Housing Market Freezes, Deleveraging, and Aggregate Demand*

Preliminary and Incomplete

Christian Bayer[†], Ralph Luetticke[‡]

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Abstract

The liquidity of the US housing market undergoes large swings that lead the business cycle. After an increase in the time to sell a house, output falls while households increase their liquid asset holdings and simultaneously lower residential investment. A model of incomplete markets and nominal rigidities can rationalize the observed behavior. When houses become less liquid assets, households maintain the capacity for consumption smoothing by demanding a larger portfolio share of liquid (paper) assets instead of houses. This leads to a demand-driven recession. The recessionary effects get stronger if the banking sector produces liquid assets from mortgaging houses.

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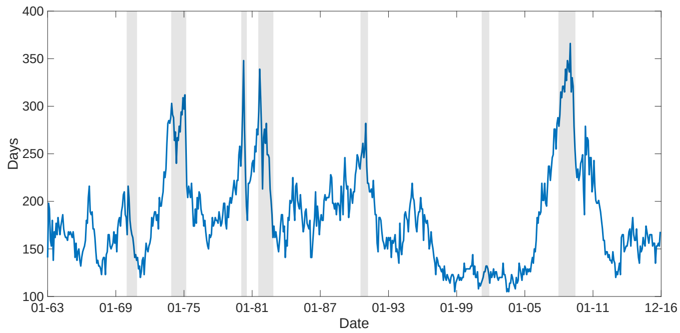
[†]Department of Economics, Universität Bonn, CEPR, and IZA. Address: Adenauerallee 24-42, 53113 Bonn, Germany. E-mail: *christian.bayer@uni-bonn.de*.

[‡]Department of Economics, University College London. E-mail: *r.luetticke@ucl.ac.uk*.

1 Introduction

The US housing market has undergone a tremendous boom-bust cycle over the past two decades with stark aggregate repercussions as **?** argue. This boom-bust cycle in housing not only regards fluctuations in the number of transactions and the prices at which houses are sold, but also the time to sell a house, c.f. Figure 1. During the housing boom in the early 2000s, the average turnover time of a house for sale was as low as 4 months. During the crisis, the time to sell a house increased by a factor of three to almost 12 months. It took until 2012 for the time on sale to return to historical average levels. In the present paper we argue that the boom-bust cycle in the liquidity of housing has strong aggregate repercussions through fluctuations in precautionary saving in liquid assets by households. What is more, we argue that fixed loan-to-value ratios aggravate the economic downturn because banks are unable to produce liquid assets out of writing mortgages in times when the demand for these liquid assets is particularly high.

Figure 1: Supply of housing relative to sales



Source: FRED Database.

Housing wealth is the most important asset for the majority of US households. This means that a decrease in the liquidity of housing threatens the ability of households to smooth income shocks. In turn, households should increase their holdings of highly liquid assets in order to maintain the ability to smooth consumption when houses become less liquid. This increased demand for liquid assets comes with a lower demand for goods and hence depresses the economy.

This is exactly what we find empirically. A one standard deviation increase in the expected time to liquidate a housing position (an increase of seven per cent, or roughly a fortnight) leads to a sharp fall in housing investment (-1.9%) but also in output (-0.8%) and house prices (-0.1%) despite a reduction in nominal interest rates.

Using a heterogeneous agent New-Keynesian model with liquid bonds and illiquid houses similar to Kaplan and Violante (2014) and Bayer et al. (2015) can reproduce the empirical findings qualitatively. In this model, households face uncertain labor productivity and decide on labor supply and consumption. They can only self-insure on incomplete markets against their productivity shocks. We assume that there are two types of assets: Houses that are illiquid and can only be traded every period with a certain probability and liquid real bonds that can be traded each period and may be issued by households up to a borrowing limit. Households hence not only face a consumption-savings and labor supply decision, but also an important decision regarding their portfolio composition in terms of illiquid houses and liquid bonds.

As houses become less liquid, households want to increase their savings in liquid assets to be able to smooth their consumption in the future. They do so by increasing their overall savings as well as by liquidating part of their housing position. This leads to a sharp and simultaneous drop in consumption and investment demand. In a representative agent, single asset model, this would show up as a simultaneous shock to investment productivity and time preferences, which is exactly what a business cycle accounting of the Great Recession produces (see Fratto and Uhlig, 2014). This drop in demand meets monopolistically competitive firms as producers that use labor for production. As monopolistic competitors they charge a markup over marginal costs, but to introduce a nominal friction, we assume them to be subject to a Rotemberg (1982) price setting friction.

This element of shocks to housing liquidity brings an important new aspect to our understanding of the role of the housing market in the Great Recession. Recent work by Justiniano et al. (2015) rejects the previously put forward micro-foundation of the zero-lower bound that relies on deleveraging by households. The authors point out that shocks to the loan-to-value ratio as in Guerrieri and Lorenzoni (2011) should only apply to new houses. Hence, the adjustment of the loan-to-value ratio is asymmetric in that all households participate in the loosening of borrowing constraints, but only new sales are subject to tighter borrowing constraints. Shocks to housing liquidity overcome this problem as they affect all households.

We highlight, however, that the deleveraging is important through another angle, namely for the supply of liquid assets. When house prices fall in response to the liquidity shock, the total volume of mortgages goes down as well. Yet, as banks write less mortgages, they also produce a smaller number of liquid deposits just in times when demand for liquid assets is highest. This aggravates the excess demand for liquid assets and thereby the recessionary effect of the liquidity shock.

The remainder of this paper is organized as follows: Section 2 looks at the data and shows that an increase in the time to sell a house has adverse business cycle consequences. Section 3 develops our model environment that adapts our earlier work Bayer et al. (2015) to a housing setup. Section **??** describes the calibration of our model, Section **??** presents the quantitative results. Section **??** concludes, an appendix follows.

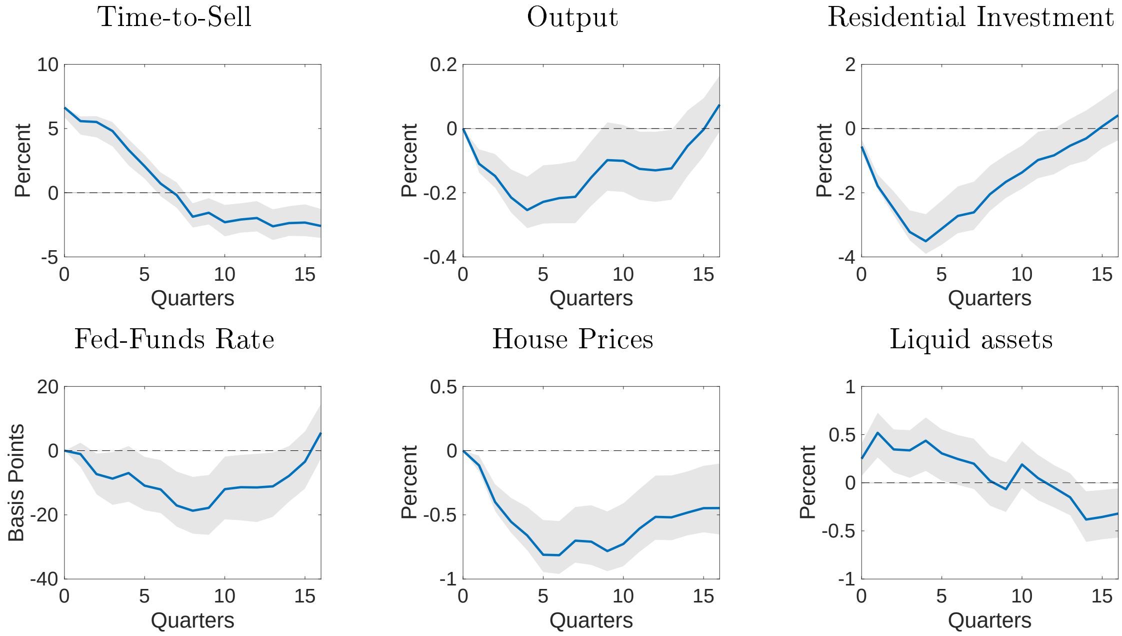
2 Empirical Findings

As Figure 1 highlights, the average number of days a house is on the market before it gets sold increases substantially *before* most recessions. In fact, the increase in time-to-sell in the housing market begins early 2005, and thus leads the house price changes and happens way before the Great Recession reaches its peak around the Lehman collapse. Notwithstanding, some of the movements in time-to sell will likely be endogenous responses to other shocks in the economy, as in **?** for example.To control for these endogenous response, we estimate the structural response to a shock in time-to-sell by estimating local projections, where we identify time-to-sell shocks by timing restrictions. To be conservative, we assume that time-to-sell is contemporaneously affected by the fed funds rate, output, and house prices.

Besides the time-to-sell data, we employ quarterly data from the national income and product accounts, the fed-funds rate, and the flow-of-funds for the time period 1981Q1 - 2016Q4. Figure 2 presents the results of this exercise. A one standard deviation increase in time to sell (a 6.5 per cent, i.e., roughly a fortnight increase of the time to sell a house) depresses output by 0.4 per cent and residential investment by more than 3 percent. The house price falls by 0.75 per cent, the fed-funds rate by 20 basis points. The total holdings of liquid assets by the household sector increase by 0.5 percent. The results are robust to an alternative ordering, moving time-to-sell first or last, i.e., assuming no contemporaneous impact on investment and liquid asset holdings.

The impulse responses point towards households increasing their liquid asset demand at the expense of the demand for investment (and consumption, not shown). This depresses aggregate demand and thus output, even though the Fed leans against the demand drop, by cutting interest rates substantially. Overall, the effects are large and they might well explain a substantial fraction of the housing boom bust cycle of the last two decades, where time to sell tripled from 2005 to 2009. Extrapolating from the impulse responses (roughly 15 standard deviations), we would estimate a decrease of the fed-funds rate of more than 3 percentage points, a 10 percent decrease in housing prices and a total collapse of residential investment.

Figure 2: Response to a one standard deviation shock to time to sell

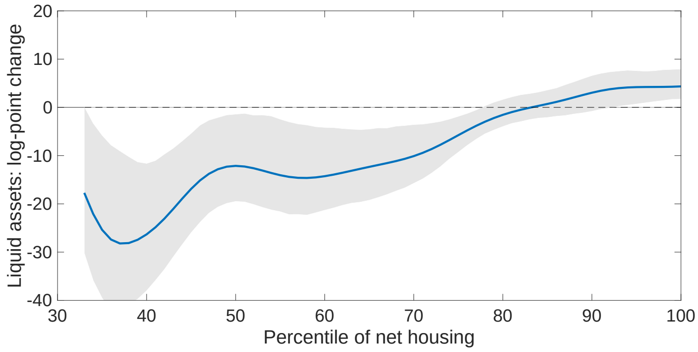


Notes: The figures display the response to a one standard deviation shock in time to sell estimated by a local projection. We identify the time-to-sell shock through a regression on contemporaneous house-prices, the fed-funds rate, and output, lags of all displayed variables, and a quadratic time trend. Output is constructed as the sum of residential investment, non-durable consumption, and government spending in line with our model. Liquid assets are taken from the household balance sheet data in the flow of funds. Grey shaded: Bootstrapped confidence bounds with 66% coverage. Aggregate data from 1981Q1-2016Q4.

The flight to liquidity and the induced recession finds its repercussion also in the distribution of household balance sheets. To show this, we first estimate the liquid asset holdings in each of the SCF waves (1983 - 2013) as a function of the percentile rank of a household in housing wealth net of mortgages. Then, in a second step, we regress these 11 functional observations on the estimated time-to-sell shock in the first quarter of the survey year.

We find that those households that hold a lot of housing wealth react with a substantial increase in their liquid assets, see Figure 3. Since these households are richer overall, they also hold on average most of the liquid assets in the economy. Poorer households in terms of housing wealth react with a decrease in their liquid asset holdings. Our model of the next section can reproduce this structure. For the housing-rich households, the loss in liquidity of housing dominates the aggregate repercussions and they want to rebalance their portfolios to maintain overall liquidity. For the poor households, the aggregate ef-

Figure 3: Cross-sectional response to a one standard deviation shock to time to sell



Notes: The figures display the response of the distribution of household portfolios to a one standard deviation shock in time to sell, see Figure 2 for further details. The liquid asset holdings are estimated by a local linear regression as a function of the percentile of net housing wealth in each SCF wave separately.

fects in terms of lower incomes and lower interest rates on liquid assets dominate. They use some of their liquid wealth to smooth consumption as incomes fall.

3 Baseline model

We keep the model close to Bayer et al. (2015) with the necessary adjustments to model homes instead of productive capital. Our model is an economy inhabited by a continuum of households that face idiosyncratic income risk. They self-insure against income fluctuations to smooth consumption. They can invest into liquid nominal bonds and into houses which are illiquid. Bonds offer a nominal and save return, houses offer a return by being rented out on a market for housing services. Illiquidity is understood in the spirit of Kaplan and Violante's (2014) model of "wealthy hand-to-mouth" consumers, where trading the illiquid asset is subject to a friction. We model this friction as a utility cost of trading houses aside a search friction. Households that want to sell or buy a house need to pay some utility cost to participate in the asset market for houses. Out of those households participating only a randomly selected fraction actually finds a trading possibility. All other households may only adjust their liquid, nominal bond holdings. We model changes in the liquidity of housing markets as changes in this trading possibility conditional on a household participating in the market.

The idiosyncratic income risks, which households face, stem from two sources. First, households stochastically transition between a "worker" and an "entrepreneur" state. Entrepreneurs obtain income from monopoly profits as they hire labor to produce and supply differentiated products. Due to their market power, they set prices above marginal costs subject to a price setting friction à la Rotemberg (1982) so that price adjustment on differentiated goods is sluggish. We calibrate the entrepreneur income to be high such that a transition from entrepreneur to worker state implies a substantial income decline. The second source of income risk regards workers, who supply labor, but are subject to idiosyncratic shocks to their idiosyncratic labor productivity. Therefore workers are heterogeneous in their incomes.

Liquid bonds are government debt as well as bonds originated by a financial sector that lends to the household sector in form of (floating rate) mortgages subject to a wasteful intermediation cost. Households can adjust the amount of outstanding mortgages up to a fixed fraction of the value of a house.

The government operates both a fiscal and a monetary authority. The former collects taxes, issues and services liquid debt, and spends on government consumption. Government consumption follows a simple rule (c.f. Bi et al., 2013). The monetary authority, the central bank, sets the interest rate on bonds according to a Taylor (1993)-type rule. We assume that all debt is real. This way, effectively the government sets prices and quantities of debt at the same time. For this to be an equilibrium, households must be willing to hold the amount of government debt at the given interest rate.[1]

3.1 Production: total output, consumption goods, and housing

We assume that households consume both goods and housing services. For simplicity, we assume that also the production of new homes requires both goods and housing services, such that we can summarize all final output, Y_t by a Cobb-Douglas aggregate over goods, Z_t, and housing, H_t

$$Y_t = Z_t^{\alpha} H_t^{1-\alpha}. \tag{1}$$

This total output is split into investment into new homes I_t and final consumption C_t.

Goods are produced by a continuum producers, all producing different varieties which are them combined into aggregate goods production according to a Dixit-Stiglitz aggregator:

$$Z_t = \left(\int z_{jt}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}.$$

[1] One way to think about this setup is that there is within period credit issued by goods producers to pay factors. If the interest rate is too high, households want to deposit more of this credit at the central bank than the government needs to buy the goods it consumes and conversely both the households and the government demand less goods than produced.

Each unit of consumption goods that is produced requires one efficiency unit of labor as an input. Housing services are produced by the aggregate stock of housing K_t where each unit of housing produces one flow unit of housing services each period $H_t = K_t$.

3.2 Firms: price setting and production of new homes

Since total output is a Cobb-Douglas aggregate of consumption goods and housing services, the price for output $P_{Y,t}$ is given by:

$$P_{Y,t} = \left(\frac{P_{Z,t}}{\alpha}\right)^{\alpha} \left(\frac{P_{H,t}}{1-\alpha}\right)^{1-\alpha},$$

where $P_{Z,t}$, and $P_{H,t}$ are the prices for consumption goods and housing services, respectively. Given the Cobb-Douglas aggregator, expenditure shares are constant and the prices for total output, consumption goods, and housing thus satisfy

$$P_{H,t} = P_{Z,t}\left(\frac{1-\alpha}{\alpha}\frac{Z_t}{H_t}\right), \tag{2}$$

$$P_{Y,t} = \frac{P_{Z,t}}{\alpha}\left(\frac{Z_t}{H_t}\right)^{1-\alpha}. \tag{3}$$

This implies that the real return on homes, r_t^K, is given by

$$r_t^K := \frac{P_{H,t}}{P_{Y,t}} - \delta = (1-\alpha)\left(\frac{Z_t}{H_t}\right)^{\alpha} - \delta, \tag{4}$$

where δ is the depreciation rate of homes.

The price for consumption goods results from the costly price setting for each variety of goods, for which our assumption of monopolistic competition yields a demand function,

$$z_{jt} = Z_t\left(\frac{p_{jt}}{P_{Z,t}}\right)^{-\eta}. \tag{5}$$

For tractability, we assume that price setting is delegated to a28. Juni mass-zero group of households (managers) that are risk neutral, consume only consumption goods, and are compensated by a share in profits. They do not participate in any asset market and have to pay a quadratic price adjustment cost if they adjust prices. Under this assumption, price setting is carried out with a time-constant discount factor. Managers face quadratic costs of price adjustment á la Rotemberg (1982) and the nominal costs of production is given by the wage rate W_t for an efficiency unit of labor.

They therefore maximize the present value of real profits under this constant discount factor:

$$E_0 \sum_{t=0}^{\infty} \beta^t Z_t \left(\frac{p_{jt}}{P_{z,t}} - \frac{W_t}{P_{Z,t}} \right) \left(\frac{p_{jt}}{P_{Z,t}} \right)^{-\eta} - \frac{\eta}{2\kappa} \left[\log \left(\frac{p_{jt}}{p_{jt-1}} \right) \right]^2 Z_t. \tag{6}$$

The corresponding first-order condition reads

$$\begin{aligned} 0 = Z_t & \left\{ \left((1-\eta) \frac{p_{jt}}{P_{Z,t}} + \eta \frac{W_t}{P_{Z,t}} \right) \left(\frac{p_{jt}}{P_{Z,t}} \right)^{-\eta-1} \right\} - \frac{\eta}{\kappa} \left[\log \left(\frac{p_{jt}}{p_{jt-1}} \right) \right] p_{jt}^{-1} Z_t \\ & + \beta \frac{\eta}{\kappa} \mathbb{E}_t Z_{t+1} \left[\log \left(\frac{p_{jt+1}}{p_{jt}} \right) \right] p_{jt}^{-1}. \end{aligned} \tag{7}$$

A symmetric equilibrium ($p_{jt} \equiv P_{Z,t}$), i.e., an equilibrium in which price-setting is homogeneous, simplifies this first-order-condition to the following Phillips curve:

$$\begin{aligned} \log(1+\pi_t^Z) &= \kappa \left(\frac{W_t}{P_{Y,t}} \frac{P_{Y,t}}{P_{Z,t}} - \frac{\eta - 1}{\eta} \right) + \beta E_t \frac{Z_{t+1}}{Z_t} \log(1+\pi_{t+1}^Z) \\ &= \kappa \left[\frac{w_t}{\alpha} \left(\frac{Z_t}{H_t} \right)^{1-\alpha} - \frac{\eta-1}{\eta} \right] + \beta E_t \frac{Z_{t+1}}{Z_t} \log(1+\pi_{t+1}^Z), \end{aligned} \tag{8}$$

where $1+\pi_t^Z$ is the gross inflation rate of consumption goods and w_t is the real wage. The inflation rate of Y-goods (total output) combines π_t^Z and the effect of housing supply

$$1+\pi_t^Y = (1+\pi_t^Z) \left(\frac{Z_t H_{t-1}}{Z_{t-1} H_t} \right)^{1-\alpha}.$$

In addition to profits from monopolistic competition in consumption goods, profits also arise from adjusting the aggregate housing stock. The entrepreneurs in the economy can transform I_t composite goods into ΔK_{t+1} additional units of housing goods (and back) according to the transformation function

$$I_t = \frac{\phi}{2} \left(\frac{\Delta K_{t+1}}{K_t} \right)^2 K_t + \Delta K_{t+1}.$$

Since they are facing perfect competition in this market, entrepreneurs will adjust the stock of housing until the following first-order condition holds for the relative price of homes holds:

$$q_t = 1 - \psi + \phi \frac{\Delta K_{t+1}}{K_t}. \tag{9}$$

Total entrepreneurial income is given by

$$\Pi_t = \left[\alpha\left(\frac{H_t}{Z_t}\right)^{1-\alpha} - w_t\right] Z_t + \frac{\phi}{2}\left(\frac{\Delta K_{t+1}}{K_t}\right)^2 K_t - \frac{\eta}{2\kappa}\log(1+\pi_t^Z)^2 \frac{Z_t}{P_{Y,t}} \tag{10}$$

3.3 Households: savings, investment, and labor supply

3.3.1 Income risks

The ex-ante identical households have measure one and are indexed by i. A fraction θ of them are entrepreneurs and obtain a share of the above profit income while the fraction $1-\theta$ are workers obtaining labor income. Both types may in addition earn income from renting out houses and from holding government bonds. Transition between these two household types is exogenous and reflects an event in which a household by chance invents a radically improved version of a differentiated product. This allows the households to take over the market power from the incumbent. It remains in this position until the product is replaced by an again better version invented by another household. Households are infinitely lived, have time-separable preferences with time-discount factor β, and derive felicity from consumption c_{it}, housing h_{it}, and suffer a disutility from work.

We assume that entrepreneurs have zero productivity on the labor market and thus will not offer labor services. Instead they obtain profit income Π_t, which is the same across all entrepreneurs. Worker households by contrast are heterogeneous in their productivity and hence all obtain different incomes on the labor market. Their labor income is $w_t\chi_{it}n_{it}$, composed of the wage rate per efficiency unit, w_t, hours worked, n_{it}, and idiosyncratic labor productivity, χ_{it}. Conditional on not transiting to the entrepreneur state, log productivity follows and AR(1)-process:

$$(\chi_{it}-\mu_\chi) = \begin{cases} \exp\{\rho_h \log(\chi_{it-1}-\mu_\chi)+\epsilon_{it}\}, & \text{if } \chi_{it-1}>0 \text{ with prob. } (1-\lambda) \\ \exp(\epsilon_{it}), & \text{if } \chi_{it-1}=0 \text{ with prob. } \zeta \\ 0, & \text{if } \chi_{it-1}>0 \text{ with prob. } \lambda \\ 0, & \text{if } \chi_{it-1}=0 \text{ with prob. } (1-\zeta) \end{cases}, \tag{11}$$

where $\epsilon_{it} \sim N(0,\sigma_h)$ and μ_χ is set such that $\mathbb{E}(\exp\chi)=1$. The first case in (11) is a worker who remains a worker. The second case captures the transition from entrepreneur to worker; the third case the reverse transition from worker to entrepreneur. Finally, the fourth case captures households that remain in the entrepreneur state.

3.3.2 Labor supply

Households have Greenwood-Hercowitz-Huffman (GHH) preferences and maximize the discounted sum of felicity over the consumption of total output (bundled consumption and housing services) and suffer a disutility from labor n. We denote the consumption of bundled goods and housing services by c. The preferences of a household are given by:

$$E_0 \max_{\{c_{it}, n_{it}\}} \sum_{t=0}^{\infty} \beta^t u\left(x_{it}\right), \tag{12}$$
$$x_{it} = c_{it} - \chi_{it} G(n_{it}), ,$$

where x bundles consumption and leisure. The felicity function has constant relative risk aversion (CRRA) with risk aversion ξ:

$$u(x_{it}) = \frac{1}{1-\xi} x_{it}^{1-\xi}, \quad \xi > 0,$$

and the disutility from labor is of the constant Frisch elasticity form:

$$G(n_{it}) = \frac{1}{1+\gamma} n_{it}^{1+\gamma}, \quad n_{it} > 0.$$

We scale the disutility of labor by labor productivity because this eases the calibration. All households work the same number of hours and income risk is equal to productivity risk. Given that we assume a constant Frisch-elasticity of labor supply and a linear tax, this is only a normalization assumption (see Bayer et al., 2015).

3.3.3 Budget constraint

Households can save in liquid bonds b_{it} and in illiquid form buying homes, k_{it}. In our baseline, we assume that both homes and bonds can only be held in positive amounts, $b_{it}, k_{it} \geq 0$. Later, we explore the effect of allowing for collateralized (mortgage) borrowing on the economy. This implies that households face the following budget constraint:

$$\begin{aligned} c_{it} + b_{it} + q_t k_{it} &= (1-\tau)\left[w_t \chi_{it} n_{it} + \mathbf{I}_{\chi_{it}=0} \Pi_t\right] \\ &+ (q_t + r_t^k) k_{it-1} + \frac{1+r_t^b}{1+\pi_t^Y} b_{it-1} \\ b_{it} &\geq 0, \quad k_{it} \geq 0, \end{aligned} \tag{13}$$

where q_t is the asset price of homes, τ is a tax rate on labor and profit income, w_t is the real wage rate per efficiency unit, $\mathbf{I}_{\chi_{it}=0}$ indicates the entrepreneur state, r_t^k is the return on housing assets, and r_t^b is the nominal net return on bonds b and π_t^Y is the inflation rate measured in the price of Y-goods.

We can simplify the households planning problem, expressing the budget equation and the planning problem only in terms of the bundle x by exploiting the first order condition for labor supply

$$\chi_{it} n_{it}^{1+\gamma} = (1-\tau) w_t \chi_{it} n_{it}, \tag{14}$$

such that all households supply the same amount of labor $n_{it} = [(1-\tau)w_t]^{1/\gamma}$, and their budget equation expressed in $x = c - \frac{\chi n^{1+\gamma}}{1+\gamma}$ becomes

$$\begin{aligned} x_{it} + b_{it} + q_t k_{it} &= \frac{\gamma}{1+\gamma} \chi_{it} \left[(1-\tau)w_t\right]^{\frac{1+\gamma}{\gamma}} + (1-\tau)\mathbf{I}_{\chi_{it}=0}\Pi_t \\ &+ (q_t + r_t^k) k_{it-1} + \frac{1+r_t^b}{1+\pi_t^Y} b_{it-1}. \end{aligned} \tag{15}$$

3.3.4 Costs of search for a trade in homes

The two assets, homes and bonds, not only differ in returns, but importantly in their liquidity. The illiquidity of homes is captured by a two-stage participation friction. First, households that would like to participate in the asset market for homes need to pay some utility cost in search of a transaction. This results in not all houses being up "for sale" every period. Second, households need to actually find a suitable trade. The latter, we model as an exogenous trading probability, ν_t, and we make this transaction probability time-varying.

We assume that the probability to find a trade conditional on paying the participation cost is common across households and follows a logit-transformed AR(1) process:

$$\nu_t = \frac{\exp \nu_t^*}{1 + \exp \nu_t^*}, \quad \nu_t^* = (1-\rho_\nu)\bar{\nu} + \rho_\nu \nu_{t-1}^* + \epsilon_t^\nu. \tag{16}$$

Therefore, all households that did not pay the utility cost of market participation or that did not find a trading opportunity hold their housing assets constant $k_{it} = k_{it-1}$. They still obtain income from renting out houses, but in terms of their portfolios, they may adjust only their bonds holdings.

3.3.5 Recursive formulation of the households planning problem

Since a household's saving decision will be some non-linear function of that household's wealth and productivity; asset prices and asset supply will therefore be functions of the joint distribution Θ_t of (b_t, k_t, χ_t). This makes Θ_t a state variable of the households' planning problem. We assume that the participation frequency ν evolves over time according to a first order Markov chain. This lets Θ_t fluctuate as well.

With this setup, the dynamic planning problem of a household is then characterized by three Bellman equations – V_a in case the household participates and can adjust its housing assets, V_{an} if the household pays the participation costs, but finds no trading partner, and V_{nn} if the household prefers not to pay trading costs. Since the cost of participation are irrelevant as long as no trade occurs, $V_{an} = V_{nn} =: V_n$ holds. The two value functions are given by:

$$\begin{aligned} V_a(b,k,\chi;\nu,\Theta) &= \max_{k',b'_a} u[x(b,b'_a,k,k',\chi)] + \beta E\bar{V}(b'_a,k',\chi';\nu',\Theta') \\ V_n(b,k,\chi;\nu,\Theta) &= \max_{b'_n} u[x(b,b'_n,k,k,\chi)] + \beta E\bar{V}(b'_n,k,\chi';\nu',\Theta') \\ \bar{V}(b,k,\chi;\nu,\Theta) &= \mathbb{E}\left\{\max\left[V_n, \nu(V_a-\kappa) + (1-\nu)(V_n-\kappa)\right]\right\} \end{aligned} \tag{17}$$

We can simplify the problem, expressing $\bar{V}$ as

$$\bar{V}(b,k,\chi;\nu,\Theta) = V_n + \mathbb{E}\left\{\max\left[0, \nu(V_a - V_n) - \kappa\right]\right\}. \tag{18}$$

For tractability, we assume that the participation costs are i.i.d. with distribution function F_κ, which we assume to be logistic with parameters $\mu_\kappa, \sigma_\kappa$, and define $\nu^*(b,k,\chi;\nu,\Theta) := \nu F_\kappa[\nu(V_a - V_n)]$ as the effective probability to trade, where $F_\kappa[\nu(V_a - V_n)]$ is the fraction of households participating in the housing market.

In line with this notation, we define the optimal consumption policies for the adjustment and non-adjustment cases as x_a^* and x_n^*, the bonds holding policies as b_a^* and b_n^*, and the housing policy as k^*.

3.3.6 Labor, bond, and capital market clearing

The labor market clears at the competitive wage, i.e.

$$w_t = N_t^\gamma/(1-\tau), \tag{19}$$

where wages need to solve the Phillips Curve (8) given current inflation and inflation expectations, which we can rewrite using the production function of consumption goods and housing services, $Z_t = N_t$ and $H_t = K_t$ as:

$$\log(1+\pi_t^Z) = \kappa\left[\frac{w_t}{\alpha}\left(\frac{N_t}{K_t}\right)^{1-\alpha} - \frac{\eta-1}{\eta}\right] + \beta E_t \frac{N_{t+1}}{N_t}\log(1+\pi_{t+1}^Z). \qquad (20)$$

The housing market clears at the competitive rental rate

$$r_t^K = (1-\alpha)\left(\frac{N_t}{K_t}\right)^{\alpha} - \delta. \qquad (21)$$

The bonds market clears, whenever the aggregate bond supply B_{t+1} is equal to the amount of bonds demanded by the households, i.e.,

$$\begin{aligned} B_t = &\int \nu^*(b,k,\chi;\mathcal{P}_t,\Theta_t,\nu_t) b_a^*(b,k,\chi;\mathcal{P}_t,\Theta_t,\nu_t)\Theta_t(b,k,\chi)dbdkd\chi \\ &+ \int \left[1-\nu^*(b,k,\chi;\mathcal{P}_t,\Theta_t,\nu_t)\right] b_n^*(b,k,\chi;\mathcal{P}_t,\Theta_t,\nu_t)\Theta_t(b,k,\chi)dbdkd\chi, \qquad (22)\end{aligned}$$

where in a somewhat sloppy notation, we highlight the role of current prices, $\mathcal{P}_t = \begin{pmatrix}\pi_t^Y & r_t^B & q_t & w_t & r_t^K\end{pmatrix}$, given the expected future prices.

Last, the asset market for homes (in short: capital market) clears when the demand for homes equals the supply K_t determined by the price q_t of homes:

$$\begin{aligned} K_t &= \frac{q_t - 1}{\phi} K_{t-1} + K_{t-1} \qquad (23)\\ K_t &= \int k^*(b,k,\chi;\mathcal{P}_t,\nu_t)\nu^*(b,k,\chi;\mathcal{P}_t,\Theta_t,\nu_t)\Theta_t(b,k,\chi)dbdkd\chi \\ &+ \int k\left[1-\nu^*(b,k,\chi;\mathcal{P}_t,\Theta_t,\nu_t)\right]\Theta_t(b,k,\chi)dbdkd\chi. \end{aligned}$$

where the first equation stems from competition in the production of homes, the second equation defines the aggregate demand for homes as assets. The goods market then clears due to Walras' law, whenever bond and capital market both clear.

3.4 Government

To close the model we need to determine the supply of bonds and the interest rate on bonds. We assume that both are set by two separate government authorities: a monetary one setting an interest rate at which households can lend and borrow from the government and a fiscal authority determining how much resources the government actually demands from the private sector net of taxes. This is equivalent to the fiscal authority determining the equilibrium amount of bonds supplied, while the monetary authority controls the real interest rate on liquid assets.

The monetary policy sets the nominal interest rate on bonds following a Taylor (1993)-type rule with interest rate smoothing:

$$\frac{1+r^B_{t+1}}{1+\bar{r}} = \left(\frac{1+r^B_t}{1+\bar{r}}\right)^{\rho_r} \left(\frac{1+\pi^Y_t}{1+\bar{\pi}}\right)^{(1-\rho_{R^b})\theta_\pi}. \tag{24}$$

The coefficient $\bar{r} \geq 0$ determines the nominal interest rate in steady state. The coefficient $\theta_\pi \geq 1$ governs the extent to which the central bank attempts to stabilize inflation around its steady-state value: the larger θ_π the stronger is the reaction of the central bank to deviations from the inflation target, $\bar{\pi}$. When $\theta_\pi \to \infty$, inflation is perfectly stabilized at its steady-state value. $\rho_r \geq 0$ captures interest rate smoothing.

We assume that the government issues bonds of real value B_{t+1} according to the rule:

$$\frac{B_t}{\bar{B}} = \left[\frac{B_{t-1}(1+r^B_t)(1+\bar{\pi})}{\bar{B}(1+\pi^Y_t)(1+\bar{r})}\right]^{\rho_B} \left(\frac{1+\pi^Y_t}{1+\bar{\pi}}\right)^{-\gamma_\pi}, \tag{25}$$

using the revenues to finance its expenditures, $G_t = \tau(w_t N_t + \Pi_t) - (1+r^B_t)B_{t-1} + B_t$.

The coefficient ρ_B captures whether and how fast the government seeks to repay its outstanding obligations $B_t \frac{1+r^B_t}{1+\pi^Y_t}$. Only if $\rho_B < 1$ the government debt is stable if the central bank follows the Taylor principle, $\theta_\pi > 1$. The coefficient γ_π captures the cyclicality of the deficit: for $\gamma_\pi = 0$ the deficit does not react to inflation (and thereby to output, hence is acyclical), for $\gamma_\pi > 0$ the deficit is countercyclical, for $\gamma_\pi < 0$ it is procyclical.

3.5 Recursive Equilibrium

A *recursive equilibrium* in our model is a set of policy functions $\{x^*_a, x^*_n, b^*_a, b^*_n, k^*\}$, value functions $\{V_a, V_n\}$, pricing functions $\{r^K, r^B, w, \pi^Y, q\}$, aggregate housing and labor supply functions $\{K, N\}$, distributions Θ_t over individual asset holdings and productivity, and a perceived law of motion Γ, such that

1. Given V, Γ, prices, and distributions, the policy functions $\{x^*_a, x^*_n, b^*_a, b^*_n, k^*\}$ solve the households' planning problem, and given the policy functions $\{x^*_a, x^*_n, b^*_a, b^*_n, k^*\}$, prices and distributions, the value functions $\{V_a, V_n\}$ are a solution to the Bellman equation (17).

2. The markets for labor, housing services, goods, as well as the asset markets for bonds, and houses clear, i.e. (19), (19),(20), (22), (23), (25), and (24) hold.

3. The actual law of motion and the perceived law of motion Γ coincide, i.e. $\Theta' = \Gamma(\Theta, v')$.

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