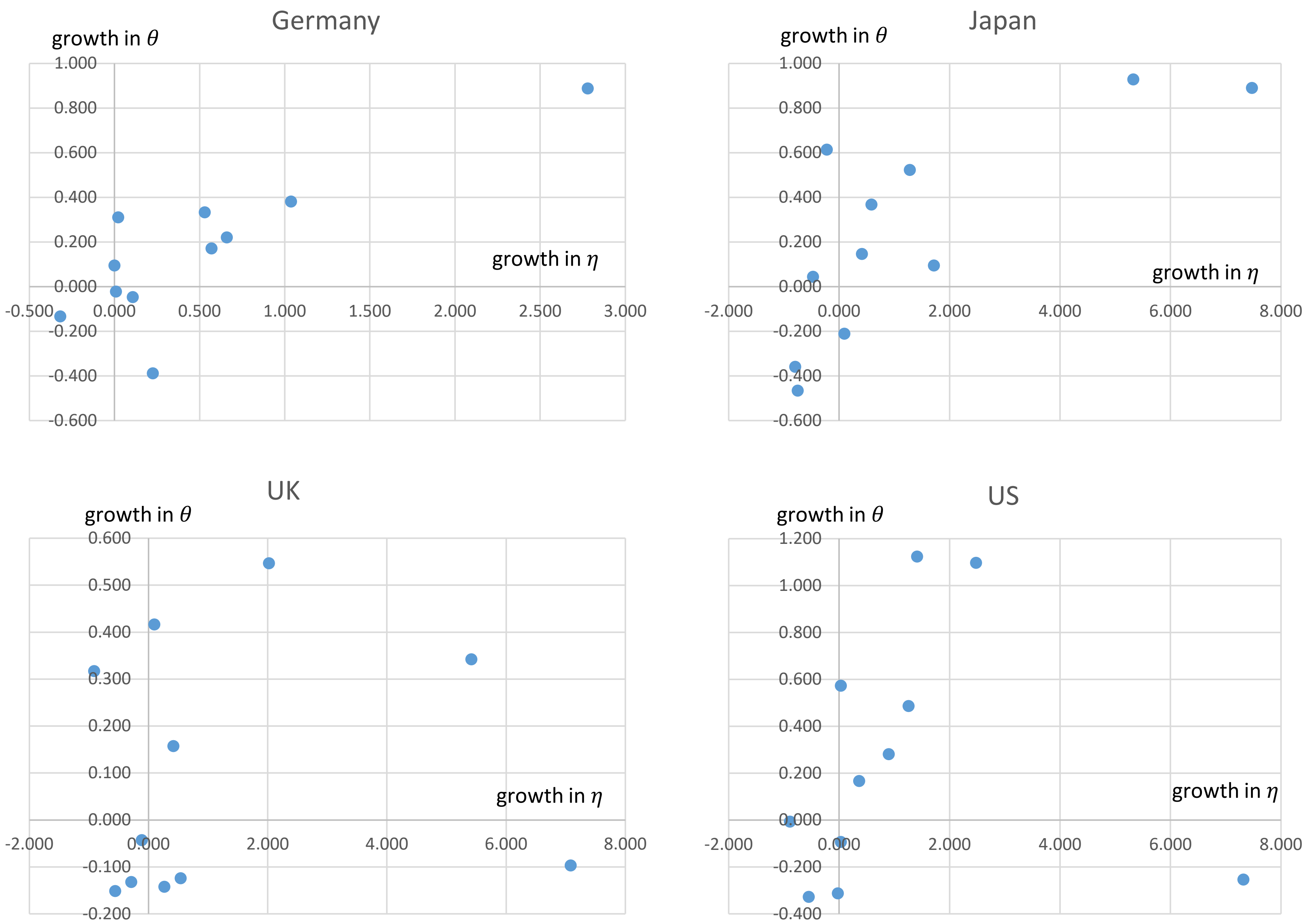


Figure 7. Growth in η and θ — Advanced Countries

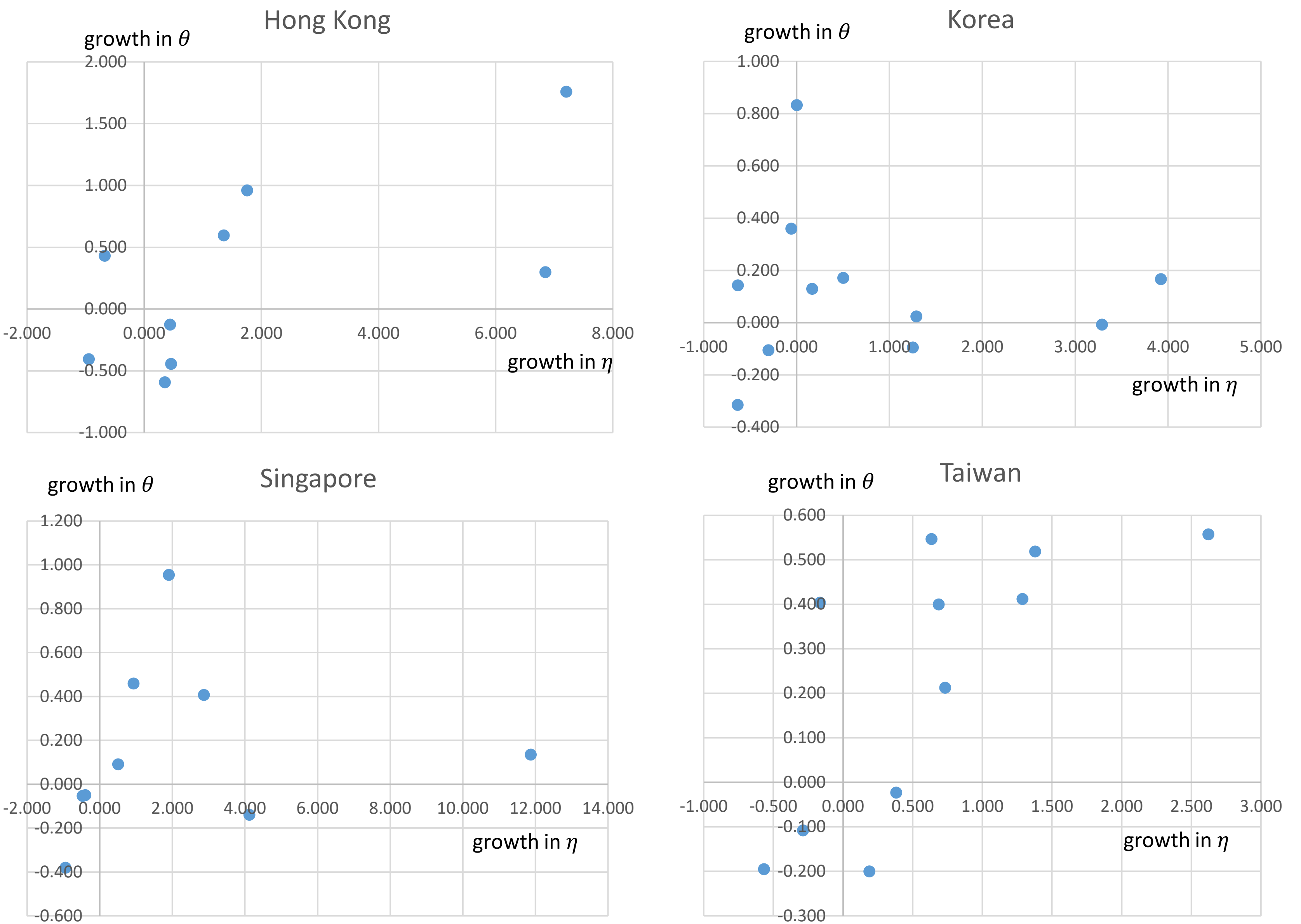


Figure 8. Growth in η and θ — Fast Growing Economies

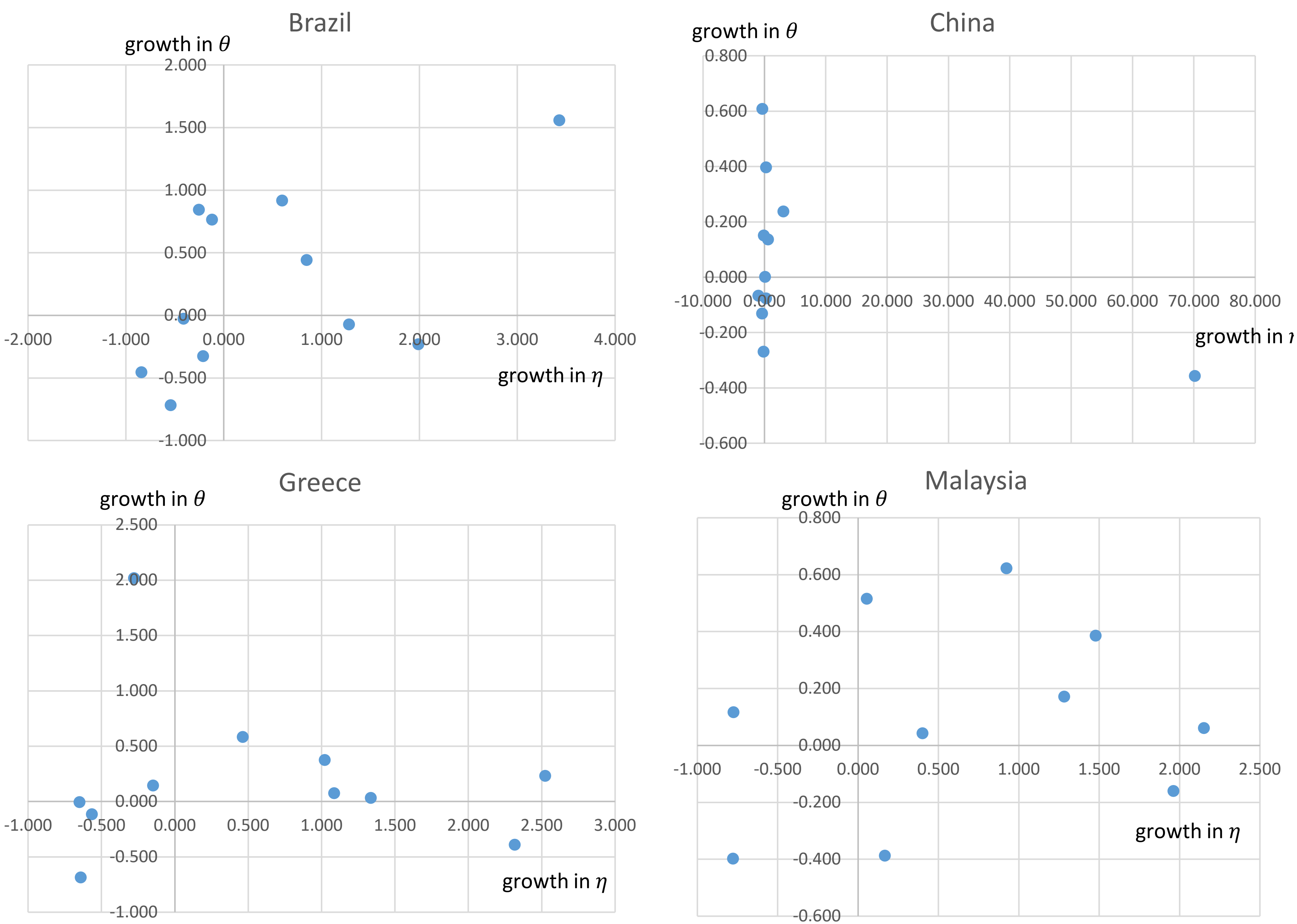
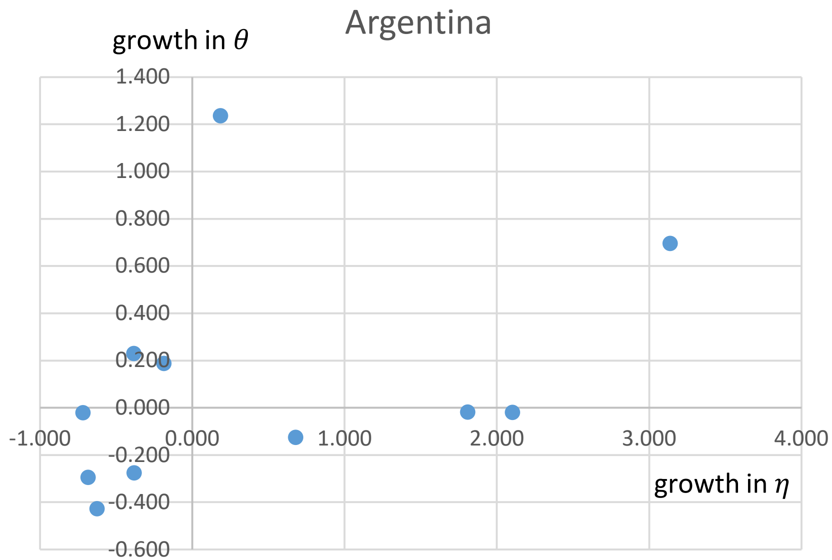
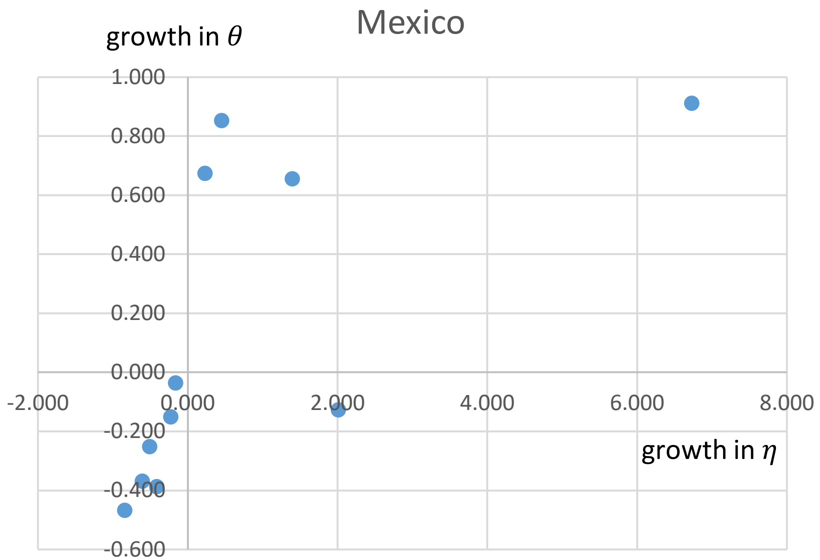


Figure 9. Growth in η and θ — Emerging Growing Economies





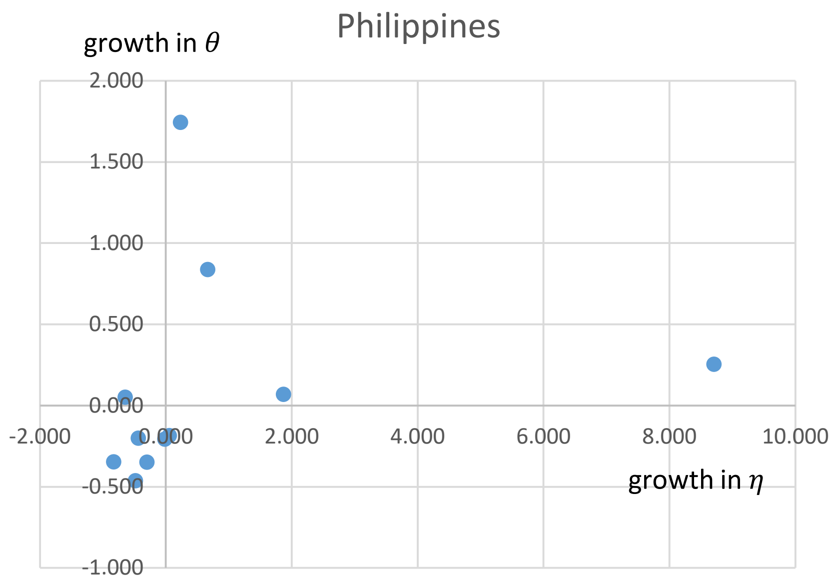


Figure 10. Growth in η and θ – Development Laggards