On the Instability of Banking and Financial Intermediation*

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Abstract

Are financial intermediaries inherently unstable? If so, why? What does this suggest about government intervention? To address these issues we analyze whether model economies with financial intermediation are particularly prone to multiple, cyclic, or stochastic equilibria. Four formalizations are considered: a dynamic version of Diamond-Dybvig incorporating reputational considerations; a model with delegated monitoring as in Diamond; one with bank liabilities serving as payment instruments similar to currency in Lagos-Wright; and one with Rubinstein-Wolinsky intermediaries in a decentralized asset market as in Duffie et al. In each case we find, for different reasons, that financial intermediation engenders instability in a precise sense.

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> Banks, as several banking crisis throughout history have demonstrated, are fragile institutions. This is to a large extent unavoidable and is the direct result of the core functions they perform in the economy. *Finance Market Watch Program @ Re-Define, Banks: How they Work and Why they are Fragile.*

Introduction

It is often said that banking, and sometimes financial activity more generally, is inherently unstable or prone to excess volatility. This seems to be based on the notion that financial institutions are somehow "special" compared to, say, non-financial corporate firms. Keynes (1936), Kindleberger (1978) and Minsky (1992) are names often associated with such views, with Williams (2015) providing a more recent perspective and additional references (see also, e.g., Akerlof and Shiller 2009 or Reinhart and Rogoff 2009). Rolnick and Weber (1986), who do not necessarily buy into the position, provide an indication of its widespread acceptance when they say: "Historically, even some of the staunchest proponents of laissez-faire have viewed banking as inherently unstable and so requiring government intervention." As a leading case, Friedman (1960) – a stalwart defender of unfettered markets in virtually all other contexts – advocated regulation (so-called narrow banking) as part of his Program for Monetary Stability. As additional evidence, consider the voluminous literature following Diamond and Dybvig (1983) studying the possibility of bank runs.[1]

Gorton and Whinton (2002) start their survey by asking: "Why do financial intermediaries exist? What are their roles? Are they inherently unstable? Must

[1]For now we are discussing banking panics, runs, financial crises etc., without defining them formally. As Rolnick and Weber (1986) say, "There is no agreement on a precise definition of inherent instability in banking. However, the conventional view is that it means that general bank panics can occur without economy-wide real shocks." Further, they say "The usual explanation... involves a local real economic shock that becomes exaggerated by the actions of incompletely informed depositors," and suggest this corresponds to Friedman and Schwartz's (1963) views of the events of 1930. In terms of models, Chari and Jagannathan (1988) capture this by having withdrawals by informed depositors lead to withdrawals by others, and Gu (2011) formalizes the process as rational herding. Our approach is different and avoids fixating on runs to the exclusion of other types of instability; still, our focus is on volatility "without economy-wide real shocks."

the government regulate them?" We want to confront the same questions. While there are different ways to proceed, our approach is to build formal models of these institutions and see if they are particularly prone to multiple equilibria, or to exotic dynamics like cyclic, chaotic or stochastic equilibria, where outcomes fluctuate even if fundamentals do not. In this approach, by models *of* intermediation, we mean more than models *with* intermediation. It does not suffice to simply assert that households lend to banks and banks lend to firms but households do not lend to firms – that may be part of a model *with* banks but not a model *of* banks.[2]

While there is much work studying financial intermediation there is no generally-accepted overarching model of financial intermediaries. This is because such institutions perform a myriad of functions that are difficult to model in a single set-up: they are safe keepers of cash and other valuables; they serve as middlemen between savers and borrowers or between asset sellers and asset buyers; they make investments on behalf of depositors; they screen and monitor investment opportunities; they issue liabilities (demand deposits) that facilitate third-party transactions; they provide liquidity insurance and maturity transformation; and they maintain privacy about their assets and customers. Different models are now used to capture these diverse activities and we consider four of those models although with a distinct formulation.

The first extends Diamond and Dybvig's (1983) model of liquidity insurance to an infinite horizon so we can highlight banker's reputation, as in Gu et al. (2013*a,b*), following models of credit based on Kehoe and Levine (1993). The second features fixed costs of finding and monitoring investment opportunities, similar to Diamond

[2] The declaration that households lend to banks and banks lend to firms but households do not lend to firms is reminiscent of monetary economics following Clower (1965), who said money buys goods and goods buy money but goods do not buy goods. While the appropriateness of an abstraction depends on the issue, it is hard to argue that Clower (cash-in-advance) constraints constitute the last word in monetary theory, and we feel similarly about banking (see Wright 2017 for more discussion). Now, not every study has to make everything endogenous – e.g., Debreu (1959) made progress using a theory *with* firms and households but did not have a theory *of* firms and households – but clearly one should prefer to have financial institutions emerge endogenously from explicit frictions in the environment when studying whether instability potentially arises as "the direct result of the core functions they perform in the economy" (from the epigraph).

(1984) and Huang (2017). The third, an adaptation of Nosal et al. (2017), puts intermediaries like those in Rubinstein and Wolinsky (1987) into an OTC (over-the-counter) asset market similar to Duffie et al. (2005). The fourth has bank liabilities serving as payment instruments, similar to currency in Lagos and Wright (2005) and Berentesen et al. (2007), building on the idea that demand deposits are about as liquid as cash, but safer in terms of loss or theft, as in He et al. (2007) or Sanches and Williamson (2010),[3] or it is less insensitive to information so it provides stable liquidity as in Dang et al. (2017) or Andolfatto and Martin (2013).

We find in all four cases that financial intermediation can indeed engender instability: economies with these institutions have multiple equilibria or display exotic dynamics for more parameters than similar economies without these institutions. In some cases the economy has a unique equilibrium without intermediation and multiple or volatile equilibria with it; in other cases both can have multiple or volatile equilibria but intermediation always expands the set of parameters where this occurs. Moreover, while the economic reasoning is different across models, in general the results are attributable to the explicit roles played – i.e., the core functions performed – by intermediaries in the models. Hence, while many existing models can produce multiple or volatile equilibria, our applications and economic interpretations are novel.

As the literature on financial intermediation is too vast to review here we refer to available sources (e.g., Freixas and Rochet 2008; Calomiris and Haber 2014; Vives 2016). We do mention Shleifer and Vishny (2010), which has a similar title and motivation, although a different approach. In general, there is much related work in finance, and we want to contrast our approach, coming mainly from monetary economics and in particular the work surveyed in Lagos et al. (2017). In finance, many studies concentrate primarily on finite-horizon models that we find ill suited for

[3]The fourth setup, where individuals worry about the safety of their payment instruments, might be most relevant for early banks, that were nothing if not safe-keeping facilities (see fn. 31), but that seems fine given the instability discussion is often couched in historical terms.

studying economic dynamics. Moreover, especially for banking, finiteness typically precludes salient elements like: (i) unsecured credit and reputational considerations; and (ii) currency.[4]

We also use general equilibrium theory, not in the sense of Debreu (1959), where there is no role for intermediation, money or reputation, but in the sense of logically closed systems without, as much as possible, exogenous assumptions on prices, contracts or behavior. This is because we want to know if instability arises from intermediation per se, and not extraneous – albeit sometimes realistic – elements of specification, like noise traders or expectations that are not fully rational. To be clear, we impose frictions like limited commitment, imperfect information or spatial separation, but these are assumptions on the environment, not prices, contracts or behavior. So, while the topics we study are similar to much research in finance, the methods are different, and hopefully they are complementary.

Model 1: Banking as Insurance

This first model extends Diamond and Dybvig's (1983) to infinite horizon. At each date in discrete time there is a $[0, 1]$ continuum of agents that live only for that period, plus some other agents that live forever, which is an obviously simple way to have agents care differently about the future. Each period is divided in two subperiods. The short-lived agents are homogeneous ex ante but face idiosyncratic preference shocks: any individual is impatient with probability π and patient with probability $1 - \pi$, where impatient (patient) ones derive utility only from consumption in the first (second) subperiod. The shock is revealed at the end of the first subperiod, and is private information. Given the shock, short-lived agents have utility $u_j(c_j)$, $j = 1, 2$, where c_j is the single consumption good in subperiod j, with $u'_j > 0$ and $u''_j < 0$.[5] Infinite-lived agents have utility $v(c)$ for c in the first or second

[4] This is obvious from backward induction: clearly no one honors obligations or accepts fiat money at terminal date T, implying no one does at $T - 1$, and so on, at least without ad hoc devices like imposing exogenous default penalties or putting dollars in utility functions.

[5] Many applications of Diamond-Dybvig assume $u_1(\cdot) = u_2(\cdot)$, but not all (e.g., Peck and Shell 2003). We like the flexibility of the general version mainly for constructing illustrative examples.

subperiod, with $v' > 0$, $v'' \leq 0$ and $v(0) = 0$.

Short-lived agents have an endowment of 1; infinitely-lived agents have 0. The technology is this: a unit of goods invested at the start of the first subperiod yields $R > 1$ units in the second subperiod; or the investment can be terminated at the end of the first subperiod to get back the input. The good can also be stored one-for-one across subperiods. As in Diamond-Dybvig, the short-lived agents can form a coalition, interpreted as a bank, to insure against the shocks. Thus, they design a bank contract, or deposit contract, (c_1, c_2) to solve

$$\max_{c_1,c_2} \{\pi u_1(c_1) + (1-\pi) u_2(c_2)\} \tag{1}$$

$$\text{st } (1-\pi) c_2 = (1-\pi c_1) R \tag{2}$$

$$c_2 \geq c_1, \tag{3}$$

where (2) is feasibility and (3) is a truth-telling constraint.[6] There are also nonnegativity constraints $c_j \geq 0$, omitted to save space.

This problem is well understood. A standard result is this: assuming $u_1'(1) > u_2'(R) R$ and $u_1'(c) \leq u_2'(c) R$ at $c = R/(1-\pi+\pi R)$, the solution satisfies $1 < c_1^* < c_2^* < R$, which means (3) is not binding and full efficiency obtains. However, its implementation requires commitment: Given each agent controls his own investment, when they learn they are patient and are supposed to deliver transfers to impatient agents, they have every incentive to renege. Without commitment there arises a role for our long-lived agents, as bankers, who accept endowments on deposit, invest them, and pay off depositors on demand at terms to be determined. We do not assume these agents can commit; this is endogenous and based on reputation, as in Gu et al. (2013*a,b*), following Kehoe and Levine (1993). Thus, bankers honor obligations lest they get identified as renegers, whence they are punished to autarky (the simplest story has them marooned on an island; in any case, no one reneges on the equilibrium path).

[6] If $c_2 < c_1$ patient agents would claim to be impatient, get c_1 and store it to the next subperiod.

As in Gu et al. (2013*a,b*), the key friction is this: if a banker misappropriates d units of his deposits – i.e., he takes more than his income as specified by the contract – he gets payoff λd, where λ is not too big, so that this is socially inefficient. The risk of this opportunistic behavior is that he might get caught, and punished, which happens with probability μ.[7] Now it is important to make the size of deposits d endogenous because it affects a banker's incentives (usually $d = 1$ is simply assumed, with exceptions, like Peck and Valipour 2018). Here depositors may want to set $d < 1$ and invest $1 - d$ on their own, to reduce a banker's incentive to misbehave. Hence, the contract now specifies d, payouts per deposit contingent on time of withdrawal r_j, $j = 1, 2$, and bankers' income $b \in [0, d]$, which he invests to consume bR. This generalizes problem (1)-(3), where $d = 1$ and $c_j = r_j$, $j = 1, 2$.

If there is more than one long-lived agent, the short-lived agents can choose any of them to act as their banker.[8] We allow them to choose no more than 1 banker, to avoid discussing the optimal number, because while that is interesting (see Huang 2017), it would be a distraction for present purposes.[9] Still, since they can choose any of the infinite-lived agents, for the usual reasons, summarized as Bertrand competition, the contract maximizes the expected utility of its depositors.

Yet the banker may still get positive surplus – a rent due to his inability to commit – since the contract must give him incentives to not to abscond with the deposits. This means it must respect the constraint

$$v(b_t R) + \beta V_{t+1} \geq \lambda d_t + \beta (1 - \mu) V_{t+1}, \tag{4}$$

[7] One interpretation is that μ is the probability that one generation of short-lived agents can communicate deviant bank behavior to the next. In fact, we could set $\mu = 1$, but that does not simplify anything, and it is known from other applications that $\mu < 1$ can be interesting.

[8] In the spirit of Diamond-Dybvig, they make this choice as a coalition, by which we mean they can commit to act in unison in designing and executing a deposit contract, even though they cannot commit to deliver transfers from patient to impatient agents after the shocks have been realized. In particular, we do not consider the possibility that subcoalitions (including those with one agent) opt out of the contract and try to do a different deal with the bank. We think this is without loss of generality, or at least one might be able to rationalize it by assumptions on information – e.g., acting as a coalition the entire group can observe total deposits, while splinter groups acting on their own cannot. Formalizing this is left for future work.

[9] This can be rationalized by assuming it is simply too costly to monitor more than 1 of them.

where V_t is a banker's lifetime utility and $\beta \in (0,1)$ the discount factor. The LHS is the equilibrium payoff; the RHS the deviation payoff, since a deviant banker gets λd for sure, and gets V_{t+1} iff he is not caught. Note that V_{t+1} is the value of a banker facing the next generation of depositors, and is taken as given when designing the contract at t. Also note that if a banker runs off, the technology does not let him take income b, which provides a deterrent over and above the risk of getting caught.[10] Using $\phi_t \equiv \beta\mu V_{t+1}$, rewrite (4) as

$$\lambda d_t - v\left(b_t R\right) \leq \phi_t, \tag{5}$$

We call ϕ_t the *franchise value* of the bank. It captures the banker's reputation for trustworthiness that is of course crucial for banking. If one believes that reputation is to some extent a self-fulfilling prophecy, this can be one channel for fragility. This channel is the subject of the following analysis.

Again omitting nonnegativity constraints to save space, the contract solves

$$\max_{d_t, r_{1t}, r_{2t}, b_t} \left\{\pi u_1\left(d_t r_{1t} + 1 - d_t\right) + (1-\pi)\, u_2\left[d_t r_{2t} + (1-d_t)\, R\right]\right\} \tag{6}$$

$$\text{st}\ \ (1-\pi)\, d_t r_{2t} = \left(d_t - b_t - \pi d_t r_{1t}\right) R \tag{7}$$

$$r_{2t} \geq r_{1t} \tag{8}$$

and (5), where (7) and (8) are similar to (2) and (3).

Substituting (7) into (6) to eliminate r_2, ignoring the t subscript for now, and taking first order conditions with respect to r_1, d and b we get

$$\begin{aligned}
r_1 &: \ d\left\{\pi\left[u_1'\left(c_1\right) - R u_2'\left(c_2\right)\right] - \eta_1\left(1-\pi+R\pi\right)\right\} r_1 = 0 \\
d &: \ \left\{\left(r_1 - 1\right)\pi\left[u_1'\left(c_1\right) - R u_2'\left(c_2\right)\right] + \eta_1\left[R - \left(1-\pi+R\pi\right) r_1\right] - \eta_2\lambda\right\} d = 0 \\
b &: \ \left[-u_2'\left(c_2\right) - \eta_1 + \eta_2 v'\left(bR\right)\right] b = 0,
\end{aligned}$$

[10] One could instead let him take b, but the case in the text is more interesting because it implies a unique stationary equilibrium, similar to the original Kehoe-Levine model (in particular, it implies $d = 0$ is not necessarily an equilibrium). Some of the models presented below have multiple stationary equilibria, and since the methods by which we generate endogenous instability differ in these cases, we want this one to have a unique stationary equilibrium.

where $c_1 = dr_1 + 1 - d$, $c_2 = dr_2 + (1-d)\,R$, η_1 is the multiplier associated with (8) and η_2 is the multiplier associated with (5). These FOC's yield two critical values, $\phi^* > 0$ and $\hat{\phi} < \phi^*$, delineating three qualitatively distinct regimes.[11]

(i) If $\phi \geq \phi^*$ then (5) does not bind, which implies $b = 0$, because the banker's future payoff is high enough that $b > 0$ is not needed to keep him honest. In this case, as in Peck and Setayesh (2019), there is a continuum of payoff-equivalent contracts that achieve (c_1^*, c_2^*): depositors can set $d \in [d^*, \max\{1, \phi/\lambda\}]$ for some $d^* < 1$, invest $1-d$ on their own, and still get full insurance. (ii) If $\phi \in [\hat{\phi}, \phi^*)$ the contract must either decrease $d < d^*$ or raise $b > 0$ to satisfy (5). Now lowering d from d^* entails less than full insurance, but this is a second-order effect by the envelope theorem, so the contract sets $d = \phi/\lambda$ and keeps $b = 0$. (iii) When $\phi < \hat{\phi}$, lowering d further is too costly, in terms of risk, so the contract sets $b > 0$ to discipline the banker. In all three regimes one can characterize other variables, like c_j and r_j, and without loss in generality we can set $r_1 = r_2 = r$.

Regime (iii) is most interesting since it has $b > 0$, which we need to get banking in stationary equilibrium.[12] In this regime (d, r, b) satisfies

$$b = d/R\left[R - (1 - \pi + R\pi)\,r\right] \tag{9}$$

$$\phi = \lambda d - v\,(bR) \tag{10}$$

$$\frac{u_2'(c_2)}{u_1'(c_1) - Ru_2'(c_2)} = \frac{\pi}{1 - \pi + R\pi}\left[\frac{(R-1)(1-\pi)}{\lambda}v'(bR) - 1\right] \tag{11}$$

with $c_1 = 1 - d + (d-b)\,R/\,(1-\pi+R\pi)$ and $c_2 = (1-d)\,R + (d-b)\,R/\,(1-\pi+R\pi)$. These conditions and the analogs from other regimes are used to draw Figure 1 in

[11] Although the formulae are not critical for understanding what follows, for the record, $\phi^* = \lambda d^*$ and $\hat{\phi} = \lambda\hat{d}$ where $d^* = 1 - (c_2^* - c_1^*)\,/\,(R-1)$ and $\hat{d}$ solves

$$\frac{\pi}{1-\pi+R\pi}\left[\frac{(R-1)(1-\pi)}{\lambda}v'(0) - 1\right] = \frac{u_2'(\hat{c}_2)}{u_1'(\hat{c}_1) - Ru_2'(\hat{c}_2)}.$$

with $\hat{c}_1 = 1 - \hat{d} + \hat{d}R/\,(1-\pi+R\pi)$ and $\hat{c}_2 = (1-\hat{d})R + \hat{d}R/\,(1-\pi+R\pi)$.

[12] It is obvious that $b = 0$ implies $V = 0$ and hence bankers would abscond with any $d > 0$.

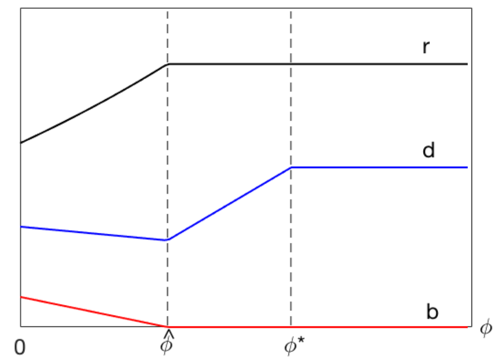

Figure 1: Model 1, bank contract vs ϕ

a case where $\hat{\phi} > 0$, which is necessary to have $b > 0$.[13] Notice $\partial b/\partial \phi < 0$ in regime (iii), a result that is useful below; as the franchise value of the bank increases, the banker has more to lose when cheating, so a lower b is enough to keep him honest. The other qualitative results are also general, except the effect of ϕ on d, which depends on parameters. Figure 1 is drawn for the following example and parameter values:

Example 1: Let $v(b) = Bb$,

$$u_1(c_1) = A_1 \frac{(c_1 + \varepsilon)^{1-\sigma_1} - \varepsilon^{1-\sigma_1}}{1-\sigma_1} \text{ and } u_2(c_2) = A_2 \frac{(c_2 + \varepsilon)^{1-\sigma_2} - \varepsilon^{1-\sigma_2}}{1-\sigma_2}.$$

Then let $B = 0.95$, $\sigma_1 = \sigma_2 = 2$, $\varepsilon = 0.01$, $A_1 = 1$, $A_2 = 0.1$, $R = 2.1$, $\mu = 0.7$, $\pi = 0.25$, $\lambda = 0.6$ and $\beta = 0.99$. This implies $\hat{\phi} = 0.3257$ and $\phi^* = 0.600$.

So far the analysis takes ϕ as given. To embed this in general equilibrium, use $\phi_t \equiv \beta\mu V_{t+1}$ to write $V_t = v(b_t R) + \beta V_{t+1}$ as a dynamical system,

$$\phi_{t-1} = f(\phi_t) \equiv \beta\mu v[b(\phi_t) R] + \beta\phi_t, \tag{12}$$

where $b(\phi_t)$ comes from the contracting problem for a given ϕ. Equilibrium is defined as a nonnegative, bounded path for ϕ_t solving (12), from which the other

[13] In terms of primitives, one can check $\hat{\phi} > 0$ iff

$$\frac{\pi[u_1'(1) - Ru_2'(R)]}{(1-\pi+R\pi)\lambda}[(R-1)(1-\pi)v'(0) - \lambda] > u_2'(R).$$

However, for our purposes, a general condition is less interesting than examples.

endogenous variables follow using the FOC's. A stationary equilibrium, or steady state, is a fixed point $\bar{\phi} = f\left(\bar{\phi}\right)$. If $\hat{\phi} \leq 0$ (conditions for this are given in fn. 13) then (12) reduces to $\phi_{t-1} = \beta\phi_t$, and the only equilibrium is a steady state with $\bar{\phi} = 0$ and no banking. If $\hat{\phi} > 0$ there is a steady state with banking and $\bar{\phi} \in (0, \hat{\phi})$. Given this, $f(0) > 0$ and $f(\hat{\phi}) = \beta\hat{\phi} < \hat{\phi}$, so $\bar{\phi} \in (0, \hat{\phi})$ exists. It is easy to show it is unique.[14] We summarize as follows:

Proposition 1 *If $\hat{\phi} \leq 0$ then steady state exists uniquely and it has $\bar{\phi} = 0$, which means banking is inoperative. If $\hat{\phi} > 0$ then steady state exists uniquely and it has $\bar{\phi} \in (0, \hat{\phi})$, which means banking is operative.*

To proceed with dynamics, note that $f(\phi_t)$, as defined in (12), has a linear term that is increasing and a nonlinear term that is decreasing, because $b'(\phi) < 0$ in the relevant range, as remarked above. Hence the system potentially can exhibit cyclic, chaotic or stochastic (sunspot) equilibria. For Example 1, Figure 2a shows f and its inverse f^{-1}, which cross on the 45^o line at the steady state $\bar{\phi} = 0.3215$. For these parameters the system is monotone and the unique equilibrium is the steady state, because that is the only bounded, nonnegative path solving (12). In this equilibrium $\bar{\phi} \in (0, \hat{\phi})$ and banking is operative. So banking is not necessarily associated with instability. But it can be, as shown by the next example.

Example 2: Same as above except $\sigma_1 = 10$ and $\mu = 1$.

Figure 2b shows f and f^{-1} for Example 2, where $f'(\bar{\phi}) < -1$, and hence these functions intersect not only on the 45^o line but off it at (ϕ_L, ϕ_H) and (ϕ_H, ϕ_L), where $\phi_H = 0.0696$ and $\phi_L = 0.0689$.[15] As is standard (see Azariadis 1993 for a textbook treatment) this means there is a two-cycle equilibrium where ϕ_t oscillates deterministically between ϕ_L and ϕ_H. Moreover, this implies there exist sunspot

[14] To prove this, first solve (12) for $\phi = \beta\mu v(bR)/(1-\beta)$ and substitute it into (10) to get $\lambda d = (1-\beta+\beta\mu)\, v(bR)/(1-\beta)$. This implies d is increasing in b. But (11) implies d is decreasing in b, so if they have a solution $(\bar{b}, \bar{d})$ it is unique. So $\bar{\phi} = \beta\mu v\left(\bar{b}R\right)/(1-\beta)$ is unique.

[15] This cycle may seem small, but of course one should not take the numbers too seriously – the goal is not to calibrate the model realistically, only to show that cycles are possible. In any case, bigger cycles are illustrated below.

equilibria where ϕ_t fluctuates randomly between values close to ϕ_L and ϕ_H (again see Azariadis 1993 and the Appendix). Hence, this model economy can exhibit volatility with banking, while it cannot without, because if banks were somehow taxed out of existence the unique equilibrium would be autarky. This does not mean banking is a bad idea: it serves a useful function by providing insurance to agents who cannot insure each other due to commitment issues. But it does mean that banking engenders instability, in the sense that there can be equilibria with volatile outcomes that coexist with a steady state equilibrium.

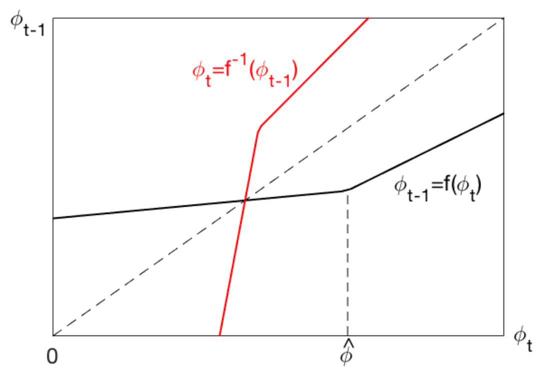

Figure 2a: Model 1, monotone f

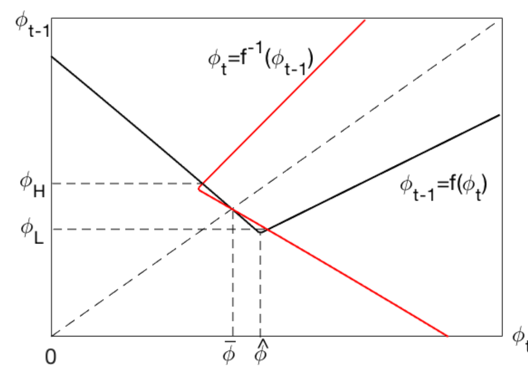

Figure 2b: Model 1, nonmonotone f

The reason is simple: a high future franchise value V_{t+1} increases ϕ_t which disciplines the banker even with a low current b_t; but that makes the current franchise value V_t and hence ϕ_{t-1} low. That means there is a tendency towards oscillations based solely on beliefs – obviously nothing fundamental is changing over time – but this effect has to dominate the linear term in $f(\phi_t)$ to get fluctuations, and therefore parameter values matter. Figure 3 plots time series of (ϕ, d, b, r) over the two-period cycle in Example 2. Notice r moves with ϕ and b moves against ϕ. Whether d moves with or against ϕ is ambiguous, in general, but in the example it moves against ϕ.

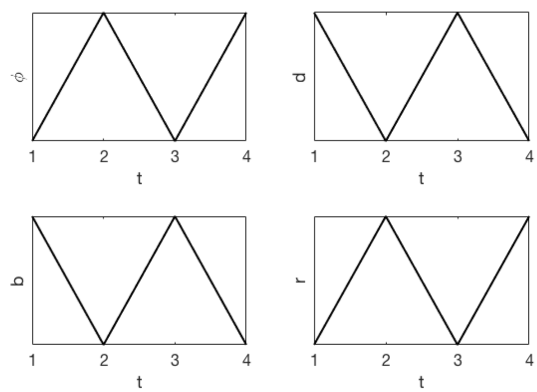

Figure 3: Model 1, time series for a two-cycle

Figure 4 displays the existence of cycles in a different way for Example 3, as described below. It plots the second iterate $f^2 = f \circ f$, which has three fixed points: the steady state $\bar{\phi}$, plus the ϕ_L and ϕ_H associated with the two-cycle. It also plots the third iterate f^3, which Example 3 has seven fixed points: $\bar{\phi}$ plus a pair of three-cycles. Standard results (again, see Azariadis 1993) tell us that the existence of a three-cycle implies the existence of cycles of any order, including chaos, interpretable as a cycle with infinite periodicity. So, for some parameters, there are very many equilibria in this model with operative banks, with deterministic, stochastic and chaotic dynamics, directly attributable to the idea that banking is critically dependent on trustworthiness, a notion that is very susceptible to self-fulfilling prophecy. We conclude that in this model banking may well engender instability.

Example 3: $B = 1$, $\sigma_1 = 14$, $\sigma_2 = 1.5$, $\varepsilon = 0.01$, $A_1 = 1$, $A_2 = 0.075$, $R = 2.2$,

$\mu = 1$, $\pi = 0.28$, $\lambda = 0.75$ and $\beta = 0.76$.

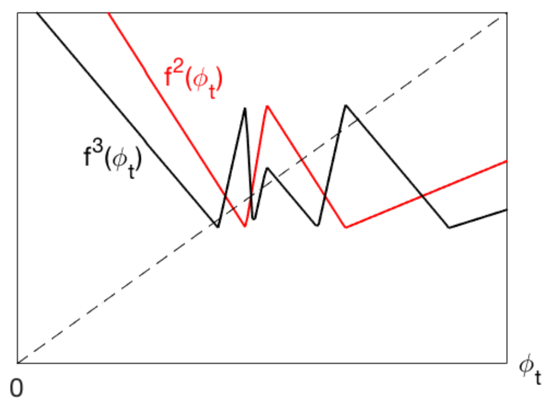

Figure 4: Model 1, two- and three-cycles

Model 2: Delegated Investment

The next formulation considers intermediation originating from economies of scale in finding, evaluating and monitoring investment opportunities. The setup is based on Diamond (1984), although it also owes something to other work mentioned below, including Huang (2017).[16] Again time is discrete and infinite, but now agents are spatially separated, say on a large number of islands. On each island there is a large number of agents all of whom are all infinitely lived, different from Model 1. Then, to avoid dealing with long-term contracting (which is interesting but complicated), suppose they are randomly relocated at the end of each period.[17]

Regarding technology, there are many projects with a constant return R per unit of investment. Any agent can find an investment project (with probability 1)

[16]Boyd and Prescott (1986) use a model of adverse selection in which financial intermediaries arise endogenously in equilibrium to screen projects and produces public information about the quality of the project. Leland and Pyle (1976) show that an agent's willingness to invest in her own project serves as a signal of her project's quality. Using this idea, they argue that a firm gathering information could become an intermediary, buying and holding assets on the basis of its specialized information.

[17]A relocation specification is useful because, without it, one ought to consider dynamic contracts that might, e.g., backload bankers payoffs (see Gu et al. 2013a). That is interesting, but would be a distraction for present purposes.

only if they pay a cost κ. This stands in for costs of evaluating and monitoring, as well as searching for, investment projects, but for our purposes, we can abstract from lending to other parties and treat these projects mechanically. The important element is that κ is a fixed cost, independent of the amount invested in projects.

Agent's period utility is $u(x) - c(d)$, where x is consumption and d is investment (e.g., time working on the project), with $u', c', c'' > 0 > u''$, plus $u'(0) R > c'(0)$, which implies agents would invest if there were no fixed cost. If one pays κ and invests on his own, the payoff is

$$\tilde{W} = \max_{x,d} \{u(x) - c(d)\} \text{ st } x = Rd - \kappa. \tag{13}$$

where we again omitted the nonnegativity constraints. If κ is too high, this is dominated by $u(0) - c(0) = 0$. Let $W_0 = \max\left\{\tilde{W}, 0\right\}$ be the autarky payoff, and consider the case $W_0 = 0$, which means κ is too high to support individual investment. Then there is room for agents to form a coalition where some, called depositors, delegate their investment to others, called bankers. This saves on fixed costs, given that paying κ once allows the coalition to pool their deposits in a project with constant return.

Bankers are chosen at random – by lottery, as in Rogerson's (1988) indivisible-labor model – with all agents having equal chance ω_t of becoming the banker given they are ex ante homogeneous. ω_t also denotes the measure of bankers when we normalize the population on an island to 1. As in Model 1, bankers cannot commit to repay depositors when the investment comes to fruition: they may instead abscond, for a payoff λ per unit of d; and with probability μ such misbehavior is detected, communicated and punished by autarky.[18] Bankers behave whenever

$$\beta V_{t+1} \geq \frac{\lambda(1-\omega_t)}{\omega_t} x_t + (1-\mu)\beta V_{t+1} \tag{14}$$

[18] To be clear, agents' histories, including their roles as bankers or depositors in the past, is private information here, except when an agent has been caught acting as a banker and absconding with the returns. Again, the simplest story has deviants marooned to autarky, but that never happens on the equilibrium path.

where V_t is again the payoff to participating in a coalition. The LHS of (14) is a banker's equilibrium continuation value. The RHS is the deviation payoff, given the deposits he controls when there is a fraction ω_t of bankers in the coalition. Although our formalization is technically different, the trade-off is similar to Huang (2017): having a small number of large banks saves on fixed costs; but then they each hold more deposits, ceteris paribus, which makes them more inclined to misbehave.

The optimal contracting problem is

$$W(\phi) = \max_{\omega,X,D,x,d} \{\omega [u(X) - c(D)] + (1-\omega)[u(x) - c(d)]\} \tag{15}$$

$$\text{st } \omega X + (1-\omega) x = R[\omega D + (1-\omega) d] - \kappa\omega \tag{16}$$

$$u(x) - c(d) \geq 0 \tag{17}$$

$$\frac{1-\omega}{\omega} x \leq \phi, \tag{18}$$

where $\phi_t \equiv \mu\beta V_{t+1}/\lambda$, (X, D) and (x, d) are the allocations of bankers and depositors, respectively, making (16) the resource constraint and (17), (18) the participation constraint and incentive constraint for depositors and bankers respectively. We also need to check the participation constraint for bankers.

Substituting (16) into (15) to eliminate X, and letting η and γ be Lagrange multipliers, we get the FOC's:

$$\begin{aligned}
D &: u'(X) R - c'(D) = 0 \\
d &: (1-\omega)[u'(X) R - c'(d)] - \eta c'(d) = 0 \\
x &: (1-\omega)[u'(x) - u'(X)] + \eta u'(x) - \gamma \frac{1-\omega}{\omega} = 0 \\
\omega &: \omega \left\{ u(X) - c(D) - [u(x) - c(d)] + u'(X) \frac{x - Rd}{\omega} + \gamma \frac{x}{\omega^2} \right\} = 0
\end{aligned}$$

If (17) binds, $u(X) - c(D) \geq 0$ is equivalant to $W(\phi) \geq 0$. If (17) does not bind, the FOCs imply $d = D$ and $X \geq x$. So the banker's participation constraint is satisfied automatically. Therefore, we only need to check if $W(\phi) \geq 0$ for banker's participation. By the envelope condition, $W' \geq 0$. It can be shown that $W(0) = 0$, which means that there is no banking at $\phi = 0$. When $\phi \to \infty$, it is obvious that the optimal choice is to let bankers grow as large as possible, $\omega \to 0$, and $W(\phi) \to W^* \equiv \max_{x,d} [u(x) - c(d)]$ st $x = Rd$, which is the highest possible W.

Example 4: Let

$$u(x) = A\frac{(x+\varepsilon)^{1-\sigma} - \varepsilon^{1-\sigma}}{1-\sigma} \text{ and } c(d) = Bd,$$

with $A = \varepsilon = 0.001$, $\sigma = 2$, $B = 0.1$, $\kappa = 230$ and $R = 1.2$.

For this example Figure 5 shows the contract given ϕ. There is a cutoff $\tilde{\phi}$, where $\tilde{\phi} = 0.0182$, below which agents are in autarky since banking is not viable, and above which banking is viable. When $\phi \in [0.0182, 0.0201]$, (17) binds, and D decreases in ϕ. For $\phi > 0.0201$, $D = d$ since (17) does not bind. As c is linear, X is constant if banking is viable. But it is hard to know how the contract (ω, D, d, X, x) varies with ϕ in general, given the trade-off between having fewer banks to reduce the fixed costs and having smaller banks to satisfy (18).

As for Model 1, we embed the optimal contract in general equilibrium using the lifetime value of an agent $V_t = W(\phi_t) + \beta V_{t+1}$. Although Models 1 and 2 are quite different, we can emulate the methods and rewrite this using the definition of ϕ_t to get

$$\phi_t = f(\phi_{t+1}) \equiv \frac{\beta\mu}{\lambda} W(\phi_{t+1}) + \beta\phi_{t+1}. \tag{19}$$

As usual, an equilibrium is a bounded, nonnegative solution to (19). Notice $f(0) = 0$. Also $W(\phi)$ is bounded by W^*, so $f(\phi) < \phi$ for big ϕ. This immediately implies the following:

Proposition 2 *There are odd number of steady states, one of which is $\bar{\phi} = 0$.*

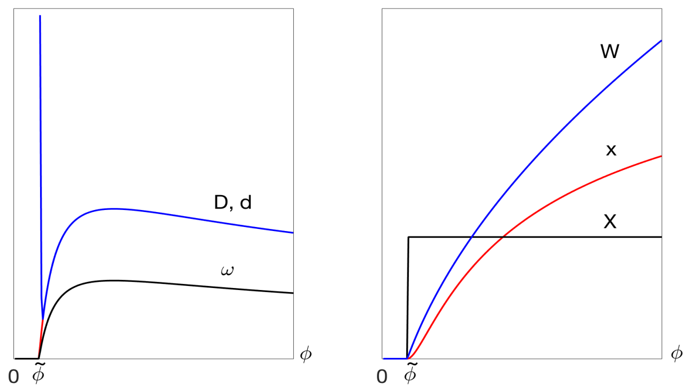

Figure 5: Model 2, bank contract vs ϕ

Example 5: Continue Example 4 with $\beta = 0.76$, $\mu = 0.95$ and $\lambda = 9$. As Figure 6 shows, for these parameters the dynamical system has exactly three steady states, 0 plus two others at $\phi_2 > \phi_1 > 0$. Clearly, ϕ_1 is stable while 0 and ϕ_2 are unstable.

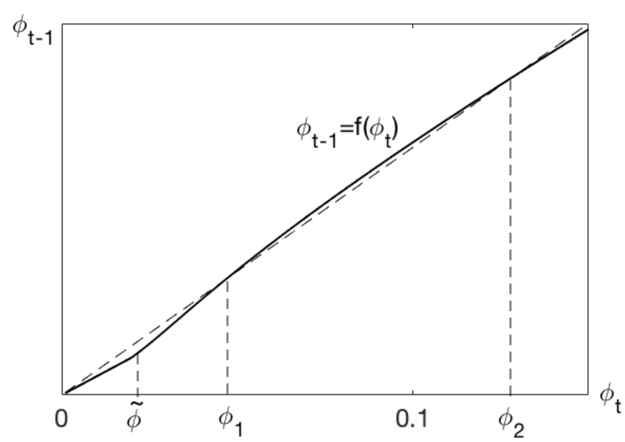

Figure 6: Model 2, monotone f with multiple steady states

This is all different from Model 1, which was designed to have a unique steady state, and relies on the nonmonotonicity of the dynamical system, since complicated dynamics emerge iff $f'\left(\bar{\phi}\right) < -1$. The dynamical system in Model 2 is monotone, so cyclic equilibria including deterministic cycles do not exist. However, since it has multiple steady states, including the stable one ϕ_1, we can construct sunspot equilibria around ϕ_1 using a different technique that works when $f'\left(\bar{\phi}\right) > 1$. Since the construction is standard from a technical perspective, we relegate details to the Appendix. The result is that if there is a stable steady state ϕ_1 in between two unstable steady states ϕ_0 and ϕ_2, as illustrated in Figure 6, then any $\phi_A \in (\phi_0, \phi_1)$ and $\phi_B \in (\phi_1, \phi_2)$ constitute a two-state, stationary sunspot equilibrium. In particular, if $\phi_A < \tilde{\phi}$, the economy switches stochastically between states with autarky and with banking. While not fixating on runs, we cannot help mention this outcome looks something like a run, with agents randomly choosing to either roll over or not roll over deposits each period as a function of some fundamentally irrelevant signal.

At this point it is important to consider a variant of the above model that incorporates bargaining because we want to emphasize how different the economic mechanism is here relative to Gu et al. (2013b). That paper contains a model that generates cycles and sunspots in credit conditions that depend on the potential nonmonotonicity of an agent's surplus when we relax the constraints in the Nash bargaining problem. This is not at all what is going on in the above models, as is demonstrated below.

To this end, suppose there are two types of agents, in a sense combining elements of Models 1 and 2: each island has a one-period lived agent born at every date, plus an infinitely lived agent.[19] To economize on the fixed cost κ, again, the pair may want to form a coalition. However, since the one-period lived agent has no concern for reputation – i.e., cannot be disciplined by the threat of future autarky – he must be the depositor, while the infinitely-lived agent might be able to play the role of banker. Again, assume κ is so high that agents do not invest by themselves, and

[19] We start with two agents because this is the natural context in which to consider Nash bargaining; a generalization to many depositors is discussed below.

a banker cannot commit to repay the depositor, as he may be tempted to abscond with the investment returns, for payoff λ per unit misappropriated.

Let the banker's bargaining power be θ. Then the Nash bargaining problem is

$$W(\phi) = \max_{X,x,D,d} [U(X) - C(D)]^{\theta} [u(x) - c(d)]^{1-\theta} \tag{20}$$

$$\text{st } X + x = R(D + d) - \kappa \tag{21}$$

$$u(x) - c(d) \geq 0 \tag{22}$$

$$x_t \leq \phi_t \tag{23}$$

where the last constraint as usual rewrite the banker's incentive condition $\beta V_{t+1} \geq \lambda x_t + (1-\mu)\beta V_{t+1}$ using $\phi_t \equiv \beta\mu V_{t+1}/\lambda$. Again, we also need to check $W(\phi) \geq 0$. Note that W is strictly increasing in ϕ if (23) binds. So there is a cutoff $\tilde{\phi}$ above which banking is viable and below which it is not. If κ is big, it is possible that $W(\phi) < 0$ even if (23) does not bind, so banking is not viable for any ϕ. To consider an interesting case, denote the solution ignoring (22) and (23) by (X^*, x^*, D^*, d^*), assume $u(x^*) > c(d^*)$, and let $\phi^* = x^*$.

Now substitute (21) into the objective function and take the FOC's to get

$$\begin{aligned}
D &: U'(X)R - C'(D) = 0 \\
d &: \theta U'(X)R[u(x) - c(d)] - (1-\theta)c'(d)[U(X) - C(D)] - \eta_1 c'(d) = 0 \\
x &: -\theta U'(X)[u(x) - c(d)] + (1-\theta)u'(x)[U(X) - C(D)] + \eta_1 u'(x) - \eta_2 = 0
\end{aligned}$$

where η_1 and η_2 are the multipliers associated with (22) and (23). From this one can see the banker's surplus does not necessarily increase with an expansion of the bargaining set, at least for ϕ close to ϕ^*:

$$\left.\frac{\partial [U(X) - C(D)]}{\partial \phi}\right|_{\phi \to \phi^*} = \frac{(1-\theta)U'c''(U-C)(R^2U'' - C'')}{(C'' - R^2U'')[C'c' + (1-\theta)c''(U-C)] - \theta R^2 U'' C''(u-c)} < 0$$

The banker's value function is

$$V_t = U(X_t) - C(D_t) + \beta V_{t+1}$$

and using $\phi_t = \beta\mu V_{t+1}/\lambda$, we have

$$\phi_{t-1} = \frac{\beta\mu}{\lambda}[U(X_t) - C(D_t)] + \beta\phi_t. \tag{24}$$

Depending on ϕ_t, (24) can be written as

$$\phi_{t-1} = \begin{cases} \beta\phi_t & \text{if } \phi_t < \tilde{\phi} \\ \frac{\beta\mu}{\lambda}[U \circ X(\phi_t) - C \circ D(\phi_t)] + \beta\phi_t & \text{if } \tilde{\phi} \leq \phi_t < \phi^* \\ \frac{\beta\mu}{\lambda}[U(X^*) - C(D^*)] + \beta\phi_t. & \text{if } \phi_t \geq \phi^* \end{cases}$$

Example 6: Let $U(x) = u(x) = Ax$ and $C(d) = c(d) = Bd^\gamma/\gamma$, where $A = 1$, $B = 0.5$ and $\gamma = 5$. For other parameters, let $R = 2$, $k = 1.5$, $\theta = 0.01$, $\lambda = 0.01$, $\mu = 1$ and $\beta = 0.35$. Figure 7 shows the dynamic system. There are three steady states including autarky. The f function crosses the 45 degree line first from below and then from above. As shown above, there are sunspot equilibria around the stable steady state. Hence, any $\phi_A \in (0, \phi_1)$ and $\phi_B \in (\phi_1, \phi_2)$ constitute a sunspot equilibrium. This is due to the fact that there is fixed search cost. The function is decreasing around ϕ_2 and the slope is less than -1. There is a two-cycle with periodic points ϕ_L and ϕ_H. As standard, two-cycle implies the existence of two-state stationary sunspot equilibria. Any $\phi_A \in (\phi_L, \phi_2)$ and $\phi_B \in (\phi_2, \phi_H)$ constitute a sunspot equilibrium. The sunspot equilibrium in this case is due to the fact that payoff is nonmonotone in Nash bargaining. As should be clear, the dynamical system with Nash bargaining shown in Figure 7 is very different from the dynamic system with the optimal contract shown in Figure 6. We conclude that in this model banking induces fragility for some parameters and the nature of fragility very much depends on details.

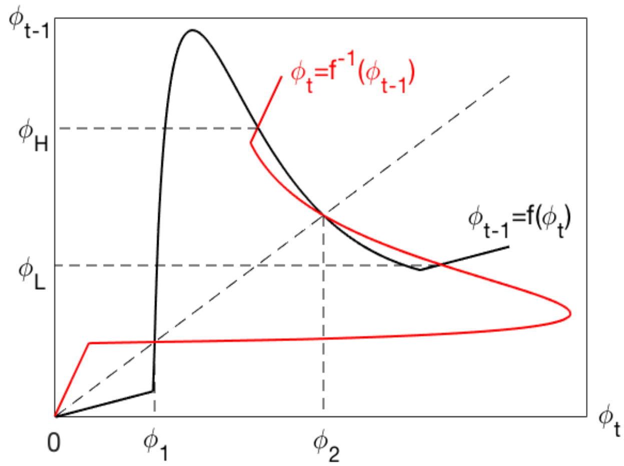

Figure 7: Delegated monitoring – Nash bargaining

Model 3: Asset Market Intermediation

We now introduce a model of intermediation in OTC asset markets using an environment that blends elements of Rubinstein and Wolinsky (1987) and Duffie et al. (2005). As in Duffie et al. (2005), at any point in time there are some agents that do not have assets but really value them, called buyers, and others that have assets but value them less, called sellers. They trade in a frictional market, which implies there may be a role for middlemen. Duffie et al. (2005) assume middlemen have a continuous access to a frictionless interdealer market. Our environment is similar, but without this frictionless interdealer market, making it more like Rubinstein and Wolinsky (1987). In that setup, middlemen can buy goods from producers, and hold them as inventories until a random time at which they contact consumers to whom they sell.[20] This occurs in equilibrium, as is efficient, if and only if middlemen have

[20]In Duffie et al. middlemen always offload their inventories in the interdealer market and do not hold any inventory. There are exceptions, e.g., Weill (2007). However, while his dealers hold inventories, he does not consider the possibility emphasized here, that they may hoard them – his dealers "hold assets only... to make some profit by buying and reselling." Also, he studies transitions after exogenous shocks, but not endogenous (belief-based) dynamics, which is our focus.

an advantage over sellers in terms of the speed with which they contact buyers.

In this model financial intermediaries do not engage in the activities usually associated with banking discussed in Models 1 and 2, like taking deposits, providing liquidity insurance, monitoring, etc. But it captures well the role of intermediaries as facilitators. With that in mind, the model is also relevant if we replace goods, middlemen, producers and consumers by, e.g., securities, dealers, low-valuation investors and high-valuation investors, or credit, banks, savers and borrowers.

In Rubinstein and Wolinsky (1987) and Duffie et al. (2005), there is a unique equilibrium and it converges to a unique steady state.[21] To study the equilibrium when agents trade assets, not goods, and middlemen use inventories, not interdealer markets, we get inspiration from Nosal et al. (2017), who generalize Rubinstein-Wolinsky to let middlemen and producers differ in bargaining powers, costs, and rates of return, not just search efficiency. To better fit our application several changes are made in the specification (see fn. 22).[22]

Our formulation has random bilateral matching, and restricts agents' asset holdings to $k \in \{0, 1\}$ for technical convenience (more on this below). In addition we highlight endogenous participation, or market composition, as also emphasized by, e.g., Nosal et al. (2017) and Farboodi et al. (2017), but we do it in a different way that is more natural for present purposes (see fn. 24). Also, we use discrete instead of continuous time, which more easily accommodates some modeling choices, and

[21] This is shown by Trejos and Wright (2016) for Duffie et al. (2005), who proved a lot, but not saddle-path stability. It is easy to verify the same for Rubinstein and Wolinsky (1987).

[22] In Nosal et al. (2017), equilibrium may or may not be efficient, and can involve too little or too much intermediation (e.g., middlemen could lower welfare but still profit from having high bargaining power, similar to Masters 2008 or Farboodi et al. 2017). There are also multiple equilibria in some versions of that model. The major differences between Nosal et al. (2017) and ours are: (1) Here time is discrete instead of continuous. It does not matter for steady-state result, but matters for dynamics. Buyers and sellers are assumed to be one-period lived in our model to rule out long-term relationship. (2) Agents enter freely instead of choosing occupation as in Nosal et al. (2017). (3) Buyers are heterogenous in terms of their productivity in our model to derive trades on the intensive margin. The middlemen choose reservation value. The frequency of trade differs in different equilibria, but market never shuts down. Technically, heterogeneity smoothes the dynamic system so it is easier to derive dynamic equilibria. (4) Goods depreciate in our model to be more natural and to avoid some technical issues.

more easily delivers interesting dynamics. Also, different from the related applications, we have heterogeneity in valuations across buyers; this requires additional work, because it becomes endogenous when potential buyers and middlemen trade, but also turns mixed- into pure-strategy equilibria and simplifies a few other things.

To begin the formalization, consider an infinite-horizon environment with a large number of agents of three types, labeled B, S and M, for buyers, sellers and middlemen. Type M agents stay in the market forever, while type S and B stay for just one period.[23] Upon exit, S and B are replaced by 'clones' to maintain stationarity, a specification borrowed from Burdett and Coles (1997), who show it tends to deliver uniqueness when alternative assumptions are more likely to generate multiplicity. Type B agents need an asset – let's call it capital – to implement a project for profit π. This is observable when agents meet, but random across type B with CDF $F(\pi)$. Realistically, in our three-sided market, B might get capital directly from S or indirectly from M.

A key component of the model is market composition and that depends on participation decisions. We consider two versions of the model: in the first, the measures n_m and n_b of types M and B are fixed, while entry by S makes n_s endogenous; in the second n_s and n_b are fixed while entry by M makes n_m endogenous. Both yield interesting results, but entry by S has this advantage: it lets us compare economies with and without middlemen while maintaining the same entry conditions; by comparison, with entry by M, going from $n_m = 0$ to $n_m > 0$ introduces intermediation and entry simultaneously.[24] To save space, we present entry by S in the main text

[23]In Rubinstein and Wolinsky (1987) type M also stay forever, while S and B exit after one trade. Nosal et al. (2015) show that having all agents stay forever simplifies the algebra without affecting the results too much. That is also true here, but we prefer to eliminate incentives to form long-term relationships. This issue, which is ignored in some earlier papers, is finessed with short-lived B and S – although, we hasten to say, it would be good to study enduring relationships in future work, perhaps along the lines of Corbae and Ritter's (2004) work in a different context.

[24]Nosal et al. (2017) take a different approach: they fix n_b and let the other agents choose to be either M or S. That would be quite awkward here because, e.g., cyclic equilibrium would have people switching back and forth between acting as intermediaries and low-valuation asset holders. In our benchmark model, instead, over the cycle successive waves of type S agents enter the market to varying degrees, while participation by M is constant.

and entry by M in the Appendix.

So for now, S agents either participate in the capital market or use their resources elsewhere, say investing abroad, or buying a new TV. Every S that enters brings to the market an endowment of 1 indivisible unit of capital. If he does not participate – say he buys a TV instead – S gets a payoff defining his opportunity cost of being in the market. Potentially he can earn a return on his capital while trying to sell it, and what matters is the opportunity cost minus the return, assumed to be $\kappa_s > 0$. Type M agents are always in the market, and at any point in time may or may not hold capital. In particular, we restrict their inventories to $k \in \{0, 1\}$, which can be relaxed, but that requires doing mostly numerical work.[25] Let n_t be the measure of type M at t with $k = 1$. Also, assume capital held by M depreciates stochastically, disappearing each period with probability $\delta \geq 0$. Type M agents earn return ρ from $k = 1$, where usually $\rho > 0$, but $\rho \leq 0$ can be interesting, too.

Each period every agent in the market contacts someone with probability α, and each contact is a random draw from the participants. If N_t is the total measure of participants at t then, e.g., types M and S both meet type B with the same probability, $\alpha n_b/N_t$. Hence, M has no advantage over S in searching for B. When B and S meet they trade for sure if $\pi > 0$, since this is S's only chance and cost κ_s is sunk. Similarly, when S meets M with $k = 0$ they trade for sure if $\rho > 0$. When M with $k = 1$ meets B they may or may not trade, depending on π, but also on endogenous considerations.

Next we discuss payoffs. All agents evaluate costs and returns linearly: if type j gives i capital k, the latter makes a payment p_{ij}, which means i produces p_{ij} units

[25] Plus there is much precedent for such restrictions. In addition to Rubinstein and Wolinsky (1987) and Duffie et al. (2005), $k \in \{0, 1\}$ is deployed to good effect in search theory going back to Diamond (1982), in monetary economics like Shi (1995) or Trejos and Wright (1995), in banking models like Cavalcanti and Wallace (1999*a,b*), and in models of partnership formation like Burdett and Coles (1997). Similarly, in labor models like Pissarides (2000), a vacancy can only be filled by $k \in \{0, 1\}$ workers, but users of those models often say a firm consists of many vacancies, some with $k = 1$ workers and others with $k = 0$; just as well we can say a financial institution consists of many M agents, some with $k = 1$ and others with $k = 0$. In any case, Nosal et al. (2017) argue that results like those derived below do not hinge on $k \in \{0, 1\}$.

of a divisible good at disutility cost $c(p) = p$ and type i consumes it for utility gain $u(p) = p$. In other words, there is transferable utility so that various standard bargaining theories (e.g., the generalized Nash and Kalai solutions) imply the same outcome. Let Σ_{ij} be the total surplus available when i and j meet; then as long as $\Sigma_{ij} > 0$, type i's surplus is $\theta_{ij}\Sigma_{ij}$, where $\theta_{ij} \in [0,1]$ is i's bargaining power when dealing with j.

Denote by $V_{s,t}$ and $V_{b,t}$ the value functions for types S and B, denote by $V_{k,t}$ the value function for M when he has $k \in \{0,1\}$, and let $\Delta_t = V_{1,t} - V_{0,t}$ be M's gain from acquiring inventory at t. Then

$$\Sigma_{bs,t} = \pi,\ \Sigma_{ms,t} = (1-\delta)\,\beta\Delta_{t+1},\ \Sigma_{bm,t} = \pi - (1-\delta)\,\beta\Delta_{t+1},$$

where $\beta \in (0,1)$ is M's discount factor.[26] Bargaining yields

$$p_{bs,t} = \theta_{sb}\pi,\ p_{ms,t} = \theta_{sm}\,(1-\delta)\,\beta\Delta_{t+1},\ p_{bm,t} = \theta_{mb}\pi + \theta_{bm}\,(1-\delta)\,\beta\Delta_{t+1}. \tag{25}$$

Hence, the probability M trades with B when the former has $k = 1$ and the latter has valuation π is $\tau_t = \tau\,(\pi, R_t)$, where

$$\tau\,(\pi, R) = \begin{cases} 0 & \text{if } \pi < R \\ [0,1] & \text{if } \pi = R \\ 1 & \text{if } \pi > R \end{cases} \tag{26}$$

and $R_t \equiv (1-\delta)\,\beta\Delta_{t+1}$ is the *reservation value* at which $\Sigma_{mb} = 0$.

Based on all this, the payoff for B with valuation π when the market opens is

$$V_{b,t}\,(\pi) = \frac{\alpha n_{s,t}}{N_t}\theta_{bs}\pi + \frac{\alpha n_t}{N_t}\tau\,(\pi, R_t)\,\theta_{bm}\,[\pi - (1-\delta)\,\beta\Delta_{t+1}]\,, \tag{27}$$

where N_t is the measure of all agents in the market, $n_{s,t}$ is the measure of type S and n_t is the measure of type M with $k = 1$. The first term is the probability B meets S, times his share of the surplus; the second is the probability B meets M with $k = 1$, times the probability they trade, times his share of the surplus; and

[26] Notice there are no continuation values or threat points for the short-lived S and B.

prices do not appear because they were eliminated using (25). Similarly, the payoff for any S in the market is

$$V_{s,t} = \frac{\alpha n_b}{N_t}\theta_{sb}\mathbb{E}\pi + \frac{\alpha(n_m - n_t)}{N_t}\theta_{sm}(1-\delta)\beta\Delta_{t+1}. \tag{28}$$

The payoffs for M depend on inventories. Using $R_t = (1-\delta)\beta\Delta_{t+1}$, we have

$$V_{0,t} = \frac{\alpha n_{s,t}}{N_t}\theta_{ms}R_t + \beta V_{0,t+1} \tag{29}$$

$$V_{1,t} = \rho + \frac{\alpha n_b}{N_t}\theta_{mb}\int_{R_t}^{\infty}(\pi - R_t)\,dF(\pi) + (1-\delta)\beta V_{1,t+1} + \delta\beta V_{0,t+1}. \tag{30}$$

Subtracting and simplifying with integration by parts, we arrive at

$$R_{t-1} = (1-\delta)\beta\left\{\rho + R_t + \frac{\alpha n_b\theta_{mb}}{N_t}\int_{R_t}^{\infty}[1-F(\pi)]\,d\pi - \frac{\alpha n_{s,t}\theta_{ms}}{N_t}R_t\right\}, \tag{31}$$

describing the evolution of the reservation value over time.

Next we need the evolution of the measure of M with $k=1$,

$$n_{t+1} = n_t(1-\delta)\left[1 - \frac{\alpha n_b}{N_t}\mathbb{E}\tau(\pi,R)\right] + \frac{(n_m - n_t)\alpha n_{s,t}(1-\delta)}{N_t}. \tag{32}$$

where $\mathbb{E}\tau(\pi,R)$ is the probability that M and a random B trade. Thus, the measure of M with $k=1$ next period is the measure this period, times the probably k neither depreciates nor gets traded, plus the measure of M with $k=0$, times the probability M acquires capital from S, and it does not depreciate.

Consider first an economy with no middlemen, $n_m = 0$. It is then routine to show there is a unique equilibrium (a formal definition of equilibrium is given below). Moreover, it is stationary, and $n_{s,t}$ is determined by $V_{s,t} = \kappa_s\ \forall t$. By design, without intermediation the outcome is basically a sequence of static equilibria and cannot display multiplicity or volatility.

Now consider $n_m > 0$. Entry by S still implies $V_{s,t} = \kappa_s$, which can be rearranged using (28) as

$$n_{s,t} = \frac{\alpha n_b\theta_{sb}\mathbb{E}\pi + \alpha(n_m - n_t)\theta_{sm}R_t}{\kappa_s} - n_b - n_m. \tag{33}$$

Inserting this as well as $N_t = n_b + n_m + n_{s,t}$ into (32) and (31), we have the two-dimensional dynamical system,

$$\begin{bmatrix} n_{t+1} \\ R_{t-1} \end{bmatrix} = \begin{bmatrix} f(n_t, R_t) \\ g(n_t, R_t) \end{bmatrix}, \tag{34}$$

where (45) and (46) in the Appendix shows the formulae for f and g.

Definition 1 *Given an initial* n_0*, equilibrium in Model 3, with entry by type* S*, is a nonnegative and bounded path for* (n_t, R_t) *solving (34). A steady state is a constant solution to (34).*

Notice that, as is standard, n is a backward- and R a forward-looking variable. Also, there is an initial condition for the former but not the latter (n is and R is not predetermined). Also notice $n \in [0, n_m]$ is obviously bounded, and when we say R is bounded we mean $\beta^t R_t \to 0$ as $t \to \infty$, the relevant transversality condition for this kind of model (e.g., see Rocheteau and Wright 2013). We can recover other variables from n and R, including prices, as well as the time between trades and the spread $p_{bm} - p_{ms}$, often suggested as good ways to measure frictions.

Consider first steady states. It is easy to verify that the locus of points satisfying $n = f(n, R)$, called the n-curve, and the locus satisfying $R = g(n, R)$, called the R-curve, both slope up in (n, R) space. Steady states obtain where they cross. As a preliminary step, consider the special case in Figure 8, where $\pi = \bar{\pi} > 0$ is the same for all type B agents. There are three possible *regimes* (kinds of steady state): (i) $R < \bar{\pi}$, which means M holding $k = 1$ and B trade with probability 1; (ii) $R > \bar{\pi}$, which means M holding $k = 1$ and B trade with probability 0; and (iii) $R = \bar{\pi}$, which means M holding $k = 1$ and B trade with probability $\tau \in (0, 1)$. The proof of the following is in the Appendix:

Lemma 1 *In Model 3, with entry by type* S *and* $\pi = \bar{\pi}$*, there exists at most one steady state in each regime.*

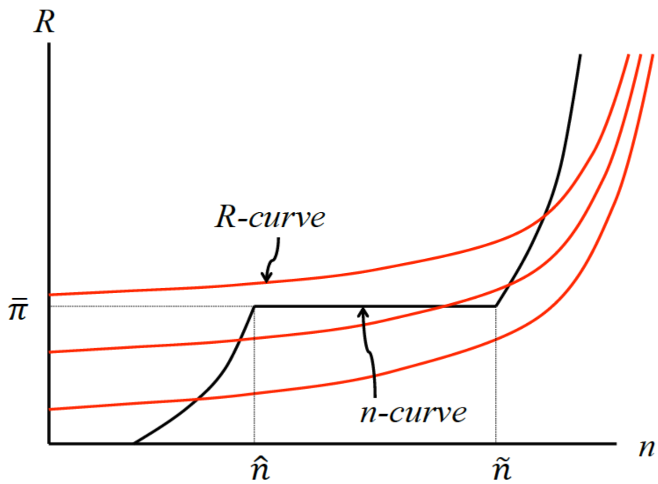

Figure 8: The n-curve and R-curve for different ρ when π is degenerate

It is easy to check the R-curve lies above the n-curve for low n and below it for high n, so steady state exists. For $\rho < \tilde{\rho}$ (defined by the point where the R-curve just touches the n-curve at $(\tilde{n}, \bar{\pi})$) in steady state M gets only low return from holding k and so M and B trade. So the curves cross where $R < \bar{\pi}$, and by Lemma 1 they cross just once. For $\rho > \hat{\rho}$ (defined by the point where the R-curve just touches the n-curve at $(\hat{n}, \bar{\pi})$), in steady state M and B never trade and this time $R > \bar{\pi}$. For intermediate levels of $\rho \in [\tilde{\rho}, r\hat{h}o]$ however, there are three possible steady states, one in each regime.

This is summarized as follows, with details of a proof in the Appendix:[27]

Proposition 3 *Consider Model 3 with entry by type S and $\pi = \bar{\pi}$. There exist $\tilde{\rho} > 0$ and $\hat{\rho} > \tilde{\rho}$ such that: (i) $\rho \in [0, \tilde{\rho})$ implies there is a unique steady state and it entails $R < \bar{\pi}$; (ii) $\rho \in (\hat{\rho}, \infty)$ there is a unique steady state and it entails $R > \bar{\pi}$; and (iii) $\rho \in (\tilde{\rho}, \hat{\rho})$ implies there are three steady states, one with $R < \bar{\pi}$, one with $R > \bar{\pi}$, and one with $R = \bar{\pi}$.*

[27]Note that $\tilde{\rho}$ and $\hat{\rho}$ always exist, but can be 0 for some parameter values; $\hat{\rho} > \tilde{\rho} > 0$ can be guaranteed under simple conditions.

Multiplicity is one manifestation of instability. To explain the intuition, first suppose type M agents trade k to B. Then the probability M has $k = 0$ is high, making it a sellers' market, so S is more inclined to enter. With more S in the market it is easier for M to get k, thus rationalizing his decision to trade with B. Now suppose type M agents do not trade k to B. Then the probability M has $k = 0$ is low, making it less of a sellers' market, so S is less inclined to enter. With fewer S in the market it is harder for M to get k, thus rationalizing his decision to not trade with B.

These results require $\rho > 0$. Without going into detail concerning $\rho < 0$, we can establish (see the Appendix) the following results.

Proposition 4 *Consider Model 3 with entry by type S, $\pi = \bar{\pi}$ and $\rho < 0$. If $|\rho|$ is too big there is no steady state where M acquires k from S. If $|\rho|$ is not too big there is a unique steady state where M acquires k from S and always trades with B,* $\Pr(R < \bar{\pi}) = 1$.

Market liquidity – the ease with which S can sell and B can buy assets – varies across regimes: it is high when $R < \bar{\pi}$, low when $R > \bar{\pi}$, and medium when $R = \bar{\pi}$. Liquidity is high when $R < \bar{\pi}$ for more than one reason: B can get the asset from M and not only S; that makes it more likely that M has $k = 0$, so S can more easily sell to M; that increases entry by S, so it is even easier for B to get the asset. Similar stories apply to $R > \bar{\pi}$ and $R = \bar{\pi}$. Since there can be multiple equilibria, we say that liquidity in intermediated asset markets is not pinned down by fundamentals.[28]

[28] This is a recurring theme in monetary theory at least since Kiyotaki and Wright (1989), but the intuition is different. In monetary economics, whether a seller accepts something as a medium of exchange depends on whether other sellers are accepting it. Here, whether an intermediary trades away something depends on whether other intermediaries are trading it away, which is different, and which we emphasize requires endogenous participation. A somewhat related effect is in Kaplan and Menzio's (2016) labor model, where higher unemployment frees up households' time for shopping, which leads to lower prices and profits, which means firms post fewer vacancies, consistent with higher unemployment. Another is Berentsen et al. (2010), where higher unemployment means fewer firms have output for sale so households hold less money, which reduces profits and again means firms post fewer vacancies, consistent with higher unemployment. While these are obviously different from our effect, they share the property that they work through entry decisions. Some other sources of multiplicity in search theory are discussed in Rocheteau and Tasci (2007).

Consider now a nondegenerate distribution $F(\pi)$. To begin, assume the support is $\mathcal{S} = [\bar{\pi} - \varepsilon_1, \bar{\pi} + \varepsilon_2]$, where ε_1 and ε_2 are not too big, with no gaps and no mass points. Then the situation looks similar to Figure but smoother and the results are qualitatively the same as Proposition 3: for intermediate values of ρ there are again three steady states, or regimes, one with $R < \bar{\pi} - \varepsilon_1$ where M and B always trade, one with $R > \bar{\pi} + \varepsilon_2$ where M and B never trade, and one with $R \in (\bar{\pi} - \varepsilon_1, \varepsilon_2)$ where M and B sometimes trade. However, these are all pure-strategy equilibria, as M and B are indifferent to trade only in the 0 probability event $\pi = R$.[29]

If the support $\mathcal{S}$ is bigger, there can be multiple equilibria of the same type – e.g., $\mathcal{S} = [0, \infty)$ implies all equilibria have trade between M and B occurring with probability strictly between 0 and 1. It is easy to construct examples where there are multiple steady states of a given type, so Lemma 1 does not hold for a general $F(\pi)$; in such cases, liquidity still varies across steady states, but more smoothly. In particular, with $\pi = \bar{\pi}$, in the steady state with $R > \bar{\pi}$, intermediation is frozen: type M adopt buy-and-hold-forever strategies (where forever means until k depreciates). In contrast, with a general $F(\pi)$, in a high R steady state M still trade with some B, but only those with very high π, and hence one might say the intermediation has gone cold but not completely frozen. The Appendix compares different steady states and presents pseudo dynamics.

We are more interested in fluctuations in real time.[30] Consider an example with a distribution that is smooth mean-preserving spread of the degenerate case:

$$F(\pi) = \begin{cases} \pi_1 \pi / \pi_0 & \text{if } 0 \le \pi \le \pi_0 \\ \pi_1 + (\pi_3 - \pi_1)(\pi - \pi_0) / (\pi_2 - \pi_0) & \text{if } \pi_0 < \pi \le \pi_2 \\ \pi_2 + (1 - \pi_3)(\pi - \pi_2) / (\pi_4 - \pi_2) & \text{if } \pi_2 < \pi \le \pi_4 \end{cases} \tag{35}$$

Example 7: Let $\pi_0 = 0.99$, $\pi_1 = 0.05$, $\pi_2 = 1.01$, $\pi_3 = 0.95$ and $\pi_4 = 2$. For the

[29] In other words, we effectively purified the mixed-strategy outcome. This is an advantage over, e.g., Nosal et al. (2017), with degenerate π and hence an intermediate equilibria involving mixed strategies, which is useful for the economics and the mathematics (more on this in fn. 30).

[30] Nosal et al. (2017) consider dynamics in continuous time, which is tricky because the system is not well behaved at their mixed-strategy steady state, where the n-curve is flat and the R-curve is kinked. Because that hinders standard local analysis, their simulations are suggestive, but the analytic properties are not known.

other parameters, let $\alpha = 1$, $\kappa_s = 0.1$, $n_b = 0.05$, $n_m = 0.5$, $\theta_{sm} = 0.5$, $\theta_{sb} = 1$, $\theta_{mb} = 0.7$, $\beta = 1/1.04$, $\delta = 0.008$ and $\rho = 0.2$. Figure 9 shows the n- and R-curves, which cross thrice at $(R, n) = (0.9098, 0.4231), (0.9997, 0.4415), (1.3963, 0.4743)$. Note that the n-curve is not flat, but almost flat, because this $F(\pi)$ is close to the degenerate case. We now show explicitly that the system admits cycles.

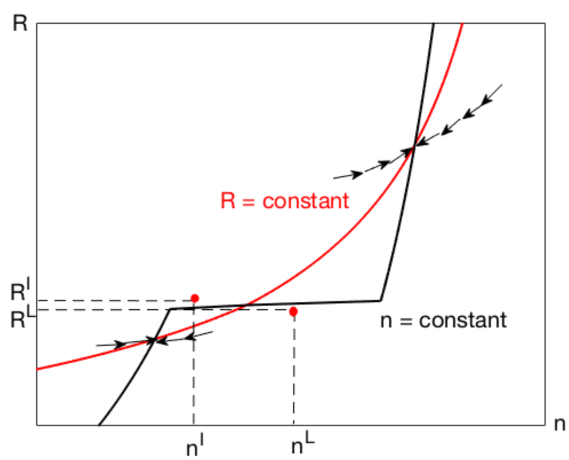

Figure 9: Phase plane in Model 3 with entry by S, including a two-cycle

Consider the simplest case of a two-cycle, where we oscillate between a liquid regime with low R and an illiquid regime with high R. If superscripts L and I denote the liquid and illiquid regimes, a two-cycle is a solution to

$$\begin{bmatrix} n^I \\ R^I \end{bmatrix} = \begin{bmatrix} f(n^L, R^L) \\ g(n^L, R^L) \end{bmatrix} \text{ and } \begin{bmatrix} n^L \\ R^L \end{bmatrix} = \begin{bmatrix} f(n^I, R^I) \\ g(n^I, R^I) \end{bmatrix},$$

with $R^L < R^I$. Thus, from $(n_t, R_t) = (n^L, R^L)$ we transit to $(n_{t+1}, R_{t+1}) = (n^I, R^I)$, then transit back. Figure 10 shows a two-cycle at $(R^L, n^L) = (0.9862, 0.4504)$ and $(R^H, n^H) = (1.0103, 0.4312)$ in Example 7, but by continuity there are similar outcomes for parameters in a nondegenerate neighborhood. Hence we say intermediated asset markets are susceptible to excess volatility, in the sense that liquidity, prices and quantities can vary over time even when fundamentals are constant.

Figure 10 shows the times series of Example 7. The liquid regime has these properties: (i) R is low, making M more inclined to trade with B; (ii) n is high, because M and B traded less last period; (iii) n_s is low, because low R and high n discourage entry by S; Next period, the illiquid regime has these opposite properties. Output y is higher in the liquid regime in this example, but that is not general: on the one hand, M pays less for the asset when its future value is low; on the other hand, lower entry raises the chance that an agent with capital meets a buyer, and M are more likely to trade with B. Of course the example is simplistic, and we do not claim that actual economic data are best explained as a cycle of period 2. The idea instead is this: if a simple model that abstracts from a great many complications can deliver equilibria where liquidity, prices and quantities vary over time as a self-fulfilling prophecy, it lends credence to the idea that this is something inherent to financial intermediation.

Prices (averaged over π in trade between M and S) are also shown in Figure 10. The (average) price B pays S is constant, since $F(\pi)$ is constant, but the price M pays S and the price B pays M move with R. The spread, which we recall is $s = \theta_{mb}\pi + (\theta_{bm} - \theta_{ms}) R$, can move either way, obviously, but here it moves against R. Although this is only an example, it is consistent with the data discussed in Comerton-Forde et al. (2010), who show the spread increases with inventories and decreases with revenues (they also offer an explanation, but different from the forces at work here). These dynamics are consistent with other stylized, e.g., inventories n are quite volatile over the cycle. While this is not a calibration, the fact that these patterns can be qualitatively consistent with observations lends further credence to the story.

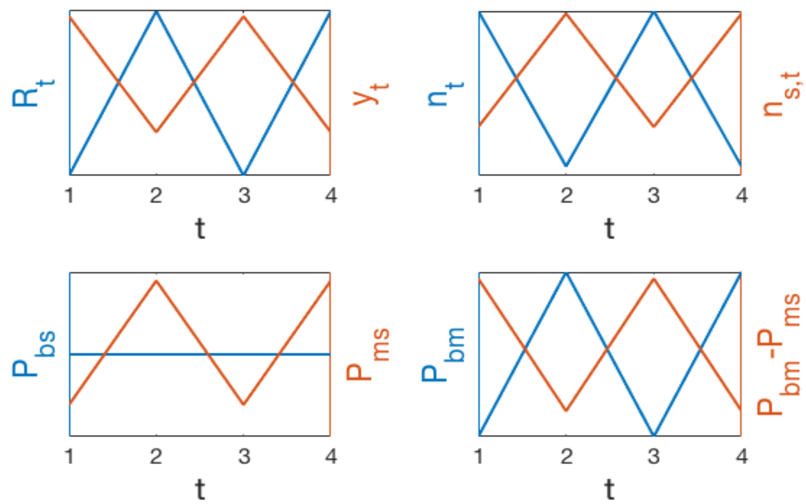

Figure 10: Time series for a two-period cycle in Model 3 with entry by S

As remarked above, $\rho > 0$ is a necessary condition for multiple steady states, at least for degenerate $F(\pi)$, because it implies M might hold inventories. Notice how endogenous market composition is crucial here, since it is entry by S that spurs M to trade with B with a greater probability, which reduces M's inventories and spurs entry by S. But all this can be self-fulfilling and we conclude that intermediation in asset markets is susceptible to self-fulfilling prophecies in ways that differ from goods markets, which may also display multiplicity, but not for this reason.

Model 4: Bankers as Safe Keepers and Secret Keepers

A very important role of banks is the issuance of circulable liabilities. Conventional wisdom has it that "banks are distinguished from other kinds of financial intermediaries by the readily transferable or 'spendable' nature of their IOUs, which allows those IOUs to serve as a means of exchange, that is, money... Commercial bank money today consists mainly of deposit balances that can be transferred either by means of paper orders known as checks or electronically using plastic 'debit' cards" (Selgin 2018). There are different reasons for banker's notes being valued: Bank notes may be more reliable as they are issued by agents with public history (Cavalcanti and Wallace 1999a,b); they may be relatively safe from loss or theft (He et al. 2005, 2007);[31,32] they may be "informationlly insensitive" thus providing stable liquidity (Gorton and Pennachi 1990; Andolfatto and Martin 2013; Andolfatto et al. 2014; Dang et al. 2017). The function of bank nots of providing more realiable liquidity is well-known in the literature. The monetary equilibrium is welfare improving but also subject to self-fulfilling prophecies. Here we use a standard model of monetary exchange in Lagos and Wright (2005) to capture bank's functions as safe keepers and secret keeper.

Safe Keepers

In this setup, in each period of discrete time, two markets convene sequentially: a decentralized market, or DM, with frictions detailed below; and a frictionless centralized market, or CM, that serves mainly to simplify the analysis. There is a set of agents called buyers, and another set of agents called sellers; they are all infinitely-lived. While in the DM their roles differ, in the CM all agents trade a numeraire consumption good x and labor ℓ, over which they have utility $U(x) - \ell$, with $U'(x) > 0 > U''(x)$. They also trade assets in the CM, formally just like the claims to 'trees' in the standard Lucas (1978) model, giving off a dividend ρ in each CM. This dividend, colorfully called 'fruit' in the literature, is here denominated

[31]It is clear that this is a key feature of banking in the context of history. To borrow from He et al. (2005,2007): "The direct ancestors of modern banks were the goldsmiths. At first the goldsmiths accepted deposits merely for safe keeping; but early in the 17th century their deposit receipts were circulating in place of money" (Encyclopedia Britannica 1954). "By the restoration of Charles II in 1660, London's goldsmiths had emerged as a network of bankers... Some were little more than pawnbrokers while others were full service bankers. The story of their system, however, builds on the financial services goldsmiths offered as fractional reserve, note-issuing bankers. In the 17th century, notes, orders, and bills (collectively called demandable debt) acted as media of exchange that spared the costs of moving, protecting and assaying specie" (Quinn 1997). Further, "They accepted deposits both on current and time accounts from merchants and landowners; they made loans and discounted bills; above all they learnt to issue promissory notes and made their deposits transferable by 'drawn note' or cheque" (Joslin 1954). Safekeeping was crucial for earlier banking, too, going back at least to the Templars, who were fierce fighters that specialized in protecting valuables moving back and forth during the Crusades (Weatherford 1997; Sanello 2003).

[32]See Chiu et al. (2018) for the most recent application of He et al (2005, 2007).

in numeraire, and captures the intrinsic value of commodity money historically, as well as the use of real assets as payment instruments or collateral in modern times. However, there is no problem with having fiat money as a special case where $\rho = 0$.

In the DM agents meet bilaterally to trade a different good q. Let α be the probability a buyer meets a seller, and a/n the probability a seller meets a buyer, where the measure of buyers is 1 and the measure of sellers is n. In any meeting, if a seller produces for a buyer the former incurs cost $c(q)$ and the latter enjoys utility $u(q)$, where $c(0) = u(0)$, $c'(q), u'(q) > 0$ and $c''(q) \geq 0 > u''(q)$. For well-known reasons – namely, goods are nonstorable, plus there is limited commitment and imperfect information about trading histories – credit is not viable. Hence, sellers only produce if they get something in exchange, which here means assets. The terms of trade are given by a generic bargaining solution: for a buyer to get q, he must transfer assets worth $v(q)$ in numeraire to a seller. A simple example is Kalai's bargaining solution, $v(q) = \theta c(q) + (1-\theta) u(q)$, with $\theta = 1$ corresponding to take-it-or-leave-it offers by buyers. We often use Kalai bargaining, but for now keep the generic notation $v(q)$, since most results apply more generally (e.g., everything goes through with Walrasian pricing, although that does not seem particularly natural in this environment).

Let $\mathbf{a} = (a_1, a_2, a_3)$ be a buyer's portfolio: a_1 denotes assets held in a safe but illiquid form; a_2 denotes assets held in a less safe but more liquid form, say in your pocket; and a_3 denotes assets deposited in a bank, which are more or less safe and about as liquid as a_2. As mentioned, safety here concerns the probably your assets get stolen, obviously hindering their use in DM payments.[33] Let δ_j be the probability a_j is stolen if you bring it to the DM, where $\delta_1 = 0$, so a_1 is perfectly safe, but generally a_2 and a_3 are not. Assume independence so that, e.g., $\delta_2(1-\delta_3)$

[33] Having assets stolen is not the only interpretation – e.g., they might get lost, the risk of which which is also insured by using Traveler's Checks. Or, a seller may trick you into accepting 'lemon' that provides no utility, which can be insured by using deposits, to the extent that one put a 'stop' on the check after discovering the trick. Also, any assets that are stolen (or lost) return to the system next period, either because the thieves (or finders) bring them to the CM, or because a new asset endowment appears, to maintain a stationary environment.

is the probability a_2 but not a_3 is stolen. If a_3 is stolen, we interpret it as a banker absconding with the deposits.[34]

Let r_j be the return on a_j, denominated in numeraire, where for simplicity $r_1 = r_2 = \rho$, while $r_3 = \iota$ is the endogenous yield on deposits. Bank's profit from having assets as deposits is

$$\Pi(a_3) = a_3(\rho - \iota) - k(a_3), \tag{36}$$

where $k(a_3), k'(a_3), k''(a_3) \geq 0$ is the cost of managing the funds. Profit maximization equates the spread $\rho - \iota$ to the marginal cost $k'(a_3)$.

The price of the asset in terms of numeraire is ϕ independent of what one does with it – i.e., whether one holds it as a_1, a_2 or a_3. Then $A = \Sigma_{j=1}(\phi + r_j)a_j$, a buyer's wealth at the start of the CM, is his state variable and $W(A)$ is his value function. However, the DM value function depends on the portfolio $V(\mathbf{a})$, not just its value. Thus a buyer's CM problem at t is

$$W_t(A_t) = \max_{x_t,\ell_t,\hat{\mathbf{a}}_t} \{U(x_t) - \ell_t + \beta V_t(\hat{\mathbf{a}}_t)\} \text{ st } x_t = A_t + \ell_t - \phi_t \Sigma_{j=1} \hat{a}_{j,t}$$

where $\hat{\mathbf{a}} = (\hat{a}_1, \hat{a}_2, \hat{a}_3)$ is a new portfolio and the CM real wage is 1 because we assume x is produced one-for-one using ℓ, merely to reduce notation. A seller's problem (not shown) is similar except, unlike buyers, they have no need for liquidity, so they only hold assets that are priced fundamentally (see below). Assuming an interior solution for buyers, we immediately have several standard results: (i) $x_t = x^*$, where $U'(x^*) = 1$; (ii) $W_t'(A_t) = 1$, so $W_t(A_t)$ is linear in wealth; and (iii) $\hat{\mathbf{a}}_t$ is independent of $\mathbf{a}_t$, so all buyers exit the CM with the same portfolio, satisfying the FOC's $\beta \partial V_{t+1}/\partial \hat{a}_{j,t} \leq \phi_t$, $= 0$ if $\hat{a}_{j,t} > 0$.[35]

[34] If it were interpreted instead as bank robbery more narrowly defined, one might ask if banks would be interested in providing insurance against such events; that won't work if the banker is in fact the culprit. It also will not help to spread your deposits around if all bankers act the same, as they do below, when $\delta_3 = 0$ or $\delta_3 = 1$. Of course, in modern times, the FDIC may help, but when we use $\delta_3 = 1$ it is only as a benchmark to see what happens without banks, and we are happy to say this represents the situation prior to FDIC.

[35] Results (ii) and (iii) especially are very useful simplifications that apply in these models, with quasi-linear utility, but that can be generalized in various ways (see the survey by Lagos et

While there is a discount factor $\beta \in (0,1)$ between the CM and next DM, without loss of generality agents do not discount between the DM and CM. Thus, a buyer's value function is

$$\begin{aligned} V_{t+1}(\hat{\mathbf{a}}_t) &= (1-\delta_2)(1-\delta_3)\left\{W_{t+1}(\hat{A}_{t+1}) + \alpha\left[u\left(q_{23,t+1}\right) - v\left(q_{23,t+1}\right)\right]\right\} \\ &\quad + \delta_2(1-\delta_3)\left\{W_{t+1}[\hat{A}_{t+1} - \left(\phi_{t+1}+\rho\right)\hat{a}_{2,t}] + \alpha\left[u\left(q_{3,t+1}\right) - v\left(q_{3,t+1}\right)\right]\right\} \\ &\quad + (1-\delta_2)\delta_3\left\{W_{t+1}[\hat{A}_{t+1} - \left(\phi_{t+1}+\iota\right)\hat{a}_{3,t}] + \alpha\left[u\left(q_{2,t+1}\right) - v\left(q_{2,t+1}\right)\right]\right\} \\ &\quad + \delta_2\delta_3 W_{t+1}[\hat{A}_{t+1} - \left(\phi_{t+1}+\rho\right)\hat{a}_{2,t} - \left(\phi_{t+1}+\iota\right)\hat{a}_{3,t}] \end{aligned}$$

where $\hat{A}_{t+1}$ is the wealth implied by $\hat{\mathbf{a}}_t$, and $q_{2,t+1}$ is the quantity transacted when paying with only $\hat{a}_{2,t}$, $q_{3,t+1}$ is the quantity when paying with only $\hat{a}_{3,t}$, and $q_{23,t+1}$ is the quantity when paying with both. In each transaction, the buyer's surplus is simply $u(q) - v(q)$, because of the result that $W(\cdot)$ is linear.

Equilibrium is described by the Euler equations, which we get from inserting the derivatives of the DM value function into the FOC's from the CM. First, to ease notation let $\lambda(q) = u'(q)/v'(q) - 1$ be the *liquidity premium* (i.e., the Lagrange multiplier on the constraint that a buyer cannot give a seller more assets than are available, which here means held as $\hat{a}_2$ or $\hat{a}_3$). Notice $\lambda(q_2) > 0$ and $\lambda(q_3) > 0$ because a buyer is always constrained after theft – intuitively, if he were to have some $\hat{a}_2$ left after a purchase when $\hat{a}_3$ is stolen, e.g., lowering $\hat{a}_2$ would be a profitable deviation as long as $\delta_2 > 0$. However, we can have $\lambda(q_{23}) = 0$ if ρ is big or $\lambda(q_{23}) > 0$ if ρ is small. In any case, the Euler equations are:

$$0 = \hat{a}_{1,t}\left[\beta\left(\phi_{t+1}+\rho\right) - \phi_t\right] \tag{37}$$

$$0 = \hat{a}_{2,t}\left\{\beta\left(\phi_{t+1}+\rho\right)(1-\delta_2)\left[1+\delta_3\alpha\lambda\left(q_{2,t+1}\right) + (1-\delta_3)\alpha\lambda\left(q_{23,t+1}\right)\right] - \phi_t\right\} \tag{38}$$

$$0 = \hat{a}_{3,t}\left\{\beta\left(\phi_{t+1}+\iota_t\right)(1-\delta_3)\left[1+\delta_2\alpha\lambda\left(q_{3,t+1}\right) + (1-\delta_2)\alpha\lambda\left(q_{23,t+1}\right)\right] - \phi_t\right\} \tag{39}$$

It is possible to have $\hat{a}_j > 0\ \forall j$, in general, because assets have heterogeneous

al. 2017). Also notice the way the environment is crafted so that buyers' income accrues in the CM but at least some of their expenditures occur in the DM. This means they need to use either credit or assets as payment instruments, and the former is precluded by commitment and information frictions (of course we can allow some credit, just not perfect credit, without changing much).

attributes: $\hat{a}_1$ is safe, illiquid and yields ρ; $\hat{a}_2$ is liquid, unsafe and yields ρ; and $\hat{a}_3$ is safe, liquid and yields ι, where it is possible that $\iota < \rho$. Moreover, buyers may want to hold both $\hat{a}_2$ and $\hat{a}_3$ for diversification – holding one partially insures against the loss of the other. We return to this below, and for now focus on cases where $\hat{a}_2 = 0$ or $\hat{a}_3 = 0$, so that buyers may use either $\hat{a}_2$ or $\hat{a}_3$ in the DM, but not both. To get $\hat{a}_3 = 0$, consider $\delta_3 = 1$, where bank deposits are *always* stolen. This can be interpreted in terms of the theory in Gu et al. (2013), where no one is trustworthy enough to be a banker, and trustworthiness depends endogenously on a potential banker's discount rate, his connection to the market, the monitoring technology that determines the probability he gets caught after absconding with the deposits, the available punishments, etc.

If $\delta_3 = 1$ then $\hat{a}_1 + \hat{a}_2 = 1$, where we normalize the aggregate asset supply to 1. Then the dynamical system implied by the model is derived as follows. At any date t, there are three possibilities or regimes: (i) agents hold nothing in their pockets, $\hat{a}_{2,t} = 0$; (ii) they hold some assets in their pockets, $0 < \hat{a}_{2,t} < 1$; and (iii) they hold all their assets in their pockets, $\hat{a}_{2,t} = 1$. In regime (i), inserting $\hat{a}_{1,t} = 1$ and $\hat{a}_{2,t} = 0$ into (37) and (38), we get $\phi_t = \beta(\phi_{t+1} + \rho)$ and $(1 - \delta_2)[1 + \alpha\lambda(0)] \leq 1$, with the latter equivalent to $\delta_2 \geq \hat{\delta}$ where

$$\hat{\delta} \equiv \frac{\alpha\lambda(0)}{1 + \alpha\lambda(0)}. \tag{40}$$

Thus, agents hold no assets in their pockets if the probability of theft is high. With Kalai bargaining and the Inada condition $u'(0)/c'(0) = \infty$, this reduces to $\hat{\delta} = \alpha\theta/(1 - \theta + \alpha\theta)$, so $\hat{\delta} = 1$ if $\theta = 1$ and $\hat{\delta} < 1$ otherwise. If (40) holds, the DM shuts down, in which case it should be obvious that the only possible outcome is $\phi_t = \phi^F\ \forall t$, where $\phi^F \equiv \beta\rho/(1 - \beta)$ is what we call the fundamental price.[36]

Now assume the theft probability is low, $\delta_2 < \hat{\delta}$, and consider regime (ii), where

[36] One might argue that ϕ^F should solve $\phi = \beta(1 - \delta_2)(\phi + \rho)$, not $\phi = \beta(\phi + \rho)$, since the asset holder only gets the return when the asset is not stolen. A rebutal is that someone gets the payoff, even if it is only the thief. This issue is purely semantic, and one can simply interpret ϕ^F as notation for $\beta\rho/(1 - \beta)$.

agents hold some but not all their assets in their pocket. Inserting $\hat{a}_{1,t}$,$\hat{a}_{2,t} > 0$ into (37) and (38), we get $\phi_t = \beta\left(\phi_{t+1} + \rho\right)$ and $(1 - \delta_2)\left[1 + \alpha\lambda\left(q_{2,t+1}\right)\right] = 1$, which means $q_{2,t+1} = \tilde{q}_2$ where

$$\alpha\lambda\left(\tilde{q}_2\right) = \frac{\delta_2}{1 - \delta_2}. \tag{41}$$

Thus, $\hat{a}_{2,t} < 1$ obtains iff $\phi_{t+1} + \rho > \hat{a}_{2,t}\left(\phi_{t+1} + \rho\right) = v\left(\tilde{q}_2\right)$, as well as $\delta_2 < \hat{\delta}$, as assumed so the DM can operate.

Finally, consider regime (iii), where agents hold all their assets in their pockets. Inserting $\hat{a}_{1,t} = 0$ and $\hat{a}_{2,t} = 1$ into (37) and (38), we now get $\phi_t \geq \beta\left(\phi_{t+1} + \rho\right)$ and

$$\phi_t = \beta\left(\phi_{t+1} + \rho\right)(1 - \delta_2)\left[1 + \alpha\lambda\left(q_{t+1}\right)\right], \tag{42}$$

where $q_{t+1} = v^{-1}(\phi_{t+1} + \rho) < \tilde{q}_2$. This last condition is equivalent to $\phi_{t+1} \leq \tilde{\phi} \equiv v\left(\tilde{q}_2\right) - \rho$. Hence the dynamic system is $\phi_t = \Phi\left(\phi_{t+1}\right)$ where:

$$\Phi\left(\phi\right) \equiv \begin{cases} \beta\left(\phi + \rho\right)(1 - \delta_2)\left[1 + \alpha\lambda \circ v^{-1}\left(\phi + \rho\right)\right] & \text{if } \phi < \tilde{\phi} \\ \beta\left(\phi + \rho\right) & \text{if } \phi \geq \tilde{\phi} \end{cases} \tag{43}$$

Definition 2 *An equilibrium is a nonnegative and bounded path for* $\phi_t = \Phi\left(\phi_{t+1}\right)$*. A stationary equilibrium is a steady state* $\bar{\phi} = \Phi(\bar{\phi})$*.*

Steady state is easy to characterize, in terms of the same three regimes, as shown in Figure 11.

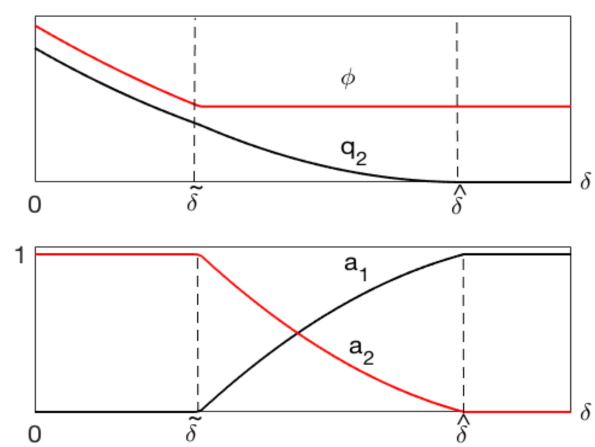

Figure 11: Regimes of Steady State

Proposition 5 *Without bankers, steady state exists, is unique, and is described as follows. Define* $\tilde{\delta} \in [0, \hat{\delta})$ *by*

$$\tilde{\delta} = \frac{\alpha\lambda \circ \upsilon^{-1}(\phi^F + \rho)}{1 + \alpha\lambda \circ \upsilon^{-1}(\phi^F + \rho)}. \tag{44}$$

Then (i) $\delta_2 \geq \hat{\delta}$ *implies* $\hat{a}_1 = 1$, $\hat{a}_2 = 0$ *and* $\bar{\phi} = \phi^F$*; (ii)* $\delta_2 \in (\tilde{\delta}, \hat{\delta})$ *implies* $\hat{a}_1 > 0$, $\hat{a}_2 > 0$ *and* $\bar{\phi} = \phi^F$*; and (iii)* $\delta_2 \leq \tilde{\delta}$ *implies* $\hat{a}_1 = 0$, $\hat{a}_2 = 1$ *and* $\bar{\phi} > \phi^F$*.*

In a regime (i) steady state the DM is inactive and $\bar{\phi} = \phi^F$, because assets are not safe enough to use as DM payment instruments.[37] In regime (ii) the DM is active at $q_2 = \bar{q}_2 > 0$, but since there is still some wealth parked in the illiquid $\hat{a}_1$, we again have $\bar{\phi} = \phi^F$. As Figure 11 shows, in regime (ii) $\partial q_2/\partial \delta_2 < 0$, because $\lambda(q)$ is decreasing in q (following Gu and Wright 2016, one can show λ must be decreasing at the equilibrium q, if not globally). Thus DM output goes down with δ_2 because it reduces output per trade, $\bar{q}_2$, as well as the number of trades, $1 - \delta_2$. The regime (iii) steady state is the most interesting, perhaps, since it maximizes 'cash in the market' with $\hat{a}_2 = 1$, and implies for $\bar{\phi} > \phi^F$ due to the liquidity premium. In this regime $\partial q_2/\partial \delta_2 < 0$ not because $\hat{a}_2$ falls but because ϕ falls with δ_2. Also notice that $\tilde{\delta}$ is decreasing in ρ, so for a given δ_2 the economy is more likely to be in regime (iii) when ρ is smaller, which means liquidity is relatively scarce.

As Figure 12 shows, the steady state can either be on the linear or the nonlinear branch of the system $\Phi(\cdot)$. In the former case, $\bar{\phi} = \phi^F > \tilde{\phi}$ and the steady is the only equilibrium (any other path for ϕ_t is unbounded). In the latter case, $\phi^F < \bar{\phi} < \tilde{\phi}$, and we could have cyclic, chaotic or stochastic equilibria where, ϕ_t oscillates around $\bar{\phi}$, which as usual happens when $F'\left(\bar{\phi}\right) < -1$. So the economy without banking may have asset prices above their fundamental value and they exhibit excess volatility, but only if $\hat{a}_2 = 1$ in steady state, which requires $\delta_2 < \tilde{\delta}$ and hence ρ not too big.

Now suppose banking becomes viable, so $\hat{a}_3 > 0$ is possible. In terms of the theory in Gu et al. (2013), this means parameters have changed so that some agents

[37] The DM can be active in regime (i) if we allow some barter or some credit, as is easy to do, but still assets would not be used in payments, and so again $\phi_t = \phi^F$ $\forall t$.

are now trustworthy enough to be bankers, e.g., the technology to monitor them has improved, or it could simply be that regulations banning their operation were removed. Also, suppose for now bankers have no cost of managing funds, $k(a_3) = 0$, which means $\iota = \rho$ (this is relaxed below). Moreover, suppose for now that banks are sufficiently trustworthy that $\hat{a}_3 > 0$ and $\hat{a}_2 = 0$, which is certainly true in the limit as $\delta_3 \to 0$, but could also be true for $\delta_3 > 0$. Then deposits pay high interest, $\iota = \rho$, and are perfectly liquid and are safe, so $\hat{a}_3 = 1$ unless buyers are satiated in liquidity, $\lambda(q_3) = 0$, in which case we can have $\hat{a}_1 > 0$ and $\hat{a}_3 = 1 - \hat{a}_1 > 0$.

Under these conditions, the economy looks similar to the one without banking except, we replace δ_2 with 0. In terms of Figure 12, this shows up as a shift upward in the nonlinear branch of $\Phi(\cdot)$. In general, this shift increases $\tilde{\phi}$, as well as $\bar{\phi}$, naturally, since agents can now park their assets in a safe place – the bank – and still be liquid. This is precisely what banking achieves, as discussed in the historical, if not the theoretical, literature. Notice that allowing banks, like reducing δ_2 in the economy without banks, increases DM output both because it increases output per trade and the number of trades. But what does it do for volatility?

Start without banks, and suppose the steady state is on the linear branch of $\Phi(\cdot)$, so there is a unique equilibrium and $\phi_t = \phi^F \; \forall t$. Then introduce banks. This could shift the nonlinear branch of $\Phi(\cdot)$ up by enough that the new steady state is now on the nonlinear branch. Then it is obvious that the introduction of banking makes possible cyclic, chaotic or stochastic equilibria that were impossible without banking. As mentioned, it could be that these phenomena were possible without banking, but the following is now clear: if the economy has a unique equilibrium with banking where $\phi = \phi^F$, the same is true without banking. We summarize as follows:

Proposition 6 *Consider an economy without banking that has a unique equilibrium, the steady state with $\bar{\phi} = \phi^F$. (i) If we introduce banking it is possible to get $\bar{\phi} > \phi^F$, which is good for DM trade in steady state, but can introduce new nonsta-*

tionary equilibria. (ii) When the economy with banking has a unique equilibrium, it is a steady state with $\bar{\phi} = \phi^F$, the same as the economy without banking.

Part (i) of the above result says that banks may engender volatility in a precise sense. Part (ii) says that in a sense they cannot eliminate volatility, since if there is a unique equilibrium with $\phi_t = \phi^F$ $\forall t$ after introducing banking, there is also a unique equilibrium with $\phi_t = \phi^F$ $\forall t$ before banking. As an explicit example, consider

$$u(q) = \frac{A}{1-\sigma}\left[(q+\varepsilon)^{1-\sigma} - \varepsilon^{1-\sigma}\right],\ c(q) = q,$$

and bargaining with $\theta = 1$.

Example 8: Let $A = 0.15$, $\sigma = 3.1$, $\varepsilon = 0.16$, $\rho = 0.033$, $\beta = 0.8333$, $\delta_2 = 0.85$ and $\alpha = 1$. Without banks, this economy has a unique equilibrium and it is the steady state with $\bar{\phi} = \phi^F = 0.1650$. If we now introduce banks with $\delta_3 = 0$ it has a steady state of $\bar{\phi} = 0.3183$ and a two-cycle of $\phi_L = 0.3193$ and $\phi_H = 0.3502$. Figure 12 shows the economy with and without bank notes.

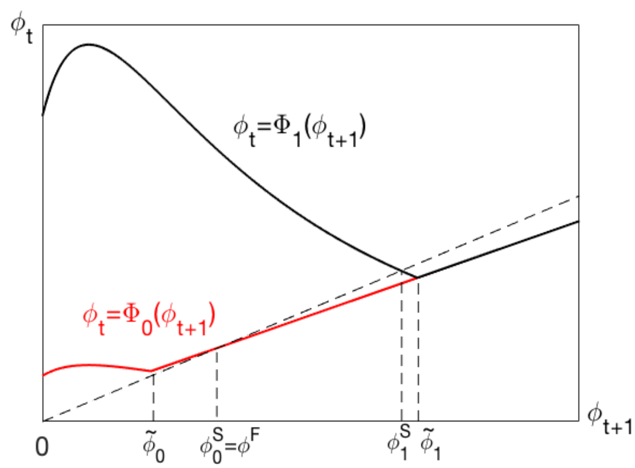

Figure 12: Dynamic equilibrium

While examples suffice to make the point that, under some conditions, banking can contribute to volatility, we also want to relax a few of the assumptions to see what else the model can generate. Here we ask if it can generate concurrent circulation of assets and bank liabilities in the DM. To this end we stick to the limit $\delta_3 \to 0$, but, so that $\hat{a}_3$ does not strictly dominate $\hat{a}_2$, we assume banks have cost function $k(a_3)$. Assume $k'(a_3) > 0$ and $k''(a_3) \geq 0 \; \forall a_3 > 0$, and impose the Inada conditions $k'(0) = 0$ and $k'(1) = \infty$. Then banks maximize $\Pi(a_3)$ by issuing deposits until $\rho - \iota = k'(a_3)$, defining a supply curve that is decreasing in $\rho - \iota$, with $a_3 \to 0$ as $\iota \to \rho$ and $a_3 \to 1$ as $\iota \to 0$. The demand for $\hat{\mathbf{a}}$ comes from the (37)-(39) with $\delta_3 = 0$ and $\delta_0 = 1 - \delta_2$. It is easy to show there exists a steady state in which both assets and bank notes are used as means of payments.

More generally, we can derive dynamics from (37)- (39). The following example gets cycles in which the issuance of bank notes fluctuates endogenously with changes in price.

Example 9: Let $A = 2.5$, $\sigma = 2.5$, $\varepsilon = 0.001$ $\rho = 0.04, \beta = 0.8$, $\delta_2 = 0.01$ and $\alpha = 1$. For bank's technology, assume $k' = 0.03 \; \forall a_3$ so $\iota = 0.01$. There is a unique steady state in which $\bar{\phi} = 1.3125$ and $\hat{\mathbf{a}} = (0, 0, 1)$. There also exists a two-cycle in which $\phi_L = 1.2128$, $\hat{\mathbf{a}}_L = (0, 0, 1)$, $\phi_H = 1.4760$ and $\hat{\mathbf{a}}_H = (0.0384, 0.2293, 0.7323)$. The price of asset fluctuates around the steady state. However, the value of deposit $(\phi_t + \iota_t) a_{3,t}$ is smaller in both periods of a cycle. Figure 13 plots the time series of asset price ϕ, the amount of deposit a_3, value of deposit $(\phi + \iota) a_3$, and trade surplus at t in this example. When ϕ is low, the return on asset is high, and agents can afford to purchase more deposit. The value of deposit moves with a_3 in this example even though ι decreases with a_3. The trade surplus moves against a_3 as when a_2 is not stolen, the total liquidity is higher, so is the expected surplus.

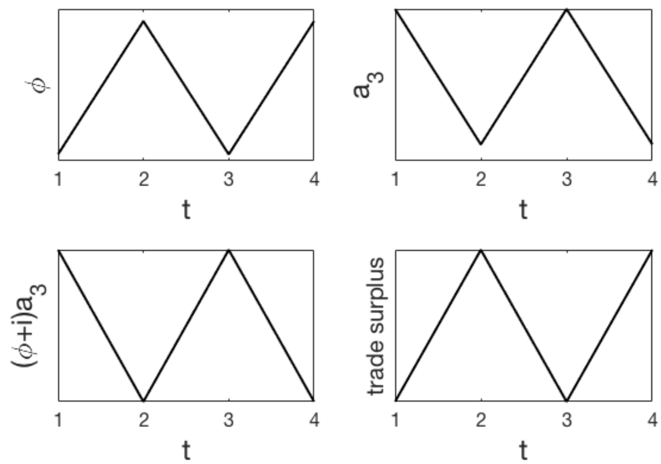

Figure 13: Time series for a two-period cycle in Model 4

Safe Keepers

We continue to use Lagos and Wright (2005) setup to study another feature of banking, banks as a secret keepers (see Dang et al. 2017 and Andolfatto and Martin 2013). The setup is the same as Model 4 except that the asset pays a stochastic dividend and that asset is the only means of payment in the DM. The dividend is realized in the beginning of the DM. The CM problem is the same as in Model 4 except that $\{\rho_t\}$ is random.

$$W_t(a_t, \rho_t) = \max_{x_t, \ell_t, \hat{a}_t} \{U(x_t) - \ell_t + \beta V_{t+1}(\hat{a}_t)\} \text{ st } x_t = (\phi_t + \rho_t)a_t + \ell_t - \phi_t \hat{a}_t$$

In the DM agents trade given the realization of the dividend. So the DM value is

$$V_{t+1}(\hat{a}_t) = \int_{\rho_{t+1}} \left\{\alpha \left[u\left(q_{t+1}\right) - v\left(q_{t+1}\right)\right] + W_{t+1}\left(\hat{a}_t, \rho_{t+1}\right)\right\} d\rho_{t+1}$$

where $v(q_{t+1}) = \left(\phi_{t+1} + \rho_{t+1}\right)\hat{a}_t$ if $\left(\phi_{t+1} + \rho_{t+1}\right)\hat{a}_t < v(q^*)$, and $v(q_{t+1}) = v(q^*)$ otherwise.

Normalize the supply of asset to 1. Derive the equilibrium from the Euler equation wrt a_t as

$$\phi_t = f\left(\phi_{t+1}\right) \equiv \beta \int_{\rho_{t+1}} \left(\phi_{t+1} + \rho_{t+1}\right) \left[1 + \alpha\lambda\left(q_{t+1}\right)\right] d\rho_{t+1}$$

where $\lambda\left(q_{t+1}\right)$ is the liquidity premium for each realization of ρ_{t+1}.

Suppose the bank has a costless technology to prevent the information of dividend from being revealed. Banks issue bank notes in the CM in exchange for asset. Bank notes are redeemed in the next CM. Using bank notes enables agents to trade in the DM by expected dividend instead of the realization of dividend. The DM value thus is

$$V_{t+1}(\hat{a}_t) = \alpha\left[u\left(q_{t+1}\right) - v\left(q_{t+1}\right)\right] + \int_{\rho_{t+1}} W_{t+1}\left(\hat{a}_t, \rho_{t+1}\right) d\rho_{t+1}$$

where $v\left(q_{t+1}\right) = \left(\phi_{t+1} + \mathbb{E}\rho_{t+1}\right)\hat{a}_t$ if $\left(\phi_{t+1} + \mathbb{E}\rho_{t+1}\right)\hat{a}_t < v\left(q^*\right)$ and $v\left(q_{t+1}\right) = v\left(q^*\right)$ otherwise. With the asset market clearing condition, the equilibrium path of ϕ_t derived from the Euler equation is

$$\phi_t = g\left(\phi_{t+1}\right) \equiv \beta\left(\phi_{t+1} + \mathbb{E}\rho_{t+1}\right)\left[1 + \alpha\lambda\left(q_{t+1}\right)\right]$$

where $\lambda\left(q_{t+1}\right)$ is the liquidity premium on $\phi_{t+1} + \mathbb{E}\rho_{t+1}$.

Notice that the price depends on the curvature of liquidity premium, not the agent's attitude toward risk. It is hard to tell in general how the introduction of bank note changes the asset price and welfare. In some special case, for example, if ρ is small for some realization but $\mathbb{E}\rho$ is large enough that $\phi = \mathbb{E}\rho/r$ and $(1+r)\,\mathbb{E}\rho/r > v\left(q^*\right)$, using bank notes ensures that DM trade achieves q^* in all periods, which is the only equilibrium. Without bank notes, q flutuates between efficient and inefficient levels. We use a less obvious example to show that using bank notes improves steady state allocation. Consider the functions and the terms of trade in Model 4. Let ρ be iid across time and take two possible values, ρ_H and ρ_L, with equal probability

$$u\left(q\right) = \frac{b_0}{1-b_1}\left[\left(q+b_2\right)^{1-b_1} - b_2^{1-b_1}\right], c\left(q\right) = q,$$

and let buyer get full bargaining power.

Example 10 Let $\beta = 0.7$, $\alpha = 1$, $b_1 = 3.5$, $b_2 = 0.1$, $b_0 = 1 - b_1$, $\rho_H = 0.5$ and $\rho_L = 0$. Without banking, $\phi = 0.8363$ $q_H = q^* = 0.9$ and $q_L = 0.8363$ in steady state. With banking, $\phi = 0.6421$ and $q = 0.8921$. In this example, the expected trade surplus is higher with bank notes.

However, it is possible that banking is subject to fragility in the sense that the economy with banking is more likely to generate multiple equilibria, cycles and sunspot. Figure 14 shows the equilibrium system in Example 10 with and without bank notes. Steady state is the only equilibrium in the economy without banking. Whereas there is a two-period cycle $(\phi_1, \phi_2) = (0.6337, 0.6553)$ if banking is introduced.

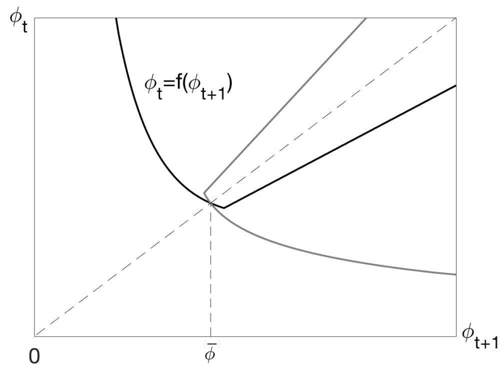

Figure 14a: Equilibrium w/o banks

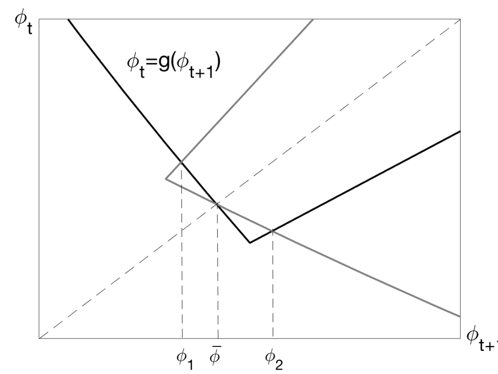

Figure 14b: Equilibrium with banks

Conclusion

We presented four models of banking/financial intermediation and compared the equilibrium sets with and without financial institution. In the first model of Diamond-Dybvig type of banking, we found that without the bank, the only equilibrium is autarky. Introducing bank improves welfare. The economy has a unique positive steady state. It can admit other equilibria, including cycles, chaos and sunspots. In the second model, bank performs the function of delegated investment.

Without banks, the economy wastes resource on fixed cost. Introducing banks improves welfare. But the economy is subject to sunspots and cycles. In the third model of asset market intermediation, the equilibrium set includes multiple steady states, cycles, chaos and sunspots whereas if the equilibrium is unique if there were no intermediation. Welfare may be improved with intermediation. In the fourth model of bank notes, introducing banking enlarges the set of parameters that admit exotic dynamics. The welfare effect is again ambiguous.

Banking or financial intermediation arises endogenously in all our models, which implies that market does not achieves the first-best. There is a long-time conjecture that in an economy where the first welfare theorem fails, there is room for sunspot equilibrium. Our finding confirms such a conjecture. Perhaps this is the major difference between models of banks and models with banks. In the latter case, there can be little room for belief-driven dynamics caused by banking.

TBD

Appendix A: Proofs

Proof of Lemma 1: Given $\pi = \bar{\pi}$, the equations for R-curve and n-curve can be written $R = g(n, R)$ and $n = f(n, R)$ defined by

$$\left(\frac{r+\delta}{1-\delta} + \alpha\theta_{ms}\right) R - \rho - \frac{\alpha n_b \theta_{mb} \tau (\bar{\pi} - R) + \alpha (n_b + n_m) \theta_{ms} R}{N} = 0 \quad (45)$$

$$\delta n + n(1-\delta)\frac{\alpha n_b \tau}{N} - (n_m - n)(1-\delta)\alpha\left(1 - \frac{n_b + n_m}{N}\right) = 0 \quad (46)$$

where

$$N = \frac{\alpha n_b \theta_{sb} \bar{\pi} + \alpha (n_m - n) \theta_{sm} R}{\kappa_s}.$$

The steady state is a fixed point of (45) and (46).

In the region of $R > \bar{\pi}$, where $\tau = 0$, combine (45) and (46) to eliminate N,

$$\left(r + \delta + \frac{\theta_{ms}\delta n}{n_m - n}\right) R = \rho (1-\delta) \quad (47)$$

This implies

$$\frac{\partial R}{\partial n} = -\frac{\theta_{ms}\delta n_m}{(n_m - n)^2 (r+\delta) + (n_m - n)\theta_{ms}\delta n} < 0.$$

Here we transform the fixed point problem of (45) and (46) to the fixed point problem of (47) and (46). As (47) is downward sloping and (46) is upward sloping, there exists at most one fixed point.

In the region of $R < \bar{\pi}$, where $\tau = 1$, combine (45) and (46) to get

$$\left(\frac{r+\delta}{1-\delta} + \alpha\theta_{ms}\right) R = \rho + \frac{n_b\theta_{mb}(\bar{\pi} - R) + (n_b + n_m)\theta_{ms} R}{(1-\delta) n_m (n_b + n_m - n)} \left[(n_m - n)(1-\delta)\alpha - n\delta\right].$$

This implies

$$\frac{\partial R}{\partial n} = -\frac{n_b\theta_{mb}(\bar{\pi} - R) + (n_b + n_m)\theta_{ms} R}{r + \delta + (\theta_{ms} + n_b\theta_{mb})(1-\delta)\alpha/N} \frac{\delta(n_b + n_m) + (1-\delta)\alpha n_b}{n_m (n_b + n_m - n)^2} < 0 \quad (48)$$

Again, since (48) is downward sloping and (46) is upward sloping, there exists at most one fixed point.

Finally, when $R = \bar{\pi}$ and $\tau \in (0, 1)$, the n-curve is flat and R-curve is upward sloping. Hence, there again there exists at most one fixed point, and that completes the proof. ■

Proof of Proposition 3: It is easily verified that R-curve and n-curve are upward sloping. At $n = 0$ the R-curve implies $R > 0$ and the n-curve implies

$n > 0$. At $R = \infty$ the R-curve implies $n = n_m$ and the n-curve implies $n = \bar{n} \equiv \alpha n_m (1-\delta) / [\delta + (1-\delta)\alpha] < n_m$. Therefore, the curves cross at least once, and generically they cross an odd number of times. By Lemma 1, there is at most one steady state of each type. Hence, if there exists a steady state at $R = \bar{\pi}$, there must exist two other steady states, one with $R < \bar{\pi}$ and one with $R > \bar{\pi}$.

Next we show R-curve shifts up with ρ. Holding n constant, we derive from (45)

$$\frac{\partial R}{\partial \rho} = \frac{1}{\frac{r+\delta}{1-\delta} + \alpha \frac{n_b \theta_{mb} \tau + n_l \theta_{ms}}{N} + \frac{n_b \theta_{mb} \tau (\bar{\pi} - R) + (n_b + n_m) \theta_{ms} R}{N^2} \frac{\alpha^2 (n_m - n) \theta_{sm}}{\kappa_s}}$$

Hence $\partial R / \partial \rho > 0$, as shown in Figure 10, and so there exist $\tilde{\rho}, \hat{\rho} \geq 0$ with the properties specified in Proposition 3. ■

Proof of Proposition 4: It is obvious that $\rho < 0$ with $|\rho|$ big implies implies M will not accept k from S. Suppose $\rho < 0$ with $|\rho|$ not so big. Then M can accept k from S, and we claim. First we establish the generalized result discussed in the text: for any $F(\pi)$ we have $\Pr(R < \pi) > 0$. Suppose not. Then from (31),

$$R \left[1 - \beta (1-\delta) \left(1 - \frac{\alpha n_s \theta_{ms}}{N} \right) \right] = \beta (1-\delta) \rho. \tag{49}$$

The LHS is positive, while the RHS is negative for $\rho < 0$. This contraction establishes $\Pr(R < \pi) > 0$ for a general $F(\pi)$. For the special case where $\pi = \bar{\pi}$, suppose $\tau = \Pr(R < \pi) < 1$. From the general case we know $\tau \neq 0$, so this means $\tau \in (0,1)$. Now (31) again implies (49), which is still a contradiction for $\rho < 0$. This establishes that $\tau = 1$ for $\rho < 1$ when $F(\pi)$ is degenerate. ■

Proof of Proposition 7 First note that $G(0) = \infty$, so $G(R)$ starts above the 45^o line. Also, $G(R) = \beta (1-\delta) [\rho + R - (1-\beta) \kappa_m]$ for R above the upper bound of π, so it is linear with $G'(R) < 1$ when R is big. Hence steady state exists. From (52) we derive

$$\Phi'(R) = \beta (1-\delta) \left\{ 1 - \frac{(1-\beta) \kappa_m n_b \theta_{mb}}{n_s \theta_{ms} R} \left[\frac{\int_R [1 - F(\pi)] \, d\pi}{R} + 1 - F(R) \right] \right\} < 1$$

Therefore, steady state is unique. If $\Phi'(R) < -1$, there exists two-period cycle and thus sunspot equilibria. ■

Appendix B: Derivation of Sunspot Equilibrium in Models 1 and 2.

If a dynamical system allows for a two-state stationary sunspot equilibrium, equilibrium is represented by

$$\phi_{s,t-1} = \rho_s f\left(\phi_{s,t}\right) + (1-\rho_s) f\left(\phi_{-s,t}\right) \tag{50}$$

where $s = A, B$ denotes two states in the sunspot equilibrium, $\rho_s \in (0,1)$ is the probability of staying in the same state, and f is the function of the dynamical system in the deterministic case (i.e., (12) in Model 1 and (19) in Model 2). We seek a pair of probabilities $(\rho_A, \rho_B) \in (0,1)^2$ satisfying (50). Rewrite (50) as

$$\rho_A = \frac{f(\phi_B) - \phi_A}{f(\phi_B) - f(\phi_A)} \text{ and } \rho_B = \frac{\phi_B - f(\phi_A)}{f(\phi_B) - f(\phi_A)}$$

WOLG, suppose $\phi_B > \phi_A$. If f is decreasing on (ϕ_A, ϕ_B), the denominator is negative. The necessary and sufficient condition for $\rho_A, \rho_B \in (0,1)$ is $f(\phi_A) > \phi_B > \phi_A > f(\phi_B)$, which implies that f crosses the 45 degree line from above and $[f(\phi_A) - f(\phi_B)] / (\phi_A - \phi_B) < -1$. Therefore, in Model 1 where f is decreasing around the steady state, there exist sunpot equilibria if $f\left(\bar{\phi}\right) < -1$.

Similarly, if f is increasing on (ϕ_A, ϕ_B), the denominator is positive. The necessary and sufficient condition for $\rho_A, \rho_B \in (0,1)$ is $f(\phi_B) > \phi_B > \phi_A > f(\phi_A)$, which implies f crosses the 45 degree line from below on $[\phi_A, \phi_B]$. Therefore, in Model 2 where f is increasing, there exist sunspot equilibria around the stable steady state ϕ_1, and any $\phi_A \in (0, \phi_1)$ and $\phi_B \in (\phi_1, \phi_2)$ satisfy the condition and constitute a two-state stationary sunspot equilibria.

Appendix C: More on Model 3

I. Steady State Comparison

Figure 15 shows the values of several variables for different ρ, as correspondences, since there are multiple steady states for intermediate values of ρ. The endogenous variables are: in the left panel are n and R; in the middle panel are welfare W and output y; and in the right panel are volume v and $1/s$, where s is the spread, two ways by which people try to measure liquidity. One point to mention is that when multiple steady states exits, W and y are both higher in ones with lower n and R, and the same is true for v and $1/s$ are both higher in ones with lower n and R. However, a more liquid steady state does not always mean a lower spread: from (25), $s = \theta_{mb}\pi + (\theta_{bm} - \theta_{ms}) R$, so lower R goes with lower s iff $\theta_{bm} > \theta_{ms}$. This should perhaps be interpreted as words of caution about interpreting liquidity in terms of spreads empirically or theoretically.

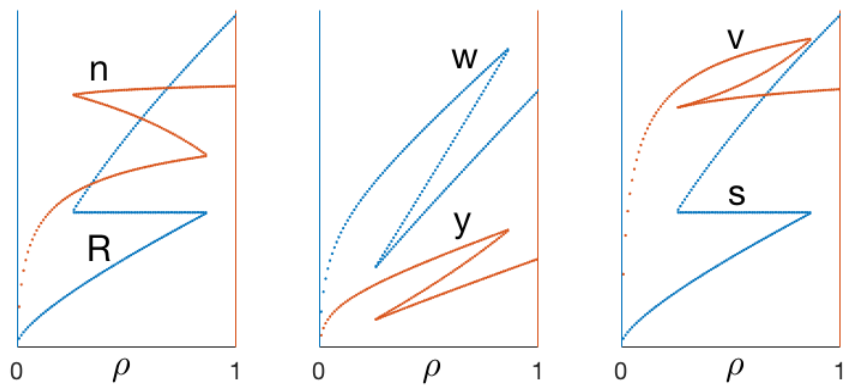

Figure 15: Steady state variables as functions of ρ

Figure 15 can also be used to illustrate a notion of fragility in terms of pseudo-dynamics – if we may be allowed to engage in that for a moment. Start with $\rho < \tilde{\rho}$, so there is a unique steady state where R and n are both low and liquidity is high. Then increase ρ, so R and N increase, until we reach $\tilde{\rho}$, at which point there are now three steady states. Continuing to increase ρ, suppose we stay on the lowest branch of the correspondences until reaching $\hat{\rho}$, at which point a further miniscule increase in ρ causes the lowest branch of the correspondences vanish, and yields a catastrophe in the mathematical and the economic sense: a quantum drop in liquidity when R jumps in response to a small change in ρ. Indeed one might reasonably call this an intermediation (or liquidity or financial) crisis, involving a big reduction in M's willingness to trade, and hence in entry by S, and hence in the rate at which B acquires the asset, and the fact that it can result from small changes in parameters is consistent with the discussion in fn. 1.

II: Entry by M or B in Model 3

Here we consider entry by M with n_b and n_s fixed, both in the interest of robustness, and to make an additional point: while multiple steady states and complicated dynamics are both consistent with the notion of instability, we do not always need multiple steady states to get multiple and complicated equilibrium dynamics. The equations (28)-(32) are the same with entry by M, instead of S, except now n_s is constant while $n_{m,t}$ can vary with t. What changes is the participation condition: $V_{s,t} = \kappa_s$ is replaced by $V_{0,t} = \kappa_m$, assuming that M must acquire his inventory after entry.

Given this, (29) yields a simple expression for N_t in terms of R_t,

$$N_t = \frac{\alpha n_s \theta_{ms} R_t}{(1-\beta)\kappa_m}. \tag{51}$$

Notice N_t depends only on R_t, while in the model with entry by S, (33) implies N_t depends on R_t and n_t. Substituting (51) into (31), after minor algebra we get $R_{t-1} = G(R_t)$, where

$$G(R) \equiv \beta(1-\delta)\left\{\rho + R + \frac{(1-\beta)\kappa_m n_b \theta_{mb}}{n_s \theta_{ms} R}\int_R^\infty [1-F(\pi)]\,d\pi - (1-\beta)\kappa_m\right\} \tag{52}$$

Notice from (52) that R_{t-1} depends only on R_t, while in the other model it depends on R_t and n_t. This make entry by M much easier.[38]

In particular, similar to Models 1 and 4, we can study the univariate system $R_{t-1} = G(R_t)$ to find equilibrium R_t paths. Then we get N_t from (51), n_t from (32), etc. If fact, given a fixed point $R = G(R)$, we get $N = \alpha n_s \theta_{ms} R_t / (1-\beta)\kappa_m$ as well as

$$\begin{aligned} n_m &= \frac{\alpha n_s \theta_{ms} R_t}{(1-\beta)\kappa_m} - n_s - n_b \\ n &= \frac{n_s(1-\delta)[\alpha n_s \theta_{ms} R - (n_b+n_s)(1-\beta)\kappa_m]}{\delta n_s \theta_{ms} R + (1-\delta)[n_b \tau(R) + n_s](1-\beta)\kappa_m} \end{aligned}$$

To be sure the fixed point is a steady state we need to check $n_m, n \geq 0$, both of which reduce to the same condition, $R \geq \underline{R} \equiv (n_s + n_b)(1-\beta)\kappa_m / \alpha n_s \theta_{ms}$. We also need to check $n \leq n_m$ but that never binds. Hence, a solution to $R = G(R) \geq \underline{R}$ is a steady state with type M in the market; if we cannot find such an R there will

[38] This is similar to many systems that are block recursive, in the language of Shi (2009). An example is the standard labor model of Pissarides (2000), where we can determine market tightness and many other variables independent of the unemployment rate.

be a steady state where the market operate without type M.

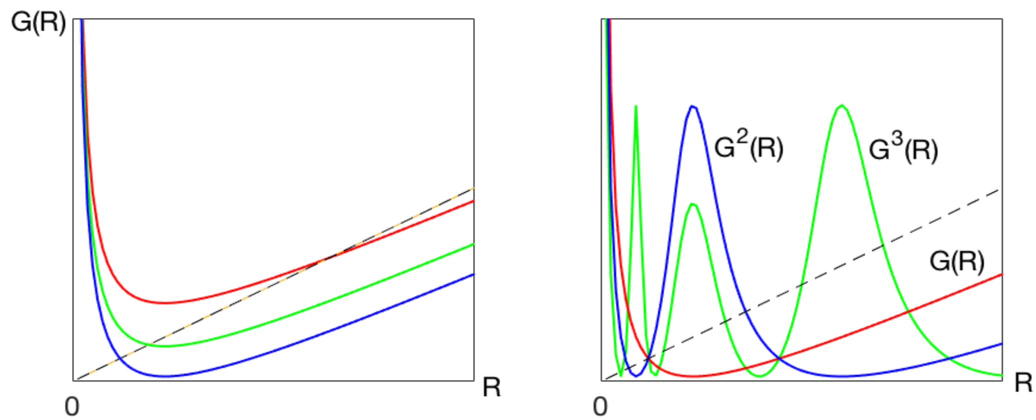

Figure 16: An example of cycles in Model 3 with entry by M

One can check $G(0) = \infty$, $G'(R) < 1$, and for $R > \max(\pi)$ G is linear with slope $\beta(1-\delta)$. Also, $G''(R) \geq 0$, so $G(R)$ is convex. This is shown in the left panel of Figure 15, from which it is clear that there exists a unique fixed point, say $\hat{R}$.[39] Notice that we can have $\hat{R} > \max(\pi)$, on the linear increasing part of $G(R)$ or $\hat{R} < \max(\pi)$, on the nonlinear part of $G(R)$. If $G'(\hat{R}) < -1$ then $\hat{R}$ is locally stable, and there exist cycles and sunspots, as in Model 1. Is $G'(\hat{R}) < -1$ possible? Yes, in fact, there is a threshold, say ρ_1, such that $G'(\hat{R}) < -1$ iff $\rho < \rho_1$. We do not know if $\rho_1 > 0$ or $\rho_1 < 0$, in general, but all our examples gave $\rho_1 < 0$. Still we have to verify $R \geq \underline{R}$ to ensure that M enter the market, as discussed above. It $G'(\hat{R}) < -1$ and $\hat{R} \leq \underline{R}$ possible? Yes, as we now show by way of example.

Example 10: Consider $\alpha = 1$, $\delta = 0.01$, $\beta = 0.99$, $n_b = n_s = 1$, $\theta_{mb} = \theta_{ms} = 0.5$, $\kappa_m = 0.1$, $\rho = -0.1$, and use the $F(\pi)$ in (35), with $\pi_0 = 4.95$, $\pi_1 = 0.05$, $\pi_2 = 5.05$, $\pi_3 = 0.95$ and $\pi_4 = 10$. This is the example used in Figure 16, where it can be easily checked that $G'(\hat{R}) < -1$ and $\hat{R} < \underline{R}$. Hence this example admits a two-cycle. Note that $\rho < 0$ in this example. If we lower ρ a little more, we can get higher-order cycles, including three-cycles, and hence chaotic dynamics. This is shown in the right panel of Figure 16, where we plot $G^3(R)$ and see that there exist fixed points

[39] We can borrow, with modification, an intuition for multiplicity vs uniqueness from Rocheteau and Wright (2005). With entry by S there are strategic complimentarities, because M choosing to trade more with B encourages entry by S, and vice versa. This is absent with entry by M, because the agents choosing whether to trade are the same ones making the entry decision.

other than $\hat{R}$, namely a pair of three cycles. In our examples the dynamics do not involve regime switching: over the cycle, M and B always trade. The dynamics over a two cycle are similar to Figure 10, from the model with entry by type S, so we do not repeat that discussion. However, here we can explicitly construct higher order cycles, and an example is shown in Figure 16. Finally, one more result is that $\rho < 0$ implies M and B must trade for some π, $\Pr(R < \pi) > 0$, if M is in the market – just like in the other version, a buy-and-hold-forever strategy is never a good idea at $\rho < 0$.

We state this formally as follows:

Proposition 7 *Consider Model 3 with entry by M. There is a unique steady state. If $\rho < \tilde{\rho}$ there exist cyclic and sunspot equilibria. Also, if $\rho < 0$ then* $\Pr(R < \pi) > 0$.

For completeness we also consider entry by B, and show that steady state is unique.

Proposition 8 *Consider Model 3 with entry by type B, $\pi = \bar{\pi}$ and $\rho > 0$. The steady state is unique.*

Proof: By the participation condition of B, we have

$$N = \frac{\alpha n_s \theta_{bs} \pi + \alpha n \theta_{bm} \tau (\pi - R)}{\kappa_b} \tag{53}$$

Notice that N is decreasing in R and increasing in n.

If $R \geq \pi$, plug $N = \alpha n_s \theta_{bs} \pi / \kappa_b$ in (31) to get

$$R = \frac{(1-\delta)\beta\rho}{1-(1-\delta)\beta\left(1-\dfrac{\theta_{ms}\kappa_b}{\theta_{bs}\bar{\pi}}\right)},$$

which is bigger than π iff

$$[1-(1-\delta)\beta]\pi \leq (1-\delta)\beta\left(\rho - \frac{\theta_{ms}}{\theta_{bs}}\kappa_b\right) \tag{54}$$

holds. Hence, when (54) holds, the R curve is a constant.

If $R < \pi$, plug $N = \alpha[n_s\theta_{bs}\pi + n\theta_{bm}(\pi - R_t)]/\kappa_b$ in (31) to get

$$[1-(1-\delta)\beta]R = (1-\delta)\beta\left[\rho + \alpha\theta_{mb}\frac{n_b}{N}(\pi - R) - \frac{\alpha n_s \theta_{ms}}{N}R\right] \tag{55}$$

The LHS of (55) is increasing, $LHS = 0$ at $R = 0$ and $LHS = \pi\left[1-(1-\delta)\beta\right]$ at $R = \pi$. Given any $n > 0$, the RHS is decreasing in R. The $RHS = (1-\delta)\beta\left(\rho + \frac{\theta_{mb} n_b \kappa_b}{n_s\theta_{bs} + n\theta_{bm}}\right) > 0$ at $R = 0$ for any $n > 0$, and $RHS = (1-\delta)\beta\left(\rho - \frac{\theta_{ms}}{\theta_{bs}}\kappa_b\right)$ at $R = \pi$. So there exists a unique fixed point iff

$$\left[1-(1-\delta)\beta\right]\pi > (1-\delta)\beta\left(\rho - \frac{\theta_{ms}}{\theta_{bs}}\kappa_b\right) \tag{56}$$

Note this argument does not depend on the value of n. So as long as $n > 0$, we can draw R as a function of n if (56) holds. It is easy to show that the R curve is increasing in n.

Next, we derive n curve. For $R > \pi$, plug $N = \alpha n_s\theta_{bs}\pi/\kappa_b$ into (32) to get

$$n = \frac{n_m(1-\delta)\kappa_b}{\delta\theta_{bs}\pi + (1-\delta)\kappa_b} < n_m$$

Hence, the steady state in which $\tau = 0$ exists iff (54) holds.

If $R < \pi$, rearrange (32) to get

$$N = \frac{\alpha(1-\delta)n_m}{\delta + (1-\delta)\alpha}\frac{n+n_s}{n} \tag{57}$$

Easy to verify that $dN/dn < 0$. Equate (57) to (53)to get the n-curve:

$$\frac{n_s\theta_{bs}\pi + \alpha n\theta_{bm}(\pi - R)}{\kappa_b} = \frac{(1-\delta)n_m}{\delta + (1-\delta)\alpha}\frac{n+n_s}{n} \tag{58}$$

Easy to verify that $dn/dR > 0$.

To see there is a unique steady state in which $\tau = 1$, we first plug (57) in (31) and show it is downward sloping. Take derivative of (31) to get

$$\frac{dR}{dn} = \frac{\left[\theta_{mb}(\pi - R)(n_m + n_s) + \theta_{ms}Rn_s\right]\frac{\alpha}{N^2}\frac{dN}{dn}}{1-(1-\delta)\beta + (1-\delta)\beta\left[\alpha\theta_{mb}\left(1 - \frac{n_m+n_s}{N}\right) + \frac{\alpha n_s\theta_{ms}}{N}\right]}$$

where N is given in (57). As the numerator is negative by (57) and the denominator is positive, $dR/dn < 0$. The steady state is the fixed point of (57), (31) and (58). As (58) is upward sloping, the fixed point is unique. ■

Supposing that $\hat{a}_j > 0\ \forall j$, we have

$$\phi_t = \beta\left(\phi_{t+1} + \rho\right) \tag{59}$$

$$\phi_t = \beta\left(\phi_{t+1} + \rho\right)(1-\delta_2)\left[1 + \alpha\lambda\left(q_{0,t+1}\right)\right] \tag{60}$$

$$\phi_t = \beta\left(\phi_{t+1} + \iota_t\right)\left[1 + (1-\delta_2)\alpha\lambda\left(q_{0,t+1}\right) + \delta_2\alpha\lambda\left(q_{2,t+1}\right)\right] \tag{61}$$

Conveniently, in steady state this system is recursive. First, (59) implies $\phi = \phi^F$. Then (60) implies $\lambda(q_0) = \delta_2/(1-\delta_2)$, or $q_0 = \tilde{q}$, the same as (41). Then (61) implies

$$\phi^F = \beta\left(\phi^F + \iota\right)\left\{1 + \delta_2 + \delta_2\alpha\lambda \circ v^{-1}\left[\left(\phi^F + \iota\right)\hat{a}_3\right]\right\}, \tag{62}$$

defining a demand curve for $\hat{a}_3$ as a function of ι.

Notice that on the demand side $a_3 = 1$ as $\iota \to \rho$, since then a_3 has the same return as a_2 but a_3 is perfectly liquid and safe. Also notice that we can have $\iota < 0$, since buyers may be willing to pay a fee (receive a negative interest rate) for safety, as they always have with travelers' checks. At $\iota = -\phi^F$, sellers do not accept bank notes, however, because the fee is so high the note itself is worthless, so $a_3 = 0$. This goes a long way towards establishing that there exists a steady state in which $a_3 \in (0, 1)$ and $\iota \in (-\phi^k, \rho)$. It remains to check the supposition that $\hat{a}_j > 0\ \forall j$, given the a_3 implied by the intersection of supply and demand for deposits. This is left as an exercise. Notice also that the steady state solved from (62) does not imply uniqueness. If multiple steady states exist, the economy can switch from one steady state to another with different $\hat{\mathbf{a}}$ even though the price stays at ϕ^F.

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