

Labor Taxes, Productivity and Tax Competition

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Abstract

Why are taxes higher in Europe than in the US? We propose that it stems from lesser competition across jurisdictions within Europe. We embed self-interested governments and tax competition into a standard neoclassical growth model with public goods. While greater jurisdictional competition reduces taxes it also reduces societal investment in public capital and thus often ends up reducing total factor productivity. We show that despite this deleterious effect on the level of productive public capital, tax competition ends up raising per capita output and welfare. We show evidence to support both our baseline assumption of lesser mobility in Europe relative to the US as well as for our predictions on productivity differences between the two.

1 Introduction

Over the past decade there has been an increasing interest in understanding the reasons behind why average work hours in Europe are lower than in the US. While still controversial, there is a gradually emerging consensus that a key factor that can explain this is the higher tax rates in Europe on average (see Prescott 2005, Rogerson 2008). An important feature of this debate is that it has left unanswered the question of why taxes are higher in Europe. This paper is an attempt at providing one candidate explanation for the differences in tax rates.

Our explanation relies on the notion of tax competition across jurisdictions. Given the more restricted labor movement across countries of Europe, there is less jurisdictional competition for

labor and business among European states relative to US states. This resulted in higher tax rates on average which, in turn, induced lower work effort.

We demonstrate our basic idea in the context of a dynamic model of tax competition across many locations. Locations are governed by sovereigns. Sovereigns have the power to tax and they can invest in public goods. Public goods are valued by private agents as they raise the productivity of private inputs. However, sovereigns are self-interested – they value their own consumption. We show that as labor becomes more mobile and competition for them increases amongst sovereigns, the average tax rate declines. While a lower tax rate raises work effort, it also induces a lower provision of public goods. This latter effect implies that TFP is lower in areas with greater jurisdictional competition even though output per worker as well as welfare of private agents is higher.

There are other potential explanations for differences in tax rates across countries. One candidate is a preference based approach. Thus, if private agents in some locations valued public goods more than in other locations, this would directly lead to higher taxes being imposed by even a planner maximizing private welfare. We have chosen not to hard-wire the answer to our central question to such a stand on preferences of private agents.

Our approach, instead, draws on a rich and older literature on tax competition across jurisdictions (see Wilson 1998 for an excellent overview of this body of work). For the most part, this literature has studied the effects of tax competition on issues such as the optimum provision of public goods and the size of government in the presence of benevolent governments. Our framework differs from this literature on two dimensions. First, we model jurisdictions as being governed by self-interested governments (often called the Leviathan government). Second, we study the question of tax competition in a dynamic context which allows us to explore the macroeconomic implications of our model.

Our paper is organized as follows. In the next section, we sketch out the basic effects of tax competition using a static environment with an exogenous stock of public capital. We show that competition reduces taxes and the size of the government, while raising labor effort. In section 3 we make the model explicitly dynamic by allowing the governments to invest in future public capital and examine the additional effects of tax competition on growth rates of per-capita output and the

steady state levels of key macroeconomic aggregates. In section 4, we relate the predictions of the model to the data. The last section concludes.

2 A Static Model of Tax Competition

We start by sketching out a simple static model of tax competition across jurisdictions. Consider a one-good economy with two locations – i and j . Each location has a sovereign and a measure N_s , $s = i, j$, of identical private subjects.

Subjects derive utility from consuming and leisure. The preferences of the representative subject are given by

$$W = \ln(c - \gamma n), \quad \gamma > 0$$

where c denotes consumption and n is labor supply by the subject.

A subject in location s who supplies n units of labor time produces output according to the technology

$$y_s = \left(\frac{G_s}{N_s} \right)^\lambda n_s^\theta, \quad \lambda + \theta < 1 \text{ and } \lambda > 0, \theta > 0.$$

where G_s is the level of public capital in location s . Hence, the level of per capita public capital in a location raises the private returns to labor in that location. Public capital is non-rival and non-excludable to all subjects in a given location. We assume that the public capital stock is given exogenously in each location.

Sovereigns impose a per-unit tax τ on the output produced by their subjects. Hence, the budget constraint of a subject in location s is

$$c_s = (1 - \tau_s) \left(\frac{G_s}{N_s} \right)^\lambda n_s^\theta.$$

Hence, the optimal level of labor supply and consumption of a subject in location $s = i, j$ given τ , N and G in his location is

$$\hat{n}_s = \left[\frac{\theta}{\gamma} (1 - \tau_s) \left(\frac{G_s}{N_s} \right) \right]^{\frac{1}{1-\theta}}, \tag{1}$$

$$\hat{c}_s = \frac{\gamma}{\theta} \hat{n}_s. \tag{2}$$

Given these solutions it is straightforward to check that the subject's maximized utility is

$$\hat{W}_s = \left(\frac{\theta}{1-\theta} \right) \ln \frac{\theta}{\gamma} + \ln(1-\theta) + \left(\frac{1}{1-\theta} \right) \left[\ln(1-\tau_s) + \lambda \ln \frac{G_s}{N_s} \right].$$

Since taxes do not have any direct or indirect benefits for the subject they are a pure deadweight loss. This is made clear by the subject's welfare \hat{V}_s above, which is maximized when $\tau_s = 0$.

Sovereigns are self-interested. They derive utility from their own consumption which we denote by X . In each location the sovereign chooses taxes to maximize

$$V_s = \ln X_s, \quad s = i, j$$

subject to his budget constraint

$$X_s = \tau_s Y_s$$

where τ denotes the tax rate and Y denotes aggregate output. Note that aggregate output is given by

$$Y_s = n_s^\theta G_s^\lambda N_s^{1-\lambda},$$

which follows from multiplying the per capita output term by N .¹ Substituting in the optimal labor supply decision of subjects (equation (1)) into the aggregate production function and solving for X_s yields the sovereign's welfare function:

$$V_s = \left(\frac{\theta}{1-\theta} \right) \ln \frac{\theta}{\gamma} + \frac{\lambda}{1-\theta} \ln G_s + \left(\frac{1-\lambda-\theta}{1-\theta} \right) \ln N_s + \ln \tau_s + \frac{\theta}{1-\theta} \ln(1-\tau_s) \quad (3)$$

This welfare function makes clear a couple of key features of the sovereign's decision problem. First, *ceteris paribus*, sovereigns benefit from having more subjects because aggregate output in a location is increasing in the number of subjects in that location. Hence, all else equal, sovereigns

¹It is useful to note that the production technology specified above implies that

$$Y = (G/N)^\lambda H^\theta N^{1-\theta}$$

where $H = nN$ which measures aggregate hours. So, the production function is constant returns to scale in private inputs, namely aggregate hours and the fixed factor ("entrepreneurial ability") which is proportional to the number of people. Hence, total factor productivity in this model is given by $(G/N)^\lambda$ while labor productivity would be the standard expression Y/N .

would try to attract more subjects to their location. Second, taxes have two opposing effects on the sovereigns welfare. Higher taxes directly raise tax revenues and thereby facilitate greater consumption for the sovereign. However, higher taxes also reduce the labor supply of subjects which reduces aggregate output and thereby reduces the tax revenues and consumption of the sovereign. The optimal tax chosen by the sovereign must then balance these opposing effects.

Clearly, each sovereign will take into account the dependence of his subject's labor supply decision on the tax τ_s while choosing taxes. However, there are two cases to analyze. The first is when labor is not mobile across locations and the second is when labor can move from one location to another. We analyze each case in turn below.

2.1 No labor mobility

When subject's cannot move across locations the sovereign's decision problem is simple since the only factor they have to take into account when choosing taxes is the distortionary effect of the tax on the subject's labor supply decision which is given by equation (1) above. Specifically, each sovereign can treat the number of subjects N_s in his location as constant.

Substituting in equation (1) into the aggregate production function and maximizing the sovereign's welfare (3) subject to the resulting constraint gives

$$\hat{\tau}_s = 1 - \theta$$

which is the optimal tax rate chosen by a sovereign who faces no competition for subjects from the other location. Clearly, both sovereign's would choose the same tax rate since the optimal tax doesn't depend on any location specific factors.

Using the optimal tax rate it follows that per capita output in location $s = i, j$ when labor is not mobile is given by

$$\frac{Y_s}{N_s} = \left[\frac{\theta^2}{\gamma} \right]^{\frac{\theta}{1-\theta}} \left(\frac{G_s}{N_s} \right)^{\frac{\lambda}{1-\theta}}.$$

Hence, per capita output across locations will depend simply on the initial distribution of G and N . Moreover, total factor productivity, which is given by $\left(\frac{G_s}{N_s} \right)^\lambda$ is also exogenously determined by the initial endowments of G and N in each location.

2.2 Mobile labor

Next we study the case where labor can move across locations upon seeing the tax announcement made by the sovereigns. Since sovereign welfare is an increasing function of the number of subjects, this is an environment where sovereigns will be competing for subjects.

We start by noting that given a G and N in any location along with the tax announcement τ , the subject in that location would choose her labor supply and consumption exactly as before. The sovereign's welfare function is also the same as before and given by equation (3). Moreover the sovereign will continue to take into account the effect of τ on the subject's labor supply decision when choosing taxes. The additional element now is that sovereigns must also factor in the effect of their tax choice on the location decision by subjects.

In order to endogenize the location decision of subjects as a function of the tax rate we proceed by first noting that along any optimum location decision by subjects they must be indifferent between locations. Hence, at an optimum we must have $W_i = W_j$. Substituting in the relevant expressions for subject welfare in the two locations gives

$$\ln\left(\frac{1-\tau_i}{1-\tau_j}\right) = \lambda \left[\ln\left(\frac{G_j}{G_i}\right) - \ln\left(\frac{N_j}{N_i}\right) \right] \quad (4)$$

We call this the “equal utility” condition. This equal utility condition yields the equilibrium measure of subjects in each location as a function of the taxes and public capital:

$$N_i = \hat{N}_i(\tau_i, \tau_j, G_i, G_j).$$

Note that since N is exogenous, determining N_i is sufficient to determine the measure of subjects in location j as well. Crucially, the relationship between N_i and the tax rates is summarized by

$$\begin{aligned} \frac{\partial N_i}{\partial \tau_i} &= -\frac{N_i N_j}{\lambda(1-\tau_i)N}, \\ \frac{\partial N_i}{\partial \tau_j} &= \frac{N_i N_j}{\lambda(1-\tau_j)N}. \end{aligned}$$

We assume that sovereigns play a Nash tax game. Each sovereign, taking the taxes chosen by the other as a given, chooses his own tax rate to maximize his own welfare (equation (3)) subject to his budget constraint, the subject's labor supply decision, and the equal utility condition for

subjects to be indifferent between locations at an optimum. This optimization exercise yields the two optimal tax rates chosen by sovereigns i and j respectively:

$$\tau_i = \frac{1 - \theta}{1 + (1 - \theta - \lambda) \frac{N_j}{\lambda N}} < 1 - \theta \quad (5)$$

$$\tau_j = \frac{1 - \theta}{1 + (1 - \theta - \lambda) \frac{N_i}{\lambda N}} < 1 - \theta \quad (6)$$

The key result here is that taxes are lower when labor is mobile relative to the case where labor is immobile across locations. Intuitively, sovereigns now also compete for subjects with the other location. This reduces the optimal tax rate. It is worth noting that the tax does not go all the way to zero (a race to the bottom of sorts) because of the congestion effect that comes from G/N . Since the level of aggregate public capital is given exogenously in this static environment, additional subjects in any location reduces the per capita provision of public capital which, in turn, reduces the private returns to labor supply. Subjects trade-off this negative congestion effect with the direct benefit of lower taxes.

Proposition 1 *If the two locations have the same initial stock of public capital, then the two locations must have the same population and the same tax rate.*

Proof. Suppose not and suppose that $N_i > N_j$. Then, from (5) and (6) it follows that $\tau_i > \tau_j$. But this implies that the right hand side of (12) is positive while the left hand side of (4) is negative. ■

Proposition 2 *If the two locations have different initial stocks of public capital, then the location with the higher level of public capital will have a higher tax rate, a higher population and a higher level of per capita public capital.*

Proof. Suppose $G_i > G_j$. Then equation (4) can be written as

$$\ln \left(\frac{1 - \tau_i}{1 - \tau_j} \right) + \lambda \ln \left(\frac{N_j}{N_i} \right) = \lambda \ln \left(\frac{G_j}{G_i} \right) < 0.$$

Equations (5) and (6) imply that $N_i \geq N_j$ as $\tau_i \geq \tau_j$. Hence, the only way for $\ln \left(\frac{1 - \tau_i}{1 - \tau_j} \right) + \lambda \ln \left(\frac{N_j}{N_i} \right) < 0$ is if $N_j < N_i$ and $\tau_j < \tau_i$. Since $G_i > G_j$ implies that $\tau_i > \tau_j$ the result on per capita public capital then follows directly from equation (4) which implies that if $\tau_i > \tau_j$ then $G_i/N_i > G_j/N_j$. ■

While this static case is instructive for understanding precisely how tax competition works in our model, it is not useful for comparing variables such as employment, per capita output and total factor productivity (TFP) across different regimes of labor mobility or jurisdictional competition. This is because under no labor mobility across regions the provision of per capita public capital, G/N , is completely exogenous. Hence, TFP is exogenous. This, in turn, implies that employment and per capita output are exogenous as well. The exogeneity of these variables under no labor mobility makes it impossible to compare these variables across regimes where sovereigns compete for labor with regimes where they don't. Most importantly, an exogenous level of public capital ignores one of the fundamental roles of governments, namely, providing public capital. The fact that public capital is valued by private citizens is one of the reasons why private subjects often willingly turn over their income to sovereigns as taxes.

3 The Dynamic Model

As we saw above the primary problem with the static model was the exogeneity of public capital provision. This made regime comparison impossible. We address this drawback in this section by extending the model to a fully dynamic setting in which public capital provision becomes an endogenous variable that is chosen by sovereigns.

Preferences of a subject are given by

$$W = \sum_{t=0}^{\infty} \beta^t \ln(c_t - \gamma n_t), \quad \beta < 1 \text{ and } \gamma > 0.$$

where n is labor supply by the subject. This is just the intertemporal version of the static utility function described previously.

The production side of the model is the same as in the static case since the technology available for production purposes is the same. The representative subject in location $s = i, j$ faces the periodic budget constraint

$$c_s = (1 - \tau_s) y_s.$$

$$W(G_s; N_s, \tau_s) = \max_{c, n} \ln(c_s - \gamma n_s) + \beta W(G'_s; N_s, \tau_s).$$

s.t.

$$c_s = (1 - \tau_s) \left(\frac{G_s}{N_s} \right)^\lambda n_s^\theta$$

$$G'_s = M(G_s; N_s, \tau_s)$$

where $M(G; N, \tau)$ is the law of motion for the provision of G . The labor supply decision is clearly a static one. For a given G and N in a location, optimal labor supply and consumption are still described by equations (1) and (2). Substituting the solution for n into the production function and multiplying by N one gets the aggregate output in location s :

$$Y_s = \left[\left(\frac{\theta}{\gamma} \right) (1 - \tau_s) \right]^{\frac{\theta}{1-\theta}} G_s^{\frac{\lambda}{1-\theta}} N_s^{1-\frac{\lambda}{1-\theta}}. \quad (7)$$

The sovereign's decision problem is two fold. A sovereign $s = i, j$ must choose the tax rate τ_s as well as the division of tax revenues into own consumption X_s and public capital for next period G'_s . We first solve for the division problem given a tax rate τ_s and a measure N_s of subjects in location s . This decision problem of sovereign s is

$$V(G_s; N_s, \tau_s) = \max_{X, G'} \{ \ln X_s + \beta V(G'_s; N_s, \tau_s) \}$$

while her constraint is

$$X_s + G'_s = \tau_s Y_s = \tau_s \left[\left(\frac{\theta}{\gamma} \right) (1 - \tau_s) \right]^{\frac{\theta}{1-\theta}} G_s^{\frac{\lambda}{1-\theta}} N_s^{1-\frac{\lambda}{1-\theta}}.$$

In order to proceed we shall guess that

$$V(G, N, \tau) = \bar{a} + a(\tau) + d \ln G + h \ln N \quad (8)$$

Substituting this guess for the value function and using the guess and verify method yields the solutions for the guessed coefficients as

$$d = \frac{\lambda}{1 - \theta - \lambda\beta}$$

$$h = \frac{(1 - \theta - \lambda)}{(1 - \beta)(1 - \theta - \lambda\beta)}$$

$$\bar{a} = \left(\frac{1}{1-\beta} \right) \left[\begin{aligned} & \left(\frac{\theta}{1-\theta-\lambda\beta} \right) \ln \frac{\theta}{\gamma} + \ln(1-\theta-\lambda\beta) + \lambda\beta \ln \lambda\beta \\ & - \left(\frac{1-\theta}{1-\theta-\lambda\beta} \right) \ln(1-\theta) \end{aligned} \right]$$

$$a(\tau) = \frac{(1-\theta)}{(1-\beta)(1-\theta-\lambda\beta)} \left[\ln \tau + \left(\frac{\theta}{1-\theta} \right) \ln(1-\tau) \right]$$

These solutions imply that optimal consumption and public investment by sovereign s for a given τ_s are

$$X_s = \left(1 - \frac{\lambda\beta}{1-\theta} \right) \tau_s \left[\left(\frac{\theta}{\gamma} \right) (1-\tau_s) \right]^{\frac{\theta}{1-\theta}} G_s^{\frac{\lambda}{1-\theta}} N_s^{1-\frac{\lambda}{1-\theta}}, \quad (9)$$

$$G'_s = \left(\frac{\lambda\beta}{1-\theta} \right) \tau_s \left[\left(\frac{\theta}{\gamma} \right) (1-\tau_s) \right]^{\frac{\theta}{1-\theta}} G_s^{\frac{\lambda}{1-\theta}} N_s^{1-\frac{\lambda}{1-\theta}}. \quad (10)$$

Observe that the right hand side of the above equation is the law of motion of $M(G; N, \tau)$ introduced earlier.

3.1 No labor mobility: dynamic case

In the event that subjects cannot move across locations then N_s is given for each location. More specifically, sovereigns do not compete for subjects. Hence, the only factor that sovereigns need to take into account while setting the tax rate for his location is the distortionary effect of the tax on the subject's labor supply. Given the optimal choices for G' and X and the solution for the value function, it is clear that the optimal tax rate τ must set $a'(\tau) = 0$. Hence, the optimal tax rate set by the sovereign is

$$\tau = 1 - \theta$$

But this is the same as the optimal tax in static case as well when there is no competition for labor across jurisdictions.

Collecting results then, in a dynamic environment with no labor mobility but where sovereigns also invest in productivity enhancing public capital, the optimal tax rate chosen by the sovereign coincides with the optimal tax chosen in the static case. Intuitively, the tax distorts labor supply by subjects. But labor supply is a static decision problem. Hence, the tradeoffs faced by the sovereign when choosing the tax rate continue to be described by exactly the same set of conditions as in the static model.

3.2 Mobile labor: dynamic case

We now turn to the case where labor can move across jurisdictions. In particular, we consider an environment in which at time 0 the levels of public capital G_i and G_j are given for the two locations. There are two sovereigns – i and j – who make announcements regarding their choices of tax rates at time 0, i.e., they announce τ_i and τ_j . After seeing the tax announcements and observing the levels of public capital in place at the two locations, subjects make their location decisions. Once the tax and location decisions are made by sovereigns and subjects respectively, all agents are committed to them permanently.

Since the choice of location depends on the stock of public capital and taxes, the population in the two locations will also depend on the stocks of public capital. These dependencies are captured through the functions $N_s(G_i, G_j)$ and $\tau_s(G_i, G_j)$, $s = i, j$.

We start by solving for the agents' optimal labor-leisure decisions contingent on the subject facing a given N_i and τ_i . Hence, the decision problem of a subject in location $k = i, j$ with G_k is

$$\begin{aligned} W(G_s; N_s, \tau_s) &= \max_{c_s, n_s} \{ \ln(c_s - \gamma n_s) + \beta W(G'_s; N_s, \tau_s) \} \\ \text{s.t.} \\ c_s &= (1 - \tau_s) \left(\frac{G_s}{N_s} \right)^\lambda n_s^\theta \\ G'_s &= M_s(G_s; N_s, \tau_s) \end{aligned}$$

This is the same problem we solved earlier. Thus optimal labor supply and consumption are still given by equations (1) and (2). Moreover, aggregate output in location s continues to be given by equation (7).

The sovereign's problem given τ_s and N_s is

$$\begin{aligned} V(G_s; N_s, \tau_s) &= \max_{G'_s, X_s} \{ \ln X_s + \beta V(G'_s; N_s, \tau_s) \} \\ \text{s.t.} \\ G'_s + X_s &= \tau_s Y_s = \tau_s \left[\left(\frac{\theta}{\gamma} \right) (1 - \tau_s) \right]^{\frac{\theta}{1-\theta}} G_s^{\frac{\lambda}{1-\theta}} N_s^{1-\frac{\lambda}{1-\theta}}. \end{aligned}$$

This problem is identical to the no-labor mobility case analyzed above. Hence, the optimal choices of X and G' are given by equations (9) and (10) as before. Also, the sovereign's value function

takes the identical form as before, i.e.,

$$V(G_s; N_s, \tau_s) = \bar{a} + a(\tau_s) + d \ln G_s + h \ln N_s$$

with the coefficients $\bar{a}, a(\tau_s), d$ and h being the same as before.

At time 0 the sovereigns have to choose their respective τ . Each sovereign chooses his τ taking the other sovereign's τ as given. Note that sovereigns recognize that their choice of τ affects the measure of people living in their location since subjects are free to move between locations. Thus, in equilibrium, subjects must be indifferent between locations. This indifference condition yields a relationship between N_s and the taxes τ_i and τ_j . Each sovereign takes this relationship as given and chooses her own tax rate taking the other sovereign's choice of tax rate and G_s , $s = i, j$, as given. Thus, at an optimum the sovereigns choose their respective τ 's such that $\frac{\partial V_s}{\partial \tau_s} = 0$ which implies that

$$a'(\tau_s) \tau_s = -h \left(\frac{\tau_s}{N_s} \frac{\partial N_s}{\partial \tau_s} \right) \quad (11)$$

If the elasticity of N with respect to τ is negative (as seems intuitive), the right hand side of the above equation is strictly positive. It follows that the equilibrium tax rate will now be at a point where $a'(\tau_s) > 0$. We can easily verify that this can only be true at a tax rate lower than $1 - \theta$. Thus, competition will lower taxes.

What is the relationship between N_s and τ_s ? Since subjects must be indifferent between locations at an equilibrium, the optimum must have

$$W(G_i, N_i, \tau_i) = W(G_j, N_j, \tau_j).$$

The subject's value function from being in location i with G_i, N_i and τ_i given is

$$W(G_i, N_i, \tau_i) = \max_{c_i, n_i} \{ \ln(c_i - \gamma n_i) + \beta W(G_i', N_i, \tau_i) \}$$

The optimal solutions for c and n imply that

$$c_i - \gamma n_i = \left(\frac{1 - \theta}{\theta} \right) \gamma n_i = \left(\frac{1 - \theta}{\theta} \right) \gamma \left(\frac{\theta}{\gamma} \right)^{\frac{1}{1-\theta}} \left[(1 - \tau_i) \left(\frac{G_i}{N_i} \right)^\lambda \right]^{\frac{1}{1-\theta}}$$

We proceed by guessing that the subject's value function takes the form

$$W(G_i, N_i, \tau_i) = \tilde{a} + \tilde{a}(\tau_i) + \tilde{d} \ln G_i + \tilde{h} \ln N_i$$

This amounts to the guess that the value functions under an arbitrary τ and the one chosen optimally by a new sovereign differ only in the term $\tilde{a}(\tau)$. Substituting the guess for W in the subject's value function and using the solution for $c - \gamma n$, one can then equate coefficients to get

$$\tilde{d} = \frac{\lambda}{1 - \theta - \lambda\theta} = -\tilde{h},$$

$$\tilde{a}(\tau_i) = \frac{1}{(1 - \beta)(1 - \theta - \lambda\beta)} [\lambda\beta \ln \tau_i + (1 - \lambda\beta) \ln (1 - \tau_i)],$$

$$\tilde{a} = \ln(1 - \theta) + \left(\frac{1}{1 - \theta - \lambda\beta} \right) \left[\lambda\beta \ln \left(\frac{\lambda\beta}{1 - \theta} \right) + \theta \ln \frac{\theta}{\gamma} \right].$$

Before proceeding further it is worth noting that the subject's value function depends on τ only through the term $\tilde{a}(\tau_i)$. Hence, the tax rate that maximizes the subject's welfare is

$$\hat{\tau}_i = \lambda\beta.$$

This is precisely the solution obtained by Glomm-Ravikumar (1994) and Chakraborty-Lahiri (2007). Any difference from this solution that we get in this problem arises due to the fact that the sovereign maximizes his own welfare which depends on his consumption X .

The indifference condition for location implies that

$$\tilde{a}(\tau_i) - \tilde{a}(\tau_j) = \tilde{d} \left[\ln \frac{G_j}{G_i} + \ln \left(\frac{N_i}{N_j} \right) \right]. \quad (12)$$

This expression defines a relationship between the tax rates τ_i and τ_j and the aggregate number of subjects N_i and N_j in each location for given levels of G_i and G_j . We also have the aggregate constraint

$$N_i + N_j = N. \quad (13)$$

Substituting (13) in (12), gives equilibrium relationships for N_i and N_j , respectively, in terms of $\tau_i, \tau_j, G_i, G_j, N$. Differentiating these implicit functions with respect to τ_i and τ_j , respectively, yields:

$$\frac{\tau_i}{N_i} \frac{\partial N_i}{\partial \tau_i} = \frac{N_j}{N\lambda(1 - \beta)} \frac{(\lambda\beta - \tau_i)}{(1 - \tau_i)}, \quad (14)$$

$$\frac{\tau_j}{N_j} \frac{\partial N_j}{\partial \tau_j} = \frac{N_i}{N\lambda(1 - \beta)} \frac{(\lambda\beta - \tau_j)}{(1 - \tau_j)}. \quad (15)$$

We should also note that

$$\frac{\partial N_i}{\partial \tau_j} = -\frac{\partial N_i}{\partial \tau_i}; \frac{\partial N_i}{\partial \tau_j} = -\frac{\partial N_j}{\partial \tau_j}.$$

Substituting in (14) and (15) into (11) and using the expression for $a'(\tau_k)$ yields:

$$\tau_i = (1 - \theta) \left[\frac{\frac{\lambda\beta}{(1-\theta)} (1 - \theta - \lambda) N_j + (1 - \beta) \lambda N}{(1 - \theta - \lambda) N_j + (1 - \beta) \lambda N} \right] \quad (16)$$

$$\tau_j = (1 - \theta) \left[\frac{\frac{\lambda\beta}{(1-\theta)} (1 - \theta - \lambda) N_i + (1 - \beta) \lambda N}{(1 - \theta - \lambda) N_i + (1 - \beta) \lambda N} \right] \quad (17)$$

It is clear from these expressions that

$$\tau_s < 1 - \theta \text{ iff } \lambda\beta < 1 - \theta, s = i, j$$

But $\lambda\beta < 1 - \theta$ is equivalent to $\frac{\lambda}{1-\theta} < 1/\beta$. This inequality is always satisfied since under our maintained restriction $\lambda + \theta < 1$

$$\frac{\lambda}{1 - \theta} < 1 < 1/\beta.$$

Hence, the optimal Nash tax rate under tax competition is less than the level chosen by a sovereign who does not face competition from other sovereigns. Furthermore, observe that we can write (16) as

$$\frac{\tau_i}{\lambda\beta} = \left[\frac{(1 - \theta - \lambda) N_j + \frac{(1-\theta)}{\lambda\beta} (1 - \beta) \lambda N}{(1 - \theta - \lambda) N_j + (1 - \beta) \lambda N} \right] > 1,$$

since $\lambda\beta < 1 - \theta$. In other words, a planner maximizing welfare of the subject would choose $\lambda\beta$ (as we showed above) while a self-centered sovereign wants to choose $1 - \theta$. Tax competition in a dynamic environment forces the sovereigns to choose a tax somewhere in between the two.

The first two results to note in the dynamic case is that Propositions 1 and 2 that were derived for the static case above continue to apply here. We restate those two propositions appropriately modified for this dynamic case.

Proposition 3 *If the two locations have the same initial stock of public capital, then the two locations must have the same population and the same tax rate at all times when labor is mobile across locations.*

Proof. Suppose not and suppose that $N_i > N_j$. Then, from (16) and (17) it follows that $\tau_i > \tau_j$. But this implies that the right hand side of (12) is positive while the left hand side of (12) is negative since $\tau > \lambda\beta$ and $\tilde{a}'(\tau) < 0$ for $\tau > \lambda\beta$. ■

Proposition 4 *If the two locations have different initial stocks of public capital, then the location with the higher level of public capital will have a higher tax rate as well as higher population at all times when labor is mobile. Furthermore, the location with the higher stock of public capital will also have perpetually higher per-capita public capital.*

Proof. Suppose $G_i < G_j$. Then equation (12) can be written as

$$\tilde{a}(\tau_i) - \tilde{a}(\tau_j) + \tilde{d} \ln\left(\frac{N_j}{N_i}\right) = \tilde{d} \ln\left(\frac{G_j}{G_i}\right) > 0.$$

Equations (16) and (17) imply that $N_i \geq N_j$ as $\tau_i \geq \tau_j$. Also, $\tilde{a}'(\tau) < 0$ for $\tau > \lambda\beta$. Hence, the only way for $\tilde{a}(\tau_i) - \tilde{a}(\tau_j) + \tilde{d} \ln\left(\frac{N_j}{N_i}\right) > 0$ is if $N_j > N_i$ and $\tau_j > \tau_i$. A symmetric outcome is not possible in this case. The result on the per-capita public capital follows from noting that the above equation can be written as

$$\tilde{a}(\tau_i) - \tilde{a}(\tau_j) = \tilde{d} \left[\ln\left(\frac{G_j}{N_j}\right) - \ln\left(\frac{G_i}{N_i}\right) \right]$$

Since $\tilde{a}(\tau_i) > \tilde{a}(\tau_j)$ and $\tilde{d} > 0$, it follows that $G_j/N_j > G_i/N_i$. ■

To solve for the symmetric Nash equilibrium set we set $\tau_i = \tau_j$. From Proposition 1 we know that this case requires that the two economies start with the same G_0 . In a symmetric Nash equilibrium we must also have $N_i = N_j = N/2$. Substituting these into the expression for τ_i derived above gives

$$\tilde{\tau} = (1 - \theta) \left[\frac{(1 - \theta - \lambda) \frac{\lambda\beta}{(1-\theta)} + 2\lambda(1 - \beta)}{(1 - \theta - \lambda) + 2\lambda(1 - \beta)} \right]$$

The solution for the Nash tax rate can now be used to determine labor productivity in the tax competition case and contrast it with labor productivity in the no labor mobility case.

3.3 Labor productivity and taxes

We start by studying the effect of taxes on steady state labor productivity. The solution for G' (equation (10)) above implies that in steady state we must have

$$\left(\frac{G}{N}\right)^{\frac{1-\theta-\lambda}{1-\theta}} = \left(\frac{\lambda\beta}{1-\theta}\right) \left(\frac{\theta}{\gamma}\right)^{\frac{\theta}{1-\theta}} \tau (1-\tau)^{\frac{\theta}{1-\theta}}.$$

Substituting this into the expression for output per person gives

$$\frac{Y}{N} = \left[\left(\frac{\theta}{\gamma}\right) (1-\tau)\right]^{\frac{\theta}{1-\theta-\lambda}} \left[\left(\frac{\rho\lambda\beta}{1-\theta}\right) \tau\right]^{\frac{\lambda}{1-\theta-\lambda}}$$

Differentiating this with respect to τ gives

$$\frac{d(Y/N)}{d\tau} = \left(\frac{Y/N}{1-\theta-\lambda}\right) [\lambda - (\theta + \lambda) \tau].$$

Clearly,

$$\frac{d(Y/N)}{d\tau} \geq 0 \text{ as } \frac{\lambda}{\theta + \lambda} \geq \tau.$$

It is straightforward to check that

$$\left.\frac{d(Y/N)}{d\tau}\right|_{\tau=1-\theta} < 0 \text{ iff } 1 > \lambda + \theta.$$

But this condition is a maintained restriction of the model. Hence, a reduction in the tax rate below the level chosen by the monopolist sovereign will raise labor productivity. Since we have already shown that $\tilde{\tau} < 1 - \theta$, it follows that labor productivity is higher under tax competition relative to the monopolist sovereign case.

Observe that if we view the aggregate production function as CRS in the private inputs, then the total factor productivity (TFP) term is simply $(G/N)^\lambda$. From the steady state expression for G/N given above, we can establish that

$$\frac{d(G/N)^\lambda}{d\tau} \geq 0 \text{ as } \tau \leq (1 - \theta)$$

Therefore, under competitive provision of the public capital, TFP is going to be lower!

4 Many Locations

The analysis above can be easily extended to a multiple location setting, i.e., where there are $J \geq 2$ sovereigns. We study this case under the assumption that the initial stock of public capital is the same in all locations. Note that the decision problem facing subjects in any location is the same as in the two sovereign case analyzed above. Moreover, for a given τ the decision problem of a sovereign in any location, say i , is also exactly the same as before. Hence, the solutions for the value functions of subjects and sovereigns that we derived in the previous section continue to apply in this case.

The main difference in the $J > 2$ case is that equation (12) – which says that in equilibrium subjects must be indifferent between locations – must now hold for all bilateral pairs of locations. Given our assumption that initial public capital is identical across locations, we shall focus on the symmetric Nash equilibrium in this case (see Proposition 1 above) wherein $\tau_i = \tau_j$ and $N_i = N_j$ for all i, j . The equality of population across locations implies that $N_i = N/J$ for all i . Then, equation (12) can be written as

$$\tilde{a}(\tau_i) - \tilde{a}(\tau_j) = \tilde{d} \left[\ln \frac{G_j}{G_i} + \ln \left(\frac{N_i(J-1)}{(N-N_i)} \right) \right]$$

where we have used the fact that under symmetry $N_j = (N - N_i) / (J - 1)$. As before, this condition can be used to obtain an expression for the elasticity of N_i with respect to τ_i :

$$\frac{\tau_i}{N_i} \frac{\partial N_i}{\partial \tau_i} = \frac{N_j(J-1)}{N\lambda(1-\beta)} \frac{(\lambda\beta - \tau_i)}{(1-\tau_i)}.$$

Using fact that $N_j = N/J$, the above equation yields

$$\frac{\tau_i}{N_i} \frac{\partial N_i}{\partial \tau_i} = \frac{(J-1)}{\lambda J(1-\beta)} \frac{(\lambda\beta - \tau_i)}{(1-\tau_i)}.$$

Using the above in (11) gives

$$\tau_i = (1-\theta) \left[\frac{\frac{\lambda\beta}{(1-\theta)}(1-\theta-\lambda)\frac{(J-1)}{J} + (1-\beta)\lambda}{(1-\theta-\lambda)\frac{(J-1)}{J} + (1-\beta)\lambda} \right] \text{ for all } i$$

Observe that this expression is consistent with what we had for $J = 2$. Furthermore, since $(J-1)/J$ is increasing in J and $\lambda\beta < 1 - \theta$, τ_i is declining in J . In the limit, as J tends to infinity, this

expression reduces to

$$\tau_i = \lambda$$

This tax rate is still in excess of $\lambda\beta$ and provides the upper bound on the efficiency gains from tax competition.

4.1 Comparing Regimes

We showed previously that that $\left. \frac{d(Y/N)}{d\tau} \right|_{\tau=1-\theta} < 0$ if and only if $1 > \lambda + \theta$. We now know that the tax rate is highest in the single sovereign case ($1 - \theta$) and falls with the number of sovereigns. Therefore, steady state labor productivity would be rising monotonically in J . Hence, the more competition there is among sovereigns the higher is labor productivity in steady state.

Proposition 5 *The growth rate of public capital is declining in the number of symmetric sovereigns.*

Proof. We showed above that τ is declining in the number of sovereigns J . Observe that we can re-write the G' equation as

$$\frac{G'}{N} = \left(\frac{\lambda\beta}{1-\theta} \right) \tau \left[\left(\frac{\theta}{\gamma} \right) (1-\tau) \right]^{\frac{\theta}{1-\theta}} \left(\frac{G}{N} \right)^{\frac{\lambda}{1-\theta}}$$

Or, defining $\ln(G/N)$ as g , we have

$$g_{t+1} = \delta(\tau) + \frac{\lambda}{1-\theta} g_t$$

where

$$\delta(t) = \ln \left[\left(\frac{\lambda\beta}{1-\theta} \right) \tau \left[\left(\frac{\theta}{\gamma} \right) (1-\tau) \right]^{\frac{\theta}{1-\theta}} \right].$$

Observe for $\tau_1 < \tau_2 \leq (1 - \theta)$, $\delta(\tau_1) < \delta(\tau_2)$. Now note that regardless of the tax regime,

$$g_{t+1} - g_t = \frac{\lambda}{1-\theta} (g_t - g_{t-1}).$$

So, it follows that if

$$(g_t - g_{t-1})|_{\tau_2} > (g_t - g_{t-1})|_{\tau_1}$$

then

$$(g_{t+1} - g_t)|_{\tau_2} > (g_{t+1} - g_t)|_{\tau_1} \text{ for all } t$$

Now, since g_0 is the same under both regimes, while $\delta(\tau_2) > \delta(\tau_1)$ it follows that

$$(g_1 - g_0)|_{\tau_2} > (g_1 - g_0)|_{\tau_1}.$$

The Proposition follows by induction. ■

The growth rate of output per worker is determined, for any given tax regime, by the growth rate of per-capita public capital stock. This follows because

$$\frac{Y}{N} = \left[\left(\frac{\theta}{\gamma} \right) (1 - \tau) \right]^{\frac{\theta}{1-\theta}} \left(\frac{G}{N} \right)^{\frac{\lambda}{1-\theta}}$$

Hence, the growth of output per worker will also decline in the number of (symmetric) sovereigns. However, we know from Proposition xx that the steady-state output per worker is a declining function of the tax rate. Thus, steady-state output per worker is rising in the number of symmetric sovereigns. Also, as the number of sovereigns rise, per capita output in the initial period also rises. Thus, as the number of sovereigns rises, both the initial and steady state per-capita output go up but the growth rate of per-capita output goes down.