

Does Informality facilitate Inflation Stability?*

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November 2018

Abstract

Informality is an entrenched structural trait in emerging market economies, despite of the progresses achieved in macroeconomic management. Informality determines the behavior of labour markets, financial access and the productivity of the overall economy. Therefore it influences the transmission of shocks and also of monetary policy. This paper develops a simple general equilibrium closed economy model with nominal rigidities, labor and financial frictions. Informality is captured by a dual labour market where the share of informal workers is endogenous. Only formal sector firms have access to financing, which is instrumental in their production process. Informality has a buffering effect on the propagation of demand and supply shocks to prices; the financial feature of the model boosts the impact of financial shocks in the formal sector while the informal sector is in principle unaffected. As a result informality dampens the impact of demand and financial shocks on wages and inflation but heighten the impact of technology shocks. Informality also increases the sacrifice ratio of monetary policy actions. From a Central Bank perspective, the results imply that the presence of an informal sector mitigates inflation volatility for some type of shocks but makes monetary policy less effective.

*The views expressed in this paper are exclusively those of the authors and not those of the BIS. This paper was written while Urrutia was visiting the BIS as research fellow under the BIS research fellowship programme.

1 Introduction

The presence of a large informal sector is a pervasive feature of labor markets in emerging economies (see Schneider (2007)). The informal sector blossoms when excessive taxes and regulations are imposed by governments which lack the capability to enforce compliance (Loayza (1996)). More restrictive labor regulations limit the process of adjustment of labor flows in response to shocks.¹ In this context, the informal sector provides flexibility to labor markets and mitigates or even offsets the impact of labor regulation on the adjustment of the economy to shocks

Given the high incidence of informality in emerging economies, it is remarkable how scarce is the literature focusing on the constraints that it imposes on monetary policy. An exception is Castillo and Montoro (2010), who use a New Keynesian DSGE model in which firms chose the mix of formal and informal workers in production. The informality option provides more flexibility to firms in expanding their output without bidding up wages, acting as a *buffer* for inflationary pressures.² Their analysis abstracts, though, from an important ingredient: the segmentation of financial markets. The formal and informal sectors differ not only on their productivity or in the ability to adjust employment in the short run, but also crucially in their access to the financial system. As shown in Levine et al. (2010), most informal firms are excluded from the financial system and have to rely on alternative and more expensive sources of funds.

In this paper, we explore the implications of the presence of the informal sector for inflation stabilization and monetary policy in emerging economies under inflation targeting regimes. Our analysis combines in a structural DSGE model two key features highlighted in the paragraphs above: (i) dual labor markets, with a frictional employment in the formal sector, modeled within the Mortensen and Pissarides (1994) search framework, coexisting with a flexible informality option, and (ii) financial exclusion in the informal sector, introducing an asymmetry in the use of credit between the two sectors. The model also features nominal rigidities and a monetary policy conducted via a Taylor rule. We use this model to evaluate the role of the informal sector in the response of the economy to different shocks and

¹See the evidence collected by Heckman and Pages (2000) for Latin American countries. Following this argument, Lama and Urrutia (2011) show that high levels of employment protection can generate potentially large misallocation effects that exacerbate output and TFP volatility along the business cycle.

²In their setup, demand shocks generate lower inflation than in an economy without informal workers. An implication of this mechanism is that the share of informal workers is pro-cyclical, as it has been shown for countries like Spain and Italy (see Conesa et al. (2002) and Bovi (2007)). However, evidence for Mexico and other Latin American countries discussed in Bosch and Maloney (2008) and Fernández and Meza (2015) suggest that the informality rate is in fact *counter-cyclical*.

in the transmission channel of monetary policy itself. Importantly, the financial asymmetry allows to explore the impact of informality when the economy is hit by financial shocks.

Our model economy features two main channels by which informality affects the propagation of shocks: a labor and a financial channel. First, the informal sector provides the economy with an employment alternative that increases the flexibility of the labor market in response to shocks, reducing inflationary pressures arising from labor demand spurs. Second, due to its lack of access to credit markets, the informal sector reduces the sensitivity of unit costs to changes in interest rates. In contrast, the formal sector borrows to cover its working capital needs, so unit costs and then inflation are affected by the interest rate through a credit cost channel (see Ravenna and Walsh (2006) and Fiore and Tristani (2013)). The combination of both channels determines how different types of shocks are propagated to the economy, shaping the dynamics of inflation.

We calibrate the economy to Mexican data. Mexico is a good example of an emerging economy with a large informal sector and for which micro data on labor flows are available. In our numerical simulations, we show that *demand* shocks lead to a pro-cyclical adjustment of the informality rate, as in Castillo and Montoro (2010), so that similar increases in aggregate demand lead to a larger increase in total employment and output in the economy with an informal sector, and therefore to a smaller increase in real wages and inflation. However, the same buffering mechanism makes inflation more responsive to *technology* shocks with informality. This is because a negative technology shock increases unit costs more with an informal sector due to the smaller reduction in wages.

In addition, our model incorporates *financial* shocks affecting the interest rate paid by firms in the formal sector. We show that these shocks drive a counter-cyclical informality rate. Through the financial cost channel, an increase in the lending interest rate increases unit costs in the formal sector, contracting output and formal employment and making the informality option more attractive. Hence, the informal sector mitigates the response to the financial shock of total employment and output, while also dampening inflationary pressures coming from increases in unit costs.

To sum up, in our quantitative model the presence of an informal sector stabilizes inflation when the main perturbations in the economy correspond to demand or financial shocks, but amplifies it under technology shocks. We show that these results are robust to changes in parameter values, with an important caveat. When the flexibility of the formal sector increases, measured by the formal labor turnover, the buffering effect of informality is less important and the financial channel plays a central role, overturning some of the results.

We also analyze the transmission of monetary policy as innovations to the Taylor rule, and show in our calibrated model that these interventions are less effective in stabilizing the economy with an informal sector (i.e., requires a larger output sacrifice to achieve a similar reduction in inflation) compared to the model without informality. Here the labor and financial channels discussed before work in the opposite direction. The buffering effect of informality implies that the contraction in labor demand associated to an interest rate hike can be absorbed by a rapid decline in informal workers together with a smaller fall in real wages. This makes the Phillips curve flatter, making disinflation harder to achieve. On the other hand, interest rates have a direct impact on unit labor costs through the financial channel, also making more difficult for monetary policy to reduce inflation; yet this effect is weaker when we add an informal sector, less sensitive to interest rate changes. Our quantitative results suggest that the labor channel is more important than the financial channel for monetary policy transmission

A wide literature has studied the role of the informal sector in shaping real business cycle models in emerging economies.³ A common theme is that while the existence of an informal sector adds flexibility to the labor market, this flexibility comes at a cost. Because the productivity of the informal sector is typically low, changes in the composition of workers during a recession can amplify changes in TFP. In two recent papers, Leyva and Urrutia (2018) and Horvath (2018) stress the role of interest rate shocks to small open economies to account for the observed cyclical properties of employment and the informality rate. We borrow from Leyva and Urrutia (2018) the main setup for the real side of the economy, adding a working capital constraint for formal firms as in Horvath (2018) and nominal rigidities.

Papers close to ours, emphasizing financial frictions are Batini et al. (2011) and Colombo et al. (2018). Batini et al. (2011) introduce financial frictions in a DSGE model with nominal rigidities and real wage frictions, in which formal jobs are rationed. Credit frictions are modeled as a financial accelerator in both formal and informal sectors, assuming an external finance premium depending positively on the leverage ratios. For the same leverage, though, the informal sector pays a higher interest rate premium, so that in equilibrium informal firms use less credit. In their model, the authors show that simple Taylor rules responding to inflation and the output gap in the formal sector are sub-optimal and welfare can be increased by adding to the rule a response to the external premium in the formal sector. They do not explore the response of the economy to shocks nor do they explicitly evaluate the transmission channel of monetary policy.

³See, among others, Bosch and Esteban-Pretel (2012), Restrepo-Echavarria (2014), Finkelstein Shapiro (2014) and Fernández and Meza (2015)

Finally, Colombo et al. (2018) analyze the impact of a financial shock in an economy with an informal sector. The main assumption is that capital is sector specific and the formal sector finances investment using bank credit, while the informal sector borrows directly from households. While the model features nominal rigidities and monetary policy conducted via a Taylor rule, the emphasis of the paper is on the propagation of financial shocks to the labor market (in particular, the reallocation of workers between formal and informal activities) and to real output, not to inflation. Consistently to our mechanism, they show that an increase in the bank lending spread due to a financial crisis affects disproportionately the formal sector, boosting the informality rate.

The paper is organized as follows. In section 2 we build a DSGE model for a closed economy, adding labor market frictions, an informal sector modeled as self-employment, nominal rigidities and a monetary policy rule targeting inflation. The calibration of the model is presented in Section 3, together with a set of impulse response functions to different supply, demand and financial shocks. Section 4 uses the model to discuss the implications of the informal sector for the transmission channel of monetary policy and its effectiveness in reducing inflation at a low output cost. Finally, we conclude.

2 A Monetary Model with Labor Market Frictions and Informality

We introduce a simple monetary model for a closed economy. The real side of the model, adapted from Leyva and Urrutia (2018), features both formal and informal sectors, labor market frictions and an endogenous participation in the labor force decision. A representative family can choose to spend part of its time endowment working in the formal sector or the informal sector, searching for formal jobs as unemployed or out of the labor force. The model captures in the formal sector search frictions for the hiring process and includes as part of the institutional environment a payroll tax. In contrast, the informal sector faces no search costs nor taxes, implying that workers can smoothly transit between non-employment and informal employment. The informal sector, however, is assumed to be less productive.

In addition, we introduce a credit market in which household's savings are channeled to formal firms through competitive financial intermediaries. Formal entrepreneurs borrow to finance their working capital needs. Informal firms, conversely, are assumed to be excluded from credit markets. There is an exogenous intermediation cost that generates a spread between the lending rate to firms and the interest rate received by households.

We embed this setup into a standard sticky price model, with nominal rigidities à la Calvo and a Central Bank that targets inflation using the nominal interest rate. The model features technology shocks, symmetric to both sectors, demand shocks to government expenditures and financial shocks to the intermediation premium.⁴

2.1 The Labor Market

The consumption side of the economy is modeled as a continuum of identical households, each comprising a continuum of ex-ante identical workers. There is perfect risk-sharing among the members of the household, so each worker has the same level of consumption and the value of leisure is also equally allocated among workers. This allows us to work within the representative agent framework.

In the production side, firms in the formal and informal sectors produce intermediate goods using linear technologies in the labor input. We assume that the two types of intermediate goods are imperfect substitutes in the production of a final good.

2.1.1 Labor Supply

Households have a constant endowment of labor $\bar{L} = 1$ each period, which can be allocated to four occupational categories: Employed in the formal sector (L_t^f), employed in the informal sector (L_t^s), unemployed (U_t) and out of the labor force (O_t):

$$\underbrace{L_t^f + L_t^s}_{\text{employed}} + \underbrace{U_t + O_t}_{\text{non-employed}} = 1. \quad (1)$$

Formal and informal workers pay the same utility cost in terms of leisure foregone, but generate labor income. Workers can freely move from inactivity (out of the labor force) to the informal sector, and in the opposite direction.⁵ In contrast, to obtain a formal job workers need to search for it, going through the unemployment state. Unemployed workers pay a search cost in utility terms, but if successful they can obtain a formal job in the current period. We define aggregate labor supply as $L_t \equiv L_t^f + L_t^s$.

⁴We focus in this section on the most relevant characteristics of the model for the purpose of the paper. Appendix A provides a rigorous definition of equilibrium for the full model and derives with some detail the optimal decision rules of consumers and firms.

⁵Our modeling of informal employment is akin to models of self-employment or home production, in which there are no frictions to entry or exit into/from these activities. The assumption is meant to capture the flexibility of the informal sector, in which there are no explicit contracts and the duration of jobs is much shorter than in the formal sector (see Leyva and Urrutia (2018)).

Preferences Household's preferences are described by the expected discounted lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log \left(C_t - \psi \Phi_t \frac{L_t^{1+\varphi}}{1+\varphi} \right) - \frac{\varsigma}{2} U_t^2 \right], \quad (2)$$

where consumption C_t is a composite basket defined over a continuum of differentiated goods:

$$C_t = \left[\int_0^1 C_t(z)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (3)$$

with elasticity of substitution $\eta > 1$. Following Jaimovich and Rebelo (2009), the disutility of labor in (2) is multiplied by a deterministic shifter factor, defined recursively by $\Phi_t = C_t^\omega \Phi_{t-1}^{1-\omega}$. This specification allow us to control for the wealth effect in labor supply by changing the parameter ω .⁶ In addition, the parameter ψ governs the disutility of labor, φ is the inverse of the Frisch elasticity of labor supply, and ς controls the quadratic utility cost of job search.

2.1.2 Matching and Labor Demand in the Formal Sector

The dynamics of employment in the formal sector includes the type of search and matching frictions in the standard Mortensen and Pissarides (1994) framework. There is a continuum of ex-ante equal formal entrepreneurs with the potential of starting an informal firm. Current unemployed workers U_t search for jobs while formal entrepreneurs post vacancies V_t . New formal matches are created through a standard, constant returns to scale, matching function $U_t^\phi V_t^{1-\phi}$. A vacancy has a cost ξ in units of the final good for the entrepreneur and only lasts for a period. If the vacancy meets a worker, the match becomes active in the current period. Formal wages are determined through repeated bargaining using the standard Nash protocol.

Job Finding and Vacancy Filling Probabilities From the matching function, the probabilities p_t of a worker finding a match and q_t of a vacancy meeting a worker are :

$$p_t = \left(\frac{U_t}{V_t} \right)^{\phi-1} \quad q_t = \left(\frac{U_t}{V_t} \right)^\phi. \quad (4)$$

At the beginning of the period, a mass L_{t-1}^f of workers are matched with a formal firm. An exogenous fraction s of formal workers are dismissed and become unemployed. New matches

⁶For instance, $\omega = 0$ corresponds to the class of non-separable preferences in Greenwood et al. (1988) that eliminate completely wealth effects, while the case $\omega = 1$ maps into the standard separable preferences.

become also active this period, so formal employment evolves according to:

$$L_t^f = (1 - s) L_{t-1}^f + q_t V_t. \quad (5)$$

Value of a Match in the Formal Sector An active match produces one unit of the formal intermediate input using one worker. We define recursively the utility value of a match for the entrepreneur as:

$$J_t \equiv \left[p_t^f - (1 + \kappa i_t^l + \tau) w_t^f \right] \lambda_t^C + (1 - s) \beta E_t J_{t+1}. \quad (6)$$

where p_t^f denotes the price of informal intermediate goods, w_t^f denotes the wage rate in the formal sector, τ is a payroll tax collected by the government, and λ_t^C is the shadow value of consumption (see Appendix A.1). Notice that formal entrepreneurs borrow within each period to pay a fraction κ of the wage bill in advance (*working capital* constraint) at the nominal interest rate i_t^l . For simplicity, we assume that $1 + i_t^l = (1 + i_t)(1 + \zeta_t)$ where i_t is the nominal interest rate for consumers and ζ_t is an exogenous intermediation cost.

Wage Determination Every period, after observing the shocks, the formal wage maximizes the Nash product $(\lambda_t^L)^\gamma (J_t)^{1-\gamma}$, where γ is the weight assigned to the worker. The value function J_t , as defined in (6), captures the value for an entrepreneur of keeping a match. We assume that the outside option for the firm has a value of zero. On the other hand, λ_t^L , defined recursively by:

$$\lambda_t^L \equiv \left(w_t^f - w_t^s \right) \lambda_t^C + \beta (1 - s) E_t \lambda_{t+1}^L, \quad (7)$$

represents the net utility value in unit of goods for the household of keeping a worker in the formal sector.⁷ Notice that the informal wage w_t^s acts as the outside option for formal workers. From this problem we obtain the standard Nash sharing rule:

$$(1 - \gamma) \lambda_t^L = \gamma J_t. \quad (8)$$

Assuming competitive entrepreneurs, a zero profit-condition for vacancy posting holds.

⁷Under this definition, λ_t^L corresponds to the optimal values of the Lagrange multiplier for the law of motion for formal employment, according to the first order conditions for household's optimization. See Appendix A.2 for details.

2.1.3 The Informal Sector

The representative firm in the informal sector hires informal workers for one period. In contrast to formal firms, this decision is static. Perfect competition in the informal sector implies that the wage of informal workers satisfy:

$$w_t^s = p_t^s \varkappa, \tag{9}$$

where p_t^s denotes the price of informal intermediate goods, relative to the final consumption goods bundle and \varkappa is a parameter denoting the productivity of informal workers in units of the informal intermediate good. While we assume that informal firms are less productive than formal matches, $\varkappa < 1$, this disadvantage is compensated by their ability to avoid payroll taxes and the lack of entry -or hiring- costs. Notice also that the informal sector is excluded by assumption from financial markets, so interest rates have no first-order effects on its dynamics.

2.2 Wholesale and Retail Production

The final wholesale good is produced using capital and intermediate inputs using a constant returns to scale technology:

$$Y_t = A_t (K_t)^\alpha (M_t)^{1-\alpha}, \tag{10}$$

where the aggregate intermediate good is itself a composite of inputs produced in the formal and informal sectors, according to the CES aggregator,

$$M_t = \left\{ \left(M_t^f \right)^{\frac{\epsilon-1}{\epsilon}} + \left(M_t^s \right)^{\frac{\epsilon-1}{\epsilon}} \right\}^{\frac{\epsilon}{\epsilon-1}}. \tag{11}$$

We assume that the final good production is carried on by a representative firm under perfect competition. The problem of this representative firm and the resulting first order conditions are discussed in Appendix A.3.

Endogenous Productivity Combining the wholesale sector production function and the linear technologies to produce intermediate inputs ($M_t^f = L_t^f$ and $M_t^s = \varkappa L_t^s$), we obtain a

simple aggregate production function for the economy:

$$\underbrace{Y_t}_{GDP} = \underbrace{\left\{ A_t \left[((1 - l_t^s))^{\frac{\epsilon-1}{\epsilon}} + (\varkappa l_t^s)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \right\}}_{TFP} (K_t)^\alpha (L_t)^{1-\alpha}, \quad (12)$$

where the term in brackets represents measured TFP and includes both an exogenous (A_t) and an endogenous component depending on the informality rate $l_t^s \equiv \frac{L_t^s}{L_t}$.

Savings and Investment Households own the capital stock K_t and hold one-period, domestic bonds B_t carrying a real interest rate ρ_t . Investing I_t units of the final good increases the capital stock according to the law of motion:

$$K_{t+1} = (1 - \delta) K_t + I_t - \frac{\vartheta}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t \quad (13)$$

featuring a quadratic adjustment cost. We assume for symmetry that investment goods are also a composite of different varieties according to the same CES aggregator as consumption, so that the relative price of investment with respect to consumption is one.

Retail Sector and Nominal Rigidities Retail firms use wholesale goods to produce final differentiated goods using a one-to-one technology. The assumptions imply that all retailers have a common real marginal cost p_t^w , the relative price of the wholesale good relative to the final differentiated good. We assume monopolistic competition in the retail sector, so that only one firm produces each variety $z \in [0, 1]$ to satisfy aggregate demand:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\eta} Y_t, \quad (14)$$

where the last expression comes from standard cost minimization using the Dixit-Stiglitz aggregator (3) and implies a constant price elasticity η .⁸ P_t is the aggregate price index, defined by:

$$P_t Y_t = \int_0^1 P_t(z) Y_t(z) dz.$$

We also introduce nominal price rigidities à la Calvo and assume that each firm faces an exogenous probability $1 - \theta$ of changing prices each period. Moreover, we focus on a symmetric solution in which all firms follow the same pricing strategy, since they are ex-ante

⁸An implicit assumption is that all agents, including the government, combine differentiated goods in the same proportions as consumers.

equal. Aggregating across retailers, the assumptions on nominal rigidities imply the following dynamics for the aggregate price level:

$$\theta \left(\frac{P_t}{P_{t-1}} \right)^{\eta-1} = 1 - (1 - \theta) \left(\frac{P_t^*}{P_t} \right)^{1-\eta} \quad (15)$$

where P_t^* is the optimal price chosen by retailers for their particular variety if allowed to change their price at t , discussed in detail in Appendix A.4.

2.3 Closing the Model

We conclude the exposition of the model by presenting some of the equations that close the model. We provide a complete definition of equilibrium in Appendix A.

Monetary and Fiscal Policy To close the nominal part of the model, we assume that the Central Bank uses the nominal interest rate as its instrument following a Taylor rule

$$1 + i_t = (1 + \iota) \left(\frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^n} \right)^{\phi_y} \nu_t \quad (16)$$

with $\phi_\pi > 1$, where ι is the long run real interest rate (pinned down by the inverse of the discount factor), ϕ_y is the weight of the output gap,⁹ and ν_t is a one shot discretionary deviation from the rule. The relation between the nominal and real interest rate is described by a Fisher equation:

$$1 + \varrho_t = \left(\frac{1 + i_t}{P_{t+1}/P_t} \right). \quad (17)$$

In the fiscal side, we simply assume that government spends a fraction of output g_t each period and finances its spending (net of payroll taxes) with lump sum taxes to consumers:

$$g_t Y_t = \tau w_t^f L_t^f + T_t. \quad (18)$$

⁹The natural product Y_t^n is defined as the level of output in an economy with flexible prices, in which money neutrality holds. The output gap corresponds to the distance between current output and its natural level. See details at the end of Appendix A.5.

Market clearing In each period, the markets for the final good and one period bonds clear:

$$(1 - g_t) Y_t = C_t + I_t + \xi V_t + \kappa (i_t^l - i_t) w_t^f L_t^f, \quad B_{t+1} = \kappa w_t^f L_t^f. \quad (19)$$

Notice that the final good is used for consumption, investment, government spending, vacancy costs in the formal sector and financial intermediation costs. Also, the supply of savings by households needs to satisfy the working capital requirements of formal firms.

Stochastic Shocks The description of the model is completed with the specification of the three sources of uncertainty: (i) a technology shock to the productivity A_t , symmetric to both formal and informal sectors; (ii), a demand shock to government expenditures as a fraction of output g_t , and (iii) a shock to the interest rate premium (lending spread) ζ_t faced by formal firms. The three shocks are assumed to follow independent $AR(1)$ stochastic processes (in logs).

2.4 Informality, Wages and Unit Labor Costs

Before moving to the quantitative exercises, it is worth to highlight the main mechanisms in the model by which the presence of an informal sector affects the adjustment of the economy, in particular the response of inflation, to different shocks.

First, the informal sector provides the economy with an employment alternative that increases the flexibility of labor supply. In contrast to formal employment, subject to search frictions, informal employment can expand and contract quickly. A more elastic labor supply implies that increases in the demand for labor (driven by technological or demand-side perturbations) lead to larger expansions in total employment and output and to smaller rises in wages.¹⁰ Informality moves pro-cyclically in response to these shocks, acting as a *buffer* for wages against labor demand spurs and hence reducing inflationary pressures arising from overheated labor markets.¹¹

¹⁰Notice that it is key for the argument that the labor market adjusts along a fixed labor supply schedule. Wealth effects can also shift the labor supply in response to these shocks, qualifying (and, perhaps, overturning) the results. We illustrate the role of wealth effects with the quantitative model in the next section.

¹¹Castillo and Montoro (2010) explore the implications of the buffer effect of informality on wages in the context of a monetary model with frictional labor markets. Although we differ in the specific modeling of the labor market, our results are consistent with their findings for demand and technology shocks. This discussion also borrows from the literature on home production and its relation to the labor supply elasticity (see Benhabib et al. (1991) and Campbell and Ludvigson (2001)).

There is, though, a second mechanism by which informality affects inflation dynamics: the sensitivity of labor costs to interest rates. Through the credit cost channel, changes in lending interest rates affect the financial component of labor costs for firms in the formal sector. For given wages, an increase in interest rates rises *unit labor costs*, defined in the baseline model as:

$$ulc_t \equiv \frac{P_t \left[(1 + \kappa i_t^l + \tau) w_t^f L_t^f + w_t^s L_t^s \right]}{Y_t},$$

i.e., the sum of labor expenses (wages plus working capital costs) incurred in the production of one unit of the final good, in nominal terms. Changes in unit labor costs, more than in wages themselves, provide a good measure of the inflationary impact of shocks propagating through the labor market.

Because the informal firms are excluded from the access to credit markets to finance working capital needs, it follows that the presence of the informal sector reduces the sensitivity of unit labor costs to changes in the interest rate. We can write:

$$ulc_t = \frac{P_t \left[w_t + (\kappa i_t^l + \tau) w_t^f (1 - l_t^s) \right]}{Y_t / L_t}, \quad (20)$$

where $w_t \equiv w_t^f (1 - l_t^s) + w_t^s l_t^s$ represents the average wage of the economy and l_t^s denotes the informality rate. If wages and interest rates move in opposite directions, as it will occur for instance in response to shocks to financial intermediation costs, the net effect of informality on inflation volatility could be ambiguous.

3 Quantitative Exercises

To illustrate the work of the model, we compute a series of impulse response functions to technology shocks, demand shocks (to government spending) and financial shocks to the lending interest rate paid by formal firms. We then compare the response of the baseline model to these shocks to the response of a counterfactual economy without the informality option. The results highlight the role of the two main channels in the theoretical model: the flexibility of labor flows from/to the informal sector and its buffer effect on wages, and the sensitivity of unit labor costs to interest rates.

3.1 The Baseline Economy

We calibrate the model to the Mexican economy, given the importance of the informal sector and the availability of detailed labor flows data. The complete list of parameters for the baseline economy can be found in Table 1. A period in the model is a quarter. Some of the parameters are based on the related work of Leyva and Urrutia (2018) or correspond to standard values in the literature.

For preferences, we chose unitary elasticity of labor supply, equal to the intertemporal elasticity, and start with a mild wealth effect parameter $\omega = 0.2$. An important parameter to measure the flexibility in the labor market is the exogenous separation rate s , governing the turnover in the formal sector. We chose a value consistent with an average duration of a job of in the formal sector of 2.8 years.¹² The elasticity of substitution between formal and informal goods is consistent with the value used in Restrepo-Echavarria (2014) and Fernández and Meza (2015). We also set the payroll tax τ to 0.25, following the estimates in Leal (2014) and Alonso-Ortiz and Leal (2016). The parameters for nominal rigidities and the Taylor rule correspond to the prototype New Keynesian model and lie in the same range as Castillo and Montoro (2010) or Colombo et al. (2018). We assume a small working capital requirement ($\kappa = 0.2$), so that 20% of the wage bill in the formal sector has to be paid in advance.

Finally, the last five parameters in the table are calibrated so that the model reproduces five targets in steady state: (i) a total employment rate (formal + informal) of 46%; (ii) an unemployment rate of 4%; (iii) a share of informal employed workers of 50%; (iv) a wage premium for formal workers of 13%, compared to informal workers; and (v) total hiring expenditures corresponding to 7% of the formal wage bill in each period.

3.2 Impulse Responses and the Role of the Informal Sector

We compute the response of the main variables in the baseline model to each of the shocks, scaled to deliver a similar increase in inflation. Figure 3.2 and the graphs in Appendix B display a set of impulse response functions for the full model and compares them to the response of an alternative economy in which there is not an informality option, only a formal

¹²Leyva and Urrutia (2018) report from the ENOE survey an average duration of the formal employment status of 2.8 years in Mexico. This number is computed using the sum of the transitions from formal employment to non-employment and the *net* transitions from formal to informal employment. The comparable duration for informal jobs is less than one year.

Parameter	Symbol	Value
Discount factor	β	0.99
Strength of wealth effect in labor supply	ω	0.2
Frisch elasticity of labor supply	$1/\varphi$	1
Capital share in production function	α	1/3
Depreciation rate	δ	1.25%
Adjustment cost of capital	ϑ	30
Separation rate (formal turnover)	s	8.8%
Elasticity of matching function	ϕ	0.4
Elasticity of substitution formal / informal inputs	ϵ	8
Elasticity of substitution between varieties	η	6
Fraction of firms not changing prices (Calvo)	θ	2/3
Weight of inflation in Taylor rule	ϕ_π	1.5
Weight of output gap in Taylor rule	ϕ_y	0.2
Payroll tax	τ	0.25
Fraction of wage bill financed through working capital	κ	0.5
Average government expenditure (fraction of GDP)	g	0.2
Persistence of productivity shock	ρ_A	0.9
Persistence of government shock	ρ_g	0.5
Persistence of financial shock	ρ_ς	0.5
Disutility of labor	ψ	2.49
Productivity informal sector	\varkappa	0.63
Search cost for unemployed	ς	114.8
Workers' bargaining power	γ	0.53
Cost of posting a vacancy	ξ	1.21

Note: The last five parameters are calibrated to reproduce in steady state a total employment rate of 46%, an unemployment rate of 4%, a share of informal workers of 50%, a wage premium for formal workers of 13% and a total hiring expenditures to the formal wage bill ratio of 6%.

Table 1: Parameters for the Baseline Economy

sector.¹³ Table 2 summarizes the responses as the cumulative deviations from the steady state values of each variable after one year (four periods in the model), with and without the informal sector.

Demand shocks

An increase in aggregate demand, modeled as a fiscal expansionary policy ($g_t \uparrow$), boosts output at the cost of inflation. This is so even though the increase in government expenditures crowds out private consumption and investment. As Figure 3.2 and the first panel of Table 2 show, the resulting expansion in output increases labor demand, total employment and the real wage, which rises unit labor costs for firms. The effect is larger on impact and fades away as government expenditures revert to their mean level.

Notice that in the full model the bulk of the expansion in employment is accounted for by the informal sector (in fact, formal employment is slightly reduced). Due to labor market frictions, employment in the formal sector is more persistent, while the static nature of the self-employment decision allows households to rapidly respond to the increase in labor demand by working informally. The mechanism is strengthened by a the pool of inactive workers that can move into the informal sector at no cost. The implication is an increase in the informality rate, which under demand shocks behaves pro-cyclically.

Without the informality option, the same aggregate demand impulse leads to a smaller increase in total employment and output and therefore to a larger increase in wages, unit labor costs and inflation. In addition, the monetary response needs to be more contractionary in the alternative model, reinforcing the previous effects through the credit cost channel (which affects now the whole economy). Notice also that the flexibility provided by the informal sector comes at a cost in terms of productivity. Increasing the share of informal workers implies a reduction in measured TFP due to a negative composition effect (see equation (12) and its discussion). This is why productivity behaves counter-cyclically under demand shocks. Removing the informality margin makes TFP a-cyclical.

Technology shocks

The second panel of Table 2 reports the response of the model to a negative technology shock ($A_t \downarrow$). Notice that the structure of the model implies a symmetric reduction in

¹³The alternative model, called “no informality” in the table, is a special case of the baseline in which the productivity χ of the technology to produce informal inputs is set to zero and the elasticity of substitution ϵ between formal and informal inputs is arbitrarily large.

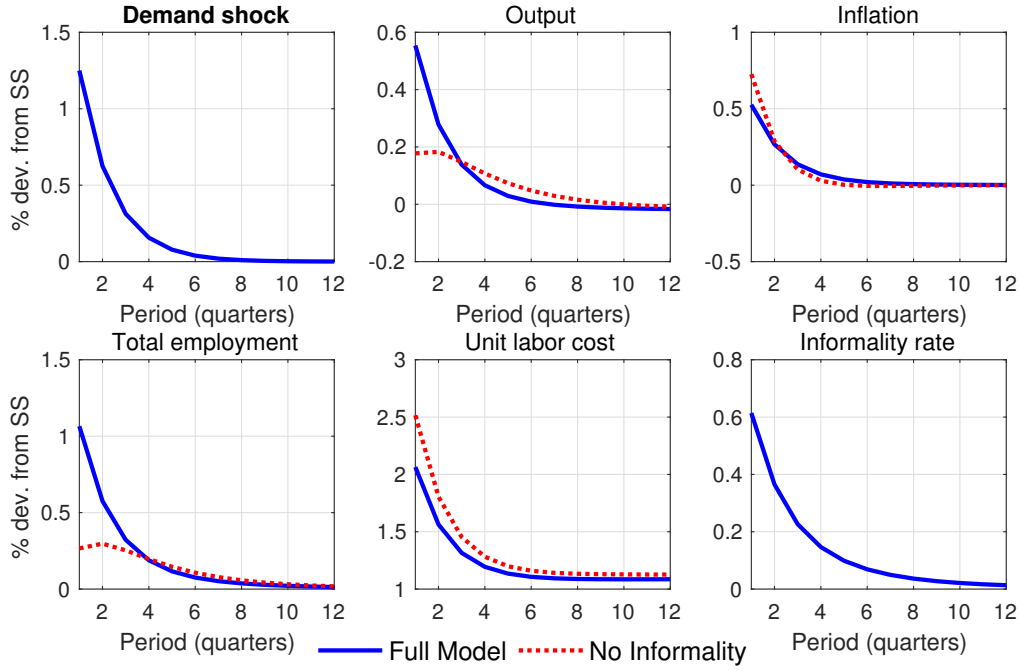


Figure 1: Impulse Response Function to a Demand Shock

	Demand ($g_t \uparrow$)		Technology ($A_t \downarrow$)		Financial ($\zeta_t \uparrow$)	
	Full Model	No Informality	Full Model	No Informality	Full Model	No Informality
Cumulative Effect First Year (%)						
Real output	1.04	0.62	-8.83	-8.59	-0.12	-0.67
Inflation rate	1.00	1.15	1.00	0.89	1.00	2.10
Nominal interest rate	1.71	1.9	1.56	1.34	1.66	3.41
Consumption	-0.94	-1.77	-6.92	-6.55	-1.88	-4.69
Investment	-5.73	-5.88	-15.8	-16.0	-7	-11.9
Total employment	2.15	1.01	-2.52	-2.16	1.47	-0.84
Formal employment	-0.39	—	-2.28	—	-6.26	—
Informality rate	1.35	—	-0.20	—	4.11	—
Average real wage	1.89	2.36	-5.69	-6.34	-1.60	-2.45
Formal real wage	2.14	—	-5.76	—	-2.84	—
Formal wage premium	0.26	—	-0.15	—	-3.71	—
Nominal unit labor cost	6.14	7.06	3.42	2.39	7.18	13.0
Measured TFP	-0.32	0.00	-6.98	-7.05	-1.02	0.00

Notes: Cumulative percent deviations from steady state after the first year (4 periods). The full model includes an informal sector, while the alternative model (“no informality”) eliminates it. Each shock is scaled to deliver in the baseline a 1 p.p. cumulative inflation boost in the first year.

Table 2: Comparing Impulse Responses with and without the Informal Sector

the productivity of both formal and informal sectors. As expected, the TFP drop reduces aggregate supply, output and increases inflation. To accommodate the shock, monetary policy becomes contractionary, i.e., interest rates increase. Total employment falls, as real wages decline due to the productivity drop.¹⁴

As in the case of demand shocks, most of the response of employment is driven by the informal sector, which is more flexible. Therefore, the informality rate is also pro-cyclical in this experiment. In the full model employment falls more and average wages decline less than in the model with only a formal sector, illustrating again the buffer effect of informality. The original TFP drop increases unit labor costs and prices, but more so in the economy with the informal sector, in which wages adjust less (and employment more). In contrast to the case of demand perturbations, an economy facing technology shocks would then experience more inflation volatility with informality than without it.

Financial shocks

Finally, an increase in the intermediation spread ($\zeta_t \uparrow$) rises the interest rate that formal firms face. Through the working capital channel, the financial component of labor costs for firms in the formal sector increases, raising prices of formal goods and inflation. The increase in the intermediation spread then reduces the value of a formal match, discouraging vacancy posting and job creation in this sector. The informal sector, conversely, is assumed to be excluded from credit markets, so interest rates do not have a first-order effect on its dynamics. The larger the size of the informal sector, the lower the impact of the lending spread on unit costs and total labor demand.

Consistently with this discussion, the last panel of Table 2 shows that formal employment and output decrease in response to the lending rate jump, and inflation rises as unitary labor costs increase through the working capital channel. The latter holds even though formal wages fall due to the contraction in the demand for labor, making the informal sector more attractive and therefore increasing the informality rate, which is now *counter-cyclical*.¹⁵ Moreover, removing the informality option amplifies the effect of the financial shock on unit labor costs (see equation (20) and its discussion) and inflation, also strengthening the negative response in total employment and output.

¹⁴The response of labor supply depends crucially on the importance of wealth effects. With our specification for the utility function, wealth effects are mitigated, so total employment is pro-cyclical under technology shocks.

¹⁵Notice that in our baseline calibration the increase in informality is large enough to offset the fall in formal employment, so total employment increases. Still, output drops due to the productivity loss.

3.3 Business Cycle Properties

The previous experiments suggest that the business cycle properties of the baseline economy depend on the relative importance of the different shocks affecting it. A full estimation of the model, using the observed time series for the main variables, would help us to uncover what are the most important shocks in Mexico and hence what are the model predictions with respect to the fluctuations in output, inflation, the informality rate and all other endogenous variables. This is, however, outside of the scope of this paper.

Short of estimating the model, we perform some back-of-the-envelope calculations for illustration purposes. We chose the following calibration strategy for the volatility of the three shocks (demand, technology and financial). First, we set the volatility of the financial spread to match the standard deviation of the EMBI spread for Mexico (0.84%), obtained using quarterly data from 2000 to 2016. Then we chose the values of the volatilities of the demand and the technology shocks to simultaneously reproduce in the full model the volatility of output and the correlation between inflation and output for Mexico, computed using quarterly hp-filtered data for the same period.

Using these parameter values, we simulate the stochastic model for 10,000 periods and compute some selected business cycle statistics for it. The results and the corresponding statistics from the data are reported in the first two columns of Table 3. Notice that in the full model (with the informal sector) these three shocks can account for about 40% of the observed volatility of inflation. Moreover, the model is broadly consistent with the observed volatility and the counter-cyclicality of the informality rate in Mexico (as reported by Leyva and Urrutia (2018)), although the size of this last correlation is less than half in the model compared to the data. This is an important check for the role of the informal sector in the model.

Finally, the last column of the table shows the effect of removing the informality option on the volatilities of output and inflation including the three calibrated shocks at the same time. We can see that informality dampens the volatility of inflation at the cost of a larger volatility in output. In other words, in our raw calibration exercise, the role of demand and financial shocks seems to be more important than the contribution of technology shocks. For inflation, demand shocks and the buffering response of informality is key, while financial shocks shape the cyclical behavior of the informality rate.

<i>(std in percent)</i>	Data Mexico	Full Model	No Infor- mality
$Std(Y_t)$	1.85	1.85	1.79
$Std(\pi_t)$	1.03	0.42	0.52
$Corr(Y_t, \pi_t)$	-0.28	-0.28	-0.35
$Std(l_t^s)$	0.53	0.64	--
$Corr(l_t^s, Y_t)$	-0.56	-0.22	--

Notes: Results from simulating the model for 10,000 periods using the parameter values in Table 1 and the volatilities of the shocks: $\sigma_A = 0.63$, $\sigma_g = 0.75$ and $\sigma_\zeta = 0.84/(1 - \rho_\zeta)$. The statistics for output and inflation in Mexico are computed using quarterly, hp-filtered data for the period 2000-2016. Data for the volatility and cyclicalilty of the informality rate are taken from Leyva and Urrutia (2018).

Table 3: Business Cycle Properties of the Model with/out the Informal Sector

3.4 Sensitivity Analysis

We now evaluate the sensitivity of the impulse responses summarized in Table 2 to changes in the values of some key parameters. Figure 2 reports the cumulative response of inflation in a year to each of the three shocks, with and without informality, as a function of the parameter values. For instance, the first graph in this figure reports the resulting yearly change in inflation following a positive demand shock for different values of the wealth effect parameter (ω) in the utility function, while keeping other parameters constant except for the last five parameters in Table 1, recalibrated to match the same steady state targets. The analysis is useful to further illustrate the mechanisms of the model and to assess how robust our results are.¹⁶

The Role of Wealth Effects

The first row in Figure 2 shows the inflation response to each shock under different wealth effect intensities. A larger wealth effect mitigates the inflationary response in the three cases. This is because in all of them the representative consumer's wealth falls (through lump-sum taxes, in the government expenditure increase experiment, or directly through wages in the other two), shifting the labor supply curve to the right. This shift dampens the increase in wages in the first experiment, where labor demand increases, reducing the inflationary

¹⁶The third figure in Appendix B reports the same sensitivity analysis to changes in additional parameters and confirms the robustness of the main results. These parameters include the degree of price rigidity, the intertemporal elasticity, the elasticity of labor supply, the substitutability between formal and informal goods, the payroll tax rate and the weight of the output gap in the Taylor rule.

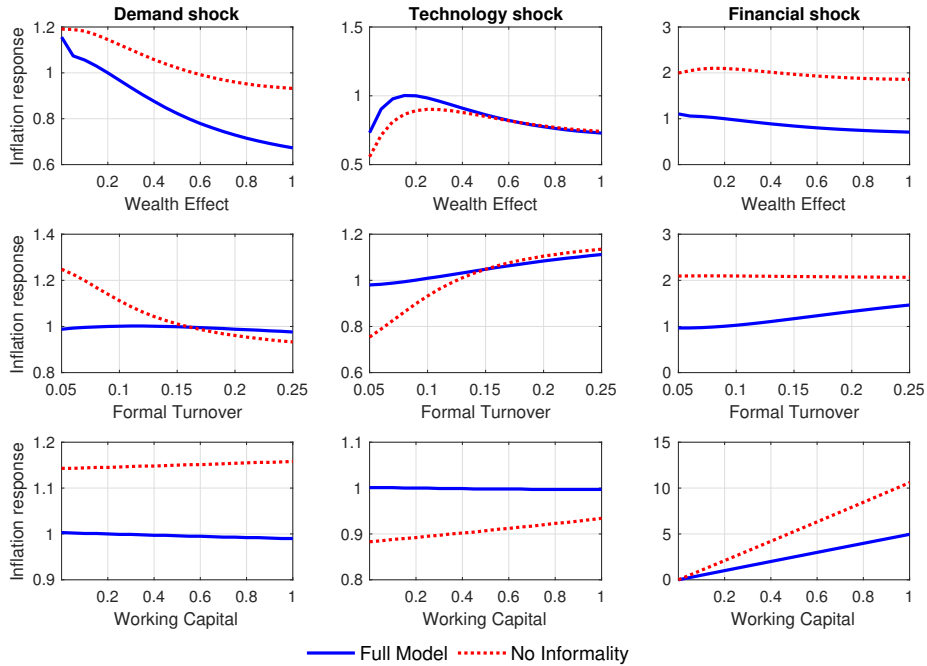


Figure 2: Sensitivity Analysis: Response of Inflation to Different Shocks

impact of positive demand shocks. The same shift in labor supply exacerbates the fall in wages in the last two experiments, in which labor demand falls, also mitigating the impact of negative technology and financial shocks on unit labor costs and inflation.

Notice that wealth effects are more important in the economy with an informal sector, where labor supply is more responsive. This implies that for a large enough wealth effect parameter the response of inflation to technology shocks might be stronger in the model without an informal sector, vanishing our original result in Table 2.¹⁷ Conversely, as the wealth effect approaches zero (corresponding to GHH preferences) the differences in the inflation response to demand shocks with and without informality also diminish.

Formal Turnover and Labor Market Flexibility

Employment turnover in the formal labor market, parametrized by the exogenous separation rate (s), provides a measure of the flexibility of the formal sector to expand and contract quickly. When this flexibility increases, wages in the formal sector need to adjust less to shocks. As discussed before, this mitigates the inflationary impact of positive demand shocks, but amplifies it in the case of negative technology and financial shocks.

¹⁷This case in which informality dampens the response of inflation under technology shocks also implies that employment moves counter-cyclically, due to the magnitude of the wealth effect on labor supply.

More labor flexibility in the formal sector reduces the relevance of the buffer effect of informality, so the distance between the inflation response in the economies with and without informality shrink. In fact, after crossing a threshold, labor flexibility is large enough for the buffer effect of informality to be muted and the working capital mechanism discussed in the next subsection to dominate, making the response of inflation to demand (technology) shocks weaker (stronger) in the model without an informal sector.

Working Capital and the Financial Cost Channel

Finally, the third row of Figure 2 shows the response of inflation under different working capital requirement values (κ). This parameter determines the sensitivity of unit labor costs in the formal sector to interest rates. Notice that in the three experiments interest rates rise (see Table 2), hence a larger working capital requirement amplifies the response of inflation. This effect is particularly strong for financial shocks, affecting directly the lending spread and the working capital costs to firms.¹⁸ Moreover, the impact of working capital is bigger in the economy without informality, assuming that only formal firms have access to credit.

4 Implications for Monetary Policy

The experiments summarized in Table 2 also document the effectiveness of the inflation targeting regime described by the Taylor rule in dampening inflation volatility under different shocks to the economy. Facing perturbations of similar sizes, the economy with the informality option achieves lower inflation volatility when the main sources of fluctuations are demand and/or financial shocks. However, if technology shocks are predominant, the economy with informality would exhibit *higher* inflation volatility.

These results depend on the relative weights of the two channels discussed in the previous section: the buffer effect of informality and the sensitivity of unit costs and job creation in the formal sector to financial costs. They reflect the ability of the economy, in particular of the labor market, to accommodate different shocks. In what follows we discuss in more detail how the presence of an informal sector affects the transmission of monetary policy itself.

¹⁸In fact, eliminating the working capital constraint ($\kappa = 0$) isolates the economy from financial shocks to the lending spread; the response of inflation to these shocks is then zero, independently of informality.

4.1 The Transmission Mechanism of Monetary Policy

The role of the informal sector in the transmission of monetary policy is analyzed in detail in Table 4. The first panel (labeled *Baseline*) compares the impact in the baseline model of a one-shot contractionary monetary surprise that raises the nominal interest rate to the same shock in an economy without the informality option.¹⁹ The remaining three panels perform the same comparison under different parameter values.

It is worth to start by revisiting the transmission channel of monetary policy in this model. An increase in interest rates have a deflationary effect through the Fisher equation, reducing the impact of nominal rigidities on the markup of monopolistic retailers. This results in an output contraction and subsequently a fall in labor demand, reducing employment and the real wage. This mechanism is summarized by the New Keynesian Phillips curve.

As the comparison between the first and second panels of Table 4 shows, this mechanism is more powerful to stabilize inflation in an economy with lower price stickiness, in the sense that the *sacrifice ratio* (in terms of output) required to achieve a similar reduction in inflation is smaller. Also, the first graph in Figure 3 reports the sacrifice ratios after one year for a wider range of values of the Calvo nominal rigidity parameter. Consistently with the previous discussion, as prices become more rigid (higher θ) the sacrifice ratios increase in a similar proportion for the economies with and without the informal sector.

There are two reasons for the presence of an informal sector to affect the transmission channel of monetary policy, working in opposite directions. The buffer effect of informality implies that the contraction in labor demand can be absorbed by a rapid decline in informal workers together with a smaller decline in real wages. This makes the Phillips curve flatter, making disinflation harder to achieve. On the other hand, interest rates have a direct impact on unit labor costs through the working capital channel, also making more difficult for monetary policy to reduce inflation; yet this effect is weaker when we add an informal sector, less sensitive to interest rate changes.

We explore each of this channels in the next subsections. For now, the first panel of Table 4 suggests that monetary policy is less powerful to stabilize inflation in an economy with an informal sector, hence the larger required sacrifice ratio to achieve a similar reduction in inflation. This implies that the buffer effect of informality dominates the working capital mechanism in our calibrated model.

¹⁹The fourth graph in Appendix B shows the complete set of impulse response functions for both economies.

Contractionary Monetary Shock ($\nu_t \uparrow$)				
	Baseline		More flexible prices ($\theta = 1/3$)	
Cumulative Effect First Year (%)	Full Model	No Informality	Full Model	No Informality
Real output	-1.72	-1.27	-0.29	-0.15
Inflation rate	-1	-1.02	-1.24	-1.26
Nominal interest rate	0.09	0.16	0.01	0.02
Sacrifice ratio	1.70	1.27	0.23	0.12
Total employment	-3.03	-1.91	-0.52	-0.23
Formal employment	-0.79	–	-0.13	–
Informality rate	-1.19	–	-0.20	–
Average real wage	-4.23	-6.28	-0.72	-0.69
Nominal unit labor cost	-9.16	-11.7	-5.85	-5.89

	Less formal turnover ($s = 0.05$)		Larger working capital ($\kappa = 0.8$)	
Cumulative Effect First Year (%)	Full Model	No Informality	Full Model	No Informality
Real output	-1.64	-0.67	-1.72	-1.31
Inflation rate	-1.02	-1.24	-1.00	-1.01
Nominal interest rate	0.08	-0.05	0.09	0.17
Sacrifice ratio	1.60	0.54	1.70	1.31
Total employment	-2.99	-1.01	-3.03	-1.97
Formal employment	-0.35	–	-0.79	–
Informality rate	-1.41	–	-1.19	–
Average real wage	-4.19	-5.90	-4.25	-6.33
Nominal unit labor cost	-9.17	-11.7	-9.16	-11.7

Note: Cumulative percent deviations from steady state after the first year (4 quarters). The full model includes an informal sector, while the alternative model (“no informality”) eliminates this option. The shock is a one-shot innovation to the Taylor rule ($\rho_\nu = 0$) scaled so that inflation *decreases* by a 1 p.p. during the first year. “Baseline” refers to the calibration in Table 1; the other three panels use same parameters except for those indicated and the last five parameters in Table 1, recalibrated to match the same steady state targets. The *sacrifice ratio* computes the percent loss in output required to reduce inflation by one percent.

Table 4: Comparing Monetary Policy Surprises with/out the Informal Sector

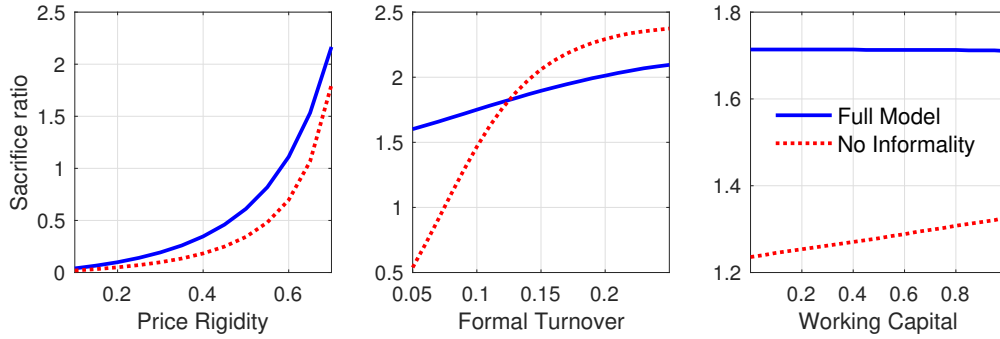


Figure 3: Sensitivity Analysis: Sacrifice Ratios for Monetary Policy Surprises

4.2 Labor Market Rigidity and Monetary Transmission

We perform an additional experiment in order to highlight the role of formal labor market rigidity in the monetary transmission. In the third panel of Table 4, we compute again the sacrifice ratios for an alternative parametrization of the model that reduces the separation rate in the formal sector. As discussed before, a lower separation rate implies a more rigid formal labor market, with less turnover and a higher duration of a match.

As expected, reducing labor market flexibility dampens the response of employment and output to monetary shocks, and hence the sacrifice ratios associated to a reduction in inflation. Moreover, as shown in the third panel of Table 4, once we increase the rigidity of the formal sector the buffer effect of informality becomes more relevant. The response of the informality rate to the monetary shock is stronger in this experiment, compared to the one in the baseline case, since a smaller fraction of the fall in employment is accounted for the drop in formal employment. Consequently, the difference in the sacrifice ratios between the economies with and without informality are now larger (more than double after a year) to the ones obtained in the baseline.

The second graph in Figure 3 also shows that more labor market flexibility, measured by the turnover in the formal sector, increases the sacrifice ratios of monetary policy but reduces the differences in these ratios with and without informality. Interestingly, after crossing a threshold, labor flexibility is large enough for the buffer effect of informality to be muted and the working capital mechanism to dominate, so that the sacrifice ratio becomes larger in the economy without an informal sector.

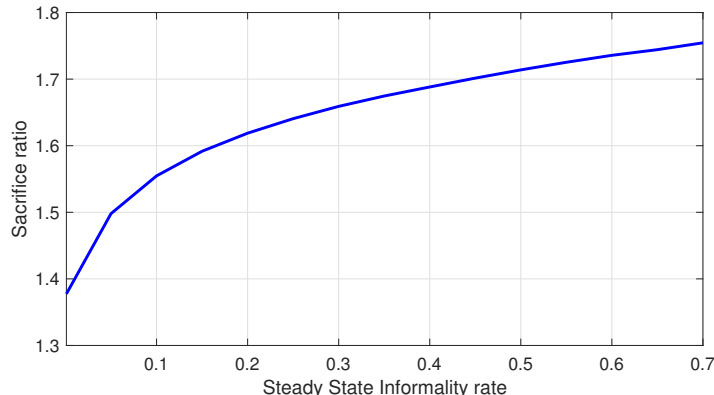


Figure 4: Sensitivity Analysis: Sacrifice Ratios for Monetary Policy Surprises (II)

4.3 Financial Cost Channel and Monetary Transmission

Finally, the last panel of Table 4 reports the sacrifice ratios after one year obtained in an economy with a larger working capital requirement (to 80% of the wage bill instead of 20%). A contractionary monetary policy has now a bigger impact on raising the financial costs of formal firms, reducing its effectiveness in deflating the economy and hence requiring a larger output drop. The smaller the size of the informal sector, the larger this counteracting effect. In contrast to the previous experiment, the impact of informality on the power of monetary policy is now weaker, as measured by the difference in the sacrifice ratios between the two economies. The quantitative differences are small, though.

To support this result, the last graph in Figure 3 shows that sacrifice ratios increase monotonically with the working capital requirement, but the distance between them with and without informality shrinks. The effect is an order of magnitude less important than in the experiment of changing labor market flexibility.

4.4 In Summary

In our setup informality has a sizable impact on the transmission channel of monetary policy, reducing its effectiveness to reduce inflation at a low output cost. As shown in the final Figure 4, the magnitude of this impact increases continuously with the size of the informal sector.²⁰ This suggests that policies aimed at curbing informality might have as a side effect an improvement in the ability of the monetary authority to reduce inflation.

²⁰The experiment involves solving the model with the parameters in Table 1 except for the last five, recalibrated to match the same steady state targets *but* the steady state informality rate, which varies at each point in the graph.

The buffer effect of informality in the labor market is important in explaining its negative impact on the monetary policy transmission. The financial cost channel has, comparatively, a minor contribution in the opposite direction. Of course these two channels are not independent in the model, so it is difficult to perform a rigorous accounting of the contribution of each. In this sense, our results should be interpreted more as suggestive examples.

5 Conclusions

The presence of a large informal sector is an important structural factor in emerging economies, although its potential impact on the dynamics of inflation and the transmission channel of monetary policy has been somewhat neglected in the academic and the policy debate. This paper attempts to fill this gap, focusing on two relatively novel features: the role of monetary policy and the impact of financial frictions, conveyed in the labour markets.

In a stylized setup that combines labor market and financial frictions, embedded in an otherwise standard monetary DSGE framework under a Taylor rule, we show that: (i) the informal sector mitigates inflationary pressures arising from demand and financial shocks, but (ii) it exacerbates the impact of technology shocks on inflation; moreover, (iii) the informal sector dampens the transmission channel of monetary policy and makes policy interventions less effective in reducing inflation, as reflected in a higher sacrifice ratio. The channels through which informality operates are the higher flexibility that it provides to the labor market (buffer effect) and the credit cost channel: as the informal sector is excluded from credit markets, the informal sector reduces the sensitivity of unit costs to changes in interest rates. The first channel explains why monetary policy is less effective with informality.

Our results are consistent with the business cycle properties of inflation, output and the informality rate. They are also robust to different parametrizations of the model, except for the parameter that conveys formal sector turnover. Higher formal turnover does overturn the effect of informality of demand and technology shocks on inflation and increases the sacrifice ratio of monetary policy. More interestingly, for high enough formal turnover the sacrifice ratio in the model without informality is higher than with informality (i.e monetary policy becomes more effective with informality). Given that formal turnover is an indicator of labour market flexibility, it follows that the flexibility provided by the informal sector is key to explaining the results of the model.

There are several venues to improve the model and the analysis. First, our economy is closed, which prevents us from addressing the impact of real exchange rates swings, external financing, etc. which are so relevant in emerging economies. Second, by modeling financial frictions through the working capital, we abstract from other relevant features, as the low levels of financial inclusion of informal workers, which are expected to have an impact on the transmission channel of monetary policy through the response of the savings rate and aggregate demand. Third, we pay no explicit attention to differential wage rigidities between formal and informal sectors. As a matter of fact, data are not very informative on this issue. But we recognize this could be another relevant channel for the transmission of shocks and monetary policy. We believe these are interesting topics for future research.

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A Defining and Solving the Equilibrium

A.1 Definition of a Stochastic Competitive Equilibrium

Given the processes for productivity, government spending, financial spread and monetary shocks, a stochastic competitive equilibrium for this economy is a set of contingent plans for aggregate quantities and prices such that:

1. Consumers maximize utility (2) subject to the budget constraint:

$$C_t + I_t + B_{t+1} = w_t^f L_t^f + w_t^s L_t^s + r_t K_t + (1 + \varrho_{t-1}) B_t + \Pi_t - T_t$$

the time allocation constraint (1), and the law of motion for formal labor:

$$L_t^f = (1 - s) L_{t-1}^f + p_t U_t$$

and for capital (13);

2. The representative wholesale producer maximizes profits each period:

$$p_t^w Y_t - r_t K_t - p_t^f M_t^f - p_t^s M_t^s$$

subject to the production function (10) and the aggregator (11);

3. Intermediate goods are produced according to the linear technologies $M_t^f = L_t^f$ and $M_t^s = \varkappa L_t^s$. The wage rate of informal workers satisfies (9);
4. Meeting probabilities are given by (4) and labor flows follow the law of motion (5);
5. Given values for the formal entrepreneur and workers satisfying (6) and (7), respectively, the Nash sharing rule (8) determines the formal wage. Vacancy posting satisfies the zero profit condition $q_t J_t = \xi \lambda_t^C$;
6. The path for the price index is consistent with the dynamic equation (15). Firms that change prices in each period maximize their expected stream of profits:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \left(\frac{\beta \lambda_{t+1}^C}{\lambda_t^C} \right) \left(\frac{P_t}{P_{t+k}} \right) \left[P_t^*(z) - \left(\frac{\eta}{\eta - 1} \right) P_{t+k} p_{t+k}^w \right] Y_t(z) \right\}$$

subject to the demand for each variety given by (14). Monetary policy is conducted according to the Taylor rule (16) and the Fisher equation (17) holds; and

7. Markets clear according to (19), total profits satisfy:

$$\Pi_t = p_t^f M_t^f - (1 + \kappa i_t^l + \tau) w_t L_t^f - \xi V_t + (1 - p_t^w) Y_t,$$

and the government's budget constraint (18) holds.

A.2 Consumer's First Order Conditions

Given initial conditions K_0 , B_0 , L_{-1}^f , prices w_t , w_t^s , r_t , ϱ_t , job finding rates p_t , profits Π_t , transfers T_t , and the stochastic process for aggregate shocks, the representative household chooses contingent plans for aggregate variables $\{C_t, \Phi_t, I_t, K_{t+1}, B_{t+1}, L_t^f, L_t^s, U_t, O_t\}_{t=0}^{\infty}$ in order to solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\log \left(C_t - \psi \Phi_t \frac{(L_t^f + L_t^s)^{1+\varphi}}{1+\varphi} \right) - \frac{\varsigma}{2} U_t^2 \right],$$

$$\text{s.to.} \quad C_t + I_t + B_{t+1} = w_t^f L_t^f + w_t^s L_t^s + r_t K_t + (1 + \varrho_{t-1}) B_t + \Pi_t - T_t \quad (\beta^t \lambda_t^C)$$

$$L_t^f = (1 - s) L_{t-1}^f + p_t U_t \quad (\beta^t \lambda_t^L)$$

$$L_t^f + L_t^s + U_t + O_t = \bar{L} \quad (\beta^t \lambda_t^O)$$

$$\Phi_t = C_t^\omega \Phi_{t-1}^{1-\omega} \quad (\beta^t \lambda_t^\Phi)$$

where the stationary Lagrange multipliers are in parenthesis. Defining,

$$U_{c,t} \equiv \left(C_t - \psi \Phi_t \frac{(L_t^f + L_t^s)^{1+\varphi}}{1+\varphi} \right)^{-1}$$

we can write the set of first order conditions as:

$$\begin{aligned}
\frac{\partial}{\partial C_t} &: \lambda_t^C = U_{c,t} + \omega \lambda_t^\Phi \left(\frac{\Phi_t}{C_t} \right), \\
\frac{\partial}{\partial \Phi_t} &: \lambda_t^\Phi = \beta (1 - \omega) E_t \lambda_{t+1}^\Phi \left(\frac{\Phi_{t+1}}{\Phi_t} \right) - \psi \frac{(L_t^f + L_t^s)^{1+\varphi}}{1+\varphi} U_{c,t}, \\
\frac{\partial}{\partial B_{t+1}} &: \lambda_t^C = \beta (1 + \varrho_t) E_t \lambda_{t+1}^C, \\
\frac{\partial}{\partial I_t} &: \lambda_t^C = \lambda_t^K \left(1 - \vartheta \left(\frac{I_t}{K_t} - \delta \right) \right), \\
\frac{\partial}{\partial K_{t+1}} &: \lambda_t^K = \beta \left\{ \lambda_{t+1}^C r_{t+1} + \lambda_{t+1}^K \left[1 - \delta + \vartheta \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} - \frac{\vartheta}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 \right] \right\}, \\
\frac{\partial}{\partial L_t^f} &: \lambda_t^L = w_t^f \lambda_t^C - \psi \Phi_t \left(L_t^f + L_t^s \right)^\varphi U_{c,t} - \lambda_t^O + (1 - s) \beta E_t \lambda_{t+1}^L, \\
\frac{\partial}{\partial L_t^s} &: w_t^s \lambda_t^C = \psi \Phi_t \left(L_t^f + L_t^s \right)^\varphi U_{c,t} + \lambda_t^O, \\
\frac{\partial}{\partial U_t} &: p_t \lambda_t^L = \varsigma U_t + \lambda_t^O, \\
\frac{\partial}{\partial O_t} &: \lambda_t^O = 0.
\end{aligned}$$

From the last four first order conditions, we obtain a condition characterizing the optimal level of unemployment (and search) for the household:

$$p_t \lambda_t^L = \varsigma U_t,$$

another for the optimal supply of self-employed labor:

$$w_t^s \lambda_t^C = \psi \Phi_t \left(L_t^f + L_t^s \right)^\varphi U_{c,t},$$

and a recursive expression for the marginal utility value of a formal worker for the household:

$$\lambda_t^L = \left(w_t^f - w_t^s \right) \lambda_t^C + \beta (1 - s) E_t \lambda_{t+1}^L.$$

A.3 Final Good Producer 's First Order Conditions

The representative firm in the wholesale sector solves the following static maximization problem:

$$\begin{aligned}
\max \quad & p_t^w Y_t - r_t K_t - p_t^f M_t^f - p_t^s M_t^s, \\
\text{s.to.} \quad & Y_t = A_t (K_t)^\alpha (M_t)^{1-\alpha} \\
& M_t = \left\{ \left(M_t^f \right)^{\frac{\epsilon-1}{\epsilon}} + \left(M_t^s \right)^{\frac{\epsilon-1}{\epsilon}} \right\}^{\frac{\epsilon}{\epsilon-1}}
\end{aligned}$$

from which we obtain the standard first order conditions for input prices:

$$r_t = p_t^w \alpha A_t \left(\frac{K_t}{M_t} \right)^{\alpha-1}, \quad p_t^f = p_t^w (1 - \alpha) A_t \left(\frac{K_t}{M_t} \right)^\alpha \left(\frac{M_t}{L_t^f} \right)^{\frac{1}{\epsilon}}, \quad p_t^s = p_t^f \left(\frac{L_t^f}{\varkappa L_t^s} \right)^{\frac{1}{\epsilon}},$$

using the linear technologies $M_t^f = L_t^f$ and $M_t^s = \varkappa L_t^s$.

A.4 Optimal Calvo Pricing

Each monopolistic firm $z \in [0, 1]$ that is allowed to change prices at date t solves the problem:

$$\begin{aligned} \max \quad & \sum_{k=0}^{\infty} \theta^k E_t \left\{ \left(\frac{\beta \lambda_{t+1}^C}{\lambda_t^C} \right) \left(\frac{P_t}{P_{t+k}} \right) \left[P_t^*(z) - \left(\frac{\eta}{\eta-1} \right) P_{t+k} p_{t+k}^w \right] Y_t(z) \right\} \\ \text{s.to.} \quad & Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\eta} Y_t \end{aligned}$$

from which we obtain the first order condition with respect to $P_t^*(z)$:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \left(\frac{\beta \lambda_{t+1}^C}{\lambda_t^C} \right) \left(\frac{P_t}{P_{t+k}} \right) \left[P_t^*(z) - \left(\frac{\eta}{\eta-1} \right) P_{t+k} p_{t+k}^w \right] \left(\frac{P_t^*(z)}{P_{t+k}} \right)^{-\eta} Y_{t+k} \right\} = 0.$$

Notice that this condition implies a *symmetric* solution, in which all monopolists changing prices at the same date behave equally. Moreover, we can write the optimal Calvo pricing rule as:

$$\frac{P_t^*}{P_t} = \frac{\left(\frac{\eta}{\eta-1} \right) \sum_{k=0}^{\infty} \theta^k E_t \left\{ \left(\frac{\beta \lambda_{t+1}^C}{\lambda_t^C} \right) \left(\frac{P_{t+k}}{P_t} \right)^\eta p_{t+k}^w Y_{t+k} \right\}}{\sum_{k=0}^{\infty} \theta^k E_t \left\{ \left(\frac{\beta \lambda_{t+1}^C}{\lambda_t^C} \right) \left(\frac{P_{t+k}}{P_t} \right)^{\eta-1} Y_{t+k} \right\}} = \frac{\Theta_t}{\Upsilon_t}$$

where Θ_t and Υ_t have a recursive representation:

$$\begin{aligned} \Theta_t &= \left(\frac{\eta}{\eta-1} \right) \lambda_t^C p_t^w Y_t + \theta \beta E_t \left[\left(\frac{P_{t+1}}{P_t} \right)^\eta \Theta_{t+1} \right], \\ \Upsilon_t &= \lambda_t^C Y_t + \theta \beta E_t \left[\left(\frac{P_{t+1}}{P_t} \right)^{\eta-1} \Upsilon_{t+1} \right]. \end{aligned}$$

A.5 Summarizing

Given the stochastic processes for A_t , ν_t , g_t and ς_t , the equilibrium is characterized by the following system of equations in the following 29 sequences: $U_{c,t}$, Φ_t , λ_t^C , λ_t^Φ , λ_t^L , C_t , I_t , K_{t+1} , O_t , U_t , L_t^f , L_t^s , Y_t , M_t , p_t^w , r_t , p_t^f , w_t^s , J_t , p_t , q_t , V_t , w_t , P_t , Θ_t , Υ_t , i_t , i_t^l , ϱ_t :

1. $U_{c,t} = \left(C_t - \psi \Phi_t \frac{L_t^{1+\varphi}}{1+\varphi} \right)^{-1}$
2. $\Phi_t = C_t^\omega \Phi_{t-1}^{1-\omega}$
3. $\lambda_t^C = U_{c,t} + \omega \lambda_t^\Phi \left(\frac{\Phi_t}{C_t} \right)$
4. $\lambda_t^\Phi = \beta (1 - \omega) E_t \lambda_{t+1}^\Phi \left(\frac{\Phi_{t+1}}{\Phi_t} \right) - \psi \frac{(L_t^f + L_t^s)^{1+\varphi}}{1+\varphi} U_{c,t}$
5. $\lambda_t^C = \beta E_t (1 + \varrho_t) \lambda_{t+1}^C$
6. $\frac{\lambda_t^C}{1 - \vartheta \left(\frac{I_t}{K_t} - \delta \right)} = \beta E_t \left\{ \lambda_{t+1}^C \left[r_{t+1} + \frac{1 - \delta + \vartheta \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} - \frac{\vartheta}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 \right]}{1 - \vartheta \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)} \right\}$
7. $\varsigma U_t = p_t \lambda_t^L$
8. $w_t^s \lambda_t^C = \psi \Phi_t \left(L_t^f + L_t^s \right)^\varphi U_{c,t}$
9. $\lambda_t^L = \left(w_t^f - w_t^s \right) \lambda_t^C + \beta (1 - s) E_t \lambda_{t+1}^L$
10. $L_t^f + L_t^s + U_t + O_t = \bar{L}$
11. $K_{t+1} = (1 - \delta) K_t + I_t - \frac{\vartheta}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t$
12. $Y_t = A_t (K_t)^\alpha (M_t)^{1-\alpha}$
13. $M_t = \left[\left(L_t^f \right)^{\frac{\epsilon-1}{\epsilon}} + (\varkappa L_t^s)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}$
14. $r_t = p_t^w \alpha A_t \left(\frac{K_t}{M_t} \right)^{\alpha-1}$
15. $p_t^f = p_t^w (1 - \alpha) A_t \left(\frac{K_t}{M_t} \right)^\alpha \left(\frac{M_t}{L_t^f} \right)^{\frac{1}{\epsilon}}$
16. $w_t^s = p_t^f \left(\frac{L_t^f}{\varkappa L_t^s} \right)^{\frac{1}{\epsilon}} \varkappa$
17. $J_t = \left[p_t^f - (1 + \kappa i_t^l + \tau) w_t^f \right] \lambda_t^C + \beta (1 - s) E_t J_{t+1}$
18. $L_t^f = (1 - s) L_{t-1}^f + q_t V_t$

19. $p_t = \left(\frac{U_t}{V_t}\right)^{\phi-1}$
20. $q_t = \left(\frac{U_t}{V_t}\right)^\phi$
21. $(1 - \gamma) \lambda_t^L = \gamma J_t$
22. $q_t J_t = \xi \lambda_t^C$
23. $\theta \left(\frac{P_t}{P_{t-1}}\right)^{\eta-1} = 1 - (1 - \theta) \left(\frac{\Theta_t}{\Upsilon_t}\right)^{1-\eta}$
24. $\Theta_t = \left(\frac{\eta}{\eta-1}\right) \lambda_t^C p_t^w Y_t + \theta \beta E_t \left[\left(\frac{P_{t+1}}{P_t}\right)^\eta \Theta_{t+1} \right]$
25. $\Upsilon_t = \lambda_t^C Y_t + \theta \beta E_t \left[\left(\frac{P_{t+1}}{P_t}\right)^{\eta-1} \Upsilon_{t+1} \right]$
26. $1 + i_t = (1 + \iota) \left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^n}\right)^{\phi_y} \nu_t$
27. $1 + i_t^l = (1 + i_t) (1 + \zeta_t)$
28. $1 + \varrho_t = \left(\frac{1+i_t}{P_{t+1}/P_t}\right)$
29. $(1 - g_t) Y_t = C_t + I_t + \xi V_t + \kappa (i_t^l - i_t) w_t^f L_t^f$

To obtain the natural product Y_t^n and then the output gap in equation (23), we solve a similar system setting $\theta = 0$ (this is, eliminating nominal rigidities). In such a system, money neutrality holds and $p_t^w = \frac{\eta-1}{\eta}$.

A.6 Solution Method

To evaluate the quantitative predictions of the model we log-linearize the previous system equations around a deterministic steady state with zero inflation. In this steady state $i = i^l = \varrho = \beta^{-1} - 1$. Moreover, we can solve for

$$p^w = \frac{\eta-1}{\eta} \quad \Theta = \Upsilon = \frac{\lambda^C Y}{1 - \theta \beta}$$

We use the algorithm proposed by Schmitt-Grohé and Uribe (2004) to solve the rational expectations model, which provides an efficient implementation of the solution method proposed by Blanchard and Kahn (1980). The final version of the code was executed using Dynare.

B Additional Figures

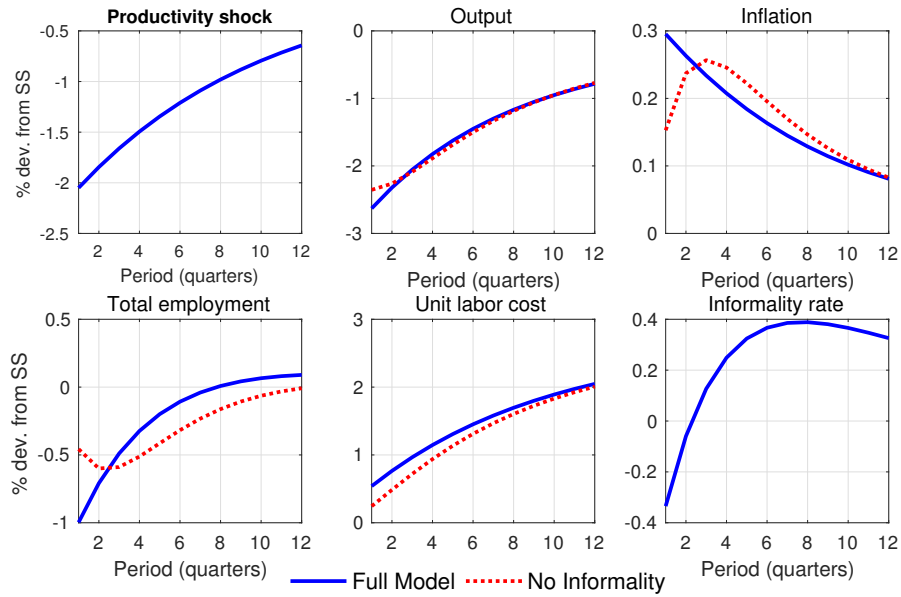


Figure 5: Impulse Response Functions to a Negative Technology Shock

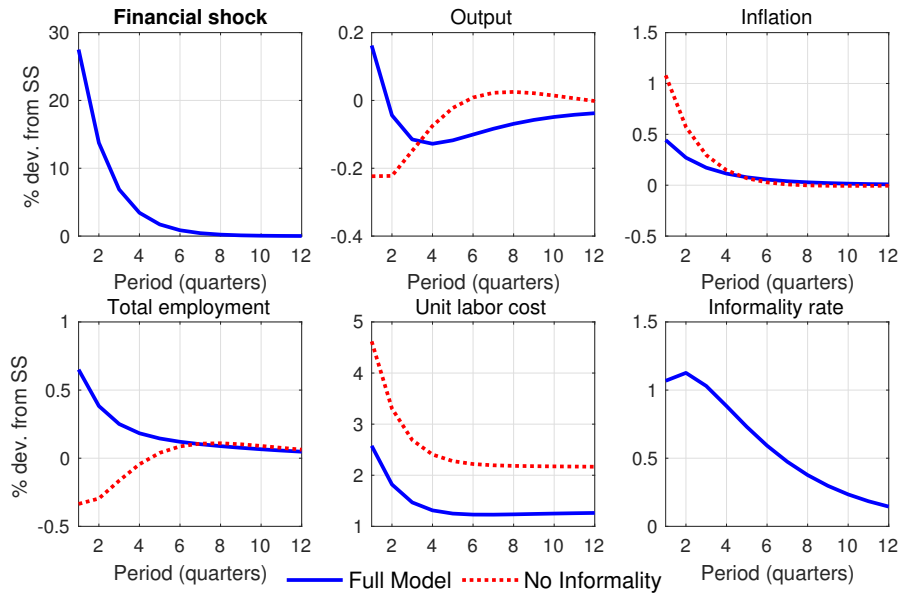


Figure 6: Impulse Response Functions to an Increase in the Financial Intermediation Cost

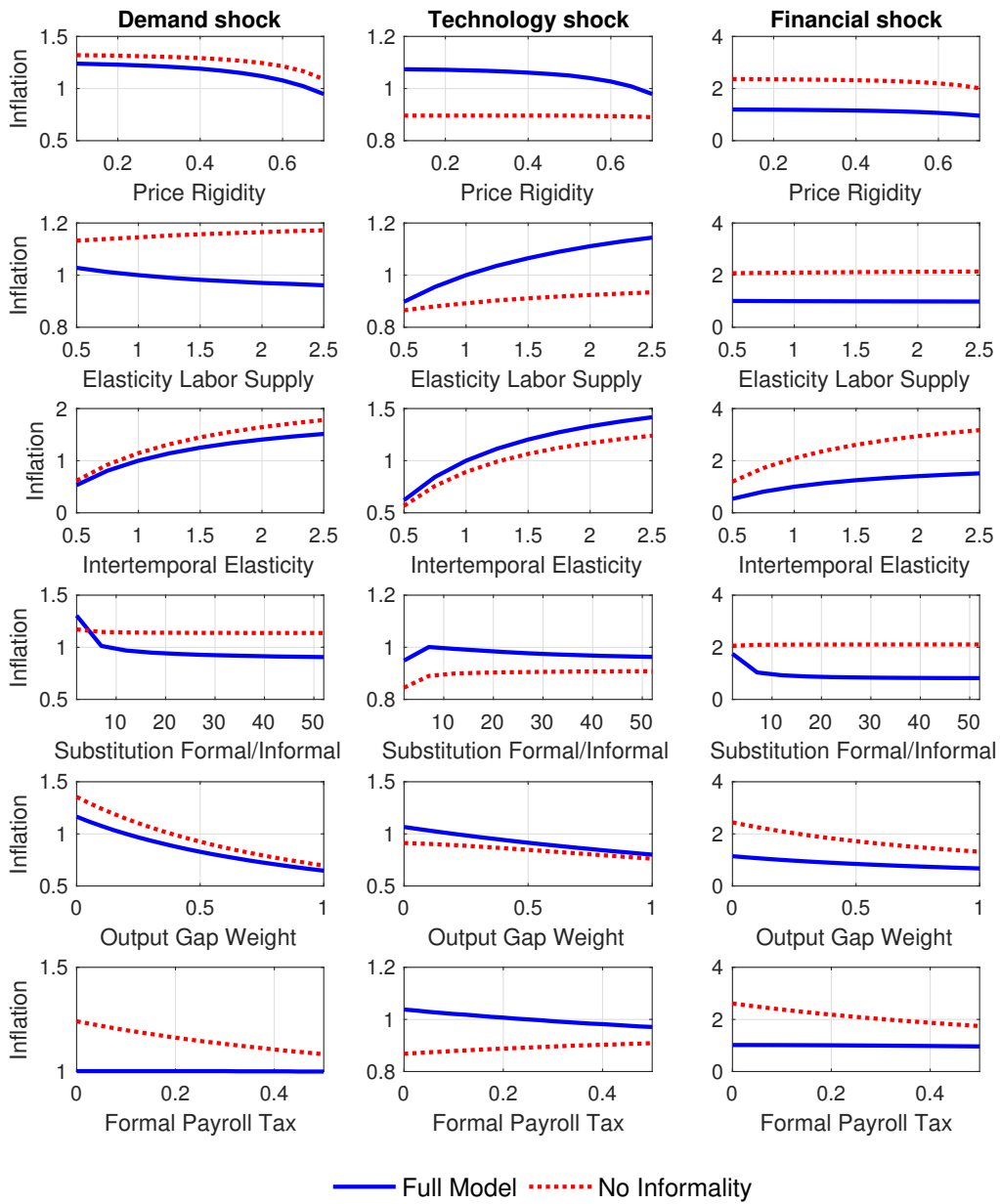


Figure 7: Sensitivity Analysis: Response of Inflation to Different Shocks

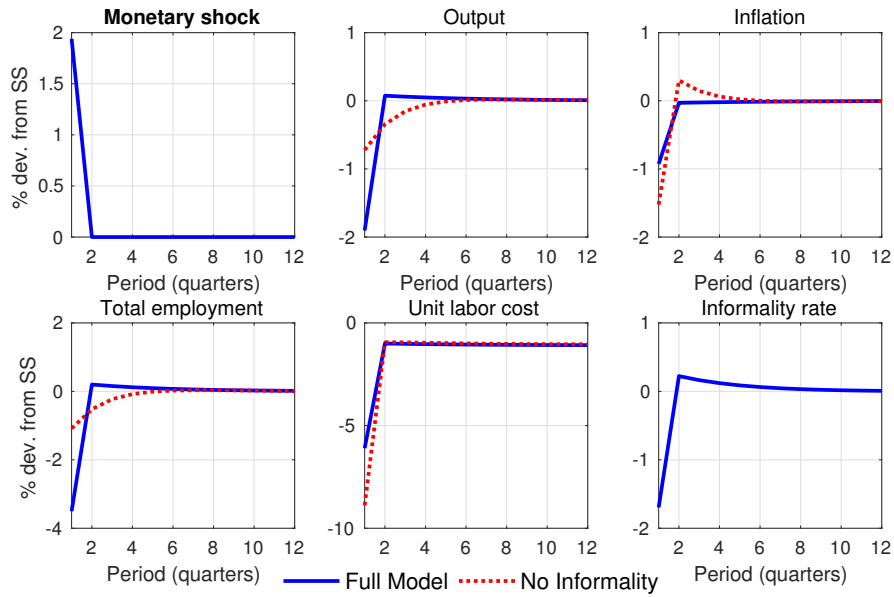


Figure 8: Impulse Response Functions to a Contractionary Monetary Surprise

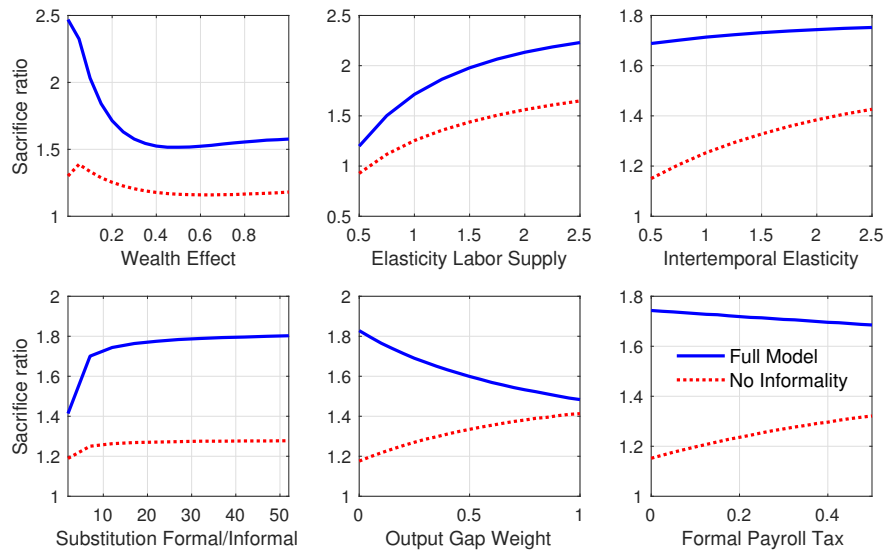


Figure 9: Sensitivity Analysis: Sacrifice Ratios for Monetary Policy Surprises